Chapter 1

Electrodynamics

1.1 Ingredients of Standard Model

We think to understand matter and the fundamental interactions on the basis of three doublets of leptons

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

which do not feel strong nuclear forces and three doublets of quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

which feel these forces due to the fact that each of them appear with three so called colours.

Even though the neutrino ν_τ has not been completely identified as different from the others, both LEP and cosmological results indicate that there are three types of neutrinos. Moreover Fermilab experiments give evidence of top quark so that the three-generation scheme seems well established. In all the fermion doublers the difference of charge between the up and down particles is one, being zero for neutrinos and 2/3 for u, c and t quarks.

Apart from gravitational forces, the interactions among these spin 1/2 fermions are given by the exchange of spin 1 bosons. They are the photon γ for electromagnetic interactions, the vector bosons W^+, W^- and Z^0 for weak interactions, and eight gluons g for the strong ones. In addition, due to the fact that the theory is built on the basis of a local symmetry i.e. for transformations in each point of the space, the mechanism to give mass to those fermions and vector bosons which have it requires the existence of at least one spin-zero boson, the still undetected Higgs particle. The problem regarding the appearance of an unwanted violation of the symmetry charge conjugation $\times$ parity (CP) in strong interactions may be solved by the inclusion of another hypothetical spin-zero boson the axion.

Since the standard model is built as a generalization of the well-known electromagnetic interactions it is convenient to begin with a description of Quantum Electro Dynamics (QED) to understand the formalism of a relativistic field theory.

1.2 Quantum Electrodynamics

When we consider high energies relativity allows creation (and annihilation) of particles[(2)] so that the non-relativistic quantum mechanics which assumes constant number of particles is no longer valid.

The relativistic generalization consists in taking a classical field theory and identifying pairs of cannonical conjugated variables on which commutators are established giving therefore the quantum theory.

For the electromagnetic radiation e.g. we may use the 4-potential $A^\mu = (\varphi, \vec{A})$ through which the electric and magnetic fields are included in

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \equiv \frac{\partial}{\partial x^\mu} A_\nu - \frac{\partial}{\partial x^\nu} A_\mu \tag{1.1}$$

The Lagrangian density for radiation is taken as

$$L_R = -\frac{1}{4} F^2 \equiv -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{1.2}$$

from which the Euler-Lagrange equations give the classical Maxwell equations. One must note that, since the field definition (1.1) is invariant under a gauge transformation

$$A_\mu \to A_\mu - \partial_\mu \wedge \tag{1.3}$$

a gauge condition must be given to fix the gauge and instead of four independent components of A_μ there are only two which can be related to the two possible linear polarizations.

Fixing canonical commutators for transverse $\vec{A}$ with the conjugate momentum which turns to be $\vec{E}$ the radiation energy which can be obtained from the Lagrangian (1.2)

$$H = \frac{1}{2} \int d\,\vec{r}\,(\vec{E}^2 + \vec{B}^2) \tag{1.4}$$

is expressed as the sum of the number of photons times $\omega = |\,\vec{k}\,|$ for each possible wave vector $\vec{k}$ with the associated two polarizations.

If we wish to describe the interactions of radiation with matter in the form of electrons we must add a Lagrangian density

$$\mathcal{L}_M = \bar{\psi}\,(i\not{D} - m)\psi = \bar{\psi}\,[i\gamma^\mu(\partial_\mu + ieA_\mu) - m]\psi \tag{1.5}$$

where ψ is the 4-component spinor, γ^μ are the Dirac matrices, $\bar{\psi} = \psi^\dagger\gamma^0$ and m and e are the electron mass and charge. The so-called covariant derivative D_μ is introduced to allow the invariance of the Lagrangian under the simultaneous gauge transformation (1.3) and that of the spinor

$$\psi \to e^{ie\wedge}\psi \tag{1.6}$$

Now the Euler-Lagrange equations give Maxwell equations with a source $J^\mu = e\,\bar{\psi}\,\gamma^\mu\psi$

The quantum theory for the Dirac field is obtaine fixing anticommutators, instead of commutators, for ψ and its conjugate momentum ψ^+. In so doing the energy for matter, disregarding its interaction with radiation, is the sum of up to one electron or positron times $E = \sqrt{\vec{p}^2 + m^2}$ for each momentum $\vec{p}$ with the associated two possible polarizations of spin.

1.3 Feynman diagrams

Because of the interaction between photons and electrons included in (1.5)

$$\mathcal{L}_{int} = -eA_\mu\,\bar{\psi}\,\gamma^\mu\psi \tag{1.7}$$

the perturbation treatment shows that in a vertex represented by i.e. γ^μ two lines of electron and one of photon join. The iteration of the interaction indicates that the electron line cannot be interrupted whereas the photon one either starts or ends in a vertex.

The probability amplitude for the quantum transition from a certain number of initial particles to the final ones, each of them with definite energy-momentum, is given by all the diagrams which may be drawn joining these particles with vertices where energy-momentum is conserved.

The Feynman rules correspond to insert the above factor for every vertex, the propagator

$$\frac{i}{\not p - m + i\epsilon} = i\frac{\not p + m}{p^2 - m^2 + i\epsilon} \tag{1.8}$$

for an internal electron line, where ϵ is a small positive number indicating how the singularity must be approached, and the propagator

$$-i\frac{g_{\mu\nu}}{k^2 + i\epsilon} + \text{gauge terms} \tag{1.9}$$

for a photon line of momentum k^μ.

If there is a closed loop, the energy-momentum circulating along it cannot be fixed and the integration on $\frac{d^4p}{(2\pi)^4}$ must be performed. If the loop is exclusively fermionic a - sign appears due to the anticommutation character of these fields and the trace over the resulting 4×4 matrix must be taken.

Building in this way the amplitude M a general expression may be given for the cross-section of $2 \to n$ particles

$$d\sigma = \frac{1}{\mid \vec{v_1} - \vec{v_2} \mid} \frac{1}{2\omega_{p_1}} \frac{1}{2\omega_{p_2}} |M|^2 \frac{d\,\vec{k}_1}{2\omega_1 (2\pi)^3} \cdots \frac{d\,\vec{k}_n}{2\omega_n (2\pi)^3}$$
$$\cdot (2\pi)^4 \delta^4 (p_1 + p_2 - \sum_{i=1}^{n} k_i)\ S \tag{1.10}$$

for bosons, where $\vec{v_1}$ and $\vec{v_2}$ are velocities of incident collinear particles and S includes 1/N! if there are N identical particles among the final ones. For fermions $\frac{1}{2\omega}$ is replaced by $\frac{m}{E}$.

Analogously the life-time τ of a boson of mass M decaying into n bosonic particles is given by

$$\frac{1}{\tau} = \frac{1}{2M} \int |M|^2 \frac{d\,\bar{k}_1}{2\omega_1 (2\pi)^3} \cdots \frac{d\,\vec{k}_n}{2\omega_n (2\pi)^3}$$
$$\cdot (2\pi)^4 \delta^4 (p - \sum_{i=1}^{n} k_i)\ S \tag{1.11}$$

where $\frac{1}{2M}$ is omitted if the decaying particle is a fermion and above quoted replacements apply for final fermions. (1.10) and (1.11) are valid for any interaction included in M with rules similar to those of QED.

1.4 Radiative corrections

The perturbative calculations without loops, the so called tree diagrams, give useful predictions on cross-sections e.g. of Compton scattering δe.

But certain loop corrections produce mathematical divergences which must be cured leaving finite physical effects.

One of these is the vacuum polarization, i.e. the electron loop correction to the photon propagator

$$\bar{w}^{\rho\nu} = -(ie)^2 \int \frac{d^4p}{(2\pi)^4} Tr(\gamma^\rho \frac{i}{\not p - m + i\epsilon} \gamma^\nu \frac{i}{\not p - \not k - m + i\epsilon}) \tag{1.12}$$

Since the integral is devergent we regularize it adding a similar contribution for a heavy fermion of mass $\wedge$ and with undertermined coefficient c

$$\bar{w}^{\rho\nu} \rightarrow \bar{w}^{\rho\nu} (k, m) + c\, \bar{w}^{\rho\nu} (k, \wedge) \tag{1.13}$$

Using the Feynman parametrization

$$\frac{i}{p^2 - m^2 + i\epsilon} = \int_0^\infty d\alpha\ e^{i\alpha(p^2 - m^2 + i\epsilon)} \tag{1.14}$$

the now convergent integral (for finite $\wedge$) may be expressed as $\bar{w}_{\rho\nu} = -i(g_{\rho\nu}k^2 - k_\rho k_\nu)\,\bar{w}$ where

$$\begin{aligned} \bar{w}\,(k^2, m, \wedge) = \frac{2\alpha}{\pi} \int_0^1 \int_0^1 & d\alpha_1 d\alpha_2 \delta(1 - \alpha_1 - \alpha_2)\alpha_1\alpha_2 \\ & \cdot \int_0^\infty \frac{d\rho}{\rho} [e^{i\rho(-m^2 + \alpha_1\alpha_2 k^2)} + c e^{i\rho(-\wedge^2)}] \end{aligned} \tag{1.15}$$

with $\alpha = e^2/4\pi$. To avoid the logarithmic divergence for $\rho \to 0$ $c = -1$, being

$$\bar{w}\,(k^2, m, \wedge) = \frac{\alpha}{3\pi} \ell n \frac{\wedge^2}{m^2} + \frac{\alpha}{3\pi} k^2 \int_{4m^2}^\infty \frac{dk'^2}{k'^2} \frac{1}{k'^2 - k^2} (1 - \frac{4m^2}{k'^2})^{1/2} (1 + \frac{2m^2}{k'^2}) \tag{1.16}$$

Considering the iteration of this correction to the propagator

$$iG_{\rho\nu}(k) = \frac{g_{\rho\nu}}{k^2[1+\bar{w}(k^2)]} + gauge\ terms \tag{1.17}$$

Since the physical mass of the photon is given by the pole of the propagator it is still zero, but putting two static well separated bare charges e_0 at each end of the propagator, the dressed charge will be

$$e^2 = \frac{e_0^2}{1+\bar{w}(0)} \equiv Z_3 e_0^2 \tag{1.18}$$

An equivalent way to interpret this result is to think that the charge is the physical one e at each order of perturbation but a quantum counteterm appears

$$\delta\mathcal{L}_R = -\frac{1}{4}(Z_3 - 1)F^2 \tag{1.19}$$

whose contribution, added to the loop correction, gives a renormalized propagator

$$iG^R_{\rho\nu} = \frac{g_{\rho\nu}}{k^2[1+\bar{w}(k^2)-\bar{w}(0)]} + \text{gauge terms} \tag{1.20}$$

Since for small k^2 $\bar{w}(k^2)-\bar{\omega}(0) \simeq \frac{\alpha}{15\pi}\frac{k^2}{m^2}$, for static situations $k^2 = -\bar{k}^2$ and the Coulomb law is modified

$$\frac{e^2}{\vec{k}^2} \longrightarrow \frac{e^2}{\vec{k}^2}(1 + \frac{\alpha}{15\pi}\frac{\vec{k}^2}{m^2}) \tag{1.21}$$

indicating the first example of running coupling constant, i.e. the charge increaes for increasing momentum as the cloud due to the polarized vacuum surrounding a charge is penetrated.

Analogously, the correction to the electron propagator due to a loop given by a photon of momentum k emitted and reabsorbed by the electron line

$$-i\Sigma(p) = (-ie)^2 \int \frac{d^4k}{(2\pi)^4}(\frac{-ig_{\rho\sigma}}{k^2+i\epsilon})\gamma^\rho \frac{i}{\not{p}-\not{k}-m+i\epsilon}\gamma^\sigma \tag{1.22}$$

has a divergence which may be regularized subtracting a similar term with a large photon mass $\wedge$. Iterating this correction Σ appears added to m. There is a twofold effect the position of the pole is changed and also its residue, i.e.

$$\Sigma(\not{p},\wedge)|_{\not{p}\sim m} = \delta m(\wedge) - (Z_2^{-1} - 1)(\not{p} - m) \tag{1.23}$$

Therefore, with the idea of keeping fixed the values of masses and residues at each perturbative order we must add two couterterms

$$\delta\mathcal{L}_D = \delta m \,\bar{\psi}\, \psi + (Z_2 - 1)\,\bar{\psi}\,(i \not{\partial} - m)\psi \tag{1.24}$$

The last divergent loop corresponds to the vertex function where an internal photon is exchanged between the two electron lines

$$-ie\Gamma^\mu(p', p) = (-ie)^3 \int \frac{d^4k}{(2\pi)^4} (\frac{-ig_{\rho\sigma}}{k^2 + i\epsilon})\gamma^\sigma \frac{i}{\not{p}' - \not{k} - m + i\epsilon}\gamma^\mu \frac{i}{\not{p} - \not{k} - m + i\epsilon}\gamma^\rho \tag{1.25}$$

which is related to the correction to the electron propagator by the Ward identity

$$\Gamma_\mu(p, p) = -\frac{\partial}{\partial p^\mu}\Sigma(p) \tag{1.26}$$

To prove this essential identity it is necessary to have a gauge invariant regularization. Defining the renormalization constant for the vertex

$$\Gamma^\rho(p', p)|_{p' \sim p} = \gamma^\mu(Z_1^{-1} - 1) \tag{1.27}$$

(1.23) and (1.26) give $Z_1 = Z_2$, and the addition of the counterterm

$$\delta\mathcal{L}_{int} = -e(Z_1 - 1)\,\bar{\psi}\not{A}\psi \tag{1.28}$$

cancels the divergence.

It is important that this finite number of counterterms is sufficient to make the results finite at any perturbation order.

1.5 Interaction with external fields

The loop corrections not only can be made finite by the renormalization procedure but also give extremely accurate predictions for atomic physics.

For the interaction with classical electric and magnetic fields, these corrections introduce the change

$$eA^\mu_c\gamma_\mu \to eA^\mu_c(\gamma_\mu + \Gamma_\mu + \bar{\omega}_{\mu\nu}\, G^{\nu\sigma}\gamma_\sigma) \tag{1.29}$$

The interaction of the spin with a magnetic field receives an additional contribution from the vertex function to give

$$-\vec{B}\cdot\vec{\mu} = -\vec{B}\cdot\frac{e}{2m}\left(1+\frac{\alpha}{2\pi}\right)\,2\int d\,\vec{r}\,\bar{\psi}\,\frac{\vec{\sigma}}{2}\psi \tag{1.30}$$

where $\frac{\alpha}{2\pi}$ corresponds to the one-loop anomalous magnetic moment.

In analogous way for the electrostatic interaction of an electron with the nucleus, the modification of the Coulomb law (1.21) due to vacuum polarization gives a decrease of energy for the state $n = 2 \quad \ell = 0$ of 27 MHz because the charge appears larger. But the vertex function gives an increase for the same state of more than 1000 MHz because the fluctuations of the radiation field on the average separate the electron from the nucleus in a s state. Both effects reproduce very accurately the Lamb shift.

Chapter 2

Electroweak Theory

2.1 Symmetry breaking

Classically, for $\mathcal{L}$ invariant under a transformation of the field φ characterized by a parameter $\delta\alpha$, the so-called Noether current

$$J_\mu = \frac{\partial \mathcal{L}}{\partial(\partial^\mu \varphi)} \frac{\delta\varphi}{\delta\alpha} \tag{2.1}$$

is conserved, i.e. $\partial_\mu J^\mu = 0$.

There are three ways to break a symmetry explicit, spontaneous and anomalous (quantum)[3], which can be described in the example of σ - model corresponding to the interaction of a pion and a nucleon

$$\begin{aligned}\mathcal{L} = & \bar{\psi}\,[i\,\not{\partial} + g(\sigma + i\pi\gamma_5)]\psi + \frac{1}{2}[(\partial\pi)^2 + (\partial\sigma)^2] \\ & -\frac{\mu^2}{2}(\sigma^2 + \pi^2) - \frac{\lambda}{4}(\sigma^2 + \pi^2)^2 + c\,\sigma\end{aligned} \tag{2.2}$$

where apart from the pseudoscalar field π there is a scalar one σ.

Writing

$$\psi = \psi_L + \psi_R = \frac{1}{2}(1 - \gamma_5)\psi + \frac{1}{2}(1 + \gamma_5)\psi \tag{2.3}$$

for $c = 0$, $\mathcal{L}$ is invariant under the $U(1) \times U(1)$ global chiral transformation

$$\psi_R \to U\psi_R, \psi_L \to V\psi_L, \sigma + i\pi \to V(\sigma + i\pi)U^{-1} \tag{2.4}$$

For $c \neq 0$ $\mathcal{L}$ is still invariant under the so called vector transformation $U = V$, but not under axial one $U = V^{-1}$. In fact the axial current is no longer conserved

$$\partial_\mu J^\mu_A = -c\pi \tag{2.5}$$

indicating an explicit breaking of $U(1)_A$ and the particles π and σ are no longer degenerate.

If $\mu^2 < 0$, $\lambda > 0$, with $c = 0$, choosing the potential minimum $\pi = 0$, $\sigma^2 = v^2 = -\mu^2/\lambda$ and expanding the fields around it we obtain a mass for the nucleon $m_N = -gv$ and

$$m^2_\sigma = -2\mu^2 \quad m^2_\pi = 0 \tag{2.6}$$

so that the pion is interpreted as a Goldstone boson.

This is an example of spontaneous breaking due to the fact that the vacuum is not invariant under $U(1)_A$ appearing a massless particle in correspondence to it.

(2.2) may be used to describe the decay $\pi \longrightarrow 2\gamma$ taking the pion coupled to a nucleon triangle emerging two photons from the other vertices. The resulting lifetime is $\tau^{-1} = 7eV$ in agreement with experiment. However calculating the axial current coupled to the triangle instead of the pion and using (2.5) for the small experimental value of c, τ^{-1} would appear to be almost zero. This occurs if this triangle were finite, but it is divergent and its regularization subtracting the loop in which a heavy fermion circulates does not respect the chiral invariance. As a consequence of this quantum effect (2.5) is modified to

$$\partial_\mu J^\mu_A = -c\pi - \frac{\alpha}{8\pi}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \tag{2.7}$$

where the second source of axial current is called "anomaly" and explains the lifetime of the pion.

If the spontaneous breaking applies to a local symmetry, the so-called Higgs mechanism appears. Taking radiation coupled to a complex scalar field

$$\mathcal{L} = -\frac{1}{4}F^2 + (D_\mu\varphi)^* D^\mu\varphi - V(\varphi^*\varphi) \tag{2.8}$$

where $V = \lambda(\varphi^*\varphi - \frac{v^2}{2})^2$
we may take φ around the potential minimum absorbing the phase at each point by a gauge transformation, i.e.

$$\varphi(x) = \frac{v + \rho(x)}{\sqrt{2}} \tag{2.9}$$

In this way from the covariant derivative D_μ the photon acquires a mass, eating the would be Goldstone boson,

$$m_A^2 = e^2 v^2 \tag{2.10}$$

and only one scalar neutral particle remains whose mass from the expansion of V is

$$m_\rho^2 = 2\lambda v^2 \tag{2.11}$$

2.2 Glashow - Salam - Weinberg model

At first sight electromagnetic interactions characterized by $\alpha = 1/137.0359840\ (51)$ seems to be very different from weak interactions where four left fermions (2.3) interact with the Fermi coupling $G_F/\sqrt{2}$, $G_F = (1.16637 \pm 0.00002)\ 10^{-5} GeV^{-2}$.

That the latter could not be the fundamental theory was indicated by the fact that a renormalization procedure similar to that described for QED in 1.4 was impossible, being necessary that the coupling constant is dimensionless.

It is clear that if we think that, in analogy with the exchange of photon in QED, weak interactions are mediated by a vector boson of mass M not far from 100 GeV, the effective Fermi coupling is compatible with a fundamental coupling of order α. But, since the massive vector boson field satisfies the Lorentz condition, its propagator is

$$G_{\mu\nu}(q) = -i \frac{g_{\mu\nu} - q_\mu q_\nu / M^2}{q^2 - M^2} \tag{2.12}$$

whose second term has a high momentum behaviour that destroys the renormalizability.

Therefore we rely on a gauge invariant theory, to ensure renormalizability, which involves four transformations to account for electromagnetic, charged and the predicted neutral weak interactions. Since weak interactions require two charged and one neutral massive vector boson we use the spontaneous symmetry breaking of three out of four transformations with the Higgs mechanism 2.1 including a couplex doublet scalar field one of whose components survives as the neutral Higgs particle.

Since only the left component of fermions takes part in weak interactions we consider a gauge symmetry SU(2), with three generators, affecting only this chirality, and a gauge symmetry U(1) acting non trivially on both chiralities to give room to electromagnetic interactions. Due to the fact that the photon is massless we look for the symmetry breaking

$$SU(2)_L \times U(1) \to U(1)_{em} \tag{2.13}$$

Generalizing the elements described in 1.2 and 2.1 we formulate the electroweak theory by

$$\mathcal{L}_{EW} = \mathcal{L}_{FG} + \mathcal{L}_{GH} + \mathcal{L}_{HF} \tag{2.14}$$

where the first fermion-gauge term contains the weak and electromagnetic interactions for leptons and quarks, the second gauge-Higgs contribution produces the symmetry breaking, and the last Higgs-fermion part gives mass to the originally massless fermions.

For the first generation of fermions

$$\begin{aligned} \mathcal{L}_{FG} = (\bar{u} \quad \bar{d})_L i \not{D} \begin{pmatrix} u \\ d \end{pmatrix}_L + \bar{u}_R \, i \not{D} u_R + \bar{d}_R \, i \not{D} d_R \\ +(\bar{\nu}_e \; e)_L \not{D} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L + \bar{e}_R \, i \not{D} e_R - \frac{1}{4} B^{\nu\mu} B_{\mu\nu} - \frac{1}{4} W_i^{\mu\nu} W^i_{\mu\nu} \end{aligned} \tag{2.15}$$

where the U(1) quantum numbers follow from the definition

$$Q_{em} = T_3 + Y \tag{2.16}$$

so that, being $T_3 = \pm\frac{1}{2}$ for the up/down component of the doublet, the hyperchange $Y = \frac{1}{6}, \frac{2}{3}, -\frac{1}{3}, -\frac{1}{2}, -1$ for the quark doublet, u_R, d_R, lepton doublet and e_R respectively to reproduce the experimental charges. Note that there is no need of right neutrino. Thus the covariant derivatives are

$$\begin{aligned} D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L = (\partial_\mu + ig'\frac{1}{6}B_\mu + ig\frac{\tau_i}{2}W^i_\mu) \begin{pmatrix} u \\ d \end{pmatrix}_L \\ D_\mu u_R = (\partial_\mu + ig'\frac{2}{3}B_\mu)u_R \\ D_\mu d_R = (\partial_\mu + ig'(-\frac{1}{3})B_\mu)d_R \\ D_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = (\partial_\mu + ig'(-\frac{1}{2})B_\mu + ig\frac{\tau_i}{2}W^i_\mu) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \\ D_\mu e_R = (\partial_\mu + ig'(-1)B_\mu)e_R \end{aligned} \tag{2.17}$$

where there are two coupling constants g' and g for $U(1)$ and $SU(2)_L$ respectively, with the corresponding gauge potentials B and W^i,and τ_i are Pauli matrices $i = 1, 2, 3$.

In terms of

$$
\begin{aligned}
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk} W^j_\mu W^k_\nu
\end{aligned}
\tag{2.18}
$$

the two last terms of (2.15) are separately gauge invariant as are each of the other ones under transformations which are obvious generalizations of (1.3) and (1.16).

$\mathcal{L}_{FG}$ contains the interaction between fermion and gauge fields

$$
\mathcal{L}_{\text{int}} = -g' J^\mu B_\mu - g J^\mu_i W^i_\mu \tag{2.19}
$$

which may be rewritten in a way that identifies the electromagnetic potentials as the one which interacts only with the electromagnetic current

$$
J^\mu_{\text{em}} = J^\mu_e + J^\mu \tag{2.20}
$$

Defining moreover the charged currents which appear in the Fermi interaction

$$
J^\mu_\pm = 2(J^\mu_1 \mp i J^\mu_2) \tag{2.21}
$$

which must be coupled to the charged vector boson fields

$$
W^\mu_\pm = \frac{1}{\sqrt{2}}(W^\mu_1 \mp i W^\mu_2) \tag{2.22}
$$

the combination of B and W_3

$$
A^\mu = B^\mu cos\theta_W + W^\mu_3 sin\theta_W, Z^\mu = W^\mu_3 cos\theta_W - B^\mu sin\theta_W \tag{2.23}
$$

allows to rewrite (2.19) as

$$
\mathcal{L}_{\text{int}} = -e J^\mu_{\text{em}} A_\mu - \frac{e}{2\sqrt{2} sin\theta_W}(J^\mu_+ W^-_\mu + J^\mu_- W^+_\mu) - \frac{e}{sin2\theta_W} J^\mu_{NC} Z_\mu \tag{2.24}
$$

where the positive electric charge must be taken as

$$
e = g' cos\theta_W = g sin\theta_W \tag{2.25}
$$

and the new interaction with Z is given by the neutral current

$$J^{mu}_{NC} = 2(J^\mu_3 - sin^2\theta_W J^\mu_{em}) \tag{2.26}$$

Note that the contribution of electrons to $J^\mu_{\rm em}$ is $-\bar{e}\,\gamma^\mu e$ as it must be for (2.24) to be correct. To relate this model to the very well measured Fermi constant we note that for $q^2 << M_Z^2, M_W^2$ from (2.12)

$$G^W_{\mu\nu} \approx i\frac{g_{\mu\nu}}{M_W^2} \quad , \quad G^Z_{\mu\nu} \approx i\frac{g_{\mu\nu}}{M_Z^2} \tag{2.27}$$

so that we may define an effective four-fermion interaction

$$i\mathcal{L}_{eff} = \frac{1}{2!}(i\mathcal{L}_{\rm int})(i\mathcal{L}_{\rm int}) \tag{2.28}$$

giving

$$\mathcal{L}_{eff} = -(\frac{e}{2\sqrt{2}sin\theta_W})^2\frac{1}{M_W^2}J^\mu_+J^-_\mu - (\frac{e}{\sqrt{2}sin2\theta_W})^2\frac{1}{M_Z^2}J^\mu_{NC}J^{NC}_\mu \tag{2.29}$$

with the identification

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 sin^2\theta_W} \tag{2.30}$$

Defining

$$\rho = \frac{M_W^2}{M_Z^2 cos^2\theta_W} \tag{2.31}$$

which is related to the particular form of symmetry breaking we may express

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}}(J^\mu_+J^-_\mu + \rho J^\mu_{NC}J^{NC}_\mu \tag{2.32}$$

To generate the masses of $W^\pm$ and Z the minimal choice is to introduce a complex doublet of scalars with $Y = -\frac{1}{2}$

$$\mathcal{L}_{GH} = (D_\mu\varphi)^\dagger D^\mu\varphi - V(\varphi^\dagger\varphi) \tag{2.33}$$

so that

$$D_\mu\varphi = (\partial_\mu + ig'(-\frac{1}{2})B_\mu + ig\frac{\tau_i}{2}W^i_\mu)\varphi \tag{2.34}$$

and a potential as that of (2.8).

Since the up component of φ is neutral and the down one negative, if we choose the vacuum

$$\varphi_{\mathrm{VAC}} = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \tag{2.35}$$

it will be invariant only under $U(1)_{em}$ so that the breaking (2.13) is fulfilled. Inserting (2.35) in (2.33) the mass term is

$$\mathcal{L}_{\mathrm{mass}} = (\frac{gv}{2})^2 W_\mu^+ W_-^\mu + \frac{1}{2}(W_3^\mu \; B^\mu) \begin{pmatrix} \frac{g^2v^2}{4} & \frac{-gg'}{4}v^2 \\ \frac{-gg'}{4}v^2 & \frac{g'^2v^2}{4} \end{pmatrix} \begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix}$$

so that, diagonalizing the 2×2 matrix the mass eigenvalues are

$$M_A^2 = 0, M_W^2 = (\frac{gv}{2})^2, M_z^2 = \frac{g^2 + g'^2}{4} v^2 \tag{2.36}$$

and according to (2.31) $\rho = 1$ which is characteristic of symmetry breaking by doublet of Higgs.

Expanding φ around the vacuum

$$\varphi = \begin{pmatrix} \frac{v+H(x)}{\sqrt{2}} \\ 0 \end{pmatrix} \tag{2.37}$$

which is general because of the local SU(2) invariance

$$\mathcal{L}_{GH} = \frac{1}{2}\partial^\mu H \partial_\mu H - \lambda(vH + \frac{1}{2}H^2)^2 + \frac{g^2}{4}(v+H)^2 W_\mu^+ W_-^\mu \tag{2.38}$$

$$+\frac{1}{8}(g^2 + g'^2)(v+H)^2 Z^\mu Z_\mu \tag{2.39}$$

so that the mass of the Higgs particle is

$$M_H^2 = 2\lambda v^2 \tag{2.40}$$

and its couplings to W and Z are proportional to their masses.

Going now to the last part of (2.14), fermion masses may be generated with the help of the doublet φ and its charge conjugate

$$\tilde{\varphi} = i\tau_2 \varphi^* = \begin{pmatrix} \varphi^+ \\ -\varphi^{0*} \end{pmatrix} \tag{2.41}$$

The former, coupled in invariant way with the quark doublet and the up right singlet, gives mass to u c and t through the breaking (2.35), and the latter coupled to fermion doublets and the corresponding down right singlets gives mass to d s b e μ τ. The most general invariant couplings for quark and lepton doublets $Q_{i_L} L_{i_L}$ and singlets $u_{i_R} d_{i_R} e_{i_R}$, where i = 1,2,3 denotes generations, is

$$\mathcal{L}_{HF} = -(h^u_{ij}\ \bar{Q}_{i_L}\ \varphi u_{j_R} - h^d_{ij}\ \bar{Q}_{i_L} \tilde{\bar{\varphi}}\ d_{j_R} - h^e_{i_j}\ \bar{L}_{i_L} \bar{\varphi}\ e_{j_R}) + h.e. \tag{2.42}$$

The mass matrices in terms of the arbitrary constants h

$$M^f_{ij} = h^f_{ij} \frac{v}{\sqrt{2}} \quad , \quad f = u, d, e \tag{2.43}$$

are not diagonal and must be diagonalized by bi-unitary transformations on generations of the fields

$$(\psi_f)_L \to U^f_L (\psi_f)_L \quad , \quad (\psi_f)_R \to U^f_R (\psi_f)_R \tag{2.44}$$

such that for each f

$$(U^f_L)^\dagger M^f U^f_R = M^f_{\text{diag}} \tag{2.45}$$

This change of basis has no effect on leptonic currents, because in the model $m_\nu = 0$ and we may always transform neutrinos as ψ_e, and on neutral currents because they are flavour diagonal. But it does affect hadronic charged currents

$$\begin{aligned} J^\mu_- = 2(\bar{u}_L \bar{c}_L \bar{t}_L)\gamma^\mu\ 1 \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} &\to 2(\bar{\psi}_u)_L \gamma^\mu (U^u_L)^\dagger U^d_L (\psi_d)_L \\ &= 2(\bar{\psi}_u)_L \gamma^\mu\ \tilde{C}\ (\psi_d)_L \end{aligned} \tag{2.46}$$

For n generations $\tilde{C}$ is a unitary matrix with $\frac{n(n-1)}{2}$ real angles and $\frac{n(n+1)}{2}$ phases. But we may redefine quark fields $\psi \to e^{i\alpha}\psi$ for both L and R parts without changing M_{diag}. Since one overall phase has no meaning we eliminate $2n - 1$ phases leaving $(n-1)(n-2)/2$ physical ones. For $n = 3$ $\tilde{C}$ is the Kobayaski-Maskawa matrix which has 3 angles θ_i and 1 phase δ which is responsible for CP violation.

2.3 Scattering of neutrinos

For a rapid determination of the parameters of the GSW model, apart from those related to fermion masses, we may use the experimental values[4]

$$M_z = 91.187 \pm 0.007 GeV, M_w = 80.10 \pm 0.27 GeV \tag{2.47}$$

from LEP and CDF/UA2 respectively.

Taking $\rho = 1$ from Higgs doublet, (2.31) gives $sin^2\theta_w = 0.2245 \pm 0.006$, so that using (2.37) together with the relation (2.25) with the electric charge we obtain $v \simeq 246$ GeV. To determine λ according to (2.40) the Higgs mass should be found experimentally.

It is important to see the consistency of the model with different pieces of information, neutrinos scattering among them. The use of nucleons as target is more convenient than electrons, even though their structure is more complicated, because the cross section is around 10^3 times higher.

The deep inelastic scattering of neutrinos on nucleons corresponding to the exchange of a vector boson with large q^2 may be described in the so called zeroth order QCD as interacting with free quarks, or partons, because its coupling decreases with increasing momentum. However the structure function, which gives the probability of having a quark with a fraction of the momentum of the nucleon, cannot be calculated from first principles.

It is lucky that with isoscaler targets, i.e. the average of proton and neutron, the dependence on the structure functions of neutral current and charged current cross sections is the same. Therefore, calculating the effective lepton-quark interaction (2.32) since $q^2 << M_w^2, M_z^2$ the ratio of total cross-sections for $\nu N \to \nu X$ and $\nu N \to eX$ is

$$R_\nu = \frac{\sigma_{NC}(\nu N)}{\sigma_{cc}(\nu N)} = \rho^2 (\frac{1}{2} - sin^2\theta_W + \frac{20}{27} sin^4\theta_W) \tag{2.48}$$

whereas for antineutrinos

$$R_{\bar{\nu}} = \frac{\sigma_{NC}(\bar{\nu}\ N)}{\sigma_{cc}(\bar{\nu}\ N)} = \rho^2 (\frac{1}{2} - sin^2\theta_W + \frac{20}{9} sin^4\theta_W) \tag{2.49}$$

Taking $\rho = 1$ and $\sin^2\theta_W = 0.23$ (2.48) and (2.49) give $R_\nu = 0.31$ and $R_{\bar{\nu}} = 0.39$ to be compared with the experimental values $R_\nu \simeq 0.31$ and $R_{\bar{\nu}} \simeq 0.37$.

The purely leptonic scattering $\nu_\mu e \rightarrow \nu_\mu e$ and $\bar{\nu}_\mu \; e \rightarrow \bar{\nu}_\mu \; e$ is theoretically simpler because there are no partonic hypothesis. Considering the Z exchange in the effective way (2.32) the ratio of both cross-sections is independent on ρ

$$R = \frac{\sigma_{\nu_\mu e}}{\sigma_{\bar{\nu}_\mu e}} = \frac{3 - 12sin^2\theta_W + 16sin^4\theta_W}{1 - 4sin^2\theta_W + 16sin^4\theta_W} \tag{2.50}$$

Recent experimental results of CHARM II with improved statistics are completey consistent with $\sin^2\theta_W = 0.23$.

2.4 Asymmetries

Considering the scattering $e^+e^- \rightarrow \mu^+\mu^-$ apart from the contribution of an intermediate γ there is also the neutral current Z intermediate diagram. Whereas for the photon one must consider its propagator, for the Z one may take the effective coupling (2.32) if the energy is not too large.

When one performs the square of the amplitude, the interference term of both diagrams gives a contribution to the differential cross-section proportional to cos θ between e$^-$ and μ^- in the centrum of mass frame.

Therefore we may define the forward-backward assymetry

$$A_{A-B} = \frac{\int_0^1 dcos\theta \frac{d\sigma}{dcos\theta} - \int_{-1}^0 dcos\theta \frac{d\sigma}{dcos\theta}}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \tag{2.51}$$

The GSW model predicts, for s square of total e.m. energy,

$$A_{F-B} = -\frac{3G_F\rho}{16\pi\sqrt{2}\alpha} s \quad , \quad s << M_z^2 \tag{2.52}$$

For PETRA energy $\sqrt{s} \sim 40$ GeV the experimental result is $\sim -10\%$ in agreement with (2.52) even though the fit prefers the inclusion of the propagator effect $M_Z^2/(M_Z^2 - s)$ with $M_Z \sim 90$ GeV. For LEP energy $\sqrt{s} \sim M_Z$ the Z width must be also included in a Breit-Wigner formula and the agreement with the standard model is again good.

Other assymmetries which are analized are the polarization assymmetry of the final particle, e.g. τ^- in $e^-e^+ \rightarrow \tau^-\tau^+$

$$P_\tau(cos\theta) = \frac{d\sigma_R(cos\theta) - d\sigma_L(cos\theta)}{d\sigma_R(cos\theta) + d\sigma_L(cos\theta)} \tag{2.53}$$

and the left-right assymmetry for polarized beams

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \tag{2.54}$$

where σ_L and σ_R are cross sections for left-handed and right-handed electrons.

2.5 Parity violation in atoms

We consider again, not only the electromagnetic interaction of electron with the nucleus, but also the weak neutral current one due to Z exchange. As a rough estimation since the weak amplitude $A_{\text{weak}} \sim G_F$ and the e.m. one is $A_{\text{e.m.}} \sim \frac{e^2}{q^2}$, the expected effect is

$$\epsilon \sim \frac{G_F}{e^2} q^2 \sim \frac{G_F}{e^2} \frac{1}{(r^2)_{\text{atom}}} \sim \frac{G_F}{e^2} (m_e \alpha)^2 \sim 10^{-14} \tag{2.55}$$

which is impossible to measure. But for heavy atoms there is an additional (nucleus charge)^3 factor which, e.g. for Bi, enhances the effect $\epsilon \sim 10^{-8}$ which is observable.

The detailed calculation for the neutral current electron-nucleus interaction involves the effective Lagrangian (2.32). We take the nucleus as static, dropping terms which mix its strong and weak Dirac components, and concentrated in the origin where we fix all the necessary quarks u and d. For the electron we use the non-relativistic approximation to relate the weak to the strong Dirac component. The resulting interaction Hamiltonian is

$$H_{PV} = \sqrt{2} G_F \rho \; Q_W \frac{\vec{\sigma} \cdot \vec{p_e}}{m_e} \delta^3(x) \tag{2.56}$$

where $Q_W \simeq -\frac{1}{2} N$ with N number of neutrons. The enchancement cubic factor quoted above comes from Q_W, p_e and $\delta^3(x)$ through the wave-function at the origin when H_{PV} is used in a non-relativistic perturbative calculation. The parity violation effect is evident in the electron spin and momentum scalar product.

The agreement with experimental results is good through both calculations and experiments in atoms are difficult.

Chapter 3

Radiative Test of Electroweak Theory

3.1 Radiative corrections

We have so far described the general agreement of GSW model at tree level, i.e. without loop corrections, with the experimental results.

The first evidence of the need of radiative corrections is offered by (2.30) because if one inserts in it the experimental weak parameters one obtains $e^2/4\pi \simeq 1/128$ instead of the expected $1/137$. The explanation of this discrepancy is that the latter value corresponds to low energy, whereas the vacuum polarization correction to the photon propagator (1.20) for $k^2 >> m^2$ is $\bar{\omega}\ (k^2) - \bar{\omega}\ (0) \simeq -\frac{\alpha}{3\pi}\ell n\frac{k^2}{m^2}$ so that

$$\frac{\alpha}{k^2} \rightarrow \frac{1}{k^2}\frac{\alpha}{1-\frac{\alpha}{3\pi}\ell n\frac{k^2}{m^2}} = \frac{\alpha(k^2)}{k^2} \tag{3.1}$$

If one takes $k^2 \approx M_z^2$ which is the scale of validity of electroweak results (3.1) gives $\alpha(M_z^2) \simeq 1/128$.

Following the example of QED, the standard model which is a gauge theory and therefore renormalizable may interpret the infinities by the replacement of bare parameters by dressed ones

$$\{g_0, g_0', v_0, \lambda_0, h_0^{ij}\} \ \rightarrow\ \{g, g', v, \lambda, h^{ij}\} \tag{3.2}$$

Instead of (3.2) a more physical set of parameters is

$$\{M_W, M_Z, M_H, m_{fi}, e^2, \theta_i, \delta\}$$

but at present since M_W is not very precisely determined it is better to replace it by the more accurate G_F.

For normal applications it is sufficient a reduced set

$$\{\ \}_{\text{standard}} \;=\; \{G_F, M_Z, M_H, m_t, e^2\}$$

The Weinberg angle, which is not included in this set, was defined at tree level in different ways

$$e_0 = g_0 sin\theta_W^0 = g_0' \; cos\ \theta_W^0 \quad , \quad sin^2\theta_W^0 = 1 - \frac{M_W^{0^2}}{M_Z^{0^2}}$$

$$\frac{G_F^0}{\sqrt{2}} = \frac{e_0^2}{8sin^2\theta_W^0 M_Z^{0^2}} \quad , \quad J_{NC}^\mu = 2(J_{3_L}^\mu - sin^2\theta_W^0 J_{em}^\mu) \tag{3.3}$$

Once we choose a relation to define the renormalized θ_W all the other will have corrections. The Sirlin choice is

$$sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \tag{3.4}$$

It is proved that G_F for μ decay is not renormalized by photons i.e. $G(M_W^2) = G_F$ so that

$$\frac{G_F}{\sqrt{2}} = \frac{e^2(M_W^2)}{8sin^2\theta_W M_W^2}$$

which may alternatively be written as

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2sin^2\theta_W M_W^2} \quad \frac{1}{1 - \triangle_r} \tag{3.5}$$

If $\triangle r$ were determined exclusively in terms of α, (3.5) together with (3.4) would allow to define θ_W in terms of the known parameters of the standard set G_F, M_Z and e^2. But $\triangle r$ includes a dependence on m_t and a smaller one on M_H.

The deep inelastis scattering of neutrinos corresponds to an energy $\mu^2 \sim 100GeV^2 << M_Z^2$ so that one needs $sin^2\theta_W(\mu) = K(\mu)sin^2\theta_W$ and

$$J_{NC}^\mu = 2(J_{3L}^\mu - K(\mu)sin^2\theta_W J_{em}^\mu) \tag{3.6}$$

The photonic corrections to the neutral current give

$$K(\mu) = 1 + \frac{\alpha}{\pi}\ell n\frac{M_W^2}{\mu^2} \tag{3.7}$$

but also in this case there is a contribution of m_t to $K(\mu)$.

3.2 Dependence on top mass

If we start from the effective interaction (2.32) where $\rho = 1$ in the minimal standard model, due to the fact that this parameter comes from the ratio of neutral to charged weak couplings which are modified by the mass corrections in the propagators of W and Z, the corrected parameter must be taken as

$$\rho = \frac{1}{cos^2\theta_W}\frac{1}{M_Z^2}(1+\frac{\Sigma_Z(0)}{M_Z^2})M_W^2\frac{1}{1+\frac{\Sigma_W(0)}{M_W^2}} \simeq 1+\frac{\Sigma_Z(0)}{M_Z^2}-\frac{\Sigma_W(0)}{M_W^2} \tag{3.8}$$

where choice (3.4) has been taken. The propagator corrections Σ are due to fermion loops and are individually divergent but with finite difference, the most relevant being due to the heaviest quark i.e. the top. It turns out

$$\rho = 1+\frac{3\alpha}{16\pi sin^2\theta_W}(\frac{m_t}{M_W})^2 \tag{3.9}$$

This dependence of ρ on m_t produces the dependence of $K(\mu)$ and $\triangle r$ on the same mass.

Taking (3.5) as coming from a relation (2.30) among bare parameters shifted with respect to the dressed ones, if we use (3.4) we obtain $\delta\, \sin^2\theta_W = -\cos^2\theta_W(\rho - 1)$ and therefore from (3.9)

$$K(\mu)|_{\rm dep\cdot m_t} = \frac{3\alpha}{16\pi sin^4\theta_W}(\frac{m_t}{M_Z})^2 \tag{3.10}$$

Analogously, being $\triangle r = -\frac{\delta\alpha}{\alpha}+\frac{\delta G_F}{G_F}+\frac{\delta sin^2\theta_W}{sin^2\theta_W}+\frac{\delta M_W^2}{M_W^2}$ and since the change of α does not depend on m_t and the change of G_F is due to that of M_W^2 so that both terms cancel in the above expression, the dependence of $\triangle r$ on m_t is the same (3.10) with change of sign

$$\triangle r \simeq 1-\frac{\alpha}{\alpha(M_W^2)}-\frac{3\alpha}{16\pi sin^4\theta_W}(\frac{m_t}{M_Z})^2 \tag{3.11}$$

One must note that in the propagator of the vector boson there is also a correction coming from a Higgs loop which is smaller because the divergence of the bosonic loop is only logarithmic and of reversed sign compared to the fermion one. The contribution to $\triangle r$ is

$$\triangle r|_{\rm Higgs} \simeq \frac{11\alpha}{48\pi sin^2\theta_W}\ell n\frac{M_H^2}{M_Z^2} \tag{3.12}$$

With these radiative corrections it is possible to compare $\sin^2\theta_W$ coming from masses according to (3.4) with the value coming from $\triangle r$ and from the ratio of neutral to charged deep inelastic scattering of neutrinos. For this last piece of information one must use (2.48) inserting $K(\mu)$ for each $\sin^2\theta_W$, taking ρ^2 coming from m_t dependence and dividing by a factor corresponding to photonic corrections of G_F^2 for charged currents. This comparison depends strongly on m_t and much smoother on M_H, being $m_t \simeq 170 GeV$ corresponding to CDF evidence near the upper bound for compatibility of results.

3.3 Physics of Z

LEP has produced extremely precise results for e^-e^+ around $s \sim M_Z^2$. Roughly speaking, considering the fermion pair production $e^-e^+ \to f\ \bar{f}$ mediated by Z, one may use a Breit-Wigner expression for the cross section in terms of total and partial widths Γ_Z and Γ_f

$$\sigma_f = \frac{12\pi\Gamma_e\Gamma_f}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \tag{3.13}$$

Since σ_e allows to determine Γ_e, from σ_μ, σ_τ and $\sigma_{\mathrm{hadrons}}$ all the other partial widths are evaluated. Writing the invisible width

$$\Gamma_{\mathrm{inv}} = \Gamma_Z - \sum_f \Gamma_f \quad , \quad f = e, \mu, \tau, \mathrm{hadrons} \tag{3.14}$$

and knowing theoretically that $\Gamma_\nu \simeq 2\Gamma_e$ the experimental Γ_{inv} is consistent with three light neutrinos with mass $m_\nu < M_Z/2$. The confirmation of the existence of three generations of neutrinos has been a very important result. Previously there was a cosmological bound $N_\nu \leq 4$ from the primordial nucleosynthesis of light elements in the universe.

The detailed analysis is more delicate since $e^-e^+ \to ff^-$ can be mediated by γ and Z and different corrections must be introduced. The most important are the propagator corrections of γ and Z and photon bremsstrahlung from initial electrons.

The correction of γ propagator implies the use of $\frac{e^2(s)}{s}$ with an effective charge, as has been seen.

Regarding the Z propagator, its correction implies a modification of the Breit-Wigner formula (3.13).

In fact, considering the relevant part of the inverse propagator

$$\triangle_Z^{-1}(s) = s - M_Z^2 + \Sigma_Z(s) \tag{3.15}$$

its real part must have a zero for $s = M_Z^2$, so that $Im\Sigma_Z(s)$ corresponds to an s-dependent width. Therefore, the resulting expression for the exchange of γ and Z amplitude is

$$\mathcal{M} = q_e q_f \frac{e^2(s)}{s} J^e_{em} \cdot J^f_{em} + \frac{\sqrt{2} G_F M_Z^2 J^e_{NC} \cdot J^f_{NC}}{s - M_Z^2 + is\Gamma_Z/M_Z} \tag{3.16}$$

As a consequence, the cross section which replaces the Breit-Wigner formula (3.13) is

$$\sigma_f = \frac{12\pi\Gamma_e\Gamma_f s/M_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} + \gamma \text{ exchange} + \text{interference} \tag{3.17}$$

Regarding the bremsstrahlung correction, it is necessary to have initially more energy than M_Z to reach the resonance. The measured cross section is expressed in terms of σ_f as

$$\sigma_f^M(s) = \int H(s, s')\sigma_f(s')ds' \tag{3.18}$$

where H takes into account the radiation from initial state giving a 30% reduction at the M_Z energy.

From the fit of the above expressions the total width is $\Gamma_Z = (2492 \pm 7)MeV$ and the partial ones $\Gamma_{\text{had}} = (1737 \cdot 1 \pm 6.7)MeV$, $\Gamma_e = (83.0 \pm .5)$ MeV, etc. The different partial widths are successfully checked by the standard model. The calculation of Γ_{had} requires the strong coupling which by the fit turns out to be $\alpha_s(M_Z) = 0.118 \pm 0.007$.

3.4 Cancellation of anomalies

The anomaly discussed in 2.1 has not bad consequences in QED because the axial current is not coupled to the gauge potential, and has the benefit of explaining the decay of π^0 . On the contrary in the GSW model the anomalous chiral currents are coupled to the gauge potentials and the fact that triangle-diagram divergences cannot be regularized in a gauge invariant way may spoil the Ward identities necessary to prove the renormalizability of the theory.

Therefore one considers necessary to cancel anomalies in the standard model. Since for non-abelian theories the anomalous source of a chiral current analogous to (2.7) has a factor

$$Tr\{\lambda_a, \lambda_b\}\lambda_c$$

where $\{\ \ \}$ means anticommutator and λ_i are group generators, for a $SU(2)$ model they would be Pauli matrices and the trace would vanish. But for the $SU(2) \times U(1)$ standard model there are contributions when λ_c corresponds to hypercharge and $a = b$ for $SU(2)$ generators. Since the anomaly arises from loops of all the fermions, its cancellation requires according (2.16).

$$\sum_{\text{ferm}} Q_{\text{em}} = 0 \tag{3.19}$$

It is remarkable that (3.19) is satisfied for each generation including the lepton doublet and the corresponding quarks remembering that the latter come in three colours each.

It is important to note that, even though the chiral anomaly is cancelled, the baryon number and lepton number currents are still anomalous in an equal amount. This nontrivial result, which allows the nonconservation of baryon number in the GSW model, has analogy in one spatial dimension. There, because of the pumping of levels of Dirac sea by electric field, if the vector currents is coupled to the gauge potential the fermion number is conserved but the chiral charge is not, and the reversed situation occurs if it is the axial curent to be coupled to the gauge potential. The latter case is analogous to the GSW model.

Chapter 4

Quantum Chromo Dynamics

4.1 Perturbative QCD

Strong interactions are due to a gauge invariant theory $SU(3)$ which transforms three possible colours for each quark by the exchange of eight spin 1 massless gluons[5], one for each group generator. Therefore the gauge invariant classical Lagrangian is

$$\mathcal{L}_{CD} = \bar{q}^{f}_{a} \; i(\not{D} q^f)_a - m_f \; \bar{q}^{f}_{a} \, q^f_a - \frac{1}{4} G^{\mu\nu}_i G^i_{\mu\nu} \tag{4.1}$$

where $a = 1, 2, 3$; $i = 1 \cdots 8$ and the flavour $f = 1 \cdots 6$ for the known quarks. The mass term, arising from the electroweak breaking, does not spoil $SU(3)$ symmetry since the transformations are not chiral. The covariant derivative is

$$D_\mu q = (\partial_\mu + i g_3 \frac{\lambda j}{2} G^j_\mu) q \tag{4.2}$$

where λ_j are the 3x3 Gell-Mann matrices which replace the Pauli ones of SU(2). The gluon fields are

$$G^i_{\mu\nu} = \partial_\mu G^j_\nu - \partial_\nu G^i_\mu - g_3 f_{ijk} G^j_\mu G^k_\nu \tag{4.3}$$

where f_{ijk} are the structure constants of $SU(3)$.

One may implement a perturbative theory analogous to QED with the additional 3-gluon vertex of order g_3 and 4-gluon vertex of order g_3^2 coming from the term G^2 of (4.1).

This produces an important change for the running coupling constant because whereas for the effective electric charge (3.1) the so called β-function

$$\beta^{\rm QED} = \frac{d\alpha(q^2)}{d\ell n q^2} = \frac{1}{3\pi}[\alpha(q^2)]^2 \tag{4.4}$$

for QCD the effective $\alpha_s = g_3^2/4\pi$ decreases for increasing momentum. This is due to the fact that in the correction of gluon propagator the gluon loop dominates over the opposite sign fermion loops if the number of flavours $f \leq 16$. This is seen by the perturbatively calculated β-function

$$\beta^{\rm QCD} = -\frac{33-2f}{12\pi}\alpha_s^2 \tag{4.5}$$

which is solved by

$$\alpha_s(q^2) = \frac{12\pi}{33-2f} \quad \frac{1}{\ell n(q^2/\wedge^2)} \tag{4.6}$$

where $\wedge$ is a free parameter which is determined to be $\sim$ 200 MeV by a fit of experiments. This means that α_s is reasonably small not only at $q^2 \sim M_Z^2$ as said in 3.3 but also in the deep inelastic scattering range $q^2 \sim 100 GeV^2$.

For the latter case we must introduce corrections due to the interaction of gluons to the partron model discussion of 2.3. The exchange and emission of gluons produces a modification of structure function, which gives the probability for finding a quark inside a nucleon, and of the cross section for the scattering vector boson-quark. This cross section receives a calculable correction of order $\alpha_s(q^2)$. The structure function for the quark q carrying the fraction ξ of the momentum of the nucleon turns out to obey the Altarelli-Parisi equation

$$\frac{df_q(\xi,q^2)}{d\ell n q^2} = \frac{\alpha_s(q^2)}{2\pi}\int_\xi^1 \frac{d\xi'}{\xi'} f_q(\xi',q^2)P(\xi/\xi') \tag{4.7}$$

where P, which can be interpreted as the probability of finding a quark inside another quark, is a calculable function. It is easier to analyze the moments of the experimental f_q

$$M_n(q^2) = \int_0^1 d\xi \xi^{n-1} f_q(\xi,q^2) \tag{4.8}$$

which, from (4.7), obey

$$\frac{d}{d\ \ell n q^2} M_n = \frac{\alpha_s(q^2)}{2\pi} A_n M_n \quad , \quad A_n = \int_0^1 d\ \xi \xi^{n-1} P(\xi) \tag{4.9}$$

so that using (4.6)

$$M_n(q^2) \propto (\ell n q^2)^{6A_n/(33-2f)} \tag{4.10}$$

behaviour remarkably checked.

4.2 Confinement

Quarks, gluons and in general coloured states are not individually observed and are said to be confined. The confinement property of QCD, i.e. the increase of attraction for large separation of components, is suggested by (4.6) where the coupling becomes truly strong for small momentum even though it cannot be taken as a proof for having been derived with perturbation theory.

If we consider that the QCD coupling is a small quantity g_S for small distances and has a large value g_L for large distances, an electrostatic analogy emerges. We may express

$$g_L^2 = \frac{g_S^2}{\epsilon} \quad , \quad \epsilon < 1 \tag{4.11}$$

in terms of a dielectric constant smaller than 1. This is impossible for a normal material where the polarization goes in the direction of the electric field, so that we may say that (4.5) characterizes a dia-electric medium. The result is that if we put a colour charge inside a hole of it, the inner surface will become polarized with the same sign and it will be energetically convenient for the hole to exist. If we now put inside the hole a colour charge and its opposite, a meson, or three different colour charges, a baryon, and take $\epsilon = 0$ outside, the chromo-electric field will remain confined. The outside medium has a sort of specular property compared to a superconductor where magnetic field cannot enter.

't Hooft and Witten have developed a procedure for the case when the QCD coupling is not small not allowing a perturbative expansion in powers of it. Calling $g_3 = g/\sqrt{N}$ where N is the number of colurs, 3 in the physical case, and representing a gluon propagator by a line of colour going in the positive direction and another colour in the negative one, it is easy to see that for large N the most important Feynman diagrams are the planar ones with any number of exchanged gluons. Very interesting consequences follow i) there are infinite meson states of definite mass interacting weakly, ii) baryons have large mass and interaction between them of the same order, iii) interaction between baryon and meson is small compared to the former mass but of the order of the latter one. This suggests the view that for large distances QCD can be described in terms of weakly interacting mesons, with baryons interpreted as non perturbative conglomerates of them, i.e. solitons.

4.3 Hadronic models

The idea of a varying dielectric constant gives way to the bag model for hadrons. Neglecting at zeroth order the gluon exchange between quarks inside the bag, the effect of ϵ is simulated by a classical static real field σ

$$\mathcal{L}_0 = \bar{\psi}\,(i \not\partial - f\sigma)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma) \tag{4.12}$$

where V has an absolute minimum at $\sigma = v$ and a relative one at $\sigma = 0$. Outside the bag one must have the vacuum $\sigma = v$ with $\psi = 0$ to avoid the quark mass fv. The quark is therefore confined inside the bag where it is massless if $\sigma = 0$.

From (4.12) one obtains the coupled equations for σ and the one-quark states of ψ. Around the bag surface defined by zero mass density $\bar{\psi}\,\psi = 0$ the solution for σ is of the kink type interpolating between 0 and v. Inside the bag the slow varying σ can be put in terms of ψ whose Dirac eigenstate equation can be therefore solved. The ground state energy depends on a parameter ξ so that the hadron mass is

$$M = n\frac{\xi}{R} + \frac{4\pi}{3}R^3 p + 4\pi R^2\gamma \tag{4.13}$$

where R is the bag radius, $n = 2$ or 3 for meson or baryon and the volume and surface contributions from σ are added. For different bags ξ is fixed and p or γ is zero, so that R is determined minimizing M in terms of a single parameter. Several predictions for static properties of hadrons can be made.

The first order gluon exchange modifies (4.12) and consequently (4.13) simply adding to ξ a constant term proportional to α_s inside the bag. From fit of masses $\alpha_s \simeq 3/8$ which is reasonable.

The $1/N$ expansion of 4.2 suggests an effective theory of only mesons. For the three pions a σ-model like that of 2.1 without fermions and fixed at the potential minimum $\vec{\pi}^{\,2} + \sigma^2 = f_\pi^2$ where f_π is the so called decay constant of pion, to eliminate the unphysical σ, allows to write the Lagrangian in terms of 3 fields

$$\mathcal{L}^{(2)} = \frac{f^2\pi}{4} Tr\partial_\mu U\partial^\mu U^\dagger \quad , \quad U = e^{i\vec{\tau}\cdot\vec{\varphi}(x)} \tag{4.14}$$

U, from which $\vec{\pi}$ and σ may be obtained, is an element of $SU(2)$ and if for static configurations varying on the 3-dimensional space we cover all the group a topological number may be defined which can be identified with the baryonic number. To have a configuration which has a minimum energy for a size R one must add a second term

$$\mathcal{L}^{(4)} = c_4 Tr[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U][U^\dagger \partial^\mu U, U^\dagger \partial^\nu U] \tag{4.15}$$

where [ ] means commutation and the solution for U is the Skyrme soliton.

Better predictions are obtained for hadrons if to a Skyrme solution for large distance one adds a bag to describe small distance details.

4.4 θ - vacuum

The vacuum structure of QCD is complicated because classically there are infinite degenerate nimina of energy for configurations with different topology. From the quantum point of view it is proved that there is tunnel from each one to the neighbour because of the existence of the so called instantons. Consequently there are infinite combinations of the minima generated by

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} G_i^{\mu\nu} \tilde{G}^i_{\mu\nu} \quad , \quad \tilde{G}^i_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G_i^{\rho\sigma} \tag{4.16}$$

where, for a particular value of the parameter θ, we have the vacuum. This term does not affect the classical equations of motion because $G\tilde{G}$ is a divergence.

Moreover another term of the type (4.16) arises from the mechanism of generation of quark masses in GSW model. In fact, even though in QCD anomaly does not spoil renormalizability because gauge potentials are coupled to vector currents, the diagonalized quark mass matrix M is not necessarily real. Therefore the phase of each diagonal element may be compensated by a chiral transformation of the corresponding quark but, in so doing, an anomaly term of the type of (4.16) appears. Thus, the additional Lagrangian will have the parameter replaced by

$$\bar{\theta} = \theta - \arg\, det\ M \tag{4.17}$$

With this replacement $\mathcal{L}_{QCD} = \mathcal{L}_{CD} + \mathcal{L}_\theta$

The new term violates CP invariance at the level of strong interactions and this is delicate because to satisfy the experimental bound of the neutron electric dipole moment it is necessary that $\bar{\theta} < 10^{-9}$. The smallness of this parameter which comes from the cancellation of two unrelated contributions is one of the last problems of the standard model $SU(3) \times SU(2) \times U(1)$ to be explained.

One might be tempted to ignore $\mathcal{L}_\theta$ hoping that this absence will be understood in the future, but the θ structure of the QCD vacuum has the virtue of solving the so called $U(1)_A$ problem. This happens because for massless u and d quarks non perturbative effects of QCD break spontaneously the global $SU(2)_A$ giving pions as Goldstone bosons and also the global $U(1)_A$ but giving an unphysical Goldstone boson explaining its experimental absence.

4.5 Axions

The negligible value of the parameter discussed in 4.4 could be understood if there would be one massless quark because in this case one might perform a chiral transformation of the corresponding field which, through anomaly, could be adjusted to cancel $\bar{\theta}$.

Peccei and Quinn found a mechanism to cancel $\bar{\theta}$ also if no quark is massless. One of its versions is to consider an exotic quark coupled to a complex scalar singlet

$$\mathcal{L} = \bar{\Psi}\, i\, \partial\!\!\!/ \Psi + \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|) - h(\bar{\Psi}_L\, \Psi_R \Phi + h.c.) \tag{4.18}$$

invariant under global chiral $U(1)$

$$\Psi_L \to e^{i\alpha/2} \Psi_L \quad , \quad \Psi_R \to e^{-i\alpha/2} \quad , \quad \Phi \to e^{i\alpha} \Psi \tag{4.19}$$

If V has a minimum for $|\Phi| = f_{PQ}/\sqrt{2}$ very large taking

$$\Phi = \frac{f_{PQ} + \rho(x)}{\sqrt{2}} e^{ia(x)/f_{PQ}} \tag{4.20}$$

$a(x)$ is the massless Goldstone boson corresponding to the spontaneous breaking and is called axion. Adjusting the parameters of V so that not only Ψ but also ρ correspond to particles with unobservable large mass, the relevant part of (4.18) will be

$$\mathcal{L}_a = \frac{1}{2} \partial_\mu a \partial^\mu a - i \frac{h}{\sqrt{2}} a\, \bar{\Psi}\, \gamma_5\, \Psi \tag{4.21}$$

Through a triangle of the heavy quark an effective coupling of axion with two gluons appears

$$\mathcal{L}_{aG} = -\frac{\alpha_s}{8\pi}\frac{a}{f_{PQ}} G_i^{\mu\nu} G^i_{\mu\nu} \tag{4.22}$$

By a shift in $a(x)$ one may absorb $\bar{\theta}$ so that the solution of the CP problem corresponds to see why $a(x)$ must be small. This is so because due to (4.22) the axion mixes with the pion and acquires a small mass

$$m_a \simeq m_\pi \frac{f\pi}{f_{PQ}} \tag{4.23}$$

so that the induced term in the potential $m_a^2 a^2$ forces $a \to 0$.

From astrophysical and cosmological bounds $10^{-5} eV < m_a < 10^{-3} eV$ so that being $f_\pi = 93 MeV$ the scale of breaking of the Peccei-Quinn symmetry is $f_{PQ} = 10^{10} - 10^{12}$ GeV.

Axions might be observed through its interaction with electromagnetic fields obtained in a way similar to (4.22)

$$\mathcal{L}_{a\gamma} = -\frac{\alpha}{\pi}\frac{a}{f_{PQ}}\, \vec{E} \cdot \vec{B} \quad , \quad \alpha = 1/137. \tag{4.24}$$

Chapter 5

Beyond the Standard Model

5.1 Grand Unified Theories

The Standard Model $SU(3)\times SU(2)\times U(1)$ (SM) has a remarkable phenomenological success without evidence of necessity to go beyond it. The only experimental motivations come from cosmology and astrophysics
i) The matter-antimatter assymmetry in universe

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10}$$

difficult, but not impossible, to explain within Standard Model which perturbatively conserves baryon number.
ii) The neutrino mass, not existent in the Standard Model, which could be convenient to explain the deficit of solar neutrinos and a part of the dark matter in the universe.

Moreover, from the theoretical point of view, it is not satisfactory to have a model with 19 parameters (3 gauge couplings, 2 Higgs parameters, 9 fermions masses, 4 Kobayashi-Maskawa parameters, 1 angle for QCD vacuum) and arbitrary assignment of multiplets and charges.

Leaving aside gravitational interactions, it has been proposed to unify the three gauge symmetries in a single one gauge group giving way to Grand Unified Theories (GUT), obtaining some predictions.

The smallest one which contains $SU(3)\times SU(2)\times U((1)$ is $SU(5)$ which has 24 generators L_i. In the 5-dimensional fundamental representation 8 of them correspond to 3-dimensional Gell-Mann matrices, 3 are 2-dimensional Pauli matrices, 1 is the diagonal hypercharge, and the additional 12 are associated to new gauge potentials called lepto-quarks because have the property of interacting with a quark transforming it into a lepton.

Regarding fermions, considering the fundamental representations $\Psi_5 \quad 5 = (3,1)+(1,2)$ in terms of $SU(3)$ and $SU(2)$ multiplets, one may put there a right quark and a right charge conjugate lepton doublet. Since the e.m. charge must be a diagonal traceless generator the quark must be d for the first generation. The interaction with vector bosons V_μ comes from the definition of covariant derivative as in the Standard Model and

$$\mathcal{L}^{(5)}_{\rm kin} = \bar{\psi}_5 \; i \not{D} \psi_5 \tag{5.1}$$

The rest of fermions can be put in the next representation $\chi_{10} \; 10 = (3,2) + (3,1) + (1,1)$ which for the first generation contains the left quark doublet, the left charge conjugate u and the left positron. Since χ_{10} may be expressed as a 5×5 antisymmetric matrix, its coupling with vector bosons comes from

$$\mathcal{L}^{(10)}_{\rm kin} = Tr \; \bar{\chi}_{10} \not{D} \chi_{10} \tag{5.2}$$

Due to the fact that W^3_μ and B_μ are associated with two $SU(5)$ generators a definite combination of which is the charge coupled to the electromagnetic potential A_μ, the Weinberg angle is predicted to be

$$sin^2\theta_W = 3/8$$

which is too high but refers to the scale M_X where $SU(5)$ symmetry holds.

One believes that there are two symmetry breakings

$$SU(5) \stackrel{M_X}{\to} SU(3) \times SU(2) \times U(1) \stackrel{M_W}{\to} SU(3) \times U(1)_{\rm em}$$

where the first must give mass to the 12 lepto-quarks leaving the rest massless up to the second where $W^\pm$ and Z get mass too.

The first breaking is obtained with scalars in an adjoint representation $\Sigma_i = i = 1 \cdots 24$. Its kinetic Lagrangian is

$$\mathcal{L}^{\Sigma}_{\rm kin} = Tr(D_\mu \Sigma)^\dagger D^\mu \Sigma \tag{5.3}$$

with

$$D_\mu \Sigma = \partial_\mu \Sigma + ig[V_\mu, \Sigma] \quad , \quad V_\mu = V^i_\mu \frac{L_i}{2} \quad , \quad \Sigma = \Sigma^i \frac{L_i}{2} \tag{5.4}$$

where g is the single $SU(5)$ coupling. The spontaneous breaking is obtained choosing the minimum of a potential V (Σ) as vacuum $\Sigma_{\rm vac} \ = \ v$ where v must be a diagonal matrix with the same eigenvalues in the triplet and in the doublet subspaces in order to commute with the 12 generators of the SM.

The second breaking is obtained by a Higgs field in the fundamental representation and a potential V(H) of whose minima the vacuum is chosen as $H_{\rm vac} = v_0$ where v_0 has non-zero value only for the neutral component of the doublet subspace.

A problem is that it is difficult to separate the effects regarding mass between the scale M_X and the scale M_W. Even though one has not included mixed terms of Σ and H they appear through radiative corrections. Therefore one must consider this additional potential $\mathrm{V}(\Sigma, H)$ and impose a very delicate cancellation among all the renormalized parameters to obtain the separation of scales M_X and M_W. This unnatural fine-tuning is called the hierarchy problem.

Regarding the generation of fermion masses, since the fermion part of the corresponding Lagrangian transforms under $SU(5)$ as $(\bar{5}\ {+}10) \times (\bar{5}\ {+}10)$ which contains the representation 5 but not the 24, to have a gauge invariant contribution the Higgs 5 H must be introduced, which is satisfactory becaused the effects will by of order M_W and not M_X.

Due to the fact that v_0 has only one non-vanishing component the equality of masses of down quarks and leptons is predicted, i.e..

$$m_d = m_e \quad , \quad m_s = m_\mu \quad , \quad m_b = m_\tau \tag{5.5}$$

As for the case of the prediction of θ_W (5.5) is not very satisfactory but it refers to the scale M_X and needs corrections.

It is interesting that radiative corrections may relate the $SU(5)$ unified coupling g to the individual three couplings at low energy. The result is

$$\begin{aligned}
\frac{1}{\alpha_3(E)} &= \frac{1}{\alpha_{GU}} + \frac{1}{6}(4N_G - 33)\ \ell n\frac{M_X}{E} \\
\frac{1}{\alpha_2(E)} = \frac{sin^2\theta_W(E)}{\alpha_{\rm em}(E)} &= \frac{1}{\alpha_{GU}} + \frac{1}{6}(4N_G - 22 + \frac{1}{2})\ \ell n\frac{M_X}{E} \\
\frac{1}{\alpha_1(E)} = \frac{3}{5}\frac{cos^2\theta_W(E)}{\alpha_{\rm em}(E)} &= \frac{1}{\alpha_{GU}} + \frac{1}{6}(4N_G + \frac{3}{10})\ \ell n\frac{M_X}{E}
\end{aligned} \tag{5.6}$$

where N_G is number of generations, $\alpha_i = \frac{g_i^2}{4\pi}, \alpha_{GU} = \frac{g^2}{4\pi}$ indicating that at energy M_X the 3 couplings should coincide. From the experimental values of $\alpha_{\rm em}$ and

α_3 (5.6) allows to determine $M_X \sim 10^{14} GeV$. Then $sin^2\theta_W(E)$ can be calculated in rough agreement with the low energy value. But with the present very precise electroweak measurements it is seen that (5.6) is not completely consistent.

In a similar way the predictions (5.5) may be corrected to low energy giving reasonable values except for the lightest components. Moreover lepto-quarks produce the transformations e.g.

$$d \rightarrow e^+ \quad , \quad d \rightarrow \bar{\nu}$$

changing baryon and lepton numbers but with conservation of $B-L$. Considering at low energy the B violation terms as an effective 4 -fermion interaction one can calculate the proton lifetime

$$\tau_p = (1 \text{ to } 5) 10^{30} \text{ years } (\frac{M_X}{5x10^{14} GeV})^4 \tag{5.7}$$

which, with previous M_X, is too small compared with the experimental bound $\tau_p > 2 \times 10^{31}$ years. Therefore we see that, though $SU(5)$ has some good features, it needs corrections.

5.2 Neutrino mass

In SM and in $SU(5)$ there is no Dirac mass

$$m(\bar{\psi}_L \, \psi_R + \bar{\psi}_R \, \psi_L)$$

because there is no ν_R.

Another possibility for neutrinos is the Majorana mass[(6)]

$$\psi_L^T C \psi_L \quad , \quad C = i\gamma^2\gamma^0$$

but in SM is again impossible because corresponds to weak isospin 1 which should be compensated by a weak Higgs triplet that does not exist. In $SU(5)$ such a triplet exists in the adjoint representation, but such scalars are not coupled to fermions so that there is no Majorana mass.

Therefore one must think in a new scale M at which $B-L$ is violated so that Majorana mass is allowed. This may be represented by an effective term

$$\frac{1}{M} \nu_L^T C \nu_L H_0 H_0$$

where H_0 is the isodoublet of the 5 H compensating the isospin 1 of neutrinos. One may expect this mass to be $\sim m^2/M$ with m quark mass.

Gell-Mann thought that the existence of ν_R may give a Majorana mass for $\nu_R\nu_R$ of order M, and also Dirac mass $\bar{\nu}_R\,\nu_L$ and $\bar{\nu}_L\,\nu_R$ of order m. Diagonalizing the matrix mass for all these contributions one of the eigenvalues m_1 decreases and the other m_2 increases with increasing M producing the so called see-saw mechanism. This may give light neutrinos of mass $m_1 \sim 10^{-3} - 10^{-6} eV$ which might explain the solar neutrinos deficit.

Whereas the addition of ν_R and the violation of $B - L$ are rather ad hoc in $SU(5)$, they are natural ingredients of another GUT based on the gauge symmetry $SO(10)$ in which one may expect neutrinos of mass $\sim 10eV$, and on the other hand also τ_p is larger than the experimental bound.

5.3 Supersymmetry

It is appealing to imagine that for any fermion there is a bosonic partner and viceversa even though there is no evidence of it from the known particles.

A motivation for supersymmetry (SUSY) is to make easier the problem of hierarchy in $SU(5)$ being necessary to establish a relation of parameters at bare level since there will be no renormalization due to cancellation of positive boson loop with the corresponding negative fermion one.

The transformation between fermion and boson is given by a fermionic generator Q such that

$$Q|F>=|B> \quad , \quad Q|B>=|F> \tag{5.8}$$

Q transforms as a Majorana spinor (2 components), must leave invariant the momentum of the state i.e.

$$[Q, P_\mu] = 0 \tag{5.9}$$

and the algebra closes with the anticommutators

$$\{Q_\alpha, Q_\beta\} = 2(\gamma^\mu)_{\alpha\beta} P_\mu \tag{5.10}$$

The minimum content of particles in a SUSY standard model is
i) To photon corresponds photino $\widetilde{\gamma}$, Majorana fermion.
ii) To gluons correspond also massless gluinos $\widetilde{g}$.
iii) To each quark or charged lepton corresponds two complex scalars (squark or

slepton). To neutrino one complex scalar sneutrino.
iv) To charged massive vector bosons $W^{\pm}$ correspond Dirac fermions winos $\widetilde{W}^{\pm}$. Since for each one there is a missing boson state we add charged Higgs particles $H^{\pm}$.
v) To Z corresponds two Majorana zinos $\widetilde{Z}_i$. Again there is a missing scalar state which may be the neutral Higgs H.
vi) Since one needs therefore two complex Higgs doublets there are still two neutral Higgs particles h(scalar) and a (pseudoscalar) with the corresponding Majorana higgsino $\widetilde{h}$.

As supersymmetric partners are not observed, SUSY must be broken. Spontaneous breaking requires vacuum non invariance, keeping (5.9),

$$Q_\alpha|0>\neq 0 \tag{5.11}$$

so that from (5.10) the vacuum energy is necessarily positive.

The SUSY modification of $SU(5)$ requires the inclusion of two quintets H and H'. Repeating the analysis of (5.6) with the modified interactions one gets the good results $sin^2\theta_W(M_W) = 0.236 \pm 0.003$ and $M_X \simeq 2 \times 10^{16} GeV$ which according to (5.7) makes the proton sufficiently stable.

5.4 Dark matter and baryogenesis

For the near future it is possible that information regarding aspects beyond SM will come from cosmology and astrophysical observations[7].

From the motion of galaxies and their structures and the primordial nucleosynthesis of light elements it seems certain that in the universe there is a 90% or 95% of non-baryonic dark matter (DM). This DM may be hot (HDM) or cold (CDM) according with their relativistic or non-relativistic motion at the time of decoupling. HDM candidate is neutrino which with a mass of 30 eV would explain all the required matter in universe. Candidate for CDM are axions, which with mass of 10^{-5} eV would account for all necessary matter and neutralinos (neutral weakly interacting SUSY partners).

From the anisotropies detected in the cosmic background radiation and the formation of structures the model of DM which seems favoured contains 70% of CDM and 30% of HDM, which would correspond to neutrinos of 7eV. But this result is sensitive to the details of primordial fluctuations in the universe.

Another much debated subject is the generation of baryon number in universe. An obvious candidate is GUT because it perturbatively violates B conservation, so that baryogenesis would have produced when the universe had a temperature $\sim M_X$ since the non-equilibrium related to a breaking symmetry phase transition is necessary. But it is not easy to accept the existence of this transition because a large amount of magnetic monopoles, not detected, would have simultaneously produced.

The alternative for baryogenesis is the transition for breaking of electroweak symmetry at $T \sim 250 GeV$. As said in (3.4) baryon current for GSW model is anomalous, and B can be created passing from one vacuum state to the next degenerate one. Even though in GSW there are no instantons the top of the barrier separating vacua, called sphaleron, may be surmounted by thermal excitations. It is necessary that CP is violated, to distinguish baryon from antibaryon, and this could be afforded by the Kobayashi-Maskowa phase or move efficiently by the two Higgs doublets required by SUSY.

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