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The Concept of Truth in Frege's Program*

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Frege's views on truth are richer and more central to his logicist program than is commonly realized. One reason these views have been underappreciated is that Frege rejected a systematic model-theoretic semantics. Another reason is that many of his views have seemed quaint, naive, or pointless because their underlying motivations have not been pursued in sufficient depth. My purpose in this talk is necessarily quite limited. I want to discuss two cases where too quick a reading of Frege's texts has led to ahistorical assimilation of Frege's views to more modern discussions, to the detriment of an appreciation of his depth as a philosopher.

Frege's argument in 'On Sense and Denotation' that a sentence's denotation is its truth value has often been seen as an elliptical, or even invalid, approximation to an argument to the same conclusion, proposed by Church, Gödel, and others. The Church-Gödel argument, which is clearly inspired by Frege's remarks, presupposes that sentences have a semantical feature that is sufficiently analogous to the central semantical feature of terms to be given the same expression. Let us call this feature "*denotation*". The argument assumes, first, that the denotation of a sentence is not changed by the exchange of co-denotational terms. Second, the argument assumes that logically equivalent expressions have the same denotation. Take any true sentences S and S' ; S is logically equivalent with sentence of the form $(\lambda x)(x = 0) = (\lambda x)(x = 0 \ \& \ S)$. So by the second premise, S and this sentence have the same denotation. But the latter sentence yields the sentence $(\lambda x)(x = 0) = (\lambda x)(x = 0 \ \& \ S')$ by substitution of co-denotational terms on the right side of the identity sign. So *these* two sentences have the same denotation, by the first premise (the substitutivity of identity). But the new sentence is logically equivalent with S' . So by the second premise, they have the same denotation. So S and S' have the same denotation by the transitivity of identity. An analogous argument shows that all false sentences have the same denotation. Frege accepted not only the conclusion of the argument, but its presupposition and premises. But in arguing for the thesis that the denotation of a sentence is its truth value, he did not advance this argument. Nor did he give an elliptical or invalid approximation to it.

* The ideas in this presentation are extracted from a paper about ten times as long, 'Frege on Truth', which is appearing in a volume of the *Synthese Library*, edited by Hintikka and Taiminen.

At the stage of motivating his logical theory that his argument is given, Frege treats the “denotation” of a sentence as *whatever is most fruitfully seen as functionally dependent on functional applications among the denotations of sentential parts*. The view that sentences have denotation is thus regarded as a corollary of the centrality of the composition principle in logical theory: the principle that the denotation of a complex expression depends wholly on functional applications among the denotations of semantically relevant parts. Frege’s argument that the denotation of a sentence is its truth value is preceded by an argument, which I shall not go into but which I regard as sound, that the sense or thought expressed by a sentence is not to be identified with its denotation.

The argument that a sentence’s denotation is its truth value is set out in this passage from *On Sense and Denotation*, which I abbreviate slightly:

The fact that we concern ourselves at all about the denotation of a part of the sentence indicates that we generally recognize and expect a denotation for the sentence itself. The thought loses value for us [at least as scientists] as soon as we recognize that the denotation of one of its parts is lacking. . . . But why do we then want every proper name to have not only a sense, but also a denotation? Because, and to the extent that, we are concerned with its truth value. . . . It is the striving for truth that drives us always to advance from sense to denotation. (“Kleine Schriften” (KS), 149)

Frege’s argument has the form: Our interest in the denotation of non-sentential expressions, particularly singular terms, derives from our interest in the truth values of sentences, or in the truth values of thoughts expressed by sentences. Our interest in the truth values of sentences or thoughts derives from our practices of assertion and judgment (“striving after truth”). And it is the business of logic to state the most general laws concerning the norm that governs assertion and judgment. In view of the normative aims of logical theory and in view of the considerations that actually motivate our interest in the denotations of terms, the appropriate feature of sentences to connect with the denotations of non-sentential expressions via the composition principle, is the sentence’s truth value. Thus the primary feature of sentences that is of interest to logic, which Frege calls sentence “denotation”, is truth value.

Frege’s argument is not a deductive argument from “first principles”. In *On Sense and Denotation* he twice calls the thesis a conjecture (KS 150, 151). Most of the article is presented as a series of tests of the conjecture – tests of whether truth value can be functionally connected to the denotation of sentential parts. Near the end of the article he suggests that the argument’s conclusion has been supported “with sufficient probability” (KS 162).

Unlike the Church-Gödel argument, Frege’s does not combine the

assumption of substitutivity of identity with the assumption that logically equivalent expressions have the same denotation. Rather it combines a generalization of substitutivity of identity, the composition principle, with the view that logical theory should analyze the basic normative notion governing judgments or assertions and that this normative notion, the notion of truth, motivates our interest in the denotation of sentence parts. Thus the argument unites the point of logic with its basic analytical tool, the composition principle.

Although Frege accepts the second premise of the Church-Gödel argument, the premise that logically equivalent expressions have the same denotation, that premise is less fundamental for him than his views about the point of logical theory and the source of our interest in term denotation. It is virtually a priori true that logical theory ought to count sentences as being the same with respect to their primary logical feature, that is their "denotation", if it counts them logically equivalent. But the notion of logical equivalence that is used in the Church-Gödel argument is explicated in terms of the notion of logical consequence, which is in turn explicated for Church and Gödel in terms of the notion of truth, e.g. necessary truth or truth in all interpretations. Since Frege's argument that the denotation of a sentence is its truth value amounts to an argument that the primary logical feature of sentences is their truth value (and thus that the central notion of logical theory is truth), the second premise of the Church-Gödel argument would be seen by Frege as less fundamental than its conclusion.

Let us turn now to our second example. In section 10 of "The Basic Laws of Arithmetic" Frege identifies the two truth values with specific extensions of concepts. He indicates that the identification is arbitrary relative to the axioms of his logical theory. Any other choice would have been equally consistent with those axioms:

... it is always possible to stipulate that an arbitrary course of values [a genus of which extensions of concepts are species] is to be the True, and an arbitrary different one, the False. Accordingly, let us lay down that ϵ ($-\epsilon$) [the extension of the concept under which only the True falls] is to be the True and that $\epsilon(\epsilon = \sim(x) (x = x))$ [the extension of the concept under which only the False falls] is to be the False.

This passage has suggested to some authors that Frege's theory of truth had a large stipulative element. Since the numbers are identified with extensions of concepts in ways that presuppose the identifications of the truth values, the remark also suggests that Frege's ontology of the numbers involves substantial and intended arbitrariness or indeterminacy. If these suggestions are correct, then the traditional view that Frege thought that the numbers were genuine abstract objects that could be identified in but one way within his logical theory must be given up. For differently

chosen identifications of the truth values with extensions of concepts would produce different accounts of which entities the numbers are. The revised interpretation of Frege based on this passage, which has recently been urged by some authors, would make Frege's position quite analogous to the common modern view that one may identify numbers with sets in any of infinitely many arbitrarily chosen ways.

The passage will not, however, submit to this modernizing interpretation. Frege's identification of the truth values is indeed arbitrary relative to the axioms of his logic – that is, relative to considerations of consistency. But Frege maintains repeatedly that consistency does not suffice for truth. (Cf. "Wissenschaftlicher Briefwechsel", 75; "Kleine Schriften", 110, 264–72; "Die Grundlagen der Arithmetik", 104–119). He indicates that philosophical considerations, particularly regarding logicism and his conception of logic have to supplement considerations of mere consistency in order to arrive at a reasonable ontology. (That these points do not appear in "Basic Laws" is consonant with the mathematical orientation of the book. Philosophical points are everywhere, excepting the introduction, severely limited.) Frege's reasoning involves several ideas, but the one most relevant to our purposes is that a semantics for mathematics should not "import anything foreign". That is, it should be in keeping with Frege's philosophical view that mathematics is a priori and reducible to logic. This stricture rules out identifying the truth value with courses of values of empirical or indeed any non-logical (such as geometrical) concepts or functions.

A second restriction emerges from Frege's view of logic as having an internal ordering. Higher-order functions presuppose first-order functions; the functional calculus rests on the propositional calculus. Since the truth values are the "objective" of every sentence of logic and since logic is an ordered elaboration of the laws of truth, the truth values should be specifiable in terms already available in the most fundamental part of logic, the propositional calculus.

Underlying this second restriction is a third. The identification of the truth values as courses of values should be derivative from Frege's concept of truth. Courses of values are derivative, in Frege's view, from functions – and in a deeper sense, from concepts. Frege held a redundancy conception of truth according to which the predication of 'is true' added nothing to the sense of the sentence to which it is applied. But such predication, though topic neutral, was held by Frege to be ubiquitous among assertions.

Frege identifies truth with the extension of the concept that maps truth onto truth and everything else onto falsity. This concept is represented within Frege's logic by the horizontal. The identification can seem to be artificial or arbitrary if one thinks of extensions of concepts set-

theoretically. From this perspective the True is its own unit class. But for a variety of reasons that I shall not go into, the set-theoretic interpretation misrepresents Frege's standpoint.

Frege's identification of the truth values derives directly from the preceding meta-logical ideas, together with the view (whose motivations I shall not describe here) that the truth values are *objects*. The first idea was that the truth values are *logical* objects. As such their specification must be derivative from specification of logical concepts. The second idea was that logic is an ordered development of the laws of truth, where truth is the aim of sentence use for purposes relevant to logic. Thus Frege takes the logically relevant aim of sentence use to be our "striving after truth", an aim revealed in our practices of assertion and judgment. The object Truth must be involved in the assertive use of every sentence of logic. The two ideas entail that truth is a logical object specifiable in terms of a logical concept that is present in the assertive use of every sentence of logic.

The only concept that is present in the assertive use of every sentence of Frege's logic is the concept denoted by the horizontal – Frege's concept of truth. In unpublished writing contemporaneous with "Basic Laws", Frege explicitly makes this very point. He writes that what distinguishes 'true' from all other predicates, and what fits it to indicating the aim of logic is that "it is asserted when anything at all is asserted" ("Nachgelassene Schriften" 140). Assertions or judgments in Frege's system are formulated by adding the vertical judgment stroke to the horizontal.

(The concept denoted by the horizontal meets an analog of a standard condition on truth predicates. Frege specifies this analog in section 5 of "Basic Laws" through the equivalence:

$$\Delta = (- \Delta)$$

where ' Δ ' varies over truth values and may take declarative sentences as substituends. The equivalence is the counterpart in Frege's system of Tarski's truth schema. In specifying the object truth Frege is nominalizing the truth concept by generalizing on his version of the truth schema.)

The critical point is that Frege forces the truth concept to be explicitly denoted whenever a sentence is assertively used in his logic. The aim and subject matter of logic are to be understood only through analysis of assertion and judgment. Frege's identification of the object truth also expresses his conception of the order within logic: The truth concept is the one in terms of which all others are explicated. Identifying truth as a logical object derived from this concept is the only identification that coheres with Frege's philosophical views about the point and epistemology of logic.

The foregoing considerations suggest a rebuttal to two venerable

criticisms, made forceful by Michael Dummett, of Frege's claim that truth values are objects. (Cf. Michael Dummett, "Frege: Philosophy of Language", 1973, and *Truth* in "Truth and Other Enigmas", 1978.) One is that once truth becomes one object among others, it is difficult to explain what it is about it that makes us want to strive after it, assert it, acknowledge it, and so forth. The other is that whereas it is part of the concept of truth that we aim at making true statements, Frege's theory of truth as an object leaves this feature of the concept of truth out of account.

I think that these criticisms cannot be sustained. According to Frege, interest in the denotation by terms of objects derives from our interest in norms governing the use of sentences in making assertions and expressing judgments. So using sentences to denote truth values derives its interest from assertion and judgment. To explain the role of these practices in terms of the features of certain objects, the truth values, would be to reverse the proper order of explanation.

With regard to the second criticism, it is just not true that Frege "leaves our aim at making true statements out of account" in his articulation of the concept of truth. The specification of the object truth is constructed in such a way as to reflect the primacy of assertion and judgment in revealing the point of logical theory. For all its oddity, Frege's thesis that sentences denote truth values, which are counted objects, is motivated by a profound conception of the epistemology of logic as rooted in analysis of our practices of assertion and judgement. I believe that understanding Frege's logicist definitions of numerical expressions – hence understanding his logicism – depends on a deeper grasp of the role of philosophical and semantical conceptions in his philosophy of mathematics than has so far been achieved.

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