Cut-off points for the rational believer

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Abstract. I show that the Lottery Paradox is just a version of the Sorites, and argue that this should modify our way of looking at the Paradox itself. In particular, I focus on what I call "the Cut-off Point Problem" and contend that this problem, well known by Sorites scholars, ought to play a key role in the debate on Kyburg's puzzle.

Very briefly, I show that, in the Lottery Paradox, the premises "ticket n°1 will lose", "ticket n°2 will lose"… "ticket n°1000 will lose" are equivalent to soritical premises of the form "~(the winning ticket is in $\{\ldots, (t_n)\}) \supset$ ~(the winning ticket is in $\{\ldots, t_n, (t_{n+1})\})$" (where "$\supset$" is the material conditional, "~" is the negation symbol, "t_n" and "t_{n+1}" are "ticket n°n" and "ticket n°$n + 1$" respectively, and "$\{\}$" identify the elements of the lottery tickets' set. The brackets in "(t_n)" and "(t_{n+1})" are meant to point out that in the antecedent of the conditional we do not always have a "t_n" (and, as a result, a "t_{n+1}" in the consequent): consider the conditional "~(the winning ticket is in $\{\}) \supset$ ~(the winning ticket is in $\{t_1\}$)").

As a result, failing to believe, for some ticket, that it will lose comes down to introducing a cut-off point in a chain of soritical premises.

In this paper I explore the consequences of the different ways of blocking the Lottery Paradox with respect to the Cut-off Point Problem. A heap variant of the Lottery Paradox is especially relevant for evaluating the different solutions.

One important result is that the most popular way out of the puzzle, i.e., denying the Lockean Thesis, becomes less attractive. Moreover, I show that, along with the debate on whether rational belief is closed under classical logic, the debate on the validity of modus ponens should play an important role in discussions on the Lottery Paradox.

Keywords. Lottery Paradox; Sorites Paradox; Belief Closure; modus ponens

1. Introduction

In the literature on rational belief and rational degrees of belief it is usually claimed that the two following principles cannot be jointly satisfied:

Belief Closure. Rational belief is closed under classical logic.

Lockean Thesis. An agent *S* should believe *P* if and only if *P* is very probable given *S*'s evidence (where "very probable" means "probable to a degree equal to or higher than a specified threshold value *t*").

Indeed, joint acceptance of Belief Closure and the Lockean Thesis gives rise to the well-known Lottery Paradox, first proposed by Kyburg (1961).

Consider a fair 1000-ticket lottery with exactly one winner. The probability, for each ticket, that it will win is very low, i.e., it is 0.001. It follows that if $t = 0.999$, then, by the Lockean Thesis, one should believe, of each ticket, that it will lose. By Belief Closure, one should also believe the conjunction "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" (where " ∧ " is the conjunction symbol). However, this conjunction has a probability 0, and for that reason one should not believe it according to the Lockean Thesis. We end up with a contradiction, viz., that one should both believe and not believe[1] the same sentence. Thus, we must conclude, Belief Closure and the Lockean Thesis are incompatible.

As Leitgeb (2014) puts it, we can classify a huge part of the classical literature on rational belief according to which principle is dropped: for instance, Isaac Levi (1967) accepts Belief Closure but rejects the Lockean Thesis, while Henry Kyburg (1961) accepts the Lockean Thesis and rejects Belief Closure.

The most widespread solution to the puzzle, however, consists in denying the Lockean Thesis: among the authors who adopt this option are Lehrer (1975; 1990), Kaplan (1981a; 1981b; 1996), Stalnaker (1984), Pollock (1995), Ryan (1996), Evnine (1999), Nelkin (2000), Adler (2002), Douven (2002), Smith (2010; 2016; 2018), and Kelp (2017).

Philosophers who believe, instead, that the Lottery Paradox puts pressure on Belief Closure include Klein (1985), Foley (1992), Hawthorne and Bovens (1999), Kyburg and Teng (2001), Christensen (2004), Hawthorne and Makinson (2007), Kolodny (2007), and Easwaran and Fitelson (2015).

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[1]Throughout this paper, as is usually the case, "not believing *P*" includes suspending one's judgement on *P* and disbelieving *P*, i.e., believing the negation of *P*.

Leitgeb's view seems to be an exception to this categorization (deniers of the Lockean Thesis vs. deniers of Belief Closure). Indeed, Leitgeb (2014; 2015; 2017) defends a form of contextualism which, he contends, allows us to keep both the Lockean Thesis and Belief Closure. I will come back to his proposal in section 5.

I will argue that the Lottery Paradox is in fact a version of the Sorites Paradox. More precisely, it is a version of the Sorites in which, unlike what happens in the classical version, an epistemic attitude is involved, namely one of rational belief. Here is, for reminder, the classical, textbook version of the Sorites. ("~" is the negation symbol and "⊃" is the material conditional.)

Consider (1) and (2).

(1) 1000 grains are a heap.[2]
(2) ~(0 grains are a heap).

Also consider the following 1000 conditionals:

(P1) ~(0 grains are a heap) ⊃ ~(1 grain is a heap)
(P2) ~(1 grain is a heap) ⊃ ~(2 grains are a heap)
(P3) ~(2 grains are a heap) ⊃ ~(3 grains are a heap)
…
(P1000) ~(999 grains are a heap) ⊃ ~(1000 grains are a heap)

Let us call these sentences "P-conditionals". (1), (2) and the P-conditionals seem true. However, multiple applications of modus ponens let us infer the following puzzling conclusion:

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[2]Here "1000" could be replaced with any sufficiently high number. This of course holds for the Lottery Paradox too: instead of a 1000-ticket lottery we could consider a 5000-ticket lottery, or a 1 million-ticket lottery, etc.

(3) ~(1000 grains are a heap). (!)

Let me stress that (1)-(3) is about truth, not rational belief: as Sorites scholars classically put it, the argument's premises are intuitively true whereas the conclusion is intuitively false. This is a fundamental difference with respect to the Lottery Paradox, as in the latter rational belief and Belief Closure are involved, instead of truth and modus ponens respectively.

In this paper, I show that the Lottery Paradox is just a version of the Sorites, and argue that this should modify our way of looking at the Paradox itself. In particular, I focus on what I call "the Cut-off Point Problem" and contend that this problem, well known by students of the Sorites, ought to play a key role in the debate on Kyburg's puzzle.

Very briefly, I show that, in the Lottery Paradox, the premises "ticket n°1 will lose", "ticket n°2 will lose"… "ticket n°1000 will lose" are equivalent to soritical premises of the form "~(the winning ticket is in {…, (t_n)}) $\supset$ ~(the winning ticket is in {…, t_n, (t_{n+1})})" (where "t_n" and "t_{n+1}" are "ticket n°n" and "ticket n°$n + 1$" respectively, and "{ }" identify the elements of the lottery tickets' set. The brackets in "(t_n)" and "(t_{n+1})" are meant to point out that in the antecedent of the conditional we do not always have a "t_n" (and, as a result, a "t_{n+1}" in the consequent): consider the conditional "~(the winning ticket is in { }) $\supset$ ~(the winning ticket is in {t_1})").

As a result, failing to believe, for some ticket, that it will lose comes down to introducing a cut-off point in a chain of soritical premises. I call the view that, for some ticket, we should not believe that it loses "the Cut-off Point View".

One important consequence of this reformulation of the Lottery Paradox is that the most popular solution to the puzzle, i.e., denying the Lockean Thesis, becomes less attractive. The reason is that keeping Belief Closure entails the (rather counterintuitive) Cut-off Point View. In order to make the counterintuitive character of this view emerge as clearly as possible I consider a heap variant of the original lottery scenario: in this scenario (which is generally used in the context of a different puzzle, viz., the Sorites) the worrying consequences of the Cut-off Point View become evident.

More precisely, I will show that we can keep Belief Closure only if we accept the Cut-off Point View as far as the heap version of the Lottery Paradox is concerned. Indeed,

as we will see, in order to provide a unified solution to the Lottery Paradox and its heap variant we only have three options:

(i) accepting the Lockean Thesis with t short of 1, which implies rejecting Belief Closure.

(ii) accepting the Lockean Thesis with $t = 1$, which allows us to keep Belief Closure, but forces us to accept the Cut-off Point View.

(iii) rejecting the Lockean Thesis across the board, which also allows us to keep Belief Closure, but forces us to endorse the Cut-off Point View.

Finally, I will demonstrate that denying Belief Closure is not enough. More precisely, it is not enough to solve a puzzle which is closely related to Kyburg's and which puts us before the following dilemma: we should either accept the Cut-off Point View[3] or reject modus ponens. That is, what is being called into question is not merely Belief Closure, but a fundamental principle of classical logic. I conclude by hinting at some results which strongly support the latter option, i.e., rejecting modus ponens, over the Cut-off Point View.

Before I start, one more preliminary remark is needed. In presenting the Lottery Paradox, I used the expression "Belief Closure". I could have been more specific, though: in Kyburg's puzzle a specific principle is applied, i.e., the closure of rational belief under conjunction introduction. From now on, I will call such a principle "conjunction introduction*", in order to distinguish this epistemic version of the conjunction introduction schema from its non-epistemic counterpart. I will do the same for the other logical principles: e.g., when referring to the closure of rational belief under

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[3]As it will become clear in what follows, in this further puzzle the expression "cut-off point" refers to something a bit different from what it denotes in Kyburg's original puzzle. Specifically, in this additional puzzle we have a cut-off point if and only if one in a series of soritical conditionals of the form "I should believe: ~(the winning ticket is in {…, (t_n)}) ⊃ I should believe: ~(the winning ticket is in {…, t_n, (t_{n+1})})" is false.

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modus ponens and modus tollens, I will use the labels "modus ponens*" and "modus tollens*" respectively.

2. *The Wide Scope Paradox*

Consider again our fair 1000-ticket lottery, with $t = 0.999$. Also assume that tickets are numbered from 1 to 1000. By the Lockean Thesis, in this scenario one should believe, for instance, "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°999 wins", as its probability is 0.999. That is, one should believe that the set which includes tickets from n°1 to n°999 contains the winning ticket. At the same time, one should not believe "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°998 wins", as this sentence only has a probability of 0.998. That is, we should suspend our judgement on whether the winning ticket is to be found between ticket n°1 and ticket n°998 (included).

In what follows, instead of saying that one should (or should not) believe the sentence "ticket n°1 wins ∨ ticket n°2 wins… ∨ ticket n°n wins" I will say that one should (or should not) believe "the winning ticket is in $\{t_1, t_2, …, t_n\}$". That is, I assume that the probability of "the winning ticket is in $\{t_1, t_2, …, t_n\}$" equals the probability of "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°n wins". This reformulation will not affect my point, and will make my presentation smoother.

Another equivalence will also be needed in what follows. "Ticket n°1 will lose" (viz., "~(the winning ticket is in$\{t_1\}$)") is just equivalent to "~(~(the winning ticket is in $\{\}$) ∧ the winning ticket is in $\{t_1\}$)". That is, "ticket n°1 will lose" (or "~(the winning ticket is in$\{t_1\}$)") is equivalent to "~(the winning ticket is in $\{\}$) ⊃ ~(the winning ticket is in $\{t_1\}$)". More generally, "ticket n°$n + 1$ will lose" is equivalent to "~(~(the winning ticket is in $\{…, t_n\}$) ∧ the winning ticket is in $\{…, t_n, t_{n + 1}\}$)", which is in turn equivalent to "~(the winning ticket is in $\{…, t_n\}$) ⊃ ~(the winning ticket is in $\{…, t_n, t_{n + 1}\}$)".

What is the point of introducing these equivalences? First, these equivalences can be used to show that starting from a lottery scenario we can generate a soritical paradox. I will call this argument "Wide Scope Paradox" (WSP), in order to distinguish it from a related argument that I will label "Narrow Scope Paradox", and that I present in section 5. I have called it "WSP" because in it the rational belief operator has wide scope over

the soritical conditionals; in the Narrow Scope Paradox instead, as we will see, the belief operator has narrow scope over the antecedent and the consequent of such conditionals.

Consider the following sentences. Remember that we have set $t = 0.999$.

(1') I should believe: the winning ticket is in $\{t_1, t_2, \ldots, t_{999}, t_{1000}\}$
(2') I should believe: ~(the winning ticket is in $\{\}$)

Also consider the following 1000 sentences:

(P'1) I should believe: ~(the winning ticket is in $\{\}$) ⊃ ~(the winning ticket is in $\{t_1\}$)
(P'2) I should believe: ~(the winning ticket is in $\{t_1\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2\}$)
(P'3) I should believe: ~(the winning ticket is in $\{t_1, t_2\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2, t_3\}$)
…
(P'1000) I should believe: ~(the winning ticket is in $\{t_1, t_2, \ldots, t_{999}\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2, \ldots, t_{999}, t_{1000}\}$)

The paradox consists in the fact that by multiple applications of modus ponens* we conclude (3'):

(3') I should believe: ~(the winning ticket is in $\{t_1, t_2, \ldots, t_{999}, t_{1000}\}$) (!)

Note that each premise is true. Indeed, "the winning ticket is in $\{t_1, t_2, \ldots, t_{999}, t_{1000}\}$" and "~(the winning ticket is in $\{\}$)" both have a probability of 1, while each of the conditionals in (P'1)-(P'1000) (viz., "~(the winning ticket is in $\{\}$) ⊃ ~(the winning ticket is in $\{t_1\}$)", "~(the winning ticket is in $\{t_1\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2\}$)"… "~(the winning ticket is in $\{t_1, t_2, \ldots, t_{999}\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2, \ldots, t_{999}, t_{1000}\}$)") has a probability of 0.999. Why 0.999? Consider, for instance, the conditional "~(the winning ticket is in $\{t_1, t_2, \ldots, t_{498}\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2, \ldots, t_{498}, t_{499}\}$): it is equivalent to "(the winning ticket is in $\{t_1, t_2, \ldots, t_{498}\}$) ∨ ~(the winning ticket

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is in $\{t_1, t_2, \ldots, t_{498}, t_{499}\})$". The reason why this disjunction has a probability of 0.999 is that its probability is calculated by adding the probability of "the winning ticket is in $\{t_1, t_2, \ldots, t_{498}\}$", i.e., 0.498, to the probability of "~(the winning ticket is in $\{t_1, t_2, \ldots, t_{498}, t_{499}\})$", i.e., 0.501; the probabilities must be added here because the sentences we are dealing with are mutually inconsistent. Another way of reaching the same result is by focusing on what would make (P'1)-(P'1000) false: as we have seen, the conditionals in (P'1)-(P'1000) are of the form "~(the winning ticket is in $\{\ldots, (t_n)\}) \supset$ ~(the winning ticket is in $\{\ldots, t_n, (t_{n+1})\})$": in order to falsify one of them, it is both necessary and sufficient that the winning ticket is exactly ticket n°$n + 1$, which has a probability of 0.001.

So (1'), (2') and (P'1)-(P'1000) are all true; nonetheless, by modus ponens*, (3') is also true, whence the problematic outcome: by modus ponens*, (3') is both true and false.

Now, it is not only the case that we can generate a soritical argument starting from the lottery scenario: it is also the case that the premises of the original puzzle by Kyburg and those of the WSP are equivalent.

Recall the equivalences I have introduced. It can be noted that (1') is equivalent, in the original version of the puzzle, to the sentence that says that we should believe that the 1000-ticket lottery has one winner (i.e., that we should believe the disjunction "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°1000 wins"). As for (2'), it corresponds to a premise which is left implicit in the original argument, viz., the (trivial) claim that we should believe that the winning ticket is not a member of the empty set. Finally, we have seen that "~(the winning ticket is in $\{\ldots, t_n\}) \supset$ ~(the winning ticket is in $\{\ldots, t_n, t_{n+1}\}$" is equivalent to "ticket n°$n + 1$ will lose" – that is, (P'1)-(P'1000) are equivalent to the premises of Kyburg's original argument which say that we should believe that ticket n°1 will lose, the ticket n°2 will lose, and so on.

The WSP, then, is just a reformulation of the standard Lottery Paradox.[4] However, one clear difference between Kyburg's original argument and the WSP is that in the original argument conjunction introduction* is used, whereas in the WSP modus ponens* is applied. Now, it turns out that it is possible to reformulate the WSP so that conjunction introduction* is used. Indeed, if we use 1000 instances of conjunction introduction*, we generate a sentence which says that we should believe the conjunction of "~(the winning ticket is in {})" and the conditionals in (P'1)-(P'1000). Then, by one further application of Belief Closure (which of course entails that if one should believe P, and if Q and P are equivalent, then one should believe Q as well), we get (3').

It could be asked what is the point of introducing this reformulation of the Lottery Paradox, i.e., of introducing the WSP. Actually, I think that the WSP is interesting in itself, as it shows that the Lottery Paradox just is an epistemic version of the Sorites (more specifically, one involving rational belief). However, this is not all there is to the WSP. The main reasons why in what follows I will use the WSP instead of the original formulation of the Paradox are matters of clarity for my current purposes. Indeed, using the WSP makes it more natural to rerun the Lottery Paradox starting from a heap scenario; as a result, the advantages of switching to a heap scenario are more apparent and the main point of the paper will emerge more clearly.

3. Setting the threshold at 1

If we reject the Lockean Thesis the Lottery Paradox is blocked. However, it is traditionally assumed that accepting $t = 1$ allows us to keep both the Lockean Thesis and Belief Closure (among the authors who argue that we should accept $t = 1$ are Gärdenfors (1986), Van Fraassen (1995), Arló-Costa (2001), Arló-Costa and Parikh (2005)). One

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[4]This strengthens a point by Dorothy Edgington (1992; 1997). Indeed, Edgington claims that the Lottery Paradox and the Sorites are structurally similar, so that a common strategy should be applied to solve both. However, unlike mine, her view presupposes the acceptance of a degree-theoretic framework (i.e., the idea that there are such things as degrees of truth). Moreover, and most importantly, my claim is stronger than Edgington's: the Lottery Paradox is not merely similar to the Sorites Paradox; the Lottery Paradox just *is* a Sorites.

main consequence of this solution is that we are forced to accept the Cut-off Point View, i.e., as specified above, we should not believe, for at least some ticket, that it will lose (e.g., in the original scenario we should not believe, for *each* ticket, that it will lose). In order to better evaluate the consequences of this view, let us put aside for a moment the standard lottery scenario and use instead a classical example from the literature on vagueness, i.e., the heap example.

Note that replacing the lottery scenario with a different scenario (and more specifically, a heap scenario) is a perfectly legitimate move. Indeed, the Lottery Paradox is not a puzzle about lotteries. On the contrary, the point of the Paradox is a general one, and consists in showing that the Lockean Thesis (with t short of 1) and Belief Closure are incompatible. Moreover, we are perfectly allowed to assign probabilities to the premises of the Sorites, based on our evidence.[5]

I have presented the classical version of the Sorites in section 1 above (see argument (1)-(3)). As already emphasized there, (1)-(3) is about truth, not rational belief ("the argument's premises are intuitively true whereas the conclusion is intuitively false"). However, it is easy to transform "the classical Sorites" into a version of the WSP: as it will become clear below, one only needs to assign probabilities to the premises of (1)-(3), set an appropriate threshold for belief, and apply modus ponens* instead of modus ponens.

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[5]Clearly, this way of looking at the Lottery Paradox is at odds with those accounts of the latter which argue that we should deny the Lockean Thesis because evidence which is "merely probabilistic" is not enough for rational belief (see, for instance, Nelkin 2000, or Smith 2010, 2016 and 2018). These accounts usually focus on the original version of the Paradox or, when they consider variants of it, they focus on cases in which the relevant evidence is statistical.

However, it can be noted that the formulation I have given of the Lockean Thesis leaves it open whether the kind of evidence the agent relies on is "merely probabilistic" or not. That is, in my formulation of the Lockean Thesis, which I take to be standard, evidence need not be "merely probabilistic". A consequence of this fact is that, as I say in the body of the paper, we should regard the original version of the Lottery Paradox as simply *illustrating* the conflict between the Lockean Thesis and Belief Closure. This is an important point, as it seems clear that the conflict does not vanish if we consider evidence which is not "merely probabilistic". In other terms, reducing the Lottery Paradox to a problem concerning statistical evidence alone does not do justice to the challenge it illustrates, which is much more general.

So how should we assign probabilities to the Sorites' premises? Keep in mind that we are concerned here with the probabilities a rational agent assigns to sentences based on the relevant evidence. Now, given that we know both that 1000 grains are a heap and that 0 grains are not a heap, it seems that we should assign a probability of 1 to (1) and (2) respectively. What about the P-conditionals (i.e., the sentences of the form "~(n grains are a heap) $\supset$ ~($n + 1$ grains are a heap)")? Clearly, we cannot assign to all of them a probability of 1. The reason is simple: consider (2):

(2) ~(0 grains are a heap).

It seems reasonable to assign (2) a probability of 1. Now, this entails that we should assign a probability lower than 1 to at least one of the P-conditionals, because otherwise, by the laws of probability, we should also assign a probability of 1 to (3), while it seems clear that its probability should be 0. So it cannot be the case that all the P-conditionals have a probability 1. Still, each of them can be assigned a very high probability (i.e., each sentence of the form "~(n grains are a heap) $\land$ $n + 1$ grains are a heap" can be assigned a very low probability). If we do so (if we assign each of the P-conditionals a probability short of 1), the WSP can be formulated starting from a heap scenario. I will call this alternative formulation "Soritical Wide Scope Paradox" (SWSP): assume that t is very high, but less than 1. By the Lockean Thesis, we should believe (1), (2), and each of the P-conditionals. However, again by the Lockean Thesis, we should not believe (3), which has a probability 0. That is, we end up having to both believe and not believe (3). (Exactly as for the WSP, we can provide a version of the SWSP in which conjunction introduction* is used, instead of modus ponens*: to this end, let us use 1000 instances of conjunction introduction* to generate a sentence which says that we should believe the conjunction of (2) and all the P-conditionals. By one last application of Belief Closure (which entails that if one should believe P, and if Q and P are equivalent, then one should believe Q as well), we get "I should believe: ~(1000 grains are a heap)".)

One further clarification is in order. We are supposing that the sentences of the form "~(n grains are a heap) $\land$ $n + 1$ grains are a heap" are all assigned low probabilities (i.e., that the P-conditionals are all assigned high probabilities). That is, the probability

distribution we are considering is a "uniform" one, viz., one in which all sentences of the form "~(n grains are a heap) ∧ $n + 1$ grains are a heap" have the same probability, or at least one in which all such sentences have a probability greater than 0. However, for the paradox to arise, we are not at all obliged to assign our probabilities this way. On the contrary, we can assume a probability distribution in which some such sentences (I will call them "negated P-conditionals") have a probability 0 (in fact, as many as we wish, provided that the probabilities of the negated P-conditionals sum up to 1).

Here we come to a crucial point. Consider again the SWSP. Exactly as the WSP, we could block it either by dropping the Lockean Thesis or by dropping Belief Closure. However, as announced in the title of this section, we could be willing to keep both principles by setting $t = 1$. However, if we assume that $t = 1$ we are forced to conclude that, for at least one n, it is not the case that one should believe "~(n grains are a heap) ⊃ ~($n + 1$ grains are a heap)". Indeed, for at least one n, the probability of "~(n grains are a heap) ∧ $n + 1$ grains are a heap" must be greater than 0 (otherwise, given that we have assigned (2) a probability 1, (3) should also have probability 1). (Of course, if there is only one n such that the probability of "~(n grains are a heap) ∧ $n + 1$ grains are a heap" is greater than 0, then, given that the sum of the negated P-conditionals' probabilities must be 1, the probability of that disjunct must be 1.)

Clearly, if our evidence is distributed uniformly over the negated P-conditionals we should neither believe the conditional "~(0 grains are a heap) ⊃ ~(1 grain is a heap)", nor any of the other P-conditionals. Conversely, if we have absolutely no evidence for some of the negated P-conditionals (i.e., if we assign a zero probability to, say, the first twenty negated P-conditionals), the cut-off point will come "later" in the distribution (e.g., we should believe "~(19 grains are a heap) ⊃ ~(20 grains are a heap)", but we should not believe "~(20 grains are a heap) ⊃ ~(21 grains are a heap)"). Anyway, what matters is that in both cases at least one of P-conditionals is not rationally believable.

So we have seen that the Cut-off Point View follows from the acceptance of $t = 1$. Now, it seems clear that in order to solve the WSP one must also solve the SWSP, which is a simple variant of the WSP, in which a heap scenario is used instead of a lottery one.

Many authors already reject the Cut-off Point View for the original lottery scenario; notably, all those who defend the Lockean Thesis with *t* short of 1. Switching to a heap scenario raises an interesting problem for those who accept the Lockean Thesis with *t* = 1, as perhaps some of them will find the outcome that at least one of the P-conditionals is not rationally believable unpalatable. Of course, "some" does not mean "all of them". Still, the fact that if we set *t* = 1 we must accept the Cut-off Point View with respect to the SWSP is something we should keep in mind when evaluating a solution to the Lottery Paradox, and this was the point I wanted to make in this section.

4. Rejecting the Lockean Thesis altogether

As we know, a possible way of solving the WSP consists in accepting the Lockean Thesis with *t* short of 1 and rejecting Belief Closure. Another possible way out of the puzzle is keeping both the Lockean Thesis and Belief Closure, while setting *t* at 1. However, we have seen that the latter option forces us to adopt the Cut-off Point View with respect to the (S)WSP.

Let us now turn to the third and last option, which consists in rejecting the Lockean Thesis across the board and accepting, instead, a different norm of belief. The alternative norms I will consider are the most popular competitors of the Lockean Thesis, i.e., the truth norm and the knowledge norm of belief. In this section, I show that accepting either of these alternative norms still forces us to endorse the Cut-off Point View with respect to the (S)WSP.

The truth norm may be defined as the norm according to which we should believe *P* if and only if *P* is true. During its history, the truth norm has been precisified in various ways; however, the subtleties of the different definitions are not relevant here.[6] As far as the WSP is concerned, this norm provides a clear verdict: the argument has one false premise. Indeed, there is a ticket (the winning one) of which we should not believe that it will lose. That is, one of (P'1)-(P'1000) is false.

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[6]Among others, Wedgwood 2002, Boghossian 2003, Shah 2003, Gibbard 2005, Bykvist and Hattiangadi 2007, Engel 2007, and Thomson 2008 contain stimulating remarks on the way the truth norm should be made precise.

Regarding the knowledge norm, i.e., the norm according to which we should believe *P* if and only if we know *P*[7], it also provides a straightforward solution to the WSP: (1') and (2') are both true, as we know that if we buy all the tickets we will win, and that if we do not buy any ticket we will lose. However, (P'1)-(P'1000) are all false. This is because we do not know, of each ticket, that it will lose.

Now consider the SWSP. If we accept the truth norm, there are only two ways out of the puzzle: one consists in embracing the Cut-off Point View, the other in denying Belief Closure.[8] The problem that faces the truth norm's advocate is the following: is there one grain such that when added to a collection of grains which is not a heap turns it into a heap? If the answer is yes, then, by the truth norm, there is one *n* such that we should not believe the conditional "~(*n* grains are a heap) ⊃ ~(*n* + 1 grains are a heap)". If, instead, she believes that there is not such a *n*, she must reject Belief Closure.

Similar remarks hold for the knowledge norm's defender, even though the problem she faces is slightly different: is there one *n* such that we know that *n* grains are not a heap but we do not know that *n* + 1 grains are not a heap? Depending on her answer, the knowledge norm's advocate will be either endorsing the Cut-off Point View or denying Belief Closure.

However, we have seen that the SWSP is an innocent variant of the WSP, and that, as a result, we should give a unified answer to the two puzzles. This means that, given that both the truth norm's and the knowledge norm's advocates endorse the Cut-off Point View with respect to the WSP, they should also endorse it with respect to the SWSP. In other words, if we accept either the truth norm or the knowledge norm of belief, we are bound to accept the cut-off point conclusion with respect to the SWSP.

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[7]The knowledge norm is adopted by a growing number of epistemologists; its most famous defender is Timothy Williamson (see Williamson 2000). Note, though, that in Williamson's work the defence of such a norm is only implicit and must be derived from the author's defence of the knowledge norm of assertion.
[8]Of course, the truth norm's advocate may also solve the SWSP by claiming that (2) is false or that (3) is true. The claim according to which (3) is true has been defended in the literature on the Sorites by Peter Unger (1979). However, I will not deal with these very unpopular options here. And anyway, as it will become clear below, these options are not relevant for my argument's purposes.

Of course, rejecting the Lockean Thesis does not automatically entail that we should endorse either the truth norm or the knowledge norm. Indeed, among the most classical proposals concerning the Lottery Paradox is that of amending the Lockean Thesis by adding a defeat clause (Pollock 1995 is a good example of this kind of approach). Other (more recent) accounts do not simply add to the Lockean Thesis a defeat condition, but propose an outright modification of the threshold constraint (see Lin and Kelly 2012a and 2012b. Actually, Leitgeb's account (2014; 2015; 2017) can also be regarded as part of this category, as Leitgeb proposes to modify the Lockean Thesis to the effect that the probability of *P* should remain higher than 0.5 conditional on any proposition consistent with it; see Staffel 2021). However, these proposals also entail the Cut-off Point View. So, if I am right in claiming that the solution to the WSP should be extended to the SWSP, the advocates of these accounts should also endorse the Cut-off Point View with respect to the SWSP.[9] More generally, given that (1') or (2') being false does not seem to be an option, it appears that if we want to preserve Belief Closure we are forced to conclude that at least one of (P'1)-(P'1000) is false.

Let us take stock: as announced in section 1, we can keep Belief Closure only if we accept the Cut-off Point View as far as the SWSP is concerned. More precisely, in order to provide a unified solution to the WSP and the SWSP we only have three options:

(i) accepting the Lockean Thesis with *t* short of 1, which implies rejecting Belief Closure.

(ii) accepting the Lockean Thesis with *t* = 1, which allows us to keep Belief Closure, but forces us to accept the Cut-off Point View.

(iii) rejecting the Lockean Thesis across the board, which also allows us to keep Belief Closure, but forces us to endorse the Cut-off Point View.

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[9]Another option consists in denying the Lockean Thesis while adopting, at the same time, an eliminativist approach to the notion of full rational belief. That is, it consists in rejecting the whole framework in which the Lottery Paradox is formulated. According to this very radical approach, which I will put aside here, talk about full belief should be entirely replaced by talk about degrees of belief. For a discussion of this option see Foley 1992.

5. The Narrow Scope Paradox

In this section I will show that, despite the appearances, option (i), i.e., accepting the Lockean Thesis with *t* short of 1, does not allow us to avoid the cut-off point conclusion (at least with respect to the argument I am going to present).

Consider again (1') and (2').

(1') I should believe: the winning ticket is in $\{t_1, t_2, \ldots, t_{1000}\}$
(2') I should believe: ~(the winning ticket is in$\{\}$)

And now consider the following 1000 sentences.

(P''1) I should believe: ~(the winning ticket is in $\{\}$) $\supset$ I should believe: ~(the winning ticket is in $\{t_1\}$)
(P''2) I should believe: ~(the winning ticket is in $\{t_1\}$) $\supset$ I should believe: ~(the winning ticket is in $\{t_1, t_2\}$)
(P''3) I should believe: ~(the winning ticket is in $\{t_1, t_2\}$) $\supset$ I should believe: ~(the winning ticket is in $\{t_1, t_2, t_3\}$)
…
(P''1000) I should believe: ~(the winning ticket is in $\{t_1, t_2 \ldots t_{999}\}$) $\supset$ I should believe: ~(the winning ticket is in $\{t_1, t_2 \ldots t_{999}, t_{1000}\}$)

Repeated applications of modus ponens lead to (3'):

(3') I should believe: ~(the winning ticket is in$\{t_1, t_2, \ldots t_{1000}\}$) (!)

I will call this puzzle "Narrow Scope Paradox" (NSP). As already explained, the reason for this label is that in the NSP the rational belief operator has narrow scope over the antecedent and the consequent of the soritical conditionals ("~(the winning ticket is in $\{\}$) $\supset$ ~(the winning ticket is in $\{t_1\}$)", ~(the winning ticket is in$\{t_1\}$) $\supset$ ~(the winning

ticket is in $\{t_1, t_2\}$)…, ~(the winning ticket is in $\{t_1, t_2, …, t_{999}\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2, …, t_{999}, t_{1000}\}$)) (whereas in the WSP it has wide scope over them).

Now, suppose that we adopt option (i), i.e., that we accept the Lockean Thesis with t short of 1: option (i) clearly implies that one of (P''1)-(P''1000) is false. That is, here too, we have a cut-off point, even though of a different kind than in the (S)WSP: what I mean by a cut-off point here is that one of (P''1)-(P''1000) is false (see fn. 3). For convenience, and in spite of the differences with the (S)WSP, I will extend the use of the expression "Cut-off Point View" to the view that one of the conditionals in the NSP is false.

Where the cut-off point falls of course depends on the value of t. If we assume, as above, that $t = 0.999$ and that the 1000-ticket lottery is fair, then the false premise will be (P''2). Indeed, by the Lockean Thesis, one should believe that ~(the winning ticket is in $\{t_1\}$). However, one should not believe that ~(the winning ticket is in $\{t_1, t_2\}$) (as the latter has a probability of 0.998). In other words, even if the defender of (i) manages to avoid the cut-off point conclusion in the case of the WSP, she cannot avoid it in the case of the NSP.

This remark can be extended to a heap variant of the NSP. As it was already the case with the WSP, we can generate the NSP starting from a heap scenario instead of a lottery one, i.e., we can generate a Soritical Narrow Scope Paradox (SNSP). One just has to replace (1') with "I should believe: 1000 grains are a heap", (2') with "I should believe: ~(0 grains are a heap)" and (P''1)-(P''1000) with sentences of the form "I should believe: ~(n grains are a heap) ⊃ I should believe: ~($n + 1$ grains are a heap)". Finally, (3') must be replaced with "I should believe: ~(1000 grains are a heap)".

Here too (i.e., in the case of the SNSP too) the advocate of (i) will be obliged to say that there is a cut-off point. Where this cut-off point is will again depend on the value of t and on the specific probability distribution associated with her evidence.

As announced in section 1, the above has some interesting consequences concerning Leitgeb's account of the Lottery Paradox.[10] According to Leitgeb (2014; 2015; 2017),

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[10]At least the one presented in Leitgeb 2014, 2015, and 2017. In a very recent paper, Leitgeb (2021) has declared rationally permissible, besides the one I present here, a different account of the Lottery Paradox,

the context in which we ask ourselves whether a given ticket *n* wins and that in which we focus on the fact that some ticket will win (i.e., that the lottery has one winner) are different and allow us to set different thresholds for rational belief.

More specifically, in a context in which we focus on the fact that some ticket will win, Leitgeb's theory of belief constrains us to set $t = 1$ (and therefore to suspend our judgement on each of the tickets). Instead, a context in which we concentrate on whether a given ticket *n* will win is one in which we can set $t = 0.999$, and this will not cause the Lottery Paradox to arise, provided that we "partition" (i.e., that we subdivide) the probabilities in our distribution as imposed by the theory. According to Leitgeb, the Lottery Paradox results from fallaciously mixing premises that come from these different contexts.

Leitgeb's view entails that we are always accepting some version of the Cut-Off Point View. In the context in which $t = 1$ we should accept the cut-off point conclusion with respect to both the (S)WSP and the (S)NSP. In the context in which, instead, we ask ourselves whether some specific ticket will win (whether some specific grain turns something that is not a heap into a heap) and we are assuming $t = 0.999$, we should accept the Cut-Off Point View with respect to the (S)NSP.

It is noteworthy that in the (S)NSP the principle that is applied is not Belief Closure, but modus ponens. This is an important fact: when I was only considering the (S)WSP, the dilemma was between accepting the Lockean Thesis with *t* short of 1 on the one hand and accepting both Belief Closure and the Cut-off Point View on the other hand. Thanks to the (S)NSP we are now aware that rejecting Belief Closure is not enough to avoid the Cut-off Point View (at least not with respect to this further puzzle): rejecting

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according to which Belief Closure should be weakened. This alternative account is only permissible given a very specific version of Leitgeb's theory of rational belief, which differs from the one I consider in this paper. For space reasons, I focus here on Leitgeb's best-known account of the Lottery Paradox, the one which arises from the most famous and most widely discussed version of his theory of rational belief (viz., the version in which, as outlined above, we should believe *P* if and only if the probability of *P* remains higher than 0.5 conditional on any proposition consistent with it).

modus ponens[11] is necessary. That is, not merely Belief Closure, but a fundamental principle of classical logic. The reason is that dropping Belief Closure would allow us to block the (S)WSP, but not the (S)NSP. Instead, giving up modus ponens would solve both the (S)WSP and the (S)NSP: if modus ponens is invalid, we should reject Belief Closure; however, the opposite direction of the conditional does not hold. In other words, if we want to solve the (S)NSP, we should either endorse the Cut-off Point View or give up modus ponens.

I will not take a stand here on which of these two very radical alternatives is the best. Of course, this new dilemma could be regarded as favouring the Cut-off Point View, i.e., as a clear indication of the fact that, puzzling as they may be, cut-off points are unavoidable. However, the validity of modus ponens has been challenged in the past. Dialetheists, for instance, argue that the derivation of Q from $P \supset Q$ and P can fail, although in very special circumstances, when both P and $\sim P$ are true (see, most notably, Priest 1979 and Beall 2009). For their part, relevant logicians have questioned the validity of disjunctive syllogism (which is just modus ponens for the material conditional modulo double negation principles; see Anderson and Belnap 1975).[12]

Anyway, I will not tackle this issue here. In this paper I wanted to show that keeping Belief Closure becomes a less appealing option when one sees what happens if instead of a lottery scenario a different case is used, notably, a heap scenario. However, it is also worth noting that rejecting Belief Closure is not enough: as we have just seen, if we want to avoid the cut-off point conclusion with respect to the (S)NSP we should embrace an even more radical solution, i.e., denying modus ponens.

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[11]Or rather, modus ponens plus at least two other principles, i.e., conjunction introduction and modus tollens. Indeed, the NSP can be generated by using indifferently modus ponens, conjunction introduction and modus tollens. I have explicitly formulated the modus ponens version, but the versions in which conjunction introduction and modus tollens are used are easy to work out. As for the conjunction introduction version: let us use 1000 instances of conjunction introduction to generate a sentence conjoining all of (2') and (P''1)–(P''1000): what we get is logically equivalent to (3'). Concerning the modus tollens version, the contradiction is generated by assuming both (1') and all of (P''1)–(P''1000), and by applying modus tollens as many times as needed.

[12]I should also mention here the advocates of the so-called "degree-theoretic view of vagueness"; indeed, many degree-theorists reject modus ponens when degrees of truth are involved in the inference (see, e.g., Goguen (1969) and Machina (1976)).

The above also teaches us something important concerning the most popular norms of belief on the market. Indeed, it can be noted that they all entail the Cut-off Point View: whether we assume the Lockean Thesis (with $t = 1$ or with t short of 1), the truth norm or the knowledge norm, we end up with a cut-off point somewhere. More precisely, if we assume the Lockean Thesis with t short of 1, we end up with a cut-off point (only) in the (S)NSP. If, instead, we assume either the Lockean Thesis with $t = 1$, the truth norm or the knowledge norm, we end up with a cut-off point both in the (S)WSP and in the (S)NSP. Indeed, accepting any of these three norms makes it the case that for some ticket we should not believe that it loses/that for some grain we should *not* believe that adding it to something which is not a heap does *not* turn it into a heap. So some soritical conditional in the (S)WSP is not rationally believable. (I have made the point about the truth norm and the knowledge norm in section 4 above; for the point about the Lockean Thesis with $t = 1$, see instead section 3.) But it follows from the fact that some soritical conditional in the (S)WSP is not rationally believable that the (S)NSP has one false conditional: for some n, we should believe "~(the winning ticket is in$\{\ldots(t_n)\}$)" ("~(n grains are a heap)"), while we should not believe "~(the winning ticket is in$\{\ldots t_n, (t_{n+1})\}$)" ("~($n + 1$ grains are a heap)").

Thus, avoiding cut-off points altogether does not only require giving up modus ponens. For while denying modus ponens would block both the (S)WSP and the (S)NSP, were we to keep any of the three aforementioned norms of belief, the cut-off points would still be there. As a result, a rejection of the Cut-off Point View would require rejecting modus ponens (in order to block the paradoxes) *as well as* a quite radical rethinking of our belief norms. To put it another way, there are only two possible unified solutions to the (S)WSP and the (S)NSP: the cut-off point solution and the cut-off point free solution. For the latter to obtain, denying modus ponens is necessary, but not sufficient: providing a cut-off point free solution to the (S)WSP and the (S)NSP requires that we reject modus ponens to block the arguments' paradoxical conclusions. However, we should also renounce (or modify) the most entrenched belief norms on the market, so that all the premises of the (S)WSP and the (S)NSP are true (i.e., that no cut-off points are generated).

Of course, this result too could be regarded as favouring the Cut-off Point View, i.e., as proof of the fact that cut-off points cannot be avoided. On the contrary, I think that the cut-off points' opponents could take up the challenge. Notably, it seems to me that the challenge can be broken into three "smaller" ones: the cut-off points' enemies should (i) propose a suitable non-classical framework in which the (S)NSP can be dealt with; (ii) come up with weaker (but still sensible) coherence constraints on rational belief (weaker than Belief Closure);[13] iii) propose a norm of belief which can be naturally associated with such constraints, and which does not entail cut-off points. These certainly are hard challenges, but hard is not impossible.

In fact, independent reasons can be provided to the conclusion that we should endorse a cut-off point free solution to both the (S)WSP and the (S)NSP: Lissia (2020) has convincingly argued that Belief Closure is to be blamed for the Lottery Paradox. I will not recall her argument here, as going through its details would require a whole paper. However, if rejecting Belief Closure is the right reaction to the (S)WSP, the cut-off point free solution is the only remaining unified solution.

Before concluding, one further clarification might be helpful. Throughout this paper I have been taking for granted that cut-off points are undesirable. However, someone could object that it is not that clear, actually, that there is something wrong with them. After all, most approaches to the classical version of the Sorites accept the existence of sharp cut-off points (even though, as in epistemicist proposals, we are unable to locate them, or, as in supervaluationist accounts, it is indeterminate where they are).

The reason why I presuppose, in this article, that cut-off points are unpalatable is that I take this to be the default assumption regarding soritical paradoxes. After all, if cut-off points were not deeply counterintuitive, we would not regard the Sorites as a paradox in the first place. And it is no coincidence, I take it, that accounts which accept the existence of cut-off points typically include sophisticated stories which are meant to justify this acceptance.

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[13]An interesting attempt to provide a compelling alternative to Belief Closure as a coherence requirement for rational belief can be found in Easwaran and Fitelson 2015.

However, I do realize that a number of (classical) Sorites scholars would probably be ok with the idea that a cut-off point is present in the (S)WSP and/or in the (S)NSP.[14] Now, their perspective is perfectly compatible with what I have said in this paper. More specifically, it seems that the most coherent choice for these authors would be to give up the Lockean Thesis (or to accept it with $t = 1$): as we have seen, if we drop it (and endorse some other belief norm among the most popular ones), or set $t = 1$, we end up with a cut-off point in both the (S)WSP and the (S)NSP.[15] (Of course, this choice would

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[14]This clearly seems to be the case of Timothy Williamson. Indeed, Williamson endorses the knowledge norm of belief (see fn. 7 above; Williamson 2000). This means that, according to him, a cut-off point will be present in the WSP: as pointed out in section 4, we know both that the winning ticket is in $\{t_1, t_2,\ldots, t_{1000}\}$ and that ~(the winning ticket is in $\{\}$), so (1') and (2') are both true. However, we do not know the conditionals "~(the winning ticket is in $\{\}$) ⊃ ~(the winning ticket is in $\{t_1\}$), ~(the winning ticket is in$\{t_1\}$) ⊃ ~(the winning ticket is in $\{t_1, t_2\}$), etc. Now, if he wants to extend this account to the SWSP, Williamson should say that a cut-off point is present in it too: we clearly know (1) 1000 grains are a heap and (2) ~(0 grains are a heap); however, there must be a first P-conditional (i.e., a first conditional of the form "~(n grains are a heap) ⊃ ~ ($n + 1$ grains are a heap)") such that we do not know it.

Note that if Williamson aims at being coherent with his epistemicist approach to the classical Sorites (Williamson 1994), he must claim that we do not (and cannot) know where the cut-off point is: there must be a first P-conditional such that we do not know it, but we cannot know which it is. Interestingly, this is perfectly compatible with Williamson's views on the way we should handle versions of the (classical) Sorites involving "know": in his perspective, "know" is a vague predicate like any other; we stop knowing at some point, but we cannot know where: we do not know that we no longer know. (For Williamson's discussion of soritical paradoxes involving "know", see Williamson 1994; for his argument against the so-called "introspection principle", or "principle KK", see, most prominently, Williamson 2000.)

[15]An anonymous reviewer asks whether it would be incoherent for someone to accept both the Lockean Thesis with t short of 1 and a supervaluationist account of the classical Sorites. In fact, one of the points I make in this section is that defending the Lockean Thesis with t short of 1 is already incoherent (quite independently of whether one also accepts supervaluationism about vagueness). The reason, as I argue, is that setting the threshold below 1 does not allow for a unified solution to the (S)WSP and the (S)NSP: the defender of the Lockean Thesis with t short of 1 is committed to accepting both a cut-off point free solution to the (S)WSP *and* the existence of a cut-off point with respect to the (S)NSP.

Now, if besides accepting the Lockean Thesis with t short of 1, one also accepts a supervaluationist account of the classical Sorites, further complications arise. In a nutshell: if she is consistent, the supervaluationist advocate of the Lockean Thesis with t short of 1 must be willing to accept a cut-off point free solution to the SWSP. (Indeed, if we accept a Lockean threshold lower than 1, in the WSP we end up

be in contrast with the result I have just mentioned (Lissia 2020), according to which Belief Closure is responsible for the Lottery Paradox, and which strongly supports the idea that we should embrace a unified cut-off point free solution to the (S)WSP and the (S)NSP.)

6. Conclusion

I have shown that the Lottery Paradox is just an epistemic version of the Sorites (one involving rational belief), and I have discussed the consequences of this result. I have focused, in particular, on the potential disadvantages of keeping Belief Closure. Namely, keeping such a principle entails what I have called "the Cut-off Point View". I have also argued that the debate on the validity of modus ponens should play an important role in discussions on the Lottery Paradox. Indeed, as we have seen, there are only two possible unified solutions to the (S)WSP and the (S)NSP: the first consists in endorsing the Cut-off Point View with respect to both arguments; the second involves rejecting modus ponens.[16] That is, in order to provide a unified account of the two paradoxes we should

with a cut-off point free solution, and so it seems that it would be incoherent to end up with a cut-off point in the heap version of the same problem (see my discussion of the (S)WSP above).)

However, this fact has consequences for the supervaluationist account of the classical Sorites: it is well known that supervaluationism claims, with respect to the classical Sorites, the existence of a cut-off point, as it maintains that one of the P-conditionals is not supertrue (and hence not true, in a supervaluationist framework). So, if she wants to be consistent with her classical Sorites account, it seems indeed that the supervaluationist cannot accept the Lockean Thesis with t short of 1, since it would be weird to end up with a cut-off point free solution to an epistemic version of the Sorites (namely, the SWSP) while simultaneously claiming the existence of a cut-off point in relation to the classical Sorites. Both paradoxes are soritical paradoxes, and a unified treatment would seem to be required. In short, then, my view is that the supervaluationist advocate of the Lockean Thesis with t short of 1 is not in a comfortable position. Given that, as I have suggested, defending the Lockean Thesis with t short of 1 is already problematic on its own, this makes her position even more problematic.

[16]Of course, as specified above (see fn. 8), we could also provide a unified account of the two puzzles by denying that we should believe "~(the winning ticket is in { })"/"~(0 grains are a heap)" or by claiming that we should in fact believe "~(the winning ticket is in $\{t_1, t_2,…, t_{999}, t_{1000}\}$)" (i.e., "the winning ticket is not in the set which contains the totality of the lottery tickets")/"~(1000 grains are a heap)". As also

either endorse a cut-off point solution or a cut-off point *free* solution to both. Moreover, I have noted that for our (unified) solution to be cut-off point free denying modus ponens is not enough; we also have to reject (or modify) the three most popular belief norms on the philosophical market. That is, we should both deny modus ponens (to block the paradoxes) and reconsider our most entrenched norms of belief. Finally, I have briefly mentioned a result supporting this last option.

However, for the time being, I take it to be the main lesson of this paper that the Cut-off Point Problem (i.e., the question whether our solution to the Lottery Paradox and its variants should allow for cut-off points) ought to play a key role in the debate on the Lottery Paradox. In the literature on the Sorites, this question has always been central. The present article is a plea for writers on rational belief and rational degrees of belief to focus on this issue, which has been neglected so far.

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mentioned in fn. 8, Peter Unger (1979) has defended the thesis that "~(1000 grains are a heap)" is true. However, I decided to put aside this very implausible thesis here. Note that in the lottery case this thesis seems even more implausible.

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