

Postscript to “Frege on Apriority” (2003)

In both “Frege on Knowing the Foundation” (Chapter 9 above)¹ and “Frege on Apriority” (Chapter 10)² I note that on Frege’s definitions of analyticity and apriority in section 3 of *Foundations of Arithmetic*, basic axioms of logic do not count as either analytic or synthetic. In both of these passages I express agreement with Michael Dummett that Frege’s definitions constitute, in this respect, a harmless oversight. I still believe that the definitions’ not covering these cases is harmless. I no longer believe that Frege was guilty of oversight.

Omission of basic logical axioms from the category *analyticity* goes back to Kant’s own formulations. Kant did not count as analytic either the principle of non-contradiction or simple, strict identities like $a = a$.

Kant counted the principle of non-contradiction as a formal mark of truth and a principle of analytic cognition.³ He seemed to regard such principles as fundamentally regulative meta-principles governing the practice of analysis. He did not count such principles themselves analytic.

Kant counted simple, strict identities tautologies. He states that tautologies are the limits of analysis.⁴ He does not count the tautologies themselves analytic. Kant writes, “Analytic judgments are grounded in identity and can therein be resolved, but they are not identical for they need analysis and serve the explanation of concept.”⁵

Kant’s remark here explains the motivation for not counting basic principles or tautologies as analytic. They are not subject to analysis. Their truth is not revealed by analysis.

Of course, Kant does not count them synthetic either. The meta-principles are canons for regulation of good thinking. In their basic form, Kant may not have regarded them as true or false. The tautologies are certainly not synthetic since the content of their predicate does not “go beyond” the content of their subject.

Frege did not follow Leibniz and Kant in believing that the principle of non-contradiction and tautologies are all that lies at the foundation of logic. His logical principles are a much richer lot.

Frege also differed from Kant in his definition of analyticity. Kant’s official characterization is in terms of containment: a truth or judgment is

¹ Cf. “Frege on Knowing the Foundation”, Ch. 9, p. 322 and n. 6.

² “Frege on Apriority”, Ch. 10, n. 18.

³ Kant, *Critique of Pure Reason*, A59/B83–4, A151/B191. I am indebted to Verena Mayer for bringing to full consciousness an awareness of these points in Kant.

⁴ Kant, *Gesammelte Schriften*, Königlich Preußischen Akademie der Wissenschaften, 29 vols. (Berlin: 1902–1983; 2nd edn. Berlin: De Gruyter, 1968, for vols. I–IX) AA IX, 111–12; AA XVI, 672–3.

⁵ *Ibid.* AA XX, 322.

analytic if its predicate is contained in its subject.⁶ Frege’s is in terms of provability from logical laws: “If in [finding a proof of a proposition, and following it back to the primitive truths] we come only on general logical laws and on definitions, then the truth is an analytic one” (*Foundations of Arithmetic*, §3). Frege does associate provability with being contained in the axioms (*Begriffsschrift*, § 13). But containment is *explained* for Frege in terms of provability—most certainly not primarily in terms of conceptual containment.⁷

Frege’s different conception of logic and his different axioms lead to his making different judgments about cases. For example, he takes statements of the form $a = a$ to be provable from general logical laws, hence both analytic and apriori.⁸

Both Kant’s definition and Frege’s, however, take analyticity to consist in being subject to analysis. In Kant’s case, analysis is partitioning concepts. In Frege’s case, it is finding a canonical proof from basic logical axioms. In each case, the starting points—the basic logical truths or principles are not subject to analysis. So they are not analytic. The motive for Kant’s terminology carries over in different form in Frege’s terminology. Both had a reason for not counting basic logical laws “analytic” in their own frameworks. Such laws are not subject to analysis—to further conceptual or proof-justificational unpacking.

Frege seems simply to have lined up his definition of apriority with his definition of analyticity. A truth’s apriority lies in the nature of its proof. The basic axioms are self-evident. It is reasonable to believe them. They yield knowledge, and belief in them is surely warranted. But there is no discursive justification for them, no proof of them. I do not use either “analytic” or “apriori” in the way Frege did. But it is interesting to see that his use, at least of “analytic”, derives from an old and well-motivated tradition.

⁶ Kant, *Critique of Pure Reason*, A6/B10. Evidently Kant did not take identity to be a type of containment.

⁷ One of Frege’s great achievements in logic was to recognize that not all logical truths are truths of containment of one concept in another. Not all logical truths are in subject–predicate form.

⁸ Cf. the opening page of “On Sense and Denotation”.