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REVIEW

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The first three volumes of Alan Turing's *Collected Works*, entitled *Mechanical Intelligence*, *Pure Mathematics*, and *Morphogenesis*, and edited respectively by D.C. Ince, J.L. Britton, and P.T. Saunders, were published in 1992. Now, forty-seven years after Turing's death, the fourth and final volume has appeared as well.

The reasons for the delay are explained in Yates's preface to this volume. There was, as he says (p. IX), "no single reason": Gandy, the original editor, began writing draft commentaries on the works that comprise parts I and II of the book in 1978, but he became bogged down in his efforts at revision. In hopes of expediting publication, Yates was invited to join him as co-editor in 1991. Four years later Gandy suddenly and unexpectedly died. And then, in 1996, the National Security Agency of the United States declassified a large amount of material on cryptanalysis that had previously been kept secret, including an item labelled "Turing's Treatise on the Enigma." Consequently it was decided to add a third part to the work "to take some new directions and tidy up some loose ends."

Unfortunately, in many respects the volume bears the marks of its difficult birth. There is a widespread lack of uniformity in the citation of references, in the ways in which different articles are reproduced, and in the amount of background information provided in the accompanying commentaries; the table of contents does not accurately reflect what is and is not included in the volume; unlike the other volumes in the series, there is no index of any kind; there are numerous instances of inexcusably slipshod editing; and, far from "tidying up" loose ends, Part III gives the impression of being the afterthought that it was—a haphazard collection of disparate and fragmentary texts that is both tantalizing and frustrating.

In any 'Collected Works' endeavor there are many critical decisions to be made, depending upon the nature of the documents to be included (essays, letters, drawings, tables, computations), their form (published or unpublished, printed or handwritten, polished or rough), availability and significance: How inclusive is the edition to be? If selective, on what basis are items to be excluded? How should the volumes, and the items within them, be organized (by date, subject, form of communication, or what)? Should articles be reproduced in facsimile or by transcription? Should they be abridged or edited? Should reference citations be made uniform? And so on. The usefulness of the resulting compilation depends critically both on what decisions are made and how well they are explained and justified to the reader.

How well, then, does Turing's *Collected Works*—and this fourth volume in particular—measure up? Certainly, given the wide range of Turing's intellect, it made sense to organize the four volumes, and the contents of the present one, thematically by subject. Readers and buyers can thus more easily find what interests them. But what to do with documents that fall into more than one category? One can either allow overlap among the volumes (preferably, as was done with Turing's paper "Solvable and unsolvable problems," which appears in both *Mechanical Intelligence* and *Pure Mathematics*), or make an arbitrary decision as to where particular items belong (as in placing one of Turing's papers on programming in the present volume and another in *Mechanical Intelligence*). But one shouldn't sometimes adopt one alternative and sometimes another.

Furthermore, since these volumes are not numbered, are edited by different individuals, and are titled according to their contents, it may not be clear to readers or librarians that the separate volumes form part of a collective edition. That should not be a problem, since all the volumes bear the designation *Collected Works of A.M. Turing* prominently on their covers and title pages; nevertheless, a number of libraries catalog them only by their individual titles. It is therefore beneficial that each of the four volumes contains a listing of the titles and contents of all of the others. (Each volume also begins with a common preface to the series and a chronology of Turing's life. Oddly, though, the obituary memoir of Turing that M.H.A. Newman wrote for the Royal Society, which contains a survey all of Turing's work, appears only at the end of this last one.)

The papers in *Mathematical Logic* are divided into three parts: "Computability and Ordinal Logics," "Type Theory" and "Enigmas, Mysteries and Loose Ends." The first consists entirely of published papers from the years 1937–38: the famous "On Computable Numbers, with

an Application to the Entscheidungsproblem"; the published correction thereto; "Computability and λ-definability"; "The p-function in λ-K Conversion"; and the long (and difficult) "Systems of Logic Based on Ordinals." The section as a whole is prefaced by an informative historical introduction by Solomon Feferman, adapted from his 1988 paper "Turing in the land of O(z)," and Feferman also provided a detailed commentary (adapted from the same source) on Turing's paper on ordinal logics. Prefaces to all of the other papers in parts I and II are unsigned. They are presumably by Gandy (or, in some cases perhaps, by Yates or by Gandy and Yates together) and, other than that for "On computable numbers," are rather brief.

Part II begins with a "General Introduction to Turing's Work on Type Theory" (also unsigned), followed by the texts of three published papers: "A formal theorem in Church's theory of types" (written jointly with M.H.A. Newman); "The use of dots as brackets in Church's system"; and "Practical forms of type theory"—the first two from 1942, the last from 1948. Then—according to the table of contents—there are three (or should it be five?) unpublished papers by Turing: "Some theorems about Church's system" (three unpublished manuscripts from 1941); "Practical forms of type theory II" (composed 1943–44); and "The reform of mathematical notation" (from 1944–45). In fact, however, only the last of those items is transcribed *in extenso* (14 extant pages of an original 18-page typescript), and its text begins on page 215, not 211, as the table of contents says, following an editorial preface that is not mentioned there; the other items are merely *described* (again, in unsigned commentary), with only the briefest of quotations from them.

The three unpublished manuscripts from 1941 are:
I. "Consequences of the Peano axioms" (comprised of pages numbered by Turing as 37–43);
II. "Finite models of Church's and Zermelo's systems" (with pages numbered 60–73 and 84); and
III. "Some theorems about Church's system" (19 consecutive unnumbered pages).
They are discussed in the order III, I, II , but only three paragraphs of III are quoted (including Turing's proof that every formula of the typed λ-calculus has a normal form), nothing at all from I, and just one paragraph from II.

As for "A practical form of type theory II," said (p. 207) to consist of "81 unnumbered pages of typescript together with 22 manuscript pages inserted in various places," the anonymous commentator has provided a

mere two pages of commentary, including quotations of only three *sentences* of text. By way of justification for this perfunctory treatment, readers are told (p. 208) that Turing regarded the manuscript as the first draft of a paper he had once intended to publish, but that as time went on he realized that "the labour of preparing a final draft would have been considerable," that "the correctness of the results was not in question" and that "the methods used were fairly straightforward" (and tedious). Presumably, the editors felt it necessary to mention the paper, as well as the three manuscripts discussed above, out of compunction to make readers aware of *all* the writings on type theory that Turing left behind. But they nowhere say so explicitly, and it is not clear, either from online information about Turing's literary remains or the typewritten inventories thereof produced in 1977 and 1985 by the Contemporary Scientific Archives Centre of the Library of King's College, Cambridge, whether they have in fact done so.

Even more must one wonder whether Part III of the volume makes reference to all that Turing wrote on the subject of cryptanalysis. Given that the first item discussed therein ("Turing's treatise on the Enigma") was only declassified in 1995, it seems likely that other material may also eventually be released. In any case, Part III is a very mixed bag (whose contents, once again, are not accurately described in the table of contents at the front of the volume). It begins with a brief preface by Andrew Hodges on Turing's work on the Enigma cipher machine, which is informative and well written, but brief. Following that are six excerpts from Turing's treatise itself (seven pages altogether, including one diagram), reproduced as reduced photographic *illustrations* (not photo-offset, as in Part I, or transcribed, like the unpublished excerpts in Part II). The accompanying commentary suggests that the excerpts were selected to illustrate certain key ideas due to Turing ("a crucial idea that defeated the plugboard connection" (p. 230), "how to 'scan' the electrical output from the Bombe to detect the possibility of a correct rotor position" (p. 234), and "the context in which Turing made his successful deduction of the bigram key-system in the naval Enigma" (p. 240)) or to G.W. Welchman (his "idea for exploiting the self-inverse property of the Enigma" (p. 238)). Because of the reduced size, Turing's messy typing, and some faintness and lack of sharpness in the photographic reproductions, however, some of the typescript pages are not readily legible. And to understand the content of the excerpts one must possess a good deal more background knowledge about the Enigma machine than Hodges' preface provides. In the very first excerpt, for example, the German term *Stecker* ("plug," or "adaptor") occurs, without prior explanation; not until the commentary to the third

excerpt (p. 234) is the reader told that the phrase "each Stecker value" refers to "each of the 26 different plugboard hypotheses." And in the preface itself Turing's "*Banburismus* method," to which great importance is ascribed, is left wholly unexplained. Only in later commentary by I.J. Good on another paper of Turing's (dealing with minimum cost sequential analysis) does the reader learn (p. 256) that *Banburismus* was "a Bayesian sequential procedure ... [that] made use of what is now called the Bayes factor method."

The tantalizing tidbits about Turing's contributions to the Enigma decipherment effort are followed by a discussion by Martin Campbell-Kelly of Turing's papers on programming—all but one of which are characterized at the outset as "an intellectual disappointment." In contrast to Turing's writings on computer hardware, only two of his papers on programming appear to have been published. One ("Checking a large routine"), dealing with program verification, appeared originally in 1949, was reprinted in 1984 in the *Annals of the History of Computing* and (as Campbell-Kelly inexplicably fails to mention) was included in the *Mechanical Intelligence* volume of these *Collected Works*. The other, entitled "Local programming methods and conventions," occupied less than a page in the proceedings of the Manchester University inaugural computer conference and is reproduced in the present volume as another reduced photographic illustration. This time the type, though miniscule, poses no serious legibility problem; but the paper is labeled only as "Fig. 3" in the midst of Campbell-Kelly's commentary, and so is not listed at all in the volume's table of contents. Instead, what is listed there as the next item is "Excerpts from an original paper" (no title given)—a reference to a four-page transcribed excerpt from Turing's "Programmer's handbook for the Manchester Electronic Computer Mark II," to which most of Campbell-Kelly's commentary refers.

The last item by Turing included in the volume is a one-paragraph abstract, transcribed from his unpublished manuscript on minimum cost sequential analysis, which is followed (not preceded, as in all other cases) by the aforementioned commentary by I.J. Good. But the volume does not end with that. Instead, there is another brief essay by Hodges, "The nature of Turing and the physical world," which serves as commentary to a letter of June 1954 from Robin Gandy to Max Newman, in which Gandy communicated "a miscellany of information" (p. 265) about Turing to Newman for the latter's use in writing Turing's obituary for the Royal Society. Both the letter and Newman's memoir are reproduced in full, followed by a Bibliography, the list of contents of the other volumes of the *Collected Works*, and finally an

Appendix listing five further items, not included in the *Machine Intelligence* volume, that the editors felt "deserve[d] mention here for the sake of completeness."

Given the fragmentary nature of most of the excerpts in Part III and the lack of any adequate explanation as to how they were chosen, such concern for completeness seems a bit hollow, to say the least. And if mere mention of all of Turing's writings is what was aimed for, why did the editors not provide a comprehensive bibliography of them?

As it is, the Bibliography at the back of the volume does not even include all of Turing's papers that are cited within the volume itself! There are, rather, no fewer than six *other* bibliographies that the reader must contend with: one following Turing's paper "Systems of logic based on ordinals" and another after his "A formal theorem in Church's theory of types"; one accompanying Hodges' preface to Turing's treatise on the Enigma; one each with the commentaries by Campbell-Kelly and Good; and one that forms part of Newman's memoir. No attempt has been made to assure uniformity of citations in cases where references overlap, nor have references to papers that were "forthcoming" in the 1940s been updated.

In part, of course, the profusion and confusion of references is a result of the decision to reproduce Turing's published works and Newman's memoir as facsimiles of the originals. Such reproduction by photographic means assures that no new errors will be introduced into the texts. But any errors, omissions, or obsolescent references in the originals will be perpetuated, and it ought to be part of the editors' task to point out such things. Here, however, only mathematical mistakes seem to have been noted.

In any case, there is no excuse for not at least incorporating all the commentators' references into the Bibliography at the back of the volume. Otherwise, what is its purpose? One might presume that it is a catch-all for everything *not* cited in any of the other reference lists. But even that is not the case: For example, Post [1965], Hilbert and Ackermann [1928], and Davis [1987] (perhaps a slip for Davis [1982]) are all cited in the preface to the computable numbers paper, but none of them appears in any of the bibliographies; and likewise for Skolem (1922), Bernays (1937–) and von Neumann (1925), all of which are cited in the commentary to "Some theorems about Church's system." (The inconsistency between the use of brackets in the former citations and parentheses in the latter is here preserved.)

References that *are* given in the Bibliography are often inaccurate or incomplete: in eight instances, page numbers for articles in books are omitted; no publisher is given for the book listed as Hilbert and

Ackermann 1950; for the items listed as Eckert 1980 and Mauchly 1980—evidently intended to refer to two articles in one and the same book—only the book title is given (*The ENIAC: A History of Computing in the Twentieth Century*), and it appears to be incorrect. (The WorldCat online bibliographic database, e.g., lists no such title.) And for the third volume of Gödel's *Collected Works* both the editors and the title are listed, wrongly, as being identical with those for the second.

It is hard to overlook such careless lapses, and they raise doubts about the accuracy of information elsewhere in the volume. What, for example, of the transcriptions from Turing's unpublished works? No apparatus of any kind is employed to indicate editorial interventions, so one cannot tell how closely the transcriptions conform to the originals. Nor can one ascertain whether errors in those texts are Turing's own or the editors'. A particularly egregious example occurs on p. 216, in the midst of Turing's paper on the reform of mathematical notation, where the phrase "Russellian *Weltanschauung*" is transmogrified to "Russelian Weltenscheung."

In sum, the volume under review brings together a variety of writings from widely scattered sources, some of which are published for the first time. Yet at the same time it is an exemplar of how not to edit a volume of collected papers. Its manifold deficiencies reflect ill on both its editors and publisher, detract from its usefulness, and certainly do not justify its high price. It is true, as the editors admit, that Turing's own life and writings exemplified an often haphazard organization and inattention to detail. But perpetuating those traits in his *Collected Works* is surely not a proper way to pay tribute to his genius.

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