6 *Frege on Sense and Linguistic Meaning (1990)*

Frege's conception of sense has fathered all the major approaches to 'meaning' that have preoccupied philosophers in our century. Some of the progeny have developed the Fregean conception. Others have rebelled against it. All have borne its mark. Certain key elements in Frege's conception have, however, disappeared in interpretations of his view. These elements derive from his rationalist predilections and from his primary concern with idealized thought. The philosophical traditions most heavily influenced by Frege have been empiricist, preoccupied with language and suspicious of the notion of thought—idealized or not. So the neglect is natural. I think that some of the neglected elements in Frege's conception are of profound philosophical importance. They are critical to a philosophical understanding of language, cognitive processes, and conceptual change. I shall not, however, be building on Frege's conception in this paper. Rather I shall approach it historically.

I

Frege introduces the notion of sense by giving it three functions.[1] The first is that of accounting for 'cognitive value'. Senses are 'modes of presentation':

The following works by Frege are cited in the text by the abbreviations that follow their titles: *The Basic Laws of Arithmetic* (*BL*), trans. and ed. M. Furth (Berkeley: University of California Press, 1967); *Begriffsschrift und andere Aufsätze* (*B*), ed. I Angelelli (Hildesheim: Georg Olms, 1964; 2nd edn. 1977); *Foundations of Arithmetic* (*FA*), trans. J. L. Austin (Evanston, Ill.: Northwestern University Press, 1968; Oxford: Basil Blackwell, 1980); *Die Grundgesetze der Arithmetik* (*GG*) (Hildesheim: Georg Olms, 1962); *Kleine Schriften* (*KS*), ed. I. Angelelli (Hildesheim: Georg Olms, 1967); *Logical Investigations* (*LI*), ed. P. Geach (New Haven: Yale University Press, 1977); *Nachgelassene Schriften* (*NS*), ed. H. Hermes, F. Kambartel, and F. Kaulbach (Hamburg: Felix Meiner, 1968; 2nd edn. 1983); *Philosophical and Mathematical Correspondence* (*PMC*), trans. B. McGuinness and H. Kaal (Chicago: University of Chicago Press, 1980); *Posthumous Writings* (*PW*), ed. H. Hermes, F. Kambartel, and F. Kaulbach, trans. P. Long and R. White (Chicago: University of Chicago Press, 1979); *Translations from the Philosophical Writings of Gottlob Frege* (G & B), trans. and ed. P. Geach and M. Black (Oxford: Basil Blackwell, 1966); *Wissenschaftlicher Briefwechsel* (WB), ed. G. Gabriel, H. Hermes, F. Kambartel, C. Thiel, and A. Veraart (Hamburg: Felix Meiner, 1976). The sign "O" marks the pagination of the original publication of the cited article by Frege.

[1] These functions are developed throughout Frege's work after 1891. But they appear in succession in 'On Sense and Reference', 57–8, 58, 58–9, in G & B, O 26–27, 27, 28. *The Journal of Philo-*

ways things are presented to a thinker—or ways a thinker conceives of or otherwise represents entities in those cases where there are no entities. Not all modes of presentations are senses. But where modes of presentation are senses, they are associated with linguistic expressions. Being a sense is not essential to the entities that are senses. Being a (possible) mode of presentation to a thinker is what is fundamental. A sense is a mode of presentation that is 'grasped' by those 'sufficiently familiar' with the language to which an expression belongs. The second function of the notion of sense in Frege's theory is that of fixing the *Bedeutung*, the denotation or fundamental semantical value, of semantically relevant expressions. The third is that of serving as the denotation of expressions in oblique contexts.

Our primary concern is with the first function. Although the third is essential to Frege's overall theory, I think it less fundamental for understanding his conception of sense, at least initially, than the first two. The second function has been widely discussed, and I shall say only a few words about it now.

Frege explicates the notion of fixing a *Bedeutung* in a purely logical way: for each sense there is at most one *Bedeutung*. It is also clear, partly from the first function, that a sense is a way of thinking of *Bedeutung*. Beyond the foregoing, Frege says little.

The urge to say more has led some to interpret this second function in verificationist terms: sense is construed as a procedure for determining *Bedeutung*.[2] Thus the sense of a name would be a means of finding or recognizing an object; that of a predicate would be a way of determining whether an object satisfies it; that of a sentence would be a method of verification or falsification. To deal with a variety of problems of interpretation, these procedures have been regarded as highly idealized—for example, as what a collection of experts or a superior being would do. I think that there

sophy, 74. They are also discussed in my 'Belief *De Re*', (1977), sec. 4. References to Frege's writings will occur in the text, using abbreviations set out in the preliminary note. Where English and German paginations differ, both will be cited, separated by a slash. I am responsible for all translations of quotations from Frege, although often there will be little or no difference from already published translations. I will not say much on the vexed question of the translation of '*Bedeutung*'. I think that the current vogue of translating the term as 'meaning' is unfortunate, despite the support from ordinary translation practice outside of philosophy. The original choice, 'reference', was also unfortunate—though less so, in my opinion. The translation I prefer and have used here is 'denotation'. The translation goes back to Church, I believe. The advantage of this choice is that the term has an ordinary meaning ('meaning') which is roughly the same as the ordinary meaning of the term '*Bedeutung*' but also has various technical meanings that are not to be presumed the same as the ordinary meaning. In English-speaking philosophy, these technical meanings bear comparison to Frege's technical meaning for '*Bedeutung*', although the analogies (especially to Russell's and Mill's technical usage of 'denotation') must be handled with extreme caution. An alternative choice would be to leave the term '*Bedeutung*' untranslated. In the light of the moral of the present paper, as applied to '*Sinn*', this choice has much to be said for it: much gets lost and distorted under translation—especially translation between philosophical cultures.

[2] Michael Dummett, *Frege: Philosophy of Language* (London: Duckworth, 1973), 488 ff.

is little justification in the texts for this procedural interpretation of Frege's notion of sense. I shall return to it later.

The main import of the second function lies in its connecting thought and judgement with truth, the *Bedeutung* of sentences. The first function indicates that the notion of sense is designed for an account of thought and judgement. The primary logically relevant function of thought and judgement is to 'strive after truth'.[3] Thus sense provides a logically relevant connection between judgement and truth. This connection grounds Frege's celebrated insistence that in construing the sense of an expression one consider only the expression's contribution to the deductive inferential potential and 'truth-conditions' of the sentences in which it is contained.

Let us now concentrate on the first function of the notion of sense in Frege's theory. The first function is to represent the way entities are presented to a thinker, or the way a thinker conceives of or otherwise represents entities in those cases where there are none. Frege's development of his theoretical notion of sense, through consideration of linguistic phenomena such as the paradox of identity and through his remark that sense is what is grasped by those 'sufficiently familiar' with a language, has led many to assimilate his conception of sense to modern notions of conventional linguistic meaning.[4] This interpretation cannot be sustained. My primary purpose is to explain in some detail why.

[3] I have discussed at some length the role of judgement and truth in Frege's system in 'Frege on Truth' (1986) (Ch. 3 above).

[4] This view was almost universally held until a few years ago. It has received its most comprehensive articulation in writings by Michael Dummett. Dummett's characterization is very complex and not to be caricatured. But despite its many strengths, it seems to me fundamentally a misrepresentation.

The notion of conventional linguistic meaning has itself received numerous characterizations. I shall not attempt another. I shall assume, however, that conventional linguistic meaning is what is understood by an ordinary speaker of a language, or by the 'most competent' speakers of a language. I shall assume that in order to understand something in this sense, a speaker must be able on reflection to articulate, or recognize, without further instruction correct explications of the meaning, at least in those cases of meaning where explications are relevant to understanding. So, for example, those who understand the conventional linguistic meaning of 'chair' could on reflection articulate or at least recognize as correct a correct dictionary explication.

I should add that Frege's conception of sense is distinct not only from conventional linguistic meaning, but also from modern conceptions of idiolectic linguistic meaning. I shall concentrate on conventional linguistic meaning, because (except in the case of proper names and indexicals) Frege's notion of sense is much more likely to be confused with a conception of meaning that has social elements. But my discussion of linguistic meaning is meant to be neutral, except where the context indicates otherwise, on issues of the relative roles of individual and community in fixing meaning. Frege's notion of sense contrasts with all modern conceptions of linguistic meaning, including those concerned with meaning in idiolects. Of course, since he conceived of sense as a sort of linguistic 'meaning', sense is something with aboutness properties which is expressed through language. To this degree, Frege's conception of sense is a conception of linguistic meaning. So when I write of 'linguistic meaning' with the intent of contrasting such a notion with Frege's notion of sense, I am using this notion in a way that associates it with modern conceptions.

II

As background for what ensues, one must bear in mind Frege's repeated claim that his primary concern in producing his logical theory is to theorize about thought and truth, not language. He says that language often obscures the actual structure and nature of thought. Of course, he also held that one could not think most of the thoughts one thinks except by means of language. So, as he says, he is forced to concern himself with language even though it is not his main interest. Thus natural language is cast in the role of a villain on which one must perforce rely, both in order to think and as a source of clues about the nature of thought. But natural language is an imperfect instrument for thought. And Frege believed that only a language yet to be fully fashioned—a 'perfect language' (one ideally suited to the expression of thought, especially *a priori* thought)—would express thought, and senses, in a perspicuous manner.

One might be inclined to assimilate this point of view to more modern conceptions according to which the form and semantical characteristics of a language can be understood only by reference to its 'deeper structure'. There is something to be said for this interpretation. But taken in the way it would normally be understood, it underestimates how far removed, for Frege, thought and sense may be from a language's surface.

I shall be arguing that no amount of investigation of the actual usage and understanding of one's language is, according to Frege, sure to reveal the 'deeper structure' and nature of sense and thought. One may have to achieve genuine advances in non-linguistic knowledge before one can fully master the senses and thoughts that one's language expresses. This point renders Frege's view and modern methodologies for studying language quite different.[5]

[5] Another background point that serves as a clue to Frege's distinction between sense and conventional linguistic meaning lies in his remarks about proper names, demonstratives and other indexicals. He ascribes sense to names, but denies that there need be any conventionally agreed-upon sense. He frequently says that the sense of expressions like 'I', 'now' and 'yesterday' shift with the context and referent. Clearly their linguistic meaning, the accepted norm for conventional and idiolectic use and understanding of the language, remains the same through these shifts. Conversely, Frege sometimes counts the sense associated with two applications of two different indexical expressions ('yesterday' and 'today') the same, whereas the linguistic meaning is clearly different. These remarks are obviously incompatible with taking the sense of these expressions to be their conventional linguistic meanings.

It would be easy, though slovenly, to think of proper names and demonstrative expressions as special cases and to dismiss Frege's remarks about language getting in the way of thought as typical expressions of a logician interested in regimentation. But there is much more behind these remarks. They are symptoms of a radically epistemic conception of sense.

These background points are discussed at some length in 'Sinning Against Frege' (Ch. 5 above), in my Review of Dummett's *The Interpretation of Frege's Philosophy, The Philosophical Review*, 93 (1984), 454–8, and in 'Frege on Extensions of Concepts' (Ch. 7 below). The latter paper contains other grounds for distinguishing conventional linguistic meaning and sense. The present paper is a development of points first made in 'Frege on Extensions of Concepts'.

We can begin to appreciate the full extent of the difference between sense and conventional linguistic meaning by developing Frege's statements about understanding and about *vagueness*.

We enter this subject by posing an interpretative puzzle. The puzzle consists of an argument that leads from Frege's statements and a plausible further premise to an inconsistency. Resolving the inconsistency as Frege would will, I think, deepen our understanding.

The puzzle arises out of the conjunction of three claims that Frege makes. The first is that vague expressions lack a *Bedeutung* (e.g. *PW* 112, 155, 179/ *NS* 133, 'Ausführungen über Sinn und Bedeutung' (1892–1895); *NS* 168, 'Begründung meiner strengeren Grundsätze des Definierens' (1897/8 oder kurz danach); *NS* 193–4, 'Über Schoenflies: die logischen Paradoxien der Mengenlehre' (1906); *PMC* 114/ *WB* 183, Frege to Peano, 9/29/1896; 'About the Law of Inertia', *Synthese*, 13 (1961), 360–1/ *KS* 122–3; O 158–160). Frege states this principle repeatedly. We need not discuss his reasons.

The second is that all of the expressions in conventional mathematics and natural science, and some of the same terms in ordinary discourse, are not sharply understood—are not backed by a sharp grasp of a definite sense—even by their most competent users (*PW* 221–2, 216–7, 211/ *NS* 239–40, 234, 228, 'Logik in der Mathematik' (1914); *FA* vii in conjunction with *FA* sections 1, 2; *FA* 81; cf. also 'About the Law of Inertia', trans. Rand, *Synthese*, 13 (1961), 360/ *KS* 122; O 158).

If the best mathematicians lack such a grasp, one might reason, conventional mathematical usage is surely, by Frege's lights, vague. And if mathematical expressions are thus vague, surely most other expressions are too. This is the premise I foreshadowed.

The third element in the puzzle is Frege's repeated citations of this or that expression from mathematics or ordinary discourse as having such and such a *Bedeutung*. Indeed, Frege consistently writes as if lacking a *Bedeutung* is an aberration, except in fiction.

These points must be properly appreciated in order to assess the relevance of attacks on Frege over the last two decades. It has been common to hold that Frege had mistaken views about the 'semantics' of proper names, demonstratives, and the like. Sometimes, with a dutiful nod toward 'the historians', a critic will say that 'Fregeans though perhaps not Frege' held certain mistaken views. I think that the historical issue is of more than historical importance. Frege held an entirely different conception of 'semantics' than do most of his modern critics. His conception of sense was introduced to serve this conception. Language was, in his view, an imperfect vehicle for thought; and thought itself was understood in a highly idealized manner that I shall try to articulate. By contrast, modern conceptions of language concentrate on explaining linguistic competence—what is standardly or normally understood by linguistic expressions.

If the philosophical interest and importance of modern conceptions of the 'semantics' of referring expressions is to be reasonably evaluated, one must be clear about what one is doing when one gives a 'semantical account' of the language. By paying too little attention to the range of possible conceptions of language (or semantics), modern critics of Frege often not only underestimate their target but fail to appreciate the nature of the philosophical problems that motivated his theory of sense. Since these problems remain philosophically important, discussions of them that fail to appreciate how they are understood in Frege's own work almost inevitably suffer substantively, as well as historically.

The three claims, together with the reasoning following the second, are jointly inconsistent. For if one holds that even the best mathematicians do not have a firm grasp of a sense that fixes a sharply bounded *Bedeutung* for concept words and concludes from this that most mathematical and other (concept) expressions are vague; and if one holds that vague concept expressions lack *Bedeutung*; one cannot go about assigning *Bedeutungen* to most of the concept expressions of mathematics and ordinary discourse. Here we have a puzzle. Did Frege find his way through it?

III

Although the puzzle centers on *Bedeutung*, much of its interest bears on sense. So in discussing it, we shall keep track of both notions. In arriving at an interpretation, one must bear in mind the fact (not seriously questionable, I believe) that Frege was committed to the view that senses of concept expressions fix definite, sharp *Bedeutungen* where they have any *Bedeutung* at all. Frege conceived of senses as eternal abstract entities that have their logical and semantical properties independently of the activity of actual minds or language-users. Neither senses themselves nor their applications to *Bedeutungen* could be vague. This in itself constitutes a difference between Frege's conception and the ordinary conception of conventional linguistic meaning. But this difference will stand out in sharper relief once we have unravelled the puzzle.

Let us begin by considering whether one could avoid attribution of the third of the three claims. One possible line would be this: 'Despite what Frege says in illustrating his doctrines, he does not really mean that our expressions have definite senses or *Bedeutungen*. For if they are vague, they do not, by his own account, have *Bedeutungen*; and if the most competent users of the expressions do not fix definite *Bedeutungen* through their linguistic practices, the expressions would seem also to lack definite senses. The fact that Frege does not speak this way could be attributed to his desire not to complicate his expositional task unduly. He conveyed his idealized picture of sense and *Bedeutung* through his examples. But he did not see imperfect natural language as embodying his idealization.'

There are two options within this general line. We might say that Frege thought that ordinary expressions in mathematics and elsewhere had a sense but lacked a *Bedeutung*. Or we might say that he thought such expressions lacked a sense as well. Let us take up the second option first.

There is an early passage in *Begriffsschrift* (1879) (*B* 27) where Frege says of a particular vague expression that it lacks a 'judgeable content'. But he does not indicate how far he intends his remark to be generalized. And since 'content' is a technical expression that Frege later thought blurred the sense–*Bedeutung* distinction, one cannot rest much weight on the

remark. More significantly, there is a letter to Peano (1896) that reads as follows:

> The fallacy known by the name of 'Acervus' rests on this, that words like 'heap' are treated as if they designated a sharply bounded concept whereas this is not the case. Just as it would be impossible for geometry to set up precise laws if it tried to recognize threads as lines and knots in threads as points, so logic must demand sharp limits of what it will recognize as a concept unless it wants to renounce all precision and certainty. Thus a sign for a concept whose content does not satisfy this requirement is to be regarded as without *Bedeutung* from the logical point of view. It can be objected that such words are used thousands of times in the language of life. Yes; but our vernacular languages are also not made for conducting proofs. And it is precisely the defects that spring from this that have been my main reason for setting up a conceptual notation. The task of our vernacular languages is essentially fulfilled if people engaged in communication with one another connect the same thought, or approximately the same thought, with the same proposition. For this it is not at all necessary that the individual words should have a sense and *Bedeutung* of their own, provided only that the whole proposition has a sense. Where inferences are to be drawn the case is different; for this it is essential that the same expression should occur in two propositions and should have exactly the same *Bedeutung* in both cases. It must therefore have a *Bedeutung* of its own, independent of the other parts of the sentence. In the case of incompletely defined concept words there is not such independence: what matters in such a case is whether the case at hand is the one to which the definition refers, and that depends on the other parts of the proposition. Such words cannot therefore be acknowledged to have an independent *Bedeutung* at all. This is why I reject conditional definitions of signs for concepts. (*PMC* 114–115/ *WB* 183, Frege to Peano, 9/29/1896)

This passage raises a number of interpretative problems that I will not go into. It could perhaps be read as supporting the view that Frege thought that virtually all words in ordinary discourse lack a definite sense. Perhaps each would be attached to a cluster of senses, no one of which it definitely expresses. But the passage certainly does not demand this interpretation. There is nothing in the passage to indicate that Frege is assimilating virtually *all* words in natural discourse to 'heap'.

The main point of the passage is that one must insist on explicit rather than contextual definitions. Frege is laying down a requirement about how we should analyse language for the purposes of logic (as contrasted with the purposes of the vernacular). Similar requirements apply to the project of defining 'number' (cf. *FA*, middle sections). But Frege repeatedly assumes that 'number' has a definite sense and denotation. For example, he writes of a certain large finite numerical expression that it has a 'perfectly definite sense' (*FA* 114).[6] Words like 'heap' seem to be treated as special cases, not as paradigms of all linguistic usage.

[6] This occurrence of 'sense' (*Sinn*) is not corrected in the letter to Husserl of May 24, 1891, that explicitly corrects a number of passages in *Foundations of Arithmetic* that blur the sense–*Bedeutung* distinction, cf. *PMC* 63/*WB* 96.

It should be noted that this passage suggests a distinction between the notion of sense and the notion of conventional or idiolectic linguistic meaning. The implication of the passage is that the relevant vague notions, like 'heap', do not have a definite sense or denotation. Different people in different contexts, or the same person in different contexts, may associate different thoughts with sentences containing the word, in such a way that there is no saying what thought component is the sense associated with the word. Sometimes such thought components will apply to 'the case at hand' (say, counting it a heap); other times, they will not. One cannot associate any particular sense with the relevant vague word on all its occasions of use, though it may have different senses on different occasions.

There is no suggestion that such words are strictly without what we would call meaning. They contribute to the fulfilment of the 'task of our vernacular languages' by facilitating communication. It might well be that despite variations in putative extension among language-users (or by a given user at different times), one could attribute a constant meaning—a constant norm for understanding—to the word. I see no reason to think that Frege would have denied that an attribution of vague linguistic meaning was possible, based on an account of vague linguistic usage. Frege's own analogy suggests that he would not have denied this. There is no suggestion that one couldn't give a theory of knots and threads. It just would not be directly relevant to geometry. Similarly, an account of vague usage would not be an account of sense.

Let us return to the option of solving the puzzle by denying that most ordinary and mathematical expressions have a definite sense. The primary trouble with the option is that there is no evidence that Frege thought that nearly all the expressions we actually use lack a definite sense. In fact, he often seems to suggest that it is relatively easy to get an expression to express a sense. He writes:

> If one is concerned with truth . . . one has to throw aside concept words that do not have a *Bedeutung*. These are . . . such as have vague boundaries . . . [But] the context cited need not lack a sense, any more than other contexts in which the name 'Nausicaa', which probably does not denote or name anything, occurs. But it behaves as if it names a girl, and it is thus assured of a sense. (*PW* 122/ *NS* 133, 'Ausführungen über Sinn und Bedeutung' (1892–1895))

Here it is clear both that senses are easily attached to words and that vague concept words do not, or need not, lack sense. It is also at least suggested that the vague words comprise a relatively small, delimited class.[7]

Frege argues in numerous places that the objectivity of science and the possibility of communication depend on sentences' expressing (objective) sense (e.g. G & B 46/ *KS* 170; 'Über Begriff und Gegenstand', O 196n; *PMC*

[7] Two pages later Frege writes that not being 'an empty sequence of sounds' is sufficient for a proper name's having a sense.

80/ *WB* 128, Frege to Jourdain, undated, *c*.1913; *LI* 32 ff., 24 ff./ *KS* 362 ff., 'Die Verneinung', O 143 ff.; *KS* 358 ff.; 'Der Gedanke'; O 74 ff.). And he repeatedly states or implies that the sense of a sentence (at an occurrence) is a function of the senses of its parts (G & B 54/ *KS* 178, 'Über Begriff und Gegenstand', O 205; *PW* 192/ *NS* 208–209, 'Einleitung in die Logik' (1906); *LI* 55 ff./ *KS* 378 ff., 'Gedankengefüge', O 37 ff.).

These general doctrines about the commonness of sense-expression are buttressed by numerous statements that this or that expression has a sense: 'the morning star', 'the celestial body most distant from the Earth', 'the least rapidly convergent series', 'the Moon', 'Odysseus' and so on are cited in 'On Sense and Denotation' alone. In fact, the general tenor of Frege's remarks is that only rationally defective expressions (like 'the concept horse', as he later decided (*PW* 177–8/ *NS* 192–3, 'Über Schoenflies: die logischen Paradoxien der Mengenlehre' (1906); *PW* 193/ *NS* 210, 'Einleitung in die Logik' (1906)), really lack a sense. Although the examples I have cited concern singular terms, many of the cited singular terms contain concept words. So by the composition doctrine, the singular terms could not have senses unless their component concept words had senses. Moreover, there are passages in which Frege explicitly speaks of concept words in conventional usage as having a definite sense (e.g. *PW* 122, 209/ *NS* 133, 'Ausführungen über Sinn und Bedeutung' (1892–1895); *NS* 225–6, 'Logik in der Mathematik' (1914)). The view that nearly all expressions lack a definite sense has no clear textual basis and runs against an overwhelming number of remarks Frege makes.

We turn now to the option of taking Frege to hold that nearly all expressions in ordinary, conventional mathematics and ordinary discourse lack a denotation without lacking a sense. This option might seem to receive support from the following unpublished passage written in 1914:

> In the first stages of any discipline we cannot avoid using the words of our language. But these words are, for the most part, not really appropriate for scientific purposes, because they are not precise enough and fluctuate in their use. Science needs technical terms that have precise and fixed *Bedeutungen*, and in order to come to an understanding about these *Bedeutungen* and exclude possible misunderstandings, we give explications. Of course, in so doing we have again to use ordinary words, and these may display defects similar to those which the explications are intended to remove. So it seems that we shall then have to do the same things over again, providing new explications. Theoretically one will never really achieve one's goal in this way. In practice, however, we do manage to come to an understanding about the *Bedeutungen* of words. Of course we have to be able to count on a meeting of minds, on others guessing what we have in mind. (*PW* 207/ *NS* 224, 'Logik in der Mathematik' (1914))

The practice of substituting technical expressions for ordinary expressions, even those like 'number', is prominent in *Grundgesetze*, written much earlier.

Here Frege does perhaps suggest that nearly all non-technical words are 'not precise enough and fluctuate in their use'.

Nevertheless, Frege does not say in this passage that non-technical words, much less ordinary mathematical expressions, are in general bereft of *Bedeutung*. In fact, the passage clearly presupposes that the relevant words *have a Bedeutung*. The problem that concerns him is that there is some difficulty in determining or understanding exactly what *Bedeutung* a person denotes with an ordinary word. The problem is lack of standardization. The remarks of this passage appear to apply primarily to expressions of ordinary language. And Frege is issuing a general indictment, not one specifically concerned with vagueness. Imprecision and fluctuation of use cover not only vagueness but ambiguity; lack of interpersonal standardization; lack of a generally agreed-upon specification, in other terms, of the *Bedeutung*; and the dependence of expressions (demonstratives or indexicals) on context.

The problem with this option for solving the puzzle is similar to the problem with the previous option. Frege repeatedly writes as if fiction and serious mistakes are the primary sources of failures of denotation. When he discusses vague, denotationless concept words, the examples he offers are always presented as if they were special cases—'heap', 'bald' and so on. By contrast, Frege gives numerous examples of concept words—both in conventional mathematies and in ordinary discourse—that are explicitly ascribed denotations (*PW* 120–121, 177, 182, 229/ *NS* 130–131, 'Ausführungen über Sinn und Bedeutung' (1892–1895); *NS* 192, 198, 'Über Schoenflies: die logischen Paradoxien der Mengenlehre' (1906); 247, 'Logik in der Mathematik' (1914); G & B 113/ *KS* 278, 'Was ist eine Funktion?', O 663; and so on).

If he were to hold that nearly all concept words in mathematics, natural science, and ordinary discourse are vague, and thus lack a *Bedeutung*, he would have to hold that nearly all sentences in actual use strictly speaking express neither truths nor falsehoods. There is no suggestion of a 'secret doctrine' to this effect. (The closest approach to it that I can find is *PW* 242–243/ *NS* 261–262, 'Logik in der Mathematik' (1914)). Indeed, again, his basic argument for the existence of sense depends on the assumption that we share knowledge, hence express true thoughts, in the sciences. Neither of the options that we have considered for denying the third step in the argument is a credible expression of the spirit or letter of Frege's writings.

There is no questioning the attribution of the first step of our argument—the claim that vague expressions lack a *Bedeutung*. Frege's remarks are frequent, straightforward and explicit. What of the second step—the claim that expressions of mathematics, science and ordinary discourse are not sharply understood, even by their most competent users? Understanding it is a complex undertaking—the central element in solving our puzzle. I shall be developing an interpretation of the second step throughout the rest of the paper.

IV

On several occasions, Frege states or implies that the sense of an expression, often a mathematical expression, is not clearly or sharply grasped even by the most competent users of the expression. This theme emerges most clearly in his late writing. But the idea informs Frege's work almost from the beginning. Let us start by considering some passages from *Foundations*:

> What is known as the history of concepts is a history either of our knowledge of concepts or of the meanings of words. Often it is only through great intellectual labour, which can continue over centuries, that a concept is known in its purity, and stripped of foreign covering that hid it from the eye of the intellect. (*FA* p. vii)

A cognate passage occurs seven years later, in 1891:

> For the logical concept, there is no development, no history . . . If instead of [this sort of talk] one said 'history of the attempt to grasp a concept' or 'history of the grasp of a concept', it would seem to me far more appropriate; for the concept is something objective that we do not form and is not formed in us, but that we try to grasp and finally, it is hoped, really grasp—if we have not mistakenly sought something where there is nothing. (*KS* 122, 'Über das Trägheitsgesetz', O 158)[8]

The significance of these two passages can be appreciated only when the context of the first one is noted. That passage occurs in the introduction to *The Foundations of Arithmetic*. It is almost immediately followed in sections 1 and 2 by a long complaint that most of the fundamental notions of arithmetic have not been 'sharply determined' or 'sharply grasped' (section 1) by mathematicians, past or present. He cites the notions of function, continuity, limit, infinity, negative and irrational numbers.

The whole book is thus cast as an attempt to produce, for the first time, a sharp grasp of concepts (later, senses and concepts through senses) that are associated with conventional mathematical usage, but that had never before been 'sharply grasped'. In the course of the book, Frege gives several further examples of applications of mathematical expressions that past usage and understanding had left indeterminate, but which his own understanding purports to fix determinately (*FA* 68, 80–81, 87, 96–98, 100–101, 102–103).

The mist metaphor recurs thrice in 1914. Here is one occurrence:

> How is it possible, one may ask, that it should be doubtful whether a simple sign [in common use] has the same sense as a complex expression if we know not only the sense of the simple sign, but can recognize the sense of the complex one [that

[8] The first of these passages was written before the development of the sense–*Bedeutung* distinction (1891). It is likely but not certain that the second one was also. The talk of 'grasping' suggests, however, that the passages concern what is thought. Frege writes in 1897: 'We might cite, as an instance of thoughts being subject to change, the fact that they are not always immediately clear. But what is called the clarity of a thought in our sense of this word is really a matter of how thoroughly it has been assimilated or grasped, and is not a property of a thought' (*PW* 138/*NS* 150, 'Logik' (1897)).

purportedly analyses it] from the way it is put together? The fact is that if we really do have a clear grasp of the sense of the simple sign, then it cannot be doubtful whether it agrees with the sense of the complex expression. If this is open to question although we can clearly recognize the sense of the complex expression from the way it is put together, then the reason must lie in the fact that we do not have a clear grasp of the sense of the simple sign, but that its outlines are confused as if we saw it through a mist. The effect of the logical analysis of which we spoke will then be precisely this—to articulate the sense clearly. (*PW* 211/ *NS* 228, 'Logik in der Mathematik' (1914))

A second occurrence appears in a critical discussion of ordinary mathematical practice. Frege makes the point that ordinary mathematicians often grasp the same senses. In particular, they 'attach the same sense to the word "number" '. But because they are so cavalier about their definitions, they do not 'get hold of the sense properly'. They do not manage to attain a definite mastery of the sense they think with. He says that the sense appears to them

in such a foggily blurred manner that when they make to get hold of it, they reach for it in the wrong place. One reaches perhaps erroneously to the right, the other to the left; and so they do not get hold of the same thing, although they wanted to. How thick the fog must be for this to be possible! (*PW* 216–217/ *NS* 234, 'Logik in der Mathematik' (1914); the third occurrence of the mist metaphor occurs later in the same essay, *PW* 241–242/ *NS* 260–261, and will be mentioned below).

The comedy of this passage is double-edged. Frege produces a deliciously absurd picture of his opponents. But the terms of the metaphor indicate a notion of understanding on Frege's part (as grabbing some elusive phantom) that inevitably seems incongruous to modern minds.

Frege goes on to blame the failure of mathematicians to 'grasp' or 'get a proper hold' of the senses that they attach to the word 'number' on a failure to lay down and abide by good definitions. There follows a lengthy discussion of Weierstrass's failures to provide clear definitions for his primary mathematical expressions. Frege is obviously taking Weierstrass as an example of one of the mathematicians who 'attach the same sense to the word number', but do not get hold of it properly. Frege thinks Weierstrass does not grasp sharply the sense that he associates with his own word. Frege ends this discussion with the remark that Weierstrass's grasp of the notion of number is very unclear. And he diagnoses the trouble as follows: 'He lacks the ideal of the system of mathematics' (*PW* 221/*NS* 239 'Logik in der Mathematik' (1914)).

The obvious implication of these passages is that the most competent users of the relevant expressions may not have achieved a sharply bounded understanding of a sense—a 'sharp grasp' of a definite sense—that they attach to the word they use. Conventional usage and understanding may not give those users a clear understanding of a definite sense that the expression already expresses.

From this remark it is natural for us to infer that the relevant expressions are vague. For if conventional use and understanding *by even the most*

competent users does not constitute mastery of a sense that fixes sharp boundaries, it is natural to conclude that the expression's 'sense' and proper application simply lack sharp boundaries. Attributing this inference to Frege is, however, the mistake that leads to our puzzle.

For Frege did not draw this inference. The clear implication of the above cited passages is that the relevant expressions *have* a definite sense, or denote a definite concept. The problem is that no-one has yet grasped it fully or clearly. The defect is in our understanding and use, not in the abstract senses themselves, nor even in our expressions' relations to their senses. There is, moreover, no suggestion that such senses lack a sharply delimited denotation.

Thus, although Frege believed something that for most modern philosophers would appear to entail that the senses of most mathematical expressions are vague, he did not accept the entailment. He might well concede that what we would call an expression's 'conventional linguistic meaning', as fixed by conventional or actual usage, is vague. (Recall the knots and threads passage.) But he thought that the expression might nevertheless have a definite *sense* with a sharply bounded denotation.

Now this fact, not the mere resolution of the puzzle, is the primary point of interest. Frege's views about vagueness are in themselves not particularly important. They are significant almost entirely because they are symptomatic of interesting aspects of his conception of thought and sense: Frege's failure to draw the inference is, I think, a sign that his conception of sense differs substantially from any ordinary conception of conventional linguistic meaning.

For all the controversy that has surrounded the term 'linguistic meaning' as a theoretical tool, I think it clear that we have a rough and ready, reasonably coherent conception that is expressed in standard dictionary entries and in intuitively acceptable explications of the meanings of terms. Conventional linguistic meaning, according to this ordinary conception, is a complex idealization of conventional use and understanding. The meaning of a term is revealed in its use and is articulated in dictionary entries and in reflective explanations of its use by competent users. If conventional use and best understanding are not sharp, then conventional meaning is certainly not sharp. An analogous point can be made for the notion of idiolectic meaning. On this common conception, it would be absurd to say that an expression had a definite, non-vague conventional linguistic meaning—or, alternatively, a definite range of application—even though all extant understanding and all actual usage failed to fix sharp boundaries for the expression's application. Similar points hold for modern conceptions of idiolectic linguistic meaning. It would be absurd to say that an expression had a definite, non-vague idiolectic linguistic meaning, even though all the individual's abilities to articulate the expression's meaning, and all the individual's actual usage failed to fix sharp boundaries for the expression's application. (Cf. n. 4.)

So it appears that for Frege the capacity of a word to express a sense is partly independent of the user's understanding and use of the word, and even of conventions about the word's usage, in a way that distinguishes his notion of sense from almost any ordinary notion of linguistic meaning—conventional or idiolectic.[9]

V

Before proceeding, I want to note that Frege does not explicitly emphasize the contrast with conventional (or idiolectic) linguistic meaning that I am highlighting. I believe that he never thought the issue through in any depth. The view that I have attributed represents a prominent strand in his thinking. And I think that he never gave it up. But there are passages that suggest that Frege was not focusing on the matter, passages in which some further specification of the view would be natural had Frege clearly formulated it to himself. There are also passages which suggest that Frege wanted to avoid questions that this conception very naturally raises.

According to the conception that we have been developing, the sense and *Bedeutung* of an expression in conventional mathematics might be quite definite even though all past and present applications and all extant abilities to explicate a word fail to fix sharp boundaries. Frege indicates, however, that the *Bedeutung* of a mathematical expression sometimes changes in the course of its history. Frege indicates in 'Function and Concept' that the *Bedeutung* of the word 'function' has changed in the course of the history of mathematics. More is now included in the *Bedeutung* than had been earlier, Frege counts inclusion of transition to the limit as an essential broadening of the original *Bedeutung* of 'function'. And he counts the admission of new arguments and values (such as negative numbers) for standard mathematical operations as evidence that the *Bedeutung* of the expression has been extended.

A similar point applies to the expression 'number'. In 1914 he writes:

> Originally the numbers recognized were the positive integers, then fractions were added, then negative numbers, irrational numbers, and complex numbers. So in the course of time wider and wider concepts came to be associated with the word 'number' . . . And the same happened with other arithmetical signs. This is a process which logic must condemn and which is all the more dangerous, the less one is aware

[9] It is important to recognize the difference between this distinctive feature, which bears on the relation between senses and words, and two other striking features of Frege's doctrine of sense. Frege conceived of senses as eternal abstract entities that have their logical and semantical properties independently of the activity of actual minds. And he held that the content of all thinking could be characterized purely in terms of such entities. These latter two views, though perhaps congenial with the conception that we are discussing, are strictly independent of it. Of the three views, I find the first (the one I am concentrating on in this paper) promising, the second deeper and more interesting than commonly thought but extremely dubious, and the third mistaken. For criticism of the third view, see 'Belief *De Re*', and 'Sinning Against Frege' (Ch. 5 above).

of the shift taking place. The history of science runs counter to the demands of logic . . . No science can master its subject matter and work it up to such transparency as mathematics can; but perhaps also, no science can lose itself in such thick mist as mathematics if it dispenses with the construction of a system. As a science develops, a certain system may prove no longer to be adequate, not because parts of it are recognized to be false but because we wish, quite rightly, to assemble a large mass of detail under a more comprehensive point of view in order to obtain greater command of the material and a simpler way of formulating things. In such a case we shall be led to introduce more comprehensive, i.e. superordinate, concepts and relations. What now suggests itself is that we should, as people say, extend our concepts. Of course . . . we do not alter a concept; what we do rather is to associate a different concept with a concept word . . . The sense does not alter, nor does the sign, but the correlation between sign and sense is different . . . If we have a system with definitions that are of some use and aren't merely there as ornaments, but are taken seriously, this puts a stop to such shifts taking place . . . In fact, we have at present no system in arithmetic. All we have are movements in that direction. Definitions are set up, but it doesn't so much as enter the author's head to take them seriously and to hold himself bound by them. So there is nothing to place any check on our associating, quite unwittingly, a different *Bedeutung* with a sign or word. (*PW* 241–242/*NS* 260–261, 'Logik in der Mathematik' (1914))

This passage is not incompatible with the passages I previously centered upon. Indeed they are complementary. This last one occurs in the same essay as the most explicit remarks about the best mathematicians' lacking a clear grasp of the senses they all attach to terms they commonly deal with. The mist metaphor even recurs in this passage.

But seen very schematically, the two groups of passages suggest different models of progress in science. On the first model, progress is a matter of obtaining a better, clearer grasp of thoughts that one is already dimly thinking and unperspicuously expressing. Better theory results in deeper understanding and clearer explication of some of one's own thoughts and senses. One might have been thinking various thoughts at different times, even though one is using a single expression. But deeper theory cuts through the mist and explicates a given sense long associated (however inconstantly and uncomprehendingly) with an expression. On the second model, theoretical improvements issue in new connections between words, on one hand, and senses and *Bedeutungen*, on the other. One replaces old concepts (and senses determining them) with new ones in one's thinking.

If the first model is to have any place at all, it must be complemented with the second. Sometimes actual usage is just too far removed from some conceptions to attribute those conceptions to the users. When the users eventually arrive at such a conception, and express it in old words, we have to see them as having altered the senses of the words. On the other hand, this point is compatible with the idea that *sometimes* a user expresses a conception that is more definite and richer than the users could articulate given his or her state of knowledge.

There remain vexed questions of practice and principle about where to draw the line. I believe that Frege never concentrated on the differences between the two models, or worried about the issues regarding change-of-belief *versus* change-of-'meaning' that have become prominent since his time. As noted, some of Frege's discussion seems fashioned so as to lay these issues aside.[10] I shall, however, continue to concentrate on the first model on the ground that it represents something philosophically important and distinctive, even if undeveloped, in Frege's thinking.

VI

How *could* an expression express a definite sense if its being related to that sense were not entirely explicable in terms of how its users actually use and understand the expression?[11] Frege does not directly confront the question. But his work seems to indicate a certain type of answer. Saying that the expression 'Number' denotes a definite concept and expresses a definite sense, despite the insufficiencies of current understanding and usage, is (for Frege) made possible by the belief that the ultimate justification of current mathematical practice supplements current usage and understanding in such a way as to explicate a definite concept and a definite sense that no one may currently be able to thoroughly understand or articulate. Weierstrass's inadequate understanding of the senses he thought with would be rectified if the 'ideal of the system of mathematics' had properly informed his thinking.

Frege's conception of sense-expression has two fundamental presuppositions. The *first* is that mathematics and other cognitive practices are founded on deeper rationally understandable aspects of reality than anyone may have

[10] For example, Frege holds that for systematic purposes it is better to introduce new mathematical signs for old ones when one gives a new definition. He largely follows this practice in *Basic Laws*. At least once he writes of a given concept expression that 'as long as it remains incompletely defined [definiert]' in a way that fails to determine for every object whether that object falls under the denoted concept, it 'must remain undecided' whether the object falls under the concept. So the concept expression is vague (*PW* 242–3/ *NS* 261–2, 'Logik in der Mathematik' (1914)). Perhaps Frege means only that as far as the incomplete definition goes, the word is left vague. Literally read, however, this passage is incompatible with the various passages, cited earlier, in which Frege speaks of the denotations of various mathematical concept expressions in conventional mathematics, even though no-one had given those expressions 'complete' definition. Frege is clearly more interested in giving an ideal system of mathematics than in providing a systematic account of the state of conventional mathematics before the ideal system has been discovered.

[11] This sort of question is something that Wittgenstein pressed in his thinking about rule-following. It is also implicit in Dummett's insistence that a theory of 'meaning' be a theory of understanding and use. I think that the question is legitimate and profoundly difficult, though I think that Dummett is not very sensitive to the elements in Frege that ignore this insistence. Indeed, he often appears to attribute acquiescence in the insistence to Frege.

presently understood. The implications of a practice can reach beyond the procedures and dispositions of the practitioners.[12]

The *second* presupposition is that mastery of these deeper rationales is to be treated as involving insight into the true senses of expressions. It is to be understood in terms of conceptual clarification. Deeper insight into the nature of things is simultaneously deeper insight into rational modes of thinking about the nature of things.

Both presuppositions derive from the rationalist tradition. They combine to form a rather special conception not only of the nature of sense-expression, but also of the enterprise of philosophical analysis. I mean this latter phrase not in a narrow sense that would apply specially to 'analytic' philosophy—the tradition that grew out of Frege and came in our century to dominate the English-speaking philosophical world. I mean by 'philosophical analysis' something that would apply to much of the philosophical activity of Socrates, Plato, Aristotle, Descartes, Leibniz and Kant, as well as to twentieth-century 'analytic' philosophers more specially concerned with language. Any activity at least partly aimed at understanding our 'conceptual scheme', our cognitive practices, is analytic in this broad sense. Frege shared with the rationalist tradition the confidence that a deep rationale underlies many of our practices. He also shared the view that this rationale makes its imprint on those practices in such a way that understanding the rationale is achieving insight into what was part of the practices even before the understanding was achieved. Since Frege develops the second of the two cited presuppositions in some depth, at least implicitly, I shall begin by discussing it. I shall return to the first afterward.

There are in Frege's writings two contrasting themes about the grasping of senses. Frege does not explicitly juxtapose and reconcile them until relatively late in his career (1914). But both themes and the makings for reconciliation are present from early on. One theme emphasizes how relatively easy it is to express and grasp a sense. We have already noted a variety of passages in which Frege suggests that normal linguistic activity is often sufficient to *express* a sense. Frege also states (G&B 57–58/ *KS* 144; 'Über Sinn und Bedeutung', O 27) that a sense expressed is grasped by everyone 'sufficiently familiar with the language'. He does not say what constitutes sufficient familiarity. But the surrounding context certainly suggests that the requisite familiarity is not an exceptional accomplishment. In the same passage, Frege cites names and (presumably) other context-dependent devices as exceptions

[12] Interpreting this point requires delicacy. Frege was primarily interested in mathematics. And he was pursuing a foundationalist programme. I think that his conception of sense should not be tied too closely to these contingencies. Frege clearly intended to apply his conception of sense to domains other than the mathematical. And neither foundationalism nor reductionism are crucial to the conception. Sense-expression is to be explicated in terms of better rationales than anyone may have grasped—regardless of whether these rationales provide a foundation in anything like the way logicism was supposed to provide a foundation for mathematics.

to the remark that knowing the language is sufficient to grasp the sense. But he indicates that people nevertheless commonly 'attach' a sense to the relevant term in a context, presumably grasping it in so doing.

There are also numerous passages in which Frege says that thinking paradigmatically involves grasping a thought, grasping the sense of a declarative sentence. There is no suggestion that thinking is especially difficult to engage in. (Frege's own virtuosity in this regard may have blinded him to difficulties that the rest of us face!) In fact, Frege bases one of his arguments for the existence of senses on the assumption that mankind commonly grasps them in common. All of these passages indicate that senses are routinely grasped in the hurly-burly of cognitive life.

On the other hand, there are passages where Frege indicates that grasping a sense or thought is a matter of degree, and that thoroughly grasping thoughts is an achievement worthy of some renown. There are, for example, the two passages from 1884 and 1891 in which Frege says that what is usually called the history of a concept is better termed the history of attempts to grasp a concept (*FA* p. vii; *KS* 122, 'Über das Trägheitsgesetz', O 158). Frege elaborates the point in a passage from 1897 (*PW* 138/ *NS* 150, 'Logik' (1897)). He is arguing that thoughts are changeless, in precise analogy to the earlier arguments that 'concepts' do not change: 'We might cite, as an instance of thoughts being subject to change, the fact that they are not always immediately clear. But what is called the clarity of a thought in our sense of this word is really a matter of how thoroughly it has been assimilated or grasped, and is not a property of a thought.' Taken together, these remarks suggest that 'thorough grasp' is a difficult matter.

The suggestion is amplified in Frege's late writing (1914). Where there is a failure to provide a satisfactory explication of mathematical terms, there is, he writes, a failure to understand the terms. Writing of Weierstrass, Frege says in an unusually smug passage:

> He had a notion of what number is, but a very unclear one; and working from this he kept on revising and adding to what should really have been inferred from his definition . . . And so he quite fails to see that what he asserted does not flow from his definition, but from his inkling of what number is. (*PW* 221/*NS* 239, 'Logik in der Mathematik' (1914))

He continues:

> But how, it may be asked, can a man do successful work in a science when he is completely unclear about one of its basic concepts? The concept of a positive integer is indeed fundamental for the whole arithmetical part of mathematics. And any unclarity about this must spread throughout the whole of arithmetic. This is obviously a serious defect and one would imagine that it could prevent a man from doing any successful work whatsoever in this science. Can any arithmetical sentence have a completely clear sense to someone who is in the dark about what a number is? This question is not an arithmetical one, nor a logical one, but a psychological one. We

simply do not have the mental capacity to hold before our minds a very complex logical structure so that it is equally clear to us in every detail. For instance, what man, when he uses the word 'integral' in a proof, ever has clearly before him everything which appertains to the sense of this word! And yet we can still draw correct inferences, even though in doing so there is always a part of the sense in penumbra. Weierstrass has a sound notion of what number is and working from this he constantly revises and adds to what should really follow from his official definitions. In so doing he involves himself in contradictions and yet arrives at true thoughts, which, one must admit, come into his mind in a purely haphazard way. His sentences express true thoughts, if they are rightly understood. But if one tried to understand them in accordance with his own definitions, one would go astray. (*PW* 222/ *NS* 239–240, 'Logik in der Mathematik' (1914))

Here Frege issues a sharp statement of the point that the best mathematicians may fail to have a clear grasp of senses or thoughts they think with. Nevertheless, he emphasizes that the expressions such mathematicians use express definite senses, definite thoughts (*Gedanken*). The presupposition of the whole passage is that the science of arithmetic and the words that Weierstrass uses already express the relevant senses and denote the relevant concepts. The lack of clarity or definiteness resides purely in the person: it is 'psychological' not 'logical' or 'arithmetical'. The mathematician is thinking with the sense, and expressing it in his writing, but not grasping it clearly.[13]

Frege articulates the same view, in a passage we quoted earlier (*PW* 211/ *NS* 228, 'Logik in der Mathematik' (1914)), at the conclusion of a long and interesting discussion of philosophical or foundational analyses—analyses of the sort that he had pursued in trying to establish logicism. The suggestion is that insofar as an analysis like the one he attempted in *Foundations of Arithmetic* is true but still subject to doubt, the difficulty resides in the person's inability to grasp the sense clearly.

These last two passages from 1914 contain the resolution of the apparent tension between the passages that suggest that grasping the sense of an expression is relatively easy and those that indicate that it is quite difficult. It is possible to express a sense and think by 'grasping' senses even though one lacks a 'clear' or 'thorough' 'grasp' of the sense. The sort of grasping that is necessary for successful communication and ordinary thinking is different from the sort (or level) of grasping necessary to articulate the senses of one's expressions through other terms.

[13] The remark comparing Weierstrass's plight with that of the ordinary mathematician when he thinks with the notion of the integral is useful in suggesting that thinking can go on without a clear, thorough grasp of the sense (the thought). On the other hand, it is important to recognize that although the notion of the integral has a fuller analysis that was coming to be grasped by the mathematical community, the notion of number that Weierstrass has not fully grapsed (and hence, presumably even the notion of the integral!) was not, in Frege's view, fully understood by any mathematician prior to Frege's own work. So the remarks that he applies to Weierstrass are applicable to the whole mathematical community.

In one respect, Frage's distinction between levels of understanding is quite ordinary. We commonly attribute thoughts and statements to people on the basis of their linguistic expressions even though those people cannot produce acceptable definitions or explications of the terms they use. That is, we interpret our attributions in standard ways even though the people to whom we are making the attributions are unable to articulate the standard meanings of the expressions they use. This practice extends even to cases where the people in question misunderstand the relevant expressions.

The striking element in Frege's view is his application of this distinction to cases where *the most competent speakers, and indeed the community taken collectively*, could not, even on extended ordinary reflection, articulate the 'standard senses' of the terms. The view is that the most competent speakers may be in the same situation as the less competent ones in expressing and thinking definite senses which they cannot correctly explicate or articulate. Definite senses are expressed and 'grasped' (with merely the weak implication that they are thought with), even though no one may be capable of articulating or explicating those senses (grasping them clearly and analytically).

Frege does not introduce a distinction between sorts or levels of sense-expression that parallels his distinction between sorts or levels of understanding or 'grasping'. Here it is clear that sense-expression is not seen as supervenient on use and individual or communal capacities in the way that conventional (or idiolectic) linguistic meaning is commonly taken to be.

As I indicated earlier, I think that Frege's conception attempts to bridge the gap between actual understanding and actual sense-expression by means of a normative concept—that of the deeper foundation or justification for actual understanding and usage. It would be incorrect to see Frege's notion of sense-expression as separated from that of actual understanding and use. But it does depend on the possibility of a projection beyond actual understanding and use.

Even the ordinary notion of conventional linguistic meaning depends on a projection. Actual usage is interpreted in terms of a standard that is drawn from such usage. But the standard depends on a certain sort of rationalized ordering of that usage. Think of how dictionary definitions are arrived at. Frege's idealization is, however, much more radical. His conception of an 'ideally competent speaker' is much further removed from that of actual competence than is the conception of an ideally competent speaker that is current in discussions of conventional (or idiolectic) linguistic meaning.

Frege's ideal speaker is not simply an ordered composite of what is—or would be, under certain fairly ordinary conditions—recognized to be the best extant usage and understanding. Frege's ideal speaker may have an understanding that is fundamentally better than, even substantially different from, anything anyone has yet achieved. Understanding in the relevant sense is seen as an achievement that brings with it insight into substantive truths. In this,

Frege is at one with the leading representatives of the rationalist tradition. (Cf. *FA* p. vii, *PW* 12–13, 222/*NS* 16–17, 'Booles rechnende Logik und die Begriffsschrift' (1880–1881); *NS* 240, 'Logik in der Mathematik' (1914).) As with Plato and Descartes, deep understanding of one's thoughts—and of the senses (forms, ideas) one thinks with—is not separable from the deepest sort of knowledge (*KS* 122–124, 'Über das Trägheitsgesetz', O 157–161; *KS* 369, 'Die Verneinung', O 150; *PW* 33/*NS* 37, 'Booles rechnende Logik und die Begriffsschrift' (1880–1881)).

Analyses that articulate the senses of expressions do not constitute degenerate knowledge for Frege, any more than logic and arithmetic themselves do. Nor are they true 'purely in virtue of sense'. Analytical insight into the nature of one's senses or thoughts may involve a legitimate feeling of having been 'committed' to the relevant knowledge all along—may produce some sense of reason's recalling its origins. But from the point of view of what informs one's explications, one's actions, and one's operative understanding, such knowledge may count as new and substantive. It is not *merely* reflection on one's thoughts or meanings in the sense that the empiricist-positivist tradition in this century would represent it. This aspect of Frege's view has been seriously neglected because it is so out of keeping with the meaning-is-use approaches that stretch from the Vienna Circle to the present.

What is original about Frege's interpretation of the relation between conceptual insight and the deepest sort of propositional knowledge is his reversal of the traditional order of priority, and his emphasis on the role of theory in attaining such knowledge. Traditionally, among some rationalists as well as empiricists, conceptual mastery was considered a precondition for judgement. And such mastery was interpreted in terms of what were presumed to be non-conceptual abilities, such as vision. This model is almost completely absent in Frege's work. There are, as I noted earlier, significant traces in Frege of the pre-Cartesian, Platonic picture of *a priori* knowledge as outer vision: vision of an eternal reality that informs and constitutes one's reason. But the traditional picture does not inform Frege's work in the traditional way. His model is not vision but theory.

Kant preceded Frege in insisting on the priority of judgement over non-propositional cognitive capacities. But Frege went beyond this Kantian view in two ways. In the first place, he developed a logical theory that demonstrated the fruitfulness of an account of judgement in analysing non-propositional parts and non-propositional cognitive capacities.[14] That is, Frege gave deep and detailed grounds for accepting the order of priority that Kant announced.

In the second place, Frege held that the analysis of judgement—and more generally, logic—is inseparable from *theoretical activity*. Unlike Kant, Frege

[14] This point is developed at considerable length in my 'Frege on Truth' (Ch. 3 above), especially the first and last sections.

was an original logician. Perhaps because of this he was more sensitive to the difficulties of arriving at a satisfactory logical theory and to the possibility that reflection on logical matters, as well as in other domains, has to be tested in a comprehensive theoretical context. For Frege, logic has to be discovered. It is not transparent to reflection. It has to be discovered through checking proposals as applied to discriminating reflection on ordinary theoretical usage in logic, mathematics and other domains. Frege thought that one could be sure of attaining thorough grasp of the senses of one's terms only through understanding their logical roles and their proper roles in a good theory. Such roles were guaranteed to be perspicuously revealed only in a logically perfect language that was ideally adequate to its subject matter.

Frege is a traditional rationalist in his belief in *a priori* knowledge, in his association of the deepest substantive knowledge with conceptual mastery, and in his modelling philosophy itself on logic and mathematics. But he goes beyond the tradition: Even *a priori* knowledge and non-propositional understanding (or conceptual mastery) were construed as grounded in logical analyses of theory, and checked against good theoretical usage. What makes Frege modern and entirely original is his combination of his traditional views about apriority, understanding and philosophy with a pragmatic understanding of the epistemology of logic. Thus Frege held a fallibilist and theory-based account of *a priori* knowledge, understanding and philosophical inquiry.[15]

Frege expresses his pragmatic perspective on understanding and *a priori* knowledge only occasionally. But it appears at all periods of his career. For example, in an unpublished manuscript from 1880–1, he writes:

> All these concepts have been developed in science and have proved their fruitfulness. For this reason what we may discover in them has a far higher claim on our attention than anything that our everyday trains of thought might offer. For fruitfulness is the acid test of concepts, and scientific workshops the true field of study for logic. (*PW* 33/ *NS* 37, "Booles rechnende Logik und die Begriffsschrift" (1880–1881))

The same point is developed at somewhat greater length in a passage of 'About the Law of Inertia' (1891). There Frege holds that thorough grasping of concepts has resulted to a large extent from recognition that the accepted delineation (*Begrenzung*) is 'blurred, uncertain, or not the one that was sought' (*KS* 123, O 159). He develops the idea (*KS* 122–124, O 157–161) that only through the development of scientific theory, whether in physics, chemistry or logic, does one achieve a thorough grasp of one's concepts. Similar remarks occur in the later period. (Cf. 'Die Verneinung', *KS* 369, O 150; *BL* 7; 25/ *GG* i. pp. x, xxvi.)

[15] This point makes it appropriate to say that for Frege philosophy is modelled on and continuous with science. But the relevant sciences are logic or mathematics, not natural science as it is for the positivists, Carnap and Quine. In this view, Frege is again a traditionalist (emulating Plato, Descartes, Leibniz, and Kant.)

Partly because of Frege's work, and the work of those such as Wittgenstein, Carnap and Quine who were influenced by it, the connection between understanding and theory has become a commonplace. What is still unassimilated, in my opinion, is the way Frege attaches ideal understanding and actual sense expression to ideal theory. It is his explication of sense-expression in terms of theorizing beyond any that may have actually been carried out that sets his conception of sense apart from more ordinary conceptions of conventional (or idiolectic) linguistic meaning.

VII

I wish now to turn briefly to the first of the two presuppositions behind Frege's view that the ultimate foundation and justification of current linguistic practice may supplement ordinary understanding in such a way as to attach it to a definite sense that no one may currently be able to articulate adequately, or thoroughly grasp. The presupposition is that some of our practices are founded on a deeper rationale or on deeper aspects of 'reality' than anyone may have presently understood.

This presupposition is, of course, bound up with deep and contentious philosophical issues. Systematic development of the idea is anything but trivial. Although I think that the idea need not be developed in foundational terms, Frege clearly regarded his senses, at least in some contexts, in the light of some final state of cognitive achievement.

One naturally thinks here of some Peircean conception. Although like Peirce Frege had a notion of ideal science, he did not construe it in terms of agreement, or in terms of any epistemological notion (such as the application of best canons of reason to all possible evidence). Such accounts would not express the notion of an ideal science for Frege because he thought that there is no conceptual guarantee that any human agreement, or any humanly applied epistemic methods, would not be mistaken. He thought it conceivable that everyone might agree on what is not correct. Where a theory is incorrect, there would always remain room for a better theory that might provide better understanding of the senses of terms in the original theory.

There are, of course, epistemically oriented accounts of ideal science that recognize these limitations and try to idealize beyond them: we should not limit the account to *human* agreement or observational capacities, or to what is available to *us* as possible evidence, or to *our* inductive methods. All these might be improved upon. But when one projects beyond human practices, the idealizations become much less clear. Moreover, they become subject to the suspicion that they are covertly relying on the assumption that they are practices that are ideal in the sense that they yield a *true theory*. Such reliance would make epistemic notions presuppose semantical notions, giving up the spirit of Peirce's idea. But short of reliance on semantical notions, it is

unclear how such epistemic explications can give a satisfactory account of the notion of ideal science—and hence of Frege's notion of sense. In any case, for Frege, the notion of truth is not to be reduced to epistemic terms.

For Frege, thoughts, the senses of declarative sentences, are conditions on truth. Conditions on truth are best understood, and senses are best articulated, from the standpoint of a comprehensive true theory. This is not to say, of course, that only expressions in a true theory have sense. The forerunners of such a theory also express senses, even if the senses are not thoroughly grasped. Moreover, non-denoting terms express senses. So even theories that are on the wrong track and have no justifying rationales express senses, possible ways of thinking. The point is rather that adequate explication of the senses of expressions is *guaranteed* only in a theory that cannot suffer further improvement or correction.

Frege did not attempt the reduction in reverse. He did not try to explicate epistemic notions by means of his notion of truth. He was as aware as anyone that the notion of truth does not provide an epistemic touchstone for judging the credibility of theories (cf. the opening pages of 'The Thought'). Thus he seems committed to the view that one has no epistemic guarantee, beyond all conceivable doubt, that one has arrived at a true theory.[16] But one is guaranteed an understanding of one's senses only from the point of view of an 'ideal science'—a true theory. So how to explicate the senses of one's terms is in principle just as subject to theoretical debate as to whether one's theory is true. The terms and methods of debate are surely different. But there is no difference as regards an epistemic guarantee that one is right.

The notion of truth provides a normative ideal that (in inevitable conjunction with concrete epistemic practices) enables us to conceive of terms' having senses that are not fully explicable in terms of usage and understanding. An ideally competent speaker is one who can give ideally thorough explications

[16] Frege does regard the basic truths of logic as self-evident. But he allowed that one might be mistaken about what is self-evident, if one has a less than thorough grasp of the thoughts involved. And the question of whether one has a thorough grasp is answerable by reference to the success of the theory in which the thoughts are embedded, cf. my 'Frege on Extensions of Concepts' (Ch. 7 below), pp. 30–4 (pp. 295–298, this volume).

It is worth pointing out here that the 'final' explications of an expression are not, at least are not in general, expressions of the same sense. Frequently they are part of fixing the sense of the expression being explicated. Frege thought that only in the context of a proposition (and more generally, a theory) could the sense of scientific expressions be fully understood. The senses of expressions could be fully grasped only by grasping equivalences given by ideal scientific explications, or by otherwise understanding the contribution of those expressions to a theory. Of course, since most of the envisioned ideal explications would come as discoveries, it is possible to doubt them (even if the doubt depends on less than full analytic mastery of the senses), while not doubting the corresponding self-identities. So by Frege's test for the identity of senses, the senses of the *explicans* and *explicandum* would be different. Of course, many expressions will express senses but will not appear in ideal science at all (e.g. denotationless expressions). The point is not that senses are expressed only in an ideal science, and certainly not that every term is incompletely understood. The point is rather that terms in science may not be fully understood, and that full understanding is guaranteed only when one has an ideal science.

and has an ideally thorough grasp of the senses of the expressions of his language. Having tied adequate explication and thorough grasp to having fundamental knowledge, and having tied fundamental knowledge to common theoretical activities (rather than indubitable insight), Frege must see full understanding as guaranteed only by completely fundamental and completely satisfactory (true) theory. It is this combination of a rationalist notion of understanding with a pragmatic epistemology that makes Frege's view unique and intriguing.

Since Frege did not develop this conception, it is important to reflect on it—and be prepared to develop it—in a flexible way. The view is most immediately attractive as applied to terms in mathematics and the empirical sciences. It is less plausible, at least initially, with respect to ordinary expressions that have no role in the sciences. The idea that one might obtain some significant improvement in our understanding of them may seem rather far-fetched. And surely, one is inclined to think, there are some expressions in ordinary discourse for whose understanding we would simply refuse to recognize any further authority than communal practice.

It is noteworthy that Frege always applies his remarks about the most competent users' incomplete understanding of their own expressions to terms that have a role in a scientific enterprise. Perhaps he would have given a different theory for non-scientific terms that are not grasped sharply. He may have held that such terms are associated with a cluster of senses. Where none of the senses is grasped sharply, no determinate thought is entertained. Or (what is more suggested by his actual remarks about vague terms) a definite sense might be assigned in the context according to how the thinker would draw a line if pressed. Perhaps he would have seen no need, in the case of ordinary non-scientific terms, for the possibility that a particular, unforeseen sharpening might be the correct one. Surely this possibility is sometimes closed.

On the other hand, the possibilities for rational improvement that extends elements already present in a practice are difficult to circumscribe. What underlies the attractiveness of Frege's conception, to my mind, is not some prior view of what a science is. It is a combination of two more general views. One is the view that full understanding of our thoughts and senses is not independent of knowledge of the subject matters about which we think. The other is that neither individually nor communally do we have infallible access to the truth about those subject matters. These two views jointly press one toward Frege's conception. And they retain some force even as applied to thoughts outside the systematic sciences.[17]

Let us return briefly to our initial puzzle about vagueness. The puzzle helped signal the distinctive character of Frege's conception of sense. The

[17] For a beginning at developing these ideas, see my 'Intellectual Norms and Foundations of Mind', *The Journal of Philosophy*, 83 (1986), 697–720.

account of sense that we have given helps, in turn, to illumine Frege's otherwise odd views about vagueness. Through theory one arrives at the sharpening of the application of expressions. This sharpening enables one to go beyond conventional or idiolectic linguistic meaning to arrive at a full understanding of one's expressions, a thorough grasp of their senses. Until a powerful theory is developed and logical analysis of its language is achieved, a full grasp of the senses of one's own expressions may be lacking.

Frege's conception of sense as deeply connected to that of a true theory also sheds light on, though it may not justify, his view that no senses have vague boundaries. Where senses are ways of thinking ideally or purportedly appropriate to a true theory, they are ways of thinking that ideally or purportedly reflect reality. Frege believed that vagueness could not infect reality itself—the objective entities which our thoughts denote. I suspect that only if one explicates reality in terms of mind or meaning does the notion of vagueness in reality make any sense. And Frege resisted such explications, partly out of allegiance to common sense, partly because of the sort of considerations that we have been rehearsing. So he concluded that senses, ways of thinking about reality, could not be vague. Only our grasp of senses could be.

This line of thought is, of course, more attractive for ways of thinking that are ideally fitted to reality than for ways of thinking that are merely purportedly fitted to reality. Some senses, as we noted, do not fit into an ideal science. They would not be expressed by a true theory. It is unclear how such an argument that senses cannot be vague could be applied to such senses. Perhaps the old charge that Frege's idealizations are too imperial still sticks. But it seems to me that his motivations are far more subtle and interesting than the traditional criticisms of his views on vagueness recognize.

I do not want to suggest that these issues are anything like as simple as I have been representing them. My primary point has been to try to indicate how Frege's odd-sounding views about vagueness are motivated by his realist conception of the world and his rationalist conception of sense.

VIII

I will close by noting in an extremely sketchy way some points about the subsequent historical fate of Frege's notion of sense.

Russell's views of understanding and knowledge began the historical process of obfuscation. From the beginning, Russell allowed little or no room for the idea that one could think with notions that one only partially understands. Russell assimilated understanding to a non-propositional, vision-like conception of knowledge, called acquaintance, which propositional abilities were supposed to presuppose. Much of the motivation for his semantical work lay in his notorious 'principle of acquaintance', which antedates

'On Denoting' *Mind*, 14 (1905), 479–493. According to this principle one has to be 'acquainted with' every constituent of a 'proposition' that one thinks. The key point for us is that acquaintance was understood in terms of an infallible, direct and absolutely complete mastery of the propositional constituents. Eventually, Russell held that the only things with which we can be acquainted are universals, present sense-data, and our selves. This approach led, of course, to much discussed difficulties with the theory of reference to individuals. But it did not allow serious consideration of the understanding of universals.

Encouraged by his interest in establishing logicism, Russell did emphasize that through analysis one could enlarge one's powers of understanding and gain insight into the foundations of notions that one is already thinking with. But because he treated understanding in terms of acquaintance rather than in terms of the use of a theory, Russell never allowed this emphasis to threaten the principle of acquaintance and the view of understanding and propositional thought that rests upon it. Thus the implicit tension between the principle of acquaintance and the natural view that with 'analysis' we obtain a better understanding of the analysanda never comes to a head in Russell's work.

Russell's communication of Frege's views to the rest of the philosophical world did nothing to call attention to the difference between his own views and Frege's on these points.

It is not clear to me to what extent the Vienna Circle reacted against Russell's vulnerable conception of understanding and to what extent it simply bypassed it. What is clear is that the Vienna Circle struck off in a very different direction. The early, more phenomenalist stage of the movement bears comparison to Russell's work in its interest in explicating understanding in terms of a construction from basic, rather specially certain types of cognition (the having of phenomenal experiences). But the prominence of this view diminished as the movement developed. Moreover, from the beginning the movement was much more ambitious about giving a detailed account of understanding than Russell was.

There seem to me to be two key ideas in the positivist programme that are relevant to our theme. One is the view that what there is to be understood, 'meaning', is to be reduced to actual procedures that express or constitute actual understanding. That is, meaning is identified with cognitive or theoretical usage. The other key idea is that both meaning and understanding are to be accounted for in terms of verification procedures.

The standard histories of the period concentrate on the second idea. But it seems to me that the first is equally momentous. Roughly speaking, it contains two moves: the reduction of what is understood to actual understanding, and the explication of actual understanding in terms of presently articulateable abilities or experiences. These moves seem to me to be fundamental, but largely unexamined in the work growing out of the Vienna Circle movement.

The influence of these moves is evident in the explications of Frege that have been taken to be authoritative until recently. Carnap's explication of Fregean sense in *Meaning and Necessity* is a prime example. He writes, 'The concepts of sense and of intension refer to meaning in a strict sense, as that which is grasped when we understand an expression without knowing the facts.'[18] Quine takes up the same line, with fewer positivist commitments, when he explains Frege's conception, without comment, in terms of linguistic meaning. Most (though not all) of Kripke's criticisms of Frege depend essentially on identifying sense with an ordinary conception of linguistic meaning. Dummett's account of Frege's notion of sense, though much more nuanced and in many respects more insightful, takes up the same theme—again influenced by preconceptions about the relation between 'meaning' and 'use'.[19]

What made it historically difficult to appreciate the difference between Frege's conception of sense and modern notions of linguistic meaning was the domineering role of the idea that originated with the Vienna Circle: the reduction of what is understood to actual understanding and the explication of understanding in terms of actual articulateable abilities or experiences. The conception of meaning as used in the later Wittgenstein, the attempt to reduce philosophical problems to questions about ordinary language, and Quine's holistic liberalizations of verificationist conceptions of meaning, are all marked by these moves.

Parallel doctrines in the philosophy of mind are equally founded on the reduction of what is thought to what the thinker can presently do. Behaviourist and functionalist theories of mental content, Wittgenstein's puzzles about the possibility of following a rule, and Quine's claims that mind and meaning are fundamentally indeterminate—all depend on the idea that what is understood is to be reduced to or accounted for purely in terms of present abilities (where these are described in such a way as not to make explicit use of intentional notions like understanding or thought).

There is, of course, considerable force—not to say epistemic comfort—in this idea. But its dominance has engendered a certain blindness to the character and, I think, power of the very different point of view that motivated Frege. Appreciating and developing that point of view may help illumine some of the dark dead-ends of twentieth-century philosophy.

[18] Rudolf Carnap, *Meaning and Necessity*, 2nd edn. (Chicago: University of Chicago Press, 1956), 125. Carnap goes on to identify sense with intension, so that logically equivalent terms have the same sense. He explains logical equivalence in modal terms, thus further obscuring Frege's view. The explication of 'sense' in modal rather than cognitive terms continued in discussions of Frege into the 1970s and recurs occasionally even now. Such explication of course has virtually nothing to do with Frege's original conception and his motivating problems.

[19] W. V. Quine, 'Two Dogmas of Empiricism', in *From a Logical Point of View*, 2nd edn. (New York: Harper Torchbooks, 1961), 21; Saul Kripke, *Naming and Necessity* (Cambridge, Mass.: Harvard University Press, 1980), e.g. 53–4; Michael Dummett, *Frege: Philosophy of Language* and *The Interpretation of Frege's Philosophy*. Some of these same points are discussed in my 'Sinning Against Frege' (Ch. 5 above).