Drainage Basins, Channels, and Flow Characteristics of Selected Streams in Central Pennsylvania

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By LUCIEN M. BRUSH, JR.

PHYSIOGRAPHIC AND HYDRAULIC STUDIES OF RIVERS

GEOLOGICAL SURVEY PROFESSIONAL PAPER 282-F

A study of the influence of the geologic character of drainage basins upon the hydraulic characteristics of 16 natural stream channels at 119 sampling stations

UNITED STATES DEPARTMENT OF THE INTERIOR

STEWART L. UDALL, *Secretary*

GEOLOGICAL SURVEY

Thomas B. Nolan, *Director*

REPRINTED 1963

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SYMBOLS

A	cross-sectional area of channel	Q	discharge
A_t	drainage area	Q_1, Q_3	quartile size
a	constant; and length of long axis of particle	$Q_{2.3}$	mean annual flood
a_ϕ	phi skewness	Q_b	bankfull discharge
b	exponent; and length of intermediate axis of particle	r	correlation coefficient
C_1	constant	S	slope of water surface
c	constant; and length of short axis of particle	S_0	Trask's sorting coefficient
d	depth of channel	S_{yx}	standard error of estimate
D_t	particle size	v	velocity
E	elevation	W	width of drainage basin
e	base of natural logarithms	w	width of stream
F	fall; and ratio of variances	x	coordinate direction
f	function; and exponent	y	coordinate direction
H	height	z	exponent
H_o	height of origin	α	exponent
K	constant	γ	specific weight of water
k	constant	ρ	constant
L	length of stream	σ_ϕ	phi standard deviation
m	exponent	τ	shear stress
N	size	ϕ	$\log_2 N$
p	constant	ψ	intercept sphericity

PHYSIOGRAPHIC AND HYDRAULIC STUDIES OF RIVERS

DRAINAGE BASINS, CHANNELS, AND FLOW CHARACTERISTICS OF SELECTED STREAMS IN CENTRAL PENNSYLVANIA

By Lucien M. Brush, Jr.

ABSTRACT

The hydraulic, basin, and geologic characteristics of 16 selected streams in central Pennsylvania were measured for the purpose of studying the relations among these general characteristics and their process of development. The basic parameters which were measured include bankfull width and depth, channel slope, bed material size and shape, length of stream from drainage divide, and size of drainage area. The kinds of bedrock over which the streams flow were noted.

In these streams the bankfull channel is filled by flows approximating the 2.3-year flood. By measuring the breadth and mean depth of the channel, it was possible to compute the bankfull mean velocity for each of the 119 sampling stations. These data were then used to compute the downstream changes in hydraulic geometry of the streams studied. This method has been called an indirect computation of the hydraulic geometry. The results obtained by the indirect method are similar to those of the direct method of other workers.

The basins were studied by examining the relations of drainage area, discharge, and length of stream from drainage divide. For the streams investigated, excellent correlations were found to exist between drainage area and the 2.3-year flood, as well as between length of stream from the basin divide and drainage area. From these correlations it is possible to predict the discharge for the 2.3-year flood at any arbitrary point along the length of the stream.

The long, intermediate, and short axes of pebbles sampled from the bed of the stream were recorded to study both size and sphericity changes along individual streams and among the streams studied. No systematic downstream changes in sphericity were found.

Particle size changes are erratic and show no consistent relation to channel slope. Particle size decreases downstream in many streams but remains constant or increases in others. Addition of material by tributaries is one factor affecting particle size and another is the parent material. Wear does not appear to account for some of the changes noted in particle size in a downstream direction. Comparison with laboratory studies indicates that at least in some streams the downstream decrease in size is much greater than would be expected from wear alone.

The type of bedrock underlying the channels included in this study appears to affect both channel slope and particle size. For a given length of stream, a stream channel underlain by sandstone tends to have a steeper slope and larger bed material than channels underlain by shale or limestone. Hence, a stream which heads in sandstone and ends in limestone tends to have a more rapid decrease in slope and particle size than a stream heading in limestone and ending in sandstone. The association of steep slopes and small particles for limestone channels implies that slope and particle size may show a vague correlation between lithologic groups although no correlation may exist within a given lithologic type.

In addition to the effect of bedrock on slope and particle size, there is some evidence that channels in limestone or dolomite have a slightly smaller cross section at bankfull stage than channels in shale or sandstone.

Near the headwaters of many of these streams, a deposit of periglacial rubble affects the slope and bed material size. Some of the debris contains residual boulders which are too large to be moved by ordinary floods and, therefore, impose larger particle sizes in the bed of the stream. The addition of this very coarse debris to the bed material is another example of the influence of geologic factors on stream channels even though the channel consists of unconsolidated debris instead of bedrock.

The influence of geologic factors noted in selected streams in central Pennsylvania may not be directly applicable to areas other than the Appalachian Mountains, but the general process is no doubt similar in most areas. In large alluvial valleys bedrock cannot be much of an influencing factor; yet large, thick alluvial deposits and terraces are in a sense "bedrock" materials upon which the stream works to form the landscape.

INTRODUCTION

Recently, considerable progress in the subject of fluvial morphology has been achieved by quantitative studies of streams, drainage basins, and geology. Some of the concepts which have evolved as a result of quantitative studies are the hydraulic geometry (Leopold and Maddock, 1953), the hypsometric integral (Strahler, 1952), and the effect of the kind of bedrock over which streams flow on their longitudinal profiles (Hack, 1957). Because these studies contain basic general ideas and quantitative data, they may be extended easily by later studies. The aim of this paper is to integrate the results and approaches of several previous investigations by earlier authors and, where possible, to extend the understanding of the relations among hydraulics, the characteristics of drainage basins, and geologic factors.

To explain the need for combining a few of the previous observations pertaining to fluvial morphology, it is necessary to review briefly portions of previous studies dealing with the hydraulic geometry of streams and longitudinal profiles.

The conception of hydraulic geometry as stated by Leopold and Maddock (1953, p. 18) is as follows:

The channel characteristics of natural rivers are seen to constitute then, an interdependent system, which can be described

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by a series of graphs having simple geometric form. The geometric form of the graphs describing these interactions suggests the term "hydraulic geometry."

The channel and flow characteristics considered were width, depth, velocity, discharge, suspended sediment load, slope of the water surface, and the Manning roughness factor. Although Leopold and Maddock were the first to express these characteristics of natural channels in terms of a hydraulic geometry, many others, including Lacey (1930), Shulits (1937), Kimball (1948), and Rubey (1952), demonstrated or implied that power function relations exist. The data collected by Leopold and Maddock were taken primarily from western streams. Wolman (1955) found similar exponential relations for Brandywine Creek, Pa., and Leopold and Miller (1956) have shown that similar relations hold for arroyos in the Southwestern United States.

The general idea of hydraulic geometry is reasonably well established, but many ramifications have not been considered. The effect of the character of the bedrock and of the particles of bed material on the hydraulic geometry has not been studied. Size of bed material has not been studied as a primary variable with the hydraulic geometry. Wolman (1955) presented some data on bed material, but the samples were too few to show any significant trends. Hack (1957) made a detailed study of the characteristics of bed material in Virginia and Maryland but did not attempt to relate these characteristics to details of the hydraulic environment. There is evidence indicating that considerable order is exhibited by the hydraulic and channel environment of streams. Few data are available, however, for determining whether the constants and coefficients utilized in describing this order vary in relation to the geologic setting. The deviation of individual points from the general empirical relations has not been studied in detail but may be related to some control other than the hydraulic characteristics of streams.

As opposed to the hydraulic approach, numerous workers have made detailed investigations of the geologic and morphologic features of drainage basins. A few of the more recent investigations which were designed to study characteristics of drainage basins and the interrelations of geology and topography are those of Strahler (1952, 1954), Miller (1953), Schumm (1954), and Hack (1957). With the exception of Hack's study, the primary aim of these papers was to present basic data and hypotheses relating the physical characteristics of the basin, longitudinal profiles, and valley slopes to other features of the basin and to its geology. Hack attempted to show that geology affects the hydraulic regimen, but the emphasis of his work

was not on hydraulics. Furthermore, where geology has been considered, concentrated study has been directed generally toward areas of uniform structure or homogeneous rocks. The relations of the various drainage basin and channel characteristics are not well known for areas of complex geology.

Investigations of basin morphology reveal a certain amount of order between variables such as slope, length of stream, size of drainage area, and kinds of rocks within each area studied. It is not known, however, whether the same order found in studies of small areas exists in larger areas. The relative importance of geology and hydraulics in determining both the characteristics of natural channels and the development of drainage basins requires much additional study.

Although many qualitative observations suggest that certain relations between the geologic and hydraulic characters exist, sufficient quantitative data are lacking to establish the nature of these relations. In an attempt to supply some of these data this study of the geology, hydraulics, and basin characteristics of 16 streams in central Pennsylvania was made.

ORDER OF DISCUSSION

The first part of this study describes the physical properties of individual streams and drainage basins. The data presented are the foundations upon which the succeeding parts are developed. Next, the composite or combined geologic and hydraulic characteristics, including hydraulic geometry, basin-form, and particle-size relations of all the basins and streams were investigated. The final part deals with the effect of the geologic character of the basins on the relations found in the preceding parts.

ACKNOWLEDGMENTS

GENERAL DESCRIPTION OF THE AREA

An area in central Pennsylvania was chosen for study, the boundaries of which are outlined in figure 88. Most of this area, covering 1,425 square miles, is in Centre County. All of it lies within the Susquehanna River basin.

The average annual precipitation for central Pennsylvania is approximately 39 inches per year[1] and

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[1] Based on the means of Weather Bureau 75-year records.

ranges from 25 to 55 inches. The average annual runoff is about 15 to 20 inches per year (Langbein and others, 1949). Heavy rainfall may be expected during any month, but most frequently occurs during March or April. The mean temperatures for January and July are 29° F and 70° F respectively (Linsley, Kohler, and Paulhus, 1949). A generally amenable climate produces the rather luxurious mixed deciduous forest cover of oak, hemlock, and pine.

Farming is the principal occupation and is confined for the most part to the valleys. Fortunately, at least for purposes of this paper, very few dams or other

FIGURE 88.—Location of area of investigation, central Pennsylvania. The shaded part represents the study area.

cultural features that might disrupt the natural flow of the streams have been constructed.

The Valley and Ridge province of the Appalachian Highlands in which the study was made is characterized by long linear ridges and valleys. The trend of these lineaments is southwest. Part of the Allegheny Plateau province is included in the northernmost portion of the area studied; the natural boundary between these two provinces is the so-called Allegheny Front which transects several of the streams investigated. Local relief is roughly 1,200 feet and the maximum relief is close to 1,800 feet. For a view of the general physiography see plate 6A.

Trellis drainage patterns prevail in this region of long linear ridges and valleys, but in some of the broad valleys a dendritic system has developed. Each drainage basin exhibits a slightly different shape (see pl. 5). Nevertheless, two shapes of drainage basins predominate. If the trunk stream parallels the valley, the basin is generally elongated in this direction; but if the trunk stream flows perpendicular to the trend of the valleys and ridges, the basin is generally short and rotund.

The streams studied range in length from 4 to 35 miles. The largest stream included is the upper 35 miles of the Little Juniata River; the smallest is Reeds Run, which measures 3.8 miles from drainage divide to

mouth. The headwaters of the streams possess steep gradients and rather coarse bed material (pl. 6B). Coarse bed material is also present in larger streams where outcrops of bedrock occur in a stream (see pl. 6C).

Flood plains composed of alluvium tend to form downstream, but are sporadic and differ appreciably in width. Terraces are rare, but at a few reaches berms do occur. The berms are not continuous for any great distance along the stream, and commonly occur on only one side of the channel.

The stabilized boulder fields and rock streams near the summits of many mountains have an important effect on the nature of flow of rivers studied. Presumably, these features (pl. 6D) are relics of intense periglacial action during the Pleistocene (Smith, 1948). The area studied is only a few miles southeast of the limits of Pleistocene glaciation and is free of glacial deposits such as moraines and outwash. Quite commonly the bouldery debris at the summits grades into finer material toward the valleys. This residuum is of considerable interest because it is possible that a portion of the stream bed may be a relic of the Pleistocene epoch. The importance of this phenomenon is discussed in a subsequent section.

Detailed geologic maps were not available, and it was beyond the scope of this investigation to map so extensive an area. Therefore, the geologic map of Pennsylvania (Stose and Ljungstedt, 1931) was used as a base map. The stratigraphic nomenclature and age designations used on the map and the columnar section do not necessarily follow current usage of the U.S. Geological Survey. The rocks range in age from Cambrian to Pennsylvanian, but the majority of the sampling stations are in areas of Ordovician, Silurian, and Devonian outcrops (Butts and Moore, 1936).

The rocks of the area are all sedimentary rocks and their weathering or erosional products. Limestone and dolomite are the predominant rocks in the valleys. Sandstone and shale tend to occur on the ridges or valley slopes, but shale underlies a considerable portion of some valleys. The most resistant detrital rocks are provided by the sandstones and conglomerates of the Tuscarora, Oswego, Pocono, and Pottsville formations and the least resistant rocks by the shales, limestones, and dolomites of the Hamilton, Trenton, Nittany, and Bellefonte formations.

Structure plainly controls the linear trends of the ridges and valleys. Most of the area is characterized by plunging anticlines and synclines that trend southwest. On the plateau, the strata are very nearly horizontal. Both normal and thrust faults are present. The strike of the thrust planes is generally parallel to the axes of the folds. There are several prominent joint systems, but these have not been studied in detail.
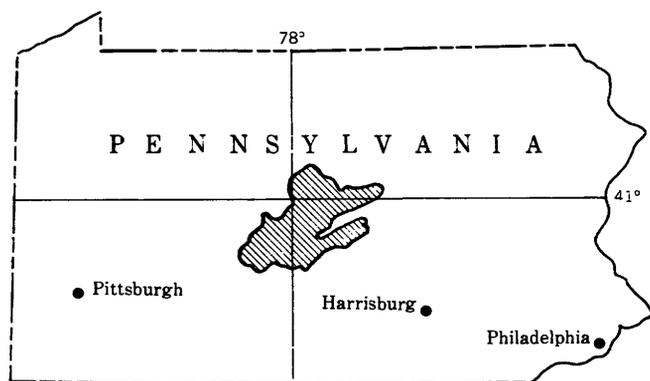
PROCEDURE

Sixteen streams were studied and 119 sampling stations were established. The procedure for locating sampling stations was arbitrary, although care was taken to spread the points fairly evenly along stream courses. Approximate positions of sampling points were first located on a map. The final field location was made by selecting a riffle near the point on the map as the sampling point. A riffle is defined here as a segment of the stream bed, generally less than 100 yards in length, possessing a gradient greater than that of adjacent segments of the stream bed and is accompanied by local increase in the slope of the water surface. Flow over the riffle is characterized by a lesser depth and greater velocity compared to conditions of flow in the pools above and below the riffle.

Stations were located at riffles for two reasons. First, if streams are to be compared on the basis of the data collected at each station, it is best to have these sampling points located in similar environments. An alternative would be to measure in pools, at meanders, and at straight reaches. Second, because a riffle usually consists of loose debris capable of being moved if discharge is sufficient, it is very likely to represent a larger percentage of bed load than would other bed segments, such as pools. However, it should be noted that some riffles are due to the presence of bedrock outcrops in the stream. These riffles were avoided in the collection of data for this study.

The final selection of a station was made in the field by finding the riffle nearest the predesignated map point; one was usually within 200 yards. In headwaters, channels are not always divided into readily discernible pools and riffles. Where it was impossible to find a distinctive riffle, the field station was located as close to the map point as possible.

After the sampling station was located, the following measurements and observations were made in the field:

1. Bankfull width was measured by stretching a measuring tape from one bank to the other.

2. Bankfull depths were measured by determining the distance from the taut tape to the channel bottom at short intervals across the channel. Hence, a mean bankfull depth could be calculated and the shape of the channel could be plotted.

3. Particle size on the riffle was measured by the Wolman (1954) method of sampling coarse material. Because the riffles in this area are usually composed of pebbles and cobbles, the method is applicable. Sampling was done by picking up individual particles at the intersections of previously established grid lines. In order that shape might be considered, three mutually perpendicular axes were recorded to the nearest millimeter. At least 60 pebbles were included in each sample.

4. Notes were made on the geologic and general characteristics of the sampling site.

In the office, measurements of the following were made: 1. Distance of the sampling station from the headwater divide. 2. Drainage area above the station. 3. Mean gradient of the reach at each station determined from topographic maps. The length of the reach used for this determination ranged from 0.5 to 2.0 miles on maps having a scale of 1:62,500.

The kind of bedrock at the station was determined from geologic maps where field data were insufficient.

CHARACTERISTICS OF INDIVIDUAL STREAMS AND DRAINAGE BASINS

To determine the influence of geologic factors on streams, it is necessary to measure certain physical characteristics of the streams. Having done that, it is possible then to study the effect of differences in geologic character of their drainage areas upon them. In comparing longitudinal profiles of channels it is logical to consider related factors such as bedrock, bed material, drainage-basin morphology, and the hydraulic characteristics of the stream.

Consideration of the hydraulic variables with regard to the longitudinal profile has been placed last, because many of the hydraulic factors, such as discharge and roughness, are, in this area, less intimately related to the profile than are the kinds of bed material and basin form.

LONGITUDINAL PROFILES OF STREAMS

To express the longitudinal profile of a stream bed mathematically, let F represent the fall, in feet, taken positive downward, and L represent the stream length, in feet, taken positive in the direction of flow. The line of the longitudinal profile then is $F=f(L)$, where f represents a function.

Except for Sixmile Creek, the streams in central Pennsylvania covered by this study possess typical concave upward profiles (fig. 89). It is evident, however, that the profiles of various streams differ in concavity and regularity. The explanation of these differences constitutes one of the basic aims of this study.

The irregularities in longitudinal profiles often prevent the use of a simple mathematical expression for the relation between fall and length. In some instances a function of the form

$$L = Ke^{aF}$$

Where L=length of stream, F=fall, K=a constant, a=a constant, and e=the base of natural logarithms, may adequately portray the profile (see fig. 90A). In other instances, an equation of the form

$$F = KL^p$$

A. Bald Eagle valley, looking toward the Allegheny Front near Port Mathilda, Pa. Strata dip gently toward the front which strikes generally northeastward.

B. Bed material, mainly Tuscarora orthoquartzite, of upper Shaver Creek at Pennsylvania State University gaging station near Charter Oak, Pa.

C. Bed material of Little Juniata River near sampling station 99, near Barree, Pa. Note the vegetation growing on the riffle and the sporadic boulders in the bed.

D. Boulder field in a water gap formed by Fishing Creek near Lamar, Pa. Note the trees growing in the debris. Maximum diameter of boulders is about 8 feet.
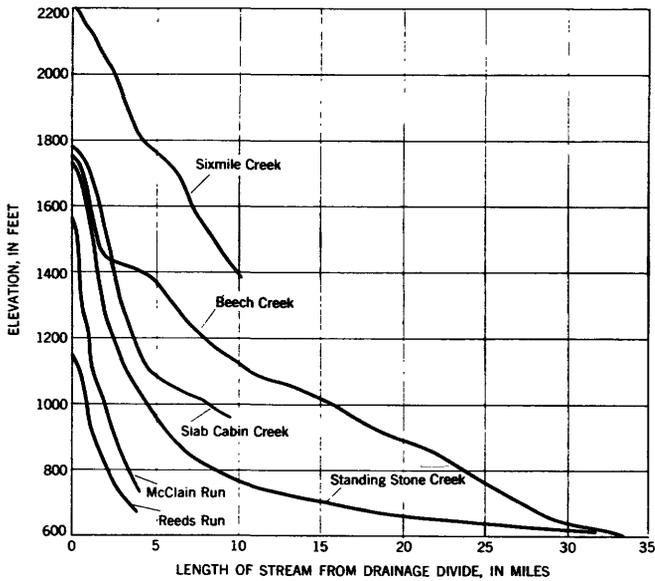
FIGURE 89.—Generalized diagrams of longitudinal profiles of six of the major streams investigated. Note large variation in shapes of the profiles. Length is used here as the distance from the headwater divide.

where K and p are constants fits the data reasonably well (see fig. 90B). It is evident that there is some scatter of points about the titled line (see fig. 90A and B), but both types of equations described the profile reasonably well. In order to standardize the results and facilitate comparisons, the type of equation adopted for this report is the simple power function, namely $F=KL^p$, in which K and p are constants.

Another way of considering features of the longitudinal profile is through the relation of slope (S) to length (L). For the general equation,

$$F=f(L) \tag{1}$$

then $dF=f'(L)dL$,

$$\text{or slope,} \quad S=\frac{dF}{dL}=f'(L) \tag{2}$$

From the power function equation,

$$F=KL^p \tag{3}$$

$$S=\frac{dF}{dL}=pKL^{(p-1)}$$

in which K and p are constants,

letting $Kp=C_1$ and $(p-1)=\rho$;

$$\text{then} \quad S=C_1L^\rho \tag{4}$$

This means that if the relation between F and L is a power function, then S also is related to L by a power function with different constants and exponents. Slope

and stream length may be determined from topographic maps; hence the relation between slope and length may be investigated without regard to elevation. However, if a power function relation exists between slope and length, the profile may be reconstructed by integration of the slope-length equation. This procedure, outlined by Hack (1957) is a useful tool.

Several plots of channel slope against length may be seen in figure 91. Straight lines were fitted to the plotted points by eye.[2] It is obvious that marked
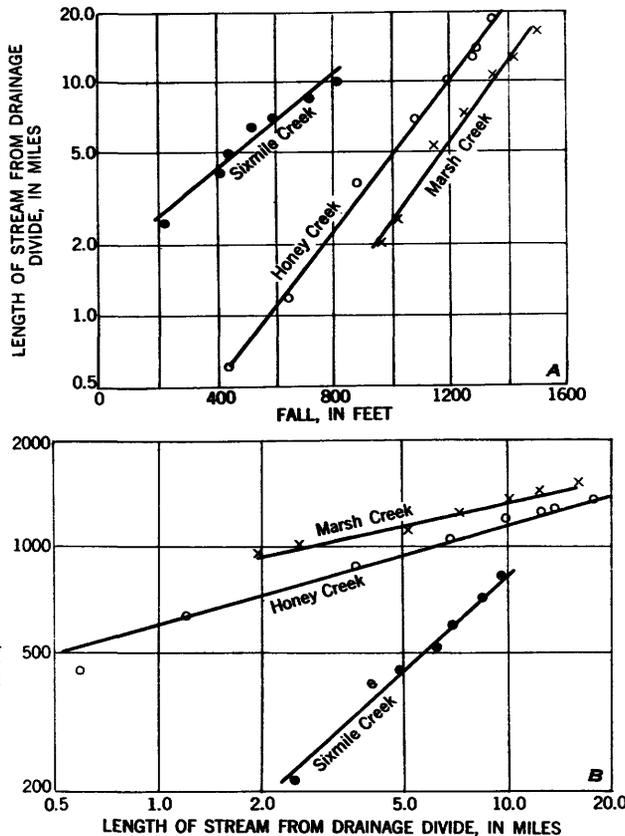
FIGURE 90.—Relation of fall to length of stream from drainage divide for Sixmile, Honey, and Marsh Creeks as plotted on A, semilogarithmic paper, B, logarithmic paper.

differences occur in the slopes of the fitted lines. Aside from these differences, there is a large amount of scatter about each line. This is an expression of the irregularities in various profiles causing deviation from the fitted power function equation, plus the errors of measurement. If these deviations can be shown to be outside the limits of error of measurement, they assume importance, because each aberrant point means that there are a set of special environmental conditions. A partial examination of the aberrant points is given in subsequent sections.

[2] To illustrate trends, lines were fitted by eye; to predict values of dependent variables, curves were fitted by the method of least squares.

FIGURE 91.—Relation of channel slope to length of stream from drainage divide for Shaver, Standing Stone, Sixmile, Bald Eagle, and Marsh Creeks.

In summary, the longitudinal profiles of 16 streams in central Pennsylvania exhibit marked variation within a homogeneous climatic and physiographic environment. This variation manifests itself in the different mathematical equations which were used to describe the profiles and in the amount of deviation from the general trends of the mathematical equations.

BED MATERIAL

In this study, size, sorting, and shape have been measured to gain an understanding of the relations between these parameters, the longitudinal profile, and the flow within the channel. Some of the irregularities noted in the longitudinal profiles may be ascribed to the variations in size of the bed material. Size alone, however, may not constitute an adequate description of the characteristics of the bed material. Mean or median particle size gives no indication of the range in size of bed material; therefore, some measure of dispersion is desirable. For this reason, sorting is also considered in relation to characteristics of the longitudinal profile. Another characteristic of the bed material which may be important is the shape of the individual particles. Two particles of different shape having identical intermediate axes may contribute different roughness to flow. Finally, it is desirable to have some explanation of why particle size or shape differs from one reach to another and how the material found on the beds of these streams behaves in terms of current hypotheses concerning wear and selective sorting.

PARTICLE SIZE

To establish a sampling technique, a test of the sampling procedure described by Wolman (1954) was made in the field. Five operators measured 60 pebbles, not necessarily the same ones, by making 6 traverses parallel to the stream channel. The analyses of variance indicated no significant differences within traverses or operators, but did show significant differences between traverses at the 1-percent level. No signifi-

cant interactions were present. On the basis of these tests, the number of samples was fixed at 60, and it was found that different operators obtained statistically similar results. The statistical results of the method of sampling indicates that the mean particle size may be used to represent a sampling point and that variations in size are not dependent on the operator.

If particle size determines the characteristics of the longitudinal profile, as was suggested by Shulits (1941), and Leopold and Wolman (1957), plots of particle size against slope or length should show a significant correlation. Figure 92 shows the relation between particle size and length for several representative streams.

A comparison of figures 91 and 92 shows certain features of the relation between particle size and slope. First, the scatter of points on these graphs, in both figures, is great. Second, the slopes of the lines for any one parameter differ from one stream to another. Third, the change of particle size with length of stream from drainage divide is not necessarily associated with a change of slope with length in the same direction. For example, Bald Eagle Creek shows a slight increase in particle size downstream, but slope decreases very rapidly. For Sixmile Creek, particle size is in effect independent of length, but slope increases with the distance. In the remaining streams both particle size and slope decrease with an increase in length of stream.

The relation between slope and particle size may be seen in figure 93. The plotted points indicate that for four of five representative streams a fairly good correlation exists between particle size and slope, and it might be concluded that slope may be approximately predicted from particle size. However, for Bald Eagle Creek, slope is independent of particle size despite the
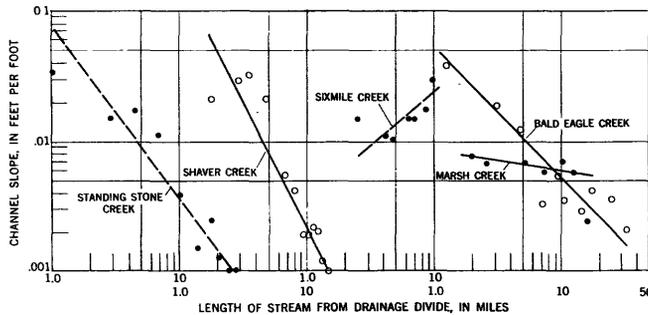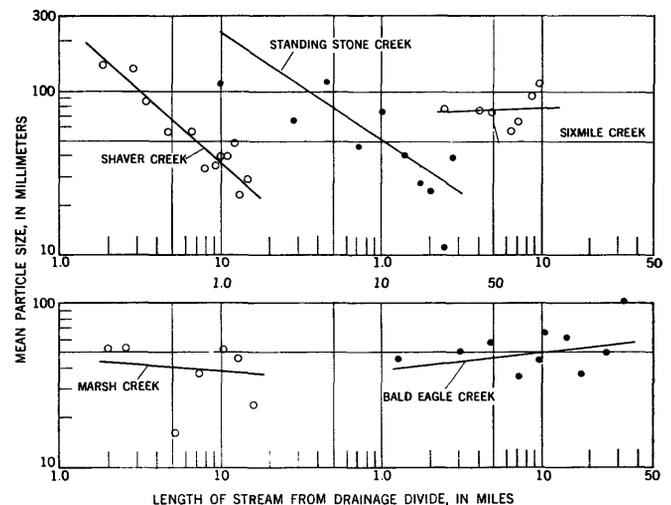

FIGURE 92.—Relation of mean particle size to length of stream from drainage divide for Shaver, Standing Stone, Sixmile, Bald Eagle, and Marsh Creeks.

close correlation because the line of best fit is approximately vertical. The data from Bald Eagle Creek suggest that at least one other variable must be used to describe the downstream changes in slope for all the streams in the area.

To investigate more fully the possibility of a relation between particle size and slope, the deviations of aberrant points from the plotted lines of particle size, channel slope, and length of stream from basin divide were studied. For example, if a point falls below the fitted regression line in the relation between slope and length, the same point would be expected to occur below a fitted regression line between particle size and slope, provided that slope depends primarily on particle size. Owing to errors inherent in sampling and the fact that chance variation occurs in employing summary statistics such as the mean, every point would not be expected to conform to the foregoing statement. However, if the deviations were measured for all the
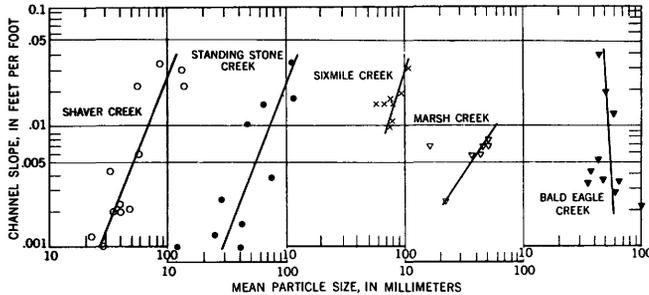
FIGURE 93.—Relation of channel slope and mean particle size for Shaver, Standing Stone, Sixmile, Bald Eagle, and Marsh Creeks. Note that slope is independent of particle size for Bald Eagle Creek and that the lines fitted to the plotted points of the other streams have very steep slopes.

points on the scatter diagram, a trend would be expected. Deviations of points on the slope-length curves were plotted against deviations on the particle size-length curves and no trends were observed.

As a result of this brief analysis of the relations between particle size, slope, and stream length, it was found that both particle size and slope vary with stream length, but particle size in general shows a closer relation to slope than to the length of the stream. However, at any given length, a slope which deviates from the line of best fit is not necessarily accompanied by a particle size which deviates from the line of best fit in the same direction. This suggests that the cause for these deviations has not been accounted for in the parameters used. There is a suggestion that the element unaccounted for may also be contributing to the general trends of the various regression lines. An attempt will be made in a subsequent section to identify the additional variable or variables necessary for describing the relations of slope and particle size.

DISTRIBUTION OF PARTICLE SIZE

In addition to calculations of the mean particle size, three other statistics were employed to estimate dispersion and skewness. The purpose of making these calculations was to determine whether any trends might be found within one stream bed or between streams and whether an aberrant particle size at a given reach might be attributed to a difference in the statistical population at this reach which would be reflected by changes in the characteristics of the distributions of particle size.

The a, b, and c axes of particles were measured in intervals of $\frac{1}{2}\ \phi$.[3] Means were calculated for each axis but only the b axis was studied in terms of sorting and skewness. (Values of the statistics for individual stations are given in appendix A.) Cumulative frequency curves were constructed for each b-axis population and were plotted on probability paper. In most instances, the fact that curves very nearly approached a straight line indicates that the populations approximate normal distributions. A few plots indicate a bimodal distribution. Figure 94 shows some examples of the distributions.

Dispersion in the form of phi standard deviation (σ_ϕ) was measured from the probability plots and was calculated in the following manner:

$$\sigma_\phi = \frac{\phi_{84} - \phi_{16}}{2} \qquad (5)$$

in which the subscript denotes the percentage of material coarser. σ_ϕ was found to range from 0.55ϕ to 2.06ϕ but usually ranged from 0.80ϕ to 0.90ϕ.

Although sorting is a function of dispersion, the standard deviation is rarely used as a basis of comparison in the geologic literature. The well-known measure of sorting usually employed is Trask's (1932) sorting coefficient, S_0. For purposes of comparison both statistics are given in appendix A.

Trask's equation for estimating sorting is:

$$S_0 = \sqrt{\frac{Q_3}{Q_1}} \qquad (6)$$

where Q_3=diameter in millimeters, of which 75 percent of the material is finer, and Q_1=diameter in millimeters, of which 25 percent of the material is finer.

For the particles on riffles in streams of central Pennsylvania, Trask's sorting coefficient ranges from 1.29 to 3.32 and has a mean of 1.57. On the basis of Trask's measure, the particles on riffles in general may be called well sorted.

[3] $\phi = -\log_2 N$, where N is the size in millimeters (Krumbein, 1938).

FIGURE 94.—Cumulative frequency curves of particle size for stations 1–6 for Shaver Creek as plotted on probability paper. Stations are numbered consecutively from the headwaters (No. 1) toward the mouth.

Skewness was studied according to the method suggested by Inman (1952) wherein,

$$a_\phi = \frac{\frac{1}{2}(\phi_{84}+\phi_{16}) - \phi_{50}}{\sigma_\phi} \tag{7}$$

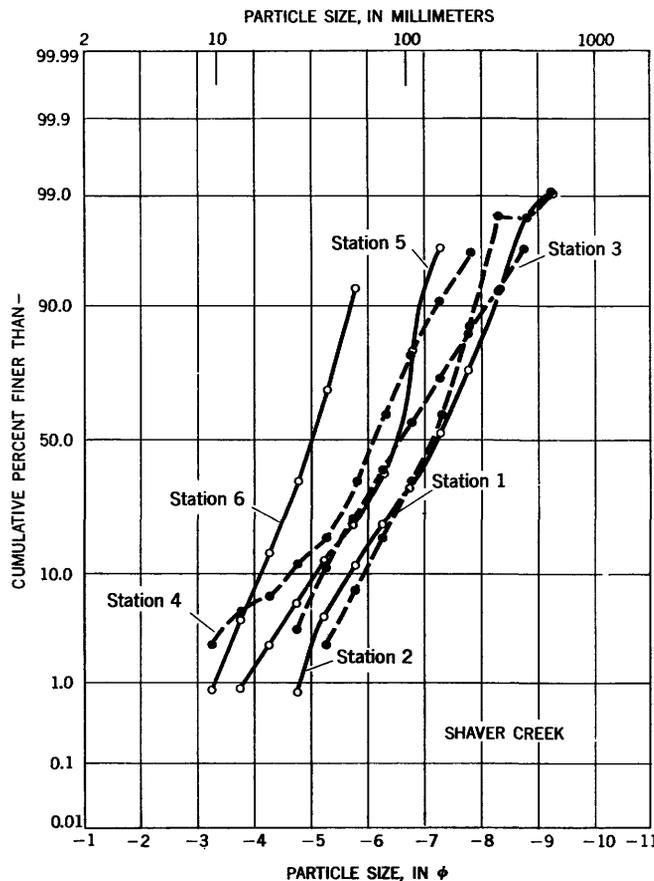where a_ϕ=phi skewness, and ϕ_{84}, ϕ_{16}, ϕ_{50}=values of phi which are larger than 84, 16, and 50 percent of the total range, respectively. The recorded values of skewness range from −0.405 to +0.762, and the mean is +0.097. The particles in general possess a slight positive skewness which, when considered in terms of size, indicates that the distributions of the samples show a small excess of particles in the larger sizes. No estimation of kurtosis was made.

In a recent paper by Emery (1955) which summarizes a portion of published data on particle size in streams from many different areas, it was shown that where streams possessed a range in particle size of from 10.4 to 355 mm, the Trask sorting coefficient ranged from 1.34 to 5.49, the median value being listed as 3.18. This value is somewhat higher than for streams of central Pennsylvania. The difference

between Pennsylvania data and Emery's data is to be expected because only riffles were measured in the former and the methods of measurement were different. Because the velocity of flow over riffles is higher than in pools, competence is greater on the riffles and small particles may pass over them. Pools tend to trap the finer debris because the velocity is less most of the time. Selective sorting in this fashion eliminates much of the finer debris from riffles and causes the material on riffles to be well sorted. The lower range of sorting given by Emery agrees with data from Pennsylvania. In general, the two sets of data are in agreement if one considers that a slightly different approach and sampling method was used in Pennsylvania.

No definite trends of changes in sorting or skewness with stream length, particle size, or slope were found. High values of skewness or sorting seem to show no systematic relation to aberrant points on plots of slope and particle size with respect to length. Buffalo Run possesses values of skewness and sorting which are much higher than for the other streams, but no logical explanation for this can be given.

In summary, it may be concluded that variations in particle-size distribution of bed material in these streams are independent of particle size, slope and length of stream, and do not differ appreciably from one stream to another.

PARTICLE SHAPE

For each pebble count, a record was made of the length of the a, b, and c axes. These data were used to study changes in shape within and between streams, and to determine any relations that might exist between particle shape and length of transport, or shape and the slope of the stream.

How to recognize a change in shape is indeed a difficult problem. In practice, certain aspects of shape are measured, usually in terms of sphericity (closeness to a sphere—see Wadell, 1932), roughness, and surface texture. Surface texture and roundness are interdependent, but the same is not true for roundness and sphericity. A perfect sphere must be perfectly round, but any shape other than a sphere may have varying degrees of roundness. For example, a cube has a high sphericity, but all of its faces meet at right angles; a right circular cylinder terminated at both ends by a hemisphere has low sphericity and high roundness. Hence, except in the limiting case, roundness is independent of sphericity. In practice, roundness is extremely difficult to measure. There are, to be sure, many methods which attempt to portray roundness, but none is entirely satisfactory.

Sphericity also contains limitations inherent in its definition, but it was studied in order to obtain at least one measure of shape. The presently accepted con-

ception of sphericity is the proximity of a particular grain to a sphere. Most commonly, sphericity is measured by means of Krumbein's (1941a) equation for intercept sphericity.

$$\psi = \sqrt[3]{\frac{bc}{a^2}} \qquad (8)$$

in which a, b, and c are the long, intermediate, and short axes, respectively.

Intercept sphericity is determined by taking the ratio of the volume of an ellipsoid defined by the dimensions of the axes of the particle to the volume of a circumscribing sphere which has a diameter equal to the longest axis of the particle. Because this equation has been used extensively in the literature, it was employed in this study.

Experimental studies of changes in sphericity with distance of travel indicate that sphericity increases with the distance of transport (Thiel, 1940; Krumbein, 1941b). Field observations do not always agree with the laboratory results. For example, Krumbein (1940) found no change in shape in the downstream direction of the flood gravels of the San Gabriel Canyon, Calif. Krumbein (1942) also noted no change in sphericity with distance of travel of the gravels in Arroyo Seco, Calif. In studying the sands of the Mississippi River, Russell and Taylor (1937) found a small decrease in sphericity in about 1,000 miles of travel. For the 16 streams studied in central Pennsylvania, it was found that sphericity remains practically constant in a downstream direction. Figure 95 shows the relation of sphericity to length of travel for three of these streams which are typical of the group.

The results are logical because there are many factors which tend either to increase or decrease sphericity in a downstream direction. Sphericity is a function of the initial shape characteristics of the particles which are introduced to the channel, the place along the stream where these particles enter, the wear of the particles in the channel, and selective transport of different shapes of particles. These opposite tendencies are reflected by the data which reveal that no significant trends exist for sphericity.

The initial shape of the bed particles is largely determined by the characteristics of the bedrock, including jointing, cleavage, and bedding from which the particles are derived. In central Pennsylvania sandstone and limestone often possess joint systems which tend to produce particles that are nearly equidimensional and therefore possess high sphericity. Outcrops of sandstone and limestone occur sporadically along the entire length of these streams. Hence, high sphericity is introduced from bedrock erosion at many points along a stream. On the other hand, the shale in the area is characterized by a pronounced cleavage, either platy cleavage or pencil cleavage, which yields low values of sphericity. The presence of shale is not restricted to a specific area along these streams, hence
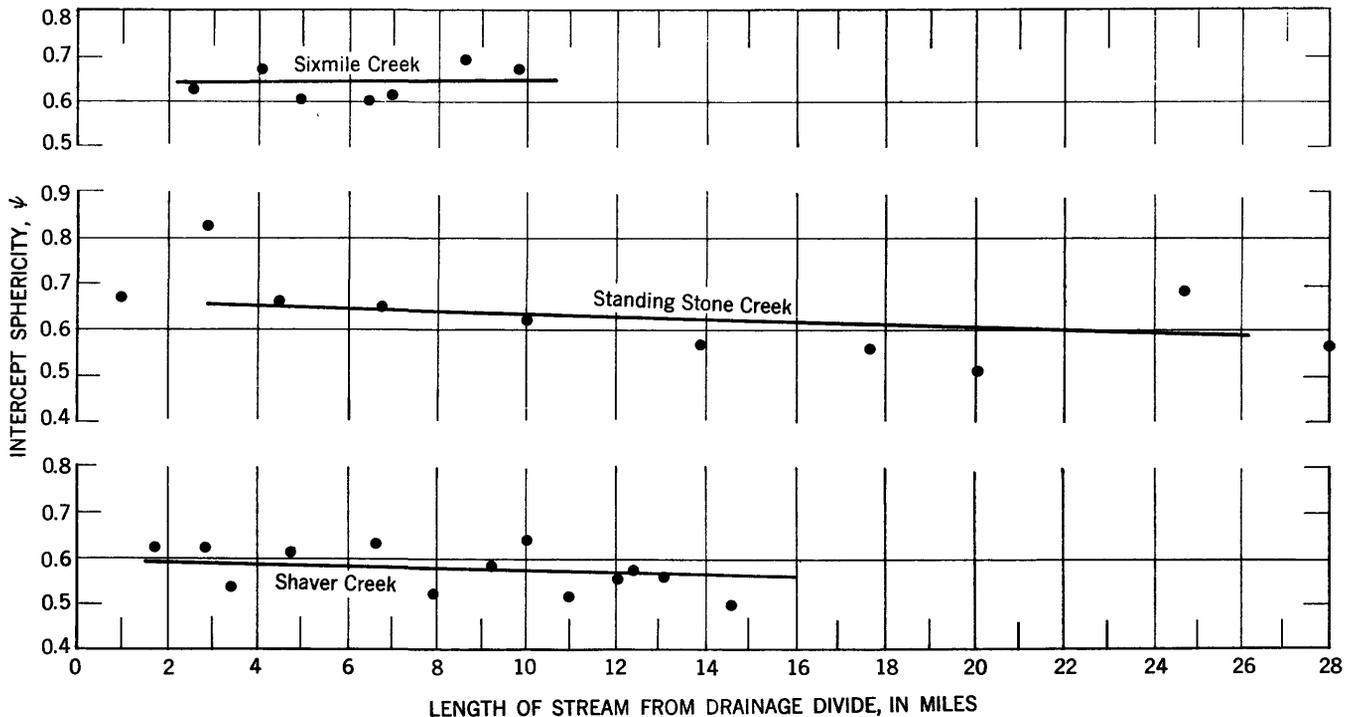

FIGURE 95.—Relation of intercept sphericity and length of stream from drainage divide for Shaver, Standing Stone, and Sixmile Creeks.

a source of low sphericity is also present along the entire length of the streams. A potential source of both high and low sphericity exists at any point along a stream, depending on the characteristics of bedrock over which the stream passes. If the channel does not erode bedrock, the shape characteristics of particles in the alluvial fill or colluvium assume the role of parent in the determination of the initial shape of the bed material. None of these initial factors of shape would tend to cause a trend to occur in sphericity in the streams of Pennsylvania.

The sphericity of material produced by tributaries is also governed by the nature and occurrence of various types of bedrock and therefore may be either high or low.

Ironically, wear may also cause high or low sphericity. Rounding of protuberances is definitely a condition leading to higher sphericity, but the breaking of a large boulder may result in two or more particles possessing high or low sphericity. The shape of the broken particles also reflects the nature of the bedding, jointing, and cleavage of the parent material. Both rounding and fracturing occur in the streams of Pennsylvania, and this results in no general trend of sphericity in a downstream direction.

The other factor, selective sorting, is the most difficult to evaluate. Although selective sorting is frequently considered to be an important process governing the distribution of particle shape along a stream, it is very difficult to understand exactly how this process works. It may be argued that selective sorting occurs because particles with high sphericity offer less resistance to flow and therefore are left upstream while particles having low sphericity and offering more resistance to flow are carried downstream. This is not a very strong argument, however, because the same particles with high sphericity also offer less resistance to rolling. Without more information on the relations between shape and the movement of particles on the bed, the role of selective sorting cannot be adequately evaluated.

In summary, it may be stated that intercept sphericity shows no systematic relation to stream length for streams in central Pennsylvania. The primary factors governing sphericity: initial shape, location of the introduction of the particle, wear, and selective sorting, all may cause particles to possess either high or low sphericity depending upon local conditions. In the streams of central Pennsylvania these factors tend to cancel all trends of increasing or decreasing sphericity with length of stream.

EFFECT OF WEAR ON PARTICLE SIZE

Studies of particle wear are often made under artificial conditions, such as those studies by Wentworth (1919), Thiel (1940), Krumbein (1941b), and Kuenen (1956).

In each of these experiments particles were placed in rotating barrels, and particle weight was measured at intervals. The assumption was made that 1 mile of barrel rotations is equivalent to 1 mile of travel in a stream.

The results of these investigations have aided in understanding certain aspects of particle wear; however, if the procedure is not examined critically it may be misleading. By measuring changes in weight, the absolute magnitudes of change in particle size are exaggerated. For example, a cube or a sphere which is reduced in diameter by a factor of 2 will be reduced by a factor of 8 in terms of weight. The large reductions in weight noted by Krumbein and Wentworth do not represent a very great reduction in particle diameter.

In order to compare the results of the tumbling barrel experiments with field data collected in Pennsylvania, a conversion of units was made for the barrel experiments. It was assumed that the limestone pebbles used in the experimental study possessed a specific gravity of 2.7 and were spherical in shape. By making these assumptions, it is possible to compare the change of mean particle size with length of travel in a barrel (see fig. 96) to the curves plotted from actual
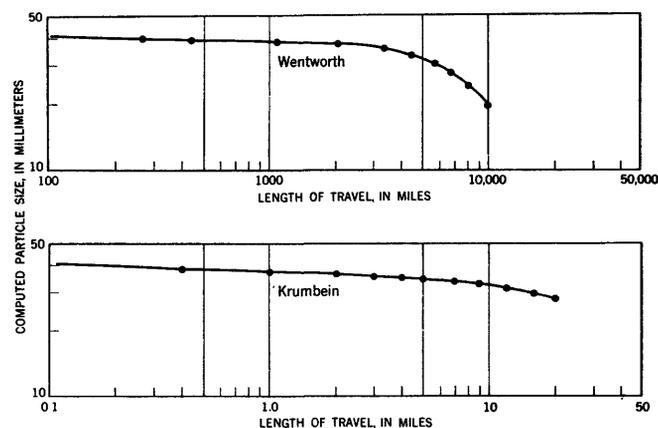
FIGURE 96.—Relation of computed particle size and length of travel by rotation in a barrel for data of Wentworth (1919) and Krumbein (1941b). Note the fact that little change takes place in size except at very great distances of travel.

field determinations (fig. 92). The rate of change of particle size with length of travel in both the Krumbein and Wentworth experiments is small except toward the ends of the runs. On the other hand, it was noted previously (see fig. 92) that the particle size decreases extremely rapidly with length for Shaver and Standing Stone Creeks. If the experimental wear curves are applicable to rates of wear in general, it must be concluded that the rapid decrease in particle size downstream in Shaver and Standing Stone Creeks cannot be accounted for by wear alone. Furthermore, many of the pebbles found in the natural streams are composed

of resistant sandstone, whereas most barrel experiments used less resistant limestone.

If wear alone cannot account for the rapid decrease in particle size in Shaver and Standing Stone Creeks, perhaps other factors such as parent material, tributary addition, and selective sorting are important because the tributaries to these streams yield large particles which would tend to keep particle size fairly high in the trunk stream. For example, Garner Run, tributary to Shaver Creek, has bed material near its mouth on the order of 76 mm, whereas Shaver Creek just below Garner Run has a mean bed particle size of 48 mm.

Selective sorting may cause a decrease in particle size in a downstream direction. In order for this to occur one may reason that the mean velocity or the velocity distribution must be the controlling factor. Empirical studies have shown that mean velocity increases in the downstream direction (see Leopold and Maddock, 1953; Wolman, 1955). A decrease in particle size associated with an increase in mean velocity is the reverse of the relation of velocity and particle size which is the basis of the definition of competence. Hence, the velocity distribution is perhaps a more reasonable factor to study. The shape of the velocity profile close to the bed may change in such a manner as to cause a decrease in effective shear despite an increase in mean velocity. Leopold (1953) suggests that a progressive change in velocity profiles downstream may cause certain particles to move while others remain in place. Proof that selective sorting operates to some extent is shown by the removal of some of the smaller material on riffles. Thus it is reasonable to believe that selective sorting may be active over the entire stream length. On the other hand, because particle size shows such a large scatter of points when plotted against length for different streams, it is hard to believe that selective sorting can account for all of the scatter.

The other important factor which might influence particle size to a large extent is the nature of the parent material from which the particles were derived. It was found for example that particle size varies with the type of bedrock. Streams flowing on sandstone tend to have larger particles than streams flowing on shale or limestone and dolomite. This relation was considered to be important enough to warrant detailed investigation and will be discussed in a subsequent section where geologic factors are considered in detail. There is no question that parent material must be included with wear in order to explain the rapid decrease in particle size for streams such as Shaver and Standing Stone Creeks.

In summary, the characteristics of parent material of the particles in the stream channels of central Penn-sylvania are probably important in determining the size and shape of these particles. Wear and selective transport also affect size and shape but are considered to be of minor importance

The characteristics of longitudinal profiles and bed material in the channels have been described. The third major division which is interrelated to the previous parameters is that of the characteristics of the drainage basins bounding the streams.

MORPHOLOGY OF DRAINAGE BASINS

Characteristics of the area from which a stream receives water determine many factors that affect streamflow. Infiltration, evaporation, and transpiration, in addition to the geometry of the basin, are the most important factors. Omitting plant and soil environment, the geometric properties of importance are size, shape, and relief. The size of the basin governs the average amount of flow or discharge of a stream; basin shape affects the timing involved in the concentration of runoff; and relief is an important factor in determining the erosion potential of the basin. In order to study the importance of some of these geometric factors, several methods of quantitative description have been used.

RELATION OF LENGTH OF STREAM TO SIZE OF DRAINAGE AREA

One of the earliest and most important quantitative studies of basin geometry was that of Horton (1945). As a part of his analysis, he was able to show simple geometric relations between such parameters as stream order, number, length, and slope. For a working definition of stream order, Horton considered the smallest unbranched tributaries as order 1. Streams which receive flow from first order streams only are classified as order 2. Order 3 consists of those receiving flow from first and second order streams, and so on.

Streams of the Susquehanna River basin conform to the scheme of classification described by Horton (see fig. 97). First order streams are taken as the smallest unbranched tributaries which possess recognizable channels. By using this definition of first order streams, the Susquehanna River becomes a tenth order stream. Most of the streams in this study range in order from four to seven. It can be seen from figure 97 that the relations among number of streams, length, drainage area, and order possess very little point scatter and suggest that these elements of the basin are in equilibrium with their environment.

The Horton analysis does not yield information on shapes of drainage basins and is somewhat restricted in the sense that it is often difficult to evaluate the physical significance of the parameters: mean length of streams and mean size of drainage areas.
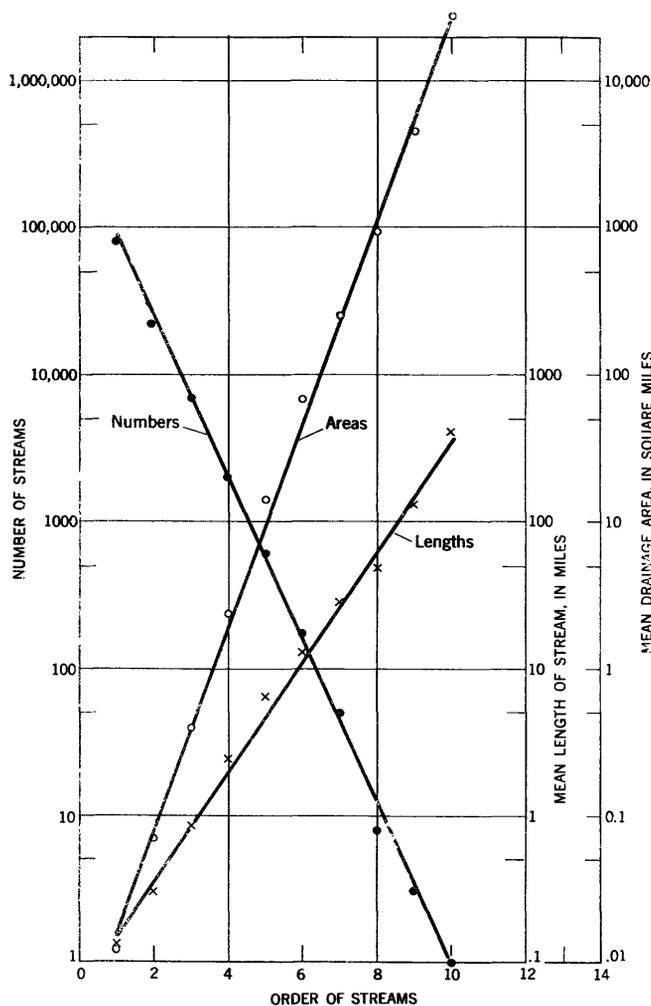
FIGURE 97.—Relation of number of streams, mean lengths of streams, and mean areas of drainage basins to stream order, for the Susquehanna River drainage basin. Data include small tributaries which are not shown on maps with a scale of 1:62,500. Horton's system of orders was employed.

Instead of using mean lengths and mean size of drainage areas, it is possible to plot the actual length of stream from the drainage divide against the size of the drainage area (Hack, 1957). This is more desirable than using averages, because the empirical relation obtained is an expression of the basin shape. This method was employed for studying each of the 16 streams of which 5 typical examples are plotted in figure 98 for illustration. Without exception among the streams studied, the relation between length of stream from the divide and size of drainage area is well defined by a small scatter of points. The power function coefficients and exponents of the equation $L=KA_d^p$ in which A_d is drainage area, K and p are constants for all streams studied are listed in table 1.

The exponents and coefficients reflect, in a very general way, the shape of the basins and depict the physiographic variations between individual streams which were previously hidden in values of mean length and mean drainage area in the Horton analysis.

Values of p range from 1.00 to 0.50, or the relation between length and drainage area ranges from $L \propto A_d$ (McClain Run) to $L \propto A_d^{\frac{1}{2}}$ (Little Juniata River). McClain Run has approximately constant width and its basin is nearly rectangular. From the equation $WL=A_d$ it is clear that if length is proportional to size of drainage area to the first power, mean width must be constant. Mean width is not a very useful parameter but is used merely to illustrate the fact that for McClain Run, the relation between length and drainage area does give some hint as to the shape of an individual basin. In contrast, the equation for the Little Juniata River reveals that because $L \propto A_d^{\frac{1}{2}}$, W must also be proportional to $A_d^{\frac{1}{2}}$. This is not immediately obvious from examination of the shape

FIGURE 98.—Relation of length of stream from drainage divide to size of drainage area for Shaver, Standing Stone, Sixmile, Bald Eagle, and Marsh Creeks.

TABLE 1.—*Summary of the power function equations relating length of stream and size of drainage area, where* $L = KA_d^p$

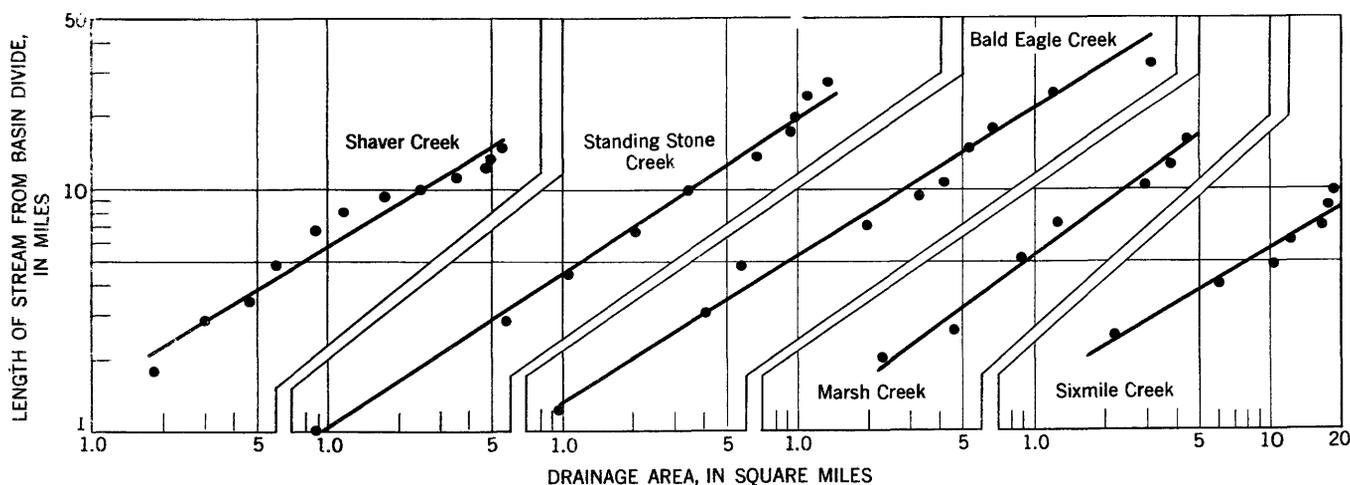| Stream	K	p
McClain Run	0. 98	1. 00
Reeds Run	1. 00	. 84
Globe Run	. 96	. 81
Beech Creek	. 50	. 80
Marsh Creek	1. 00	. 70
Standing Stone Creek	1. 10	. 62
Bald Eagle Creek	1. 30	. 60
Sixmile Creek	1. 50	. 60
Fishing Creek	1. 90	. 59
Honey Creek	1. 70	. 59
Shaver Creek	1. 50	. 59
Weiker Run	2. 50	. 57
Warriors Mark Creek	1. 40	. 55
Slab Cabin Creek	2. 00	. 52
Buffalo Run	1. 40	. 51
Little Juniata River	1. 80	. 50

of the Little Juniata River basin (see pl. 5), but the basin has the general appearance of being short relative to width, with width increasing downstream. In fact from the equation it can be shown that $W \propto L$. Values of exponents intermediate between 0.5 and 1.0 indicate basin shapes which lie between these two extremes although the irregularities in outline of these basins are extremely large.

In general, drainage basins that possess nearly constant average widths tend to have high values of the exponent p in the expression relating length to area. Streams that show a linear relation between width and length tend to have low values of p. Streams generally show curvilinear relations between length and width and have intermediate values of p which range from about 0.55 to 0.75. The common basin shape, approximately an inverted pear in form, appears on the basis of this study to have a value of p approximately equal to 0.6. Finally, the erosional stream patterns that determine basin shapes depend almost entirely on the topography which, for this area, is largely governed by the geology. For example, McClain Run has a p equal to 1 and its drainage basin closely resembles a rectangle. The reason the drainage divides for McClain Run are parallel to the stream is that it flows along the axis of a tightly folded syncline in the Clinton formation (mostly shale) and is bounded laterally by limbs composed of Tuscarora sandstone. The shape of this basin is almost entirely determined by structure and although it is just one example, it clearly illustrates one way in which the mean width of a drainage basin may remain constant.

Large synclines and anticlines which have been breached by erosion do not exhibit the same characteristics because the drainage networks have space to develop. This allows tributary divides to follow minor stratigraphic controls in addition to structure. Never-

theless, the outlines of the major basins generally parallel the strike of the rocks or of the fault planes and may be considered as structurally controlled.

The exponents and coefficients of the equation relating length of stream and size of drainage area give some indication of the general shapes of the basins although they do not depict the irregularities in outline. The scatter of points about each plot may be the result of several factors. First, the scatter may be due to irregularities in the outline form of the basin. Second, part of the scatter may be due to the effect of tributaries entering the main stream. Despite these reasons for scatter, the relations between length of stream and size of drainage area plot with such little scatter that length of stream may prove to be a useful parameter for studying landforms or hydrology.

RELATION BETWEEN MEAN ANNUAL FLOOD AND DRAINAGE AREA

The relations between length of stream and size of drainage area were obtained for each of the 16 streams studied. If either of these variables can be related to a hydraulic variable, a link between longitudinal profiles, drainage basins, and channel hydraulics will be formed. Assuming similar climatic and geologic conditions drainage basins of equal size should have approximately equal discharge.

In order to study the relation between drainage area and discharge, it is necessary to utilize a flow frequency of some hydrologic significance. Leopold and Maddock (1953) have demonstrated that the selection of a constant flow frequency is extremely important if comparisons are to be made within and between streams.

The selection of the flow frequency to be used was governed by several considerations. First, presumably moderately large flows are required to move coarse bed material. Second, if parameters such as channel shape are to be considered, it would be advantageous to deal with discharge at stages which nearly fill the entire channel, and this points again to large flows. Third, an investigator must be able to locate objectively within the channel some characteristic which may be related to the chosen frequency of flow. The last requirement considerably limits the possibilities. However, if the bankfull stage can be shown to approximate an equal frequency of flow among and within drainage basins, it would be an ideal choice because there is less subjectivity involved in identifying the bankfull channel. Furthermore, stages accompanying bankfull discharge also fulfill the requirement of possessing a high discharge with a high erosion potential.

In order to evaluate the bankfull discharge for any stream, it is necessary either to measure the bankfull discharge at each reach to be studied or to determine

the frequency at which bankfull discharge occurs by analyzing gaging station records covering a period of years. It was both impractical to wait for a storm capable of producing bankfull flow and impossible to synchronize measurements of these values at 119 stations. As a result, bankfull discharge was measured at only two stations, neither of which was a regular gaging station. However, at five gaging stations within the area of study it was possible to measure the gage height of the bankfull stage and to obtain the discharge from the rating curve for each station. The recurrence intervals of these flows were calculated by use of the partial duration series and were found to range from about 1.9 to 10 years.

The partial duration series was selected for use in the study because the values of discharge for various recurrence intervals take into account secondary floods within 1 year; these are not included in the annual flood series (see Langbein, 1949).

A typical flood frequency curve is shown in figure 99. Because of the apparent range in recurrence intervals at bankfull flow, flood-frequency analyses were made by use of the partial duration series and were plotted against drainage area for various recurrence intervals. The recurrence interval which best fit the measured and estimated flows was about 2.3 years.

A plot of the 2.3 year-discharge against drainage area may be seen in figure 100.

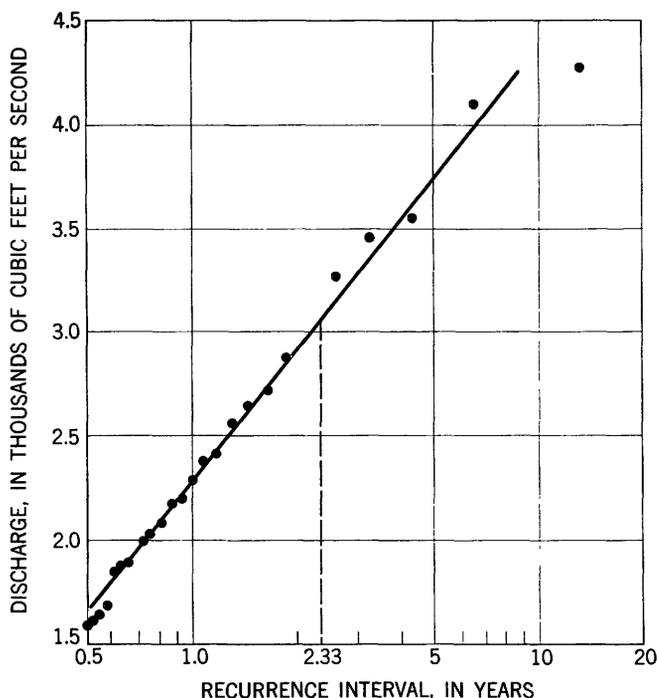Figure 100 includes a graphic representation of the

standard error of estimate, the equation of the line of best fit, and the correlation coefficient which is significant at the 1 percent level.

The significance of the relation between bankfull discharge and size of drainage area is worth noting. For example, the magnitude of the 2.3-year discharge is related to the size of the drainage area, but the length of stream is also related to area. Hence, it is possible to estimate roughly the bankfull discharge of any stream in the area of study by measuring the length of the stream. Again, in order to emphasize the unity of these relations, it must be stated that an element of the longitudinal profile therefore may be roughly related to discharge, as well as to characteristics of the channel and the general morphology of the basin. The addition of discharge to the known basin and profile characteristics leads to the possibility of relating other hydraulic variables to the characteristics of the basin.

HYDRAULIC GEOMETRY

It has been shown that the discharge at any point in a stream may be estimated from the drainage area, and it is therefore possible from the measurement of the cross-sectional area to estimate the mean velocities of these streams at the bankfull stage because

$$v = Q/A. \qquad (9)$$

Sufficient data are available through computation or field measurements to describe the downstream hydraulic geometry of streams in central Pennsylvania, following the general pattern set by Leopold and Maddock (1953). This is a partial discussion of hydraulic geometry because only the downstream changes will be considered, leaving out, because of insufficient data, the at-a-station relations.

HYDRAULIC GEOMETRY BY INDIRECT APPROXI-MATIONS

Although hydraulic geometry has not been studied by indirect methods, all of the necessary components for making the study are present. For each station, bankfull depth, bankfull width, and slope were measured. Discharge was calculated from size of drainage area and velocity computed from the relation $v=Q/A$. The parameters mentioned above pertain only to bankfull frequency assumed to be 2.3 years. The plots of downstream values of bankfull width, depth, and velocity against $Q_{2.3}$ for several streams are given in figure 101.

These plots represent the downstream portion of the hydraulic geometry for each stream. It is noteworthy that width and bankfull discharge are very closely related, for the scatter about regression lines fitted by eye is relatively small. On the other hand, most plots

FIGURE 99.—Relation of discharge to recurrence interval for gaging station on Standing Stone Creek near Huntingdon, Pa., for water years 1939–50.
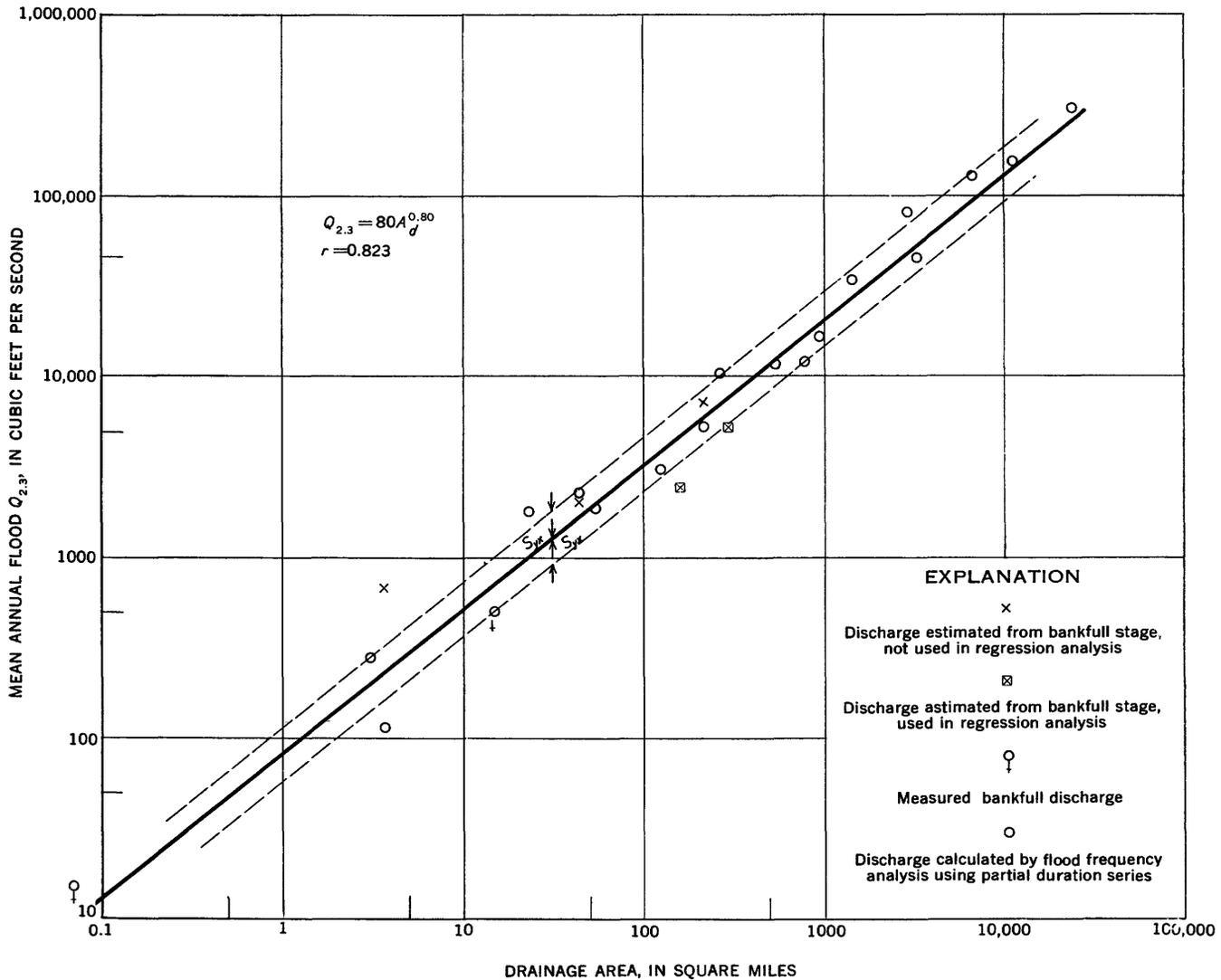
FIGURE 100.—Relation of mean annual flood and drainage area for gaging stations in the vicinity of the field area. Mean annual flood was assumed to have a recurrence interval of 2.3 years. Two points represent measured discharge, five points represent the mean annual flood as determined by flood frequency analyses of gaging station records. The equation for the line of best fit is shown along with the standard error of estimate (S_{yx}) and the correlation coefficient (r).

of velocity against bankfull discharge show a large amount of scatter. For these particular streams it may be argued that the scatter may be partly due to the fact that velocity was computed from Q/A and, in turn, discharge was calculated from drainage area. However, other workers (Leopold and Maddock, 1953; Wolman, 1955) have also discovered that points of measured velocity scatter more than points of width or depth where plotted against discharge.

For many streams, changes in bed configuration during the rising and falling stages may account for a range in velocity at a given discharge. Backwater from ice or debris dams may also affect velocity. Without more information about the above factors for a given discharge it is impossible to explain the scatter of points on the plots of velocity against discharge for the streams in this area.

EXPONENTS OF POWER FUNCTIONS FOR DOWNSTREAM RELATIONS

Power function relations between bankfull width (w_b), depth (d_b) velocity (v_b), and discharge (Q) in a downstream direction are represented by straight lines on log paper. The exponents must total 1.0 because $wdv=Q$. Then

$w_b d_b v_b = Q_{2.3} = aQ^b_{2.3} \cdot cQ^f_{2.3} \cdot kQ^m_{2.3}$ (see Leopold and Maddock, 1953) where b, f, and m are regression coefficients of straight lines on log plots of w, d, and v, respectively, against $Q_{2.3}$. Thus

$$b+f+m=1 \qquad (10)$$

Failure of the exponents to total 1.0 can be explained by two reasons. First, error may arise from fitting a line to scattered points on graph paper. A discrepancy

TABLE 2.—*The exponents of power function relations in the equations for width, depth, and velocity*

Stream	b	f	m	(b+f+m)
Weiker Run	0.67	0.55	−0.18	1.04
Beech Creek	.65	.36	0	1.01
Sixmile Creek	.51	.33	.16	1.00
Marsh Creek	.89	.63	−.51	1.01
Fishing Creek	.64	.38	0	1.02
Reeds Run	.48	.56	−.06	.98
Shaver Creek	.47	.34	.21	1.02
Standing Stone Creek	.65	.25	.10	1.00
Honey Creek	.43	.31	.28	1.02
Bald Eagle Creek	.47	.21	.25	1.01
Globe Run	.36	.30	.29	.95
Little Juniata River	.55	.38	.07	1.00
McClain Run	.66	.38	0	1.04
Warriors Mark Creek	.77	.32	−.05	1.04
Buffalo Run	.30	.70	0	1.00
Slab Cabin Creek	.42	.29	.29	1.00

of this nature has no bearing on hydraulic characteristics, but the possibility of error must be noted. Second, the addition of $b+f+m$ may not yield a value of unity if the relation between w_b, d_b, and v_b plotted against $Q_{2.3}$ is not truly a simple power function. In practically all cases a simple power relation seems to fit the scatter of points, but it must be remembered that the only requirement is that a given discharge must equal the product $w_b \cdot d_b \cdot v_b$. In table 2 the values of b, f, and m for the streams in central Pennsylvania are given.

The results of table 2 support the hypothesis that the exponents add up to one in the downstream direction at the bankfull stage. However, it is important to note that individual values of b, f, and m have a considerable range from one stream to another. Furthermore, the fact that m, the exponent relating velocity to discharge, is negative for four streams implies that mean velocity decreases downstream in each of these streams. This is unusual, according to the results of Leopold and Maddock (1953), Wolman (1955), and Leopold and Miller (1956).

It is difficult to explain why velocity appears to decrease in a downstream direction in four streams. Velocity was computed, but width and depth were measured in the field. If the size of the cross-sectional area at any station is small, a large computed velocity would result. If the size of the channel is unusually large, the equations used for computing velocity would yield small velocities. Consider a channel which is undergoing active erosion from the mouth toward the headwaters in response to a lowering of base level. Although the channel might be much too deep at the mouth, it is likely that at some point upstream the change in base level would not affect the bottom of the channel. A stream undergoing this type of change might cause an apparent decrease in bankfull velocity in a downstream direction because of the formation of

an over-deepened channel which is no longer directly related to the 2.3-year flood. In other words, this example respresents what might happen if a knickpoint developed and migrated upstream. In the case of Reeds Run, Marsh Creek, and Warriors Mark Creek, the idea of a knickpoint deserves consideration because qualitative field observations and topographic maps suggest that faint knickpoints may be present. Unfortunately, the evidence is not precise enough to warrant definite conclusions, but the suggestion of this possibility is worth mentioning in attempting to explain the downstream decrease in computed velocity in 4 of the 16 streams studied.

VALUES OF THE COEFFICIENTS FOR DOWNSTREAM RELATIONS

By means of the equation $wdv=Q$, it can also be shown that $ack=1$, in which a, c, and k are intercept values of the power relation at $Q_{2.3}=1$. The values of these coefficients for central Pennsylvania streams are given in table 3.

TABLE 3.—*Coefficients of the power function equations relating bankfull width, depth, and velocity to discharge*

	a	c	k	ack
Weiker Run	0.52	0.088	20.0	0.82
Beech Creek	.41	.15	12.0	.74
Sixmile Creek	1.00	.30	2.8	.84
Marsh Creek	.14	.042	220.0	1.29
Fishing Creek	.26	.18	18.0	.84
Reeds Run	1.55	.12	4.8	.89
Shaver Creek	1.50	.29	1.9	.83
Standing Stone Creek	.41	.42	4.30	.74
Honey Creek	1.95	.29	1.55	.88
Bald Eagle Creek	1.90	.43	1.25	1.02
Globe Run	2.00	.40	1.80	1.44
Little Juniata River	1.15	.28	2.60	.84
McClain Run	.64	.31	4.30	.85
Warriors Mark Creek	.14	.29	18.5	.75
Buffalo Run	2.40	.023	20.0	1.10
Slab Cabin Creek	1.65	.34	1.75	.98

Variation from unity in the product ack shows a marked difference in magnitude from one stream to another. It is evident that the magnitude of deviation is much larger than the deviation from unity in the summation of the exponents. However, this variation is to be expected because the value of the coefficient depends to a certain extent on the value of the exponent. Owing to this interrelation of the values of the coefficients and exponents, and to the fact that velocity was not actually measured, it is impossible to explain the variation of coefficients from one stream to another.

The general method of studying the hydraulic geometry by indirect approximations, that is, by calculating discharge from size of drainage area and
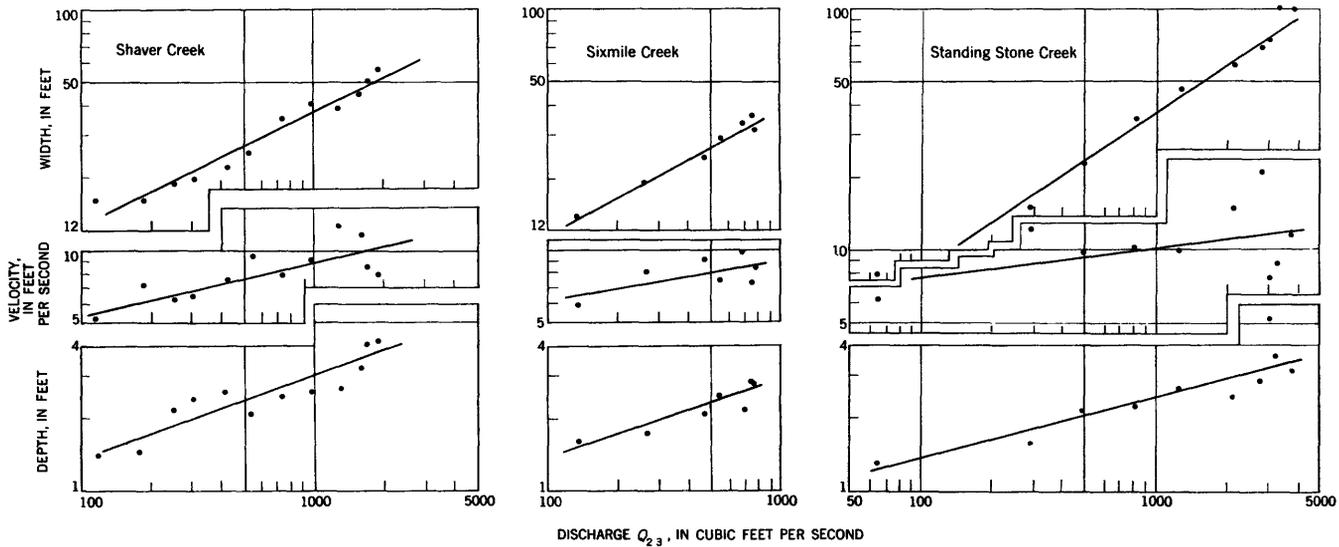
FIGURE 101.—Relation of bankfull depth, bankfull width, and computed bankfull velocity to bankfull discharge, which was assumed to be equal to the mean annual flood, for Shaver, Standing Stone, and Sixmile Creeks.

then computing velocity, appears to yield reasonable results. Although the absolute values may be in error, the relative rates of change designated by the exponents b, f, and m agree for the most part with results obtained by field measurement in other areas. The coefficients of the power function equations are not independent of the value of the exponents and do not yield independently derived supporting evidence of variations from one stream to another.

The discussion to this point has been aimed at describing the characteristics, including aberrancies and differences, of individual streams in terms of the longitudinal profile, basin morphology, and hydraulics. A degree of order has been found in many of the plotted graphs, which manifests itself in the amount of point scatter and the slopes of the lines of best fit. Although these relations for individual streams are important, it would be advantageous to determine whether any generalizations can be made.

COMPOSITE CHARACTERISTICS OF STREAMS

COMPOSITE RELATIONS

In order to visualize the picture for the entire area covered by 16 streams, it is desirable to summarize the relations of variables obtained at each stream. One method for doing this is to find the mean values for the exponents and coefficients of the different equations relating the variables. This would not be the most desirable procedure because not all streams have the same number of sampling stations. Hence, streams with only a few stations would give less reliable estimates of the exponents and coefficients. In order to avoid unequal weighting because of sampling, the data

at each station were treated as if they represented one stream flowing at the bankfull stage. By plotting the points of all streams on one graph, the composite or general picture of relations of the pertinent variables may be studied. The data have been divided into two categories. Longitudinal profile and basin characteristics have been placed in one group and the hydraulic characteristics in another.

The composite relationships of drainage basins and profiles which were plotted are slope against length of stream, length of stream against size of drainage area, mean particle size against slope, and sphericity against length of stream.

The plot of slope against length (fig. 102A) indicates that there is a definite trend toward a decrease in slope with length of stream despite the fact that some individual streams have a very wide range in relations involving the same variables. The scatter of points is very large but there is no question that a trend exists. The plotted points of length and drainage area (fig. 102B) fit very well with the line drawn by eye. The scatter is so small in these data that there seems to be no doubt that the lengths of the streams studied in the area are closely related to the size of the drainage basin to about the 0.6 power. On the other hand, the plot of particle size against slope (fig. 102C) shows much more scatter, indicating that the variation within individual streams is too large to permit an estimate of trends with any degree of certainty. As might be expected from the individual stream plots, sphericity bears no relation to stream length (fig. 102D). The fact that length and size of drainage area are closely related supports the initial assumption that the composite plot of all sampling stations represents one hypothetical stream.
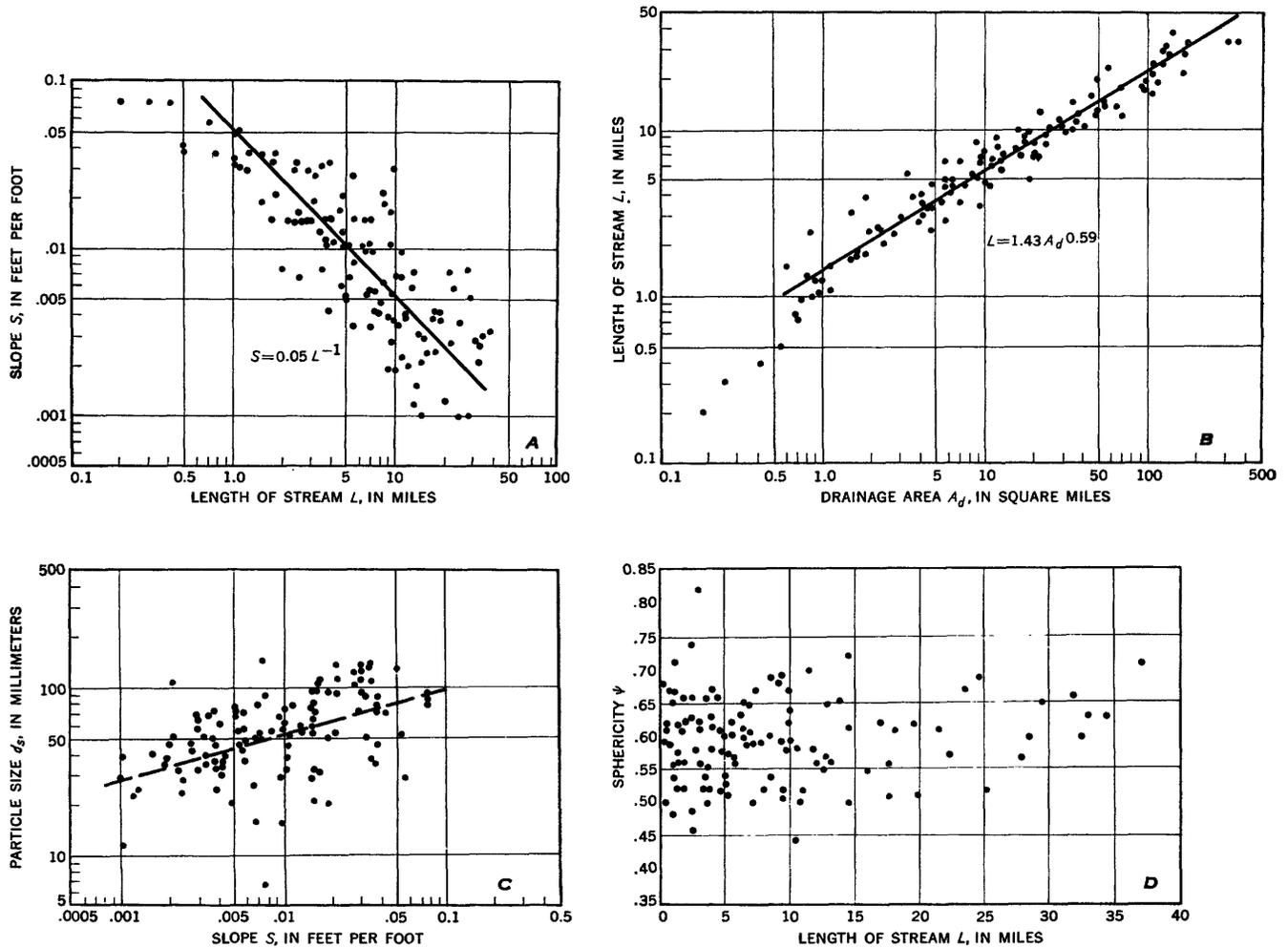
FIGURE 102.—Relation at all sampling points of: *A*, channel slope to length of stream from drainage divide, *B*, length of stream from drainage divide to size of drainage area, *C*, mean particle size to channel slope, and *D*, intercept sphericity to length of stream from drainage divide.

The equation obtained for the relation of length to size of drainage area compares very well with similar observations made by Hack (1957) for streams in the Appalachian Mountains further south. Hack's equation for these same variables is

$$L = 1.43 A_d^{0.6} \qquad (11)$$

Further information about the hypothetical stream may be obtained by studying the hydraulic geometry of the composite plots of w, d, and v shown in figure 103. The scatter of points is large for velocity plotted against discharge, but the relations of width and depth to discharge indicate that a definite trend is present. Furthermore, the exponents in the power function equations which are drawn through the points compare very favorably with the average values of these exponents for other streams as listed by Leopold and Maddock (1953).

Pennsylvania streams	*Average streams*
	(Leopold and Maddock, 1953, p. 16)
$b = 0.55$	$b = 0.5$
$f = .36$	$f = .4$
$m = .09$	$m = .1$

These results emphasize the fact that the variation between individual streams is not so great as to mask the overall relation for all the stations plotted as one stream. Furthermore, the curves can be used for other purposes. For example, the scatter of points about a trend line can be plotted against other variables with the hope of delimiting the cause of the scatter of points.

EFFECT OF KIND OF BEDROCK ON LONGITUDINAL PROFILES AND BASIN CHARACTERISTICS

Erosion surfaces in the stratigraphic column indicate that streams erode all types of bedrock, though in a given period of time the total amount of erosion is not necessarily the same for all rocks. An interplay of hydraulic and geologic forces occurs but it is difficult to estimate the relative magnitudes of these forces. In

order to evaluate certain of geologic influences on longitudinal profiles, kinds of bedrock, stratigraphic position, and structure were considered. If geologic factors can be shown to be unimportant, the conclusion would be that the prime determinant of profile characteristics is of a hydraulic nature. However, if geologic factors are important, they may also influence other aspects of hydrologic environment in addition to merely the longitudinal profile.

GENERAL GEOLOGY

Because most streams in mountainous areas flow close to the bedrock, it is reasonable to suppose that the characteristics of the bedrock beneath the stream might influence the channel shape and the absolute position of the channel in space. Two of the bedrock characteristics which may be important are composition and relative ability to resist erosion. These two factors are intimately related for most climates, although variations between the relations occur from one area to another.

Detailed geologic mapping of the quadrangles has been completed for only 2 of 11 quadrangles included in the present study. Plate 7 is a reproduction of a portion of the geologic map of Pennsylvania produced

by Stose and Ljungstedt (1931), which was used as a base map for this study.

Stratigraphy was also considered, in terms of the relative positions of various formations and the rock types in the stratigraphic column. For reference purposes the column produced by Butts and Moore (1936) is included (see pl. 8).

BEDROCK LITHOLOGY AND THE LONGITUDINAL PROFILE

Three main kinds of bedrock were recognized: sandstone, shale, and limestone-dolomite. No attempt was made to break the sandstone category into components such as graywacke, arkose, and orthoquartzite. Limestone and dolomite were considered to be sufficiently equivalent in compositional properties to warrant combining them into one category. Hence, the studies of bedrock are of a general nature and only as important as the variations within and between categories.

In the previous section it was demonstrated that all the stations on different streams may be thought of as stations along one large, hypothetical stream. The scatter of points about regression lines which may be fitted to variables of the imaginary stream is quite large in some cases and small in others. Those graphs which show the largest scatter may actually yield the most information if some of the deviations can be explained. If the type of bedrock at each station is designated, the large, hypothetical stream may be thought of as a series of different streams each of which flows on one kind of rock. Stations which lie upon or slightly above formations which were composed of interbedded sandstone and shale, as well as other mixtures of the three categories, were not considered in this discussion. However, the general principles which will be derived may be applicable to other rock types as well. After each point had been labelled as to kind of bedrock present, it was found that the streams at 20 stations flow on or slightly above sandstone, 14 on shale, and 15 on limestone-dolomite. The assignment of a station to a rock category depends on the bedrock at the local site. The other 70 stations are located at reaches of streams which flow on combinations of the major rock types.

RELATION OF SLOPE TO LENGTH OF STREAM FOR DIFFERENT KINDS OF BEDROCK

Figure 104 shows the relation of channel slope and length in areas having three types of bedrock.

The equations of best fit derived from regression analyses are:

$$\text{Sandstone:} \quad S = 0.046 L^{-.67} \tag{12}$$

$$\text{Shale:} \quad S = 0.034 L^{-.81} \tag{13}$$

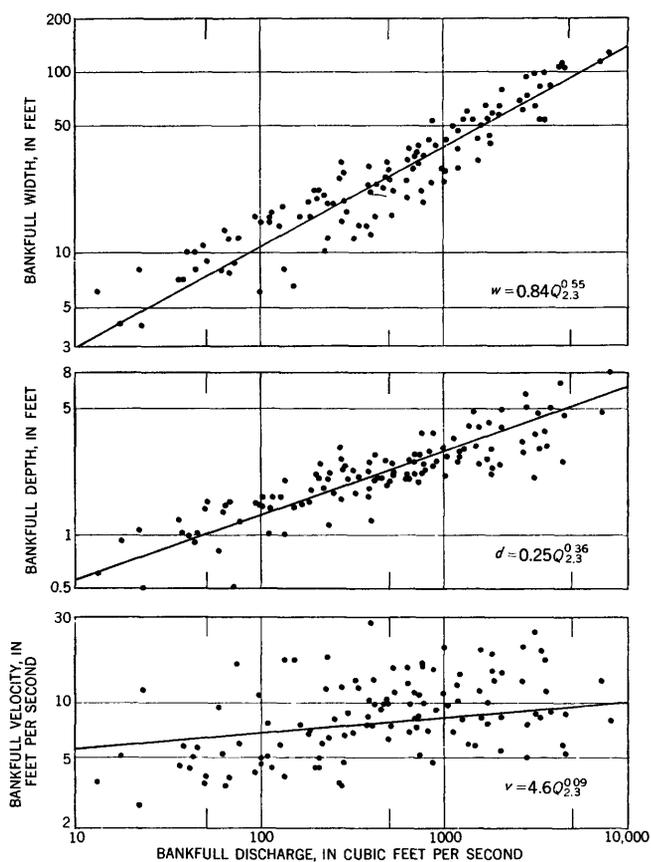$$\text{Limestone-dolomite:} \quad S = 0.019 L^{-.71} \tag{14}$$

FIGURE 103.—Relation of bankfull width, bankfull depth, and computed bankfull mean velocity to bankfull discharge for all sampling stations.
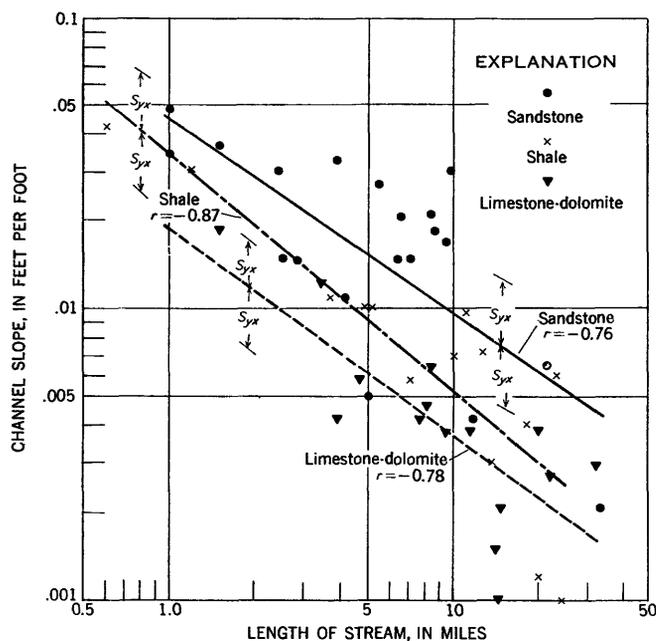
FIGURE 104.—Relation of channel slope and length of stream from drainage divide in areas of sandstone, of shale, and, of limestone-dolomite. The standard errors of estimate (S_{yx}) are shown graphically and the values of the correlation coefficients (r) are given.

Plots of these equations, along with the graphical representation of the standard error of estimate, which is a measure of the standard deviation of the scatter of points about the regression line, are shown on the figure. Correlation coefficients for the regression of slope on length are also shown. Each is significant at the 1 percent level.

To investigate whether the three equations are statistically different, an analysis of covariance was made by standard statistical methods (Snedecor, 1946, p. 318).

The first test consists of an analysis of variance of slopes and rock types. An outline of the form used is shown in table 4.

The results of the initial analysis of variance indicate that there are significant differences at the 1 percent level between slopes for sandstone, shale, and limestone. This test does not determine whether these differences are caused by the fact that most limestone tends to occur at larger values of length or that sandstone occurs only at small values of length. In order to determine

TABLE 4.—*Analysis of variance of channel slopes for three rock types*

Source of variation	Degrees of freedom	Sum of squares	Mean square	Value of F	Level of signifi-cance (per-cent)
Within rock types____	46	57. 100	1. 24	_____	
Among rock types____	2	29. 443	14. 721	11. 87	1
Total_____	48	86. 543			

whether these differences are independent of length, an analysis of covariance was made. This test may be described as a test of significance of differences among adjusted group means. The results of the analysis are shown in table 5.

This analysis shows that length does not account for differences among slopes which were found in the original analysis of variance.

The relative positions of means of slopes for the different rock types have been established; however, it has not been determined whether the slopes of the regression lines within rock types (regression coefficients, fig. 104) are signicantly different. In order to test this, the outline shown in table 6 was used.

This test indicates that the differences between the regression coefficients for the rock types are significant at the 5 percent level. Furthermore, the relation of slope to length for shale was found to be the one which is causing the significant difference.

The following statements may be made about the three types of rocks in the area:

1. The variation of slope with length is significantly different for the three rock types.
2. The length of the stream does not account for these differences.
3. The regression coefficients are significantly different at the 5 percent level.
4. The regression coefficient for the streams in shale was found to be the primary factor contributing to this difference.

Each rock type has a unique equation expressing the relation between slope and length. The exponents in the power function equations are nearly the same for all rocks, although shale does possess a slightly larger

TABLE 5.—*Analysis of covariance of slope among adjusted group means of rock types*

Source of variation	Degrees of freedom	Sum of S^2	Sum of SL	Sum of L^2	Total sum of squares of estimate	Degrees of freedom	Mean square	Level of signifi-cance (percent)
Within rock types_____	46	57. 100	−50. 749	69. 408	19. 994	45	0. 444	
Among rock types_____	2	29. 443	−13. 244	6. 000	12. 243	2	6. 12	1
Total_____	48	86. 543	−63. 993	75. 408	32. 237	47		

TABLE 6.—*Analysis of errors of estimate from average regression within groups of rock types*

Source of variation	Degrees of freedom	Sum of squares	Mean square	Level of significance (percent)
Deviation from average regression within rock types	45	19. 994	--------	
Deviation from individual regressions	43	16. 694	0. 388	
Difference among regressions for different rock types	2	3. 300	1. 650	5

(significant at the 5 percent level) negative value. This means that changes in slope in the downstream direction are fairly similar regardless of the lithologic character of the channel. In contrast, the absolute values of slope for any given length of stream are quite different, depending on the lithology. For the range in stream length considered, the average slope is greater on sandstone than on shale, which in turn is greater than the slope on limestone-dolomite for any given length.

The relationships found in these streams appear to agree with general knowledge of the resistance of similar rock types. In temperate humid areas, it is generally believed that sandstone is more resistant than shale, which in turn is more resistant than limestone-dolomite. Visual proof of this conclusion abounds in the Appalachian Mountains. Sandstone is almost invariably the ridge former of the mountains, whereas limestone and dolomite usually occur in the valleys.

If all mountain ridges were composed entirely of sandstone and all valleys of limestone and dolomite, the analysis of covariance would have indicated that the differences between slopes for different rock types were attributable to the length of the stream. The analysis did not; hence the ideas pertaining to relative resistance to erosion must be thought of as average relationships in which many exceptions may occur. For example, the thickness of the individual rock types would alter the general relationships. It takes a fairly thick sandstone to form a large mountain, and wide valleys require a great thickness of limestone and dolomite. Another important exception to the general rule may be related to stratigraphic position. For example, if the distance separating two resistant formations is small, only one mountain with two ridges may result; but if the distance is large, two mountains may be formed. The fact that there are variations in the distance between units and in the stratigraphic arrangement of limestones, shales, and sandstones tends to complicate the general relations. Nevertheless, order

exists within rock types as was shown by the relation between slope and length in figure 104.

There is no question that a large amount of scatter is exhibited on these plots of slope against length, but it must be remembered that the rock types are extremely broad and include within them considerable lithologic variation.

In summary, it may be stated that slope tends to decrease in a downstream direction almost independently of bedrock character, but the absolute values of slope at any given length depends on the bedrock. The geologic and hydraulic controls each exert an influence on the slope of the longitudinal profile, and the effect of each can be detected.

LONGITUDINAL PROFILES DETERMINED BY INTEGRATION OF THE EQUATIONS RELATING SLOPE AND LENGTH OF STREAM

Another method by which the effects of geologic factors on the longitudinal profile may be studied is to reconstruct the longitudinal profiles of the hypothetical streams previously considered. This may be done by integrating the equations which express the relation between slope and length (Hack, 1957).

Integration of the slope-versus-length equations for sandstone, shale, and limestone-dolomite in central Pennsylvania yields the following results,

$$\text{Sandstone } H = H_0 - 736 L^{.33} \qquad (15)$$

$$\text{Shale } H = H_0 - 945 L^{.19} \qquad (16)$$

$$\text{Limestone-dolomite } H = H_0 - 346 L^{.29} \qquad (17)$$

where height, H, is expressed in feet, and length, L, is expressed in miles.

The summit of the arbitrarily selected datum was chosen so that $H_0 = 0$ and the profiles were plotted in figure 105. The curves were not extended to lengths less than 0.2 mile because it is believed that the equations are not applicable to such short lengths in which recognizable channels are absent. The profiles (fig. 105) represent hypothetical longitudinal profiles of streams flowing on homogeneous bedrock and beginning at a common datum. It should not be forgotten that the basic data for the original equations were obtained from 16 different streams and these values were subsequently combined in order to study different rock types.

If the profiles were taken to represent the conditions which actually prevail in streams, it is obvious that streams flowing on limestone would create valleys with very small total relief compared to the valleys created by streams flowing on sandstone and shale. However, the relative positions of the integrated longitudinal profiles cannot be used as an index of the rate of erosion, for it is probable that the erosion rate would
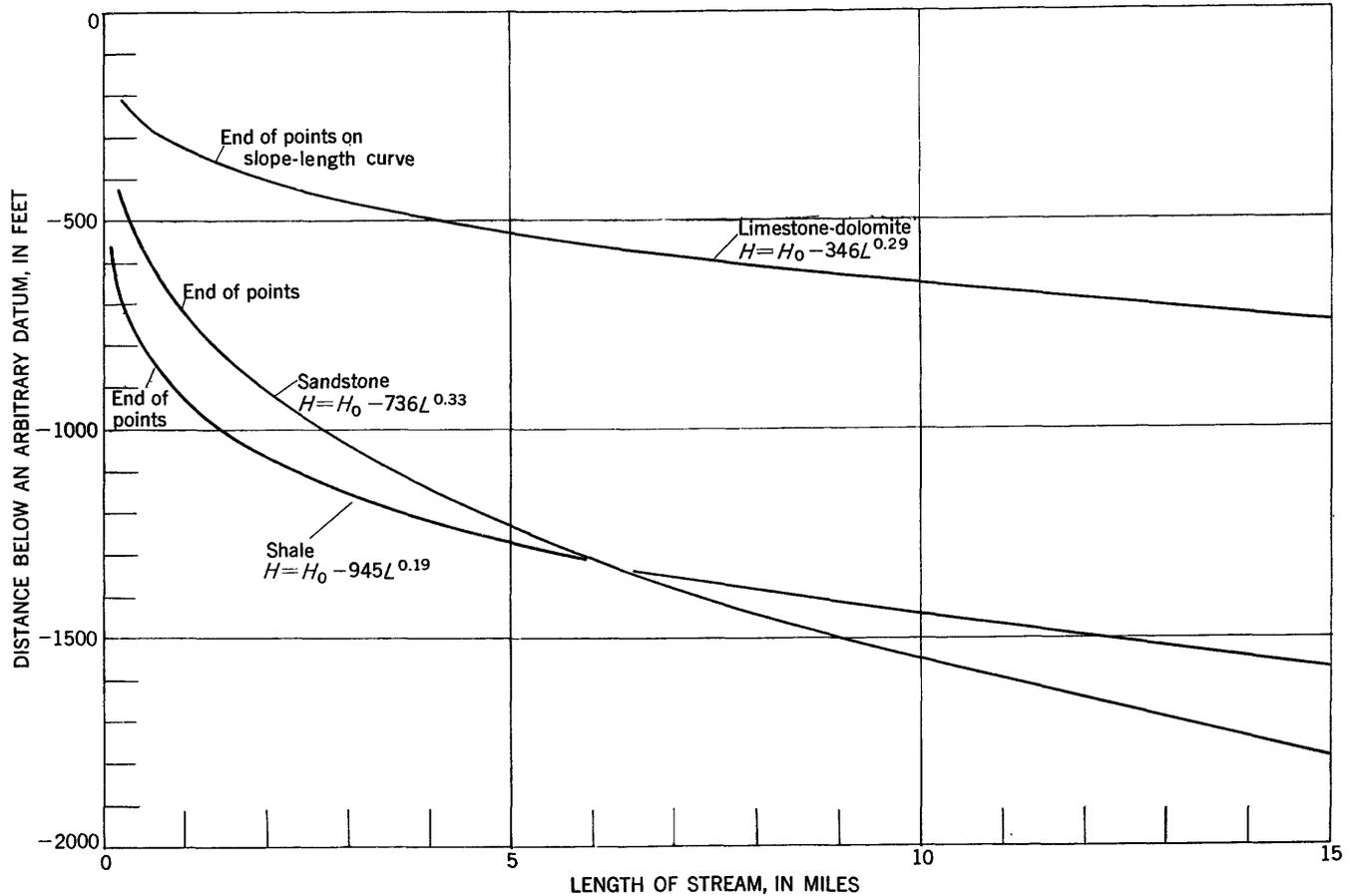
FIGURE 105.—Relation of hypothetical longitudinal profiles beginning at an arbitrary datum and length of stream for three rock types in areas of sandstone, of shale, and of limestone-dolomite. The equation for each profile is shown.

be much faster on limestone-dolomite than on sandstone, although the total relief is low for a stream flowing on limestone-dolomite.

After recognizing these limitations, one can utilize the profiles for other purposes. This may be done by understanding that the integrated profiles may be compared in a relative sense. For example, if a stream flows upon sandstone for a length of 2 miles, it would possess a high overall slope, but at specific points the slope would still decrease in magnitude with length. Consider what would happen where a stream passes from sandstone to limestone. A large decrease in slope would be expected to occur. For as many miles as the stream flows on limestone, the profile would be that of the one shown for limestone for the length range which is applicable. A return to flow upon sandstone should be accompanied by an increase in slope although the absolute value of slope would be less than for sandstone streams at smaller lengths. By means of these diagrams it is possible to devise various lithologic combinations which would cause the longitudinal profiles to become very irregular in concavity and general appearance, but be similar, in fact, to conditions in natural streams.

EFFECT OF KIND OF BEDROCK ON BED MATERIAL IN THE CHANNEL

RELATION OF PARTICLE SIZE AND LENGTH OF STREAM

By use of the same procedure as before, the relation of particle size to length was investigated. Particle size was plotted against length and the kind of bedrock at each point was noted (fig. 106). A regression analysis of the three rock types was made. The results are tabulated below:

$$\text{Sandstone } d_s = 116L^{-0.09} \qquad (18)$$

$$\text{Shale } d_s = 90L^{-0.30} \qquad (19)$$

$$\text{Limestone-dolomite } d_s = 24L^{0.18} \qquad (20)$$

The correlation coefficients for sandstone and for limestone-dolomite are not significantly different from zero, but the correlation coefficient for shale is significant at the 5 percent level. Furthermore, none of
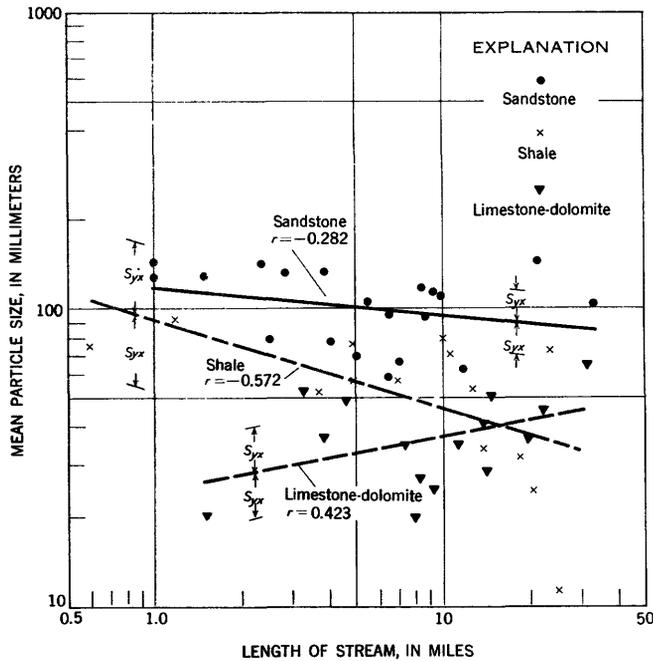
FIGURE 106.—Relation of mean particle size and length of stream from drainage divide for three rock types in areas of sandstone, of shale, and of limestone-dolomite. The standard errors of estimate (S_{yx}) are shown graphically and the values of the correlation coefficients (r) are given.

TABLE 7.—*Analysis of variance of particle size for three rock types*

Source of variation of particle size	Degrees of freedom	Sum of squares	Mean square	Level of significance (percent)
Within rock types_____	46	12. 829	0. 2789	_____
Among rock types_____	2	16. 768	8. 384	1
Total_____	48	29. 597	_____	_____

These analyses show that particle size differs among sandstone, shale, or limestone-dolomite at the 1 percent level of significance. Particle size may or may not decrease in the downstream direction in the hypothetical stream channel of uniform bedrock.

Some explanation is needed to account for the fact that in the hypothetical channels in any given bedrock type (1) the particle size which occurs on the bed appears to be independent of stream length, and (2) different particle sizes occur on different types of bedrock. One plausible suggestion is that the properties of the various bedrock types are different and result in large sandstone particles, smaller shale particles, and even smaller limestone particles. The relationship between sandstone with large particles and limestone with small particles implies that a complete mixing of the bed material does not occur. Otherwise, the differences in size with respect to bedrock would not be obvious. The lack of mixing implies that at least a fraction of the bed material does not move except at high flows or that the supply of new material is very large.

In either case the pebbles in the bed material should be related to the bedrock. Unfortunately only qualitative estimates of the rocks of the bed material were made and these show no conclusive results except that in channels cut on sandstone few other rock types were noted. A detailed study of the lithologic character of bed material in Virginia and Maryland was made by Hack (1957). His results indicate that a very large percentage of the coarse bed material reflects the underlying bedrock and that many of the boulders in the bed material rarely move. It seems reasonable to assume that the same thing is true for the streams in central Pennsylvania.

the regression coefficients is significantly different from zero. Thus, despite the fact that the equations of best fit appear different, statistically they might all just as well be horizontal lines. This, of course, results from the large scatter and the rather small number of samples.

To determine whether a significant difference in particle size existed between the three rock types, an analysis of variance was made. Table 7 summarizes this analysis.

From this table it may be seen that the difference in particle size between rock types is significant at the 1 percent level. Hence, the noted differences in particle size would be expected less than 1 percent of the time if random samples were drawn from a population possessing a normal distribution.

An analysis of covariance showed that the variation between particle size for the various bedrock types is not accounted for by stream length at the 1 percent level. A summary of this test is given in table 8.

TABLE 8.—*Analysis of covariance of particle size for three rock types*

Source of variation	Degrees of freedom	Sum of L^2	Sum of d_*L	Sum of d_*^2	Sum of squares	Degrees of freedom	Mean square	Level of significance (percent)
Within rock types_____	46	12. 829	−7. 555	68. 408	11. 995	45	0. 267	_____
Among rock types_____	2	16. 768	−10. 027	7. 000	13. 503	2	6. 752	1
Total_____	48	29. 597	−17. 582	75. 408	25. 498	47	_____	_____

COMPETENCE

Because hydraulic data are available for the streams in Pennsylvania, it is possible to investigate the probability of movement of particles present on the bed by calculating the competence of the stream. One method is to compute the so-called competent velocity necessary to move a particle, such as was determined by Hjulstrom (1939) and Menard (1950). Although this approach is valid, the method does not take into account differences in slope. Another method which does involve the use of slope is that used by Lane (1953), in which the shear stress at the bed of the channel is used as a criterion of competence. The shear stress (τ) at the bed of a stream is equal to γdS, in which γ is the specific weight of water, d is the depth, and S is the slope.

The maximum shear stress computed for a station in central Pennsylvania is about 6 pounds per square foot, which, extrapolating from Lane's graph of limiting fractive forces (1953, fig. 7), would just move a particle 250 mm in diameter. Table 9 is a summary of some headwater stations which have large fractions of coarse bed material. Included in the table is the maximum particle size which would move for the given shear stress as given by the empirical plot of Lane.

The data listed in table 9 indicate that many riffles contain large percentages of particles greater than 150 mm in diameter. There is reason to believe that a sizeable (perhaps 10–20 percent) portion of the bed material in the headwaters is stationary except during rare floods, because the shear stress at many of the headwater stations would not be sufficient to move many of the particles resting on the bed.

Several factors may cause bed material that is too large to be moved by the 2.3-year flood to remain on the bed. One is that bedrock outcrops in the channel are common and these outcrops may contribute large boulders to the stream, depending upon the weathering and jointing characteristics of the bedrock. Some joint systems are present in the area and can be seen in many outcrops. The distance separating joint planes was estimated to range from a few inches to several feet. If an outcrop which possesses a joint system of widely spaced joints occurs in a stream bed, it is obvious that large boulders could be contributed to the stream. Another source of large boulders is the mantle of coarse debris which occurs in the headwater areas. This mantle of debris is believed by Smith (1948) to have been caused by intense periglacial activity during the Pleistocene. Rock streams and boulder fields are common, but in addition, a blanket of unconsolidated boulders, sand, and silt occurs over wide areas. Some of these boulders fall into the streams because of undercutting of the banks and mass movement on the adjacent slopes. Other large boulders which the streams are not competent to move may be residual and occur in the channels because the finer debris has been removed.

In addition, the fact that many particles in the bed do not move is of considerable importance because this is one mechanism, though only a portion of the bed material is affected, by which the bed material may reflect the local lithology, as was suggested in the previous section.

HYDRAULIC GEOMETRY AND KINDS OF BEDROCK

If the geologic environment is an important factor in determining absolute values of slope and particle size in any length of channel, it is reasonable to believe that differences may be reflected in other channel characteristics. In order to determine whether this assumption is true, the hydraulic geometry was treated in a way similar to the analyses of slope, particle size, and length.

Composite graphs of width, depth, and bankfull velocity were plotted against size of drainage area. The points were labelled according to the type of bedrock. Kinds of bedrock had no obvious effect on width or depth in these plots, but in the plots of bankfull velocity against drainage area, the points representing streams that flow on limestone-dolomite plot above most of the points for sandstone and for shale (fig. 107).

This graph shows that for a given drainage area, the bankfull velocity of a stream flowing on limestone-dolomite is greater than the bankfull velocity of streams flowing on sandstone or shale. It is difficult to say which of several plausible hypotheses may explain this fact. If bankfull discharge truly represents an equal flood frequency for all stations for a given drainage

TABLE 9.—*Frequency of occurrence of large particles in headwaters of streams in central Pennsylvania*

Stream	Station No.	Percent >150 mm [1]	Percent >200 mm [1]	Percent >250 mm [1]	Approximate maximum size of particle expected to move for given bankfull shear (mm)
Honey Creek	107	5.0	?	?	80
Do	108	21.0	5.0	?	85
Shaver Creek	1	40.0	17.0	9.0	90
Do	2	47.0	28.0	15.0	140
Standing Stone Creek	13	30.0	11.0	6.0	140
Do	14	6.5	1.8	?	80
Slab Cabin Creek	87	19.0	7.5	5.0	130
Weiker Run	55	49.0	25.0	16.0	55
Globe Run	48	7.5	3.7	3.0	230
McClain Run	66	25.0	11.7	8.0	200
Little Juniata River	92	37.0	23.0	17.0	160
Reeds Run	73	8.0	1.4	?	130
Sixmile Creek	24	14.0	7.0	5.0	80
Bald Eagle Creek	40	2.0	?	?	90

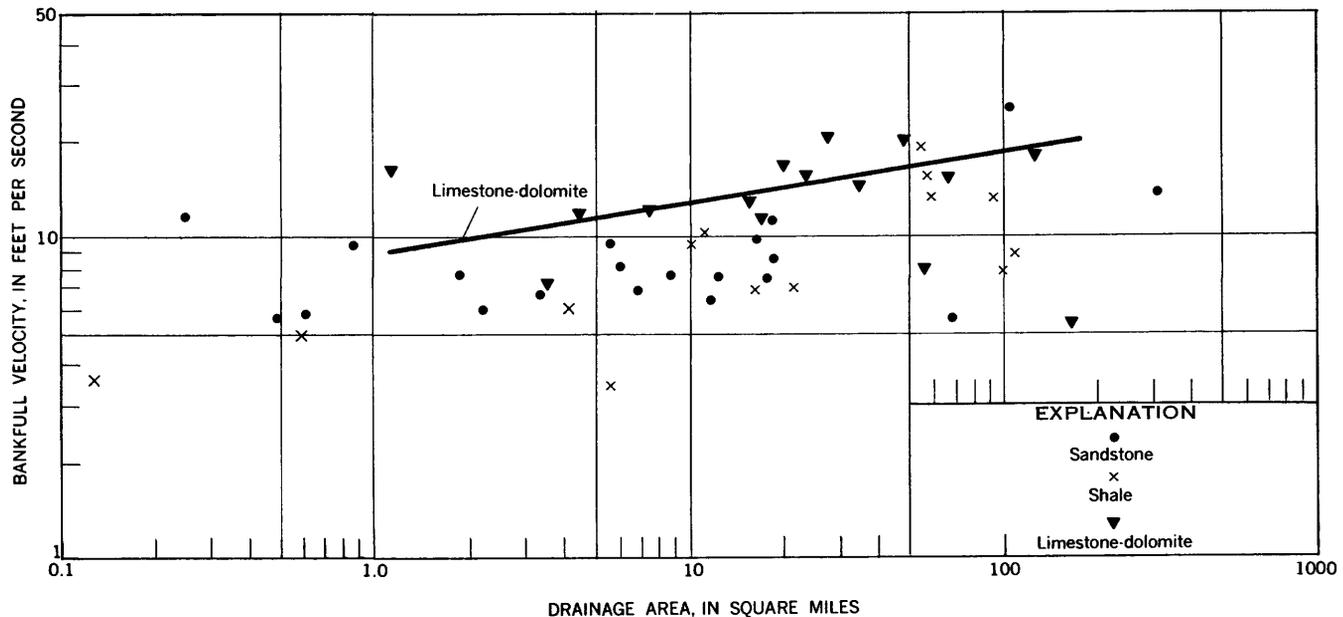[1] ? denotes details of distribution not determined.

FIGURE 107.—Relation of computed bankfull velocity and size of drainage basin in areas of sandstone, of shale, and of limestone-dolomite. The line of best fit is shown for limestone-dolomite.

area, a small channel may exist provided its roughness is low compared to the larger channels. The only measure of roughness made in the field was of particle size. It showed that small particles are present in the channels of limestone-dolomite bedrock and, hence, that roughness is lower. On the other hand, equal frequency of bankfull flow is an assumption which can not be proved absolutely. It is conceiveable that limestone-dolomite channels may overflow their banks at a frequency somewhat more often than once every 2.3 years. If this is so, it is equally difficult to explain the reason for it. Underflow or leakage is not believed to be greater in limestone-dolomite channels and cannot be used to explain the smallness of cross sections of the streams measured. Nevertheless it appears that the type of bedrock also influences to some extent the size of the channel.

Less weight must be given to the points on figure 107 for larger drainage areas because the bedrock is in most cases not in immediate contact with the channel but is separated from it by alluvium.

SUMMARY AND DISCUSSION

The 10–20 percent of almost stationary material present in the channels of the headwaters does not explain why different sizes of particles occur in channels which flow on different types of bedrock. Particle size was found to be largest in streams flowing on sandstone, smaller on shale, and smallest on limestone-dolomite. Because sandstone usually weathers into large blocks, it is to be expected that particle size is large in channels on sandstone. Shale tends to weather

into two shapes: plates and rods, which are usually smaller than the weathering products of sandstone. On the other hand, limestone and dolomite are found to weather into rather large blocks at some places along a stream. However, many of these large blocks appear to be practically stationary. Hack (1957) supports the observation that many large limestone blocks do not move but are reduced in size in place until the particle is small enough to be moved by the available velocity. There are many long reaches of streams which flow over limestone or dolomite in which the bed material is not large. These reaches in which the channel has small pebbles outnumber the reaches that have large blocks and tend to make the mean particle size in channels on limestone-dolomite smaller than in channels on shale or sandstone.

The relations of particle size to length are in accord with ideas of competence. If it is recalled that for a given length of stream, slope was greater in channels on sandstone than on shale and limestone-dolomite, it follows that competence ·is greater for channels on sandstone than on shale or limestone-dolomite. Thus, particle size would be expected to be greater on sandstone than on shale or limestone-dolomite, as it is in central Pennsylvania.

It was suggested that slope is determined by the amount of discharge, which is related to channel length and to the relative resistance to erosion of the bedrock. Particle size appears to be related to bedrock and does not correlate significantly with length. Competence determines the particle size at some reaches, and in other reaches the weathering character-

istics of the bedrock or particle size of the parent material, such as periglacial debris, determines the size of the particles which enter the stream channels. Thus, slope and particle size in the streams of central Pennsylvania respond to discharge, weathering, and general resistance to erosion. The relative magnitudes of these forces in a particular stream determine whether there is a simple relation between particle size and slope. The suggestion that slope and particle size respond to some controls other than hydraulic reduces the probability that there is a direct cause and effect relation between particle size and slope in these streams.

Finally, the data which were derived from a study of channel characteristics, such as width, depth, and velocity, indicate that the bedrock may indirectly affect channel size.

RELATIONS OF SLOPE, PARTICLE SIZE, LENGTH, AND KIND OF BEDROCK

It has already been shown that there is no relation between particle size and slope in streams flowing on sandstone, shale, and limestone-dolomite. This is true because particle size was found to be independent of length, and slope was noted as being related to length. There are two obvious and likely places along a stream that might tend to obscure the relation of particle size and slope. One, of course, is the headwater area and the other, the mouth. Active slope movement or interplay between mass wasting and streamflow may hide relations which are present in other reaches. In contrast, toward the mouth of a steam backwater from the main channel may alter the relation between particle size and slope.

PARTICLE SIZE AND SLOPE FOR COMMON LENGTHS

To test the possibility that headwater reaches and the extreme downstream portions of these streams might obscure relations between particle size and slope, the data were arbitrarily divided into three categories of length on a logarithmic scale. The categories are 1 mile to 3.2 miles, 3.2 miles to 10 miles, and 10 to 32 miles. The results of this separation may be seen in figure 108. The stations have been labelled according to the type of bedrock present. It is obvious that the separation was fairly successful insofar as the plot for the intermediate length category indicates that a relationship exists between particle size and slope. There is no trend for the headwater category, and there is a very doubtful relationship for the longer streams.

Although the original boundaries for the three categories of length were arbitrarily defined so that an approximately equal division of the points would result, it is possible to increase the range in the intermediate category from 2 miles to 15 miles and not seriously disrupt the general relationship previously noted.

The initial conclusion reached from figure 108B is that for streams in this length range, particle size and slope are related. If this statement were approximately true, it would be desirable to investigate whether all stations regardless of geologic factors, between lengths of 3.2 to 10 miles show a relation between particle size and slope independent of the bedrock lithology. A plot of these data did not indicate a trend. In fact, the relation was less well defined than the original particle size-slope diagram (fig. 102C) which was based on all the stations measured. From figure 102C, one must conclude that particle size is independent of slope for lengths of 10 to 32 miles. Some explanation must be offered for the fact that in figure 108B particle size and slope appear to be related. The answer may rest with the fact that the station points were selected to represent definite kinds of rocks. From figure 106 it was shown
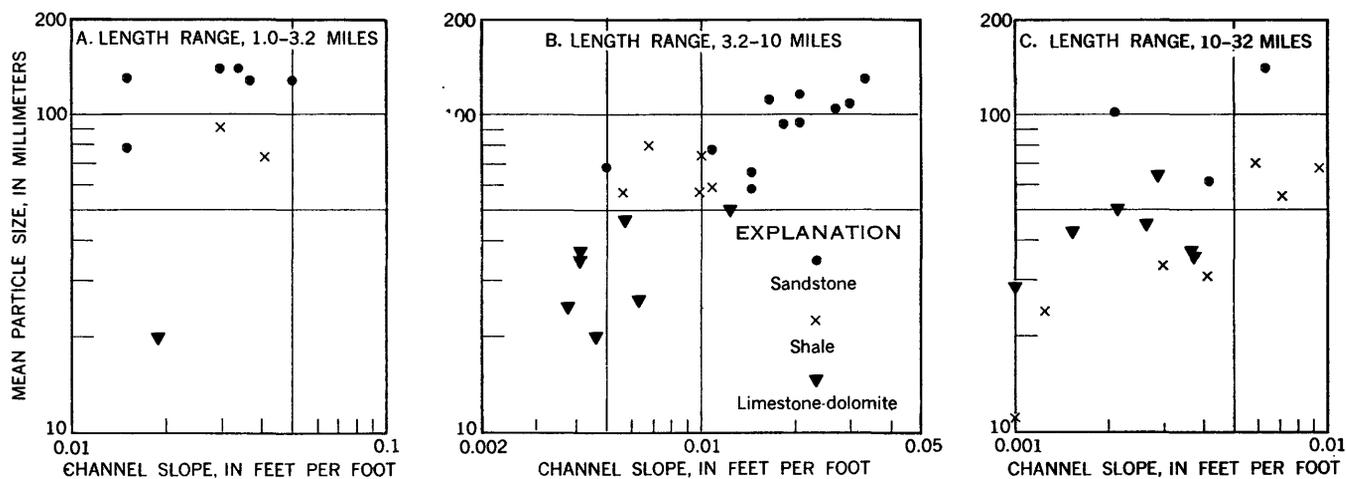

FIGURE 108.—Relation of mean particle size and channel slope for three categories of length. Lengths of streams are divided into groups on the basis of a logarithmic scale. Bedrock lithology is listed for each point.

that particle size was independent of length for each of these kinds of rock and is in effect constant. Particle size is different for each type of bedrock in such a fashion that particle size decreases from sandstone to shale to limestone-dolomite respectively. In figure 104, it was shown that slope was proportional to length for each type of rock, but the relation was different for each rock type in such a manner that for any given length, slopes on sandstone are greater than slopes on shale, which, in turn, are greater than slopes on limestone-dolomite. It follows then that for a given length category, particle size and slope for sandstone should be greater than for shale which is greater than for limestone-dolomite, and that a general relation between particle size and slope might be expected. Particle size and slope are dependent upon other factors, such as weathering characteristics of the bedrock, stream competence, and presence of residual boulders in the channel, in such a fashion that a correlation between particle size and slope may exist in some streams and not in others.

ROCK TYPE, SLOPE, AND PARTICLE SIZE FOR INDIVIDUAL STREAMS

Although plots of slope against particle size of bed material for individual stations do not show precise relations, perhaps trends exist between particle size and slope for whole streams. A plot of the exponents in the equations relating slopes to drainage area (A_d) and particle size (d_s) to drainage area (A_d) was used to investigate this possibility (see fig. 109). z is the exponent in the equation relating slope to drainage area,

$$S=KA_d^z \tag{21}$$

and α is the exponent in the equation relating particle size to drainage area,

$$d_s=KA_d^\alpha \tag{22}$$

From figure 109 it is apparent that a linear relation exists between α and z, and it may be expressed as,

$$\alpha=0.08+0.64z \tag{23}$$

This equation may be interpreted to mean that the faster log S decreases with log A_d, the faster log d_s decreases with log A_d. In terms of an actual stream, the results indicate that the faster slope decreases in a downstream direction, the faster particle size decreases.

Because previous results indicate that the kind of bedrock controls to a large extent the absolute values of particle size and slope at any given length of stream, it is desirable to investigate the possibility that bedrock may also control the rates of change of the variables, slope and particle size. This was done by making
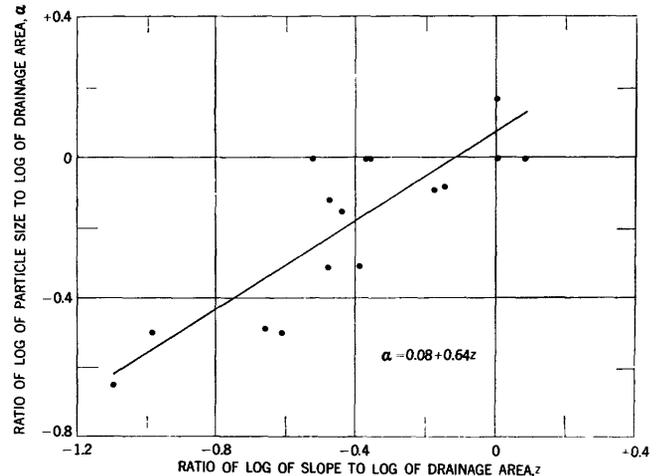

FIGURE 109.—Relation of the rate of change of log channel slope to log drainage area and rate of change of log particle size to log drainage area. Each point represents one stream.

detailed examinations of three of the streams represented by points on figure 109.

Shaver Creek, Little Juniata River, and Sixmile Creek were the streams selected. Shaver and Sixmile Creeks have extreme values of z, namely, -1.10 and $+0.08$, respectively, and the Little Juniata River, an intermediate value of z, -0.48.

The values of z and α for Shaver Creek indicate that particle size and slope decrease very rapidly with an increase in drainage area or length. The rocks underlying the various stations listed from head to mouth are shown in figure 110. The creek flows from most resistant bedrock at the head to least resistant at the mouth. According to previously established principles, particle size would be expected to be greater for shale with some sandstone than for limestone and shale or for limestone alone. Also, slopes would be expected to respond in a similar fashion. Hence, it would be expected that particle size and slope would correlate very well for this stream, which in fact they do (fig. 93). Furthermore, because slope also decreases with length or discharge, it would be expected that the rate of change of slope with drainage area would be exceptionally rapid, as in fact it is.

The Little Juniata River behaves somewhat differently because the bedrock over which it flows offers a different pattern of resistance to erosion. Although slope tends to decrease with length, it is not accompanied by an orderly decrease in relative resistance of the bedrock and slope does not decrease as markedly as for Shaver Creek. Furthermore, the change in particle size in a downstream direction is variable because the bedrock changes. The total reduction in particle size between the uppermost and lowest station is merely the difference between particle size for sandstone and the particle size for shale with some interbedded sandstone.
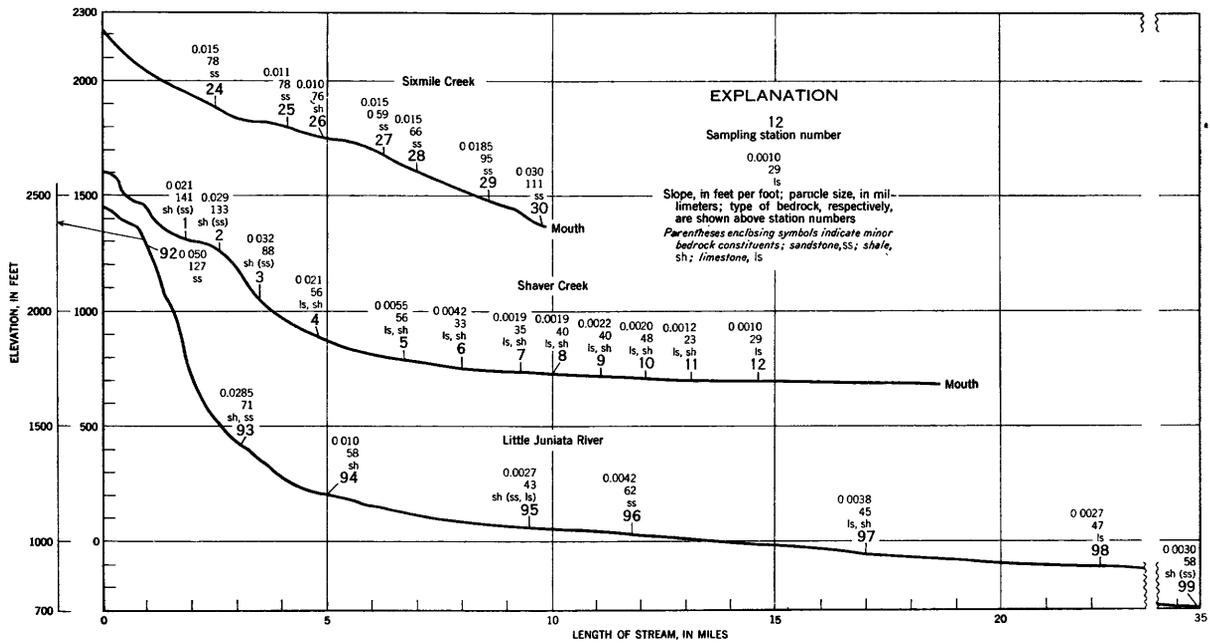
FIGURE 110.—Longitudinal profiles of Sixmile Creek, Shaver Creek, and the Little Juniata River, including rock type, channel slope, and mean particle size for each sampling station.

Sixmile Creek represents another extreme in which the observed relations are converse to the relations found in Shaver Creek. Slope is everywhere (all 7 stations) extremely steep and particle size is large at almost all stations. In addition, at 6 of the 7 stations the stream bed is on sandstone in which steep slopes and large particle size are to be expected (see fig. 110). The exceptional feature of Sixmile Creek is that slope actually increases slightly in a downstream direction.

For each of the other streams in the area, most of the relations of slope and particle size may be explained in terms of the expected weathering characteristics of the bedrock. This is especially true of particle size but must be qualified for slope when it is considered that slope tends to decrease downstream regardless of the kind of bedrock. Therefore, it is suggested that the relation of α and z (fig. 109) exists because slope and particle size respond to other variables, one of which is the kind of bedrock.

COMPARISON WITH RESULTS OF OTHER AUTHORS

Hack's (1957) findings in Virginia and Maryland on the effects of geologic factors on stream properties may be compared with the results of this study of streams in Pennsylvania. Although there are slight differences in procedure, results, and interpretations, in general the similarities outweigh the differences. Hack sampled streams which flow on sedimentary, igneous, and metamorphic rocks and on unconsolidated Coastal Plain material. Many of the streams which he selected for study were not characterized by large variations in kind of bedrock along individual streams. His composite plots of data are, for the most part, combina-

tions of main streams and tributaries, both of which have very few major lithologic differences along their courses. In contrast, the study in central Pennsylvania was centered on streams which have many lithologic differences along their courses.

Hack was able to segregate successfully the relations of slope to length of stream for many types of bedrock including the igneous, metamorphic, and sedimentary rocks. Furthermore, he found that for a given length, streams on sandstone have a greater slope than those on limestone or shale. Separations of the latter types were not so pronounced as in Pennsylvania; that is, there was considerable overlap of points on the curves. Nevertheless, the fact that he could distinguish between relations of slope and length of stream for various kinds of rocks attests to the general validity of the assumption made for Pennsylvania; namely, that all stations from various streams common only in rock type might be grouped and their data treated as representing one stream.

Correlations which Hack made involving particle size of bed material were not quite so successful as were those for streams in central Pennsylvania, although he found several correlations of particle size and length of travel. Hack (1957) concludes that particle size is a contributing determinant of slope. This is brought out in his relation which takes the form,

$$S=18\left(\frac{d_s}{A_d}\right)^{0.6} \qquad (24)$$

In the data for central Pennsylvania, a plot of slope against drainage area was slightly improved, as reflected by a small decrease in scatter, by introducing particle

size into the equation. The difference was not nearly so great as found by Hack but nonetheless a slight improvement was evident. If it may be assumed for the purpose of illustration that length of stream is proportional to drainage area to the 0.6 power (refer to fig. 103), then it can be shown that particle size will decrease in the downstream direction only where slope is related to length with an exponent more negative than minus one.

$$L = 1.43 A_d^{0.6} \qquad (11)$$

$$S = \frac{(1.4)(18) d_s^{0.6}}{L} = \frac{25 d_s^{0.6}}{L} \qquad (25)$$

$$\text{If } S \propto \frac{1}{L}, \; d_s = \text{a constant} \qquad (26)$$

$$\text{If } S \propto \frac{1}{L^2}, \; d_s^{0.6} \propto \frac{1}{L} \qquad (27)$$

$$\text{If } S \propto \frac{1}{L^{0.5}}, \; d_s^{0.6} \propto L^{0.5} \qquad (28)$$

Eight of the 16 streams in central Pennsylvania that were studied show a relation of $S \propto \frac{1}{L}$, but show a decrease in particle size with length. For this reason it is believed that although the empirical equation found by Hack (1957) may be used to summarize the composite relations of slope, drainage area, and particle size for the entire area, the relations found in individual streams are somewhat different. This does not detract from the merits of his relation but may indicate that one or more additional parameters are needed to describe the interaction of particle size, slope, and drainage area. The data from streams in Pennsylvania suggest that one of these additional parameters is kind of bedrock.

SUMMARY OF RELATIONS BETWEEN KIND OF BEDROCK AND THE LONGITUDINAL PROFILE

Relations between particle size, slope, and length were demonstrated by studying bedrock types. Slope is related to length, insofar as the rate of change of slope is proportional to the rate of change of length (exponent in slope-length relation). The absolute value of slope depends on the type of bedrock. Particle size was found to be independent of length for each of the same bedrock types, hence, independent of slope, but very much dependent upon the type of bedrock material upon which the stream flows. The size of the particles was in turn ascribed to the weathering, jointing, and residual debris (in the case of periglacial debris) of the bedrock. Relationship exists between particle size and slope where rock types change in a

downstream direction from more resistant to less resistant in coincidence with the hydraulically controlled change of slopes with length or discharge. Comparison of results with those of Hack (1957) indicate that slope depends upon rock types in both areas. Particle size was also found by Hack to differ with rock types, but their relations were not so well defined as in central Pennsylvania.

A large part (10–20 percent) of the bed material in the headwater reaches is believed to be stationary. This conclusion is based on consideration of the competence of the stream to move bed material and comparison with field studies made by Hack. Tributary action also contributes to the change of particle size downstream. No relations of sphericity and sorting to kind of bedrock were noted.

The effect of the type of bedrock in the area upon the characteristics of the longitudinal profile was found to be very great. In fact, the relationships involving particle size and slope fit so well with the accepted ideas of relative resistance of sandstone, shale, and limestone-dolomite in humid areas, that the longitudinal profiles may be thought of as being quite similar to the general profiles of the mountains present in the area, which have steep slopes on ridge-making sandstones and gentle slopes on valley-forming limestones.

Finally, the results and interpretations listed above are not expected to apply to dissimilar areas. Certainly the bedrock under rivers flowing in large alluvial valleys cannot affect appreciably the slopes, particle sizes, and shapes of the channels; however, the valley fill in these areas replaces the solid bedrock and this replacement becomes the lithologic type which in turn influences the longitudinal profile. In alluvial valleys, the stream may move the bed material according to its slope requirements, but the same is not true for headwater areas except over very long periods of time. In fact, the process must be slow or the differences in slope and particle size for various types of bedrock would not exist.

SUMMARY AND CONCLUSIONS

Obviously there are many facets of this investigation which could not be evaluated precisely because of insufficient data. Nevertheless, many relations between hydraulic and geologic factors, basin morphology, and longitudinal profiles were studied. Some of these findings apply only to streams less than 35 miles long in central Pennsylvania but a few may have general application. For the purpose of brevity and clarity, the principle conclusions are outlined below.

1. Hydraulic and geologic factors contribute to the differences found between the various longitudinal profiles. The absolute value of slope at any point

along a stream depends in part on the type of bedrock, but slope decreases in a downstream direction for all types of bedrock. The decrease in slope with length of stream from watershed for common rock types correlates with the increase in discharge in a downstream direction. The basic slope-length equations for points in similar rock types may be integrated to obtain the longitudinal profiles which would be expected if the streams flowed continuously within regions of homogeneous bedrock. The profiles derived by integration compare well with those found by Hack (1957) for natural streams in the Appalachian Mountains in Virginia and Maryland.

2. Particle size is related to length of stream from watershed and slope for many of the streams studied but not in others. The distributions of the bed particle sizes indicate that the material on riffles is well sorted and contains a slight skewness indicated by a lack of finer material. This skewness is thought to represent selective erosion of fine material at riffles. Particle shape is independent of the longitudinal profile and is thought to be primarily a reflection of shape characteristics which were inherited from the parent bedrock, alluvium, or colluvium. Wear of particles in transport seems to be of minor importance in determining particle size.

3. In the selected streams of central Pennsylvania, particle size is related to the type of bedrock and is not always dependent on discharge or the length of stream. It is believed that particles constituting 10–20 percent of the bed material in the headwaters do not move; this belief is based on a study of competence and on the work of Hack (1957).

4. Slope and particle size are related to kinds of bedrock. Slope is also related to length. In many streams slope and particle size are related because each is related to the type of bedrock. Sandstone tends to produce steep slopes and large particle size, shale tends to produce gentler slopes and smaller particle size, and limestone-dolomite tends to have the most gentle slopes and smallest particles. Thus, the distribution of rock types and their relative positions along the length of a stream tend to govern the rate of change of slope and particle size and to determine the correlation between these variables if one exists. The relations between particle size, slope, and bedrock are similar to those which would be expected by differential erosion in areas with humid temperate climates.

5. Orderly geometric relations exist between certain characteristics of drainage basins, such as length of stream, size of drainage area, and the mean annual flood. Relations between length of stream and drainage area describe the general shape of the drainage basins but not the irregularities. Analyses of the type

suggested by Horton (1945) reveal that drainage development and pattern follow regular geometric progressions independent of the diversity of the bedrock throughout the area.

6. By means of the relation between this 2.3-year flood and the size of the drainage area, the hydraulic geometry of streams may be studied by indirect methods. The power function equations relating bankfull width, depth, and velocity to bankfull discharge in a downstream direction are similar to those reported by Leopold and Maddock (1953). There is slight suggestion that the bedrock may influence the hydraulic geometry of streams. The effect is seen in small cross-sectional areas of streams flowing in limestone and dolomite.

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APPENDIX A.—*Summary of data*

Station No.	Bankfull width, w (ft)	Bankfull depth, d (ft)	Bankfull velocity,[1] v (ft per sec)	Drainage area, A_d (sq miles)	Mean annual flood, $Q_{2.3}$ (ft³ per sec)	Slope, S (ft per ft)	Length of stream, L (mi)	Station elevation, E (ft)	Mean b-axis particle size d_s (mm)	Trask sorting coefficient, S_0	Phi standard deviation, σ_ϕ	Phi skewness, a_ϕ	Mean a-axis particle size, a (mm)	Mean c-axis particle size, c (mm)	Intercept sphericity, ψ	Kind of bedrock[2] under station
						Shaver Creek										
1	16	1.40	5.1	1.83	115	0.0210	1.8	1,315	141	1.42	0.75	+0.160	209	74.1	0.62	sh(ss)
2	16	1.44	7.4	3.01	171	.0290	2.9	1,260	133	1.59	.99	+.212	242	103	.62	sh(ss)
3	19	2.19	6.3	4.71	246	.0320	3.5	1,160	88	1.69	1.16	−.017	116	24.8	.54	sh(ss)
4	19	2.42	6.6	6.11	300	.0210	4.8	900	56	1.43	.90	+.056	72	21.0	.61	ls & sh
5	22	2.54	7.5	9.14	420	.0055	6.7	790	56	1.30	.73	+.342	77	26.3	.63	ls & sh
6	26	2.05	9.7	11.92	520	.0042	8.0	760	33.3	1.33	.61	+.098	57.8	14.1	.52	ls & sh
7	35	2.55	8.1	17.54	720	.0019	9.3	740	35	1.80	1.23	+.138	56.2	19.1	.59	ls & sh
8	40	2.59	9.2	25.41	960	.0019	10.1	730	40	2.22	1.52	+.211	60.7	24.4	.64	ls & sh
9	38	2.56	12.9	35.70	1,260	.0022	11.0	720	40	1.54	1.04	−.077	76.4	19.9	.52	ls & sh
10	43	3.20	11.6	47.60	1,600	.0020	12.1	705	48	1.54	.98	+.097	68	17.0	.56	ls & sh
11	50	4.07	8.3	49.38	1,680	.0012	13.1	690	23	1.42	.82	+.097	39	11.6	.56	ls & sh
12	55	4.27	7.8	55.01	1,840	.0010	14.6	685	29	1.46	.76	+.053	40	6.8	.50	ls
						Standing Stone Creek										
13	8	1.30	6.2	0.88	64	0.0340	1.0	1,520	112	1.39	0.71	+0.127	156	66	0.67	sh(ss)
14	15	1.58	12.15	5.78	290	.0150	2.9	1,125	65.5	1.51	.88	+.136	75	48	.82	sh(ss)
15	23	2.17	9.82	10.73	490	.0170	4.5	995	117	1.67	1.15	+.130	171	73	.66	sh(ss)
16	35	2.29	10.2	20.69	850	.0110	6.8	850	46	1.29	.55	+.091	72	30	.65	ls & sh
17	47	2.66	10.1	34.27	1,260	.0038	10.0	770	74.5	1.61	.92	+.065	114.5	43	.62	ls & sh
18	58	2.47	15.1	66.01	2,150	.0015	13.9	720	41.4	1.49	.85	+.292	68.7	20	.57	ls
19	68	2.92	21.4	92.63	2,800	0.0024	17.7	680	28.0	1.40	0.84	−.405	54	18.3	0.56	sh(ss, ls)
20	75	5.19	7.7	99.88	3,000	.0012	20.1	660	24.6	1.47	.81	−.086	45.7	12.3	.51	sh
21	100	3.70	8.9	109.72	3,300	.0010	24.7	635	11.5	1.55	1.02	+.059	17.4	8.7	.69	sh
22	100	3.18	12.0	133.30	3,800	.0010	28.0	620	40.2	1.49	.96	0	62.1	17.5	.57	sh & ss
						Sixmile Creek										
24	14	1.6	5.9	2.20	133	0.0150	2.5	1,990	78	1.45	0.92	+0.054	110	40	0.63	ss
25	19	1.7	8.1	6.01	260	.0110	4.1	1,800	78	1.55	.95	+.042	106	46	.67	ss
26	24	2.1	9.2	10.03	462	.010	4.9	1,760	76	1.46	.95	+.042	113	37	.60	sh
27	29	2.5	7.5	12.12	540	.0150	6.4	1,690	59	1.35	.59	+.017	88	30	.60	ss
28	33	2.1	9.8	16.15	680	.0150	7.0	1,610	66	1.84	1.19	0	121	50.5	.61	ss
29	36	2.8	7.3	17.55	740	.0185	8.6	1,490	95	1.49	1.05	+.152	122	52	.69	ss
30	32	2.8	8.5	18.42	760	.0300	9.8	1,390	111	1.51	.97	+.155	163	72	.67	ss
						Fishing Creek										
31	25	2.1	20.4	28.09	1,070	0.0038	11.5	1,205	36	1.49	0.90	+0.089	51	24.5	0.70	ls & dol
32	30	2.9	14.7	34.48	1,280	.0021	14.5	1,160	50	1.60	1.14	+.271	65	12.7	.61	ls & dol
33	33	2.5	20.0	48.04	1,650	.0038	19.7	1,090	37	1.44	.78	0	62.5	24.8	.62	ls & dol
34	41	3.0	15.5	57.08	1,900	.0059	23.0	1,000	72	1.39	.78	+.205	110	52	.67	sh
35	55	3.1	20.5	121.88	3,500	.0050	29.5	765	76	1.38	.72	+.208	120	54	.65	ls & sh
36	55	3.8	17.7	126.50	3,700	.0029	31.8	710	65	1.44	.81	+.037	96.	40.5	.66	ls & dol
37	85	5.2	9.1	141.08	4,000	.0032	37.2	610	51	1.54	.92	+.043	61.5	26.3	.71	dol(ss)

See footnotes at end of table.

APPENDIX A.—*Summary of data*—Continued

Station No.	Bankfull width, w (ft)	Bankfull depth, d (ft)	Bankfull velocity,[1] v (ft per sec)	Drainage area, A_d (sq miles)	Mean annual flood, $Q_{2.3}$ (ft³ per sec)	Slope, S (ft per ft)	Length of stream, L (mi)	Station elevation, E (ft)	Mean b-axis particle size d_s (mm)	Trask sorting coefficient, S_0	Phi standard deviation, σ_ϕ	Phi skewness, a_ϕ	Mean a-axis particle size, a (mm)	Mean c-axis particle size, c (mm)	Intercept sphericity, ψ	Kind of bedrock[2] under station
						Bald Eagle Creek										
38	13.5	1.42	3.44	0.96	66	0.0380	1.25	1,510	46	1.62	1.34	+0.477	57	14.2	0.58	sh & ss
39	22	2.04	4.90	4.05	220	.0190	3.1	1,200	51	1.67	.72	+.028	74	18.8	.56	sh & ss
40	27	2.33	4.70	5.79	295	.0125	4.8	1,055	58	1.49	.81	+.111	78	21.2	.58	sh(ss, ls)
41	40	3.71	5.25	19.91	780	.0033	7.1	1,000	36	1.61	.97	+.052	48	8.2	.50	sh(ss, ls)
42	50	3.50	6.90	32.27	1,200	.0054	9.4	950	45	1.42	1.06	+.302	67	19.4	.58	sh(ss, ls)
43	60	4.06	5.83	40.59	1,420	.0035	10.5	905	67	1.59	.94	−.362	120	44.0	.58	sh(ss, ls)
44	65	2.68	10.30	53.42	1,800	.0029	14.5	850	62	1.51	.87	+.103	78.5	37.5	.72	sh(ss, ls)
45	65	3.95	8.40	66.36	2,160	.0042	17.7	815	37.5	1.46	.86	−.116	55	11.8	.51	sh(ss, ls)
46	85	4.93	8.35	120.99	3,500	.0036	25.1	710	50	1.48	.81	−.012	70	14.3	.52	sh & ss
47	113	4.84	13.90	311.09	7,600	.0021	33.0	610	104	1.45	.81	−.161	144	51.5	.63	ss
						Globe Run										
48	8	1.05	2.7	0.25	22	0.0760	0.3	1,420	65	2.00	1.33	−0.150	104	35.2	0.59	sh(ss)
49	6.5	1.7	9.1	1.55	100	.0150	1.7	1,060	63	1.80	1.45	+.228	99	35.7	.61	sh(ss)
50	15	2.0	9.5	5.60	285	.0150	2.8	1,030	131	1.72	1.20	+.250	231	70.4	.55	ss
51	18	1.8	10.6	7.02	342	.0150	3.7	1,005	97	1.89	1.40	+.071	158	45.4	.56	sh(ss)
52	16	2.0	12.5	8.50	400	.0150	5.5	800	98	1.32	1.13	+.292	147	51.0	.61	sh(ss)
53	16.5	2.75	9.6	9.46	438	.0100	6.5	730	63	1.92	1.45	+.145	111	41.1	.59	ls & sh
54	17	3.0	8.7	9.72	445	.0095	7.0	715	16	1.40	.72	+.083	24	6.2	.55	ls & sh
						Weiker Run										
55	4	0.5	11.8	0.25	23.5	0.0340	1.0	1,940	142	1.37	0.66	−0.017	205	53.5	0.56	ss
56	8	.8	9.5	.84	61	.0300	2.4	1,720	141	1.43	.76	+.025	192	67	.63	ss
57	15	1.0	7.7	1.82	115	.0330	3.9	1,510	132	1.49	.86	+.059	176	60	.63	ss
58	19	1.5	6.7	3.40	190	.0275	5.5	1,300	105	1.52	.87	−.034	160	52.5	.60	ss
59	21	2.3	6.8	6.94	330	.0210	6.5	1,140	95	1.54	.94	+.064	115	37.5	.65	ss
60	30	1.8	7.6	8.75	410	.0210	8.4	960	117	1.52	.93	+.054	176	58	.60	ss
61	32	2.6	6.3	11.75	520	.0170	9.3	850	112	1.54	.94	+.032	165	77	.68	ss
						Buffalo Run										
62	9	0.52	16.2	1.13	76	0.0190	1.5	1,295	20	3.32	2.06	+0.387	43	12.5	0.52	ls
63	12.5	1.20	28.2	9.15	422	.0076	3.5	1,080	6.6	1.51	.90	+.133	9.6	4.1	.66	dol(ss)
64	16	2.16	15.9	12.37	550	.0034	5.5	1,000	39	2.53	2.01	+.762	58	19.3	.60	dol(ss)
65	19	2.51	16.9	20.02	810	.0063	8.4	910	27	2.01	1.28	+.297	49	13.6	.54	dol
						McClain Run										
66	4	0.9	5.0	0.19	18	0.0760	0.2	1,430	96	1.51	0.90	+0.067	138	65	0.68	sh(ss)
67	7	1.2	4.4	.43	37	.0760	.4	1,335	86	1.80	1.16	+.009	120	38	.61	sh(ss)
68	9	1.5	3.9	.71	53	.0570	.7	1,240	28.5	1.59	1.08	−.056	46.5	16.2	.59	sh(ss)
69	12	1.5	3.8	.97	69	.0520	1.05	1,150	51	1.64	1.18	+.076	92	26.6	.54	sh(ss)
70	15	1.6	4.5	1.68	107	.0380	1.80	1,010	72	1.45	.75	−.040	103	42.5	.66	sh(ss)
71	18	2.0	3.9	2.33	140	.0325	2.45	900	52	1.57	.92	−.120	76	32.3	.66	sh(ss)
72	20	2.5	4.4	4.05	220	.0150	4.00	715	21.5	1.54	.93	0	39.5	16.4	.61	ls & sh
						Reeds Run										
73	10	0.98	4.3	0.54	42	0.0380	0.5	1,055	78	1.42	0.73	+0.096	151	35.8	0.50	sh(ss)
74	11	1.33	3.5	.69	51	.0365	.75	990	86	1.47	.84	+.107	122	52.0	.67	sh(ss)
75	12	1.14	5.8	1.17	79	.0305	1.1	920	36.5	1.75	1.16	+.121	74.5	17.1	.48	sh(ss)
76	15	1.47	4.8	1.65	105	.0345	1.75	830	38.5	1.53	.91	−.099	58	15.4	.56	sh(ss)
77	17	1.59	4.4	1.91	120	.0165	2.5	750	32.0	1.38	.68	+.147	56.5	12.0	.49	ls & sh
78	22	2.18	4.4	3.86	210	.0150	2.7	735	29.5	1.46	.79	+.114	41.5	11.8	.58	ls & sh
79	26	3.03	3.6	5.48	280	.0105	3.7	670	32.5	1.45	.80	+.112	57.5	12.6	.50	ls & sh

See footnotes at end of table.

APPENDIX A.—*Summary of data*—Continued

Station No.	Bankfull width, w (ft)	Bankfull depth, d (ft)	Bankfull velocity,[1] v (ft per sec)	Drainage area, A_d (sq miles)	Mean annual flood, $Q_{s.s}$ (ft³ per sec)	Slope, S (ft per ft)	Length of stream, L (mi)	Station elevation, E (ft)	Mean b-axis particle size d_s (mm)	Trask sorting coefficient, S_0	Phi standard deviation, σ_ϕ	Phi skewness, a_ϕ	Mean a-axis particle size, a (mm)	Mean c-axis particle size, c (mm)	Intercept sphericity, ψ	Kind of bedrock[2] under station
								Marsh Creek								
80	8	1.00	17.5	2.31	140	0.0076	2.0	1,140	52	1.65	1.04	+0.048	85	19.6	0.52	sh & ss
81	12	1.12	18.2	4.68	245	.0067	2.6	1,090	52	1.57	.83	+.096	85	13.7	.46	sh & ss
82	24	1.66	10.3	8.81	410	.0067	5.2	940	16	1.89	1.36	+.382	35.5	10.4	.51	sh & ss
83	22	2.21	11.5	12.63	560	.0057	7.3	850	37	1.57	.99	−.131	60	25.2	.67	sh & ss
84	41	2.70	9.9	29.34	1,100	.0069	10.3	750	51	1.52	.91	+.088	81	11.2	.44	sh & ss
85	55	2.97	8.1	37.12	1,330	.0058	12.7	680	45	1.42	.75	+.147	60	13.7	.55	sh & ss
86	55	4.91	5.7	44.38	1,550	.0024	16.0	595	23.5	1.64	.98	+.102	36	9.0	.55	sh(ls, ss)
								Slab Cabin Creek								
87	8	1.0	5.75	0.60	46	0.0370	1.5	1,600	127	1.35	0.61	−0.213	212	62.1	0.56	ss
88	16	1.74	7.0	3.54	195	.0042	3.9	1,115	37	1.53	.83	+.060	66	23.3	.58	dol
89	17	2.05	8.9	6.22	310	.0052	5.0	1,085	82	1.56	.94	+.043	131	40.8	.58	dol(ss)
90	25	2.08	12.9	15.73	670	.0042	7.6	1,030	35	1.32	.61	+.082	56.5	19.0	.59	dol
91	29	2.20	11.3	17.11	720	.0038	9.4	955	25	1.72	1.02	+.255	35.5	16.9	.69	dol
								Little Juniata River								
92	7	1.0	5.6	0.49	39	0.0500	1.0	2,370	127	1.54	0.90	+0.011	177	70.0	0.65	ss
93	16	1.5	4.1	1.51	98	.0285	3.1	1,425	71	1.52	.84	+.298	103	35	.61	sh & ss
94	32	2.6	3.5	5.74	290	.0100	5.0	1,200	58	1.45	.82	+.043	106	28.3	.53	sh
95	53	3.7	4.7	23.48	920	.0027	9.5	1,060	43	1.54	.93	+.011	77	19	.52	sh(ss, ls)
96	80	5.0	5.5	67.09	2,180	.0042	11.8	1,030	62	1.78	1.12	+.080	101	33	.58	ss
97	95	6.2	5.1	97.07	3,000	.0038	17.0	940	45	1.41	.73	−.041	67.5	23.7	.62	ls & sh
98	110	7.1	5.4	166.39	4,600	.0027	22.2	870	46.5	1.43	.84	+.048	77	23	.57	ls
99	130	8.0	8.2	354.92	8,500	.0030	34.4	700	58.0	1.55	.95	+.021	88	33.7	.63	sh(ss)
								Warriors Mark Creek								
100	6.5	1.4	17.5	2.80	160	0.0150	2.3	1,185	33	1.49	1.02	+0.333	43.5	23.3	0.74	dol(ss)
101	10	1.98	12.0	4.46	237	.0125	3.4	1,105	56	1.53	.91	+.231	111	30.3	.52	dol(ls)
102	14	2.10	12.2	7.42	358	.0058	4.7	1,060	48	1.44	.86	+.151	82	20.2	.52	ls
103	20	2.12	15.8	12.79	670	.0084	5.6	1,020	55	1.84	1.20	+.350	101	35.0	.57	dol(ss)
104	22	2.28	16.1	20.21	805	.0095	6.7	980	29	1.67	1.07	+.121	42.5	18.7	.67	dol(ss)
105	25	2.40	15.3	23.66	920	.0047	8.0	955	20	1.60	.94	+.170	35.5	15.2	.62	dol(ls)
106	30	3.02	11.5	27.42	1,040	.0105	9.4	900	38.5	2.27	1.64	+.500	90	27.4	.51	dol(ss)
								Honey Creek								
107	6	0.60	3.6	0.13	13.2	0.0410	0.6	1,520	74	1.49	0.74	+0.324	119	45.5	0.62	sh
108	10	.90	5.0	.58	45	.0300	1.2	1,320	91	1.54	.90	+.078	120	56	.71	sh
109	21	1.82	6.0	4.25	230	.0110	3.7	1,080	52	1.46	.80	+.125	86	20.5	.52	sh
110	26	1.92	10.2	11.13	510	.0057	7.0	880	58	1.42	.77	+.324	85	26	.59	sh
111	38	2.60	6.9	16.09	680	.0069	10.1	760	80	1.56	.89	+.213	138	49.5	.59	sh
112	42	2.91	7.0	21.48	860	.0072	12.9	680	54	1.39	.72	0	85	37.5	.65	sh
113	45	2.20	19.2	55.80	1,900	.0030	13.8	675	33.5	1.41	.78	+.077	39	12.2	.65	sh
114	62	3.40	13.4	92.96	2,820	.0042	18.1	610	31	1.43	.82	+.049	52.3	19.8	.61	sh
								Beech Creek								
115	35	1.98	11.1	18.54	770	0.0050	5.0	1,360	68	1.38	0.70	−0.286	96	21	0.54	ss
116	60	2.37	13.7	59.03	1,950	.0095	10.8	1,115	69	1.48	.85	−.200	127	29	.50	sh
117	64	2.06	25.0	106.16	3,300	.0064	21.4	860	145	1.43	.76	+.079	203	64.8	.61	ss
118	108	2.58	5.9	167.64	4,700	.0076	28.4	760	91	1.48	.88	+.182	126	38	.60	sh & ss
119	115	4.67	8.9	178.44	4,800	.0027	33.5	580	44.3	1.39	.73	+.178	66	22.0	.60	ls & sh

[1] Calculated from $Q=wdv$.
[2] Ss, sandstone; sh, shale; ls, limestone; dol, dolomite. Parentheses indicate minor amounts.

APPENDIX B.—*Summary of flood-frequency analyses and measured bankfull discharge*

Stations in Pennsylvania	Mean annual flood $Q_{2.3}$ (cfs)	Bankfull discharge, Q_b		Drainage area A_d (sq mi)
		Rating curve (cfs)	Current meter (cfs)	
Susquehanna River at Harrisburg	291, 000			24, 100
West Branch Susquehanna River at Karthaus	33, 500			1, 462
West Branch Susquehanna River at Lewisburg	122, 500			6, 847
West Branch Susquehanna River at Renova	79, 000			2, 975
Susquehanna River at Danville	150, 000			11, 220
Little Fishing Creek at Eyers Grove	1, 860			56. 5
North Bald Eagle Creek at Beech Creek Station	11, 500			559
Shaver Creek near Charter Oak	117	[1,2] 680		[1] 3. 7
Frankstown Branch of Juniata River at Huntingdon	11, 800			816
Fishing Creek near Bloomsburg	10, 300			274
Raystown Branch of Juniata River near Huntingdon	16, 250			957
Grafius Run at Williamsport	281			3. 14
Juniata River at Newport	44, 000			3, 354
South Bald Eagle Creek at Tyrone	2, 250	[2] 2, 080		45. 1
Muncy Creek near Sonestown	1, 800			23. 8
Standing Stone Creek near Huntingdon	3, 070			128.
Little Juniata River at Spruce Creek	5, 250	[2] 7, 150		220
Penns Creek between Middleburg and Mifflinburg		5, 370		301
Kishacoquillas Creek at Reedsville		2, 530		164
Slab Cabin Creek near Lemont			502	15. 2
Crooked Creek near Pine Grove Mills			14. 5	. 7

[1] 1938 rating, possible leakage prior to 1950.
[2] Not used for regression analyses.

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