Cosmological parameters

Linnea Hjalmarsdotter

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Abstract

Cosmology of today has developed into a precision science with a well-established theoretical framework including predictions and tests testable by astronomical observations. In this report we outline the basics of the theory behind the standard model of present day cosmology and discuss the impact of the mass and energy content of the Universe on its geometry as well as its historic and future evolution. We describe the different methods of observationally determining H_0, Ω_M and Ω_Λ. Finally, we discuss the latest results and their implications.

1 Introduction to curved spacetime

Our Universe, like any other homogeneous and isotropic four-dimensional curved space-time, can be described by the metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \tag{1}$$

where r is dimensionless, $a(t)$ denotes the time-dependent scale factor, k is a curvature term that can be chosen as $+1, 0$, or -1 depending on whether the constant curvature is positive, zero, or negative respectively, and we set $c = 1$. This metric is called the *Robertson-Walker line element*, and we note that for $k = 0$, it reduces to flat Minkowski space. For $k = +1$, the Universe will be closed with $a(t)$ as the radius at time t. For $k = -1$, the Universe will be open and plausibly of infinite extent.

The effect of matter and energy on the curvature of space-time is given by *Einstein's equations* of general relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor, R the Ricci scalar, $T_{\mu\nu}$ the energy-momentum tensor, and Λ the so-called cosmological constant. By choosing as the metric the Robertson-Walker line element above, and calculating the metric connections we obtain from the 00 component

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_{rad}) - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{3}$$

or by defining the vacuum energy density

$$\rho_{vac} = \rho_\Lambda = \frac{\Lambda}{8\pi G}, \tag{4}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{tot} - \frac{k}{a^2} \tag{5}$$

with $\rho_{tot} = \rho_m + \rho_{rad} + \rho_{vac}$. This is the *Friedmann equation* and describes the evolution of the scale factor a depending on the contributions from matter and radiation as well as from the cosmological constant.

Without the cosmological constant, Einstein's equations do not permit static solutions, only expanding or contracting. The concept of an expanding or contracting universe being too bold at the time - it was generally taken for granted that the Universe would be static and eternal - this was considered a failure which prompted Einstein to include the term. Later, when Hubble discovered the expansion of the Universe, Einstein is said to have considered the introduction of the cosmological constant as his biggest mistake. Today, however, it seems as there might be a need for a non-zero cosmological constant after all. This will be further discussed in Section 3.5.

1.1 The expansion of the Universe

The predictions made by Friedmann (1922) and Lemaître (1927) of non-static solutions to Einsteins equations were confirmed with the discovery that distant galaxies are in fact moving away from us with velocities proportional to their relative distances. The relationship $v = H_0 d$ is known as *Hubble's law*, after its discoverer Edwin Hubble (1929). The Hubble constant H_0 is one of the most fundamental parameters of modern cosmology. The value of H_0 defines the present observed value and is actually more correctly expressed as $H(t_0)$, where t_0 is the present time and $H(t)$ for all times t determined by

$$H(t) = \frac{\dot{a}(t)}{a(t)}. \tag{6}$$

As we can see from Friedmann's equation, the evolution of the scale factor is governed by two additional parameters, the mass density and the cosmological constant. The expansion of the Universe is an expansion of space-time itself and thus does not involve any motion in coordinate space. If we assign a set of coordinates (r_i, θ_i, ϕ_i) to each galaxy i, these coordinates will not change when the Universe evolves and the pattern of galaxies will stay the same. All the cosmological distances will, however, be stretched by a factor $a(t)$. A popular analogy is that of a 'raisin bread', where the distances between the raisins increase as the bread is raising. Nevertheless, using just ordinary Newtonian gravity, one can show that a massive particle outside a spherical piece of the Universe expanding with a velocity $v = H_0 d$ will escape the gravitational attraction only if the density of space is

$$\rho \leq \rho_{crit} = \frac{3H_{0^2}}{8\pi G}. \tag{7}$$

This means that to stop the expansion and make the Universe contract a mass density equal or higher than this critical density is required. The value of the critical density is time-dependent, and the present value is

$$\rho_{0,crit} = 1.9 \cdot 10^{-29} h^2 g \text{ cm}^{-3}, \tag{8}$$

where

$$\rho_0 = \rho_m \frac{a(t)^3}{a_0^3} \tag{9}$$

is the matter density for the present scale factor (i.e. matter density now), and

$$h = \frac{H_0}{100 \text{ km s}^{-1}\text{Mpc}^{-1}}. \tag{10}$$

Today's contribution from all kinds of matter and radiation to the critical density can be expressed as

$$\Omega_0 = \frac{\rho_0}{\rho_{crit}}. \tag{11}$$

By expanding the scale factor $a(t)$ around the present time t_0, we find

$$a(t) = a(t_0)\left(1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + ...\right), \tag{12}$$

which defines the decceleration parameter

$$q_0 = -\frac{\ddot{a}}{aH_0^2}, \tag{13}$$

or using the critical density,

$$q_0 = \frac{1}{2}\,\Omega_0 + \frac{3}{2}\sum \Omega_i \alpha_i, \tag{14}$$

where α gives the equation of state as $p_i = \alpha\rho_i$ (for baryons $\alpha = 0$, for radiation $\alpha = 1/3$, and for vacuum energy $\alpha = -1$).

2 Cosmological models

The standard model of present day cosmology is built on the cosmological principle of a homogeneous and isotropic Universe, the Robertson-Walker line element and the Friedmann equation, constituting the *Friedmann-Lemaitre-Robertsson-Walker* model, or FLRW for short.

It is also customary to express the present day contributions to the critical energy density from matter and the cosmological constant separately as

$$\Omega_M = \frac{\rho_m}{\rho_{crit}} = \frac{8\pi G}{3H_0^2}\rho_0, \tag{15}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}} = \frac{\Lambda}{3H_0^2}. \tag{16}$$

Defining

$$\Omega_K = \frac{-k}{a_0^2 H_0^2}, \tag{17}$$

we can express the Friedmann equation as

$$\Omega_M + \Omega_\Lambda + \Omega_K = 1. \tag{18}$$

We note that there are only two independent contributions to the energy density. The curvature is determined by the total density of matter and energy and vice versa. (In the early Universe, a contribution from the radiation density has to be included. This term vanishes for the present or future times, reflecting the fact that radiation plays no major role today.)

The full solutions to the Friedmann equation for the scale factor a at all times t can be quite complicated due to the different a-dependence of the individual terms contributing to ρ_{tot}, but let us look at a few simple cases and examine the long-time behaviour as $t \to \infty$.

The simplest cases to consider are for $\Lambda = 0$. The Friedmann equation reduces to

$$\Omega_M + \Omega_K = 1. \tag{19}$$

The solution is

$$\dot{a}(t) = \frac{8\pi G \rho_{m a_0^3}}{3a(t)} - k. \tag{20}$$

For the (somewhat unrealistic) case of a Universe without matter, real value solutions for $a(t)$ require $k = -1$, an open geometry, and

$$a(t) = t. \tag{21}$$

This is the so-called Milne model and escribes a linearly expanding (or contracting) Universe.

Including the matter density term, the solutions for large t will depend on k. For $k = 0$,

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}. \tag{22}$$

This is the so-called Einstein-de Sitter model. For $k = -1$, $\dot{a}^2$ is always positive, which means an ever increasing $a(t)$. For $k = +1$, $\dot{a}^2$ will be positive up to a critical largest value, given by

$$a_{crit} = \frac{8\pi G \rho_m a_0^3}{3}. \tag{23}$$

For larger t, the Universe will start to contract and eventually come to a 'Big Crunch'.

Including both matter and a non-zero cosmological constant we obtain a richer variety of cosmological models. We have to solve

$$\dot{a}(t) = \frac{8\pi G \rho_m a_0^3}{3a(t)} - k + \frac{\Lambda\ a(t)^2}{3}. \tag{24}$$

If $\Lambda < 0$, $a(t)$ can not become arbitrarily large to allow real solutions for $\dot{a}$. For the largest possible value, $\ddot{a} < 0$, and we have an oscillating Universe.

For $\Lambda > 0$ and $k = 0$ or -1 we observe that for large $a(t)$ the Universe will enter a period of exponential expansion as the vacuum energy takes over. This situation can actually have occurred in the past, during the period of inflation when the scale of the Universe was blown up exponentially during a short period of time.

The energy densities and thus H_0, Ω_0, Ω_M, and Ω_Λ are measurable by observations and the focus of today's modern observational cosmology is to measure these parameters with ever better precision. In the next section, we will outline some of the most important methods and present latest results. In the last section, we will compare results from different measurements and discuss the implications of the current best estimates.

3 Measuring the parameters

3.1 The Hubble parameter, H_0

Hubble's law states that distant galaxies are moving away from us with velocities proportional to their relative distances, $v = H_0 d$. The value of H_0 is found by measuring the recession velocities of galaxies, whose distances are independently known by other measurements. The most reliable distance indicators are obtained from the observed relation between the rotational velocity (distance independent) and the apparent luminosity (related by the distance to intrinsic luminosity) of spiral galaxies. Another distance indicator is the relation between observed and intrinsic luminosities of so-called astronomical standard candles, objects with constant luminosity. One such example are type Ia supernovae, which will be discussed more in detail in section 3.5. The present best estimate of $H_0 = 65 \pm 5$ km sec^{-1} Mpc^{-1}, combining different types of measurements.

3.2 The total matter and energy density, Ω_0

The cosmic microwave background radiation, or CMBR, provides a snapshot of the Universe at the so-called last scattering surface, when photons scatter for the last time off free electrons, before the electrons recombine with protons to form neutral matter (with a lot smaller cross section) at t_{ls}, about 300 000 years after the Big Bang. At that time, photons were still not decoupled from baryons. As the baryons were falling into the potential wells of the density fluctuations eventually making up the structure of the Universe, the photons acted as a restoring force, which led to gravity-driven acoustic oscillations. These oscillations show up in the CMBR as acoustic peaks in its power spectrum. The wavelength of the lowest frequency acoustic mode $\lambda_{max} \sim v_s t_{ls}$ and thus provides a standard ruler on the last scattering surface. Both λ_{max} and the distance to the last scattering surface depend on the curvature of the Universe and a value of Ω_0 can be derived from the location of the first acoustic peak. Since this is a geometrical method it assumes nothing about, and is insensitive to, the composition of matter and energy and is therefore a measure of the curvature (in some sense rather Ω_K than Ω_0). CMBR anisotropy measurements have now been carried out by more than twenty experiments, the most precise ones being COBE (1989) and WMAP (2001). All the measurements have defined the position of the first acoustic peak at a value consistent with $\Omega_0 \approx 1$ (and $\Omega_k = 0$, implying either $\Omega_M = 1$ and $\Omega_\Lambda = 0$ or both > 0).

3.3 Baryonic matter, Ω_B

The most precise determination of the baryon density in the Universe comes from a comparison with measured primordal abundances of the light elements, D, $^3He, ^4He$, and ^7Li to predictions from Big Bang nucleosynthesis. The agreement of measurements and theoretical predictions of the primordial abundances is one of the strongest observational evidence for the Big Bang scenario, but the values also give upper limits on Ω_B, since the formation of the primordial elements depend strongly on the baryon density. The best baryon density probe is deuterium since it can not be produced by any known astrophysical processes, and the evolution of its abundance since Big Bang is therefore simple. Measurements of the deuterium abundance from high-redshift gas-clouds, seen in absorption against distant quasars, have yielded $\Omega_B h^2 = 0.019 \pm 0.0012$ or $\Omega_B \approx 0.05$ with $h = 0.65$.

The results are consistent with those from two other measurements, based on entirely different physics. By comparing measurements of the opacity of the Ly-α forest toward high-redshift quasars with high-resolution hydrodynamical simulations of structure formation, it is possible to obtain a lower limit for Ω_B through the intensity of the ionizing field. Measurements indicate $\Omega_B h^2 \geq 0.015$.

The CMBR anisotropy also provides a determination of the baryon density from the amplitudes, or more precisely from the difference between the amplitudes of the first two acoustic peaks in the power spectrum, described in the section above. The value from the most recent and accurate experiment WMAP is consistent with $\Omega_B \approx 0.05$.

3.4 Dark matter, $\Omega_M - \Omega_B$

The baryonic matter thus represents only a tiny fraction of the total mass density of the Universe (assuming $\Omega_0 = 1$). It has, however, been known from other forms of mass measurements of for example the rotational velocities of galaxies, that most of the matter in galaxies and galaxy clusters is in the form of non-luminous matter. It is likely that the same situation applies to the Universe as a whole and that the bulk of the contribution to the matter density should be searched for in the form of dark matter. The classical approach to weighing the dark matter involves the use of mass-to-light ratios. The idea is to extrapolate ratios found for galaxies or galactic clusters to the Universe as a whole and divide by the critical mass-to-light ratio to obtain Ω_M. Best estimates suggest $\Omega_M = 0.20 \pm 0.04$. The problem with this method, however, is that the extrapolation from a structure such as a cluster of galaxies to the entire Universe might not hold, especially since the luminosity density of the Universe itself evolves strongly with redshift.

Another method, involving less assumptions, is based on determining the ratio of baryon to total mass density in galactic clusters. Most of the baryons in a cluster reside in the hot, x-ray emitting intracluster gas. The mass of this gas can be determined by either measuring its X-ray flux, and/or mapping the Sunyaev-Zel'dovich CMBR distortion caused by CMBR photons scattering off hot electrons in the gas. The total mass of the cluster can be measured by its dynamics or by mapping using gravitational lensing. Assuming that the baryon density of the Universe is the same fraction of the total mass as the baryons in the gas of the clusters to the total cluster masses, one can then derive a value by making use of the baryon density from section above. Best estimates suggest $\Omega_M = 0.4 \pm 0.1$, significantly larger than the values derived from calculating mass-to-light ratios, and are supported by several other different methods as well as theoretical considerations for structure formation. A value of $\Omega_M \gg \Omega_B$ is required for the evolution of the structure that we see today. If the mass had been in form of baryons only density fluctuations seen in the CMBR today would have begun to grow only at the time of decoupling ($z = 1000$), and would not have had time to evolve and produce all the structure we see today. There are therefore no viable models for structure formation without a significant amount of dark matter.

Such considerations also rules out a significant fraction of the dark matter as being in the form of dark baryons. We know that only about 10 percent of baryons are in form of luminous matter or stars. The reminding 90 percent is made up of dark baryonic matter most of which probably resides in intracluster gas as described above but also in 'dark stars' such as white dwarfs, neutron stars, black holes or objects of mass below the hydrogen burning limit. But the fact that there seems to be a lot more matter than there are baryons, including this dark fraction, require most of the dark matter to be in some other exotic form of particles. The currently most favoured candidates are relic elementary particles left over from the Big Bang, heavy stable particles with very weak interactions, such as e.g. the neutralino. No such particle has however yet been detected, neither in space nor in particle physics experiments on Earth.

3.5 Dark energy, Ω_Λ

Accepting the notion of most of the matter in the Universe being in the form of some unknown exotic particle or particles, there is still a discrepancy between Ω_M= 0.4 and the curvature measurements implying that the total matter/energy density of the Universe is equal to the critical one, i.e. $\Omega_0 = 1$. There is obviously a need for an additional component, some kind of 'dark energy'. This component has to be smoothly distributed not to have been detected in measurements of matter density. Its properties are also severely constrained by structure formation, the age of the Universe, and the CMBR anisotropy. In order not to have interferred with matter domination during the period from matter-radiation equality until very recently, and thus hampering structure formation, the dark-energy component must have been much less important in the past than it is today. The simplest example of such a component would be vacuum energy described by Einstein's cosmological constant. A property of vacuum energy is that its equation of state is $p = \alpha\rho$ with $\alpha = -1$ and thus a contribution to the energy density by the vacuum energy gives a negative pressure. Inserting in the expression for the decceleration parameter q_0, we observe that a consequence of $\Omega_\Lambda > 0$ is a negative decceleration parameter an thus accelerated expansion.

An accelerated expansion should be measurable and in 1998 two groups (Perlmutter et al. and Reiss et al.) presented the first results favouring a non-zero Ω_Λ, based on measurements of apparent magnitudes vs. redshifts for type Ia supernovae (SNeIa). The results are shown in Fig.1. The method is based on the notion of SNe Ia as so-called standard candles. Since they are all the result of an explosion of a white dwarf exceeding the Chandrasekhar mass of approximately $1.4 M_\odot$, they are thought to have the same absolute luminosities. The difference in apparent (observed) luminosities is then only a function of distance which is related to redshift via Hubbles law, $v_0 = H_0 d$. If the expansion rate was constant, the measured luminosities vs. redshift should give a perfectly linear relation. The fact that distant galaxies have lower redshift and thus are moving slower than predicted by Hubble's law means that the expansion is speeding up. The first break-trough results were followed up by even more precise measurements of even more distant supernovae and the best fit to the current data (Supernova Cosmology Project, Knop et al. 2003) corresponds to a flat universe with Ω_M=0.25 and Ω_Λ=0.75.

3.6 Cosmic concordance

The results of the best values and confidence intervals of the cosmological parameters as measured by the different methods are shown grafically in Fig 1, right panel. We find that the values of the total Ω_0 define a concordance region in the Ω_M-Ω_Λ plane with Ω_Λ from the supernova measurements that is

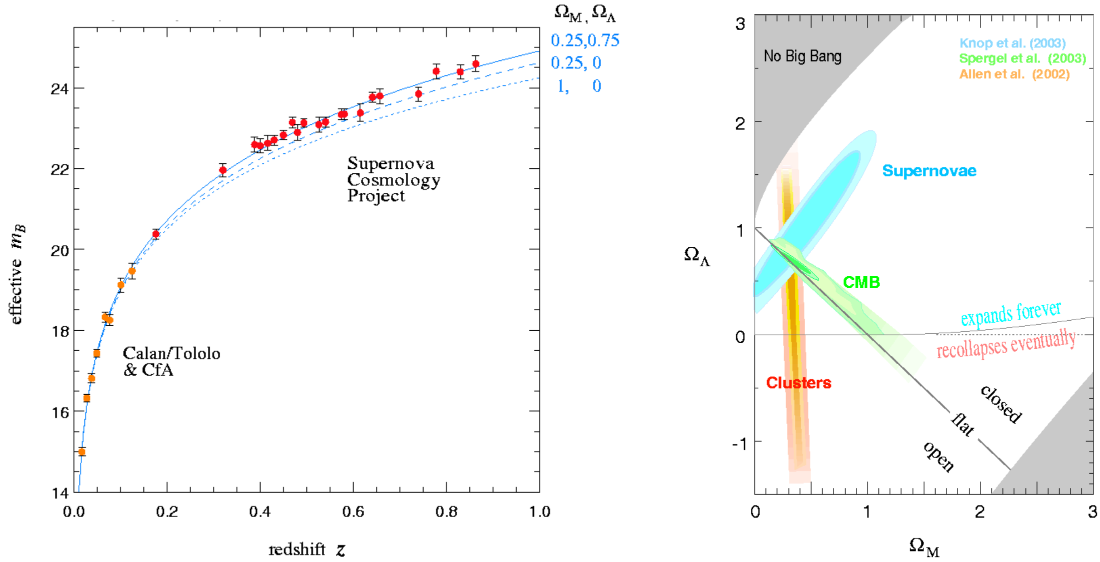

Figure 1: Figure 1. *left* Figure from the Supernova Cosmology Project (Knop et al. 2003) showing the relation between apparent magnitude and redshifts for distant supernovae. The observations favour a flat Universe with a non-zero cosmological constant and must be interpreted as an accelerated expansion. *right* Concordance in Ω_M-Ω_Λ space between results from different methods (from the Supernoova Cosmology Project).

further constrained by the results for Ω_M. This means that not only are we now able to measure these cosmological parameters with reasonable error estimates, the results from totally different experiments also seem to agree with each other. The values of $\Omega_M + \Omega_\Lambda$(or Ω_K) of approximately 1.0, $\Omega_B \approx 0.05$, $\Omega_M \approx 0.3$, and $\Omega_\Lambda \approx 0.75$ seem to imply that we live in a flat universe with a non-zero comological constant with the energy density made up of about 5 % baryonic matter, 25 % non-baryonic or dark matter and 75 % vacuum energy.

The results, however, have some strange implications. The cosmological parameters are strong functions of time. How can it be that we today happen to live in a Universe which is so close to the critical density? And what is the meaning of $\Omega_\Lambda \neq 1$? In natural units, the vacuum energy density for $\Omega_0 = 1$ equals $\pm\rho_{vac} \approx 10^{-46}$ GeV^4. This is about a factor of 10^{122} smaller than what one could expect to emerge from a quantum theory of gravity using the Planck mass as a mass scale. The smallness of the cosmological constant is a problem to both cosmology and theoretical physics. Some particle physicists believe that when the problem is understood, the answer will be exactly zero. Others have tried to resolve the issue by invoking a dynamical cosmological constant, evolving with time and the scale factor. In such a scenario, even if the true vacuum is zero, it is possible that not all fields have evolved to their state of minimum energy, but are still 'rolling' towards it. An evolving cosmological constant is sometimes referred to as 'quintessence'.

4 Summary

To summarize, recent experiments, especially perhaps the most recent measurements of the CMBR anisotropy together with the results from the distance vs. redshift measurements of distant supernovae, have provided accurate determinations of the cosmological parameters H_0, Ω_0, Ω_M, and Ω_Λ. The results between different methods of measurements also seem to coincide very well with each other thus confining the values to a small parameter space in the Ω_M-Ω_Λ plane. Not only do measured values provide observational evidence for the correctness of the Big Bang scenario. They are now also accurate enough to provide constrains on the evolution of the expansion and the ultimate fate of our Universe. Future results from the second year of the WMAP project as well as the launch of the Planck mission (in 2007), will further refine our understanding of these cosmological parameters governing the evolution of space-time.

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