Turbofan Aero-Engine Efficiency Evaluation: An Integrated Approach Using VSBM Two-Stage Network DEA

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Abstract

The application of Data Envelopment Analysis (DEA) has been wide, especially for the purpose of evaluating efficiency among similar production processes within enterprises belonging to particular industries. Although research pertinent to DEA has primarily focused on efficiency of production systems or corporate entities/organizations (e.g., terminals, hospitals, universities/schools, banks), fairly little attention has been given to efficiency evaluation among engineering systems featuring common configurations (e.g., automobiles, power plants). Furthermore, the limited previous literature involving efficiency evaluation of engineering systems has implemented DEA methodologies with limited discriminatory power, i.e. there is a quite increased portion of efficient Decision Making Units (DMUs). In the current paper, a methodological framework deploying Variable intermediate measures Slacks-Based Measure (VSBM) Two-Stage Network DEA is implemented, in order to evaluate the efficiency of turbofan aero-engines, currently utilized by active-duty commercial and military aircraft. Apart from exploring the positive correlation of DEA efficiency with engineering efficiency, we also develop a methodology evaluating the features of near-future turbofan designs in terms of DEA efficiency, thus comprising a potential tool for efficiency assessment of any turbofan aero-engine being in the conceptual or preliminary design stage.

Keywords: *turbofan aero-engines, efficiency benchmarking, data envelopment analysis*

1 Introduction

The development of effective and efficient production methods in several industrial sectors constitutes a major aspect of the effort conducted by industry professionals worldwide. In this framework, researchers have dealt with the production efficiency evaluation by deploying a great variety of methodologies. Data Envelopment Analysis (DEA) has been so far designated as a quite prominent methodology for evaluating efficiency of production systems. In essence, DEA enables the evaluation of relative efficiency of entities possessing similar features otherwise referred to as Decision Making Units (DMUs), with minimal prerequisites pertinent to relations among inputs and outputs within these units [23]. Although DEA has been introduced by Charnes et al. [7] for the purpose of evaluating non-profit organizations like schools, Liu et al. [37] claim that relevant research deploying DEA methodology lies mainly in four sectors, namely the banking sector, the health care sector, the agricultural sector, and the transport sector.

The use of DEA in research pertinent to engineering systems is quite limited, while very few scholars and especially engineers have implemented efficiency measurement concepts for the purpose of assessing and improving performance of engineering products. In this context, it is vital to highlight the fact that in the framework of the conceptual or preliminary design stage of any engineering system/asset, engineers tend to implement a "stand-alone" approach (i.e. develop a design by merely adopting engineering principles) in order to optimize the particular system/asset, thus omitting to adopt a more holistic approach which encompasses benchmarking their design against any existing systems/assets of similar configuration [56].

The current paper primarily aims to evaluate the efficiency of turbofan aero-engines utilized by contemporary commercial and military aircraft implementing an advanced two-stage network DEA approach, thus further extending the previous research conducted by Bulla et al. [6]. The research contribution of the current paper is threefold, which is described as follows:

- Assess the efficiency of engineering systems/assets in an advanced DEA framework, subsequently conducting comparison with efficiency assessment within an engineering framework. While Bulla et al. [6] implement a single-stage DEA model, the current paper deploys a two-stage network DEA model in order to better represent the functional concept of turbofan aero-engines and concurrently possess enhanced discriminative power.

- Evaluate the DEA efficiency of commercial and military turbofan aero-engines with respect to technological progress over time and certain distinctive technological features, which are not investigated in the previous research of Bulla et al. [6].

- Create a fundamental framework for the development of methodology appropriate for benchmarking turbofan aero-engines being in the conceptual/preliminary design stage or entering service in the near future.

The structure of the rest of the paper is as follows: Section 2 describes the technological aspects of commercial and military turbofan aero-engines, concurrently highlighting contemporary design trends. Section 3 attempts to conduct a thorough literature review, with the first part dealing with previous research pertinent to engineering systems' efficiency evaluation with DEA, while the second part deals with previous research specifically investigating turbofan aero-engine performance optimization within an engineering framework. Section 4 has also two parts. In the first part description of the implemented DEA methodological approach is given, simultaneously providing a thorough justification of the two-stage network model. In the second part of Section 4, the turbofan aero-engine sample and the associated quantified features are presented, subsequently numerically assessing efficiency via the VSBM Two-Stage Network DEA methodology. In Section 5, post-hoc analysis of the efficiency results is conducted, thus designating DEA efficiency determinants and subsequently comparing them with the engineering efficiency determinants. Moreover, in Section 5 results of the post-hoc analysis are being discussed, taking into account the current and near-future technological limitations, concurrently setting the fundamentals for a method which could evaluate efficiency of any turbofan aero-engine design being in the conceptual/preliminary design stage or development stage. Finally, Section 6 makes a summary of the concluding remarks, while it additionally discusses some recommendations for future research.

2 Turbofan Aero-Engine Description & Performance Measures

Turbofan aero-engines comprise the most common propulsion system for modern era commercial and military aircraft, alternatively referred to as "bypass" engines. Unlike turbojet engines, which do not divert any portion of suctioned air mass, turbofans divert suctioned air in two different streams, commonly quoted as "cold" stream (not passing through the high-pressure compressor, combustion chamber, and turbine) and "hot" stream (passing through the high-pressure compressor, combustion chamber, and turbine) [17,46]. In addition, it should be mentioned that the arrangement of high-pressure compressor (HPC), combustion chamber, and turbine is frequently quoted as "core engine" [46].

In terms of number of spools utilized by turbofan aero-engines on a global scale, the two-spool configuration is the most widespread among commercial turbofan aero-engines in service, as well as in a significant number of military turbofan aero-engines [46]. In this configuration, the fan, low-pressure compressor (LPC) and low-pressure turbine (LPT) are connected by a single shaft [alternatively referred to as low-pressure (LP) shaft], thus rotating with the same speed. Simultaneously, high-pressure compressor (HPC) and high-pressure turbine (HPT), which are located immediately before and after the combustion chamber respectively, are also connected by a shaft [alternatively referred to as high-pressure (HP) shaft), which is concentric with the LP shaft. In Figure 1 we can observe a two-spool, high-bypass ratio (BPR), unmixed flow commercial turbofan aero-engine, while in Figure 2 we can observe a two-spool, low-BPR, mixed flow military turbofan aero-engine.

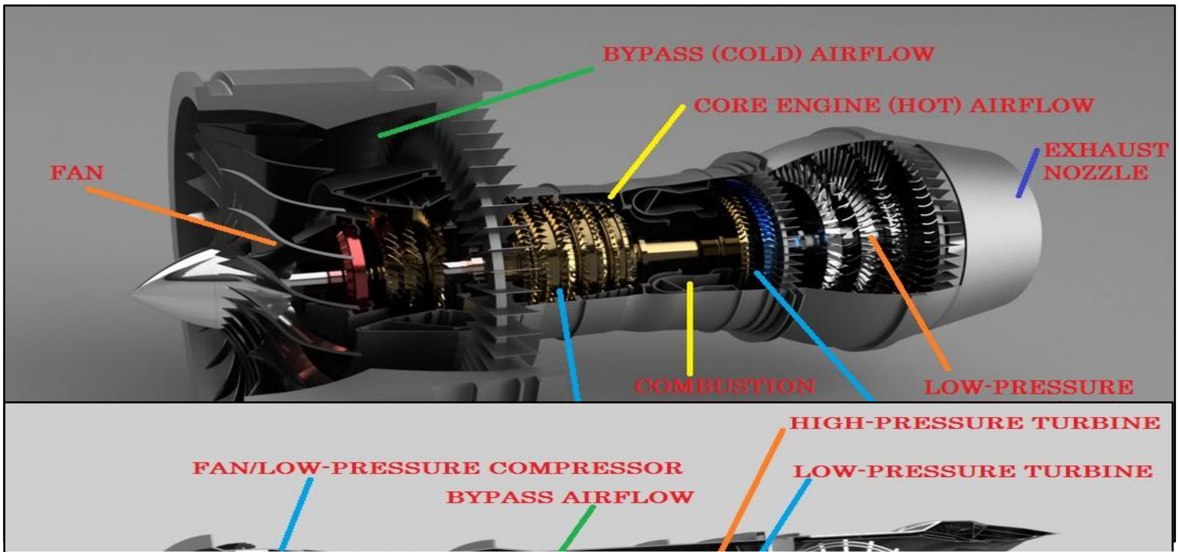

Figure 1: Two-Spool, High-BPR, Unmixed Flow Commercial Turbofan Aero-Engine
(Source: https://grabcad.com/library/pratt-whitney-turbofan-engine-1)

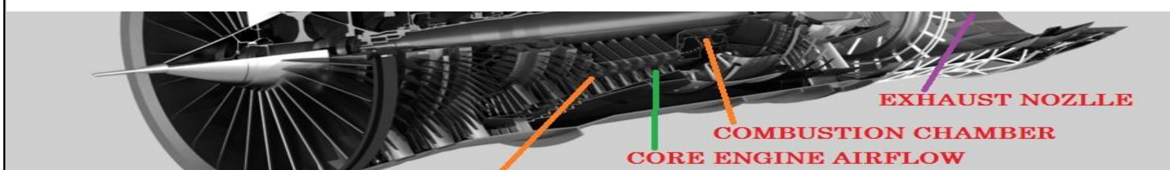

Figure 2: Two-Spool, Low-BPR, Mixed Flow Military Turbofan Aero-Engine
(Source: https://grabcad.com/library/low-bypass-turbofan-pratt-whitney-f100)

As a general rule, commercial turbofan aero-engines tend to adopt fairly high BPR (even reaching values of 7-8), while military turbofan aero-engines mostly adopt low BPR (usually values ranging from 0.3 to 0.5). Moreover, military turbofan aero-engines tend to add augmentor (alternatively referred to as "reheat" or "afterburner"), in which bypass and engine core flows are mixed and subsequently additional fuel is injected to create a supplemental combustion process, thus enhancing thrust [17,48].

As far as concerning the performance of turbofan aero-engines, thrust is the most widely used metric. Nevertheless, thrust alone is considered as inadequate to reflect the efficiency of any aero-engine, taking into account the fact that greater thrust can be achieved by simply making the aero-engine bigger, which is essentially deemed inefficient [39]. In this context, we shall refer to the main types of aero-engine efficiency, which are as follows [16,17,21]:

- Propulsive Efficiency: It comprises the efficiency of the conversion of kinetic energy of airflow passing through the engine into thrust power. It is considered an external efficiency measure, which is expressed by the following equation:

$$n_p = \frac{Thrust\ Power}{Power\ imparted\ to\ engine\ airflow} \quad (1)$$

For different types of propulsion systems, the propulsive efficiency is greatly differentiated with respect to flight airspeed, hence each propulsion system type is more efficient than other propulsion system types in certain airspeed ranges. As a general rule, higher propulsive efficiency can be attained when airflow velocity increase across the propulsion system is the smaller possible. A representative depiction of the aforementioned statements is displayed in Figure 3, which clearly designates the superior propulsive efficiency of turbofan propulsion systems compared to turboprop and turbojet propulsion systems in the high subsonic/transonic flight speed region.

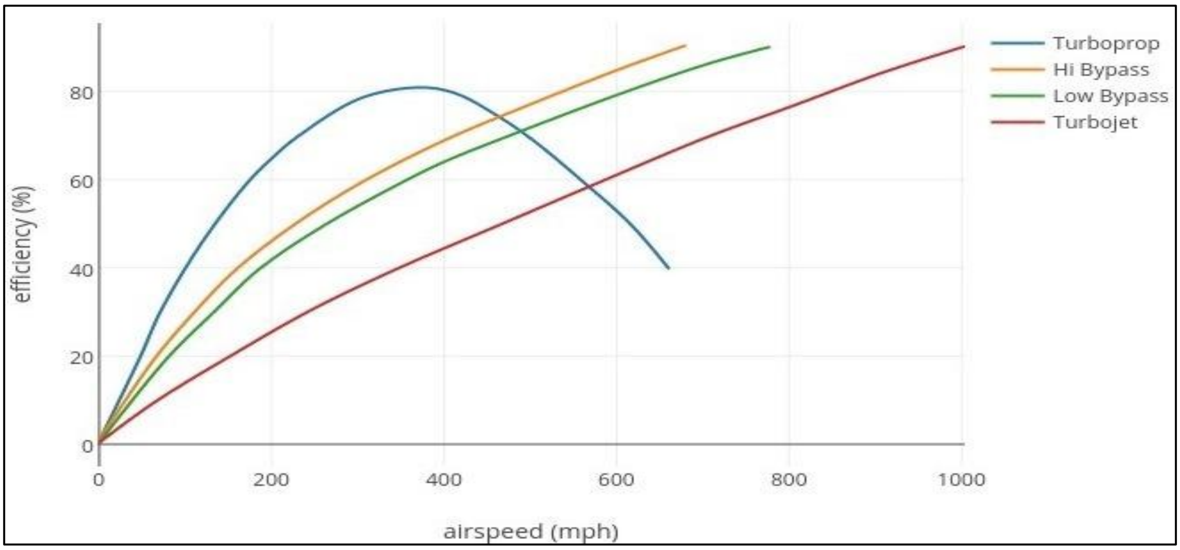

Figure 3: Propulsive Efficiency versus Airspeed for Different Propulsion System Types (Source: https://aviation.stackexchange.com/questions/48221/is-a-turboprop-or-a-turbofan-more-eco-friendly)

- Thermal Efficiency: It comprises the ratio of the kinetic energy obtained by the airflow during the whole engine cycle (compression, combustion, expansion) divided by the thermal energy inherently existent in the fuel provided. It is deemed as an internal efficiency measure expressed by the following equation:

$$n_{th} = \frac{Power\ imparted\ to\ engine\ airflow}{Rate\ of\ energy\ supplied\ in\ the\ fuel} \quad (2)$$

- Overall Efficiency: It constitutes the portion of the thermal power provided by fuel combustion, which is converted into thrust power by the propulsion system. It is expressed as the product of propulsive efficiency multiplied by the thermal efficiency ($n_p \times n_{th}$). For all types of propulsion systems overall efficiency can be expressed by the equation:

$$n_o = \frac{T \times u}{\dot{m}_f \times Q_R} \quad (3)$$

Where (T) is the thrust, (u) is the aircraft speed, ($\dot{m}_f$) is the fuel flow, and (Q_R) the fuel calorific value. Given the fact that overall efficiency depends on aircraft speed and produced thrust (which balances the aircraft drag), it is deduced that overall efficiency is not a performance parameter solely dependent on the engine, but influenced by the propulsion system/airframe combination and the state of flight operating conditions.

- Specific Fuel Consumption: For the case of turbofan propulsion system, the particular performance parameter is defined per unit of thrust force, otherwise referred to as *Thrust Specific Fuel Consumption* (TSFC), which is expressed by the fraction of fuel flow divided by the produced thrust i.e.

$$TSFC = \frac{\dot{m}_f}{T} \quad (4)$$

As far as commercial aviation is concerned, TSFC represents maybe the most important parameter of a propulsion system, as fuel expenditures constitute the biggest part of airline operating costs.

- Take-Off/Specific Thrust: Take-off thrust comprises a major determinant of propulsion system performance, as it represents its ability to propel an aircraft during take-off run and render it airborne. In certain instances, the parameter of specific thrust is implemented as it designates the capability of the propulsion system to produce thrust per unit of mass of inducted air, thus expressed as the fraction of the produced thrust divided by the inducted air mass flow ($\frac{T}{\dot{m}_{air}}$) . It is important to stress the fact that optimum TSFC is feasible through lower values of specific thrust, thus comprising a vital aspect associated with reduced fuel consumption in high-BPR turbofan propulsion systems like the ones utilized by commercial aircraft [48].

The above measures are used to evaluate performance and efficiency solely within an engineering context. Though, it is evident that overall efficiency depends on flight speed. Concurrently, flight speed affects the aircraft drag and subsequently the required thrust in order to maintain level flight [42]. Due to the fact that every individual aircraft type has different drag, it is deduced that even the same engine in different aircraft should produce different thrust performance in order to maintain level flight. As a result, aero-engine comparison is inherently very subjective and inaccurate if it is assumed that aero-engines under comparison are installed on aircraft.

On the other hand, take-off thrust represents thrust in static conditions, i.e. conditions where there is zero speed, thus being independent whether the aero-engine is installed on aircraft or not. However, although comparison of aero-engines in static conditions would be less subjective, it ignores the important parameter of engine weight. Hence, the aforementioned drawbacks which render aero-engine comparison vague when using engineering approach, designate the need to establish a benchmarking concept sufficient enough to perform efficiency evaluation in a more integrated manner.

3 Literature Review

3.1 Engineering Systems Efficiency Evaluation with DEA

As previously stressed, application of DEA for the purpose of evaluating the comparative efficiency of engineering systems is fairly scarce in the research conducted so far. Adoption of an engineering approach for the purpose of developing an engineering product usually encompasses optimization of sub-systems and/or components. In certain cases, the engineering approach ignores certain aspects which can have an impact on overall performance. Consequently, engineering design and development phases preceding the production of any engineering asset should implement methods to evaluate engineering systems' performance in a broader manner [57]. In the framework of the current paper, an extensive literature search has been conducted in order to pinpoint previous research related to engineering systems' evaluation using DEA. The results of the search confirmed the scarcity and subsequently the gap in scholarly research, pertinent to DEA implementation for benchmarking engineering assets. As observed in Table 1 of the current paper, 20 instances of research relevant to engineering asset benchmarking deploying DEA methodology were found. All research efforts incorporate single-stage basic DEA models, except for the research of Zhao et al. [60] which incorporates multistage network DEA.

The earliest piece of relevant research is the one by Färe et al. [20] which evaluates the efficiency of 22 steam-electric utility plants in the Western United States for the time period 1977-1979. Subsequent research conducted by Doyle and Green [15] benchmarks 37 computer printers. Like Färe et al. [20], Golany et al. [24] conduct efficiency assessment of power plants belonging to the Israel Electric Corporation for the time period between September 1981 and November 1987. The research efforts of Hjalmarsson and Odeck [29] and Odeck [43] are deemed quite unique, with the former evaluating the efficiency of heavy construction trucks belonging to the Norwegian public roads administration, while the latter evaluates the efficiency of rock-blasting units also belonging to the Norwegian public roads administration. The research of Lo Storto [38] also involves vehicle efficiency evaluation, namely efficiency evaluation of 29 Italian market passenger cars. In a similar framework to Lo Storto [38], Papahristodoulou [44] assesses the comparative efficiency of 121 passenger cars being on sale for the year 1996.

Electrical power-generating units' efficiency evaluation is revisited by Chitkara [9], thus comparing the performance of 37 coal-based generating units belonging to National Thermal Power Corporation of India for the period from 1991 to 1995. Sarkis and Talluri [49] conduct a quite novel efficiency assessment of Flexible Manufacturing Systems (FMS) incorporating cardinal and ordinal data, thus employing an evolved form of the methodology introduced by Cook et al. [10]. Subsequently, Braglia and Petroni [4] evaluate the efficiency of industrial robots by implementing a sophisticated single-stage model embedding restricted weights and sensitivity analysis with varying weight threshold values, while they concurrently compare the obtained results with the results obtained by implementing the methodology of Sarkis and Talluri [49].

Prominent research effort regarding the efficiency evaluation of engineering systems is the one conducted by Bulla et al. [6], which evaluates the efficiency of 29 turbofan jet engines used on commercial aircraft. The prominence of the particular research lies in the fact that DEA efficiency is compared with engineering efficiency, thus attempting to assess whether those types of efficiencies are correlated. The research of Sun [52] is also of interest, evaluating the efficiency of 21 Computer Numerical Control (CNC) machines.

The evaluation of power-generating units is repeatedly conducted in the research of Cook and Zhu [11] along with goal-programming. In a similar manner, the subsequent research of Meenakumari et al. [41] also deals with efficiency evaluation of 29 state-owned electric utilities in India. Efficiency evaluation of Flexible Manufacturing Systems (FMS) is revisited by Karsak [34] utilizing single-stage DEA model, albeit taking into account crisp, ordinal, and fuzzy data.

The research of Lee and Lee [36] is also deemed quite unique, as it attempts to evaluate the energy efficiency of 47 government buildings in Taiwan. In addition, the research of Zhao et al. [60] comprises a quite notable effort regarding engineering systems' evaluation, thus assessing the efficiency of a novel Downtown Space Reservation System (DSRS) for 28 scenarios, with each scenario representing an individual Decision Making Unit (DMU).

Automobile efficiency is also evaluated by Voltes-Dorta et al. [58], thus implementing VRS (Variable Returns to Scales) methodology along with Malmquist index for car models sold in Spain during the years 2004 to 2010. In a similar vein, Hampf and Krüger [28] evaluate automobiles sold in the German market, concurrently taking into account CO_2 emissions. Finally, Lozano et al. [40] apply their own-developed novel MMF (Multiple Modes of Functioning) DEA methodology, in order to assess technical, cost, and allocative efficiency of 14 Reconfigurable Machine Tools (RMTs) producing 7 different part types.

Table 1 designates that previous research dealing with engineering systems' evaluation using DEA, has mainly implemented single-stage CRS (Constant Returns to Scale) or VRS (Variable Returns to Scales) models. However, the fairly recent research of Zhao et al. [60] implements a multistage network DEA model, thus incorporating the functional relationships among the major segments of the engineering system. Consequently, the implemented DEA model represents the internal processes and their interdependencies within DMUs in a more accurate manner, compared to the single-stage basic DEA models where the conversion process of inputs to outputs essentially comprises a "black box".

In the current paper, a two-stage network DEA model is implemented, in order to better reflect the functions within DMUs (i.e. turbofan aero-engines) and subsequently obtain an improved outlook regarding the relative performance of DMUs under evaluation. Compared to the DEA model implemented in the previous research of Bulla et al. [6], the two-stage network DEA model deployed within the framework of current research seeks to attain the following advantages:

- Represent the turbofan aero-engine operation in a more accurate manner, when compared to its actual operation principles. More specifically, the energy which is exogenously provided by the fuel is transformed into another form of energy i.e. of a compressed air mass (aero-engine "cold" section). The compressed air which has certain flow and pressure magnitude is subsequently entered into the combustion chamber and converted into thrust within the so-called "hot" section of the aero-engine. Hence, our model concurrently assesses the efficiency of the "cold" section and the "hot" section of the turbofan aero-engine. However, as in Bulla et al. [6] aero-engine weight is also considered as an exogenous input, which is explained by the actual advantage that is gained by an aero-engine which adopts weight-saving technologies.

- Improved discriminative power, which usually comprises a major drawback of the single-stage basic DEA models. As a matter of fact, the DEA model of Bulla et al. [6] results in 8 out of 29 DMUs being efficient (or 27.58 percent of the sample), which is in any case deemed quite increased. Also, lack of ranking methods among efficient DMUs renders post-hoc statistical analysis quite subjective, thus significantly affecting conclusions.

Table 1: Previous Research Summary of Engineering Systems' Benchmarking Using DEA

Author(s), Year	**Engineering System Type**	**Number of DMUs**	**DEA Model Type**	**DEA Model Inputs**	**DEA Model Outputs**
Färe et al. [20]	Steam-electric Utility Plants	22	CRS & VRS (Input-oriented)	Labor, Fuel Consumed, Installed Generating Capacity	Kilowatt-Hours
Doyle and Green [15]	Computer Printers	37	CRS (Input-oriented)	Acquisition Cost	Throughput, Print Quality, Reliability, Disruption through Noise, Delay through Occupancy
Golany et al. [24]	Powerplants	87	CRS & VRS (Input-oriented)	Installed Capacity, Fuel Consumed, Manpower	Generated Power, Operational Availability, Deviation from Operational Parameters, Sulphur Dioxide Emissions
Hjalmarsson and Odeck [29]	Heavy Construction Trucks	72	VRS (Input & Output-oriented)	Driver Wages, Fuel Consumed, Tire Costs, Maintenance Costs	Total Transport Distance, Effective Hours in Production
Odeck [43]	Rock-Blasting Units	170	VRS (Input & Output-oriented)	Capital (transport & machines), Material Costs, Labor	Physical Volume Produced
Lo Storto [38]	Cars (Italian Market)	29	CRS (Input-oriented)	Acquisition Price, Fuel Consumption	Max Speed, Mass, Engine Capacity, Max Torque, Specific Power, Specific Torque, Acceleration, Pick-Up, Noise, Braking, Safety, Quality
Papahristodoulou [44]	Cars (1996 Models)	121	CRS, VRS, NIRS (Input-oriented)	5 inputs (not specified)	9 outputs (not specified)
Chitkara [9]	Power-Generating Units	37	CRS (Output-oriented)	Specific Coal Consumption, Specific Oil Consumption, Auxiliary, Vintage, Part Load, Outage, Availability, Capacity, Coal Quality	Power Generated

Table 1 (contd): Previous Research Summary of Engineering Systems' Benchmarking Using DEA

Author(s), Year	Engineering System Type	Number of DMUs	DEA Model Type	DEA Model Inputs	DEA Model Outputs
Sarkis and Talluri [49]	Flexible Manufacturing Systems	12	Single-stage capable of incorporating cardinal and ordinal data	Capital & Operating Costs, Floor Space	Improvements in Work-in-Process (ordinal), Percentage of Tardy Jobs, Yield (Throughput minus scrap & rework)
Braglia and Petroni [4]	Industrial Robots	12	CRS (Input-oriented) along with Weight Restrictions, Sensitivity Analysis with Varying Weight Restriction Thresholds, and Cross-Efficiency	Price	10 Operational Performance Parameters
Bulla et al. [6]	Turbofan Jet Engines (Commercial Aircraft)	29	CRS (Input-oriented)	Fuel Consumption, Engine Weight, Drag	Airflow, Cruise Thrust
Sun [52]	Computer Numerical Machines (CNC)	21	VRS (Input-oriented)	Acquisition Cost	Spindle Speed Range, Number of Tool Capacity, X-Axis Traverse Rate, Z-Axis Traverse Rate, Maximum Machine Turning Diameter, Maximum Machine Turning Length
Cook and Zhu [11]	Power-Generating Units	40	CRS (Input-oriented) along with Goal Programming	Labor & Materials Total Expenditures, Total Occupied Hours	Equivalent Full-Capacity Operating Hours, Number of Forced & Sudden Outages, Number of Forced Deratings due to Equipment Failure
Meenakumari et al. [41]	Power-Generating Units	29	CRS & VRS (Input-oriented)	Installed Capacity, Distribution Line Length, Power Losses	Energy Supplied, Number of Consumers (as profit)

Table 1 (contd): Previous Research Summary of Engineering Systems' Benchmarking Using DEA

Author(s), Year	**Engineering System Type**	**Number of DMUs**	**DEA Model Type**	**DEA Model Inputs**	**DEA Model Outputs**
Karsak [34]	Flexible Manufacturing Systems	15	Single-Stage capable of incorporating crisp, ordinal, and fuzzy data	Capital & Operating Cost, Required Floor Space, Work-In-Process (fuzzy)	Product Flexibility (ordinal), Quality Improvement (ordinal), Lead Time Reduction (fuzzy)
Lee and Lee [36]	Government Office Buildings	47	CRS & VRS (Input-oriented)	Floor Area, Occupants Number, Climate-Adjusted Energy Consumption	Total Energy Consumption
Zhao et al. [60]	Downtown Space Reservation System	28 (as scenarios)	Radial Network DEA & Slacks-Based Network DEA (multistage)	Operation Cost, Fuel Cost, Travel Time	Vehicle Miles Travelled (also used as interstage input), Average Speed (also used as interstage input), Revenue, Person Miles Traveled
Voltes-Dorta et al. [58]	Automobiles sold in Spanish market during year 2004-2010	281	VRS (Input-oriented) & Malmquist Index	Vehicle Mass, Fuel Consumption	Engine Power, Vehicle Volume, Range
Hampf and Krüger [28]	Automobiles sold in German market for year 2010	3961	CRS & VRS (Ouput-oriented) along with Frontier Separation (VRS) and Efficiency Scores' Bootstrappin		
Lozano et al. [40]	Reconfigurable Machine Tools	14 (each producing 7 part types)	MMF (Multiple Modes of Functioning) DEA	Modules/tools Usage, Labor, Energy Consumption	Number of Units Produced of Each Type

3.2 Turbofan Aero-Engine Performance Optimization Implementing Engineering Approach

Although the research of Bulla et al. [6] is the only effort to date evaluating turbofan aero-engine efficiency both implementing DEA and engineering approaches, research dealing with turbofan aero-engine comparative efficiency evaluation exclusively implementing engineering approach has not really developed. Instead, researchers of the gas turbine engineering scientific field primarily tend to optimize various turbofan aero-engine components or integrated configurations taking into account thermodynamic aspects and features of the aircraft that shall utilize the prospective aero-engine. Consequently, they lack a coherent outlook regarding the performance of their design, compared to already in-service turbofan aero-engine designs.

For example, Guha [26] develops a systematic methodology to optimize commercial aircraft turbofan aero-engines from a thermodynamic perspective on an individual major component basis and subsequently on an integrated turbofan aero-engine basis. Thus, Guha [26] determines optimum fan pressure ratio, optimum exhaust stream speed, and optimum BPR for given specific thrust. Additionally, optimum specific thrust with respect to certain BPR is determined, along with optimum specific thrust optimization with respect to direct engineering cost. On the other hand, Guha [27] attempts to exclusively optimize a single major component, thus defining the optimum fan pressure ratio for turbofan aero-engines incorporating separate or mixed exhaust streams.

Following a similar pattern to Guha [27], the research of Von der Bank et al. [59] determines the optimum design features for compressors incorporated in ultra-high-pressure-ratio turbofan aero-engines, from structural, aerodynamic, and thermodynamic perspectives. Though, the research of El-Sayed at al. [18] determines the optimum thermodynamic features on an individual major component and integrated engine basis, for the purpose of maximizing efficiency and absolute performance. Similarly, the research of Dik et al. [14] performs thermodynamic analysis for the purpose of determining optimum pressure ratio, jet velocity ratio, and temperatures in order to subsequently achieve minimum specific fuel consumption (SFC).

Nevertheless, we should also refer to the research effort of Zhu et al. [61] who deploy a mathematical approach in order to evaluate aero-engine efficiency for particular aircraft types. More specifically, Zhu et al. [61] utilize fuzzy mathematical theory in order to designate the most suitable aero-engine for diverse aircraft types (commercial, military) by taking into account certain features like thrust, BPR, thrust-to-weight ratio, fuel consumption rate, and total engine life.

4 Turbofan Aero-Engine Efficiency Evaluation

4.1 Network DEA Model Formulation

Before proceeding with the DEA model that shall be implemented for the purpose of benchmarking in-service commercial and military aircraft turbofan aero-engines, a brief description of the turbofan aero-engine operation shall be given. More specifically, a brief architectural and functional description of a two-spool commercial turbofan aero-engine shall be given, in order to justify the structure of the developed network DEA model, along with the implemented inputs, intermediate inputs/outputs, and final outputs of the model.

In essence, air is initially fed to the turbofan engine through the fan, which compresses the air. Subsequently, the major portion of the suctioned air, i.e. the bypass (cold) airflow is expanded to produce thrust (otherwise referred to as “cold stream thrust”), while a significantly smaller portion (depending on the BPR) is routed into the compressor section (also referred to as “core airflow”) to be further compressed. After exiting the compressor section, core airflow is driven into the combustion section, where compressed air is mixed with fuel and combustion takes place by ignition. The combustion gases are then routed into the turbine section, thus providing kinetic energy in order to rotate the multistage high- and low-pressure turbine (HPT and LPT). Finally, the combustion gases are expanded to the atmosphere through an exhaust nozzle, producing additional thrust. Due to mechanical connection of the HPT with the compressor section and of the LPT with the fan unit section, more air is suctioned by the fan and the aforementioned sequence is rendered continuous [17]. Considering the previous turbofan engine operation description, it is evident that both cold stream and hot stream thrust are essentially produced by the HPT and LPT, which are part of the so-called “core engine” [50].

Another aspect that is seriously taken into consideration by companies designing and manufacturing turbofan aero-engines and propulsion systems in general, is the engine weight. As a matter of fact, weight-saving practices are of paramount importance in the aviation industry sector. Given the maximum take-off weight (MTOW) of an aircraft, its payload-carrying capability can be significantly enhanced if the empty weight (or structure weight) is reduced [22]. Consequently, reducing the propulsion system weight is a desirable practice by designers and manufacturers, thus dictating the integrated approach of concurrently optimizing performance, mechanical robustness and reduced weight [39]. In this framework, thrust-to-weight ratio has been established as a key design parameter among propulsion systems designers and manufacturers [18].

Taking into account the aforementioned operational, design and manufacturing aspects, we construct the functional model depicted in Figure 4, in order to subsequently implement it for the purpose of benchmarking commercial and military turbofan engines.

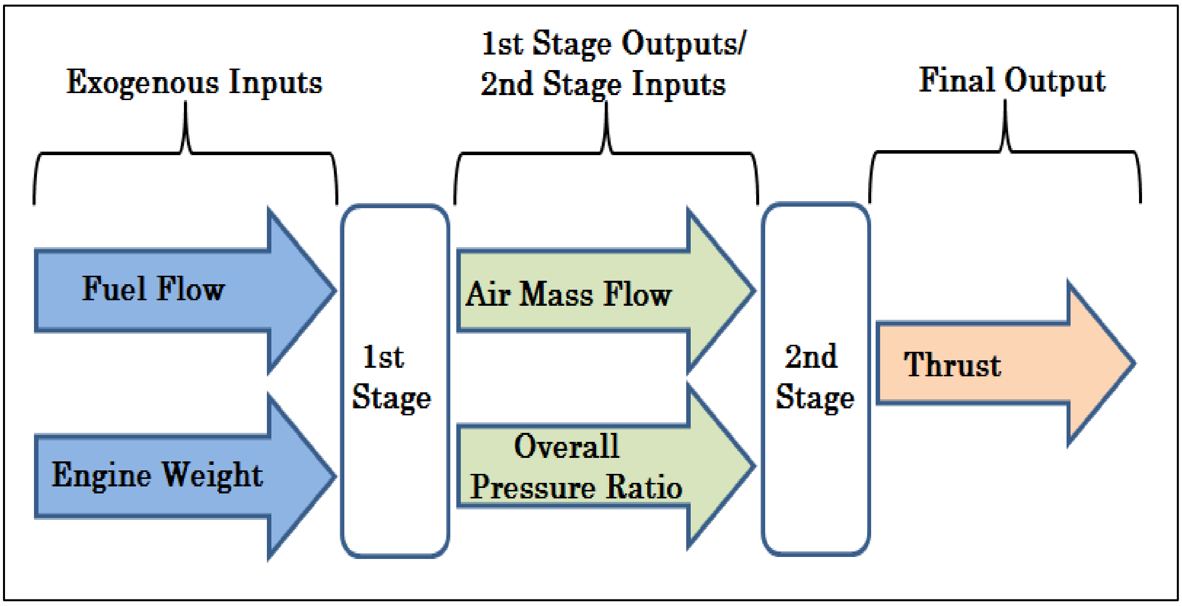

Figure 4: Network DEA Functional Model for Turbofan Aero-Engine (Source: Own elaboration)

We next proceed with the justification of the implemented exogenous inputs, intermediate measures (1st stage outputs, 2nd stage inputs), and final output of the proposed network DEA model, which is reported as follows:

<u>Exogenous Inputs</u>

- <u>Fuel Flow</u>: Fuel is stored in the aircraft tanks, thus pressurized and subsequently fed to the engines through pumps. Depending on the thrust setting, fuel flow is properly regulated in order to sustain the desired speed, climb rate etc. It is considered an exogenous input, assuming that it is not an element pre-existing in the engine. In the current paper, fuel flow is recorded in the same way as in aviation, i.e. as pounds per our (lb/hr).

- <u>Engine Weight</u>: As previously stated, weight is an important aspect for propulsion system designers and manufacturers. In fact, aero-engine weight is essentially the sum of surrounding structure weight plus the weight of the rotating modules and the accessories. Irrespective of the thermodynamic efficiency and thrust of a turbofan engine, increased weight negatively affects the payload-carrying capability of the aircraft and subsequently the marketability of the engine [19]. Hence, weight is regarded as an exogenous input, assuming that excess weight is translated into excess resource consumption in the form of materials and energy, along with the fact that engine weight results from manufacturing processes, which do not take place within the aero-engine. The units utilized for this input are recorded as pounds (lbs).

Intermediate Measures

- Air Mass Flow: In order for a turbofan engine to produce thrust, sufficient air mass should be suctioned, regardless of the portion that shall be routed through the fan and the core. Hence, air mass flow comprises a representative measure of an aero-engine's potential to provide thrust. We consider air mass flow as an intermediate measure, taking into account that in the turbofan engine self-sustained continuous operation the power consumed to induct air into the engine comes from the turbine section (HPT and LPT). The units for this measure are pounds per second (lb/sec), which are the most frequently used by scientific community dealing with propulsion systems.

- Overall Pressure Ratio (OPR): It literally comprises the ratio of the pressure of the compressed air after exiting the HPC (before entering the combustion chamber), to the pressure of the ambient air (before entering the fan). In a similar manner to air mass flow, OPR also represents an aero-engine's potential to produce thrust, taking into account that higher pressure exhaust gases possess higher energy, consequently leading to higher energy extraction by the HPT and LPT. OPR was selected instead of fan pressure ratio and HPC pressure ratio, due to the fact that both fan and HPC are driven by the LPT and HPT respectively, thus rendering the adoption of OPR more proper for our study. Moreover, it should be stated that OPR has no units, as it comprises a ratio.

Final Outputs

- Thrust: It comprises the final desirable product of a turbofan aero-engine. Produced thrust propels the aircraft during ground and flight operations, i.e. taxi, take-off, climb, cruise, descend, and landing. Regarding the cruise phase, alternatively referred to as *design point*, engine designers assign increased gravity to efficient engine operation during this phase, taking into account that it consists the biggest portion of an engine's operational life [25]. Regarding the utilized units, thrust is usually expressed in pounds (lbs).

Furthermore, it is essential to refer to a similar previous research effort dealing with turbofan aero-engine efficiency evaluation using DEA, namely the one conducted by Bulla et al. [6]. In the particular research effort, fuel flow, engine weight, air mass flow, and thrust (i.e. cruise thrust) are also implemented using the very same units. However, the current paper deploys two-stage network DEA instead of single-stage CRS DEA, while all inputs, intermediate measures, and output refer to static conditions at sea level instead of cruise conditions. It should be stressed that the rationale behind opting for static sea level conditions is described as follows:

- Engine data sources mostly contain figures for static conditions at sea level, otherwise referred to as take-off at sea level (T-O @ SL). Especially for the case of air mass flow, OPR, and thrust, reported figures almost solely refer to T-O @ SL conditions.

- Figures referring to cruise conditions and especially thrust, depend on a vast degree to the type of aircraft that the engine is installed. In particular, the type of aircraft defines the resulting drag and consequently the required thrust at any flight speed. Hence, it is deemed more appropriate to evaluate DEA efficiency of turbofan engines in T-O @ SL conditions, where all figures are independent of the engine/aircraft combination. The above statement is further augmented by formula (3) of the current paper, which denotes that overall efficiency i.e. the combination of propulsive and thermal efficiency does not solely depend on the engine features, but it directly depends on speed (u) and thrust (T) balancing aircraft drag, thus confirming its high degree of correlation with the aircraft/engine combination [17].

After extensive study regarding Network DEA models that have been developed for evaluating efficiency of multistage systems and taking into account the special features of turbofan aero-engines, the current paper implements the *variable intermediate measures slacks-based measure* (VSBM) model developed by Chen et al. [8]. More specifically, the variable returns to scale (VRS) approach of the VSBM model is adopted for the purpose of evaluating commercial and military turbofan aero-engines.

The justification for implementing the VRS approach of the VSBM model developed by Chen et al. [8] is reported as follows:

- It comprises a non-radial model, thus excluding the assumption of proportional changes in inputs and outputs, dealing directly with slacks [54,55]. This property fits well in engineering systems' evaluation, having in mind that processes within these systems are not necessarily linear.

- It is non-oriented, i.e. the model is structured to both minimize the inputs and maximize the outputs [55]. Hence, it is rendered ideal for engineering systems' evaluation, where designers apply optimization techniques which simultaneously reduce utilized resources and maximize performance.

- The VRS approach aims to calculate pure technical efficiency as claimed by Ramanathan [45], which is by definition the efficiency of a production process in converting inputs into outputs, independent of the operations' scale and the associated prices and costs [2]. Consequently, the specific type of efficiency is the exact type of efficiency incorporated within engineering systems, where resource consumption and subsequent performance output are dealt with absolutely technical terms and conditions.

- The particular model enables the simultaneous assessment of stage efficiency and overall system efficiency, while it additionally makes feasible to compute the frontier projections for inputs, intermediate measures, and outputs. Hence, the researcher can locate where inefficiencies are occurring and quantify the excess inputs and the deficit of outputs of each evaluated DMU (i.e. engineering system). However, as cited by Tone [53] the excess in inputs and deficit in outputs that are expressed by slacks, may not always be feasible in actual situations, thus designating the need to further investigate to which degree each evaluated DMU is able to improve its relative efficiency.

From the above justification, it is evident that current research comprises a major leap compared to the early research effort of Bulla et al. [6] dealing with the very same topic, i.e. turbofan aero-engine efficiency evaluation deploying DEA. In specific, the adoption of the VRS VSBM DEA model of Chen et al. [8] along with the network DEA functional model depicted in Figure 4, is expected to enhance discriminative power and subsequently the objectivity of the undertaken efficiency evaluation. The additional features of VSBM model, namely the dual orientation (input & output), stage efficiency evaluation, and frontier projection, certainly comprise differentiating factors over the CCR input-oriented DEA model adopted by Bulla et al. [6], which assumes proportional changes of outputs with respect to inputs. Moreover, it should be stressed that the steady-state continuous operation of turbofan aero-engines does not encompass multiple time periods with carry-overs, thus rendering incompatible in our case the deployment of DEA models dealing with DMUs incorporating multiple divisions and multiple time periods as those adopted by Kao [33] and Kou et al. [35].

In accordance with Chen et al. [8], envelopment form of their VSBM model is deployed for the purpose of evaluating overall system efficiency and frontier projections, while the multiplier form VSBM model is deployed to obtain efficiency decomposition, i.e. to determine efficiency of each individual stage. The envelopment form of the deployed VRS VSBM DEA methodology of Chen et al. [8] has the following formulation:

$$\min\ e_o = \frac{1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}}$$

s.t.

$$\sum_{j=1}^{n}\lambda_j^1 x_{ij} = x_{io} - s_i^-, \ \ i = 1,2,\dots,m$$

$$\sum_{j=1}^{n}\lambda_j^1 z_{dj} = \widetilde{z_{do}}, \ \ d = 1,2,\dots,D,$$

$$\sum_{j=1}^{n}\lambda_j^2 z_{dj} = \widetilde{z_{do}}, \ \ d = 1,2,\dots,D,$$

$$\sum_{j=1}^{n}\lambda_j^2 y_{rj} = y_{ro} + s_r^+, \ \ r = 1,2,\dots,s,$$

$$\sum_{j=1}^{n}\lambda_j^1 = \sum_{j=1}^{n}\lambda_j^2 = 1,$$

$$s_i^-, s_r^+ \geq 0, \quad i = 1,2,\dots,m; \quad r = 1,2,\dots,s, \quad (5)$$

$$\lambda_j^1, \lambda_j^2 \geq 0, \quad j = 1,2,\dots,n$$

Where:

- (e_o) is the overall system efficiency
- (n) is the number of DMUs under evaluation
- (m) is the number of exogenous inputs of each DMU
- (D) is the number of intermediate measures of each DMU
- (s) is the number of final outputs of each DMU
- (x_{ij}) is the i-th input of the j-th DMU
- (x_{io}) is the i-th input of the DMU_o under evaluation
- (s_i^-) is the slack of the i-th input of the DMU_o under evaluation
- (z_{dj}) is the d-th intermediate measure of the j-th DMU
- ($\widetilde{z_{do}}$) is the d-th variable intermediate measure of the DMU_o under evaluation

- (y_{rj}) is the r-th final output of the j-th DMU
- (y_{ro}) is the r-th final output of the DMU_o under evaluation
- (s_r^+) is the slack of the r-th final output of the DMU_o under evaluation
- (λ_j^1) and (λ_j^2) are the intensity variables of the first and second stage respectively

4.2 Turbofan Aero-Engine Sample & Efficiency Evaluation Results

The turbofan engines comprising the sample under evaluation are all in-service (except for GE9X) either installed on commercial (passenger) aircraft or military aircraft. More specifically, the sample consists of 34 commercial turbofan engines and 21 military turbofan engines. As stressed in Section 2, commercial and military turbofan aero-engines are architecturally the same, except for the addition of the augmentor (alternatively referred to as "afterburner") in military engines. However, in our paper the function of the augmentor is fundamentally excluded, thus solely adopting unaugmented ("dry") thrust for all military turbofan aero-engines existent in our sample. In addition, military turbofan aero-engines usually adopt low BPR (bypass ratio), due to the fact that are mainly installed inside the aircraft fuselage. On the other hand, commercial turbofan aero-engines tend to adopt high BPR (usually greater than 5), thus exclusively installed underwing or adjacent to the vertical tail.

As previously stated, the data collected for the whole sample refer to take-off static conditions, for the purpose of ensuring a common benchmarking base and concurrently ensuring data availability. Primary source of data is Jane's Aero Engines 2018-2019 [13], while alternative sources of a quite high portion of the acquired data are the following:

- ICAO Aircraft Engine Emissions Databank, Version 25a [30]
- AIR International Magazine [1] (for GE9X engine)
- El-Sayed et al. [18]
- Farokhi [21]
- Roux [47]
- Jet Engine Specification Database (http://www.jet-engine.net/) [32]

It should be clarified that with respect to fuel flow data, these are either obtained by ICAO [30] by the figure relevant to fuel flow at take-off conditions or by multiplying the thrust specific fuel consumption (TSFC) by thrust. Also, for the purpose of benchmarking an engine that shall enter service in the near future, the General Electric GE9X was added. The specific engine is going to enter service soon with the Boeing 777X commercial aircraft. Some data figures are obtained from Jane's Aero Engines [13], while others are obtained from AIR International [1]. Especially for the fuel flow figure, its value is obtained based on the claim cited by AIR International [1] and Jane's Aero Engines 2018-2019 [13] that it achieves 10 percent lower specific fuel consumption than the General Electric GE90-115B used in Boeing 777-300ER aircraft. The data for exogenous inputs, intermediate measures, and output of the DEA model represented by Figure 4, are presented in Table 2. In addition, data pertinent to the year each engine entered service, type of aircraft each engine is installed on, and bypass ratio are also collected, thus being presented in Table 3. Also, Table 3 contains data pertinent to TSFC and thrust-to-weight ratio, for the purpose of acquiring a more detailed overview of each engine.

Table 2 : Sample Turbofan Aero-engines

Manufacturer	Model	1st Stage Inputs		1st Stage Outputs/2nd Stage Inputs		Final Output
		Weight (lbs)	Fuel Flow (lbs/hour)	Overall Pressure Ratio	Air Mass Flow (lbs/hour)	Thrust (lbs)
Commercial						
Avco Lycoming	ALF502R-6	1,376	2,814	13.8	192	6,700
CFE Company	CFE738-1-1B	1,325	2,201	23.0	210	5,918
CFM International	CFM56-3C	4,301	9,150	30.6	710	23,500
CFM International	CFM56-5B3/P	5,250	11,340	34.4	968	33,300
CFM International	CFM56-7B26	5,300	9,681	32.7	783	26,300
Engine Alliance	GP7270	13,416	20,910	38.6	3,000	70,000
General Electric	CF34-80C1	2,403	4,791	28.0	441	12,670
General Electric	CF6-80C2B6F	9,790	20,141	31.4	1,759	60,030
General Electric	CF6-80E1A3	11,225	23,701	34.8	1,926	69,800
General Electric	GE90-110B1	19,316	32,456	42.0	3,618	110,100
General Electric	GE90-115B1	19,316	37,189	42.0	3,618	115,540
General Electric	GE90-85B	17,250	25,232	36.9	3,120	84,700
General Electric	GE90-94B	17,250	27,864	39.6	3,234	93,700
General Electric	GE9X	20,000	29,560	60.0	3,850	102,000
General Electric	GEnx-1B70	19,856	13,540	44.3	2,545	69,800
Honeywell	TFE731-60	988	2,025	22.0	187	5,000
IAE	V2533-A5	5,200	11,307	33.4	858	33,000
Ivchenko Progress	D-18T	9,001	17,823	25.0	1,687	51,660
Pratt & Whitney	PW2040	7,295	13,964	29.9	1,340	41,700
Pratt & Whitney	PW4056	9,420	19,419	28.4	1,705	56,750
Pratt & Whitney	PW4084	14,920	27,048	34.2	2,400	84,600
Pratt & Whitney	PW4090	15,740	31,131	38.6	2,720	91,790
Pratt & Whitney	PW4098	16,260	35,456	42.8	2,850	99,040
Pratt & Whitney	PW4462	9,420	22,663	32.3	1,800	62,000
Pratt & Whitney	PW6122A	5,041	8,464	26.6	660	22,100
Rolls-Royce	AE3007A3	1,586	2,841	21.0	240	7,040
Rolls-Royce	BR715-C1-30	4,597	7,803	32.0	625	21,000
Rolls-Royce	RB211-535C	7,294	14,274	21.1	1,142	37,400
Rolls-Royce	Tay 620-15	3,185	6,930	15.8	410	13,850
Rolls-Royce	Trent 1000C	11,924	19,887	47.7	2,659	73,000
Rolls-Royce	Trent 556	10,660	17,762	36.3	1,939	56,000
Rolls-Royce	Trent 700	10,467	25,374	35.5	2,030	71,100
Rolls-Royce	Trent 895	13,100	31,956	41.6	2,664	95,000
Rolls-Royce	Trent 970	14,190	20,617	38.5	2,655	70,000
Military						
Aviadvigatel	D-30F6	5,326	15,080	21.2	331	20,944
Eurojet	EJ200	2,180	9,983	26.1	170	13,490
General Electric	F101-GE-102	4,460	9,554	26.8	352	17,000
General Electric	F110-GE-100	3,920	12,972	30.4	269.8	17,530
General Electric	F110-GE-129	3,980	10,880	30.7	270	17,000
General Electric	F110-GE-132	4,150	12,224	33.3	275.6	19,100
General Electric	F118-GE-100	3,200	12,730	35.1	287	19,000
General Electric	F404-GE-400	2,195	9,010	26.0	146	10,600
General Electric	F414-GE-400	2,470	10,683	30.0	172	14,756
Klimov	RD-33	2,683	8,569	21.7	169.8	11,128

Table 2 (contd): Sample Turbofan Aero-engines

Manufacturer	Model	1st Stage Inputs		1st Stage Outputs/2nd Stage Inputs		Final Output
		Weight (lbs)	Fuel Flow (lbs/hour)	Overall Pressure Ratio	Air Mass Flow (lbs/hour)	Thrust (lbs)
Military (contd)						
Lyul'ka Saturn Inc	AL-31F	3,373	11,525	23.5	247	17,305
Lyul'ka Saturn Inc	AL-37FU	3,660	12,687	25.0	264	18,740
Pratt & Whitney	F100-PW-220E	3,245	10,651	25.0	228	14,590
Pratt & Whitney	F100-PW-229	3,795	12,923	32.4	248	17,800
Pratt & Whitney	F119-PW-100	3,900	16,470	35.0	305	27,000
Pratt & Whitney	TF33-P-103	3,900	8,585	13.0	460	17,000
Rolls-Royce	F402-RR-408	4,260	18,088	16.3	461	23,800
SNECMA	M88-2	1,978	8,760	24.5	143.3	10,950
SNECMA	M88-3	2,013	10,665	26.6	158.1	13,500
Turbo-Union	RB199 Mk105	2,185	6,273	24.5	166	9,650
Volvo Aero Corp.	RM12	2,326	10,198	27.5	152	12,140

After obtaining the data contained in Table 2, we incorporate them in the VRS VSBM network DEA model, in order to evaluate the efficiency of each engine of the sample. The "envelopment form" model is used to determine overall efficiency and frontier projections, while the "multiplicative form" model is used to determine the first and second stage individual efficiencies. The gained results regarding overall efficiency, stage efficiency, and frontier projections are contained in Table 4.

Discriminative power of the implemented VRS VSBM network DEA can be considered as very satisfactory, regarding both overall and stage efficiency. In particular, as far as overall efficiency is concerned, no DMU out of 55 is overall efficient (i.e. overall efficiency equal to unity). Also, the discriminative power for both the first and second stage is also satisfactory, as four DMUs are efficient regarding the first stage and five DMUs are efficient regarding the second stage (7.27 and 9.09 percent of total DMUs respectively). The attained efficient DMU portion is significantly low and consequently the achieved discriminative power is at any case acceptable, as stipulated by Avkiran [2] who considers that acceptable discriminative power is achieved when the portion of efficient DMUs is less than one third of the total number of DMUs.

The most efficient engine of the sample is General Electric GE90-115B1, which is solely used by the Boeing 777-300ER. The second most efficient engine is the General Electric GE90-110B1, comprising along with GE90-115B1 the engine options for Boeing 777-200LR and the sole engine option for 777F (freighter version of Boeing 777). It should be mentioned that GE90-110B1 is considered a decreased thrust variant of the GE90-115B1, with both engines comprising an evolutionary improvement of the General Electric GE90-94B, which also belongs to the current engine sample. In particular, major improvements over GE90-94B comprise the larger fan with toughened composite fan blades, along with design and material enhancements in HPT and LPT blade aerofoils [13].

Table 3: Aero-engine Additional Data

Model	Entry into Service (Year)	Aircraft Type(s)	Thrust-Specific Fuel Consumption ($lb_f/lb{\cdot}hr$)	Bypass Ratio	Thrust-to-Weight Ratio
Commercial					
ALF502R-6	1984	BAe 146-200/300	0.42	5.6	4.87
CFE738-1-1B	1993	Dassault Falcon 2000	0.372	5.3	4.47
CFM56-3C	1986	Boeing 737-400	0.389	5.0	5.46
CFM56-5B3/P	1994	Airbus A321-200	0.34	5.4	6.34
CFM56-7B26	2001	Boeing 737-900ER	0.368	5.1	4.96
GP7270	2005	Airbus A380-800	0.299	8.7	5.70
CF34-80C1	1995	Bombardier CRJ700	0.378	4.8	5.27
CF6-80C2B6F	1985	Boeing 767-300ER	0.336	5.1	6.13
CF6-80E1A3	2001	Airbus A330-200	0.34	5.3	6.22
GE90-110B1	2004	Boeing 777-200LR	0.311	8.9	5.70
GE90-115B1	2004	Boeing 777-300ER	0.322	8.9	5.98
GE90-85B	1995	Boeing 777-200	0.298	8.3	4.91
GE90-94B	2000	Boeing 777-300	0.2974	8.4	5.43
GE9X	Beyond 2020	Boeing 777-8X/-9X	0.2898*	10.3	5.10
GEnx-1B70	2014	Boeing 787-9	0.2845	9.1	5.16
TFE731-60	1995	Dassault Falcon 900	0.405	3.9	5.06
V2533-A5	1995	Airbus A321-200	0.342	4.5	6.35
D-18T	1993	Antonov An-124-100	0.345	5.6	5.74
PW2040	1987	Boeing 757-200/757F	0.335	5.5	5.72
PW4056	1997	Boeing 747-400	0.342	4.9	6.02
PW4084	1995	Boeing 777-200	0.32	6.4	5.67
PW4090	1997	Boeing 777-300	0.339	6.3	5.83
PW4098	1999	Boeing 777-300	0.358	5.8	6.09
PW4462	1993	MD-11	0.366	4.8	6.58
PW6122A	2007	Airbus A318-100	0.383	5.0	4.38
AE3007A3	1999	Embraer ERJ-135	0.403	4.8	4.44
BR715-C1-30	1999	Boeing 717-200	0.372	4.55	4.57
RB211-535C	1983	Boeing 757-200	0.382	4.4	5.13
Tay 620-15	1987	Fokker 70	0.5	3.04	4.35
Trent 1000C	2014	Boeing 787-9	0.273	11.0	6.12
Trent 556	2002	Airbus A340-500/600	0.317	7.6	5.25
Trent 700	1995	Airbus A330-200	0.357	5.0	6.79
Trent 895	2000	Boeing 777-200ER	0.336	5.8	7.25
Trent 970	2007	Airbus A380-800	0.295	8.7	4.93
Military					
D-30F6	1980	MiG-31	0.52	0.52	3.93
EJ200	2003	EF2000	0.74	0.4	6.19
F101-GE-102	1985	B-1B	0.562	2.1	3.81
F110-GE-100	1986	F-16C/D	0.74	0.76	4.47
F110-GE-129	1992	F-16C/D & F-15	0.64	0.76	4.27
F110-GE-132	2003	F-16E/F	0.64	0.68	4.60
F118-GE-100	1993	B-2A	0.67	0.76	5.94
F404-GE-400	1981	F/A-18A/B/C/D	0.85	0.27	4.83
F414-GE-400	2001	F/A-18E/F	0.724	0.27	5.97
RD-33	1981	MiG-29	0.77	0.55	4.15

Table 3 (contd): Aero-engine Additional Data

Model	Entry into Service (Year)	Aircraft Type(s)	Thrust-Specific Fuel Consumption ($lb_f/lb \cdot hr$)	Bypass Ratio	Thrust-to-Weight Ratio
Military (contd)					
AL-31F	1985	Su-27	0.666	0.57	5.13
AL-37FU	1995	Su-30MK	0.677	0.65	5.12
F100-PW-220E	1985	F-16 & F-15	0.73	0.6	4.50
F100-PW-229	1991	F-16 & F-15	0.726	0.36	4.69
F119-PW-100	2005	F-22A	0.61	0.45	6.92
TF33-P-103	1962	B-52H	0.505	1.36	4.36
F402-RR-408	1986	AV-8B	0.76	1.2	5.59
M88-2	1999	Rafale	0.8	0.3	5.54
M88-3	2005	Rafale	0.79	0.3	6.71
RB199 Mk105	1999	Tornado ECR	0.65	1.1	4.42
RM12	1986	JAS39	0.84	0.31	5.22

*Assuming 10 percent improvement over GE90-115B1 [1], [13]

In addition, it is important to stress that if the GE90-110B1 engine is excluded from the sample, GE90-115B1 is rendered efficient. By observing the stage efficiencies in Table 4, it is observed that GE90-110B1 has 1st stage efficiency equal to one while GE90-115B1 is not efficient (efficiency score equal to 0.936370). With respect to 2nd stage efficiency, the situation is reversed with the GE90-115B1 obtaining unity efficiency score and GE90-110B1 achieving efficiency score equal to 0.84959. Hence, it is evident that the GE90-110B1 and GE90-115B1 influence each other in both stages, which can be explained by the respective lambda values obtained by VSBM ″envelopment form″ model.

As far as frontier projections are concerned, it is evident from Table 4 that they significantly deviate compared to actual values of inputs, intermediate measures, and final output. However, deviations are relatively smaller for the case of overall pressure ratio and thrust. Especially regarding thrust, there are 29 instances (mainly concerning commercial aircraft turbofan aero-engines) where actual engine thrust coincides with the frontier projection calculated from the VSBM Network DEA model.

Also, it should be noted that nearly all military aircraft turbofan engines (except for F119-PW-100 and F402-RR-408) have identical frontier projections for all inputs, intermediate measures, and final output. More specifically:

- Fuel flow frontier projections lie in the range between 3,014.7 and 3,034.4 lbs/hour
- Engine weight frontier projections lie in the vicinity between 1,593.8 and 1,605.9 lbs
- Overall pressure ratio frontier projections take certain distinctive values of either 23.4 or 23.5.
- Air mass flow frontier projections lie in the vicinity between 323.9 and 326.7 lbs/sec.
- Thrust frontier projections lie within the value range between 21,757 lbs and 21,951 lbs.

Table 4: Overall Efficiency, Stage Efficiencies, and Frontier Projections

Model	Overall Efficiency	Stage Efficiencies		Frontier Projections*				
		1st Stage	2nd Stage	Fuel Flow	Weight	Overall Pressure Ratio	Air Mass Flow	Thrust
Commercial								
ALF502R-6	0.35014	0.45837	0.89177	2,657.4	1,376.0	22.9	274.7	18,601
CFE738-1-1B	0.38181	0.94221	0.43960	2,201.5	1,101.9	22.2	212.8	14,194
CFM56-3C	0.38278	0.73280	0.64998	3,275.1	1,753.4	23.8	360.0	23,500
CFM56-5B3/P	0.52680	0.80122	0.72558	5,380.1	3,042.1	26.8	651.3	33,300
CFM56-7B26	0.39114	0.71387	0.67726	3,792.5	2,070.1	24.5	431.6	26,300
GP7270	0.63064	0.98121	0.64944	13,696.4	8,133.9	38.8	1,802.3	70,000
CF34-80C1	0.37565	0.89518	0.48047	3,034.3	1,605.9	23.5	326.7	21,951
CF6-80C2B6F	0.62867	0.70239	0.92628	11,438.0	6,751.8	35.5	1,489.8	60,030
CF6-80E1A3	0.64905	0.67122	0.97783	13,651.1	8,106.1	38.7	1,796.0	69,800
GE90-110B1	0.84959	1	0.84959	27,009.4	16,747.1	42.3	3,364.8	110,100
GE90-115B1	0.93637	0.93637	1	32,456.3	19,316.0	42.0	3,618	115,540
GE90-85B	0.63619	0.82903	0.80716	16,953.7	10,363.4	40.6	2,307.0	84,700
GE90-94B	0.68004	0.82145	0.85859	18,935.8	11,739.5	41.5	2,619.0	93,700
GE9X	0.69864	1	0.69864	21,280.6	13,559.7	41.7	2,989.6	102,000
GEnx-1B70	0.64309	0.89209	0.75100	13,651.9	8,106.9	38.7	1,796.1	69,800
TFE731-60	0.40219	1	0.40219	2,025	988.0	22.0	187.0	12,424
V2533-A5	0.52327	0.72595	0.79732	5,312.7	3,000.8	26.7	641.9	33,000
D-18T	0.57809	0.71313	0.86496	9,544.1	5,591.5	32.8	1,227.5	51,660
PW2040	0.54910	0.75917	0.78992	7,283.0	4,207.1	29.6	914.6	41,700
PW4056	0.60947	0.69364	0.91583	10,698.9	6,298.9	34.5	1,387.3	56,750
PW4084	0.65962	0.67099	0.98863	16,930.5	10,347.8	40.6	2,303.4	84,600
PW4090	0.66097	0.69562	0.96535	18,514.0	11,446.6	41.3	2,552.7	91,790
PW4098	0.67973	0.68122	0.99852	20,401.1	12,783.7	41.7	2,854.1	99.040
PW4462	0.63498	0.70084	0.93414	11,883.2	7,023.7	36.2	1,551.3	62,000
PW6122A	0.34131	0.62403	0.71728	3,057.6	1,620.4	23.5	329.9	22,100
AE3007A3	0.33705	0.74335	0.59371	2,841.0	1,524.9	22.7	308.5	20,479
BR715-C1-30	0.35312	0.69952	0.65360	3,033.5	1,605.5	23.5	326.6	21,941
RB211-535C	0.46843	0.59399	0.87445	6,309.4	3,611.3	28.2	779.9	37,400
Tay 620-15	0.29720	0.47487	0.82233	3,026.5	1,604.0	23.4	326.2	21,888
Trent 1000C	0.72058	1	0.72058	14,368.3	8,569.4	39.5	1,900.8	73,000
Trent 556	0.58662	0.80296	0.78366	10,523.9	6,191.5	34.2	1,363.2	56,000
Trent 700	0.67064	0.71373	0.95690	13,948.9	8,288.8	39.1	1,837.2	71,100
Trent 895	0.75641	0.75641	1	19,222.3	11,938.2	41.6	2,664.0	95,000
Trent 970	0.61876	0.86287	0.75589	13,697.9	8,134.7	38.8	1,802.5	70,000
Military								
D-30F6	0.23984	0.23984	1	3,034.1	1,605.8	23.5	326.6	21,948
EJ200	0.31976	0.35895	0.96081	3,014.7	1,593.8	23.4	323.9	21,757
F101-GE-102	0.26241	0.38635	0.87606	3,034.3	1,606.0	23.5	326.7	21,951
F110-GE-100	0.25699	0.33296	0.92403	3,033.5	1,605.4	23.5	326.6	21,942
F110-GE-129	0.26424	0.35593	0.90831	3,032.9	1,605.1	23.5	326.5	21,937
F110-GE-132	0.27635	0.35552	0.92084	3,034.4	1,605.9	23.5	326.7	21,950
F118-GE-100	0.32035	0.44028	0.88007	3,034.3	1,605.9	23.5	326.7	21,950
F404-GE-400	0.25797	0.33736	0.92061	3,034.1	1,606.1	23.4	326.7	21,947
F414-GE-400	0.31400	0.36923	0.94477	3,034.2	1,605.9	23.5	326.7	21,950
RD-33	0.24148	0.27945	0.96203	3,034.3	1,605.9	23.5	326.7	21,951

Table 4 (contd): Overall Efficiency, Stage Efficiencies, and Frontier Projections

Model	Overall Efficiency	Stage Efficiencies		Frontier Projections*				
		1st Stage	2nd Stage	Fuel Flow	Weight	Overall Pressure Ratio	Air Mass Flow	Thrust
Military(contd)								
AL-31F	0.29145	0.30036	0.99109	3,034.3	1,605.9	23.5	326.7	21,951
AL-37FU	0.28939	0.30141	0.98798	3,034.3	1,605.9	23.5	326.7	21,950
F100-PW-220E	0.25915	0.31360	0.94555	3,033.5	1,605.9	23.5	326.6	21,939
F100-PW-229	0.26678	0.33941	0.92737	3,034.3	1,605.9	23.5	326.7	21,950
F119-PW-100	0.39777	0.39777	1	3,951.7	2,167.5	24.8	453.6	27,000
TF33-P-103	0.29632	0.40430	0.89202	3,030.7	1,603.7	23.4	326.0	21,894
F402-RR-408	0.30095	0.30095	1	3,321.6	1,781.8	23.9	366.4	23,800
M88-2	0.28890	0.34019	0.94871	3,034.2	1,605.9	23.5	326.7	21,939
M88-3	0.33281	0.36358	0.96923	3,033.4	1,605.3	23.5	326.5	21,939
RB199 Mk105	0.26789	0.39447	0.87342	3,034.3	1,605.9	23.5	326.7	21,951
RM12	0.27320	0.33668	0.93653	3,034.3	1,605.9	23.5	326.7	21,951

*Underlined numerical values designate frontier projections equal to actual values

As depicted in Table 5, the frontier projections designate that in order for a military turbofan aero-engine to become efficient, the optimal thrust specific fuel consumption (TSFC) should be approximately 0.1383 lb_f/lb·hr, while the optimal thrust-to-weight ratio (T/W ratio) should take an approximate value of 13.65. Consulting Table 3 of the current paper, it can be concluded that the aforementioned optimal TSFC and T/W ratio values vastly differ from the actual values, taking into account that typical actual TSFC values for military turbofan aero-engines are within the 0.65 - 0.75 lb_f/lb·hr range, while typical actual thrust-to-weight ratios are within the 4.5 - 6.0 range.

By observing the column of Table 5 named “Air Mass Flow Change Percentage”, it can be deduced that frontier projections regarding air mass flow for military aircraft turbofan aero-engines designate the requirement for substantially increasing the air mass flow for the majority of these engines (except for D-30F6, F101-GE-102, TF33-P-103, and F402-RR-408). The aforementioned frontier projection pattern for air mass flow coincides with the engineering approach which is associated with propulsive efficiency. In particular, military turbofan aero-engines usually propel a relatively small air mass to a high velocity (compared to commercial turbofan aero-engines), thus resulting in a fairly low propulsive efficiency within the subsonic speed region. In general, propulsion systems’ theory suggests that higher propulsive efficiency is typically obtained when a large mass flow is given a slightly small speed increase. Hence, it is evident that the VSBM Network DEA efficiency score pattern follows the propulsive efficiency pattern for the subsonic speed region (see Figure 3 of the current paper), which is confirmed by the relatively low efficiency scores (mean value equal to 28.66 %) of military turbofan aero engines, when compared with the efficiency scores of the commercial turbofan aero-engines (mean value equal to 56.49 %).

Table 5: DEA Optimal Values & Value Change Percentages

Model	**DEA Optimal TSFC**	**DEA Optimal Thrust-to-Weight**	**Weight Change Percentage**	**Air Mass Flow Change Percentage**	**Thrust Change Percentage**
<u>Commercial</u>					
ALF502R-6	0.1429	13.52	**0%**	+43.07%	+177.62%
CFE738-1-1B	0.1551	12.88	-16.84%	+1.32%	+139.84%
CFM56-3C	0.1394	13.40	-59.23%	-49.29%	**0%**
CFM56-5B3/P	0.1616	10.95	-42.05%	-32.72%	**0%**
CFM56-7B26	0.1442	12.70	-60.94%	-44.88%	**0%**
GP7270	0.1957	8.61	-39.37%	-39.92%	**0%**
CF34-80C1	0.1382	13.67	-33.17%	-25.91%	+73.25%
CF6-80C2B6F	0.1905	8.89	-31.03%	-15.30%	**0%**
CF6-80E1A3	0.1956	8.61	-27.79%	-6.75%	**0%**
GE90-110B1	0.2453	6.57	-13.30%	-7.00%	**0%**
GE90-115B1	0.2809	<u>5.98</u>	**0.00%**	0.00%	**0%**
GE90-85B	0.2002	8.17	-39.92%	-26.06%	**0%**
GE90-94B	0.2021	7.98	-31.950%	-19.02%	**0%**
GE9X	0.2086	7.52	-32.20%	-22.35%	**0%**
GEnx-1B70	0.1956	8.61	-40.13%	-29.43%	**0%**
TFE731-60	0.1630	12.58	**0%**	**0%**	148.48%
V2533-A5	0.1610	11.00	-42.29%	-25.19%	**0%**
D-18T	0.1847	9.24	-37.88%	-27.24%	**0%**
PW2040	0.1747	9.91	-42.33%	-31.74%	**0%**
PW4056	0.1885	9.01	-33.13%	-18.63%	**0%**
PW4084	0.2001	8.18	-30.64%	-4.02%	**0%**
PW4090	0.2017	8.02	-27.28%	-6.15%	**0%**
PW4098	0.2060	7.75	-21.38%	+0.14%	**0%**
PW4462	0.1917	8.83	-25.44%	-13.82%	**0%**
PW6122A	0.1384	13.64	-67.86%	-50.01%	**0%**
AE3007A3	0.1387	13.43	**-3.85%**	+28.54%	+190.89%
BR715-C1-30	0.1383	13.67	-65.08%	-47.75%	+4.48%
RB211-535C	0.1687	10.36	-50.49%	-31.71%	**0%**
Tay 620-15	0.1383	13.65	-49.64%	-20.44%	+58.04%
Trent 1000C	0.1968	8.52	-28.13%	-28.51%	**0%**
Trent 556	0.1879	9.04	-41.92%	-29.70%	**0%**
Trent 700	0.1962	8.58	-20.81%	-9.50%	**0%**
Trent 895	0.2023	7.96	-8.87%	**0%**	**0%**
Trent 970	0.1957	8.61	-42.67%	-32.11%	**0%**
<u>Military</u>					
D-30F6	0.1382	13.67	-69.85%	-1.31%	+4.79%
EJ200	0.1386	13.65	-26.89%	+90.54%	+61.28%
F101-GE-102	0.1382	13.67	-63.99%	-7.19%	+29.12%
F110-GE-100	0.1382	13.67	-59.05%	+21.04%	+25.17%
F110-GE-129	0.1382	13.67	-59.67%	+20.92%	+29.04%
F110-GE-132	0.1383	13.67	-61.30%	+18.53%	+14.92%
F118-GE-100	0.1382	13.67	-49.81%	+13.82%	+15.53%
F404-GE-400	0.1382	13.66	-26.83%	+123.76%	+107.50%
F414-GE-400	0.1382	13.67	-34.98%	+89.92%	+48.75%
RD-33	0.1382	13.67	-40.14%	+92.39%	+97.26%

Table 5 (contd): DEA Optimal Values & Value Change Percentages

Model	DEA Optimal TSFC	DEA Optimal Thrust-to-Weight	Weight Change Percentage	Air Mass Flow Change Percentage	Thrust Change Percentage
Military(contd)					
AL-31F	0.1382	13.67	-52.39%	+32.26%	+26.85%
AL-37FU	0.1382	13.67	-56.12%	+23.74%	+17.13%
F100-PW-220E	0.1383	13.67	-50.53%	+43.23%	+50.37%
F100-PW-229	0.1382	13.67	-57.68%	+31.72%	+23.32%
F119-PW-100	0.1464	12.46	-44.42%	+48.73%	**0%**
TF33-P-103	0.1384	13.65	-58.88%	-29.13%	+28.79%
F402-RR-408	0.1396	13.36	-58.17%	-20.51%	**0%**
M88-2	0.1382	13.67	-18.81%	+127.97%	+100.46%
M88-3	0.1383	13.67	-20.25%	+106.53%	+62.51%
RB199 Mk105	0.1382	13.67	-26.50%	+96.79%	+127.47%
RM12	0.1382	13.67	-30.96%	+114.92%	+80.81%

5 Post-Hoc Analysis & Discussion

5.1 Non-Parametric Post-Hoc Analysis

The theoretical distribution of DEA efficiency is generally deemed as unknown, consequently designating the need to implement non-parametric statistical tests, fundamentally dealing with cases where observations are considered as statistically independent [5,12]. In order to perform the necessary statistical tests in the framework of the current paper, the IBM SPSS Version 22 software is used.

Initially, it is investigated whether statistically significant mean efficiency differences exist between commercial aircraft and military aircraft aero-engines. After consulting Table 3 of the current paper, it is deduced that there is a group of 34 commercial aircraft aero-engines and a group of 21 military aero-engines. The mean DEA efficiency of those groups is as follows:

- Mean efficiency of commercial aircraft aero-engines = 56.49% (n_1 = 34)
- Mean efficiency of military aircraft aero-engines = 28.66% (n_2 = 21)

For the purpose of assessing whether the mean efficiency difference between the two aforementioned groups is statistically significant, the Mann-Whitney rank-sum test is deployed. The Mann-Whitney non-parametric statistical test evaluates the hypothesis whether two groups belong to the same population or they differ in a significant grade. It is important to stress that the validity of Mann-Whitney rank-sum test is enhanced for the case where at least 10 observations are contained into the investigated groups [5,12]. In addition, the hypothesis shall be examined at 0.05 significance level.

The hypothesis investigated in order to evaluate whether the mean efficiency differentiation between commercial and military aircraft aero-engines is statistically significant, is stated as follows:

H_1: Commercial and military aircraft aero-engine have identical mean efficiency.

The Mann-Whitney rank-sum test provides a Z-value equal to -5.942 and a p-value far less than 0.01 (see Table 6 for further details). Hence, hypothesis H_1 is rejected at 0.05 significance level, consequently designating that the mean efficiency difference between commercial and military aero-engines is statistically significant.

Table 6: Mann-Whitney statistical test for hypothesis H_1

Group	N	Mean Rank	Sum of Ranks
1	34	38.09	1,295.00
2	21	11.67	245.00
Total	55		
Mann-Whitney U	14.000		
Wilcoxon W	245.00		
Z	-5.942		
p-value	**$2.8135 \cdot 10^{-9}$**		

Observing the numerical values regarding BPR in Table 3, it is evident that the great majority of military aircraft aero-engines incorporate a significantly low BPR (usually less than 1) compared to commercial aircraft aero-engines. Hence, the obtained result for hypothesis H_1 coincides with the superior propulsive efficiency of high-BPR over low-BPR turbofan aero-engines in the subsonic speed region, which is qualitatively expressed in Figure 3 of the current paper. Subsequently, it is deduced that the resulting efficiency scores of the VSBM Network DEA model follow the same pattern as the propulsive efficiency dictated by the propulsion systems' fundamental theory (as also noted in sub-section 4.2 of the current paper).

Another turbofan aero-engine feature that shall be investigated how it is associated with VSBM Network DEA efficiency is the Entry Into Service (EIS) year. The scope of investigating how EIS year is associated with DEA efficiency, is to evaluate whether technological progress achieved by aero-engine manufacturers has resulted into significant efficiency enhancement. Taking into account the data in Table 3, we divide the sample into three groups. In particular, the first group consists of turbofan aero-engines with EIS year in the 1980s decade and earlier, the second group consists of turbofan aero-engines with EIS year in the 1990s decade, while the third group consists of turbofan aero-engines with EIS year spanning from 2000 to near-future i.e. 2020.

The mean DEA efficiency scores for each of the formed group were calculated as follows:

- Mean efficiency of turbofan aero-engine group with EIS year in the 1980s decade and earlier = 33.48% (n_1 = 16)
- Mean efficiency of turbofan aero-engine group with EIS year in the 1990s decade = 46.32% (n_2 = 21)
- Mean efficiency of turbofan aero-engine group with EIS year in the 2000s decade and later = 56.35% (n_3 = 18)

In order to assess whether there are statistically significant differences regarding the mean DEA efficiency scores among the aforementioned turbofan aero-engine groups, we shall deploy the Games-Howell statistical test. The specific statistical test has been designated as the most appropriate for the purpose of conducting pairwise comparisons among multiple groups, given that the samples' size and associated variance is not equal. As stated by Shingala and Rajyaguru [51], Games-Howell statistical test possesses the best capability with respect to pairwise comparisons among multiple groups. In a similar concept with the previous Mann-Whitney rank-sum test, any pairwise comparison with significance level (p-value) less than or equal to 0.05 shall be considered as statistically significant. The results of the conducted Games-Howell statistical tests are shown in Table 7.

Table 7: Games-Howell Test for pairwise comparison of turbofan aero-engine groups based on Entry Into Service (EIS) year

(I) Turbofan Aero-engine Group	(J) Turbofan Aero-engine Group	Mean Difference (I-J)	p-value	95% Confidence Interval	
				Lower Bound	Upper Bound
1	2	-0.1284417	0.020659*	-0.2397012	-0.0171821
	3	-0.2287411	0.000964*	-0.3673748	-0.0901074
2	1	0.1284417	0.020659*	0.0171821	0.2397012
	3	-0.1002994	0.223214	-0.2458241	0.0452252
3	1	0.2287411	0.000964*	0.0901074	0.3673748
	2	0.1002994	0.223214	-0.0452252	0.2458241

*Statistically significant at 0.05 significance level

By observing the pairwise comparison results provided by the Games-Howell statistical test, it is deduced that there are statistically significant differences between the first group (i.e. turbofan aero-engines which entered service in 1980s decade and earlier) and both other groups (i.e. turbofan aero-engines which entered service in 1990s and beyond). Hence, it can be stated that a major technological leap has been achieved from 1990s decade and onwards. However, it would be potentially of great interest to form a group consisting of turbofan aero-engines with EIS year from 2010 to date, in order to better assess the efficiency difference compared to the groups consisting of turbofan aero-engines having EIS date within the 1990s and 2000s decades.

In addition, we attempt to evaluate the effect of BPR in DEA efficiency, in order to confirm whether the DEA approach coincides with the engineering approach. More specifically, we divide the sample into the following groups with respect to BPR [19] (along with their respective mean DEA efficiency):

- Low-BPR engines, i.e. engines featuring BPR ≤ 2 with mean DEA efficiency score equal to 28.78% ($n_1 = 20$)
- Medium-BPR engines, i.e. engines featuring $2 < \text{BPR} \leq 5$ with mean DEA efficiency score equal to 43.53% ($n_2 = 13$)
- High-BPR engines, i.e. engines featuring $5 < \text{BPR} \leq 8$ with mean DEA efficiency score equal to 56.91% ($n_3 = 13$)
- Ultrahigh-BPR engines, i.e. engines featuring BPR > 8 with mean DEA efficiency score equal to 71.27 % ($n_4 = 9$)

In a similar way to the previous efficiency differences' evaluation with respect to EIS year, we conduct the Games-Howell statistical test, taking into account that it is considered the best-performing in terms of pairwise comparison. Moreover, although the population of the last group can be deemed as quite small, Shingala and Rajyaguru [51] state that Games-Howell statistical test can be deployed for samples with population greater than five. The results of the Games-Howell statistical test evaluating whether there are statistically significant mean efficiency differences among the aforementioned aero-engine groups are shown in Table 8.

Table 8: Games-Howell Test for pairwise comparison of turbofan aero-engine groups based on bypass ratio (BPR)

(I) Turbofan Aero-engine Group	(J) Turbofan Aero-engine Group	Mean Difference (I-J)	p-value	95% Confidence Interval	
				Lower Bound	Upper Bound
1	2	-0.1474897	0.008954*	-0.2591569	-0.0358226
	3	-0.2813090	0.000013*	-0.3865049	-0.1761130
	4	-0.4248761	0.000007*	-0.5424199	-0.3073322
2	1	0.1474897	0.008954*	0.0358226	0.2591569
	3	-0.1338192	0.067098	-0.2747984	0.0071599
	4	-0.2773863	0.000202*	-0.4238899	-0.1308828
3	1	0.2813090	0.000013*	0.1761130	0.3865049
	2	0.1338192	0.067098	-0.0071599	0.2747984
	4	-0.1435671	0.047898*	-0.2860546	-0.0010796
4	1	0.4248761	0.000007*	0.3073322	0.5424199
	2	0.2773863	0.000202*	0.1308828	0.4238899
	3	0.1435671	0.047898*	0.0010796	0.2860546

*Statistically significant at 0.05 significance level

The results obtained by performing pairwise comparisons via the Games-Howell statistical test can be characterized as quite interesting. More specifically, it is clearly evident that the mean efficiency differences are nearly all statistically significant. Subsequently, it is deduced by observing the p-values of Table 8 that the statistical significance is increasing as the gap in terms of BPR range increases. In particular, with respect to the ultrahigh-BPR engine group the p-values of the pairwise comparisons between the first and the second group (0.000007 and 0.000202 respectively) are significantly lower to the p-value when compared to the third group (0.047898).

The results of the aforementioned Games-Howell statistical test designate the positive association of BPR and turbofan aero-engine DEA efficiency. The particular positive association is fully aligned with the results of previous research using the engineering approach. A typical example is the research of Guha [26] which clearly demonstrates the positive effect of ultrahigh BPR (usually between 8 and 10) on specific fuel consumption, namely reduced specific fuel consumption. Also, it is of paramount importance to stress that in accordance with El-Sayed [16] high BPR is positively associated with increased propulsive efficiency and reduced specific fuel consumption, especially for BPR within 8 to 10 value range. Consequently, the aforementioned engineering research conclusions constitute a strong indication of the validity of VSBM Network DEA methodology, as an alternative approach to evaluate turbofan aero-engine efficiency without incorporating complex engineering calculations.

5.2 Regression Analysis

Apart from non-parametric statistical analysis, regression comprises a quite popular method for assessing the effect of various exogenous factors on DEA efficiency. The regression methodologies implemented by researchers dealing with DEA efficiency post-hoc analysis are diverse and in certain cases pretty sophisticated (e.g., truncated regression).

In the current paper, we adopt the following regression methodologies:

- Generalized Linear Model (GLM) equipped with the quasi-binomial family and logit link function. The GLM is applied to separately test the null hypothesis that DMU efficiency is independent of the DMU grouping in terms of BPR and EIS.

- Standard Linear Model (LM), which is particularly deployed separately on raw BPR and EIS data for the purpose of testing the null hypothesis that DMU efficiency is independent of BPR and EIS respectively. For the particular model, it should be stressed that before proceeding with the aforementioned LM methodologies, an assessment for the linear model assumptions has been conducted by using the global test.

After applying the GLM and LM regression methodologies on the resulting DEA efficiency scores with respect to BPR and EIS data, it is highly evident that both BPR and EIS are positively correlated with increased DEA efficiency. The regression analysis output is summarized in Table 9, while Figure 5 and Figure 6 display the boxplots and scatterplots corresponding to prediction models regarding BPR and EIS respectively.

Table 9: Regression Analysis Results

Model	Explanatory Variable	Intercept (p-value)	Slope (p-value)	Linearity Assumption (p-value)	Null Deviance Reduction	Adjusted Coefficient of Determination
GLM	BPR-based Groups	$-1.492(2.95 \cdot 10^{-15})$	$0.598(4.44 \cdot 10^{-15})$	Not Applicable	69.93%	Not Applicable
GLM	EIS-based Groups	$-1.125(6.44 \cdot 10^{-5})$	$0.468(2.12 \cdot 10^{-4})$	Not Applicable	22.88%	Not Applicable
LM	BPR	$0.251(4.13 \cdot 10^{-16})$	$0.051 (< 2 \cdot 10^{-16})$	0.844	Not Applicable	72.57%
LM	EIS	$-17.868(1.76 \cdot 10^{-4})$	$0.009 (1.25 \cdot 10^{-4})$	0.502	Not Applicable	23.01%

More specifically, GLM assessing the four BPR-based engine groups (as previously determined in sub-section 5.1) designates the remarkable reduced null deviance reduction value of 69.93 percent, while the LM for assessing DEA efficiency with respect to BPR features a relatively increased adjusted coefficient of determination value of 72.57 percent. However, EIS demonstrates fairly low explanatory potential when regressed against DEA efficiency. In particular, for the case of LM assessing DEA efficiency with respect to EIS, although the adjusted coefficient of determination gets a fairly low value of 23.01 percent, EIS is found to be significantly correlated with DEA efficiency at the 5 percent significance level. In addition, it is vital to stress that for LM the assumption of linearity is not rejected at the 5 percent significance level, which is clearly designated by the p-value of 0.844 for DEA efficiency against BPR and the p-value of 0.502 for

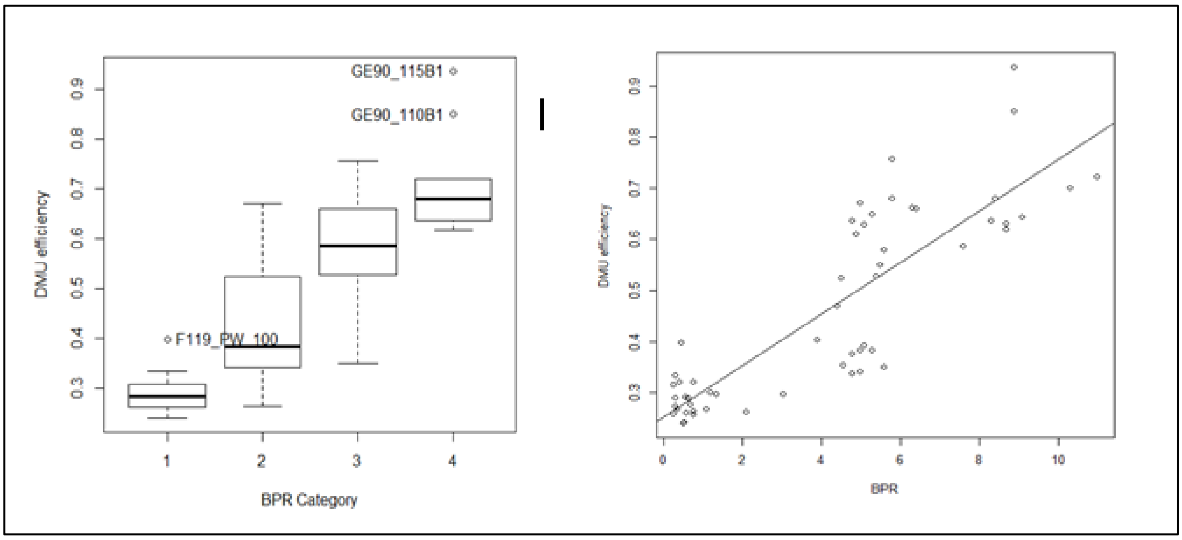

Figure 5: Boxplot and Scatterplot of DEA Efficiency versus BPR (Source: Own elaboration)

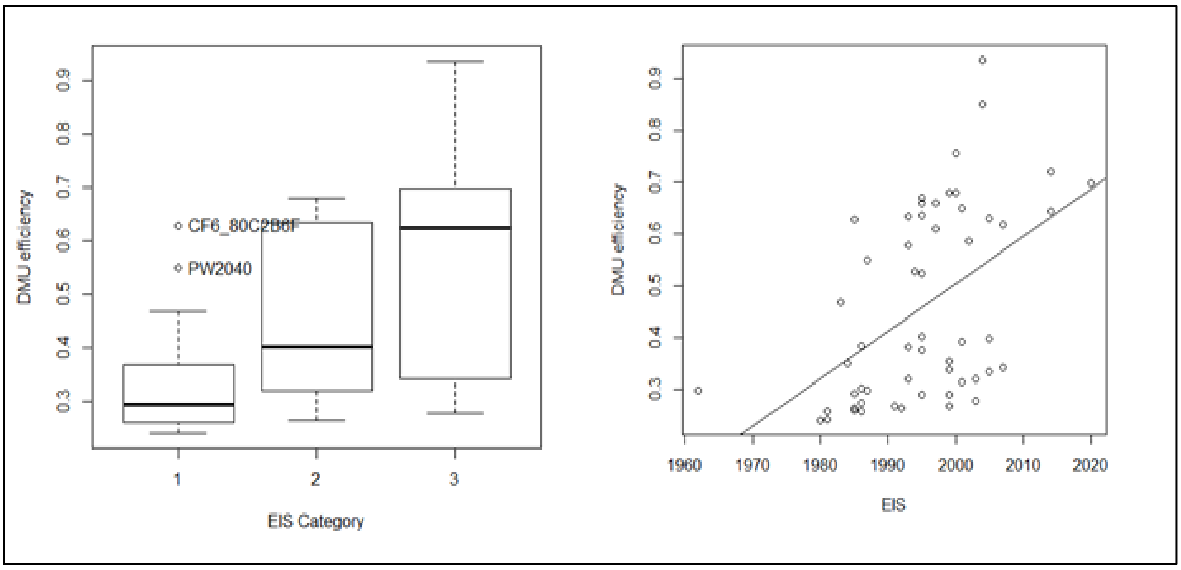

Figure 6: Boxplot and Scatterplot of DEA Efficiency versus EIS (Source: Own elaboration)
DEA efficiency against EIS.

5.3 Discussion

By observing the numerical values in Table 4 and Table 5 regarding the frontier projections and the associated optimal values/change percentage respectively, it is notably evident that for the vast majority of the engines/DMUs the frontier projections do not correspond to figures achievable in the near future. Especially for the case of TSFC, although progress in aerodynamics, thermodynamics, and combustion technology has contributed to significantly reduce TSFC, the TSFC figures contained in Table 5 are certainly unrealistic.

With respect to future TSFC reduction due to technological progress, Birch [3] stresses that by year 2020 an annual reduction of 0.5% in TSFC per year shall be feasible, taking into account the current and near-future technological constraints. Hence, considering the lowest TSFC values in Table 3 (Trent 1000C and GE9X) and the claim of Birch [3], it can be deduced that the TSFC threshold for commercial turbofan aero-engines at the end of the next decade (i.e. year 2030) is predicted to lie within the 0.255 – 0.275 value range. Consequently, the optimal TSFC values in Table 5 which result from the frontier projections can be characterized as technologically infeasible.

Also, frontier projections designate extensive weight reduction for the majority of the engine sample, which is observable in “Weight Change Percentage” column of Table 5. Simultaneously, frontier projections for several engines stipulate for significant thrust increase, which in some cases are well above 50% or even 100%. Similarly to TSFC, it is highly doubtful whether near-future technology shall render the weight and thrust figures stipulated by frontier projections achievable. Although the frontier projections’ viability is not stressed in the research effort of Chen et al. [8], the earlier research of Tone [53] expresses the need to impose bounds on the slacks in order to render reduction of inputs and increase of outputs to lie within a technically feasible spectrum. Hence, in the framework of evaluating efficiency of production or engineering systems, the developed methodologies implementing slacks-based measures should additionally investigate whether the resulting slacks represent technically feasible frontier projections.

Commercial and military aircraft take-off field requirements are directly associated with the take-off thrust-to-weight ratio, i.e. the total engine take-off thrust divided by the aircraft take-off weight. For the case of Boeing 777-8X/9X, although it has the same MTOW of 775,000 lbs as its predecessor 777-200LR/300ER [31], the reduced take-off thrust of GE9X engine is definitely a compromise in terms of required take-off field distance, compared to the GE90-110B1 and GE90-115B1. As a matter of fact, the take-off thrust-to-weight ratio for the 777-300ER/GE90-115B1 at MTOW is 0.2982, while the take-off thrust-to-weight ratio for the 777-8X/GE9X(102,000 lbs version) at MTOW is 0.2632, which is certainly a quite notable reduction. Besides the reduced take-off thrust, the 3.5 percent more weight of GE9X compared to GE90-115B1 poses an additional feature, which negatively affects the DEA efficiency of the GE9X engine. Although the effort of the aero-engine manufacturers is concentrated on weight reduction, GE9X is not fulfilling the prospect of a lighter aero-engine, consequently leading to a payload reduction compared to GE90-115B1, taking into consideration that Boeing 777-300ER and 777-8X/-9X feature exactly the same MTOW.

Based on the previous two paragraphs, it is evident that incorporating weight as an exogenous input renders the current efficiency evaluation more objective. Existing research pertinent to performance optimization of turbofan aero-engines via an engineering approach mainly incorporates aerodynamic and thermodynamic aspects, thus omitting including weight considerations in the optimization process. Hence, turbofan aero-engine designers should include weight estimation techniques like the one developed by Lolis [39], after completing the aerodynamic and thermodynamic calculations and before finalizing their conceptual/preliminary designs.

For the purpose of assessing to which degree current sample turbofan engines could improve their efficiency given the technological boundaries, we subsequently impose additional constraints to the slacks of the VRS VSBM Network DEA formulation, albeit in an alternative manner to the one dictated by Tone [53]. In accordance with Birch [3], where a 5 percent decrease in TSFC is technologically feasible till the end of the next decade, we numerically approximate the optimal efficiency within a feasible range of exogenous inputs and final output.

More specifically, for each DMU_o under consideration we assume fuel flow (X_{1o}), engine weight (X_{2o}), and take-off thrust (Y_{1o}) are uniformly distributed on $[0.95x_{1o}, x_{1o}] \times [0.95x_{2o}, x_{2o}] \times [y_{1o}, 1.05y_{1o}]$. Subsequently, we use simulation to generate 10^6 sets of pseudo-random variables that follow the aforementioned multivariate uniform distribution. Through an iterative implementation of the multiplier form model, we obtain a range of technologically feasible DMU_o efficiencies. The set that maximizes the DMU_o efficiency is considered optimal. The VRS VSBM Network DEA has been duly implemented and executed in the Wolfram Mathematica v10.3 computation environment. The obtained results are reported in Table 9.

By observing the numerical values within Table 9, it is evident that only GE90-110B1 and GE90-115B1 can attain ideal efficiency by simultaneously improving the exogenous inputs and final output within the aforementioned technologically feasible ranges. In addition, it is worth mentioning that for both the above engines the ideal efficiency is achieved by simultaneously improving the exogenous inputs and final output by less than 5 percent, thus designating the feasibility of such attempt by resorting to fairly minor enhancements. Another notable example is TFE731-60, where maximum attained revised efficiency, which is lower than unity, is achieved by simultaneously improving exogenous inputs and final output by less than 5 percent.

Table 10: Revised Efficiency & Corresponding Exogenous Inputs/Final Output Values

Model	Maximum Attained Revised Efficiency	Efficiency Improvement (% of original value)	Corresponding Fuel Flow* (% of actual value)	Corresponding Weight* (% of actual value)	Corresponding Thrust* (% of actual value)
Commercial					
ALF502R-6	0.39147	11.8 %	95.00 %	95.00 %	105.00 %
CFE738-1-1B	0.42629	11.6 %	**95.05 %**	**95.18 %**	105.00 %
CFM56-3C	0.42709	11.6 %	95.00 %	95.00 %	105.00 %
CFM56-5B3/P	0.59520	13.0 %	95.00 %	95.00 %	105.00 %
CFM56-7B26	0.44605	14.0 %	95.00 %	95.00 %	105.00 %
GP7270	0.68187	8.1 %	95.00 %	95.00 %	105.00 %
CF34-80C1	0.41520	10.5 %	95.00 %	95.00 %	105.00 %
CF6-80C2B6F	0.70193	11.6 %	95.00 %	95.00 %	105.00 %
CF6-80E1A3	0.71736	10.5 %	95.00 %	95.00 %	**104.81 %**
GE90-110B1	1	17.7 %	**95.90 %**	**96.22 %**	**104.96 %**
GE90-115B1	1	6.8 %	**98.46 %**	**95.56 %**	**104.09 %**
GE90-85B	0.70894	11.4 %	95.00 %	95.00 %	105.00 %
GE90-94B	0.76654	12.7 %	95.00 %	95.00 %	105.00 %
GE9X	0.82276	17.7 %	95.00 %	95.00 %	105.00 %
GEnx-1B70	0.71737	11.5 %	95.00 %	95.00 %	105.00 %
TFE731-60	0.42251	5.0 %	**97.48 %**	**96.82 %**	**104.99 %**
V2533-A5	0.59139	13.0 %	95.00 %	95.00 %	105.00 %
D-18T	0.64676	11.8 %	95.00 %	95.00 %	105.00 %
PW2040	0.61668	12.3 %	95.00 %	95.00 %	105.00 %
PW4056	0.68098	11.7 %	95.00 %	95.00 %	105.00 %
PW4084	0.71014	7.6 %	95.00 %	95.00 %	105.00 %
PW4090	0.73222	10.8 %	95.00 %	95.00 %	**104.85 %**
PW4098	0.71754	5.4 %	95.00 %	95.00 %	**104.48 %**
PW4462	0.70875	11.6 %	95.00 %	95.00 %	105.00 %
PW6122A	0.38092	11.6 %	95.00 %	95.00 %	105.00 %
AE3007A3	0.37586	11.5 %	95.00 %	95.00 %	105.00 %
BR715-C1-30	0.39064	10.6 %	95.00 %	95.00 %	105.00 %
RB211-535C	0.52744	12.6 %	95.00 %	95.00 %	105.00 %
Tay 620-15	0.32849	10.4 %	95.00 %	95.00 %	105.00 %
Trent 1000C	0.77153	7.1 %	95.00 %	95.00 %	105.00 %
Trent 556	0.65549	11.7 %	95.00 %	95.00 %	105.00 %
Trent 700	0.74924	11.7 %	95.00 %	95.00 %	105.00 %
Trent 895	0.79622	5.3 %	95.00 %	95.00 %	**104.60 %**
Trent 970	0.69028	11.5 %	95.00 %	95.00 %	105.00 %
Military					
D-30F6	0.25493	6.3 %	95.00 %	95.00 %	105.00 %
EJ200	0.35342	10.3 %	95.00 %	95.00 %	105.00 %
F101-GE-102	0.29004	10.5 %	95.00 %	95.00 %	105.00 %
F110-GE-100	0.28404	10.5 %	95.00 %	95.00 %	105.00 %
F110-GE-129	0.29206	10.5 %	95.00 %	95.00 %	105.00 %
F110-GE-132	0.30544	10.5 %	95.00 %	95.00 %	105.00 %
F118-GE-100	0.35408	10.5 %	95.00 %	95.00 %	105.00 %
F404-GE-400	0.28512	10.5 %	95.00 %	95.00 %	105.00 %
F414-GE-400	0.34705	10.5 %	95.00 %	95.00 %	105.00 %
RD-33	0.26690	10.5 %	95.00 %	95.00 %	105.00 %

Table 10: Revised Efficiency & Corresponding Exogenous Inputs/Final Output Values (contd)

Model	Maximum Attained Revised Efficiency	Efficiency Improvement (% of original value)	Corresponding Weight* (% of actual value)	Corresponding Fuel Flow* (% of actual value)	Corresponding Thrust* (% of actual value)
<u>**Military(contd)**</u>					
AL-31F	0.31796	9.1 %	95.00 %	95.00 %	105.00 %
AL-37FU	0.31856	10.1 %	95.00 %	95.00 %	105.00 %
F100-PW-220E	0.28643	10.5 %	95.00 %	95.00 %	105.00 %
F100-PW-229	0.29486	10.5 %	95.00 %	95.00 %	105.00 %
F119-PW-100	0.43897	10.3 %	95.00 %	95.00 %	105.00 %
TF33-P-103	0.32751	10.4 %	95.00 %	95.00 %	105.00 %
F402-RR-408	0.32738	8.8 %	95.00 %	95.00 %	105.00 %
M88-2	0.31931	10.5 %	95.00 %	95.00 %	105.00 %
M88-3	0.36784	10.5 %	95.00 %	95.00 %	105.00 %
RB199 Mk105	0.29609	10.5 %	95.00 %	95.00 %	105.00 %
RM12	0.30196	10.5 %	95.00 %	95.00 %	105.00 %

*Numerical values in bold designate improvement below the 5 percent threshold

6 Concluding Remarks

The current paper attempts to evaluate turbofan aero-engine efficiency by deploying an advanced benchmarking methodology, namely the VRS VSBM Two-Stage Network DEA. The implemented approach comprises an alternative pathway to the engineering approach, which is predominantly of a thermodynamic texture. In any case, the engineering approach is by definition assuming that the aero-engine design process has the following main features:

- It takes place in isolation, i.e. there are no subsequent comparisons with similar designs.
- It excludes certain traits which would negatively affect the usability or the marketability. For example, an oversized/overweight aero-engine would affect usability, while an aero-engine with high emissions (pollutant/noise) is highly unlikely to be marketable.

Vital part of the implemented benchmarking approach comprises the selection of the appropriate exogenous inputs, intermediate measures, and final output. In addition, adherence to the functional concept of the turbofan aero-engine has rendered the adopted Network DEA model sufficiently representative, while incorporating the engine weight as an input enhances the objectivity of the benchmarking results.

Another novelty of the conducted research lies in the fact that without resorting to extremely complex engineering calculations, it has been rendered feasible to adequately assess the relative efficiency of in-service and near-future turbofan aero-engines. As previously reported, the resulting efficiency scores follow a pattern similar to the propulsive efficiency pattern implementing an engineering approach (see Figure 3). Moreover, with respect to the GE9X turbofan aero-engine which is planned to enter service soon aboard the Boeing 777-8X/-9X, the current paper designates the additional effort that should be undertaken from General Electric regarding weight reduction and take-off thrust enhancement.

Additionally, the concurrent deployment of non-parametric statistics and regression analysis for the purpose of conducting efficiency scores' post-hoc analysis is another novel feature of the present research effort. Consequently, the convergence of both post-hoc analysis methods corroborates the positive association of BPR with increased DEA efficiency, which is also the case regarding the engineering approach. Moreover, post-hoc analysis results designate the advances in turbofan aero-engine technology by confirming the positive association of EIS with increased efficiency. However, lack of data pertinent to new-generation turbofan aero-engines like Rolls-Royce Trent XWB, Pratt & Whitney GTF (1100G/1500G/1700G Series), and CFM LEAP rendered infeasible to gain a more accurate outlook of the turbofan aero-engine evolution over time. Hence, it comprises a challenge for future research to evaluate the particular aero-engines within a similar benchmarking framework, thus acquiring a more coherent view regarding the association of technological leaps with enhanced efficiency.

From a purely technical standpoint, the present paper introduces a methodological framework that enables turbofan aero-engine designers to evaluate conceptual/preliminary designs against existent in-service designs. In particular, after defining the turbofan aero-engine features deploying engineering calculations, benchmarking the resulting overall configuration against existing designs with similar configurations can comprise a complementary tool in the process of design optimization. Consequently, adopting the aforementioned design evaluation framework could substantially reduce developmental effort along with minimizing the need to reconsider design features. As a matter of fact, non-oriented VRS VSBM Network DEA has a clear advantage over alternative benchmarking methodologies taking into account that:

- It does not assign weights to the inputs, intermediate measures, and final outputs in a predetermined manner (e.g., Analytic Hierarchy Process).
- It simultaneously attempts to reduce inputs and increase outputs in a non-linear way, similarly to the engineering approach.
- It designates the areas requiring improvement, thus avoiding unnecessary modifications to the design features.

Finally, it is worth mentioning that the devised simulation method for defining the optimum DEA efficiency within a certain improvement threshold of the incorporated exogenous inputs and final outputs, further augments the potential for implementing the herewith integrated approach for objectively assessing both existing relative efficiency and optimum efficiency enhancement within technological limitations. Hence, the particular simulation method could comprise a tool for turbofan aero-engine designers and manufacturing enterprises, in order to evaluate whether it is preferable to invest in using an existing engine as design baseline or develop a “white-paper” design.

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List of References

[1] AIR International, December 2017 Issue, Key Publishing Ltd.
[2] Avkiran NK. Productivity Analysis in the Service Sector with Data Envelopment Analysis. 3rd ed. Queensland, AU: University of Queensland; 2006.
[3] Birch NT. 2020 vision: the prospects for large civil aircraft propulsion. The Aeronautical Journal 2000; 104(1038):347-352.
[4] Braglia M, Petroni A. Evaluating and selecting investments in industrial robots. International Journal of Production Research 1999;37(18):4157-4178.
[5] Brockett PL, Golany B. Using rank statistics for determining programmatic efficiency differences in data envelopment analysis. Management Science 1996;42(3):466-472.
[6] Bulla S, Cooper WW, Wilson D, Park KS. Evaluating efficiencies of turbofan jet engines: a data envelopment analysis approach. Journal of Propulsion and Power 2000;16(3):431-439.
[7] Charnes A, Cooper, WW, Rhodes, E. Measuring the efficiency of decision making units. European Journal of Operational Research 1978; 2(6):429-444.
[8] Chen Y, Li Y, Liang L, Salo A, Wu H. Frontier projection and efficiency decomposition in two-stage processes with slacks-based measures. European Journal of Operational Research 2016;250(2):543-554.
[9] Chitkara P. A data envelopment analysis approach to evaluation of operational inefficiencies in power generating units: a case study of Indian power plants. IEEE Transactions on Power Systems 1999;14(2):419-425.
[10] Cook WD, Kress M., Seiford, LM. Data envelopment analysis in the presence of both quantitative and qualitative factors. Journal of the Operational Research Society 1996;47(7):945-953.
[11] Cook WD, Zhu J. Within-group common weights in DEA: An analysis of power plant efficiency. European Journal of Operational Research 2007;178(1):207-216.
[12] Cooper WW, Seiford LM, Tone, K. Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software. 2nd ed. New York: Springer Science & Business Media; 2007.
[13] Daly M, editor. Jane’s Aero Engines 2018-2019. Jane’s Information Group; 2018.
[14] Dik A, Bitén N, Zaccaria V, Aslanidou I, Kyprianidis KG. Conceptual Design of a 3-Shaft Turbofan Engine with Reduced Fuel Consumption for 2025. Energy Procedia 2017;142:1728-1735.
[15] Doyle JR, Green RH. Comparing products using data envelopment analysis. Omega 1991;19(6):631-638.
[16] El-Sayed AF. Aircraft Propulsion and Gas Turbine Engines. Boca Raton: CRC Press; 2008.
[17] El-Sayed AF. Fundamentals of Aircraft and Rocket Propulsion. London: Springer-Verlag; 2016.
[18] El-Sayed AF, Emeara MS, El-Habet MA. Performance Analysis of High Bypass Ratio Turbofan Aeroengine. International Journal of Development Research 2016;6(7):8382-8398.
[19] Encyclopaedia Brittanica. Jet Engine, https://www.britannica.com/technology/jet-engine/Medium-bypass-turbofans-high-bypass-turbofans-and-ultrahigh-bypass-engine/; 2015 [accessed 23 February 2019].
[20] Färe R, Grosskopf S, Logan J. The comparative efficiency of Western coal-fired steam-electric generating plants: 1977–1979. Engineering Costs and Production Economics 1987;11(1):21-30.
[21] Farokhi S. Aircraft Propulsion. 2nd ed. Chichester: John Wiley & Sons; 2014.
[22] Gohardani AS, Doulgeris G, Singh R. Challenges of future aircraft propulsion: A review of distributed propulsion technology and its potential application for the all electric commercial aircraft. Progress in Aerospace Sciences 2011;47(5):369-391.
[23] Golany B, Roll Y. An application procedure for DEA. Omega 1989;17(3):237-250.
[24] Golany B, Roll Y, Rybak D. Measuring efficiency of power plants in Israel by data envelopment analysis. IEEE transactions on engineering management 1994;41(3):291-301.

[25] Greatrix DR. Powered flight: the engineering of aerospace propulsion. London: Springer-Verlag; 2012.
[26] Guha, A. Optimisation of aero gas turbine engines. The Aeronautical Journal 2001a;105(1049):345-358.
[27] Guha A. Optimum fan pressure ratio for bypass engines with separate or mixed exhaust streams. Journal of Propulsion and Power 2001b;17(5):1117-1122.
[28] Hampf B, Krüger J. Technical efficiency of automobiles–A nonparametric approach incorporating carbon dioxide emissions. Transportation Research Part D: Transport and Environment 2014;33:47-62.
[29] Hjalmarsson L, Odeck J. Efficiency of trucks in road construction and maintenance: an evaluation with data envelopment analysis. Computers & Operations Research 1996;23(4):393-404.
[30] International Civil Aviation Organization (ICAO). Aircraft Engine Emissions Databank, Version 25a, 25 May 2018, https://www.easa.europa.eu/easa-and-you/environment/icao-aircraft-engine-emissions-databank/ ; 2018 [accessed 9 December 2018].
[31] Jackson P, Peacock LT, editors. Jane's All the World's Aircraft 2017-2018: Development & Production. IHS Global; 2017
[32] Jet Engine Specification Database. http://www.jet-engine.net/ [accessed 15 September 2018].
[33] Kao C. Dynamic data envelopment analysis: A relational analysis. European Journal of Operational Research 2013;227(2):325-330.
[34] Karsak, E. Using data envelopment analysis for evaluating flexible manufacturing systems in the presence of imprecise data. The International Journal of Advanced Manufacturing Technology 2008;35(9-10):867-874.
[35] Kou M, Chen K, Wang S, Shao Y. Measuring efficiencies of multi-period and multi-division systems associated with DEA: An application to OECD countries' national innovation systems. Expert Systems with Applications 2016;46:494-510.
[36] Lee WS, Lee KP. Benchmarking the performance of building energy management using data envelopment analysis. Applied Thermal Engineering 2009;29(16):3269-3273.
[37] Liu JS, Lu LY, Lu WM, Lin BJ. A survey of DEA applications. Omega 2013;41(5):893-902.
[38] Lo Storto CL. Technological benchmarking of products using the data envelopment analysis: an application to segments' A'and'B'of the Italian car market. In: Innovation in Technology Management-The Key to Global Leadership. IEEE; PICMET'97: Portland International Conference on Management and Technology; 1997, p. 783-788.
[39] Lolis P. Development of a preliminary weight estimation method for advanced turbofan engines. Doctoral Thesis. Cranfield: Cranfield University; 2014.
[40] Lozano S, Villa G, Eguía I. Data envelopment analysis with multiple modes of functioning. Application to reconfigurable manufacturing systems. International Journal of Production Research 2017;55(24):7566-7583.
[41] Meenakumari R, Kamaraj N, Thakur T. Measurement of relative operational efficiency of SOEUs in India using data envelopment analysis. International Journal of Applied Decision Sciences 2009;2(1):87-104.
[42] Oates GC, editor. Aircraft pro**p**ulsion systems technology and design. Washington, DC: American Institute of Aeronautics and Astronautics; 1989.
[43] Odeck J. Evaluating efficiency of rock blasting using data-envelopment analysis. Journal of Transportation Engineering 1996;122(1):41-49.
[44] Papahristodoulou C. A DEA model to evaluate car efficiency. Applied Economics 1997;29(11):1493-1508.
[45] Ramanathan R. An Introduction to Data Envelopment Analysis: A Tool for Performance Measurement. New Delhi: Sage Publications; 2003.
[46] Richter H. Advanced Control of Turbofan Engines. New York: Springer Science & Business Media; 2012.
[47] Roux, E. Turbofan and turbojet engines: database handbook. Blagnac: Editions Elodie Roux; 2007.

[48] Saravanamuttoo HI, Rogers GFC, Cohen H, Straznicky PV, Nix AC. Gas Turbine Theory. 7th ed. Harlow: Pearson Education; 2017.
[49] Sarkis J, Talluri S. A decision model for evaluation of flexible manufacturing systems in the presence of both cardinal and ordinal factors. International Journal of Production Research 1999;37(13):2927-2938.
[50] Sforza PM. Theory of aerospace propulsion. Cambridge, MA: Butterworth-Heinemann; 2017.
[51] Shingala MC, Rajyaguru A. Comparison of Post Hoc Tests for Unequal Variance. International Journal of New Technologies in Science and Engineering 2015;2(5):22-33.
[52] Sun S. Assessing computer numerical control machines using data envelopment analysis. International Journal of Production Research 2002;40(9):2011-2039.
[53] Tone K. A slacks-based measure of efficiency in data envelopment analysis. European Journal of Operational Research 2001;130(3):498-509.
[54] Tone K, Tsutsui M. Network DEA: A slacks-based measure approach. European Journal of Operational Research 2009;197(1):243-252.
[55] Tone K. Non-Radial DEA Models. In: Tone K, editor. Advances in DEA Theory and Applications: With Extensions to Forecasting Models, Hoboken, NJ: John Wiley & Sons; 2017, p. 11-19.
[56] Triantis KP. Engineering Applications of Data Envelopment Analysis. In: Cooper WW, Seiford LM, Zhu J, editors. Handbook on Data Envelopment Analysis, New York: Springer Science & Business Media; 2011, p. 363-402.
[57] Triantis KP. Network Representations of Efficiency Analysis for Engineering Systems: Examples, Issues and Research Opportunities. In: Cook WD, Zhu J, editors. Data Envelopment Analysis: A Handbook on the Modeling of Internal Structures and Networks, New York: Springer Science & Business Media; 2014, p.569-584.
[58] Voltes-Dorta A, Perdiguero J, Jiménez, JL. Are car manufacturers on the way to reduce CO2 emissions?: A DEA approach. Energy Economics 2013;38:77-86.
[59] Von der Bank R, Donnerhack S, Rae A, Poutriquet F, Lundbladh A, Antoranz, A, Ruzicka M. Compressors for ultra-high-pressure-ratio aero-engines. CEAS Aeronautical Journal 2016;7(3):455-470.
[60] Zhao Y, Triantis K, Murray-Tuite P, Edara P. Performance measurement of a transportation network with a downtown space reservation system: A network-DEA approach. Transportation Research Part E: Logistics and Transportation Review 2011;47(6):1140-1159.
[61] Zhu R, Liang Q, Zhan H. Analysis of aero-engine performance and selection based on fuzzy comprehensive evaluation. Procedia engineering 2017;174:1202-1207.