

LIVING WAGES OF *SINN*\*

Frege's concept sense (*Sinn*) is better understood now than it was fifty years ago. Its value and distinctiveness as a tool for theorizing about language and thought remain, however, underrated in many quarters. Here, against some recent doubts, I argue again for distinguishing sense from any modern notion of linguistic meaning. I renew some criticisms of Frege's *conception* of sense and his employment of the concept sense. I then advocate the power of his concept for understanding thought, and some aspects of language.

## I

I assume that there is a concept sense, and that there are senses. I do not assume that everything that Frege thought about senses is true. I impute his mistakes to the conception that he associates with his concept. I take senses to ground certain explanations that he gave the concept a role in. He gave the concept sense four explanatory roles.<sup>1</sup>

First, Frege introduces sense to account for scientific thought expressed in language. Every passage that centers on the concept emphasizes this role. Frege's interest in thought expressed in language is motivated by interest in scientific knowledge. His use of sense does not aim to account for rules of linguistic usage, or literal meanings of words, at least when his conception of scientific thought does not square with ordinary linguistic rules or usage.<sup>2</sup>

Helping to account for scientific thought and cognition is the primary explanatory role of Frege's concept. Senses are representational thought contents. They are ways of thinking as of entities in a subject matter. They are ways that such entities are presented in thought—cognitive routes to them. Senses of declarative sentences

\*A shorter version of this paper was given as the first annual Saul Kripke Lecture in May 2011 at City University of New York. The paper benefitted from audience comments on that occasion.

<sup>1</sup>I made these points in Tyler Burge, "Belief *De Re*," this JOURNAL, LXXIV, 6 (June 1977): 338–62; reprinted in my *Foundations of Mind* (New York: Oxford, 2007). I elaborate them in Introduction to my *Truth, Thought, Reason: Essays on Frege* (New York: Oxford, 2005), pp. 31–39.

<sup>2</sup>I made these points in "Sinning Against Frege," *The Philosophical Review*, LXXXVIII, 3 (July 1979): 398–432; reprinted in *Truth, Thought, Reason*. See also Introduction to *Truth, Thought, Reason*, pp. 35–40. In "Sinning Against Frege," I wrote that 'linguistic meaning' is multi-purposed and adaptive to theory. One could use a notion of linguistic meaning that is the same as Frege's notion of sense. But current notions of linguistic meaning are not, I think, notions of sense.

are propositional thought contents. Sub-propositional thought contents associated with structural, inferentially relevant parts of sentences are also senses. So senses are contents of thought that are thought with or through language.

Second, senses figure in semantics of scientific thought and language. Denotations are entities in a subject matter that linguistic expressions refer to or otherwise represent. Senses of sentences set truth conditions: if the subject matter meets the condition, the sense is true. Senses of relevant sentential parts *determine* denotations in this sense: if there is a denotation of a linguistic expression of the sense, there is exactly one denotation. For every sense there is at most one denotation. Or in the propositional case, if the sentence or its utterance has a truth value, the sense of the sentence, or sentence utterance, has exactly one truth value.

Third, senses can be denotations when they are subject matter. When I discuss certain thought contents, I refer to senses. I shall do this by underlining the expression of the sense. Thus by '2 + 3 = 5' I refer to the representational content of a belief that  $2 + 3 = 5$ . And '2 + 3 = 5' denotes the representational content  $2 + 3 = 5$ . I call nonpropositional thought contents that are not identical with and do not contain occurrence-based applications '*concepts*'.<sup>3</sup> '3' and '≡' name the concept 3 and the identity concept, respectively.

Fourth, senses are what are understood by users of scientific language. In understanding language, one associates the language with thought or components of thought. This construal of understanding is to be aligned with the approach to language as a vehicle for thinking, inferring, and knowing, and to the requirement that senses determine denotations. By understanding senses, one normally understands something about denotations. They are ways denotations are presented in thought and ways by which denotations are determined. The language denotes a subject matter because it is associated with thought that represents the subject matter.<sup>4</sup>

Frege's concept sense differs from any ordinary modern concept of linguistic meaning. The concepts are complementary, overlapping

<sup>3</sup> For discussion of applications—occurrence-based representations—and their fundamental importance to perception and thought, see Burge, "Belief *De Re*"; "Postscript to 'Belief *De Re*,'" in *Foundations of Mind*; and "Five Theses on *De Re* States and Attitudes," in Joseph Almog and Paolo Leonardi, eds., *The Philosophy of David Kaplan* (New York: Oxford, 2009), pp. 246–316.

My use of 'concept' differs from Frege's. Frege uses the cognate term '*Begriff*' for functions denoted by predicates. I use 'concept' for certain ways of thinking that may be senses.

<sup>4</sup> Senses determine denotations only because other thinker-subject-matter relations are in place.

tools for theorizing. The distinctiveness of Frege's concept sense is signaled most clearly in his discussion of indexicals.<sup>5</sup> Consider this text:

(A) If someone wants to say today what he expressed yesterday using the word 'today', he will replace this word with 'yesterday'. Although the thought is the same, the verbal expression must be different to compensate for the change of sense that would otherwise be brought about by the different time of utterance (*Sprechens*). The case is the same with words like 'here' and 'there'. In all such cases, the mere wording, as it can be fixed in writing, is not the complete expression of the thought—but one further needs for its correct apprehension also the knowledge (*Kenntnis*) of certain circumstances accompanying the utterance, which are used as means of thought expression (*Gedankenausdrucks*). Fingerprintings, gestures and glances can belong here too. The same wording (*Wortlaut*) containing the word 'I' will express different thoughts in the mouths of different people, of which some may be true and others false.<sup>6</sup>

On any ordinary notion of linguistic meaning—communal or idiolectic—the meanings of sentences (and sentence utterances) containing 'today' and 'yesterday' are different. This point would have been plain to Frege. Frege states, however, that the senses, hence the thoughts expressed, 'will be' the same. So sameness of sense is compatible with difference of linguistic meaning.

Saul Kripke objects to distinguishing Fregean sense from modern notions of linguistic meaning.<sup>7</sup> He builds his interpretation on statements that occur immediately before (A):

(B) If a time indication (*Zeitangabe*) should be made with the present tense, one must know when the sentence was uttered in order to grasp

<sup>5</sup> Modern notions of linguistic meaning became prominent when attention focused on rules embedded in natural language use. This focus involved a double shift—away from scientific language, and away from thought expressed in language, to natural language and to communication. Early Russell uses the term 'meaning'. But he is primarily interested in knowledge and thought. The positivists' notion of meaning also did not center on understanding rules of natural language usage. Frege's notion of sense and modern notions of ordinary linguistic meaning do overlap. Many (cognitive) linguistic meanings are senses.

<sup>6</sup> Gottlob Frege, "Der Gedanke" (1918–1919), in *Kleine Schriften*, ed. Ignacio Angelelli (Hildesheim, Germany: Georg Olms, 1967), p. 348. Translations are my responsibility. See also Frege, "Thought," in *The Frege Reader*, ed. Michael Beaney (Cambridge: Blackwell, 2009), p. 332. I cite pagination in Frege's original in brackets. Passage (A) is from [64].

<sup>7</sup> Saul Kripke, "Frege's Theory of Sense and Reference: Some Exegetical Notes," *Theoria*, LXXIV, 3 (September 2008): 181–218; cf. p. 201. Kripke thinks that if sense were *not* meaning, it would be odd for Frege to omit introducing a technical term for meaning. Since Frege was not theorizing about ordinary language, or ordinary meaning, this omission is not odd at all. (But see note 44.) Kripke thinks that, on my view, Frege's notion of sense is close to David Kaplan's notion of content, and

the thought correctly. Therefore the time of utterance is then part of the expression of the thought.<sup>8</sup>

Kripke correctly reads (B) as expressing the view that apart from the time of the utterance, the words ‘Today is Thursday’ do not express a complete thought. He takes the time itself as well as the verbiage ‘Today is Thursday’ to be part of the expression of the thought, and even part of the sentence. He regards the time as ‘an unrecognized piece of language’.<sup>9</sup> Thus he takes the full sentence to consist of both the verbiage ‘Today is Thursday’ and the time *t* at which the verbiage is uttered. *t* is taken to refer to itself, and the sense of *t* is ‘that of autonomous designation’. Kripke further takes the verbiage ‘Today is Thursday’ to denote a function from times to truth values, and he takes ‘Today’ to denote a function mapping each time onto the day containing it.<sup>10</sup>

On this view, the senses of ‘Today’ and ‘Today is Thursday’ are what nearly everyone would take to be their linguistic meanings. The only oddity is taking utterance times as self-denoting linguistic expressions. I will not center on this oddity. I discuss it only in the Appendix.

Kripke’s interpretation is incorrect. His assumption that Frege thinks that ‘today’ denotes a function, determined by the sense (meaning) of ‘today’—a function from a time to the day containing it—is incompatible with (A), which immediately follows (B).<sup>11</sup> For if, as Frege claims, the utterances, on successive days, of ‘Today is Thursday’ and ‘Yesterday was Thursday’ have the same sense, the senses of the utterances cannot be or include their linguistic meanings. For the meanings of the two sentences, and sentence utterances, are plainly different.<sup>12</sup>

that Frege, oddly, omits any technical notion analogous to Kaplan’s notion of character. Again, I think that the omission is not odd. In fact, I think that Frege’s notion of sense is not much like *either* of Kaplan’s notions. Cf. David Kaplan, “Demonstratives,” in Almog, John Perry, and Howard Wettstein, eds., *Themes from Kaplan* (New York: Oxford, 1989), pp. 481–563.

There is much in Kripke’s article that I agree with. Although he does not note the point, his view of Frege’s hierarchy of senses is very similar to mine. Cf. my “Postscript: ‘Frege and the Hierarchy,’” in *Foundations of Mind*. As stated below, I also agree with the core of his important criticism of Frege on the sense of names.

<sup>8</sup> Frege, “Der Gedanke,” in *Kleine Schriften*, p. 348 [64]; “Thought,” in *The Frege Reader*, p. 332.

<sup>9</sup> Kripke, “Frege’s Theory of Sense and Reference,” pp. 201–02.

<sup>10</sup> *Ibid.*, pp. 204, 212.

<sup>11</sup> It is also directly incompatible with my interpretation—the very interpretation that Kripke argues against. Thus the assumption begs the question.

<sup>12</sup> Kripke writes that although on the relevant occasions ‘Today’ and ‘Yesterday’ denote the same day, ‘they plainly pick it out in different ways, paradigmatic cases of

Kripke notes that his interpretation is incompatible with (A). He blames Frege for ‘confusion’. He writes, ‘If Frege really means that we have expressed literally the same thought again, it is very hard for me to see how to reconcile this assertion with his other doctrines.’<sup>13</sup>

Kripke holds that the view stated in (A) is incompatible with two Fregean doctrines. One is that ‘today’ expresses a sense that determines a function from the time of utterance to the day containing it, and that ‘yesterday’ expresses a sense that determines an analogous function. Frege nowhere in his writings expresses this “doctrine.” He states a view incompatible with it in the very paragraph on which Kripke bases his interpretation. So it cannot be invoked in interpreting that paragraph.

The other doctrine that Kripke takes to be incompatible with (A) is that ‘the sense of a sentence (the thought) is composed of the senses of its parts’. Kripke reasons that the senses of the parts of ‘Today is Thursday’ and ‘Yesterday is Thursday’ differ, because the senses of ‘Today’ and ‘Yesterday’ differ; so the senses of the sentences (and sentence utterances) differ. Kripke asks, ‘How could anyone argue that the two sentences in question have the same sense?’<sup>14</sup>

As follows: Frege holds the sense-composition principle. But Kripke’s application of the principle is not Frege’s. As is well known, Frege took his principle to be applicable only once relevant sentence parts are isolated under logical analysis. Logical analysis aims at revealing thought structure. Not just any natural-language word can be assumed to be a sentential part under logical analysis, or to have a sense. Frege repeatedly criticizes natural languages as misleading regarding the structure of thought. He does so in this very passage.

Kripke’s argument that ‘today’ and ‘yesterday’ sentence utterances cannot have the same sense, because their senses are compounded

difference in sense’, *ibid.* Kripke goes on to claim that on his interpretation, Frege believes that a present-tense thought (or today thought) expressed at time  $t$  ‘...cannot be recaptured at any later time. This is simply because the earlier time is never with us again, and immediate acquaintance with it is supposed to be essential to the expression of the thought’, *ibid.*, p. 205. But passage (A) indicates that Frege believed that the same today thought *can* be expressed, and thought, again on the next day—by thinking ‘Yesterday was Thursday’. Kripke’s belief that recapture is impossible depends not only on his misinterpretation of Frege’s account of thought expression, but also on undefended views, which I think questionable, about the relation between modes of presentation in memory and modes of presentation through present acquaintance. Memory can have an anaphoric function in preserving earlier modes of presentation. I will not pursue these issues here.

<sup>13</sup> *Ibid.*, p. 204.

<sup>14</sup> *Ibid.*, p. 205.

out of the senses of their parts, *assumes* that ‘today’ and ‘yesterday’ are parts that in themselves, *across contexts*, express senses. He states: “‘Today’ denotes a function which when applied to  $t_0$  (used autonomously) gives the day containing  $t_0$ ...”<sup>15</sup> He takes this rule to give the sense of ‘today’, on Frege’s view. Thus Kripke’s application of the second doctrine depends on assuming the alleged first doctrine.

Frege does not say that these indexicals *in themselves* express senses or denote anything. He writes of people expressing senses (thoughts) by using sentences that contain indexicals. In (B) he writes of a ‘time indication made with present tense....’ But he does not say that the present tense in itself denotes a function, or anything else. Frege never writes that any indexical on its own, cross-contextually, expresses a sense or denotes a function. Yet indexicals have linguistic meaning, analogous to the one Kripke gives for ‘today’ through the denotation rule.

Frege held the views stated in (A) for over two decades—making it further implausible that they are confused or in conflict with his principle of sense composition. In the cognate passage in “Logic” 1897 that became “Thought” 1918–1919, Frege writes that relevant *words* (he cites ‘here’, ‘now’, ‘I’) ‘only acquire their full sense always through the circumstances in which they are used.’<sup>16</sup> He targets *words*, not sentence types that contain the words. He states that these words, apart from uses in contexts, do not have *their* full senses. So they cannot denote functions on their own, or across contexts of use, as Kripke’s reading requires. Temporal indexicals are not, for Frege, used to contextually determine a time *in a way that corresponds to their linguistic meanings*. Yet they clearly have linguistic meanings across contexts.

So the incompleteness of indexical sentences is *not* that they need a singularly indicated time as argument for a function that, when completed, would yield a value (a time) as denotation, if the sentence utterance is to express a complete thought with a truth value. The contextually used singular term that denotes the time (for example, the present day) is not the result of filling in a functional expression with a singular term that denotes a time of utterance. For Frege,

<sup>15</sup> *Ibid.*, p. 204.

<sup>16</sup> Frege, “Logik” (1897), in *Nachgelassene Schriften*, ed. Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach (Hamburg, Germany: Felix Meiner Verlag, 1983, 2<sup>nd</sup> edition), p. 146; in English: Frege, “Logic” (1897), in *Posthumous Writings*, ed. Hermes, Kambartel, and Kaulbach (Chicago: University Press, 1979), p. 135; Frege, “Logic” (1897), in *The Frege Reader*, p. 235. I discuss other relevant passages in the Appendix. Note that indexicals do not need contexts to have their full linguistic meanings. What it is to be an indexical is to have the same meaning across contexts, a meaning that allows shifts in referent.

indexical expressions, taken with their cross-contextual linguistic meanings, do not contribute a sense to the complete thought. He thinks that by using 'today' in one context and 'yesterday' in another, an individual can express the same sense and think the same thought.

Frege tends toward treating indexicals as used demonstratives. He takes uses to be backed by a denotation-determining sense, which he takes to be an eternal mode of presentation associated with the "demonstrative" in a context. The mode of presentation, the component of thought content, is linguistically constrained only by the singularity of the "demonstrative" and, with temporal indexicals, by their applying to a time. Such used "demonstratives" take on senses through perceptual belief, or contextual association with definite descriptions, or in other context-dependent ways.<sup>17</sup>

I think that the foregoing points decisively undermine the only serious objection that Kripke raises to taking passage (A) literally. (A) is incompatible with Kripke's interpretation. Frege neglects the linguistic meanings of indexicals because he thinks that they are not relevant to the thought expressed. This position serves his view of senses as elements in thought *ideal for science*. I believe that he thought that indexicals and demonstratives do not occur in a scientifically ideal language.<sup>18</sup> That is why the meanings of indexicals are not, for him, senses.

Frege's emphasis on the timelessness of truth implies that truth is not relative to a time—that *truth*, not truth at a time (or place, or in a model), is the fundamental category of logic. Frege's ontological Platonism about thoughts, and senses, extends this point. Thoughts are in a third realm—dependent on neither the mental nor the physical.<sup>19</sup> It is plausible, I think, that Frege understood his ontological Platonism about thought content, and his conception of scientific thought, in an even stronger way—a way that excluded indexicals from scientific language.

In "Logic" 1897, Frege makes these remarks: (1) 'Thought is impersonal'; (2) 'It is of the essence of a thought to be non-temporal and non-spatial'; (3) '...the thought is either true, in which case it is

<sup>17</sup> Thus not only the rules that Kripke states, but the rules Kaplan states for determining content and character, do not determine the senses of indexicals for Frege. See note 7 above. I believe that Frege's approach to the senses of indexicals makes his view largely irrelevant to modern projects that aim to give linguistic rules governing indexicals.

<sup>18</sup> See Frege, "Logik in der Mathematik" (1914), in *Nachgelassene Schriften*, p. 230; *Posthumous Writings*, p. 213.

<sup>19</sup> See my "Frege on Knowing the Third Realm," *Mind*, n. s., CI, 404 (October 1992): 633–49; reprinted in *Truth, Thought, Reason*.

always, or better, timelessly, true, or it is false and in that case it is false without qualification. This holds of any particular historical fact: *if it is true, it is true independently of the time at which it is judged to be true*.<sup>20</sup> In “Thought” (1918–1919), he writes,

A property of a thought will be called inessential if it consists in or follows from the fact that this thought is grasped by a thinker.<sup>21</sup>

These passages allow various interpretations. I will develop one way of reading them.

It is plausible that by the rules for the *meaning* of ‘today’ or present tense, it follows that only *uses* of sentences containing those devices can be true. For example, only uses (either in thought or in public discourse) of ‘Today is Thursday’ or ‘Spain is currently world champion in soccer’ can be true. Strictly speaking, only an instantiated use of the sentence is true. And uses require that the meaning is *thought* or *understood* by some thinker.<sup>22</sup> The *meanings*, as units in truth conditions, seem to require *uses* if their truth conditions are to be fulfilled. On this view, although any such truths would not change

<sup>20</sup> Frege, “Logik” (1897), pp. 146, 146, and 147, respectively; “Logic,” in *Posthumous Writings*, pp. 134, 135, 135; “Logic,” in *The Frege Reader*, pp. 234, 235, 236. My italics.

<sup>21</sup> Frege, “Der Gedanke,” in *Kleine Schriften*, p. 361 [76]; “Thought,” in *The Frege Reader*, p. 344.

<sup>22</sup> I am aware that many formal semantical theories for indexicals do not—at least on their face—accept such an implication. They postulate that relevant tensed or indexical *sentence types* are true relative to times (and other parameters), regardless of whether anyone uses the sentences at those times (or other parameters). I think that such postulations are useful idealizations. But I think that truth relative to times (*et alia*) is not the basic semantical notion. I think that truth and falsity are the basic semantical notions. Truth, full stop, is fundamental. Frege himself clearly maintained this view.

Truth is fundamental, I think, because truth-conditional semantics is ultimately a psychological theory, and the theory should illuminate truth conditions for belief and judgment, which are among the fundamental psychological states. Belief states and judgment events are true or false, not true-at or false-at. Semantics is a branch of psychology. Frege’s motive for holding truth to be fundamental is analogous. He took semantics to be a theory of thought/judgment, but with a difference. He took thoughts and judgments to be idealized event types that meet certain norms in an idealized science, not actual psychological states. Still, he held that in this idealized sense, semantics is ultimately a theory of truth conditions for judgments. And judgments are true or false, full stop—not true-at. So I believe that his reasons for taking truth, not truth-at, to be the fundamental semantical concept are parallel to mine.

I think—and I think Frege thought—that only used tokens of indexical sentences are true or false. Particular times are semantically associated with tensed sentence types through uses of those sentence types at particular times. Thus the natural semantics of indexicals seems to entail that (necessarily) if any structure that contains an indexical element is true, the indexical is used—tokened. Use entails the existence of judgment, or at least belief. This is not to say, of course, that each use is an exercise of judgment or belief. It is to say that there is a necessary, constitutive relation between there being uses and there being judgments or beliefs.

truth value, they would not be essentially nontemporal. Further, they would *not* be independent of times at which they are judged true. The meanings of true sentence-occurrences containing 'I' plausibly require uses by an utterer, if the occurrences are to be true. Analogously, 'I's meaning rule might be taken not to be "impersonal."

Frege may have reasoned in these ways. In his ontological Platonism, he advocated what I believe to be an unnecessarily strong construal of the nonrelativity of truth to time and to thinkers. He may have thought that including indexical meanings in the contents of scientific thought would compromise the trans-temporality and objectivity of scientific truth.

Contrary to this Frege-like view, the requirement that truths be true timelessly can be understood *not* to require expulsion of the ordinary meanings of indexicals from the contents of the truths.<sup>23</sup> It is not clear exactly what Frege meant by the four quotations. He does not conceive the meanings of indexicals as part of the senses of sentences or sentence-uses that contain them. I have outlined a motivation that he may have had—a motivation that I do not myself accept.

How to apply modern notions of linguistic meaning to *proper names* is not obvious. So contrasting sense and linguistic meaning in the case of proper names is less straightforward than it is in the case of indexicals. Frege treats proper names and indexicals similarly. He takes the senses of names to shift with context, even for a given name and idiolect. He thinks of such senses in cognitive terms, not in terms of rules governing linguistic usage.<sup>24</sup>

Frege illustrates senses of proper names with senses of definite descriptions. I doubt that Frege held a general theory on the matter. Most definite descriptions in his examples contain names. If he held a general theory, one would expect him to have noted the incompleteness of his examples in illustrating it. Also, in *Begriffsschrift* Frege cites a name guided by intuition to a position in space.<sup>25</sup> He contrasts

<sup>23</sup> Taking truth not to vary with time does not entail taking all truths to be timeless entities. For some of my reasons for rejecting Frege's view that true thoughts are in general timeless, mind-independent entities, see *Truth, Thought, Reason*, pp. 50–54.

<sup>24</sup> Kripke seems to think that I deny that Frege holds that communal natural languages are, strictly speaking, amalgams of idiolects. See Kripke, "Frege's Theory of Sense and Reference," p. 210. I do not. See Burge, *Truth, Thought, Reason*, Introduction, pp. 38–39; "Sinning Against Frege," pp. 232–34; "Wherein Is Language Social?" in Alexander George, ed. *Reflections on Chomsky* (Cambridge: Blackwell, 1989), reprinted in *Foundations of Mind*; and "Gottlob Frege: Some Forms of Influence," forthcoming in Beaney, ed. *Oxford Handbook of the History of Analytic Philosophy*.

<sup>25</sup> 'Intuition' in a Kantian sense. Frege, *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens* (Halle, Germany: L. Nebert, 1879), section 8. English

the intuitive way of determining the position with a descriptive way. I doubt that Frege would have thought that definite descriptions express ways of thinking built on intuitions. The passage suggests that on his later theory of sense, Frege may not have agreed that every name has a descriptive sense, much less a purely descriptive sense.

In any case, Frege thinks that senses of many proper names in contexts of use are those of definite descriptions, though perhaps not pure definite descriptions. I think that he took ordinary proper names, as well as indexicals, to be unsuited for an ideal language of scientific thought. He regarded contextual variability of sense as a defect in an expression, for scientific purposes.

I have been writing about Frege's conception of sense. I have not implied that I accept it. I do not.<sup>26</sup> The way to object to Frege's views is to object to his theory of thought, not to claim that he failed to understand linguistic meaning or rules of use in natural language—communal or idiolectic. It is misguided to claim, as some have, that names lack a sense, but then introduce a mode of presentation, a way of thinking associated with them. Modes of presentation associated in certain ways with expressions—even contextually associated with them—are senses.

Ordinary linguistic meaning is not confined to, and sometimes hardly includes, the way a user thinks in using language. The meaning of demonstrative constructions commonly does not include the way of thinking that guides a demonstrative, beyond indicating that the way of thinking is in some way context-bound. (Think of perceptually guided demonstratives such as 'that' in 'that is a sphere'. The way of

translation: *Begriffsschrift, A Formula Language, Modeled upon that of Arithmetic, for Pure Thought*, in Jan van Heijenoort, ed., *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931* (Cambridge: Harvard, 1981). There is also the famous passage in "Der Gedanke," in *Kleine Schriften*, p. 350 [66]; "Thought," in *The Frege Reader*, p. 333, in which Frege ascribes to some uses of 'I' a sense private to the speaker that is almost surely not descriptive, or at least not purely descriptive.

<sup>26</sup> Burge, "Sinning Against Frege," aims to show that certain objections to Frege's account of the senses of names and other context-dependent devices contain anachronistic elements. Some of these objections are undermined by the anachronism. I was specific about which objections were undermined. I did not and do not think that all objections contain anachronism, or even that all those that do, are mistaken in their conclusions. Kripke and Donnellan showed on epistemic and psychological grounds that reference (or denotation) by names and other context-dependent devices is not semantically determined by description or cognitive resources available to thinkers. Their main points against Frege are, I think, decisive. For elaboration, see Burge, *Truth, Thought, Reason*, Introduction, pp. 35–43ff.; *Origins of Objectivity* (New York: Oxford, 2010), pp. 145–46; "Frege: Some Forms of Influence," section 6.

thinking that guides ‘that’, perhaps a perceptual way, is vastly richer than the meaning expressed by ‘that’.) Despite the criticisms, advanced below, of Frege’s applications of his concept sense, I think (a) that senses that are thought in uses of language are often different from the linguistic meanings expressed, and (b) that there are senses (especially perceptually based senses) that are different from any meanings that are actually expressed in the thinker’s language. Senses are ways of thinking that are at least contextually associated with language in certain explanatorily circumscribed ways that I will discuss later. Frege’s concept has a valuable cognitive orientation, not centered on rules of linguistic usage.

As noted, I do not accept Frege’s ontological Platonism about senses. Representational content—certainly empirical representational content—depends for its nature on events in space and time. It depends for its nature on mental operations (the psychological counterparts of uses) and, more particularly, on relations of occurrence-based referential applications to particulars. It is unclear whether Frege intended to give a general theory of thought expressed in language, or just a theory of thought ideally suited to science. I have been inclined to attribute the former view. Perhaps, though, he did not exclude indexical meanings and occurrence-bound referential applications from all thought content—only from the content of scientific thought.

Either way, Frege’s views on sense cannot be right. For occurrence-based contents (applications) cannot be eliminated from empirical scientific thought, because they cannot be eliminated from perceptual belief. Occurrence-based representational contents depend for their natures on occurrence in time and on being used in occurrent thinking. (See note 3.)

There are further difficulties with Frege’s view, conceived as a general theory of thought. For any definite description *the d* and almost any proper name *n*, a thought expressed by a sentence of the form  $n = \underline{\textit{the d}}$  is not the thought expressed by a sentence of the form  $n = \underline{\textit{n}}$ .<sup>27</sup>

In *Naming and Necessity*, Kripke established this point. He showed that one can use proper names without having a definite description

<sup>27</sup> I am referencing Frege’s account in “Über Sinn und Bedeutung” (1892), in *Kleine Schriften*, p. 144n [27n]; in *The Frege Reader*, p. 153. ‘=’ is read ‘is identical with’. Instances of the forms are: ‘Aristotle is identical with the teacher of Alexander’ and ‘Aristotle is identical with Aristotle’. I write ‘almost any proper name’ to allow for stipulations that names abbreviate definite descriptions. I am inclined to say that these are not genuine cases of proper names. I assume that to be a definite description, an expression’s logical form must be governed by a wide-scope uniqueness operator, *not* by a demonstrative or other context-dependent element.

that even purports to be sufficient to denote what the name names. And he showed various epistemic ways in which speakers' uses of a name and any associated definite description can come apart.<sup>28</sup>

The point does not depend on name and description coming apart epistemically. A thought  $n = n$  is almost never the same as a thought  $n = (\text{the entity that} = n)$ , even assuming that the rigidity of the name exactly matches the rigidity of the definite description. Thinking the pure identity thought with any ordinary proper names engages different cognitive capacities than does thinking any definite-description thought.<sup>29</sup> Thinking of someone as Plato differs from thinking of that person via *any* definite description. The name-type figures essentially in the thought content, and no generalized definite description operator figures in thought with a proper name.<sup>30</sup> Thought contents type-identify cognitive capacities. Since the capacities differ, the thought contents differ. Frege's reflections on identity can be used to show that, in a general theory of thought, his examples of proper names' having the senses of definite descriptions cannot be right.

The same point goes for indexicals. Meanings of indexicals figure in thought. Thinking of a time as today differs from thinking of it via a definite description, or via perception of events linked to the day. The occurrent thoughts today is the one and only  $F$  and today is the time of [perceptually presented] event  $E$ , are not the thought today is today (assuming all are true and thought on the same day, and assuming the rigidities match). Testing a sense against a pure identity thought is a powerful tool. What makes the test robust and effective is its connecting sense and thought content with the typing of psychological states and competencies.

<sup>28</sup> Kripke, *Naming and Necessity* (Cambridge: Harvard, 1980). Similar points were made by Keith Donnellan, "Proper Names and Identifying Descriptions," *Synthese*, XXI, 3/4 (October 1970): 335–58.

<sup>29</sup> I mean by '*pure identity thought*' any thought of the form of an identity, with the same representational content in each argument place of the identity concept. By '*ordinary proper name*', I intend to exclude "names" stipulated to abbreviate a definite description, where the definite description is perhaps rigidified in certain ways.

I believe that the point that  $n$  is never epistemically equivalent to the entity that =  $n$  holds even if one construes the description—as I think one can—in such a way that it cannot come apart from the name *modally*. See my "Postscript to 'Sinning Against Frege,'" in *Truth, Thought, Reason*. My basic point is psychological: names do not involve use of the identity predicate; the relevant definite description does involve such use.

<sup>30</sup> The same point applies to the quantificational apparatus that Russell thought played the role of the definite description operator (see note 27). Most individuals learn proper names before mastering such a generalized logical apparatus.

A similar test applies to indexicals like tense, without forming identity thoughts.

(a) the leaf is [present tense] green; so the leaf is [present tense] green

does not have the same inferential form as

(b) the leaf is [present tense] green; so the leaf is green at the time I think this very thought.

Assume that, in (a) and (b), citing the times in the conclusions uses anaphora. So the argument cannot be invalid on account of a shift in times. Then (a) and (b) are necessarily and apriori truth preserving. Thinking (a) is not thinking (b). (b) types resources to denote oneself and to think about thought. (a) does not. The inferences differ—and are marked by different logical forms. As with proper names, the sense of an indexical is almost never (see note 27) that of a definite description.

Let me summarize what I have said so far about relations between sense and modern notions of linguistic meaning. The sense of a *demonstrative* on an occasion of use often differs very significantly from the linguistic meaning of the demonstrative on that occasion. Let us bracket the context-bound applications of demonstratives. Applications are part of the representational content of a thought and part of the truth condition of an utterance. For present purposes, I will count such applications neither part of the linguistic meaning nor part of the sense. An ability-general mode of presentation in thought that is appropriately associated with a demonstrative *is* a sense. The ability-general mode of presentation is, very frequently, vastly richer than the linguistic meaning of the demonstrative-governed phrase on the occasion of use.

Occasions on which a demonstrative is backed by perceptual modes of presentation make the point very vividly. Linguistic meaning grounds accounts of communal or idiolectic rules for understanding a linguistic item. Sense is invoked, with certain restrictions to be discussed later, in an account of the ability-general aspect of what a thinker thinks. In the case of use of demonstratives partly backed by perceptual modes of presentation, the two notions usually diverge significantly. For example, the linguistic meaning of 'that cat' (applied in a context in which the utterer is looking at and thinking about a particular cat) centers on rules for understanding the expression. The linguistic meaning is not nearly as rich as the mode of presentation, the sense, that the thinker uses to determine the case visually on a particular occasion. That sense includes a particular perceptual conception of the cat as it appears from a certain angle and distance in a certain type of light.

The sense is vastly richer than the linguistic meaning, even the linguistic meaning in the utterer's idiolect. Since such uses of demonstratives are so common, this difference is substantial and theoretically important.<sup>31</sup>

Frege's account of *indexicals* is unacceptable, unless one both focuses on scientific thought and grants that meanings of indexicals are to be banned from scientific thought. In a general account of thought, it must be acknowledged that the linguistic meaning of indexicals is usually a part of the ability-general aspect of what a thinker thinks in using an indexical. (Again, I bracket the occurrent application.) I leave open whether the ability-general aspects of what is thought on particular occasions of use are sometimes richer than linguistic meanings of the indexicals.

Frege's account of *proper names* is mistaken where it assigns them descriptive senses. I think that proper names have an attenuated idiolectic and communal linguistic meaning—what individuals master by mastering rules for using the name. It is clear that uses of names express for individuals cognitive, ability-general modes of presentations—senses. It is clear that these senses are not to be associated with ordinary descriptions. The ability-general aspect of the name is very thin and is associated with the “property” of having the name—being one of the Aristotles or Bobs. (I believe that uses of names, like uses of demonstratives and indexicals, involve context-bound applications. Again, I bracket applications.)<sup>32</sup> I leave open whether the senses of proper names coincide with their linguistic meanings, pending better understanding of the linguistic meaning of names. It seems to me possible that they commonly do coincide.

Distinguishing sense from linguistic meaning helps orient one's approach to *indirect discourse* and *reports of propositional attitudes*. Linguistic rules for such reports allow immense contextual variation with the reporter's purposes. If, however, one focuses on uses in which the reporter's purposes are scientific, or otherwise specifically concern what an individual's psychological states are, one can discern in such uses something approaching reports that designate an individual's senses. On Frege's view, of course, the oblique senses of expressions in (ground-level) oblique contexts determine their ordinary senses; oblique senses are used in attributing the ordinary

<sup>31</sup> I discuss use of the terms ‘context-bound’, ‘ability general’, and ‘application’ in “Five Theses on *De Re* States and Attitudes.” Here I think that the intuitive meaning of the terms will suffice. (See note 3.) I make the point that applications are not easily assimilable to Fregean senses in “Belief *De Re*.” See also *Truth, Thought, Reason*, pp. 50–54.

<sup>32</sup> See my “Reference and Proper Names,” this JOURNAL, LXX, 14 (Aug. 16, 1973): 425–39.

senses to the individual reported on. In other words, one can use such constructions to designate senses or representational thought contents. In most cases of ordinary reports, however, the meanings of the reports fall far short of determining the senses in the thought of the individual who is reported on. Again, Frege was focused on scientific uses of language. He took his account to be a first step in explicating a language for a science of thought. It was.

In the next sections, I develop a further respect in which common uses of the notion of linguistic meaning vary, at least in emphasis, from appropriate uses of the notion of sense. That variation has to do with certain cases of incomplete understanding. To get to that point, I must discuss in more detail Frege's conception of the individuation of senses.

## II

Fregean reflection on identity thoughts ought not be applied mechanically. As Frege emphasized, differences in words do not automatically indicate differences in thought content. One cannot read an individual's thought contents off sincere assertions in response to pieces of language. But often, one can use Frege's form of reflection to evoke fine-grainedness in modes of presentation—which mark fine-grainedness in competencies in individual psychologies. What we know about cognitive competence helps in determining the senses of words.

Expositions of Frege's reflections on identity thoughts in "On Sense and Denotation" often conflate points that he makes about his examples with the main thrust of his reflections, missing their power. It is often said that his point is to distinguish empirical or synthetic thoughts from apriori or analytic ones. Frege does make these distinctions. His most famous examples (The Morning Star is identical with the Evening Star; The Morning Star is identical with the Morning Star) illustrate them. But these distinctions are not his main point. In writings from the same period, and elsewhere, Frege cites pairs ( $2^2 = 4$ ,  $2 + 2 = 4$ ) both members of which are, by his lights, apriori and derivable from truths of logic, analytic in his sense. He uses the pairs to illustrate differences in sense.<sup>33</sup>

<sup>33</sup> Frege, *Die Grundgesetze der Arithmetik* (1893) (Hildesheim, Germany: Georg Olms, 1962), [7]; partly translated in *The Basic Laws of Arithmetic: Exposition of the System*, trans. and ed. Montgomery Furth (Berkeley: California UP, 1964), p. 35. A similar point ( $2^4 = 4^2$  and  $4 \times 4 = 4^2$ ) is made in Frege, "Funktion und Begriff" (1891), in *Kleine Schriften*, p. 132 [13]; translated as "Function and Concept" in *The Frege Reader*, p. 138. There are other passages in which Frege contrasts the senses of arithmetic truths, both of which are for him apriori and, at least until late in his career, both of which are for him analytic: ( $3^2 = 3^2$  and  $2^3 + 1 = 3^2$ ) Frege to Russell 12/28/1902, *Wissenschaftlicher*

Another common misreading holds that the examples in “On Sense and Denotation” are meant to contrast statements having cognitive value (*Erkenntniswert*) with statements lacking it. Frege never writes that statements of the form  $a = a$  lack cognitive value.<sup>34</sup> He writes only that they have different cognitive value from statements of the form  $a = b$ .

An even more common error is to offer a criterion for difference in sense along this line:

A sentence  $S$  has a different sense from a sentence  $S'$  if and only if it is possible to understand the senses (thought contents) of  $S$  and  $S'$  while having different cognitive attitudes (such as judgment) toward those senses as thought contents.<sup>35</sup>

*Briefwechsel*, ed. Gottfried Gabriel et al. (Hamburg, Germany: Felix Meiner Verlag, 1976), p. 235; *Philosophical and Mathematical Correspondence* (Oxford, UK: Blackwell, 1980), p. 152; ( $7 = 3 + 4$  and  $7 = 7$ ) Frege to Russell 11/13/1904, *Wissenschaftlicher Briefwechsel*, pp. 245–48; *Philosophical and Mathematical Correspondence*, pp. 163–65; ( $5 + 2 = 4 + 3$  and  $5 + 2 = 5 + 2$ ) Frege to Peano (undated), *Wissenschaftlicher Briefwechsel*, p. 197; *Philosophical and Mathematical Correspondence*, p. 128; ( $5 = 2 + 3$  and  $5 = 5$ ) “Logik in der Mathematik,” in *Nachgelassene Schriften*, p. 242; *Posthumous Writings*, p. 224.

<sup>34</sup> Frege calls  $a = a$  statements ‘boring’, Frege to Peano undated, *Wissenschaftlicher Briefwechsel*, p. 195; *Philosophical and Mathematical Correspondence*, p. 126. He says of certain statements of the form  $a = b$  that they *extend* our knowledge, whereas statements of the form  $a = a$  do not, since they are self-evident: “Logik in der Mathematik,” in *Nachgelassene Schriften*, p. 242; *Posthumous Writings*, p. 224. But he never writes that the latter statements do not constitute knowledge, or lack cognitive value. Cf. also Frege to Jourdain undated, Frege to Russell 12/28/1902, and Frege to Russell 11/13/1904, *Wissenschaftlicher Briefwechsel*, pp. 128, 234–35, 246–47; *Philosophical and Mathematical Correspondence*, pp. 80, 152, 164–165, respectively. I think that there is no good evidence to believe that he held either of these views.

<sup>35</sup> In view of incomplete understanding, which I discuss shortly, I take ‘understand’ here to require that the understanding is not defective or incomplete. Close analogs of the principle cited in the text are stated by Michael Dummett and Gareth Evans, except that they write of attitudes toward sentences’ being true rather than attitudes toward thought contents. (Frege uses both locutions.) See Michael Dummett, *The Interpretation of Frege’s Philosophy* (Cambridge: Harvard, 1981), p. 323; Gareth Evans, *Varieties of Reference* (New York: Oxford, 1982), pp. 19–20. Dummett cites the first part of (C), cited below in the text. But he fails to note that the latter part shows that Frege did not accept the criterion for sense difference that Dummett proposes. In interpreting Frege, Evans rightly emphasizes the psychological relevance of sense. But he moves, incautiously, from passages in which Frege states that possible differences in attitude toward contents are sufficient for *differences* in sense to an analog of the criterion stated in the text, an analog that also claims that necessary sameness in attitudes toward thought contents is sufficient for *sameness* of sense. Examples of passages in which Frege makes analogs of the weaker, sufficient-for-difference claim are “On Sense and Denotation,” [25–26, 32]; Frege to Russell 12/28/1902, *Wissenschaftlicher Briefwechsel*, p. 236; *Philosophical and Mathematical Correspondence*, p. 153; “Kurze Übersicht meiner logischen Lehren,” in *Nachgelassene Schriften*, p. 212; *Posthumous Writings*, p. 197. [Note continued next page.]

Frege did not accept the ‘only if’ direction. He thought that if two thoughts are self-evident, understanding them compels sameness of all cognitive attitudes.<sup>36</sup> Perhaps  $2^2 = 4$  and  $2 + 2 = 4$  cannot attract different cognitive attitudes, if one understands both. Yet they constitute different thoughts. Frege may not have believed that these arithmetical truths are self-evident. But in *Basic Laws* he clearly holds that there are plural basic logical laws, canonically expressed by different sentences with distinct senses. He clearly holds that all basic logical laws, and even some of their immediate consequences, are self-evident.<sup>37</sup> So he holds that sentences with different senses can

Dummett and Evans rightly require that the sentences be understood. As noted, I take this to mean *fully understood*. Consider this inadequate sufficient-for-sense-difference variant:

- (I) If it is possible for an individual with *ordinary* linguistic understanding of *S* and *S'* to take one as true and the other not to be true, then *S* and *S'* express different senses and have different thoughts.

An important theme in Frege’s philosophy is to insist that even expert language users can use a sentence with ordinary understanding and lack complete understanding of its sense. Then different attitudes by ordinarily competent language users toward *sentences* that express the same senses would be possible. This point is implied by my “Frege on Sense and Linguistic Meaning,” in David Andrew Bell and Neil Cooper, eds., *The Analytic Tradition: Meaning, Thought, and Knowledge* (Oxford, UK: Blackwell, 1990); reprinted in *Truth, Thought, Reason*.

In the literature there is some misreading me both as holding that Frege accepted (I) and as holding that Frege *never* thought that an analysis preserves sense—which is contrary to a passage in Frege, “Logic and Mathematics” (1914), in *Nachgelassene Schriften*, p. 228, *Posthumous Writings*, p. 211, which I certainly did not overlook since I discussed it in “Frege on Sense and Linguistic Meaning,” in *Truth, Thought, Reason*, pp. 252–53. Neither claim about Frege can be found in my writing.

As I wrote in *ibid.*, I think that many “analyses,” including most successful “analyses” in the history of sciences other than semantics—even those culminating in definitions—are explications in which *explicans* and *explicandum* express different senses, different ways of thinking. (I think that Frege probably underestimated the importance of this point.) Explications and definitions are usually illuminating not because they produce sameness of thought content, or sameness of sense, but because they illuminate applications of the thought content of the *explicandum*. Some might paraphrase these remarks as entailing that there are few interesting “true” conceptual analyses, where ‘true analysis’ is misleadingly used in the paraphrase to apply to an explication that “unpacks” a complex concept and expresses it more perspicuously.

<sup>36</sup> Frege, “Gedankengefüge” (1919), in *Kleine Schriften*, p. 393 [50]; *Collected Papers on Mathematics, Logic, and Philosophy*, ed. Brian McGuinness (New York: Blackwell, 1984), p. 405. Note that this point shows that another common reading—that the point of Frege’s examples is to distinguish informative from uninformative statements—is mistaken. Many statements that express genuine knowledge, but do not ‘extend our knowledge’—and are *a priori* and self-evident—are uninformative in the ordinary sense, and uninformative in the technical sense that accepting them does not affect our subjective probabilities. But many such statements differ in sense.

<sup>37</sup> Frege takes senses of pure identity statements to be self-evident (note 34). These are immediate consequences of the basic logical law of identity. For more on Frege’s

compel the same cognitive attitudes. Contrast with pure identity thoughts, or pure identity inferences, is a more powerful differentiating tool than possible contrast in judgment.

The possible-difference-in-cognitive-attitude test is inadequate because it focuses on separability of judgment of whole thoughts, not on thought type. Difference in thought type can hinge on thought-component differences, which mark differences in competencies. Competence with a name is different from competence with a definite description. Competence with squaring is different from competence with addition. These differences remain even if in individual cases they do not yield possible differences in judgment.

The passages that express Frege's sharpest grip on his own concept sense are the one in "On Sense and Denotation" just discussed and the following passage:

(C) Now two sentences *A* and *B* can stand in such a relation to one another that anyone who recognizes the content of *A* as true must also recognize that of *B* straightway [*ohne weiteres*] as true, and, also conversely, that anyone who accepts the content of *B* as true must immediately [*unmittelbar*] accept that of *A* (*equipollence*), whereby it is presupposed that the grasp of content of *A* and *B* does not raise any difficulty....I assume that there is nothing in the content of either of the two equipollent sentences that would have to be at once immediately [*sofort unmittelbar*] recognized as true by anyone who had grasped it correctly.<sup>38</sup>

Frege requires that the content of neither sentence contain a self-evident element (an element that commands immediate acceptance). He does so to block collapse of the senses of all expressions of self-evident thoughts, or thoughts that depend for their truth on

notion of self-evidence see Burge, "Frege on Knowing the Foundation," *Mind*, n. s., cvii, 426 (April 1998): 305–47; reprinted in *Truth, Thought, Reason. Frege's belief in a plurality of basic logical laws is constant through his career. Cf. Begriffsschrift*, section 13. (For a caveat, see note 41.) Further evidence that Frege took the senses of sentences expressing different axioms or basic logical laws to be different derives from combining his sense-composition doctrine with remarks about the senses of expressions for logical constants, for example, "Gedankengefüge" (1919), in *Kleine Schriften*, p. 381 [39]; *Collected Papers on Mathematics, Logic, and Philosophy*, p. 393.

<sup>38</sup>Frege, "Kurze Übersicht meiner logischen Lehren" (1906), in *Nachgelassene Schriften*, p. 213; *Posthumous Writings*, p. 197. Eva Picardi, "A Note on Dummett and Frege on Sense-Identity," *European Journal of Philosophy*, 1, 1 (April 1993): 69–80 (see p. 75), recognizes both points about this passage made in the text. She also correctly notes that immediacy has to do with the form and type of a transition, not its temporal speed. The idea is that there is no inference other than one of the form  $\underline{p}$ ; so  $\underline{p}$ —which Frege probably would not have counted as an inference.

self-evident components, into a single thought content.<sup>39</sup> Note the crucial role of immediacy, which is additional to universal co-acceptance. The immediacy condition requires that an individual can recognize the sense of *B* to be true *in* recognizing the truth of *A*, and vice versa, *without inference or reflection*.<sup>40</sup>

This principle is a close analog of the condition that Frege sets out in “On Sense and Denotation.” If one cannot recognize the truth of a thought *T'* expressed by sentence *B* from the truth of a thought *T* of the form  $\underline{a = a}$ , expressed by sentence *A*, without inference and without reflection, then thought *T'* has form  $\underline{a = b}$ , and the senses of *B* and *A* differ—assuming that one understands both thoughts rightly. These are, I think, Frege’s best thoughts on sense identity.<sup>41</sup>

Again, applying such principles requires theoretical acumen. Application is fallible. I believe that whether inference or reflection is required to get from thought *T* to thought *T'* hinges on the psychological capacities that are type-individuated by the senses. We are often entitled to presume that our judgments on these matters have some

<sup>39</sup> This point comes clear by comparing (C) with the cognate passage in Frege to Husserl 9/12/1906, *Wissenschaftlicher Briefwechsel*, pp. 105–06; *Philosophical and Mathematical Correspondence*, p. 70, which explicitly discusses self-evidence.

<sup>40</sup> Thus *any* inferential transition (other than  $\underline{p; \text{so } p}$ ) suffices to differentiate senses. I believe that this test is the inferential corollary of Frege’s account in “On Sense and Denotation.”

<sup>41</sup> It is well known that not all of Frege’s remarks accord with these ideas. In Frege to Husserl 9/12/1906, Frege states a criterion on which all logical truths would have the same sense. Cf. *Wissenschaftlicher Briefwechsel*, pp. 105–06; *Philosophical and Mathematical Correspondence*, p. 70. There are a few other remarks, mostly in unpublished writings, that do not accord with (C) and are congenial with the Husserl letter. But the criterion in that letter is incompatible with “Function and Concept” (1891), [13]—the difference in sense between ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’; *Die Grundgesetze der Arithmetik* (1893), [7]—the difference in sense between ‘ $2^2 = 4$ ’ and ‘ $2 + 2 = 4$ ’. As noted, note 33, there are many other such passages.

Frege may have unconsciously used two conceptions of sense—a psychologically realistic one and a very idealized one. Carlo Penco so argues in “Frege, Sense and Limited Rationality,” *The Review of Modern Logic*, ix, 1–2 (November 2001–November 2003): 53–65. I think it more likely that Frege made a mistake. Within a month or so after the Husserl letter, he engages the same issue, using similar language even, but proposes (C)’s restrictive account of sense identity, seemingly correcting the criterion in the Husserl letter. Similar views of the matter are stated by Heijenoort, “Frege on Sense Identity,” *Journal of Philosophical Logic*, vi, 1 (January 1977): 103–08; Dummett, *The Interpretation of Frege’s Philosophy*, pp. 324–25—‘it is hard when a philosopher is held to account for remarks made in purely private correspondence’ [it should be noted, however, that not *all* the deviations from (C) are in private correspondence]; and Picardi, “A Note on Dummett and Frege on Sense-Identity,” who plausibly regards (C) as *correcting* the criterion in the Husserl letter. I agree with Picardi that Frege was perhaps sometimes too ready to count an obvious inference as no inference at all. In any case, I regard the approach in “On Sense and Denotation,” “Function and Concept,” *Basic Laws*, and (C), as much the more valuable.

weight. But ultimately, they are matters for philosophically informed psychology and psycholinguistics.

Frege cites at least four ways in which determining sense is open to complication.

First, although Frege's assimilation of indexicals and proper names to demonstratives and his taking their senses sometimes to be those of definite descriptions are mistaken, his idea that used demonstratives express senses beyond linguistic meaning (say, in perceptual beliefs) is correct. Determining such senses has no recipe. Asking a speaker how he or she conceives the referent is a start. But finding thought content that both type-individuates the relevantly used psychological capacity and determines the demonstrative's referent is a theoretical matter.

Second, Frege's method of formalization requires reflection on patterns of inferences.<sup>42</sup> Formalization seeks to determine the logical structure and inferential capacities of thinkings underlying formalized sentences. Formalization helps individuate senses. Frege showed that one cannot read the formal structure of a thought off ordinary grammar. He claimed that active and passive structures have the same logical structure and sense. Similarly, for interrogative and declarative structures and for other pairs of grammatically different expressions ('Jupiter has four moons'; 'the number of moons of Jupiter is four').<sup>43</sup> Formalization can yield understanding of sense identity or difference that is unobvious to language users.

Third, Frege's distinction of sense from coloring complicates sense determination. Sense sets truth conditions, yields a cognitive route to denotation, figures essentially in recognition of truth, and contributes to logical form. Call these '*cognitive aspects*' of language and psychology. Colorings are psychological elements associated with the expression, or the thinking, of a sense that do not fill these roles. Frege holds that 'and' and 'but' differ in coloring, but not sense.<sup>44</sup>

<sup>42</sup> For extended discussion of Frege's method, see Burge, Introduction, *Truth, Thought, Reason*, pp. 7, 11–26.

<sup>43</sup> See Frege, *Grundlagen der Arithmetik* (1884), translated as *The Foundations of Arithmetic*, trans. J. L. Austin (Evanston, IL: Northwestern, 1968), section 57; cf. also "Über Begriff und Gegenstand" (1892), in *Kleine Schriften*, [199–200]; "On Concept and Object," in *The Frege Reader*; "Der Gedanke," [62]. I am not committed to agreeing with him on these cases.

<sup>44</sup> Frege, "Gedankengefüge," [43]. Frege sometimes counts coloring part of a sentence's 'content' (*Inhalt*), but not part of its sense. Cf. "Kurze Übersicht meiner logische Lehren," in *Nachgelassene Schriften*, pp. 213–14; *Posthumous Writings*, pp. 196–97; *The Frege Reader*, p. 300. Here Frege does distinguish an ordinary notion of linguistic meaning (which he terms '*Inhalt*', 'content') and sense. Of course, many modern notions of linguistic meaning follow him—distinguishing a notion of cognitive or literal meaning

'But' entails some contrast between the connected clauses. 'And' does not. Despite these psychological differences, their contributions to what is judged true about the subject, as opposed to what is expected, and their contributions to formal inference are, he thinks, the same. The sense/coloring distinction is real. Applying it is a theoretical matter.<sup>45</sup>

Fourth, incomplete understanding, which is closely connected to the second and third points, complicates determination of sense.<sup>46</sup> Frege takes *fully* understanding expressions to be a condition on individuating their senses. Of course, a foreigner's inept usage may not indicate much about his or her thoughts. Frege's requirement goes deeper. He held that the sense of an expression might not be fully understood even by expert users.<sup>47</sup> I will dwell on this fourth point.

Frege's discussions of incomplete understanding focus on aspects of language use that are largely ignored in discussions of ordinary notions of linguistic meaning. Frege holds that even the most expert users may not fully understand the sense, communal or idiolectic, of an expression.<sup>48</sup> The view that the most competent and expert users may lack full understanding is at least somewhat unusual.

from a broader notion of linguistic meaning. On this broader notion, 'but' and 'and' do not mean the same. Distinctions between sense and ordinary linguistic meaning remain. These are most prominent in accounting for used demonstratives/indexicals—a hugely important area of understanding the use of language—and, at least in emphasis, in accounting for incomplete understanding.

<sup>45</sup>As Grice's development of the idea in his distinction between literal meaning and conversational or conventional implicatures made clear. Paul Grice, *Studies in the Way of Words* (Cambridge: Harvard, 1989). The theoretical nature of disputes over whether indicative 'if-then' is the material conditional plus implicatures (a prominent sort of coloring) illustrates the complexity of applying the distinction.

<sup>46</sup>I discuss Frege's notion of incomplete understanding in "Sinning Against Frege"; "Frege on Sense and Linguistic Understanding"; and "Frege on Extensions of Concepts, from 1884 to 1903," *The Philosophical Review*, xciii, 1 (January 1984): 3–34, reprinted in *Truth, Thought, Reason*. I also discuss incomplete understanding more substantively in *Foundations of Mind*; see the Introduction and subject index. Note that formalization and distinguishing coloring from sense are ways of improving incomplete understanding. Frege's view is more radical than might appear from ordinary cases like these, however. He thought that no one had fully understood the concept number until he discovered his logicist analysis.

<sup>47</sup>Frege's view was not new. It is an application of traditional rationalist views of thought. See Burge, "Frege on Sense and Linguistic Understanding."

<sup>48</sup>Such a view runs contrary to most thinking about ordinary linguistic meaning. It is sometimes allowed in the literature that the idiolectic meaning of an individual's expression may depend on meanings of expressions used by others on whom an individual relies. But it is rarely allowed, or at least rarely maintained, that it is possible that no one in a community fully understands the *meanings* of their expressions. Whether Frege's point can be accommodated by modern notions of linguistic meaning is a matter that I shall not try to decide.

This view opens a possible gap between how a thinker explains or identifies a sense and what sense the individual expresses. For example, by not fully understanding a simple-seeming expression, one may fail to understand complexity in the expression's sense.<sup>49</sup> In applying a sense identity test that relies on pure identity thoughts or on lack of inference, one must presume that the test is not undermined by incomplete understanding of the thoughts. One is often entitled to the presumption. But one must remain open to correction by theory.

This issue has tempted some to propose a normal form that specifies *thinking* as a three-place relation—thinker *I* thinks thought content *c* via way *w* of thinking *c*. A *way of thinking* the thought content is added as a parameter over and above the thought content, which is itself a way of thinking its subject matter. On this non-Fregean proposal, modes of presentations of modes of presentations are specified in every specification of thought. The new layer of mode of presentation is taken to aim at sense, the thought content, which is taken to be an object presented in thought.<sup>50</sup>

I believe that this move is misguided. It fails to appreciate the point and power of the idea that senses are thought contents, associated in certain ways with language. And it misconstrues the nature of incomplete understanding. Thought is a real-world psychological phenomenon. Along with the mode of the thought (belief, supposition), thought contents, including their components, type-identify psychological states and events in ways that ground very basic levels of explanation. Senses are aspects of certain psychological kinds. Normally, they are not objects aimed at in thought. Thought content suffices to play the role of individuating cognitively relevant psychological states. The proposed move is epicyclic.

The error of taking thought contents as aimed-at objects of thought is one of the oldest and most persistent in philosophy.<sup>51</sup> Avoiding the

<sup>49</sup> Frege considers such a situation in “Logik in der Mathematik,” in *Nachgelassene Schriften*, p. 226; *Posthumous Writings*, p. 209.

<sup>50</sup> Ideas that motivate this move occur in Felicia Ackerman, “Analysis, Language, and Concepts: The Second Paradox of Analysis,” *Philosophical Perspectives*, iv (1990): 535–43. I have seen and heard canonical introduction of modes of presentation (whether linguistic or not) of modes of presentation proposed by various others.

<sup>51</sup> Frege occasionally courts the error himself. Cf. “Der Gedanke,” in *Kleine Schriften*, p. 359 [75]; “Thought,” in *The Frege Reader*, p. 342—where he writes of elements in individuals’ consciousness “aiming” at thought contents. Moreover, his own metaphor of incomplete understanding as seeing a thought content, or a sense, through a mist invites the error. Cf. *Grundlagen der Arithmetik*, Preface, [vii–viii]; “Logik in der Mathematik,” in *Nachgelassene Schriften*, p. 228; “Logic in Mathematics,” in *Posthumous Writings*, p. 211. Frege’s theoretical work—his distinguishing sense from denotation, understanding from reference—provides some of the deepest resources for recognizing and rejecting the error.

error is facilitated by firmly taking thought contents to be an aspect of kinds of psychological states. Avoiding the error is also facilitated by reflecting on the psychology of incomplete understanding.

I will try to explain in more detail why introducing a second layer of mode of presentation is misguided. It is unobjectionable to speak colloquially of someone who incompletely understands a sense as thinking it in a “different way” from someone who fully understands it. But the role of this “way” must be sharply distinguished from the role of senses. This “way” is not a *relatum* in the normal form of specifications of thoughts.

One psychological element that has been thought to play a “second layer” role is the linguistic expression of a sense. Incomplete understanding of expressions can yield mistakes about their sense-relations to other expressions. Incomplete understanding of structural complexity underlying a simple-seeming expression can even lead to conflating what is in fact a pure identity thought or argument with one of another form. Perhaps an example is a mistaken belief that ‘Cain hit Abel; so there was a hitting at some time before now, and it was by Cain, and it was of Abel’ does not express an inference of the form  $\underline{p}; \text{so } \underline{p}$ .<sup>52</sup> There can be structural complexity hidden in an apparently simple expression (‘hit’). The surface simplicity of ‘hit’ can be psychologically significant, even though the word is not part of the sense, and may obscure it. In such cases, the triviality of the argument form and the nature of the underlying cognitive competence are masked by a mistaken meta-linguistic belief. Such errors can be corrected by formalization—the second way in which determination of sense is open to complication. It has been claimed that accounting for the incomplete understanding can be aided by specifying the thinking through the normal form: *I* thinks thought content *c* via  $e_1$ —and *I* thinks *c* via  $e_2$ .

The proposed normal-form specifications do not correspond to a difference between defective and nondefective understanding. Here the proposed normal form is idle in explaining the incomplete understanding. In both incomplete- and full-understanding cases, the individual thinks the same sense through the same two expressions. The defective understanding can easily be represented in Frege’s

<sup>52</sup> I assume, purely for the sake of argument, that the Davidsonian analysis of action sentences, elaborated by Terence Parsons and James Higginbotham, is correct. Donald Davidson, “The Logical Form of Action Sentences” (1967), in *Essays on Actions and Events* (New York: Oxford, 2001, 2<sup>nd</sup> ed.); Terence Parsons, *Events in the Semantics of English* (Cambridge: MIT, 1990); James Higginbotham, “On Events in Linguistic Semantics,” in Higginbotham, Fabio Pianesi, and Achille C. Varzi, eds., *Speaking of Events* (New York: Oxford, 2000), pp. 49–79.

normal form. The defective understanding consists in a mistaken meta-linguistic belief: *I* thinks:  $e_1$  does not express  $c$ , and  $e_2$  does.

Another psychological element that has been thought to play a “second layer” role is Fregean coloring.<sup>53</sup> As with expressions, colorings do not play the role of determining sense in the way that senses determine denotations. Colorings do not represent senses or thought contents. They are collateral psychological elements that have some other psychological association—systematic or contextual—with senses. Frege often assimilated colorings to images or feelings, not thought contents. Thus a feeling of disgust might be systematically associated with ‘cur’ but not ‘dog’, in an idiolect that otherwise uses the two words identically. Or an image of a rose might first accompany uses of ‘flower’. Then the association might lapse. But as Frege occasionally notes, colorings can consist in associated *thoughts* either systematically or contextually associated with the senses of linguistic expressions. Thus thoughts of contrast are colorings systematically associated with ‘but’, and not ‘and’. Thoughts of heroism might be contextually connected with ‘steed’, but not ‘horse’. Such colorings could lead an individual to judge that the senses of the expressions are different, when they are, let us suppose, the same.

Colorings can be colloquially termed ‘ways of thinking’. But they are collateral psychological elements. They are not modes of presentation that determine senses as denotations or *representata* in anything like the way senses are modes of presentation of their subject matters. Colorings are not even parts of literal, full understanding of senses. They bear essentially adventitious relations to them.<sup>54</sup> So they can be specified separately.

Frege never took sense to type-identify every psychologically significant state associated with language, or even every significant aspect of thinking associated with language. In fact, he emphasizes the psychological significance of words in ordinary language—coloring—that does not line up with sense. He stresses that the ‘contents of consciousness’ are not an immediate guide either to the structure of complex senses or to individuation of noncomplex senses.<sup>55</sup>

<sup>53</sup> Frege usually writes of colorings as ideas—tokens in individuals’ minds. But one can, as I will, think of colorings as psychological types—hence as shareable, as most senses are.

<sup>54</sup> I believe that other candidates for playing the role of “second layer” modes of presentation are subject to similar points.

<sup>55</sup> Frege, “Über Sinn und Bedeutung,” in *Kleine Schriften*, pp. 145–46 [29]; “On *Sinn* and *Bedeutung*,” in *The Frege Reader*, p. 154: “[An idea] is often drenched [*getränkt*] with feelings; the clarity of [an idea’s] individual parts varies and fluctuates. The same idea is not connected with the same sense, even in the same person.”; “Über Begriff und

One cannot infallibly isolate senses or thought contents just by introspecting what is present to consciousness.<sup>56</sup> Senses and thought contents type-individuate psychological competencies, which in turn can be delineated only by reflecting on patterns of use and by applying theoretical concepts of sense and thought content. These concepts center on *cognitive aspects* of psychological kinds—those that are relevant to setting truth conditions, judging a thought true, finding routes to and determining denotations, and making formal inferences.<sup>57</sup>

Frege made a bold theoretical postulation. He postulated that thought, conceived in his regimented way, is a psychological kind that grounds powerful explanations. He thought that one could find explanatorily important psychological kinds by reference to *cognitive aspects* of psychological kinds. He thought that normative issues regarding truth conditions, judgment, knowledge, and formally valid

Gegenstand," in *Kleine Schriften*, p. 170 [196]; "On Concept and Object," in *The Frege Reader*, p. 184; "Logik" (1897), in *Nachgelassene Schriften*, pp. 150–55; "Logic" (1897), in *Posthumous Writings*, pp. 138–43; *The Frege Reader*, pp. 239–44; "Einleitung in die Logik" (1906), in *Nachgelassene Schriften*, p. 209; "Introduction to Logic," in *Posthumous Writings*, pp. 192–93; *The Frege Reader*, p. 295; "Kurze Übersicht meiner logische Lehren" (1906), in *Nachgelassene Schriften*, p. 213; "A Brief Survey of My Logical Doctrines," in *Posthumous Writings*, p. 197; *The Frege Reader*, p. 300; Frege to Husserl 10/30–11/1/1906, *Wissenschaftlicher Briefwechsel*, p. 102; *Philosophical and Mathematical Correspondence*, p. 67; "Der Gedanke," in *Kleine Schriften*, pp. 347–48 [63–64]; *The Frege Reader*, pp. 330–31. In all these passages except the first, Frege links coloring with the specific linguistic expression used, and with the problem of formalization.

<sup>56</sup> This remark, and reflection on what senses and thought contents type-individuate, show that requirements that senses and thought content be "transparent" to the mind through introspection need careful qualification. It has been said that anyone who grasps a sense must be able to differentiate it accurately from any other sense. Assuming that one brackets representational contents of occurrence-based demonstrative applications (as not being senses), such a requirement is plausible for ideal, full understanding. It is unacceptable for minimal understanding, the sort sufficient to think with a sense. One can have the self-knowledge and the control over one's reasoning that can be reasonably expected, while still allowing for vulnerabilities to error regarding senses and thought contents as objects of individuation, and in special cases of environmental or contextual shift. See my "Individualism and Self-knowledge," this JOURNAL, LXXXV, 11 (November 1988): 649–63; "Memory and Self-Knowledge," in Peter Ludlow and Norah Martin, eds., *Externalism and Self-Knowledge* (Stanford: CSLI, 1998); both reprinted in *Foundations of Mind*. For a different but largely congenial view, see also Mikkel Gerken, "Conceptual Equivocation and Warrant by Reasoning," *Australasian Journal of Philosophy*, LXXXIX, 3 (September 2011): 381–400.

<sup>57</sup> It has been suggested, contrary to Frege, that different proper names naming the same thing always differ merely in coloring, not in sense. I think that this view is psychologically very implausible. With Frege, I have been ignoring it. Different names correspond to different procedural and epistemic routes to a referent. They have different determination conditions (systematically different causal relations to a referent). They enter into different cognitively or scientifically relevant inferences. Some of their psychological differences are clearly cognitive.

inference could be used to help isolate significant psychological kinds. His anti-psychologism can be fruitfully regarded as a protest against understanding not only logic but the psychology of thought in terms that disregard what we know about truth conditions and competence for making valid inferences—regardless of coloring.

Frege's postulation has been empirically vindicated. Powerful explanations in semantics, psycho-linguistics, and cognitive psychology are grounded in psychological kinds that are isolated by his conception of sense and thought content.<sup>58</sup> Modern notions of meaning have made use of some of the key guidelines that he used in applying his own concept *sense*.

I think that “ways of thinking” thought contents involved in incomplete understanding can always be assimilated to collateral, separately specifiable psychological states—conceptual or nonconceptual, cognitive or not—that accompany the relevant thoughts. Incomplete understanding can always be accounted for as failure to accept thoughts—either thoughts about relations between expressions and senses, or first-order thoughts that bear on constitutive explications. Frege's normal form for specifying thoughts need not be altered to account for incomplete understanding.

### III

I turn, from here on, to constructive remarks about incomplete understanding. They further support the point of the preceding paragraph. There are many kinds of incomplete understanding, just as there are many kinds of understanding. There are two generic types that share some instances. One—which I call ‘*linguistic incomplete understanding*’—consists in misapprehension of the relation between an expression and its sense, or inability to explicate an expression's sense. This type may or may not accompany incomplete understanding of the *sense*—that is, of the thought content—*itself*. The other type is incomplete understanding of the sense or thought content *itself*. It does not consist in misapprehension of the relation between an expression and the expression's sense. Call this type ‘*conceptual incomplete understanding*’.

*Linguistic incomplete understanding* divides into *two species*. In one, the incompleteness can be ameliorated by making conscious what is already fully formed and available in the individual's psychology. In the other, incomplete understanding cannot be improved in that way.

<sup>58</sup> As nearly arbitrarily chosen examples, see Steven Pinker, *The Language Instinct: The New Science of Language and Mind* (London, UK: Penguin, 1994); Lance J. Rips, *The Psychology of Proof: Deductive Reasoning in Human Thinking* (Cambridge: MIT, 1994); Gennaro Chierchia and Sally McConnell-Ginet, *Meaning and Grammar: An Introduction to Semantics*, 2<sup>nd</sup> ed. (Cambridge: MIT, 2000).

In cases of this *second species* of linguistic incomplete understanding, there is something deficient in the individual's understanding, at all psychological levels, of the meaning of his own linguistic expression. The deficiency lies in not having a veridical, or otherwise fully adequate, general explication of the cases that the meaning in fact applies to. The defect may not be correctable by reflection alone. Since this species often occurs when conceptual incomplete understanding occurs, I do not discuss it separately.

The *first species* of linguistic incomplete understanding divides into several partly overlapping sub-cases. One is failure to distinguish an expression's sense from its coloring. An individual may mistakenly insist that 'and' and 'but' have different senses in the individual's idiolect. The sense type-individuates a different psychological kind from the coloring. The individual needs theory and reflection to draw relevant distinctions. Some differences in coloring center in associated feelings. Others may reside in differing entailments that mark linguistic meanings but not senses. These latter colorings—close to Gricean conventional implicatures—are contents of collateral thoughts, but not cognitive aspects of the use of the relevant word, in the sense indicated above.

Another sub-case is not recognizing logical forms that reflection on usage would reveal. An individual may mistakenly deny that two sentences have the same sense because structural elements in one are not evident in its surface grammar. Not recognizing the putative structure of 'hit' sentences, or of interrogative and declarative forms, may illustrate this sub-case.

A further sub-case involves failure to recognize sense synonymies in certain definitions. An individual may forget a stipulative abbreviation and mistakenly deny sense synonymy purely because of the expressions' surface structural differences, or differences in coloring between *definiendum* and *definiens* that accrued since the abbreviation was established. On some classical philosophical views, abbreviative definitions model definitions in general. I shall argue, shortly, that this model has been overused.

A fourth sub-case involves failure to accept a definition or other constitutive explication, where the *definiens* concept is unconsciously fully formed and guides the *definiendum* concept in the unconscious psychology, and where expressions for *definiens* and *definiendum* differ in sense. An individual may evince this type of incomplete understanding through erroneous stabs at a definition, but may eventually bring to consciousness the *definiens* concept via reflection.

In all these cases, linguistic incomplete understanding consists in an individual's conscious judgments' failing to accord with actual

competence. In a sense, these are cases in which performance (conscious judgment that involves incomplete understanding) does not match competence (the understanding that resides in unselfconscious competence, which itself is not incomplete). The nature of linguistic competencies is often not evident to immediate consciousness. Incomplete understanding in these cases is naturally expressed in meta-linguistic form: the individual mistakenly believes that two expressions differ in sense; but the underlying psychology shows a single sense, just differences in coloring.

*Conceptual incomplete understanding* consists either (a) in the lack, anywhere in the individual's psychology, of a fully formed, adequate conceptual explication, where having such an explication is a relevant kind of understanding, or (b) in the lack of an understanding of the concept's boundaries, vague or otherwise.<sup>59</sup> The individual has the minimal competence-understanding to think with a concept (and sense). But the individual either lacks an adequate explicative conception or lacks an understanding of the scope and limits of the concept's domain of application. Here incompleteness does not consist merely in failing to match sense with expression. Incomplete understanding is of the concept, or sense, itself. There is a deficiency in conceptual understanding. It is not simply a performance problem in connecting expressions to an adequate underlying understanding.

In the sciences—outside semantics—and perhaps in ordinary life, this type of incomplete understanding is the most common and interesting. In nearly all interesting explications, definitions, and “analyses” that remedy incomplete understanding in those sciences that are not mainly concerned with language, the incomplete understanding is not of expressions that express pure identity thoughts, or pure identity inferences. The incomplete understanding does not consist in mistaken meta-linguistic belief. Nor can the incomplete understanding be remedied just by making conscious a fully formed thought content that explicates a sense or thought content.

<sup>59</sup> The notion of a *fully formed* conception must be understood carefully. In cases where an individual has the component concepts of a conception, or explication, or definition, there is always a sense in which the individual has the conception—because the individual has the competence to put together component concepts in any logically or grammatically admissible way. In this sense, the individual trivially “has the conception” as part of his psychological competence. But in my sense, the individual lacks a fully formed conception. For a conception to be fully formed, it must have an actual psychological function, as a unit, for the individual, or the individual's unconscious. To be “implicit,” the conception must meet this condition. And it must be connected in the individual's psychology (perhaps unconsciously) with the concept or expression that the conception provides an understanding of, so that the conception guides that concept's, or that expression's, applications.

Conceptual incomplete understanding consists in failure to accept, even unconsciously, certain informative sound inferences, or certain informative, true identity thoughts. Such inferences or identity thoughts are those central to explicating thought contents or senses.

I discuss four examples of incomplete understanding, reflecting on their relations to sense identity tests and the psychology of thought. Each exemplifies both *conceptual* incomplete understanding and second-species linguistic incomplete understanding.

First, suppose that an individual cannot—without trial and error—think out a definitional explication of his or her chair concept (or term ‘chair’).<sup>60</sup> The individual *could* have a fully formed, fully adequate, unconscious *definiens* conception of chairs. Bringing it to consciousness would then be purely a performance problem. Then we have first-species, fourth-case linguistic incomplete understanding. It is more likely that, even if the individual has the component concepts in an adequate, complex explicational *definiens* concept,<sup>61</sup> they have not been composed into that concept—even unconsciously. Often in finding definitions of familiar words, individuals put together a general explicational concept that was not originally fully formed. The sense and meaning of the *definiendum* expression and the nature of the concept are not precisely the same as those of the *definiens* expression or the *definiens* concept. Antecedent uses of the *definiendum* concept are guided by very generic concepts, perceptual templates, and senses of similarity that were not unified into a complex explicational concept.

In such cases, the competence typed in the *definiens* concept is different from the competence typed by the *definiendum* concept—for example, the concept chair. The chair concept is used before the complex explicational *definiens* concept is used. Applications of ‘chair’, and of the concept *chair* that is its sense, need not rely on a general *definiens* concept. It is often fairly clear that the chair concept and the complex *definiens* concept type-individuate different psychological states and competencies. Reflection may form a concept that explains and unifies applications of the chair concept. The explicating (*definiens*) concept provides a constitutive understanding of the chair concept. Incomplete understanding is remedied by coming

<sup>60</sup> I discuss this case from a different angle in “Intellectual Norms and Foundations of Mind,” this JOURNAL, LXXXIII, 12 (December 1986): 697–720; reprinted in *Foundations of Mind*.

<sup>61</sup> One can have a chair concept without having certain concepts, like furniture, function, intention, normal, that are probably in a good *definiens* conception. This point in itself would suffice to show that such a chair concept (the ‘chair’ sense) is not identical with the corresponding complex *definiens* concept/sense.

to believe the definitional explication. The definitional explication is not a pure identity thought, and the explicating *definiens* concept is not identical with the chair concept.

Second, an individual might have incomplete understanding of arthritis, and of ‘arthritis’, that is not remediable by the individual’s reflection. The individual believes arthritis is arthritis but does not believe—and may even doubt—arthritis is rheumatoid inflammation of the joints.<sup>62</sup> There may be nothing in the individual’s inferential patterns that calls for taking ‘arthritis’ to express inflammation of the joints, and nothing in his or her knowledge base that allows thinking of anything as rheumatoid. Thought contents help type-individuate psychological competencies. Attributing the more complex *definiens* sense to ‘arthritis’ to account for the individual’s thinking may have no psychological basis. The two thoughts contents are different.<sup>63</sup> The individual’s understanding of his concept (and sense) is incomplete because rheumatoid inflammation of the joints is central to explicating arthritis and unifying and justifying the individual’s applications—including those that rely on others. The individual lacks an adequate conceptual explication. As with the ‘chair’ example, the definition does not involve a sense identity. In contrast to the chair example, the individual cannot correct the incomplete understanding purely by reflection.

A third example illustrates incomplete understanding that is unremediable by mere reflection, even by a community. Suppose that a community believes this explication of the concept tomato: the tomato is the most common red vegetable that normally has such and such an appearance. The explication betrays incomplete understanding. Until the community learns that tomatoes are fruits, the error of thinking that tomatoes are vegetables cannot be remedied. After the community corrects its error, the community continues to think of tomatoes as tomatoes, and continues to believe tomatoes are tomatoes. But it gives up its former explication. Nothing supports the view that thinking of tomatoes as fruits guided thoughts containing the concept tomato during the period of incomplete understanding. So taking the new explication (the most common red fruit

<sup>62</sup> I discuss this case in “Individualism and the Mental,” *Midwest Studies in Philosophy*, iv, 1 (September 1979): 73–121; reprinted with Postscript in *Foundations of Mind*.

<sup>63</sup> Whether the individual has the same arthritis concept that more knowledgeable members of the community do is not important for the case—or for my use of the case to argue for anti-individualism. See Burge, “Wherein Is Language Social?” In fact, I think that individuals can and often do share incompletely understood concepts with more knowledgeable communal members. Contents are often preserved in reliance on others. I will not develop the point here.

that normally has such and such an appearance) to be sense-identical with the sense of 'tomato' and to be a correct articulation of the internal structure of the concept tomato is incompatible with our knowledge of the individuals' psychologies.<sup>64</sup>

In these three cases, accounting for incomplete understanding—and its remedy—does not require introducing modes of presentation of senses in a new normal form for specifying thoughts. The minimal understanding needed to think with the concept or sense is accompanied by conceptual incomplete understanding. The incomplete understanding consists in lack of a good, object-level (not meta-linguistic) explication anywhere in the psychology.

Frege discusses incomplete understanding mainly in connection with technical theorizing, especially in mathematics. Here the matter is more complex, but a similar picture emerges. A fourth example concerns Newton and Leibniz's incomplete understanding of concepts in the calculus.<sup>65</sup> Each man favored a different concept of limit, and of the derivative. Newton favored concepts that center on approaching a limit value. Call Newton's *approaching-the-limit limit concept* ' $L_N$ '. Call Newton's *approaching-the-limit derivative concept* ' $D_N$ '. Leibniz favored concepts  $L_L$  and  $D_L$  that invoke infinitesimals. I believe that each mathematician had a further ur-concept of limit and an ur-concept of the derivative that are less specific than  $L_N$ ,  $D_N$ ,  $L_L$ , and  $D_L$ . Their respective approaching-the-limit concepts and infinitesimal concepts are elaborations of such ur-concepts. The elaborations involve more specific conceptions, and they presage

<sup>64</sup> Hilary Putnam, "Is Semantics Possible?" (1970) and "Explanation and Reference" (1973), both reprinted in his *Mind, Language and Reality: Philosophical Papers, Volume 2* (Cambridge, UK: University Press, 1975), made a deep contribution to philosophical understanding of explications of common nouns in everyday life and in scientific theory. He claims in "Is Semantics Possible?" p. 148, that the meaning of such nouns change if their stereotypical explications change. This may be so for one notion of linguistic meaning. I think that it is not so for sense. And for at least one notion of linguistic meaning (what I have called 'translational meaning'), I think that it is not so. The community's translational meaning of 'tomato' and concept tomato would not change after such a change in dictionary explication. For discussion of translational meaning, see Burge, "Wherein Is Language Social?"; and "Concepts, Definitions, and Meaning," *Metaphilosophy*, xxiv, 4 (October 1993): 309–25; reprinted in *Foundations of Mind*.

<sup>65</sup> I have discussed this case in "Concepts, Conceptions, Reflective Understanding: Reply to Peacocke," in Martin Hahn and Bjørn T. Ramberg, eds., *Reflections and Replies: Essays on the Philosophy of Tyler Burge* (Cambridge: MIT, 2003), pp. 383–96, at pp. 385–86; and "Postscript to 'Individualism and the Mental,'" in *Foundations of Mind*, pp. 170–72. Newton had infinitesimal-oriented limit and derivative concepts as well as the approaching-a-limit-oriented limit and derivative concepts. There is evidence that Newton may have had other orientations as well. He came to favor his approaching-a-limit concepts. I will follow custom in focusing on his favored versions, and in associating the infinitesimal versions with Leibniz.

specific (different) definitional explications. Such ur-concepts agree with all elaborations on the central applications, but they are unspecific enough to allow different elaborations. Indeed, the ur-concepts commonly have less sharp extensions than their elaborations.  $L_N$  and  $D_N$  eventually received clarifying explication in Weierstrass's definition.  $L_L$  and  $D_L$  later received clarifying explication in Robinson's definition.

I focus on Newton and Weierstrass. Newton's incomplete understanding of  $D_N$  showed in failed, even incoherent, explications of his concept.<sup>66</sup> For example, he could not explain why use of  $D_N$  does not lead to division by 0 in certain equations. Weierstrass's definition explicates Newton's concepts  $L_N$  and  $D_N$ , and justifies Newton's applications of them. Let us suppose, for the moment, that  $D_N$  and Weierstrass's *definiens* concept  $D_{W-d}$  are coextensive.  $D_{W-d}$  provides a generalized explication and justification of the derivative and of approaching limits.

Is the first-order form of Weierstrass's definitional explication a pure identity thought? Is  $D_N$  identical with  $D_{W-d}$ ? I think not.  $D_N$  is an element in Newton's thought. It helps mark a kind of psychological state of Newton's. Newton had the component concepts in  $D_{W-d}$ .<sup>67</sup>  $D_{W-d}$  yields a *mathematical* explanation and justification of Newton's applications of  $D_N$ . But these two points are not enough for  $D_{W-d}$  to be  $D_N$ . It is not enough that Newton *could have* done the reflective mathematical work that Weierstrass did, without learning new concepts. It is not enough that  $D_{W-d}$  *mathematically* unifies and justifies Newton's usage

<sup>66</sup> Newton had an explication of derivative in terms of approaching a limit that is, at least in modern understandings, correct: the derivative of a function  $f(x)$  is the limit as  $i$  goes to 0 of  $(f(x + i) - f(x)) \div i$ . This explication shows something about the nature of Newton's concepts of both derivative and limit. But it was, and was known by Newton to be, an unsatisfying explication. It was unsatisfying because the notion of approaching a limit was notoriously disputed and not well understood—in need of mathematical explication—and because it failed to provide the clarity that would either justify practice or provide a basis for proofs. Satisfying, acceptable mathematical explications must use terms that are at least provisionally sources of illumination, and must be useable in proof. Explications of the concept limit in Newton's day did not meet this standard. Our current use of this explication is backed by a deeper understanding of a limit concept—an understanding provided by Weierstrass's explication.

<sup>67</sup> As noted in Burge, "Postscript to 'Individualism and the Mental'," Leibniz had a concept of infinitesimal that was later explicated by Robinson. But Leibniz lacked some component concepts in the explication. One might think of this case as a fifth example—one in which the community and greatest experts have a concept, but cannot explicate it adequately through mobilizing concepts that they already have. This case is like the Newton case in that the explication was not present in the user's psychology, at *any* level. It differs from the Newton case in that even some *component* concepts in the relevant explication are not present in the user's psychology. Leibniz could not have composed the Robinson explication by using concepts that he already had, either consciously or unconsciously.

of  $D_N$ . For  $D_{W-d}$  to be  $D_N$ ,  $D_{W-d}$  must type the same *psychological* competencies and ground the same psychological explanations that  $D_N$  does.  $D_{W-d}$  must be fully formed as a unified, used conception in Newton's unconscious psychology, *and* it must be the content of all Newton's thinking expressed with 'derivative'. In scientific explications like Weierstrass's, the sense of the *explicans* almost never does the psychological work that the sense of the *explicandum* did before the explication's discovery.

That Newton had a consciously unarticulated but unconsciously ("implicitly") complete competence with  $D_{W-d}$  is not plausible. Distinguishing  $D_N$  and  $D_{W-d}$  is demanded by recognizing the roles of sense and conceptual content in type-individuating psychological states.

The psychology is often obscured by a classical view of apriori reflection.<sup>68</sup> This view holds that in reflection (the sort that Weierstrass did and that Newton could have done), the individual discovers structure in his concepts, previously hidden below the level of full consciousness.<sup>69</sup> It is not plausible, however, that all Newton's uses of the derivative

<sup>68</sup> This classical conception derives from a particular reading of Plato's *Meno*. It appears in early modern rationalism and in Kant. As applied to definitions and explications in empirical science, the conception is even less plausible than in the mathematical case.

<sup>69</sup> A modern philosopher who relies on this conception of reflection is Christopher Peacocke, "Implicit Conceptions, Understanding, and Rationality," in *Reflections and Replies: Essays on the Philosophy of Tyler Burge*, pp. 117–52; also in Enrique Villaneuva, ed., *Philosophical Issues 9: Concepts* (Atascadero, CA: Ridgeview, 1998). I have criticized this view for misrepresenting the psychology underlying concept use in the calculus case. See my "Concepts, Conceptions, Reflective Understanding: Reply to Peacocke," in *Reflections and Replies* (see also note 64). Peacocke expounds substantially the same view, without directly addressing the criticism, in *Truly Understood* (New York: Oxford, 2008), chapter 4. Most of the exposition there is committed to holding that thinkers have tacit *knowledge* of a definition or other generalized explication that is intensionally equivalent with terms or concepts they use. (See, for example, page 133.) Peacocke qualifies this view at one place. There he allows that 'the content of an implicit conception could equally be grounded in the operation of a processor which does not involve, at the subpersonal level, explicit representation of the content of the implicit conception' (page 142). I believe that it is a mistake to take such a case as a case of implicit conception, or a case involving a kind of representation 'operative in' (142) a given thinker. Moreover, if there is no representation of the content at the sub-personal level, there is no tacit knowledge. The case in which a representational content is in the psychology of the thinker (implicitly, tacitly, operatively) and the case in which the content is merely in the laws or principles that govern the operation of the thinker's contentful psychological states are fundamentally different, and cannot be reasonably assimilated to different types of implicit conception, much less tacit knowledge. In the latter case, the content is not a conception by the thinker, or in the thinker, in any sense. It is a conception used in explaining and justifying laws or transitions that occur among the thinker's actual psychological states—whether these are implicit, tacit, sub-personal, unconscious, or not. For discussion of the distinction between implicit representation and contents in the rules or laws governing the operations in a psychological system, see Burge, *Origins of Objectivity*, pp. 403–07, 488–90.

symbol that are on the track toward Weierstrass consisted in unconscious uses of the complex *definiens* concept that Weierstrass assembled. Newton's reasoning can be psychologically accounted for by lower-level rules of thumb, such as apply the same algebraic rules in differentiation as one would with the rational numbers, the derivative of  $x^2$  is  $2x$ , and the derivative of  $x^2$  is the limit as  $i \rightarrow 0$  of  $((x+i)^2 - x^2) \div i$ . I know of no evidence that Newton unconsciously applied Weierstrass's complex concept with stacked quantifiers wherever he used his concept of the derivative, or even that his uses were psychological shorthand for that complex concept. The contrast with the 'Cain hit Abel' case is palpable. The concept that Weierstrass discovered as a mathematical unification and explanation of previous applications of  $D_N$  was very probably never even an *unconscious* unit in Newton's psychology.

Mathematical justification and unification are not to be confused with reflective psychological discovery. Apriori reflection is not, in most theoretically interesting cases, assimilable to psychological discoveries of structures already assembled in the unconscious. Mathematical explicative definitions tend to be synthetic apriori, not pure identity thoughts revealed through bringing to consciousness antecedent structures within the explicated concepts.

Is Weierstrass's derivative concept  $D_W$  identical with  $D_{W-d}$ ? I will assume, for the sake of keeping exposition simple, that at least before his discovery, Weierstrass's derivative concept was Newton's. Let us call his derivative concept after the discovery ' $D_W$ '. I will discuss the question that I posed in terms of hypothetical possibilities—mainly to bring out considerations that bear on answering it. After the discovery, Weierstrass may have acquired a new concept of the derivative, one that is on the same track as Newton's, but that is strictly speaking different. Let us assume that his explication  $D_{W-d}$  and his post-discovery derivative concept  $D_W$  are at least coextensive. Weierstrass's acquisition of a concept  $D_W$  different from Newton's  $D_N$  might have derived from his utilizing  $D_{W-d}$  to settle special issues (differentiate functions) that Newton's concept might not have been fitted for. Given that  $D_W$  is coextensive with  $D_{W-d}$ , it could not then be coextensive with  $D_N$ . So  $D_N$  and  $D_W$  would be different concepts.

Hypothetical cases in which  $D_{W-d}$  settles issues that Newton's concept  $D_N$  might not have settled fall into at least two categories. A case (for example, whether a given function is differentiable) might be definitely beyond the domain of application of Newton's concept. (Certain functions might definitely not be in that concept's extension, whereas Weierstrass's definition gives them values.) Then I will say that Weierstrass's concept  $D_W$  is a *broadening* of Newton's concept  $D_N$ . It differentiates all functions that Newton's concept clearly applies

to and adds further ones. Alternatively, a case settled by  $D_{W-d}$  might be indeterminate as to whether it is in the domain of application of Newton's concept. Then I will say that Weierstrass's concept  $D_W$  is a *sharpening* of Newton's  $D_N$ . Any such broadening or sharpening of  $D_N$  is the result of  $D_W$ 's being coextensive with  $D_{W-d}$ . If Weierstrass's own post-discovery concept of the derivative  $D_W$  is coextensive with  $D_{W-d}$  and Newton's concept  $D_N$  is not coextensive with  $D_{W-d}$ , then  $D_W$  is not  $D_N$ .  $D_N$  may be broadened or sharpened (or both) as well as explicated and justified by  $D_{W-d}$ .  $D_W$  is explicated and justified by  $D_{W-d}$  but not broadened or sharpened by it.

I take it to be an empirical, historical question whether  $D_W$  is identical with  $D_N$ . If  $D_W$  is a broadening or a sharpening of  $D_N$ , they are not identical. There may be other reasons why they are not identical, even assuming that  $D_{W-d}$  explicates both  $D_N$  and  $D_W$ . I am not committed on whether  $D_N$  and  $D_W$  are identical. I know of no decisive reasons on either side. I take it that showing them different requires showing a difference in the constitutive minimal competence displayed in applications or inferences in which they were used, other than those applications or inferences that use explication  $D_{W-d}$  itself.<sup>70</sup> I think it quite possible that  $D_N$  and  $D_W$  are identical. My main point is that they need not be identical, even though  $D_{W-d}$  explicates both. It explicates both because it provides clarifying understanding and a mathematically acceptable justification for their core uses. It could explicate a concept either while also providing a broadening or sharpening of it or while providing a mathematically equivalent *definiens*.

Let us suppose, for the sake of exposition, that  $D_N$  and  $D_W$  are not identical. That leaves it open whether  $D_W$  is identical with  $D_{W-d}$ . Of course, in his mathematical discovery, Weierstrass, unlike Newton, did put together his explicative concept  $D_{W-d}$  and used it as *a priori* coextensive with his concept  $D_W$ . On this basis, one might think that  $D_W$  is identical with  $D_{W-d}$ .

<sup>70</sup> I say 'other than those applications or inferences that use explication  $D_{W-d}$  itself' because I think that explications are responsible for unifying and justifying *antecedent* usage, in applications and inferences (with allowances for broadening and sharpening). Of course, having another person's concept, thinking with it, does not require making all the other person's applications or inferences. There is some minimum usage, comprised of applications and inferences, that constitutes minimum competence with the concept. This minimum might be any of various subsets from a family of uses.

Once one achieves a correct definition, and once one has acknowledged an explication as a definition, as Weierstrass did, there is a commitment to make all usage accord with the definition, at least other things equal. Such a commitment is associated with uses of  $D_W$ , but is not associated with any of Newton's uses. I take it that, in formal work, such a commitment is regarded as a normative aspect of the use of the concept. But I doubt that such a commitment is psychologically constitutive to having the concept.

Even after successful definitional (equivalent) explications, however, the form of the first-order explication is rarely that of a pure identity thought. I believe that  $D_W$  is not identical with  $D_{W-d}$ . There are two complementary reasons.

One depends on the epistemic status of definitional explications. Such explications are first-order, non-meta-linguistic beliefs. That is, they do not have a form like ' $D_W$  expresses the same concept as  $D_{W-d}$ '. They are also not epistemically trivial. Weierstrass's discovery was not primarily a discovery about language. And the explicational equivalence was in principle open to scrutiny and criticism, even after it was discovered, and even after it was taken to be definitional. Pure identity thoughts are not open to such scrutiny or criticism. Counterexamples to the explicational definition can take the form: function  $f$  is differentiable and is within the natural domain in which the *explicandum* concept is applicable, but is given no derivative by the *explicans* concept. Weierstrass's explication is a remarkable mathematical claim, not a piece of trivial logic or a piece of linguistic analysis. The explication is not a pure identity thought.

A second reason why  $D_W$  is not the same concept as  $D_{W-d}$  is that, psychologically, the two are differently grounded. Whether or not  $D_W$  is identical with  $D_N$ ,  $D_W$  is associated with the tradition of rule-of-thumb problem solving that predated the explication. These uses realize a competence with  $D_W$  that is psychologically independent of the *definiens* concept.<sup>71</sup> (See note 69.) That independence grounds epistemic scrutinizing of the explication, just discussed.<sup>72</sup> The explication unifies and justifies an antecedent usage.

<sup>71</sup> Although Frege's discussions of explicative definitions and stipulative definitions are mostly deep and insightful, his citing the definition of 'integral' as a case of a simple word's having a complex sense is, I think, mistaken. He seems to take the sense of 'integral' to be that of its complex definition. Cf. "Logik in der Mathematik," in *Nachgelassene Schriften*, p. 226; *Posthumous Writings*, p. 209; *The Frege Reader*, p. 315. Except as a stipulative definition, the non-meta-linguistic form of the standard definition of 'integral' is not the form of a pure identity thought. (It must be remembered that if there is a genuinely stipulative or abbreviative definition in an individual's idiolect, the *definiens* must express a sense that type-individuates the *same* cognitively relevant psychological state that the sense of the *definiendum* does, since the senses are the same: they are the same thought contents. One can use the abbreviating expression as a kind of anaphoric shorthand in thought. But supposing a sense identity, the idea that the individual thinker never thought as a unit the complex sense that the abbreviating expression allegedly abbreviates would be incoherent.) As indicated in the text, I do not say that pure identity thoughts are never the product of informative definitions. Informativeness can involve revelation of unconscious structure in use of and competence with a word. Then the incomplete understanding is primarily linguistic. As a matter of fact, most informative and significant definitions are not of that sort.

<sup>72</sup> These points elaborate the important work by Quine on definition in general and by Putnam on scientific definitions. See W. V. Quine, "Two Dogmas of Empiricism," in

This picture is rendered more complicated by the issue of concept broadening and sharpening, noted some paragraphs back. In mathematics, there is a natural progression of finding different, often broader, domains of application for an ur-concept and its successors. Moreover, I think that many natural concepts, including many mathematical concepts, have fuzzy boundaries. That is, there are cases that the concept is neither true of nor false of. Or for concepts applying to functions, there are arguments for which the function provides no value. Concepts commonly apply within a given domain. When new hard cases arise, a concept can undergo elaboration that consists in development of a new concept that is either a broadening of a predecessor or is given a more precise extension (or both). In mathematics, such elaborations usually involve broadening domains of application—wider extensions.

Early concepts of number that applied only to the natural numbers were elaborated to include 0, negative numbers, reals, imaginary numbers, and so on. At each stage, there is a number concept, but number concepts are increasingly generalized, and perhaps also made more precise. All the elaboration concepts are on a track with an ur number concept. Each elaboration concept is developed to satisfy a need to apply a number concept in a domain that is suggested by previous numerical operations, but to which earlier number concepts were not clearly applicable.

The development of differentiation concepts manifests this phenomenon. In fact, ur-concepts for derivative and limit underwent several tracks of elaboration, of which those that initially led to Weierstrass and Robinson are just the most famous. Weierstrass's explication fails to differentiate functions that subsequent, broader explications of derivative concepts, in the Weierstrassian tradition, do differentiate.<sup>73</sup> The subsequent *definiens* concepts are not coextensive with  $D_{W-d}$ . They provide derivatives for functions that Weierstrass's definition does not. The derivative concepts that they define and are coextensive

*From a Logical Point of View* (Cambridge: Harvard, 1953), section 3; Putnam, "The Analytic and the Synthetic" (1962), "Is Semantics Possible?" (1970), and "Explanation and Reference" (1973), all reprinted in *Philosophical Papers, Volume 2*. Putnam's work reinforces Quine's point that there are many sorts of definitions and "analyses," and that theoretically successful ones in the sciences (including mathematics) are subject to revision—in ways that pure identity thoughts are not.

<sup>73</sup>I have benefitted especially here, but also elsewhere, from reading Sheldon Smith's "The Derivative and Incomplete Understanding" in draft, and from discussing these matters with him. His marvelous knowledge of post-Weierstrassian developments and of alternatives to Robinson's explication of Leibniz's infinitesimal concept was illuminating for me.

with are broadenings or sharpenings of  $D_W$ . Since the broader or sharper *definiens* concepts are not coextensive with  $D_W$ , they are not identical with it.  $D_W$  has Weierstrass's explication  $D_{W-d}$ . The more general explications explicate  $D_W$  in a different way. They justify it, provide a broadened elaboration of it, unify cases it applies to with further cases, provide insight into its limits, and show how to transcend those limits. The more general explications yield an understanding of  $D_W$  that is complementary to, but fuller than, the understanding provided by  $D_{W-d}$ .<sup>74</sup> Unifying, broadening, sharpening, and generalizing are standard functions of scientific explications, even those that provide definitions. A broader or sharper concept  $D_W$  is explicated, though not broadened or sharpened, through *its* coextensive *definiens* concept.

Although Weierstrass's definition is not a fully general explication of the approach to differentiation first conceived in  $D_N$ , it is a sound definition because it illuminatingly applies to a natural mathematical domain. There are wider (or different) domains that other *definiens* concepts, and other (perhaps broadened or sharpened) derivative concepts apply to.

Of course, in a formal mathematical system a *term* for a derivative can be *stipulated* to abbreviate the *definiens* expression. Then the object-level analog of the definition is a pure identity thought. Such stipulations are usually not explications of antecedent derivative concepts.

So Weierstrass's *definiens* concept  $D_{W-d}$  has several uses. It can explicate Newton's concept  $D_N$ . Such explication may or may not be a broadening or a sharpening. If it is, the *definiens* is not coextensive with the *explicandum*; and then the *definiens* defines and also explicates a sharpening derivative concept  $D_W$  that is (necessarily) coextensive with  $D_{W-d}$ , but not with  $D_N$ . If it is not, then  $D_N$  and  $D_W$  may be identical; and the *definiens* is coextensive with both and provides an explication of both, where the senses of the *definiens* expression and the *definiendum* expression differ. In either case, one can also use the *definiens*  $D_{W-d}$  in a stipulative abbreviative definition of 'derivative'. Then the definition correlates with a pure identity thought and does not explicate any concept, though it simplifies a formal mathematical system. In mathematical thinking, such abbreviative definitions are contextual, passing affairs—rarely, if ever, long-term revelations of the natures of explicated concepts.<sup>75</sup>

<sup>74</sup> All these points leave open whether  $D_W = D_N$ , or whether  $D_W$  is instead a sharpening of  $D_N$ .

<sup>75</sup> Quine, "Two Dogmas of Empiricism," in *From a Logical Point of View*, section 3; "Carnap and Logical Truth," in *The Ways of Paradox* (New York: Random House, 1966), section v.

The explicated concept(s) are *loci* for different cognitive associations. The explicated concept is expressed by a common noun backed by various conceptions. The *explicans* or *definiens* conception functions to unify and justify the other conceptions, and the applications of the *definiendum* concept. Whether it does so successfully is a substantive mathematical question. Theoretical concepts expressed by common nouns like 'derivative' are like concepts expressed by proper names in that, commonly, no descriptive contents (like  $D_{W-d}$ ) are sense-identical with them. In accord with Kripke's and Putnam's work, the psychology and cognitive function of common-noun-type concepts are almost never those of concepts that explicate them, define them, or give constitutive accounts of their applications. The common-noun-type concepts are open to broadening and sharpening.<sup>76</sup> Some broadenings or sharpenings yield fuller understanding of unbroadened, unsharpened concepts. A track-like continuity often connects elaborations with antecedent concepts. Such continuity and the fact that all the concepts cover clear, basic cases of differentiation justify counting certain noncoextensive concepts all concepts of the derivative.

Incomplete understanding takes many forms. It can be poor recognition of structure, remediable through formalization. It can be failure to distinguish different types of meaning or content. It can be meta-linguistic or object-level. It can be lack of a correct unifying, justifying explication. It can reside in use of a concept associated with procedures that do not suffice to settle relevant cases, so that the concept needs explication that not only unifies and justifies existing usage but broadens or sharpens such usage, so as to cover unforeseen cases. These latter two types of incomplete understanding are not meta-linguistic. They are types that involve failure to hold explicating beliefs that illuminate constitutive aspects of a concept. Correct explication beliefs are neither meta-linguistic beliefs nor pure identity beliefs. As with all conceptual understanding, they involve understanding connections and joints in the world.

Newton and his community incompletely understood his ur-concept of the derivative and his approaching-the-limit concept of the derivative,  $D_N$ . Weierstrass's explication definition yielded fuller understanding. Newton was perhaps able to produce rules of thumb that fully

<sup>76</sup> Jim Pryor pointed out that some proper-name concepts may be open to broadening or sharpening. The issue arises, for example, in determining how to think about changing city limits that are not merely a matter of growth or contraction, say, for Berlin. The fundamental distinction between ordinary common nouns (like 'derivative' and 'sofa') and proper names is that the primary representational function of names is to single out a bearer, whereas the primary representational function of ordinary common nouns is to attribute attributes (including relations and kinds).

account for what guided his conceptual and linguistic usage: they constitute the relevant linguistic rules in his idiolect and the conceptions in his psychology that guided his applications of his concept. They might codify his linguistic and conceptual competence in that respect. What was incomplete about his understanding was not (merely) a codification of the linguistic rules governing his usage, or a codification that captured the conceptions that guided his concepts. Incompleteness of understanding lay in his lack of a *correct* general explication that covered and justified the cases that the meaning and concept in fact applied to.

In both the derivative and tomato cases, neither the individual's nor the community's guiding rules of thumb—even the true ones—amount to full understanding of the meaning or concept. Here we see two aspects of conceptual and linguistic understanding. The first aspect concerns codifying rules that guide linguistic usage, and constitute the general conceptions that guide conceptual applications. In the tomato and derivative cases, there was no incompleteness in this aspect of understanding. In those cases, incompleteness lies in the second aspect—the one that concerns not merely codifying usage and the guiding conceptions, but providing a *correct* unifying, potentially guiding, justifying, general explication that *gets right* the nature of the instances to which the concept and the meaning apply.

In the derivative and tomato cases, the senses (the concepts) do not differ from the meanings. But what it is to fully understand the senses (or meanings) is different, depending on whether one focuses on (a) codifying usage or competence—providing explications that are in principle available to the users that best match the general, guiding, rule-like aspects of their competence, or (b) producing a correct, ideally general, potentially guiding, unifying, justifying explication of the senses (or meanings)—one that may invoke knowledge not available to the users, but that is potentially useful and that correctly captures the senses' extensions. Linguistics and modern theories of linguistic meaning are oriented to (a). Scientific explications—and, as the tomato case illustrates, even ordinary nonscientific explications that are influenced by new knowledge—that are concerned with the subject matter of the concepts (senses, meanings) are oriented to (b). Frege's concern with sense, as a component in knowledge and cognition in thought, encouraged (b). It provides a valuable conception of understanding that supplements the tendency in modern theories of meaning to concentrate on codifying and matching actual linguistic explicational or rule-based competence.

Frege's concept *sense*, as applied in the philosophies of language, logic, and psychology, continues to repay use and reflection, despite

a century of censure and misunderstanding. These wages have earned *Sinning* a long and productive life.

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#### APPENDIX

I have criticized Kripke's interpretation of passage (A). I want to make three further points. I begin by re-citing passage (B) together with material just preceding it:

(B') ...the mere wording, which can be made permanent by writing or the gramophone, does not suffice for the expression of the thought. The *present tense* is used in two ways: first, in order to make a time-indication....If a time indication should be made with the present tense, one must know when the sentence was uttered in order to grasp the thought correctly. Therefore the time of utterance is then part of the expression of the thought.<sup>77</sup>

First, Kripke holds that, in an utterance of 'Today is Thursday', the linguistic *utterance* consists of the just-quoted words together with the time *t* when they are uttered:  $\langle W, t \rangle$ . *W* is supposed to denote a function from times to truth values. Time *t* is supposed to denote itself.

Later in "Thought," regarding present tense, Frege writes:

(D) To be sure the same verbal expression (*Wortlaut*) can take on another sense on account of the changeability of language with time; but the change then concerns (*betrifft*) [only] the linguistic (*das Sprachliche*).<sup>78</sup>

As quoted earlier, in the 1897 draft "Logic," an ancestor to "Thought," Frege writes,

(E) Words like 'here' and 'now' only acquire their full sense always only through the circumstances in which they are used.<sup>79</sup>

(D) can be interpreted in various ways. But it can be naturally read as indicating that linguistic elements change senses with time. (E) states very clearly that particular indexical words take on full senses only through contextual circumstances.

<sup>77</sup> (B') is directly followed by (A). Both passages come from Frege, "Der Gedanke," in *Kleine Schriften*, pp. 348–49 [64]; "Thought," in *The Frege Reader*, p. 332.

<sup>78</sup> Frege, "Der Gedanke," in *Kleine Schriften*, p. 361 [76]; "Thought," in *The Frege Reader*, p. 344.

<sup>79</sup> Frege, "Logik" (1897), in *Nachgelassene Schriften*, p. 146; "Logic" (1897), in *Posthumous Writings*, p. 135; "Logic" (1897), in *The Frege Reader*, p. 235.

Kripke's proposal is not in accord with this latter point. On his proposal, no expressions, in the relevant cases, ever change senses, as a matter of their ordinary uses. Take the ordered pair  $\langle W, t \rangle$ , where  $W$  comprises the 'mere wording' as it can be written down. Suppose that the time is an additional 'unrecognized linguistic expression'. If times are part of the linguistic expression, denoting themselves, they do not change senses. With a change of time, a new expression is used with a new sense.  $W$  also does not change sense on Kripke's proposal. It always expresses a sense that determines a function from times to truth values. And on that proposal, the component of  $W$  that consists in present tense, or some other indexical, always denotes the same function in the same way—perhaps denoting a function from times to times. It does not change sense either.

Certainly in (E), and arguably in (D), Frege implies not only that the mere sentential wording  $W$  (exemplified by the *type* 'Today is Thursday') can be used to express different complete thoughts. He also implies that indexical words change their senses in different contexts. He thinks that *on their own apart from context*, indexicals never have a full sense.

Kripke's proposal is incompatible with (E) as well as (A).<sup>80</sup> For Frege, expressions regularly change *sense*. Indexicals do not change linguistic *meanings* with context. Indexicality is not ambiguity in linguistic meaning.

Second, in (B') Frege writes of present tense as making a time indication—presumably the time of utterance. We have already seen that present tense does not indicate the time of utterance by denoting a function that maps any given time onto the time of utterance. Such a view, combined with passage (A), would be incompatible with Frege's sense-composition doctrine. Nevertheless, present tense is used to indicate the time of utterance, seemingly in something like the way 'this time' or 'that time' would do so, where the difference between 'this' and 'that' (like that between 'today' and 'yesterday') helps the hearer get on to the relevant sense, but does not contribute to the sense. Given that Frege takes present tense to be used as a demonstrative to designate a time, it would be odd for him to take the time as an expression denoting itself. There would be two designators (denoters) of the time: the occurrent use of present tense and the time itself. This point casts some doubt on Kripke's proposal that times denote themselves.

Third, in isolation, the last two sentences of (B')—that is, the sentences in (B)—are naturally read as supporting Kripke's view that

<sup>80</sup> There is a similar text that involves 'I' that I will discuss below.

times are literally parts of expressions of thoughts, and denote themselves. But much of the surrounding text suggests a different reading.

Frege writes, 'one must know when the sentence was uttered in order to grasp the thought correctly'. Then he states, apparently as the result of an inference, that the time of utterance is part of the thought expression. The exact relation between these remarks is not obvious. He could be thinking of an acquaintance relation to the time, where the time is a symbol denoting itself, as Kripke proposes. Then the first remark would be understood in terms of direct acquaintance with the time—'know the time'. But alternatively, 'the time of the utterance' in the second remark could be shorthand repetition of, anaphorically going back to, 'when the utterance occurred'. Such a reading might only require any circumstance that could be used to specify what time it is. Direct acquaintance with the time would not be required.

I think it plausible, as Kripke says for his proposal, that specification of the time would reveal what time it is. 'Knowing when the sentence was uttered' (Frege's phrase) seems to entail something stronger than having just any way of uniquely denoting the time. But on the present view, the requirement is weaker than on Kripke's: no direct acquaintance with the time is required. All that is required is some specification that reveals when the sentence was uttered, assuming that the thinker uses the sense of the specification in thinking the thought. The time of the utterance would be 'part of the thought expression' only in the loose sense that it is specified (hence denoted) by the thinker's use of contextual circumstances to express a mode of presentation that appropriately determines the time. The time could be known through any number of senses or thought components, depending on context.

I see this reading as more in the spirit of the paragraph in which (B') and (A) are contained. Frege seems interested not just in the speaker's grasping the thought, but third-person auditors' grasping it. (The word 'correct' in (A'), just below, suggests this point.) Frege seems more eclectic about what sort of expression can designate the time—or place, in the case of 'here' or 'there'—than invoking the time as a self-designator would suggest. Recall that in (A) after having discussed 'here' and 'there', he summarizes at the end of the paragraph:

(A') In all such cases, the mere wording, as it can be fixed in writing, is not the complete expression of the thought—but one further needs for its correct apprehension also the knowledge (*Kenntnis*) of certain circumstances accompanying the utterance, which are used as means of thought expression (*Gedankenausdrucks*). Fingerpointings, gestures, and glances can belong here too. The same wording (*Wortlaut*)

containing the word 'I' will express different thoughts in the mouths of different people, of which some may be true and others false.<sup>81</sup>

In giving examples of thought expressions that supplement indexicals, Frege does not invoke the time or place itself as the means of thought expression. He seems to have in mind knowledge of various other circumstances that might be used to determine time or place. With 'in all such cases', Frege seems to allow any sort of circumstances (including the gestures) that can be understood in determining (and specifying) time or place, as expressions of the thought.

As noted earlier, tokens of indexical words tend to be part of the circumstances that determine time or place. In "On Sense and Denotation" Frege assumes that present tense can be used to designate the present time.<sup>82</sup> In the 1897 passage (D), he takes 'now' to be part of the linguistic expression that singularly denotes a time. I see no reason to doubt that 'today', 'yesterday', 'here', and 'now' in (A) are parts of the language used in specifying time or place. These points and examples suggest that Frege understood time determination not to be effected purely by direct acquaintance with a time, considered as a piece of language.<sup>83</sup>

Later in "Thought" after (B)–(A), Frege writes again that the time of utterance 'belongs to' the thought expression.<sup>84</sup> He infers that the time determination (*Zeitbestimmung*) is thus given. In the same passage, he remarks about a sentence utterance containing 'I' that 'the identity of the speaker is essential to the sense'. I think that his writing that the time is part of the thought expression is to be taken analogously to his remark that the identity of the speaker is essential to the sense of a sentence utterance. The point is not that the speaker is literally part of the sense, or that the time is literally part of the expression of the thought. The point is that the sense must determine the speaker, and the thought expression must denote and specify the time.

Similarly, in the 1897 "Logic," in passage (C) where he comments that words like 'here' and 'now' only acquire their full sense through

<sup>81</sup> Frege, "Der Gedanke," in *Kleine Schriften*, p. 349 [64]; "Thought," in *The Frege Reader*, p. 332.

<sup>82</sup> Frege, "Über Sinn und Bedeutung," in *Kleine Schriften*, pp. 155–56 [42–43]; "On Sinn and Bedeutung," in *The Frege Reader*, p. 165. Frege takes the present tense to be used as a singular term to designate a time. He does not say in this passage that (unused) present tense denotes a function, as one might expect him to if he believed that it does. He does not say enough to make it *clear* what his view of the semantics is. But what he writes is certainly consistent with what he writes in 1897 and 1918–1919.

<sup>83</sup> See *ibid.*, note 10, [43].

<sup>84</sup> Frege, "Der Gedanke," in *Kleine Schriften*, [76]; "Thought," in *The Frege Reader*, p. 344.

the circumstances in which they are used, he writes that the time and place 'must be supplied'. The time seems to be 'part of the thought expression' only in the sense that it is supplied, by being specified by the sense of some thought expression. Frege indicates in several places that this "supplying" can take various forms—through present tense, 'now', gestures, indications of events that are associated with the time.<sup>85</sup>

Although (B)–(A), the later passage in "Thought" (see note 84), and the passage in "Logic" 1897 (note 85) are compatible with Kripke's reading, I think that Frege probably means that the time is 'part of the thought expression' only in the loose sense that the expression expresses a sense that specifies the time. In these passages, and in the one that discusses present tense in "On Sense and Denotation" (note 82), Frege writes that time determination can be effected in various ways (for example, by specifying events that occur at the time—'the time of Schleswig Holstein's separation'). So I doubt that Frege thought of times as parts of language. I doubt that he invoked direct acquaintance with times as symbols for themselves. But this issue is not central to the basic point. The basic point is that Frege's notion of sense is not that of linguistic meaning.

<sup>85</sup> Frege, "Logik" (1897), p. 146; *Posthumous Writings*, p. 135; "Logic," in *The Frege Reader*, p. 235. The remark in "Der Gedanke," [76] that the identity of the speaker is essential to the sense also occurs in this 1897 passage.