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Truth and Singular Terms

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Non-denoting singular terms have been a prime stimulus, or irritant, to students of the use and formal representation of language. Only one other subject (modal logic) has provoked so many differences among logicians over which sentences should be counted valid. It may be that in the case of singular terms these differences are not fully resolvable, especially in light of the various purposes that logical analyses serve. But there are, I think, means of narrowing our options in choosing logical axioms. This paper is devoted to exploring one such means.[1]

I will be viewing a logic of singular terms in the context of a truth-theoretic account of natural-language sentences under their intended interpretations. As a consequence, I shall take native intuitions about truth conditions and truth values as evidence in framing a semantical theory for native sentences and for framing a logic underlying that theory. Within this context, I shall argue that some proposed logics for singular terms are unsatisfactory because they lead from true premises to untrue conclusions and that others are faulty because they are too weak to justify transformations needed for an adequate theory of truth. I shall conclude by arguing that one otherwise plausible logical axiom is incompatible with a straightforward means of avoiding these difficulties. I begin by motivating and sketching an account of singular terms which will set the stage for these points.

I. THE MOTIVATION FOR FREE LOGIC

The most clearly semantical problem which non-denoting singular terms raise is that of saying how the truth conditions of sentences containing them may be determined on the basis

NOÛS 8 (1974)

of the logical roles of the parts of the sentences. The classical treatments of the problem are, of course, Russell's and Frege's. (Cf. [6], [18], [25].) Russell's elimination of singular terms pays well-known dividends but fails to account for natural language as it is actually used. Frege's stipulation that intuitively non-denoting singular terms denote the null set forces one to formalize certain negated existence statements in syntactically unnatural ways. Moreover, by thus "identifying" all non-denoting singular terms, the theory counts sentences like 'Pegasus is the smallest unicorn' true. Although some analytical enterprises can perhaps overlook such results, an account of truth in natural languages cannot.

In the last decade or so, a number of free logics have been developed to account for, rather than do away with, non-denoting singular terms. (Cf. [10], [21], [13], [14], [11], [20], [23], [19], [15], [8].) These logics seem to me to be on the right track. But there has been little agreement over precisely which inferences such logics should validate or block. The restriction on existential generalization and universal instantiation which is common to all free logics has frequently been justified by reference to various purportedly true singular sentences, like 'Pegasus is winged', from which one cannot existentially generalize. Literally taken, these sentences are, I think, untrue. In so far as they are counted true, they are best seen as involving an implicit intensional context: '(A well-known myth has it that) Pegasus is winged'. The strategy is an extension of Frege's approach to apparent failures of substitution. In our sample sentence (regarded as true), we cannot substitute for 'Pegasus' other singular terms which do not differ in denotation ('the tallest unicorn'), and still preserve truth. So we regard the context as oblique.

One might object that different "non-existents" are denoted by these singular terms in *all* their occurrences. But as regards the intended interpretation of non-denoting singular terms, this way of speaking is, I think, misleading. Currently, the temptation to speak this way seems to arise only in the face of sentences which are easily seen to be related to one of the standard sorts of intensional contexts (indirect discourse, subjunctives, psychological contexts). The point is clear in our example. When native speakers are asked whether 'Pegasus is winged', is true, they rely on common knowledge and contextual clues to determine what the questioner intends. It is now common knowledge among people who use 'Pegasus' in the relevant contexts that the name is part of, and is meant to be related to, a mythical story. The prefix to the sentence that we supplied above will generally be accepted as producing a paraphrase. But if asked whether the myth itself is true—whether it is a matter of fact (rather than a matter of fiction) that Pegasus is winged—native speakers will reply that the sentence is not literally or "factually" true. And they will justify this by saying that Pegasus does not exist, "except in the myth." I take this behavior as evidence for embedding the sentence in an intensional context when it is regarded as true. Non-denoting singular terms simply do not have anything as their "ordinary" (non-oblique) denotation. It is just that the oblique reference of some singular terms is the reference they most often have in their everyday uses. Talk of "non-existents" in contexts like the above can perhaps be assimilated to the strategy of finding the oblique reference for singular terms in intensional contexts.[2]

Implementing the strategy and providing an account of oblique contexts, is, of course, beyond our present purpose. It is enough to note here that the motivation for free logic may be regarded as independent of issues about apparent substitution failures of singular terms. Consider the sentence '$(x)(x = x)$' from identity theory. By universal instantiation, we derive 'Pegasus = Pegasus'; and by existential generalization we arrive at '$(\exists y)(y =$ Pegasus$)$', which is clearly false. Unless we regress to Russell or Frege, we must either alter identity theory or restrict the operations of instantiation and generalization. Experimentation with the latter two alternatives indicates that the restriction strategy is simpler and more intuitive.

II. A SKETCH OF A THEORY OF SINGULAR TERMS

We now characterize a logic underlying a formalized metalanguage *ML* and a theory of truth couched in that language for the sentences of a natural object-language *OL*. The grammar of *ML* is that of first-order quantification theory with predicate constants, identity, function signs, and the definite-description operator. The logical axioms and rules underlying *ML* are as follows:

(*A*1) If *A* is a tautology, $\vdash A$.

(*A*2) $\vdash (x)(A \rightarrow B) \rightarrow ((x)A \rightarrow (x)B)$.

(*A*3) $\vdash (x)(x = x)$.

(*A*4) $\vdash t_1 = t_2 \rightarrow (A(x/t_1) \leftrightarrow A(x/t_2))$.

(*A*5) $\vdash (x)A \,\&\, (\exists y)(y = t) \rightarrow A(x/t)$.

(*A*6) $\vdash (x)(x = (\iota y)A \leftrightarrow (y)(A \leftrightarrow y = x))$, where variable $x \neq$ variable y, and x is not free in A.

(*A*7) $\vdash (x)(\exists y)(x = y)$.

(*A*8) $\vdash (x)(x = t_1 \leftrightarrow x = t_2) \rightarrow (A(y/t_1) \leftrightarrow A(y/t_2))$, where x is not free in t_1 or t_2.

(*A*9) $\vdash At_1 \ldots t_n \rightarrow (\exists y_1)(y_1 = t_1) \,\&\, \ldots \,\&\, (\exists y_n)(y_n = t_n)$, where A is any atomic predicate, including identity, and where y_i is not free in t_i.

(*A*10) $\vdash (\exists y)(y = f(t_1, \ldots, t_n)) \rightarrow (\exists y_1)(y_1 = t_1) \,\&\, \ldots \,\&\, (\exists y_n)(y_n = t_n)$, where y is not free in $t_1, \ldots, t_n$, and y_i is not free in t_i.

(*R*1) If $\vdash A$ and $\vdash A \rightarrow B$, then $\vdash B$.

(*R*2) If $\vdash A \rightarrow B$, then $\vdash A \rightarrow (x)B$, where x is not free in A.

'A' and 'B' range over well-formed formulas of *ML*; 't', 't_1', ..., 't_n', over terms (including variables); and 'x', 'y', 'y_1', ..., 'y_n', over variables. '$A(x/t_1)$' signifies the result of substituting t_1 for all occurrences of x in A, rewriting bound variables where necessary.

Axioms (*A*3), (*A*5), and (*A*8) are non-independent. (For the details, see [1].) They are included for the sake of clarifying our motivation. Alternatively, one might take (*A*3), (*A*4), and (*A*5) as non-independent and the others as primitive, adding the symmetry and transitivity axioms of identity. The value of this formulation is that it focuses on (*A*8) instead of (*A*4). As will be seen, (*A*8) constitutes the main principle of interchange in the system.

The logic of *ML* is nearly classical. If (*A*7) were changed to '$(\exists y)(x = y)$', and if singular terms other than variables were excluded from the language, the logic would revert to classical quantification theory with identity. The chief motivation for (*A*7) is that, unlike '$(\exists y)(x = y)$', it allows some of the free variables (like some of the other terms) to be uninterpreted.

"Non-denoting" free variables are useful in representing sentence utterances which involve failure of reference with demonstrative constructions.[3]

Axiom (*A*9) differentiates the syntax of *ML* from that of Scott [20]. It expresses a deep and widely held intuition that the truth of simple singular sentences (other than those implicitly embedded in intensional contexts) is contingent on the contained singular terms' having a denotation.[4] The pre-theoretic notion seems to be that true predications at the most basic level express comments on topics, or attributions of properties or relations to objects: lacking a topic or object, basic predications cannot be true. Given that *ML* is bivalent, simple singular predications containing non-denoting terms are counted false, and negations of such sentences are true. ('Pegasus is an animal' is false. 'It is not the case that Pegasus is an animal' is true.) Within *ML*, logical operations such as negation should be intuitively seen as working on simpler sentences as wholes, not as forming complex comments on purported topics or complex attributions to purported objects. This remark would admit of exceptions if we were to provide for singular terms with wide scope ('Pegasus is such that he is not an animal').[5] Then negation operates on an open sentence rather than on a closed one. Non-denoting singular terms with wide scope should cause the sentences they govern, no matter how complex, to be untrue.

Axiom (*A*9) rests weight on the notion of atomic predicate. As just indicated, I think that the weight has intuitive support, support associated with semantical intuitions about truth and with the pre-theoretic notions of property and relation. The axiom should be regarded as a methodological condition on investigations of predication in natural language: count an expression an atomic predicate in natural language only if one is prepared to count simple singular sentences containing it untrue whenever they also contain non-denoting singular terms.[6] Scott's and Lambert's systems show that it is possible to arrange a logically coherent language with atomic predicates that violate our condition. But it is another question whether such predicates have natural-language readings that are best construed as having the logical form of atomic predicates. In numerous cases, intuition backs our condition; the present proposal is that the condition should be used to guide intuition. Needless to say, it must be judged by the quality of its guidance.

Axiom ($A9$) enables us to derive the Russell equivalence

$$B(\iota x)Ax \leftrightarrow (\exists y)((z)(Az \leftrightarrow z = y) \& By),$$

where B is atomic. (This latter restriction amounts to the proviso that the iota operator always takes smallest scope. Cf. note 5.) The present system thus captures Russell's intuitions without using his means of doing so. Whereas we agree with Russell about truth conditions, we disagree with him about logical form. Rather than regarding singular terms on the model of abbreviations for *other* language forms, we take them as primitive in natural language and in formal languages whose purposes include representing natural language. Consequently, rather than give a semantical analysis for singular terms only indirectly, as Russell did, via a semantical analysis for the grammar of quantification theory with identity, we do so directly. (Cf. Kaplan [12].)

It is worth remarking that in languages where some singular terms fail to denote, ($A9$) is inconsistent with $\ulcorner t = t \urcorner$. Since some free logics have included this principle, ($A9$) will be discussed at greater length in Section IV.

Axiom ($A10$) complements ($A9$): If n-ary function signs are to be regarded as potentially explicable in terms of (n + 1)-ary predicates in the usual way, then where function signs are given primitive status (as they are here), ($A10$) must be added if ($A9$) is. Axiom ($A10$) is Fregean in motivation. The value of a function was, on his view, the result of completing the function with an argument—where 'argument' is understood to apply to objects rather than to substituted linguistic items (terms). (Cf. [5]: 24–25; [7]: 33–34, 84.)

The model theory for the logic is straightforward. The domain may be empty. Under each interpretation, all sentences are either true or false. Variables, function signs, and complex singular terms are defined by the interpretation function, if at all, on the domain. Only values identical and within the domain satisfy the identity predicate. The clauses for other atomic predicates are as usual. Completeness is provable. (For details, see [1].)

We turn now to a theory of truth in *ML* for a natural object-language (or, better, a canonical reading of a natural object-language) *OL*. We assume that *ML* has resources capable of describing the syntax of *OL*. Further, we assume a general correspondence between the vocabulary of *OL* and a sub-vocabulary of *ML*. This correspondence may be understood in terms of inclusion or in terms of translation. Details of such a translation relation conceived generally are, of course, difficult to state and well beyond the scope of this paper.

I shall first indicate the postulates of the theory of truth and then explain how to read the indications:

(*T*1) $\vdash (\exists\alpha)(v)(\exists x)(\alpha(v) = x)$.

(*T*2) $\vdash (\alpha)(v)(x)(\exists\beta)(\alpha \overset{v}{\underset{x}{\approx}} \beta)$.

(*T*3) For each atomic function sign $\bar{f}^n_j$,
$\vdash (x)(x = \alpha(\bar{f}^n_j(\bar{t}_1, \ldots, \bar{t}_n)) \leftrightarrow$
$x = f^n_j(\alpha(\bar{t}_1), \ldots, \alpha(\bar{t}_n)))$.

(*T*4) $\vdash (x)(x = \alpha(iota(v,\bar{A})) \leftrightarrow$
$x = (\iota y)((\exists\gamma)(\alpha \overset{v}{\underset{y}{\approx}} \gamma \mathbin{\&} \gamma \text{ satisfies } \bar{A})))$.

(*T*5) For each atomic predicate $\bar{A}^n_j$,
$\vdash \alpha$ satisfies $\bar{A}^n_j(\bar{t}_1, \ldots, \bar{t}_n) \leftrightarrow$
$A^n_j(\alpha(\bar{t}_1), \ldots, \alpha(\bar{t}_n))$.

(*T*6) $\vdash \alpha$ satisfies $nega(\bar{A}) \leftrightarrow -(\alpha$ satisfies $\bar{A})$.

(*T*7) $\vdash \alpha$ satisfies $condit(\bar{A},\bar{B}) \leftrightarrow (\alpha$ satisfies $\bar{A} \rightarrow$
α satisfies $\bar{B})$.

(*T*8) $\vdash \alpha$ satisfies $unquant(v, \bar{A}) \leftrightarrow$
$(\beta)(x)(\alpha \overset{v}{\underset{x}{\approx}} \beta \rightarrow \beta$ satisfies $\bar{A})$.

Greek letters 'α', 'β', and 'γ' vary over sequences. '$\alpha(v)$' is written for 'the assignment of α to v'; analogously for other uses of 'α' in function-sign position. 'v' ranges over variables of *OL*; '$\bar{A}$' and '$\bar{B}$', over wffs; and '$\bar{t}_1$', ..., '$\bar{t}_n$', over terms. '$\alpha \overset{v}{\underset{x}{\approx}} \beta$' is read '$\beta$ agrees with α in all assignments except that it assigns x to v'. Schematically, we use '$\bar{A}^n_j$' as a name of an *OL* predicate which translates into *ML* as 'A^n_j'; analogously for the schematic function-sign name '$\bar{f}^n_j$'. '$\bar{A}^n_j(\bar{t}_1, \ldots, \bar{t}_n)$' is read, 'the result of applying the predicate $\bar{A}^n_j$ to any singular terms $\bar{t}_1, \ldots, \bar{t}_n$ in the n-place predicative way'. Functional application is analogous. The operation sign '*nega*' is read 'the negation of'. The readings of the other signs will be obvious. The various styles of variables can be eliminated in favor of a single-sorted, first-order quantification theory.

We omit the usual relativization of the quantifiers to the domain of *OL*. If we were attempting an *explicit* definition of truth for *OL*, this omission would lead to inconsistency. But since we are content with a finitely axiomatized recursive characterization of truth, the omission may be tolerated. The dividend is that we may intuitively think of the quantifiers of *OL* (a fragment of our natural language) as ranging over all that there is.

The material adequacy of the theory is shown by proving that $\vdash Tr(\bar{A}) \leftrightarrow A$, for all closed wffs $\bar{A}$ of *OL*, where $Tr(\bar{A}) =_{df} (\alpha)(\alpha \text{ satisfies } \bar{A})$. (Intuitively, '*Tr*' is the truth predicate for *OL*.)[7] The proof of adequacy is reasonably straightforward and can be largely ignored here. (For details, see [1].) What is important for our purpose is the treatment of singular terms.

An aim of the proof is to derive biconditionals like:

(1) α satisfies $\overline{\text{Is-a-number}}\ \overline{\text{(the successor of}}\ \overline{\text{(the successor}}$ $\overline{\text{of}}\ (\bar{0})))$
$\leftrightarrow$ the successor of (the successor of (0)) is a number.

If all singular terms denoted something, the steps would be quite ordinary. We would begin by deriving:

(2) $\alpha(\overline{\text{the successor of}}\ \overline{\text{(the successor of}}\ (\bar{0})))$
$=$ the successor of (the successor of (0)).

(The assignment of every sequence α to 'the successor of the successor of 0' is the successor of the successor of 0.) Then we would obtain (1) by using Leibniz's law and (2) to substitute on the right side of this instance of axiom schema ($T5$):

($T5$a) α satisfies $\overline{\text{Is-a-number}}\ \overline{\text{(the successor of}}\ \overline{\text{(the successor}}$ $\overline{\text{of}}\ (\bar{0})))$
$\leftrightarrow \alpha(\overline{\text{the successor of}}\ \overline{\text{(the successor of}}\ (\bar{0})))$ is a number.

The derivation of (2) would utilize an axiom for '0' (taken as a 0-place function sign):

(3) $\alpha(\bar{0}) = 0$.

(The assignment of every sequence α to '0' is 0.) And it would utilize an axiom for 'the successor of':

(4) $\alpha(\overline{\text{the successor of}}\ (\bar{t})) =$ the successor of $(\alpha(\bar{t}))$.

(The assignment of every sequence α to the result of applying 'the successor of' to any term $\bar{t}$ is the successor of the assignment of α to $\bar{t}$.) These two axioms together with Leibniz's law would suffice to derive (2) and thence (1).

But since some singular terms do not have a denotation (or a sequence assignment), we cannot follow this route. For the use of axioms like (3) and (4) would undermine the truth of the semantical theory in the metalanguage. Axiom (4) is false because, say, 'the successor of $\alpha(\overline{\text{the Moon}})$' is improper (since there is no successor of the Moon). Though (3) is probably true, other axioms relevantly like it are not. Thus

(5) the assignment of every sequence α to 'Pegasus' = Pegasus

is intuitively untrue because the terms on both sides of '=' are improper.

Axioms of the form of ($T3$) and ($T4$) circumvent this problem. For example, we have instead of (5):

($T3$a) $(x)(x = \alpha(\overline{\text{Pegasus}}) \leftrightarrow x = \text{Pegasus})$.

This axiom is true despite the fact that there are non-denoting singular terms in it. Instead of (4), we have

($T3$b) $(x)(x = \alpha(\overline{\text{the successor of}}\ (\bar{t})) \leftrightarrow x = \text{the successor of } \alpha(\bar{t})))$.

Whereas in the case of (3) and (4) we could rely on Leibniz's law to make the substitutions needed to prove sentences like (1), that law is not strong enough to make recursive transformations using ($T3$a) and ($T3$b). This is where ($A8$) is required. It enables us to substitute *different* non-denoting singular terms (e.g., 'Pegasus' and '$\alpha(\overline{\text{Pegasus}})$') without relying on false identities like (4) or (5).

III. CRITICISM OF OTHER ACCOUNTS

I make no claims of final acceptability for the account set out in the previous section. But I shall put it to normative use in judging other accounts. Roughly speaking, my view is that the published accounts with relatively strong logical axioms yield falsehoods, and that accounts with relatively weak axioms cannot justify substitution of the relevant non-denoting singular terms in the adequacy proof of a truth theory.[8]

The description theories which, I think, are most interesting from a semantical viewpoint are those of Scott [20]; Lambert

[13], [23] (*FD*2); and Grandy [8]. Given uncontroversial empirical assumptions, each of these logics imply sentences which are uncontroversially untrue under their intended interpretation. Thus, Lambert uses the axiom $\ulcorner(x)(x \neq t_1 \; \& \; x \neq t_2) \rightarrow t_1 = t_2\urcorner$, and Scott invokes $\ulcorner -(\exists y)(y = t) \rightarrow t = *\urcorner$, where '*' is a constant denoting an object outside the domain of the object-language. Since it is not the case that either the present King of France or the only unicorn on the moon exists, we derive from each axiom

(6) The present King of France is identical with the only unicorn on the moon.

Grandy, who takes these intuitive difficulties with the Lambert and Scott systems seriously, utilizes the rule

(7) If $\vdash C \rightarrow (x)(Ax \leftrightarrow Bx)$, then $\vdash C \rightarrow (\iota x)Ax = (\iota x)Bx$.

This rule makes the equality of $(\iota x)Ax$ and $(\iota x)Bx$ contingent on which principles are included among the *non-logical* axioms of a theory.[9] In a theory which added

(8)(x)(Present King of France $(x) \leftrightarrow$ Unicorn on the Moon (x))

as an axiom—an axiom which is surely true—(6) would become provable. If (8) is not added to the theory, then (6) is unprovable as well as untrue. These consequences seem arbitrary and unintuitive. In addition to the rule just discussed, Grandy employs in his truth theory two axiom schemas (numbered 'T3' and 'T4') which have untrue instances. For example, they yield the sentences:

(9) What any sequence α assigns to 'the successor of the Moon' is identical with the successor of what α assigns to 'the Moon'.

(10) What any sequence α assigns to 'der Vater von Pegasus' is identical with the father of what α assigns to 'Pegasus'.

(11) What any sequence α assigns to 'the only unicorn on the moon' is identical with the unique object assigned to the variable v by some sequence β which satisfies 'is a unicorn on the moon'.

Sentence (10) is untrue because there is no father of what every sequence α assigns to 'Pegasus', there is no assignment by α to 'Pegasus', and there is nothing which α assigns to 'der

Vater von Pegasus'. Analogously for (9) and (11). To give another, slightly oversimplified, example, we can derive in Grandy's truth theory:

(12) The present King of France is the denotation of 'The present King of France'.

It should be emphasized that the standard defense of consequences like (6) and (9)–(12) (which I think is doubtful in any case) is clearly inappropriate in the present context. It is not sufficient to say that such sentences are unimportant to most cognitive sciences and that a smoother theory is obtained by counting them true. For from the present viewpoint—that of a semantical theory which takes native intuitions as part of its evidence—sentences like the above are not unimportant. And as ordinarily intended, they are untrue. Nor is it evident that significant differences in smoothness of theory are at issue.

The logical principles which lead to untrue conclusions should not be dismissed without taking account of their purpose. Scott notes as reason for "identifying" all non-denoting singular terms the resulting ability to derive the following principle of extensionality:

(13) $(x)(A \leftrightarrow B) \rightarrow (\iota x)A = (\iota x)B.$

Together with Leibniz's law, (13) provides substitutivity for non-denoting singular terms as well as for singular terms that denote. As we have seen in the previous section, the availability of substitutivity for different non-denoting singular terms is critical in the recursion steps of a theory of truth. But the logic of Section II yields a principle which together with (*A*8) seems to justify all the reasonable substitutions which (13) and Leibniz's law justify, without leading to untrue sentences like (6), and without depending on axioms like (9)–(12) for their usefulness in a truth theory. This principle is

(14) $(x)(A \leftrightarrow B) \rightarrow (x)(x = (\iota x)A \leftrightarrow x = (\iota x)B).$

Several free logics which have been regarded as having the advantage of producing no untrue consequences are too weak to provide the substitutivity needed in a theory of truth. For example, Van Fraassen and Lambert's *FD* ([23] and [24]) and most of the very early proposals do not allow for substituting *any* two (different) non-denoting singular terms. Even slightly stronger logics (e.g., Lambert's *FD*1, [13]) do not appear capable of combining with true semantical axioms such as ($T3$) and ($T4$) to yield the needed substitutions. Of course, it is conceivable that one might find semantical axioms other than ($T3$) and ($T4$) which are intuitively true and yet strong enough to combine with relatively weak logics to derive the biconditionals, like (1), which are the touchstones of a truth theory. But the prospects for most of the published logics are, I think, dim.

IV. SELF-IDENTITY AND EXISTENCE

By translating each occurrence of '*' as '$(\iota x)\ (x \neq x)$' and each occurrence of $\ulcorner t_1 = t_2 \urcorner$ as $\ulcorner (z)(z = t_1 \leftrightarrow z = t_2) \urcorner$, we can prove that a sentence is a theorem of Scott's system if and only if its translation is a theorem of ours. By translating each occurrence of $At_1 \ldots t_n$ as $\ulcorner (\exists y)(y = t_1)\ \&\ \ldots\ \&\ (\exists y)(y = t_n)\ \&\ At_1 \ldots t_n \urcorner$ (where A is atomic), we can prove that a sentence is a theorem of our system if and only if its translation is a theorem of Scott's. (I ignore function signs since Scott's system does not contain them.)

The latter result serves simply to place in a different perspective the view of natural-language predication I urged in Section II. The former result indicates that from the viewpoint of our system, Scott's system is sound (truth preserving) if and only if his $\ulcorner t_1 = t_2 \urcorner$ is read not as $\ulcorner t_1$ is identical with $t_2 \urcorner$, but as $\ulcorner$anything is identical with t_1 iff it is identical with $t_2 \urcorner$. On our view, Scott's $\ulcorner t_1 = t_2 \urcorner$ says that t_1 and t_2 do not differ in denotation, not that their denotations are the same. In Section III, I argued from the assumption that Scott's $\ulcorner t_1 = t_2 \urcorner$ was read $\ulcorner t_1$ is identical with $t_2 \urcorner$ to the conclusion that the system was unsound. What can be said for characterizing identity our way?

In the first place, our representation satisfies the minimum restrictions on any identity predicate—the logical laws of identity, ($A3$) and ($A4$). The fact that $\ulcorner x = x \urcorner$ is not valid in our system does not show that the self-reflexive law fails, since formulations of the law as $\ulcorner x = x \urcorner$ in classical systems presuppose that variable x always receives a value. That is, the law is standardly interpreted as ($A3$).

Analogous remarks apply to the Hilbert-Bernays method of simulating identity within a language. (Cf. Quine [16]: 230.) Insofar as the method has been regarded as relevant to understanding identity, it has been seen as a means of expressing indiscernibility of *objects* x and y from the viewpoint of the predicates of a given language. Our identity predicate is co-extensive with the Hilbert-Bernays open sentence (for any given language)—it is true of just the same objects. If, however, non-denoting singular terms are attached to the identity predicate, one will get a falsehood, whereas if they are substituted into the Hilbert-Bernays open sentence, one will get a truth. Insisting on equivalence of the closed sentences (in addition to the above-mentioned co-extensiveness) would commit one to holding that sentences like (6) or (12) are true in the fragments of English that we have been discussing. I know of no good reason for such insistence. It cannot be justified by the claim that the Hilbert-Bernays open sentence *expresses* or *characterizes* identity (as opposed to merely simulating it by being co-extensive with it within the given language). For such a claim is quite unintuitive: indiscernibility via a given stock of predicates just does not seem to give the intended interpretation of identity. (Cf. Quine [17]: 63.)

The main ground for characterizing identity our way, of course, is intuitive. It avoids the unattractive results of the other systems and accords with those intuitions which are held generally. The intuitions of some, though by no means all, will be crossed by the fact that (A9) contradicts

(15) $t = t$

in a language containing non-denoting singular terms.[10] It would be easy to dismiss acceptance of (15) as the result of misguided applications of universal instantiation to (A3). But a deeper consideration of the matter is worthwhile.

Testing instances of (A9) and (15) on intuition does not resolve the question of which to take as valid. Whereas native speakers are clear and nearly unanimous in their rejection of sentences like (6), they react differently to sentences like

(16) The present King of France is identical with the present King of France.

Some find them clearly untrue. Others take them to be just as clearly acceptable. Hesitations (on both sides) can be elicited by further discussion. Negative reactions seem to become somewhat more widespread in the face of sentences like

(17) The only square circle is identical with the only square circle.

But on the whole, the evidence from intuition as to whether or not all instances of (15) are true is unclear. The decision must rest on more general considerations.

One reason for doubting the validity of (15) is an application of the Fregean considerations raised in Section I. We cannot preserve the purported truth of (16) if we substitute for one occurrence of 'the present King of France' other singular terms (e.g., 'the only unicorn on the moon') which do not differ in denotation. Together with (*A*8) and (8) we can use (15) to derive (6)—a sentence rejected by native speakers in unison. One may, of course, wish to doubt (*A*8) rather than (15).[11] But (*A*8) served an important purpose in justifying transformations needed for proving the material adequacy of a theory of truth. It seems fair to ask of anyone who rejects the principle to provide a replacement which effects the relevant transformations without leading to untrue consequences or relying on untrue semantical axioms. Since (*A*8) is non-independent in the logic of Section II largely because of (*A*9), we may regard the latter as tentatively preferable to (15).

There is a further consideration against (15)—one that is vaguer and less compelling but nonetheless philosophically interesting. An extremely intuitive feature of Tarski's theory of truth is that it explicates what it is for a sentence to be true in terms of a relation (satisfaction) between language (open sentences) and the world (sequences of objects). The notion of correspondence which had always seemed so integral to truth came clean in Tarski's theory. (This point is forcefully made by Davidson in [3].) It is difficult to see how the purported truth of, say, (16) can be explicated in terms of a correspondence relation.

As mentioned earlier, some may want to give correspondence a toehold by assigning 'Pegasus' an unactualized possible. But quite apart from questions about the propriety, clarity, and credibility of the move, it does not apply to (17). Assigning 'Pegasus' itself is not very satisfying either, because it encounters difficulty in explicating '$-(\exists y)(y = \text{Pegasus})$'. The fault is the one we found with Frege's account of singular terms: there is too much correspondence rather than too little.

Loosely speaking, self-identity is a property of objects and all objects have it; sentences expressing identities are true or false by virtue of the relation that the identity predicate and its flanking singular terms bear to the world—never merely by virtue of the identity of the singular terms. Philosophical questions regarding identity seem bound to the notions of existence and object. The point is summed up more austerely by the principle

$$(18) \qquad (\exists y)(y = t) \leftrightarrow t = t,$$

which is easily derived in the logic of *ML*.

In their focus on the intended interpretation of the symbols we employ, our truth theory and its underlying logic help clarify how with respect to singular terms we can use the language we use and in the same language believe in the world we believe in. Alternative combinations of theory and logic should be required to do at least as much.

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NOTES

[1] I am grateful to Alonzo Church, Donald Kalish, Dana Scott, and especially Richard Grandy for criticisms and suggestions regarding earlier drafts.

[2] Some of what Meyer and Lambert say in [15] is congenial with these remarks. However, their distinction between "nominal truth" and "real truth" appears superfluous. "Nominal truth" may be assimilated to "real truth" as applied to sentences involving oblique contexts. One advantage of making explicit the oblique contexts in sentences like the above is that by doing so one uncovers grounds for confirming or disconfirming otherwise puzzling sentences. For example, to confirm or disconfirm fictional sentences, we look at the relevant fiction. Whereas some authors have felt that a sentence like 'Pegasus had fewer than 7 million hairs' should be counted truth-valueless because they could find no plausible reason to count it true or false, our view counts it false (even taken as implicitly oblique) because it is easily disconfirmed.

[3] Application of this idea is discussed briefly in [2]. The axioms governing the proper name 'Pegasus' which we discuss below ignore for the sake of brevity the considerations of that paper.

[4] Donnellan in [4], esp. pp. 295–304, may seem to be in disagreement with this intuition. But I think that the disagreement is only apparent. It should be noted that the bivalence of *ML* and the account of negation in the object language *OL* (cf. below) are incompatible with some treatments of presupposition in terms of truth-value gaps. I think that the intuitions backing these accounts can be explicated in other ways. But this issue may be left aside here.

[5] Provisions for scope distinctions will be important in a full account of the logical behavior of singular terms in natural languages, especially in treating certain ambiguities which occur with non-denoting terms and in dealing with singular terms in and out of intensional contexts. Such provisions can be added to the present system, but the matter is tricky and will not be carried out here. For a detailed discussion of the problem and an attempt to solve it from a different standpoint than ours, see Grice [9].

[6] It is tempting but mistaken to suppose that the condition prohibits taking both a predicate and another predicate understood as its "contradictory" or

"negation" as primitive. In such cases, the condition may be seen as forcing us merely to construe the singular term as having wider scope than the "negative element" attributed to the predicate. As long as the predicate is atomic, it is hard to imagine the situation any other way.

[7] Cf. Tarski [22]. In our formulation, relativizations of '*Tr*' to a canonical reading, a person, and a time are suppressed for the sake of brevity.

[8] An exception is Schock [19]: 94. Subsequent to arriving at the theory of Section II, I found that he uses axioms very like (*A*9) and (*A*10). Schock gives a Frege-type model theory for his logic using the empty set as the denotation of intuitively non-denoting singular terms.

[9] It would be a mistake to think that Grandy "identifies" only logically equivalent singular terms. In order to prove the adequacy of his truth theory for the iota case, he must derive an identity sentence containing two non-denoting definite descriptions. The identity is derived from an equivalence established via a truth-theoretic axiom (as opposed to a logical axiom) together with the above-mentioned rule.

[10] Smiley [21] and Hintikka [11] are in accord with us on this matter. So, with qualifications, is Russell [25]: 184.

[11] (*A*8) is derivable in both Scott's system and Lambert's *FD*2. It is not derivable in Grandy's system or in the weaker ones.