Optics in the Relativistic Regime

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Abstract

The advent of ultra-intense laser by the technique of chirped pulse amplification (CPA) and by the development of high fluence laser materials has opened up an entirely new field of optics, that in the relativistic regime. The electromagnetic field intensity in excess of 10^{18} W/cm^2 leads to the relativistic electron motion in the optical fields. The CPA method is reviewed and the future growth of laser technique is previewed, including the ultimate power of zettawatt. A number of consequences of relativistic optical fields are surveyed. In contrast to the non-relativistic regime, the laser fields are capable of moving matter much more effectively, including in the direction of laser propagation. One of the consequences of this is the effect of wakefield generation, a relativistic version of optical rectification, where the longitudinal field could be as large as the transverse one. In addition, relativistic focusing, relativistic transparency, nonlinear modulation and multiple harmonic generation, and strong coupling to matter and other fields (such as high frequency radiation) do occur. A proper utilization of these phenomena and effects can lead to the new technology of relativistic engineering, where both matter and laser can be manipulated to move and be modified in a relativistic fashion and further the technologies of accelerators and lasers may be cross-fertilized.

A number of prominent applications are cited, including the fast ignition of the inertially confined compressed fusion target by a short-pulsed laser energy delivery and bright

sources of energetic particles (electrons, protons, other ions, positrons, pions, etc.). The intense laser field coupling also reveals the kind of mechanism of highest energies in astrophysics such as the ultra-high energy cosmic rays (UHECR) in excess of 10^{20} eV of energy. The laser fields can be so intense as to make the accelerating field so huge and thus general relativistic effects (via the Equivalence Principle) be examined in the laboratory in strong limits, as well as to explore the collective regime of quantum electrodynamics. In such regimes, the effect of radiative damping is no more negligible. Further, when the fields are close to the Schwinger value, vacuum now begins to behave as if it is a nonlinear medium in much the same way that the early laser a few decades ago has forced the ordinary dielectric matter to behave so.

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I. Introduction

Over the past fifteen years we have seen optics on the threshold of a new scientific adventure similar to the one experienced in the 60's. Soon after the advent of the laser in 1960's the first nonlinear optics effect, was demonstrated**.** For the first time the laser field could challenge the Coulomb field that binds the electrons to their nucleus. The laser could produce new radiations (Franken, 1961) or be rectified (Bass, 1962). It could change the index of refraction of optical media (Mayer, 1964; Bloembergen and Lallemand, 1966). Raman modes in molecules could mix with the laser field to produce stimulated Raman scattering (SRS) **(**Woodbury, 1962**).** The electrostrictive effect could induce acoustic waves to produce Stimulated Brillouin Scattering (SBS) **(**Chiao et al, 1964**).** Also, higher order, optical nonlinearities that could involve the simultaneous absorption of several photons were demonstrated opening the field of Multiphoton Ionization (MPI) **(**Voronov and Delone, 1965; Agostini et al., 1968**).** The examination of **figure 1** highlights the strong correlation between the rapid intensity increase in the 1960's and the discovery of all the major effects in nonlinear optics. The rapid intensity evolution was due to the introduction of Q-switching (Hellwarth, 1961) and mode-locking (Mocker and Collins, 1965). In fact during this period the intensity increase was so rapid that physicists were already predicting new types of optical nonlinearities dominated by the relativistic character of the free electrons (Howard and Reiss, 1962, Litvak 1969, Elberly, 1969, Sharachik and Schappert, 1970, Max, et al., 1974) or vacuum nonlinearity (Brezin and Itzykson, 1970) .

The key of high and ultrahigh peak power / intensity is the amplification of ultrashort pulses in the picosecond and femtosecond time scale. Over the past 40 years the laser pulse duration has continuously decreased from the microsecond with the free running, to the nanosecond with the Q-switching, and finally to the picosecond and few femtosecond regime with mode-locking Brabec and Krausz, 2000 **(Figure 2).** With mode locking, the laser pulse duration became so short that they could not be amplified without producing unwanted nonlinear effects. This is the reason of the power and intensity plateau seen in **figure 1**. For reasonable size systems, i.e. with beam diameter of the order of 1cm the maximum obtainable power stayed around 1GW and focused intensity at about 10^{14}W/cm^2. More power can be obtained if we can. firstly, use amplifying media that can accommodate the short pulse spectrum. Secondly if we can use high energy storage amplifying media that is media with low transition cross section σ_a. However this will imply to use high laser fluence (J/cm^2). To extract the stored energy it is necessary that the input laser energy be of the order of the saturation fluence, i.e. $F_{sat}=h\nu/\sigma_a$. (Superior energy storage media will have large saturation fluence. This fluence delivered over a short time will lead to prohibitively large intensities that can exceed the TW/cm^2) way above the limit of ~ GW/cm^2 imposed by nonlinear effects and optical damage in the amplifiers and optical components. The only way we could meet these conditions was to use low energy storage materials (dyes and excimers) and increase the laser beam cross section, leading to unattractive, large, low repetition rate, high price tag systems. Because of the laser size, high intensity research physics was limited to few systems such as CO2 (Carman et al., 1981), Nd: glass (Bunkenburg, 1981) and excimer lasers (Luk, et al., 1989, Endoh, et al., 1989).

The state of affairs dramatically changed when laser physicists at the University of Rochester, in 1985 (Strickland and Mourou, 1986, Maine and Mourou, 1988), demonstrated a way to simultaneously accommodate very large beam fluence necessary for energy extraction in superior storage materials while keeping the intensity and the nonlinear effects to a minimum level. The technique was dubbed Chirped Pulse Amplification or CPA by one of the author (GM). This technique revolutionized the field in three major ways. First table top systems became capable of delivering intensities almost 10^5-10^6 times higher than in the past. Second the CPA technique could be readily adapted to existing large laser fusion systems at a relatively low cost. Today the CPA techniques are incorporated to all the major laser chains, Japan (Yamakawa et al., 1991), France (Rouyer, 1993), United Kingdom, United-States (Perry, et al., 1999) etc.. mainly for Fast Ignition research (Tabak et al, 1994). Third, because of their reduced sizes they could be married with large particle accelerators such as synchrotrons **(**Schoenlein, 2000; Wulff, 1997; Larson, 1998**)** to time-resolve x-rays diffraction or with linear collider such as SLAC to produce fields higher than the critical field (Bula, et al., 1996) and observe nonlinear QED effects as pair generation. At the moment all the colliders are planning to incorporate CPA technology to produce γ-rays for photon-photon, i.e. $\gamma-\gamma$ collider (Telnov, 1990, 2000, 2001).

As we will see later, the availability of these ultrahigh intensity lasers has extended the horizon of laser physics from the atomic and condensed phase physics to plasma physics, nuclear physics, high energy physics, general relativity and cosmology, up to the edge and beyond the standard model. It had also a major effect in bringing back to the university laboratory science performed with large instruments.

For the studying of the relativistic radiation interaction with matter it is characteristic that we meet wide range of the complexity to provide a detailed description: in the experiment due to microscopic in scale and short living in time entities; in the theory due to the high dimensionality of the problem, to the lack of symmetry and to the importance of nonlinear and kinetic effects. On the other hand, powerful methods for investigating the laser-plasma interaction have become available through the advent of modern supercomputers and the developments of applied mathematics, Dawson and Lin,(1984) and Tajima (1989). In the case of ultra-short relativistically strong laser pulses, simulations with 3D Particle-in-Cell codes provide an unique opportunity for describing adequately the nonlinear dynamics of laser plasmas, including nonlinear wave breaking, the acceleration of charged particles up to high energy and the generation of coherent nonlinear structures such the relativistic solitons and vortices. In this case the role of three dimensional computer simulations can not be understated.

II. Ultrahigh Intensity laser: The Chirped Pulse Amplification Technique

A. The amplification energy extraction condition

Before the technique of 1985 all amplifier systems where based on direct amplification.

As stressed in the introduction, a general rule in laser amplification is that the maximum energy per unit area that can be extracted by an amplifier must be of the order of F_{sat} the saturation fluence of the materials. This value is given by

$$F_{sat} = \frac{\hbar\omega}{\sigma_a}, \qquad (1)$$

here $\hbar$ is the Planck constant, ω the laser frequency and σ_a the amplifying transition cross-section. F_{sat} is 0.9 J/cm^2 for Ti:sapphire and 4J/cm^2 for Nd:Glass. It can be shown (Siegman, 1986) that the output fluence F_{out} is given by

$$F_{out}(t) = F_{sat} \times \ln\left[\frac{G_0 - 1}{G(t) - 1}\right] \qquad (2)$$

Where G_0 is the initial gain and

$$G(t) = \exp[\sigma N_{tot}(t)] \qquad (3)$$

the amplifier total gain. Here $N_{tot}(t)$ is the total population inversion. The amplifier efficiency η is given by the expression

$$\eta = \frac{\ln G_0 - \ln G_f}{\ln G_0} \qquad (4)$$

where, the gain G_f at the end of the impulsion is given by:

$$G_f = 1 + (G_0 - 1)\exp\left[-\frac{F_{pulse}}{F_{sat}}\right] \qquad (5)$$

From (4) and (5) we draw the conclusion that to reach an efficiency close to one the laser input fluence must correspond to few times F_{sat} .

Figure 3 illustrates this point for two different initial gain G_o of 10 and 10^3.

B. Amplification- Propagation condition

Prior to CPA the amplifyng media were exclusively, dye (Migus, et al., 1982), excimers (Luk, et al, 1989, Endoh, et al, 1989. Typical cross section for these media are very large in the range of 10^{-16}cm^2 implying a F_{sat} of only few mJ/ cm^2, of beam or a power density of 1GW/cm^2 for subpicosecond pulses. Above this power density level, the index of refraction becomes intensity dependent according to the well known expression

$$n = n_0 + n_2 I. \qquad (6)$$

Due to the spatial variation of the laser beam intensity this effect will modify the beam wave-

$$B = \frac{2\pi}{\lambda}\int_0^L n_2 I dx \qquad (7)$$

front according to B represents in λ, the amount of wavefront distortion due to intensity dependent index of refraction, accumulated by the beam over a length L. For a perfectly Gaussian beam, B will cause the whole beam to self –focus at a critical power given by the expression

$$P_{cr} \cong 17(\omega/\omega_p)^2 GW. \qquad (8)$$

This effect is strictly power dependent. In the case where the laser beam exhibits some spatial intensity modulations, n_2 will cause the beam to break up in filaments. In practice the small scale self-focusing represents the most severe problem in an amplifier system. The maximum growth rate (Bespalov and Talanov, 1966) will occur for spatial frequencies K_m given by

$$K_m = \left(\frac{2\pi}{\lambda}\right)\left(\frac{2n_2 I}{n_0}\right)^{1/2} \qquad (9)$$

$$g_m = \left(\frac{2\pi}{\lambda}\right)\left(\frac{2n_2 I}{n_0}\right) . \qquad (10)$$

For intensities of I~1 GW/cm^2, K_m~200 cm^{-1}, corresponding to 50 μm these irregularities will grow at a rate of g_m~3 cm^{-1}. Note that the growth rate G_m over the gain length L is exactly equal to B

$$G_m = B. \quad (11)$$

For laser fusion system, the beam is cleaned through spatial filters, every time B reaches 3. In CPA systems where the beam quality requirement is much higher, B must be kept bellow 0.3 corresponding to a wavefront distortion of $\lambda/20$.

C. The CPA concept

As seen above going to amplifying media with low cross-section offers some obvious benefits. For instance Nd:glass has a cross section of 10^{-21}cm^2 which means that we can store thousand to ten thousand times more atoms per unit volume and consequently get thousand to ten thousand times more energy before it self oscillates, than dye of excimers with cross-section ~10^{16}W/cm^2. However to extract this large amount of energy would require a beam with a fluence F_s of the order of 1J/cm^2 or an intensity of 10^{12}W/cm^2 corresponding to a B of few thousands, i.e. thousand times the limit established in the previous paragraph!

In order to utilize superior energy storage materials, the laser scientist is confronted with the seemingly unsolvable problem, to increase the input energy necessary for energy extraction, while keeping the input intensity at an acceptable level. This quandary is solved with CPA. The technique is simple. Instead of using the direct amplification, the pulse is first stretched by a factor of a thousand to hundred thousands. This step does not change the input pulse energy, and therefore our energy extraction capability, but decreases the input intensity by the stretching ratio and keeps to a reasonable level the B. Once the pulse is amplified from six to twelve orders of magnitude i.e. from the nJ to the millijoule-kilojoule level it is recompressed by the same stretching ratio back to a duration close to its initial value.

D. The key element: The Matched Stretcher-Compressor

In the first CPA embodiment (Strickland and Mourou, 1986) the laser pulse is stretched in an optical fiber that has positive group delay dispersion and recompressed by a pair of parallel gratings as shown by Treacy (Treacy, 1969) which can have a negative group delay dispersion. Although this first embodiment had led to a spectacular 100 times improvement in peak power it had the problem that both stretcher and compressor were not matched over all orders. Therefore, after recompression the pulse exhibited unacceptable prepulses and post pulses. Following the first CPA demonstration the Rochester group started to look for the ideal "matched stretcher-compressor". It came about when in 1987 Martinez (Martinez, 1987) proposed for communication applications a new grating arrangement with positive group dispersion as shown in **figure 5.** Note that in communication the wavelength of choice is 1.5μm, a region where the fiber exhibits negative group velocity dispersion. After propagation in a fiber the communication bits exhibit a negative chirp. It is therefore necessary to use a dispersive delay line with a positive group delay dispersion to recompress the pulse. After examining this device the Rochester group came to the conclusion that the Martinez "compressor" was in fact the matched stretcher of the Treacy compressor they were looking for. This can be easily shown by considering **figure 6.** Using a telescope of magnification 1, the input grating located at f from the first lens will be imaged at the same distance f of the second lens to form an "imaginary "grating. The second grating can be placed at a distance b from the imaginary grating. Note that b can be positive or negative according to the second grating position.

To stretch the pulse we impart a frequency-dependent phase shift $\phi(\omega)$ that can be expended in a Taylor series around the central frequency ω_0:

$$\phi(\omega) = \phi_0 + \phi_1(\omega - \omega_0) + \phi_2(\omega - \omega_0)^2 + \phi_3(\omega - \omega_0)^3 + \ldots \quad (12)$$

Here

$$\phi_n = \frac{1}{n!}\frac{d^n\phi}{d\omega^n}\bigg|_{\omega_0} . \quad (13)$$

The quadratic phase ϕ_2 is also known as the second- order dispersion or SOD. It is responsible for stretching the pulse. The higher order terms ϕ_3 and ϕ_4 TOD and FOD will distort the pulse shape and create wings. If ϕ_{str}, ϕ_{comp} are the frequency dependent phase of the stretcher and compressor a matched stretcher compressor fulfills the condition

$$\phi_{str} = \phi_{com}. \quad (14)$$

The Treacy compressor is composed of a grating pair. It acts as a dispersive delay line that will produce negative SOD. The value of the SOD can be shown as

$$\phi_2 = -\frac{m^2\lambda^3}{2\pi c^2 d^2 \cos^2\theta} b, \quad (15)$$

where c is the speed of light and m is the diffraction order, d the groove spacing.

$$b = -\frac{G}{\cos\theta(\lambda_0)}. \quad (16)$$

Here G is the perpendicular grating separation and θ the diffraction angle. TOD and FOD can be easily derived using (13) and their values are given by:

$$\phi_3 = -\phi_2 \frac{\lambda}{2\pi c}\left[1 + \frac{m\lambda}{d}\frac{\sin\theta}{\cos^2\theta}\right], \quad (17)$$

$$\phi_4 = -\phi_3 \frac{3\lambda^2}{4\pi^2 c^2}\left\{4 + 8\frac{m\lambda}{d}\frac{\sin\theta}{\cos^2\theta} + \frac{\lambda^2}{d^2}\left[1 + \tan^2\theta(6 + 5\tan^2\theta)\right]\right\}, \qquad (18)$$

Because all these orders are strictly proportional to *b* and its sign, the condition (14) can therefore be fulfilled by locating the second grating in a stretcher at a position –*b* from its image.

The phase conjugation of the two systems was demonstrated for the first time by Pessot et al (1987) by stretching a pulse of 80 fs,1000 times using Martinez arrangement and compressing it back exactly to the same value using the Treacy compressor. This demonstration represented a major step in Chirped Pulse Amplification. This matched stretcher compressor was for the first time integrated into a CPA system to produce a Terawatt pulse from a Table Top system -so called T^3- by Main et al. for subpicosecond pulses **(**Maine et al., 1988; 1989) and for pulse duration of 100fs by Pessot et al. (Pessot, et al., 1989)**.** This arrangement has become the standard architecture used in all CPA systems.

For shorter pulse systems with large bandwidth an additional phase term $\phi_{mat}(\omega)$ due to material dispersion in the amplifier, Faraday rotator, Pockels cells, etc.. must be added to (14) to produce the new matching condition

$$\phi_{str}(\omega) + \phi_{comp}(\omega) + \phi_{med}(\omega) = 0. \qquad (19)$$

To calculate $\phi_{mat}(\omega)$ we will use the familiar Sellmeier expression

$$n^2(\lambda) = 1 + \sum_j \frac{b_j}{\lambda^2 - \lambda_j^{\,2}}, \qquad (20)$$

where b_j, λ_j are materials constant. From (20) SOD, TOD and FOD can be calculated using the expressions (13) to produce

$$\phi_2 = \frac{\lambda^3 L}{4\pi c^2}\frac{d^2 n}{d\lambda^2} \qquad (21)$$

$$\phi_3 = -\frac{\lambda^4 L}{24\pi^2 c^3}\left[3\frac{d^2 n}{d\lambda^2} + \lambda\frac{d^3 n}{d\lambda^3}\right], \qquad (22)$$

$$\phi_4 = -\frac{\lambda^5 L}{192\pi^3 c^4}\left[12\frac{d^2 n}{d\lambda^2} + 8\lambda\frac{d^3 n}{d\lambda^3} + \lambda^2\frac{d^4 n}{d\lambda^4}\right], \qquad (23)$$

where *L* is the material length.

Fulfilling the condition (19) over a wide spectrum has become one of the most important topics of ultrafast optics. A number of matched stretcher-compressor arrangements have been demonstrated (Lemoff and Barty, 1993; White, et al., 1993; Tournois, 1993; Cheriaux, et al., 1996; Banks, 2000).

Very often all the terms can not be ideally compensated. Higher order corrections need devices such as the acousto-optic temporal phase corrector known as Dazzler introduced by the company Fastlite (Tournois, 1997)

1. New materials for CPA and Gain Narrowing

CPA was demonstrated initially with the only two broadband amplifying media that were available Nd: glass and alexandrite (Pessot, et al., 1989). It was shortly after that the concept was extended to Ti:sapphire (Vaillancourt, et al., 1990, Squier , et al., 1991, Kmetec, et al., 1991, Sullivant, et al., 1991) as well as Cr: $LiSrAlF_6$ **(**Ditmire and Perry, 1993, Beaud et al., 1993**)** and Yb: glass **(Nees 1997**). Among all these materials Ti:sapphire has the great properties to have the largest bandwidth very good thermal conductivity that is enhanced at cryogenic temperature (Backus, et al., 1997). Parametric amplifiers has been also proposed (Dubeis, et al., 1992) and mainly developed at Rutherford (Ross, et al., 1997). This technique called OPCPA has the advantage if the nonlinear propagation effects are kept under control to provide first an extremely large bandwidth and second to be pumped by large scale laser systems and therefore to be a companion of any large laser fusion system. One of the limitations in pulse duration comes from the gain narrowing. Because of their wide spectrum short pulses can be amplified only by materials with a gain bandwidth greater than their spectrum. Let's mention that superior energy storage materials have a low transition cross section and broad gain bandwidth. However large gain will lead to a reduction of the laser spectrum as it gets amplified. In the unsaturated regime-the linear regime- the laser spectrum will be subjected to a narrowing. The gain narrowing effect will be in the linear or unsaturated regime given by

$$\Delta\omega = \Delta\omega_a \sqrt{\frac{3}{G(\omega_a) - 3}}, \qquad (24)$$

where $\Delta\omega_a$ is the gain bandwith, $G(\omega_a)$ the exponential gain. A gain of 10 orders of magnitude will narrow the gain by a factor 3 to 4. A fraction of this gain can be recovered in the saturated section oft the amplifier.

2. The Petawatt

As soon as the CPA concept was demonstrated at the millijoule and joule levels, it became clear to us that the it could be extended to much higher energies by simply using laser fusion systems already built to amplify nanosecond pulses to the 100 -1000 joules. With a remarkably small amount of alterations, that is by chirping the pulse at the input and compressing at the output, a laser chain built to produce TW pulses could now produce petawatt –PW pulses (Maine, Mourou 1988). This work was undertaken at Livermore under the direction of M. Perry. The first petawatt pulse was demonstrated in 1996 (Perry, et al., 1999) ten years after the first terawatt. One of the impressive hurdle overcame by Perry's group has been the fabrication of meter size gratings. Today there are around 20 petawatt systems in the planning stage or being built in the world.

Parallel to the Nd:based petawatt systems we have today a number of Ti: sapphire-based systems. They have much shorter pulses in the 20-30fs range and energy in the 5-10 J therefore producing peak power in the 100TW. 100 TW class Ti:sapphire laser has been first demonstrated, at the university of California in San Diego (Barty et al., 1994). The leading laboratories in this area are the Advanced Photon Reserch Center (APRC) in Japan with around 500TW (Aoyama, 2002), 200TW with the Janus System at Lawrence Livermore, 100TW at the Laboratory d' Optique Appliquée in France (Pittman, 2002), 100 TW at the Max-Born Institute in Germany and 30TW at the University of Lund in Sweden. Also at the University of Michigan a PW class system is under construction.

E. The Optical Parametric Chirped Pulse Amplification (OPCPA) (Dubeis, et al., 1992, Ross, et al., 1997)

Figure 7 shows the concept of OPCPA. Like in straight CPA the laser is stretched up to a nanosescond then amplified to the Joule and higher energy levels by optical Parametric Amplification and then recompressed close to its initial value. Note that here the stretching is important not only to keep the *B* integral down but also to extract the energy. Only during the stretched pulse duration the light can be transferred from the pump beam to the signal beam.

The pros of this technique are:

1) Large bandwidth that could accommodate few cycle pulses.
2) Benefit from very large KDP crystals available (100cm x100 cm) that have been already developed for laser fusion.
3) Well adapted to existing laser fusion chains, that benefit from single frequency nanosecond well collimated laser pulses at 532nm.
4) No heat dissipation in the OPA crystal itself.
5) No transverse ASE, which is a major problem for large aperture Ti:sapphire systems.
6) Can use Iodine laser as pumping source.
7) Very simple amplification system.

The cons are:

1) Low efficiency compared to straight CPA. For a regular Ti:sapphire CPA the efficiency can be 50% from a long green pulse say of 50ns. The energy storage time of Ti:sapphire is 2μs leading to much smaller pump energy, by a factor ten.
2) The very large stretching ratio in the range of 10^6 to 1(<10fs to 5ns) necessary for energy extraction will make difficult pulse compression down to the ten femtosecond regime.
3) Gain is a significant function of the intensity. The pump beam profile may affect the beam quality.
4) The ultimate bandwith will be a function of the grating bandwidth. At this moment no large gratings have the efficiency and the bandwidth required for efficient pulse compression below 30fs.

1. Overall Comparison between CPA and OPCPA

The beam quality in the CPA has been demonstrated to be excellent and needs to be demonstrated in the OPCPA case. The possibility to reach large powers seems to be more straightforward with the OPCPA because it can benefit from kJ, ns fusion laser already installed. However, the pulse duration will be limited by the grating bandwidth. The CPA must wait for large Ti:sapphire crystals that can be grown to 20cm x 20cm dimensions. Larger dimensions could become available with the demand. In the mean time, Ti:sapphire matrix could be used. However, the crystal positions will need to be interferometrically controlled. For large Ti:sappire systems, a problem to circumvent is the transverse ASE. Let's note that CPA and OPCPA work both near damage fluence threshold for the stretched pulse. Consequently, both systems should produce the same output energy for the same beam cross section.

2. Temporal Quality: Prepulse Energy Contrast

The characterization of the pulse duration by its full-width-at-half- maximum only is in the ultrahigh intensity field far from adequate. The peak intensity can be at present10^{20} W/cm^2 and in the future as high as 10^{23}W/cm^2. Six to ten orders of magnitude below the peak, that is at 10^{12}-10^{14}W/cm^2 plasmas can be created that will modify the target physical condition. **Figure 8** represents for the case of solid target interaction, the intensity laser as a function of pulse duration not to be exceeded.

There are mainly three sources of prepulse energy. The first is the Amplified Stimulated Emission (ASE). It is due to the amplifier gain and incomplete Pockels cell switching. It lasts around 10ns. The second originates from the oscillator background and the third from incomplete compression due to high orders effects and spectral clipping. It is important that the prepulse energy stays at a manageable level. For the long ASE (ns) pulse the energy level can not exceed .1J/cm^2 for a metallic target and few joules/cm^2 for a dielectric one. For the short prepulse component, it should be less than 1J/cm^2 for short prepulses and 10J/cm^2 for long prepulses.

It is not easy to study an optical pulse over ten decades with femtosecond resolution. Standard detectors like streak cameras, neither have temporal resolution nor thenecessary dynamic range. The only adequate technique is based on third order autocorrelation (Auston 1971; Albrecht 1981). It is described in figure 9. We first make a clean pulse by frequency doubling the pulse understudy. For instance in the case where the main pulse at ω has a contrast of 10^6 to 1, the 2ω pulse will have a contrast of around 10^{12} to 1. This temporally clean pulse at 2ω, will now be mixed with the pulse at ω in a third harmonic crystal. By varying the time delay between the ω pulse with respect to the 2ω pulse a replica of the ω pulse at 3ω will be constructed. The resulting 3ω radiation can be easily isolated from the ω and 2ω signals, so we canproduce the pulse replica at 3ω with an extraordinary large dynamic range covering more than 10 orders of magnitude (see **figure 10).** Note that this technique requires many shots it can be done only with the front end of the system that can operate at a higher repetition rate.

3. Pulse cleaning

Pulse cleaning is essential to achieve the contrast compatible with laser solid interaction at intensities > 10^{19}W/cm^2. A number of techniques have been tried based on frequency doubling, saturable absorber, plasma mirrors. However all these techniques being intrinsically nonlinear in intensity deteriorate the beam quality and are marginally adequate. The mechanism of **polarization rotation in a single mode fiber (Tapié and Mourou, 1992) has been demonstrated to be the most promising way to temporally clean pulses while preserving the laser beam quality.** When a high intensity laser propagates in a single mode birefringent fiber its polarization rotates. The rotation is a function of the intensity and it is therefore possible with a polarizer to discriminate the high intensity part from the low intensity part of the pulse.

This technique has been demonstrated with microjoule level pulses. It was used at the front end of a table-top terawatt (T^3) laser system. Recently the same concept was demonstrated in a hollow core fiber to the 20μJ level (Homoelle, 2002**).** They showed a contrast enhancement of 3 orders of magnitude. Here also the beam quality is preserved.

F. Spatial quality. Deformable mirrors.

High intensity CPA laser systems unlike laser fusion systems that are working at relatively low intensity (10^{14}W/cm^2) on target require very high quality wavefronts. Trying to express beam quality in terms of diffraction limit is not adequate. For example a 1.1 diffraction limit beam can have only 50% of its energy contained in the main spot. The rest is being dispersed in a background surrounding the focal spot.

A better criterion is the Strehl ratio that gives the fraction of the intensity on axis of the aberrated image over the intensity on axis for a Gaussian image point. Marechal (Born and Wolf 1980) has developed an expression that expresses the Strehl ratio R as a function of the mean-square deformation $\Delta\phi^2$ of the wavefront:

$$R = 1 - \left(\frac{2\pi}{\lambda}\right)^2 \overline{\Delta\phi^2}. \qquad (25)$$

We can see that $\Delta\phi^2$ has to be maintained in the range of $\lambda/8$ to get 80% of the theoretical limit. To restore the wavefront after amplification, compression and propagation it is necessary to use a deformable mirror. With deformable mirrors, not only the laser but also the focusing optics can be corrected to produce the highest intensities. As we will see relativistic intensities in the so called λ^3 limit was obtained by using only mJ energy focused with a NA=1 parabola to one single wavelength spot size **(Figure 11) (Albert, 2000).**

A very important technical aspect very often ignored by CPA builders is highlighted in **figure 12**. It shows how important it is to use of holographic gratings as opposed to ruled ones (**Tapié: Thesis 1991)**. Ruled grating are not sinusoidal and experience some dephasing between grooves (ghosts) during the long ruling fabrication process. This will produce a far from ideal beam profile. This is completely absent in holographic gratings where all the grooves are nearly sinusoidal and strictly in phase.

G. Theoretical Power and Intensity Limits (Mourou 1997)

In CPA and OPCPA systems, the pulse maximum energy that can be produced is ultimately limited by the damage threshold F_{str} of the stretched pulse and or the saturation fluence F_{sat} **whichever comes first.** In the nanosecond regime the damage threshold scales like $T^{1/2}$ **(Bloembergen, 1974)** where T is the pulse duration. F_{sat} is of the order of 20J to 50J/cm^2 for surface or bulk and depends on laser wavelength, material (energy gap) its purity and preparation. Note that F_{sat} is 0.9J/cm^2 for Ti:sapphire and 40J/cm^2 for Yb: glass. We have seen in equation (5) that to extract the energy the input fluence must be of the order of F_{sat}. On the other hand, the minimum pulse duration τ_p is imposed by the relation

$\Delta\omega_a \tau_p \approx 2$, where $\Delta\omega_a$ is the medium gain bandwidth. Therefore the maximum power that can be produced by unit area is given by

$$P_{th} = \frac{\hbar\omega}{2\sigma} \Delta\omega_a. \quad (26)$$

From this expression, we can easily derive the maximum obtainable intensity by focusing this power on a spot size limited by the laser wavelength.

$$I_{th} = \frac{\hbar\omega^3}{8\pi^2\sigma} \frac{\Delta\omega_a}{c^2}. \quad (27)$$

The intensity limit presented in **Figure 1 and Figure 13**, represents the theoretical power per unit area of beam (cm^2)that could be obtained for different types of materials. Ti:sapphire and Yb:glass, P_{th} is 200TW and 3000 TW respectively per cm^2 of beam size and I_{th} of the order of 0.3 and 3 $10^{23}/cm^2$ for Ti:sapphire and Yb:glass

H. The λ^3 laser -the smallest relativistic laser (Albert, 2000)

Pulses with mJ energy and sub-10 fs duration when focused over a single wavelength can produce intensities over 10^{18}W/cm^2 well in the relativistic regime. This type of laser has just being demonstrated **(Albert, 2000)** and has the advantage to work at kHz repetition rates. We called this laser λ^3 laser, because all the energy is concentrated within an paraboloid with single wavelength dimension, i.e. one wavelength in transverse dimension and few wavelengths (cycles) along the propagation direction. It has a number of significant advantages. First, the laser being very stable and with a high repetition rate, we can investigate relativistic effects by observing small perturbations with lock-in detection. Second, the small spot size will offer a cut off to instabilities with feature sizes larger than the laser wavelength. Third, x and $\gamma-$rays, electron, proton etc. sources will have a higher spatial coherence, since spatial coherence scales with the inverse of the spot area. This quality is important for most applications such as x-ray, electron imaging, diffraction, x-ray holography, electron and proton injection, etc. Also, it is expected that the shortness of the pulse will produce a more coherent interaction between the laser field and the electrons, leading to a more efficient laser–particle coupling and higher quality sources.

I. The Largest Relativistic Laser: The Zettawatt Laser (Tajima & Mourou 2002)

Considering today's technology what could be the most powerful laser that we could be built. The power of such a laser would be limited by the available pump source. The largest laser that could be used as a pump, at present is the National Ignition Facility (NIF) in the US and the Laser Megajoule in France. Working at 2ω and with 10-20ns long pulses this laser could produce 5MJ of pump light. Using Ti:sapphire as amplifying medium and working at few times the saturation fluence we could expect a 30% overall efficiency, or 1.5MJ before compression. The beam cross-section at few J/cm^2 would be around 10m. Assuming that we could compress the beam over 10fs, with a 70% efficiency compressor we would obtain a power close to $0.1\cdot10^{21}$W or 0.1 zettawatt. If focused by a well corrected parabola of the same type as the Keck telescope that has also a ≈10 m diameter, a micrometer spot size with a power density of $\approx10^{28}$W/cm^2 (an intensity level very close to the critical field (Schwinger field) could be produced. We are in a situation similar to where we were fifteen years ago when the first table top terawatt laser was demonstrated. At that time we announced in a paper "En route vers the Petawatt" (**Maine, 1988)** that by using the largest developed laser at the time like Nova at LLNL or Omega at LLE, Rochester, petawatt pulses could be produced. Ten years later the Petawatt was demonstrated by M. Perry and his colleagues at LLNL and today twenty Petawatt lasers have been built or scheduled to be built.

J. New amplification techniques: Plasmas Compression

New ideas are being proposed as ways to overcome the limit of few joule/cm^2 imposed by the saturation fluence of the amplifying components and/orthe dielectric breakdown of materials in CPA systems. May be the most elegant scheme has been plasma compression by stimulated Raman backscattering (**Malkin et al, 1999; Shvets et al, 1998**). In this concept a long pulse transfers his energy to a contrapropagating one through the process of stimulated Raman backscattering **(Figure 14**). Because the medium, a plasma is already broken down, it will not be limited by damage considerations and will be able to accept much higher fluences as high as few 1000J/cm^2 instead of few joules/cm^2 with conventional CPA). Such system also does not require large and expensive gratings.

K. Average Power

Ultimately most ultra-high intensity applications will require high average powers. CPA laser combined with materials with excellent thermal conductivity such as Ti: sapphire has improved laser average power by two to three orders of magnitude (see figure 12. Table top femtosecond excimer and dye lasers had typical average powers in the mW range. CPA systems have been demonstrated over a wide range of repetition rates from the MHz **(Norris, 1992)** to the mHz for the petawatt. Their average power is almost independent of their repetition rates and is typically of the order of one watt **(Figure 15).** Using thermal lens **(Salin, 1997)** and cryogenic cooling of the amplifier **(Backus, 1997**) much higher average power in the 10 W regime has been demonstrated. At cryogenic temperature the thermal conductivity of Ti: saphire becomes as good as copper. Also, at that power level the absorption in the grating becomes significant. The thermal effect deforms the grating surface, producing a deterioration of the beam quality. Applications in high energy physics for instance, i.e. neutrino beams production, γγ collider will require average power in the MW range. With the impressive advances in laser diode power, high efficiency gratings, and new broadband materials MW average power, one day could be envisaged.

1. Ultrahigh Intensity laser regimes: Extending the field of laser physics form the eV to the TeV

As mentioned previously, the progresses in high intensity lasers has been so rapid that it is necessary to redefine the different regimes of intensities. We refer to high intensity when the laser field *E* fulfills the following condition:

$$\hbar\omega < m_e c^2\left[\sqrt{1+a_0{}^2}-1\right] < m_e c^2 \tag{28}$$

Here $m_e c^2\sqrt{1+a_0{}^2}$ is the ponderomotive potential, in the limit $a_0 \gg 1$ it is equal to $eE_0\lambda/2\pi$ and for $a_0 \ll 1$ it is $e^2E_0{}^2/2\omega^2 m_e$, $\hbar\omega$ the photon energy and mec2 the rest mass energy of the electron, $\lambda = 2\pi c/\omega$ the laser wavelength, *e* and m_e the electron charge and mass. This regime corresponds to intensities between $5{\cdot}10^{14}$ W/cm^2 and 10^{18}W/cm^2 for 1µm wavelength.

The ultrahigh intensity regime will be defined as the one above the 10^{18}W/cm^2 limit. That is where

$$eE_0\lambda > 2\pi m_e c^2 \tag{29}$$

For excimer wavelength at 248 nm the relativistic limit will be at 10^{19}W/cm^2 while for 10.6 µm for CO2 this limit will be at 10^{16} W/cm^2. Finally displayed on the figure 1 is the nonlinear QED limit reached for the laser field *E* such that

$$eE\lambda_c/2\pi > 2m_e c^2, \tag{30}$$

where $\lambda_c = \hbar/m_e c$ is the Compton length. Relation (30) shows that the work that the field has to produce over a Compton length λ_c set by the uncertainty principle to separate an electron-positron pair, must be greater than $2m_e c$. This regime corresponds to intensities greater than 10^{30} W/cm^2 for 1µm light. Let's recall that the laser field *E* is related to the intensity *I* by

$$E^2 = Z_0 I, \tag{31}$$

where $Z_0 = 377$ is the vacuum impedance in ohms.

The physics in the high intensity regime includes, high harmonic generation, multiphoton ionization, etc… It deals with bound electron nonlinear optics. This regime has been covered extensively by a number of excellent reviews (Bloembergen, 1965; Shen, 1984) and won't be addressed in this article.

The ultrahigh intensity regime has already produced a wealth of scientific results that are all related to the relativistic character of the electrons (Lindman, 1977). In laser-atom interactions the work at high intensity has been based on the non-relativistic Schrödinger equation and dipole approximation. Trying to extend the laser-atom interaction in the relativistic regime would require solving the time-dependent Dirac equation (Keitel, 2001; Milosevic, Krainov, Brabec, 2002; Chirila, 2002; Bandrauk, 2003). The laser-plasma interaction in the ultrahigh intensity regime leads to a panoply of new phenomena like x-generation (Kiefer et al., 1992; Kmetec et al., 1992; Beg et al., 1997) and γ-ray generation Norreys, 1999, relativistic self focusing (Max, 1974; Sprangle, 1987; Borisov, et al., 1992; Gibbon, 1995; Chen, 1998; Fuchs, 1998), high harmonic generation (Bulanov et al., 1994; Lichters et al., 1996; Zepf et al., 1998; Von der Linde, 1997, Tarasevich, et al., 2000), electron (Clayton, 1993; Modena, et al., 1995; Nakajima et al, 1995; Umstadter 1996; Wagner, 1997; Gordon 1998; Chen 1998; Malka et al., 2002) and proton (Krushelnik et al., 1999; Sarkisov, 1999; Zhidkov 1999; Esirkepov 1999; Bulanov 2000; Maksimchuk 2000; Clark 2000; Snavely et al., 2000) acceleration, neutron (Pretzler 1998; Disdier, et al., 1999) and positron (Gahn et al., 2000) production, as well as the demonstration of nonlinear QED: Bula et al., 1996, Burke et al., 1997.

III. On the similarities and differences between bound-electron and relativistic nonlinear optics

The classical treatment of classical linear and nonlinear optics deals with the electron displacement *x(t)* around the nucleus. This displacement gives rise to the polarizibility

$$\mathbf{P}(t) = Ne\mathbf{x}(t), \tag{32}$$

where *N* is the electron density. The force applied to the electron is the Lorentz force

$$\mathbf{F}(t) = e\mathbf{E}(t), \tag{33}$$

in which in the classical limit we neglected the magnetic field part due to smallness of the *v/c* ratio. In the linear regime, **F***(t)*is proportional to the displacement **x***(t)*. As the displacement increases the proportionality between **x**(t) and **E***(t)* is not respected anymore and is at the origin of all the nonlinear optical effects, harmonic generation, optical rectification, etc… mentioned above. As the laser intensity is increased to the ultrahigh intense level, the electron velocity will approach the speed of light and in the Lorentz force

$$\mathbf{F}(t) = e\left[\mathbf{E}(t) + \frac{1}{c}\mathbf{v}\times\mathbf{B}(t)\right] \tag{34}$$

the term $\mathbf{v}\times\mathbf{B}/c$ can not be neglected. Because of the combined action of the **E** and **B** fields the electron will follow a complicated trajectory. For a linearly polarized light this trajectory is a figure eight in the frame moving at the average electron velocity as it is explained in the *Classical Theory of Fields* by Landau and Lifshits (1975). The normalized vector potential quantity

$$a_0 = eA / m_e c \tag{35}$$

means the quivering momentum normalized to $m_e c$. Here A is the electromagnetic vector potential. The longitudinal displacement is proportional to a_0^2 whereas the transverse displacement scales as a_0. In the reference frame where the charged particle initially is rest for $a_0<1$ its transverse momentum is larger than the longitudinal one whereas for $a_0>1$ the situation is obviously reversed and the longitudinal momentum becomes much larger than the transverse one. This complicated electron motion is the source of the relativistic nonlinear effects like relativistic rectification, relativistic self-focusing, harmonic generation, etc.

A. Relativistic rectification

This effect is known in the literature as plasma wake field effect. It was introduced by Tajima and Dawson, 1979. The idea is to introduce a stable method of exciting large amplitude fast waves, whereas the previous collective acceleration methods (see pioneering works on the collective acceleration mechanisms by Budker, 1956, Veksler, 1956) seffer from instabilities involving ions (Mako and Tajima, 1984). It was further theoretically studied by Gorbunov and Kirsanov (1987), by Bulanov, Kirsanov and Sakharov (1989), and by Sprangle et al., 1988. By analogy with the well-known optical rectification process in conventional nonlinear optics it is tempting to call it relativistic rectification.

In the plasma, the electrons are strongly pushed forward, due to the $\mathbf{v}\times\mathbf{B}$ force. They drag behind the much more massive ions setting up a large electrostatic field parallel to the laser propagation direction. This field is extremely large and of the order of magnitude of the laser transverse field. The $\mathbf{v}\times\mathbf{B}$ term "transforms" the laser field into ***a longitudinal electrostatic field with an amplitude equivalent to the laser transverse field.*** This is a remarkable result if we consider that for the longest time laser researchers recognized the enormous amplitude of the laser transverse field and tried to flip a fraction of this field along the transverse direction using various schemes (see e.g. Byer, 2002; Schaechter et al., 2002; Apollonov et al., 1998). In the relativistic regime this conversion done in plasma is automatic and efficient. If we consider that harmonic generation is the hallmark of bound electron nonlinear optics, relativistic rectification seems to be the most prominent effect of relativistic optics. In addition, it is interesting to note that optical rectification in classical nonlinear optics is not often used. It occurs only in non-centrosymmetric crystal and is not very efficient. Because of the noncentro symmetricity the charge are pushed unequally in the direction perpendicular to the propagation axis to produce a net electrostatic field perpendicular to the propagation axis. In relativistic optics it is the converse. The rectified field is longitudinal. It is produced in centro-symmetric media – plasmas – and is efficient.

Relativistic intensities can produce extremely large electrostatic fields. For example for I=10^{18}W/cm^2 we could produce- see equation (34) - an electrostatic field up to 2TV/m and 0.6PV/m for 10^{23}W/cm^2. These values are gargantuan. To put them in perspective they correspond to SLAC (50GeV) on 100μm. It is mesmerizing to think that if we were able to maintain this gradient over one meter, a PeV accelerator using conventional technology as discussed by Fermi in 1954 that would circumvent the earth could fit on a table. One direct consequence of electron acceleration is proton/ion acceleration resulting from the electron pulse pulling the positively charged ions behind them to make a short proton pulse. Further details on relativistic rectification are developed in the sections devoted to Nuclear Physics and High Energy Physics.

B. Opportunity of the Relativistic Regime

There are two kinds of interaction between photons and charged particles. The first is well-known single particle interaction. In its most basic way, it takes the form of collision between a photon and an electron. The other is collective interaction between photons and particles, or between an intense laser and matter. This may be considered as the stream of photons and a collection of charged particles such as electrons. Both interactions become more intense, as the intensity of the laser is increased. It becomes particularly so when the intensity enters the relativistic regime. These two kinds of interaction may have a good analogue in the interaction between the wind and the water of a lake. When the wind is slow or gentle, the surface of the lake water is gently swept by the wind to cause a slow stream in the surface water via the molecular viscosity of water by the shearing wind molecules. This interaction arises from collisions between the flowing water molecules and originally stationary water molecules. When the wind velocity picks up, the wind begins to cause ripples on the surface of the lake. This is because the shear between the velocity of the wind and the originally stationary surface water becomes sufficiently large so that a collective instability (see e.g. Lamb, 1932, Chandrasekhar, 1961 and Timofeev, 1979) sets in. More detail studies of the wave generation on a water surface by wind (see Vekstein, 1998) show an analogy between the Landau damping of plasma waves and the resonant mechanism of wave generation on a water surface by wind. Due to this instability, the wind and water self-organize themselves in such a way to cause undulating waves on the surface, which cause a greater friction (which is called anomalously enhanced viscosity or in short anomalous viscosity) between the wind and water. When this commences, momenta of wind molecules are much more effectively transported to those of water molecules and the water stream becomes more vigorous.

In the single particle interaction for the stream of a large number of photons, they collide with electrons via a collision between the photon and the electron, which is called the Thomson scattering. Within the framework of the classical physics model the electron scatters the incident electromagnetic wave without any change in the frequency of the radiation in the reference frame where the electron is rest. The cross section of the scattering, the Thomson cross section, is given by

$$\sigma_T = \frac{8\pi}{3} r_e^{\,2} = 0.665 \cdot 10^{-24} cm^2, \qquad (36)$$

where $r_e = e^2 / mc^2 = 2.82 \cdot 10^{-13}$ cm is the classical radius of electron. The quantum theory, using the conservation lows of the energy and momentum, shows that the frequency and the wave vector of the scattered photons change as $\lambda\!\!\!^{-} = \lambda\!\!\!^{-}_0 + l_c(1-\cos\theta)$. Here $\lambda_0 = 2\pi\lambda\!\!\!^{-}_0$ and $\lambda = 2\pi\lambda\!\!\!^{-}$ are the wavelength before and after scattering, θ is the scattering angle, and $l_c = \hbar / mc = 3.86 \cdot 10^{-11}$ cm is the Compton length. The scattering cross section in this limit is given by the Klein-Nishina-Tamm formula (see Beresteskii, Lifshitz, Pitaevskii, 1982). When the flux of laser is shone on an electron, this causes a force on it

$$F \approx \frac{\sigma_T}{4\gamma^2} \frac{E_0^{\,2}}{4\pi}, \qquad (37)$$

where γ is the Lorentz factor of the electron, i.e. the electron energy grows as

$\mathcal{E} \propto (W \sigma_T t)^{1/3}$ (see Landau and Lifshits, 1980).

IV. Theory of relativistically strong electromagnetic and Langmuir waves in the collisionless plasma

In the small amplitude limit electromagnetic and Langmuir waves propagate through the collisionless plasma with their frequency independent of the amplitude. The frequency of the longitudinal Langmuir wave in the cold plasma $\omega_{pe} = \sqrt{4\pi \mathrm{ne}^2 / m_e}$, does not depend also on the wave vector of the wave, i. e. the Langmuir wave phase velocity is equal to $v_{ph} = \omega_{pe} / k$ and its group velocity $v_g = \partial\omega / \partial k$ is equal to zero. For the frequency of transverse electromagnetic wave we have $\omega = \sqrt{k^2c^2 + \omega_{pe}{}^2}$, i.e. its group and phase velocity are related each to other as $v_{ph}v_g = c^2$. In the case of finite amplitude waves the frequency depends on the wave amplitude as it was demonstrated in the paper by Akhiezer and Polovin (1956), where the exact solution to the problem of the propagation of relativistically strong electromagnetic waves in collisionless plasmas was found.

Using an assumption an unbounded cold collisionless plasma is described by Maxwell's equations and by the hydrodynamic equations of an electron fluid, we find that coupled electromagnetic and Langmuir waves are given by equations (see e.g. Kozlov et al., 1979, Farina, Bulanov 2001)

$$\phi'' = \frac{\beta_g}{1-\beta_g{}^2}\left(\frac{\psi_e}{R_e} - \frac{\psi_i}{R_i}\right), \tag{38}$$

$$a'' + \omega^2 a = a\frac{\beta_g}{1-\beta_g{}^2}\left(\frac{1}{R_e} - \frac{\rho}{R_i}\right). \tag{39}$$

Here the waves are assumed to depend on the coordinate and time via the variables $\xi = x - v_g t$ and $\tau = t - v_g x$. The normalized on $m_e c^2 / e$ electromagnetic and electrostatic potentials depend on ξ and on τ as $A_y + iA_z = a(\xi)\exp(i\omega\tau)$ and $\phi = \phi(\xi)$. A prime in equations (38, 39) denotes a differentiation with respect to the variable ξ. In these equations normalized group velocity of the electromagnetic wave (it is a phase velocity of the Langmuir wave) is $\beta_g = v_g / c$, the electron to ion mass ratio $\rho = m_e / m_i$, functions $\psi_e = \Gamma_e + \phi$, $\psi_i = \Gamma_i - \rho\phi$, $R_e = \sqrt{\psi_e{}^2 - (1-\beta_g{}^2)(1+a^2)}$, $R_i = \sqrt{\psi_i{}^2 - (1-\beta_g{}^2)(1+\rho^2 a^2)}$. Constants Γ_e and Γ_e must be specified by the boundary conditions at infinity. If the amplitude of the electromagnetic wave at $x \to \pm\infty$ is finite ($a = a_0,\ \phi = 0$), and the plasma is at rest, then we have $\Gamma_e = \sqrt{1 + a_0{}^2}$ and $\Gamma_i = \sqrt{1 + \rho^2 a_0{}^2}$. The density and the energy of the α –species (α=e,i) particles are equal to

$$n_\alpha = \beta_g \frac{(\psi_\alpha - \beta_g R_\alpha)}{R_\alpha(1-\beta_g{}^2)}, \qquad \gamma_\alpha = \frac{\psi_\alpha - \beta_g R_\alpha}{1-\beta_g{}^2}. \tag{40}$$

The set of equations (38) – (39) admits the first integral

$$\frac{1-\beta_g^{\,2}}{2}(a'^2+\omega^2a^2)+\frac{1}{2}\phi'^2+\frac{\beta_g}{1-\beta_g^{\,2}}\left(R_e-\beta_g+\frac{R_i-\beta_g}{\rho}\right)=\text{const}. \qquad (41)$$

For $a=a_0=0$, the set of equations (38) – (39) describes a longitudinal plasma wave. In this case the integral (41) gives the relationship between the electric field and the particle energies: $E^2+2(\gamma_e+\gamma_i/\rho)=\text{const}$. The amplitude of the Langmuir wave can not be arbitrarily large. It is limited by the condition $R_\alpha>0$. At $R_\alpha=0$ the particle density tends to infinity. This is the wave breaking point. As it has been shown by Khachatryan, 1998 and Gorbunov, Mora and Ramazashvili, 2002, the ion influence on the wave breaking limit is small being given by the terms of the order of ρ. When the wave is slow, i.e. $\beta_g\ll 1$, the wave-breaking amplitude is equal to $E_m=\beta_g$, as it has been discussed by Dawson, 1959. In the generic case when $\gamma_g=1/\sqrt{1-\beta_g^{\,2}}$ can be arbitrarily large, the maximum value of the electric field in the wave is

$$E_m=\sqrt{2(\gamma_g-1)}. \qquad (42)$$

This field is called the Akhiezer-Polovin limiting electric field. It was this that Tajima and Dawson (1979) recognized that the fast wave does not break (easily) because electron momentum increases while its velocity is still at *c*. At the wave-breaking the electron velocity becomes equal to the Langmuir wave phase velocity. This condition is equivalent to the equality $\gamma_e=\gamma_g$. The role of the thermal motion of the electrons on the Langmuir wave breaking has been discussed by Katsouleas and Mori, 1988 and by Khachatryan, 1998.

Another important characteristic of nonlinear wave is the dependence of its frequency (and the wavelength) on the wave amplitude. In cold plasma the wavelength of weak Langmuir wave is $\lambda_p=2\pi\beta_g c/\omega_{pe}$. In the ultra-relativistic case ($\gamma_e,\gamma_g\gg 1$) the wavelength is about $4\lambda_p\sqrt{2\gamma_e}$, where $\gamma_e\le\gamma_g$. We see that the relativistic nonlinearity effects lead to the increase of the wavelength. However, the effects of the ion motion decrease the Langmuir wave wavelength as it has been discussed by Khachatryan, 1998, Bulanov et al., 2001, and Gorbunov, Mora and Ramazashvili, 2002.

As we have seen above, the Langmuir wave break occurs when the quiver velocity of the electrons becomes equal to the phase velocity of the wave. In a plasma with inhomogeneous density, the Langmuir wave frequency depends on the coordinates, as a result, the wavenumber of the wave depends on time through the well known relationship (see Whitham, 1974) $\partial_t k=-\partial_x\omega$. The resulting growth in time of the wavenumber results in decrease of the wave phase velocity and leads to the break of the wave at the instant of time when the electron velocity becomes equal to the wave phase velocity, even if the initial wave amplitude is below the wave break threshold. In this case the wave break occurs in such a way that only a relatively small part of the wave is involved. We can use this property to perform a gentle injection of electrons into the acceleration phase as it was shown by Bulanov et al, 1998 (see also Refs. Hemker, et al., 2002;). In a similar way the Langmuir wave breaking may occur in the non-one-dimensional configurations (see Dawson, 1959, Bulanov et al., 1997) and due to the dependence of the relativistically strong Langmuir wave frequency on its amplitude analyzed by Drake et al., 1976 and by Bulanov et al., 1997.

For a circularly polarized transverse electromagnetic wave with $a = a_0$ and $\phi = 0$ we can easily obtain from equation (39) that the frequency as a function of the wave amplitude and velocity is given by $\omega^2 = \gamma_g^2(1/\Gamma_e + \rho/\Gamma_i)$. This expression may be rewritten in the form containing the wavenumber k : $\omega^2 = k^2 + 1/\sqrt{1+a_0^2} + \rho/\sqrt{1+\rho^2 a_0^2}$. We see that the relativistic effects and the ion motion modify the plasma frequency. The electron in the transverse electromagnetic wave moves along a circular trajectory with the energy $\sqrt{1+a_0^2}$. Its longitudinal momentum is equal to zero and the transverse component of the momentum is equal to a_0.

In the linearly polarized waves in plasma the transverse and the longitudinal motion of electrons are always coupled as was shown by Akhiezer and Polovin, 1956 and Chian, 1981. In the small but finite amplitude a_0 linearly polarized wave the transverse component of the electric field oscillates with the frequency $\omega \approx k^2 + 1 - a_0^2 / 4\sqrt{k^2+1}$, the longitudinal component of the electric field oscillates with the double frequency and its amplitude is of the order of a_0^2.

A. Wake Field Generation and Relativistic Electron Acceleration

On the other hand, just like a sufficiently strong wind has induced the instability at the surface of the water and its subsequent waves and anomalous viscosity, a sufficiently intense laser pulse (or photon flux) induces a wave in the plasma, the plasma wave (or the Langmuir wave, the longitudinal wave, that has been mentioned above as a relativistic rectification). In this case the photon flux can now cause a ‘ripple’ in the plasma that can now act as a collective force to drag (accelerate) electrons. This wave is called the wakefield (or laser wakefield), as it is induced as a result and in the wake of (i.e. behind) the laser pulse.

The wakefield excitation within the framework of the approximation of given laser pulse is described by equation (38), where the terms ψ_α and R_α in the right hand side contain a given function $a(\xi)$. The wake filed is excited by the nonlinear force of the laser electromagnetic fields, which is called the ponderomotive potential:

$$\Phi = m_e c^2 a_0^2 / e. \qquad (43)$$

in the case when $a_0 \ll 1$. In this 'weak field' limit the reason why the ponderomotive force is proportional to the square of the laser field (a_0^2) is that the force $\mathbf{v} \times \mathbf{B}$ is proportional to $\mathbf{v} \times \mathbf{E}$, where $\mathbf{E}$ and $\mathbf{B}$ are the laser electromagnetic fields. When a_0 is sufficiently large (or arbitrary),

$$\Phi = m_e c^2 \gamma_e / e. \qquad (44)$$

As we can see from equation (38) in the case of immobile ions ($\rho \to 0$) the electrostatic potential in the wakefield wave is bounded by $-1 < \phi < a_m$ with a_m the maximum value of the laser pulse amplitude (see Bulanov, Kirsanov and Sakharov, 1989). On the contrary, from equation (38) we see that the effect of the ion motion restricts the potential ϕ between the two bounds $-1 < \phi < \min\{a_m, \rho^{-1}\}$. From this equation we can also find that behind a short laser pulse the wavelength λ_{W-F} of the wake wave and the maximum value of the electric field E_{W-F} and of the potential ϕ_{W-F} scale as

$$\lambda_{W-F} = 2^{3/2} a_m, \quad E_{W-F} = 2^{-1/2} a_m, \quad \phi_{W-F} = a_m^2. \qquad (45)$$

for $-1 < a_m < \rho^{-1/2}$, and as

$$\lambda_{W-F} = 2^{1/2} / (\rho a_m), \quad E_{W-F} = 2^{-1/2} a_m, \quad \phi_{W-F} = \rho^{-1} \qquad (46)$$

for $a_m > \rho^{-1/2}$.

As we have seen above, the ion motion effects come into play and modify the transverse electromagnetic wave, when its amplitude becomes larger than ρ^{-1}. For the electron-proton plasma and the $1\,\mu m$ laser this corresponds to the radiation intensity $I = 4.7 \cdot 10^{24} W / cm^2$. However, in the wakefield generation and evolution the ion motion becomes important at much lower laser pulse intensity, when $a_m > \rho^{-1/2}$. It results to the wake field wavelength decrease with increasing laser pulse amplitude. This limit corresponds to the substantially lower laser intensity $I = 2.5 \cdot 10^{21} W / cm^2$.

In dimension units the excited wakefield is

$$E_{W-F} = \frac{m_e c \omega_{pe}}{e} f(a_m, \gamma_g). \qquad (47)$$

Here $f(a_m, \gamma_g)$ is a function that depends on the laser pulse form and the amplitude as well as on the plasma density. The field $E_{W-F,0} = m_e \omega_{pe} c / e$ is the Tajima-Dawson field at which a wave with a nonrelativistic phase velocity would break (resulting in so-called ‘whitewaves’, as captured in Hokusai’s immortal Ukiyoye) and a wave acquires density modulations near 100% or more (Tajima and Dawson, 1979). Because of the above, sometimes $E_{W-F,0}$ is called the wave breaking field, but it really does not the case of relativistic regimes, where the wavebreaking is alleviated by the relativistic phase velocity of the wave. This is one of several points that have been suggested by Tajima and dawson(1979). In this case since the phase velocity of the wakefield is equal to the laser pulse group velocity ($v_g = c\sqrt{1-\omega_{pe}{}^2/\omega^2}$) we have for the parameter γ_g : $\gamma_g = \omega/\omega_{pe} = \sqrt{n_{cr}/n_0}$, where n_0 is the electron density of the plasma and the critical density is $n_{cr} = \omega^2 m_e / 4\pi e^2$. Thus the intensity of collective accelerating field is immense and for fixed amplitude of the laser pulse and below the wave-break limit the wakefield scales as

$$E_{W-F,0} = (n_0 / 10^{18} cm^{-3}) GeV / m. \tag{48}$$

When the laser pulse amplitude is larger than the wavebreak limit, i.e. it is larger than $2\sqrt{\gamma_g - 1}$, a stationary wakefield does exist. However, in this regime for a finite time the laser pulse can generate in the plasma the electric field substantially higher than the field given by expressions (45) and (48). This corresponds to the electron acceleration behind the laser pulse in the near-critical plasma as it has been discussed by Bulanov, Kirsanov and Sakharov, 1991, Tseng et al., 1997, Gordon et al., 1998, Liseikina, Califano, Vshivkov, et al., 1999, Nagashima, Kishimoto and Takuma, 1999, Trines et al, 2001.

It should be noted that the ponderomotive potential of the laser pulse can exert strong force on electrons either directly (i.e. by the electromagnetic fields of the laser themselves) or via the electrostatic field such as the wakefield. For relativistically strong laser fields ($a_0 > 1$) the accelerating field increases in proportion to the square root of the laser intensity *I*. At the same time, the interaction time between the laser and electron gets longer, as the electron velocity along the laser propagation (the *x*-direction) approaches *c*, which is proportional to a_0 . As a result the energy (or momentum) gain $\Delta\mathrm{E}$ in the laser-electron interaction in the relativistic regime in a homogeneous plasma an ultrarelativistic particle in a moderately strong plasma wave acquires an energy of the order of

$$\Delta\mathrm{E} = eE_{W-F} l_{acc}, \tag{49}$$

where l_{acc} is the acceleration length, Tajima and Dawson, 1979,

$$l_{acc} = \frac{2c}{\omega_{pe}} \gamma_g{}^2 = \frac{2c}{\omega_{pe}} \left(\frac{\omega}{\omega_{pe}} \right)^2 . \tag{50}$$

This length is approximately $\left(\omega / \omega_{pe}\right)^2$ times larger than the plasma wave length. We note that this result has been obtained in the limit of a small amplitude wakefield. In the case of a relativistically strong wakefield the acceleration length is $l_{acc} = (2c / \omega_{pe})\gamma_g{}^2 a_0$. The maximum energy of accelerated particles is limited by the constraint imposed due to the plasma wave breaking: it is $\Delta E_{max} = 4m_e c^2 \gamma_g{}^3$ (Esarey and Pilloff, 1995).

The wakefield acceleration of the electrons has been observed in the experiments by Nakajima et al., 1995, Modena et al., 1995. State-of-the-art on 2002 fast electron energy detected in Ref. Malka et al., 2002.

The plasma inhomogeneity, depending on its form, can either detoriate or improve the acceleration conditions. In an inhomogeneous plasma with a density that depends on the coordinate as $n_0(x) = n_0(L / x)^{2/3}$; $L \approx (c / 3\omega_{pe})(\omega / \omega_{pe})^2$, the acceleration length becomes formally infinite and the particle energy growth is unlimited,

$$\Delta E = m_e c^2 \left(\frac{\omega}{\omega_{pe}}\right)^2 \left(\frac{x}{L}\right)^{1/3} . \tag{51}$$

As we have seen above the electron energy gain in the regime, when the wakefield is under the wave-breaking threshold, scales as

$$\Delta E = m_e c^2 a_0 . \tag{52}$$

The snowplow acceleration discussed in Tajima (1985) finds in fact the energy gain is proportional to $a_0{}^2$. This scaling, as explained above, arises from the ponderomotive potential and sometimes called the direct acceleration (Landau and Lifshits, 1971, Felddman and Chiao, 1971, Lai, 1980, Hartemann et al, 1995, Rau, Tajima, Hojo, 1997, Pukhov et al. 1997; Salamin and Faisal, 1997, Quesnel and Mora, 1998, Hartemann et al. 1998, Narozhny and Fofanov, 2000), the Dirac acceleration (Nakajima et al. 2002), and other names as well, but the basic acceleration kinematics is the same. This scaling is a tremendous blessing when we increase the intensity of laser and enter the relativistic regimes. Instead of the laser quivering energy, which scales as $m_e c^2 a_0$, the longitudinal electron energy scales as $m_e c^2 a_0{}^2$. This is one of the opportunities that the relativistic laser-matter interaction brings to us. Similar acceleration mechanism of has been considered by Gunn and Ostriker, 1969, as to be responsible for the production of the ultra-high energy cosmic rays during their interaction with the strong electromagnetic radiation generated by the rotating neutron stars. The underlying reason for this sharply favorable energy gain tendency is the following. In the relativistic regime, electrons begin to move as a stretched coil (deviating from the Figure 8 orbit mentioned above in weakly or mildly relativistic regimes) and proceed more and more forward in the direction of the light propagation. This allows the electron motion more coherent with photons. This in turn makes the interaction more efficient and stronger, as the laser intensity increases in this ultrarelativistic regime. In an infinite plane geometry, even a strong electromagnetic wave interaction amounts to no energy gain of an electron according to the Woodward-Lawson theorem (Woodward, 1979; Lawson 1947). We notice that the vacuum acceleration of the electrons up to MeV energy has been observed in the experiments by Malka et al., 1997. However, the condition for the theorem may be broken for a variety of ways, such as radiation damping due to the intense acceleration, due to the external magnetic field (Davydovskii, 1963, Roberts and Buchsbaum, 1964, Kolomenskij and Lebedev 1966, Apollonov et al, 1988), or due to extraction of the fast particles in the appropriated phase by means of the thin foil (Vshivkov et al., 1998). It is also worth noting that until the laser intensity exceeds 10^{22} W/cm^2, ions are left out in the action in the interaction with the laser, as they are too inertial. This leaves the laser-matter interaction in our problem nearly solely due to the electron dynamics. This is radically different from typical plasma physics situations in which both ions and electrons are allow to move simultaneously. It is this simultaneous motion of these two species that bring in a host of destructive plasma instabilities (Mikhailovskii, 1992). Instead, the plasma instabilities in our regime, as we shall see below, are, more often than not, self-organizing in nature. This is another very significant feature of the relativistic regime of intense short pulse laser-plasma interaction.

B. Relativistic self-focusing

Probably the most impressive nonlinear phenomenon in an underdense plasma is the self-focusing of the laser radiation. The self-focusing, discovered by G. A. Askar'yan in 1962, appears due to the nonlinear change of the refractive index of the medium in the region where a high intensity electromagnetic wave propagates. In the relativistic laser pulse - plasma interaction self focusing appears due to the relativistic increase in the electron mass and to the plasma density redistribution under the action of the ponderomotive force. This effect has been predicted in the 60’s and 70’s by Litvak, 1969, Max et al., 1974, Schmidt and Horton, 1985, but had to wait the advent of ultrahigh intensity lasers to be demonstrated **(**Borisov et al, 1992**)**. The threshold (critical) power for relativistic self-focusing is, Sun et al, 1987; Barnes et al., 1987,

$$P_{cr} = \frac{m_e c^5 \omega^2}{e^2 {\omega_{pe}}^2} \simeq 17 \left(\frac{\omega}{\omega_{pe}} \right)^2 \mathrm{GW}. \qquad (53)$$

The laser pulse can be self-focused over a distance much larger that the Rayleigh length

$$Z_R = \pi {w_0}^2 / \lambda. \qquad (54)$$

Here w_0 is the laser pulse waist at the focus.

The self-focusing of initially almost homogeneous wave field corresponds to the development of the filamentation instability. If the wave amplitude is initially slightly modulated in the transverse direction, then the modulation of the refractive index causes the wave fronts to curve. This results in the redistribution in the electromagnetic field energy in the transverse direction so that the modulation amplitude increases, i.e., the instability develops. The filamentation instability can be described by linearizing the set of relativistic electron hydrodynamics – Maxwell equations and assuming the variables to be in the form $\sim \exp[-i(\omega_0 + \Omega)t + ik_0 x + iQ_\perp r_\perp]$, where the unperturbed wave frequency and wavenumber are related as $\omega_0 = \sqrt{k_0{}^2 c^2 + \omega_{pe}{}^2}$. As a result we obtain for the instability growth rate the dispersion equation

$$\Omega = \frac{Q_\perp}{2k_0}\sqrt{Q_\perp{}^2 c^2 - \omega_{pe}{}^2 |a_0|^2}. \tag{55}$$

Here $Q_\perp$ is the transverse wavenumber of the perturbation. The instability develops (i.e., the perturbation frequency Ω is imaginary) if $Q_\perp < Q_{\perp,max} = |a|\,\omega_{pe}/c$. For $Q_\perp > Q_{\perp,max}$, diffraction prevails and the instability is suppressed.

The relativistic filamentation instability leads to the relativistic self-focusing of the laser-beam. In the weakly relativistic case ($|a| \ll 1$), the condition for the relativistic refraction to dominate over diffractive spreading is $P > P_{cr}$ (see (53)). It is easy to verify that this condition is the analog of the above condition for the filamentation instability with $Q_{\perp,max} \approx 1/w_{p,0}$, (where $w_{p,0}$ is the initial laser spot size). For $P = P_{cr}$, diffractive spreading of the laser beam is balanced by the radial inhomogeneity of the plasma refractive index caused by the relativistic increase in the electron mass. For $P > P_{cr}$, relativistic self-focusing overcomes diffractive spreading and, in the cubic-nonlinearity approximation, the axially symmetric beam is focused into a field singularity (the transverse size of the laser beam tends to zero and the amplitude of the laser field tends to infinity) in a finite time

$$t_{s-f} = \frac{Z_R}{c}\sqrt{\frac{P}{P_{cr}} - 1}, \tag{56}$$

where Z_R is the Rayleigh length (54). If $P \gg P_{cr}$, depending on the initial radial intensity profile, the laser beam can split into several filaments, each of which can undergo catastrophic self-focusing.

The propagation of a relativistically strong ($|a| \geq 1$) short pulse (or of a long pulse with a sharp leading edge), is accompanied from the very beginning by the excitation of a strong wakefield. In this more involved situation self-focusing cannot be studied separately from other dynamical processes, including pulse self-modulation, generation of a strongly nonlinear wakefied, erosion of the leading edge, etc. At present, a consistent analytical theory of relativistic self-focusing and filamentation of ultrashort superintense laser pulses is still lacking, because the nonlinear evolution of an electromagnetic wave in an underdense plasma has been studied under various simplifying assumptions, such as pulse circular polarization, quasistatic approximation and weak nonlinearity, Litvak, 1967, Sun et al., 1987, or within the framework of the paraxial approximation, Barnes et al., 1987; Bulanov and Sakharov, 1991. Particularly, linearly polarized pulses are more complex to study because the analytic simplifications that follow in the case of circularly polarized pulses from their lack of harmonic content do not apply. In addition the intensity of petawatt power laser pulses is so high that we cannot take advantage of the weak nonlinearity approximation. Appreciably large amount of information on the dynamics of self-focusing of such pulses is provided by computer simulations (see, e.g., Askar'yan et al., 1994, Askar'yan et al., 1995, Pukhov and Meyer-ter-Vehn, 1996, Tzeng, Mori, Decker, 1996, Tzeng and Mori, 1998, Chessa and Mora, 1998, Naumova et al., 2002).

As is well known, in 3-D plasma configurations the role of nonlinearity becomes more important than in 1-D and 2-D cases because in 3-D configurations the phenomenon of wave collapse results in the development of a 3-D singularity, Zakharov, 1972, Kuznetsov, Rubenchik, Zakharov, 1986, Kuznetsov, 1996.

To illustrate specific features of the laser light plasma interaction in three dimensional regimes, in **Fig. 16** we present the results of 3D-PIC simulations with a three-dimensional PIC code REMP (Esirkepov, 2001) of the laser beam propagation in an underdense plasma (Naumova et al., 2002). Some of these features were described by Honda et al., 1999. Pukhov and Meyer-ter-Vehn, 1996, have shown that the magnetic interaction, discovered in 2-D configurations by Askar'yan et al, 1994, plays an important role during relativistic self-focusing also in the 3-D case for circularly polarized light.

We consider the relativistic self-focusing of a linearly polarized semi-infinite laser beam in an underdense plasma with electric field in the *y*-direction. The dimensionless amplitude of the laser pulse is $a=3$, which corresponds, for a $1\mu m$ laser, to the intensity $I=1.25\cdot 10^{19} W/cm^2$. The pulse width is 12λ. The plasma density corresponds to $\omega / \omega_{pe} = 0.45$. The ion to electron mass ratio is equal to $m_i / m_e = 1836$. **Fig. 16** shows the relativistic self-focusing of a linearly polarized semi-infinite. We see the formation of a narrow self-focusing channel in the region between the leading part of the pulse, with pronounced filamentation, and the wide rear part of the pulse. The laser pulse distortion is asymmetric. This anisotropic self-focusing is illustrated by the projections, shown in **Fig. 16,** of the surface of the constant value of the electromagnetic energy density on the x, z − plane (a) and on the x, y − plane (b). In the x, z − plane (which corresponds to the s-polarization plane) the distribution of the electromagnetic energy density is up-down symmetric with three filaments in the leading part of the pulse. The self-focusing in the s-plane is very similar to the self-focusing of the s-polarized laser pulse in the 2-D case, Askar'yan et al., 1994. On the contrary, in **Fig. 16 (b),** the projection on the x, y − plane (in the *p*- polarization plane) is asymmetric and we see that the leading part of the pulse starts to bend. The pulse bending mechanism is discussed Naumova, Koga, Nakajima et al., 2001.

The asymmetry of the self-focusing leads to a quite complicated internal structure of the laser pulse channel, as shown in **Fig. 16**. Here we present two-dimensional cross sections of the distribution of the *y*-component of the magnetic field (a), of the electron density in the x, y − plane (b) and in the x, z − plane (c) and of the ion density in the x, y − plane (d) and in the x, z − plane (e). The self-generated magnetic field (Fig. 16 (a)) changes sign in the symmetry plane, as discussed by Askar'yan et al 1994. In the distribution of the electron and ion densities shown in Figs. 16 (b-e), we see a high density thin plasma sheet.

The quasi-static magnetic fields have been observed in laser produced plasmas for moderate intensity of the laser radiation, Korobkin and Serov, 1962, Askar'yan et al., 1967, Stamper et al., 1971, Daido et al., 1986. They can affect the thermal conductivity and the long time range plasma dynamics (see e.g. Bell, 1994). Several mechanisms of magnetic field generation are discussed in the literature, including linear and nonlinear processes in plasma waves, Khachatryan, 2000, Gorbunov, Mora, and Antonsen, 1996, baroclinic effects Shukla, Rao, Yu, Tsintsadze, 1986, anisotropic electron pressure, Bychenkov, Silin and Tikhonchuk, 1990, spatial nonuniformity or time variation of the ponderomotive force, Sudan, 1993, inverse Faraday effect in a circularly polarized pulse, Steiger and Woods, 1971, Berezhiani, Mahajan, Shatashvili, 1997, Gorbunov and Ramazashvili, 1998, and the effect of the current produced by the electrons accelerated inside the self-focusing channels of the electromagnetic radiation, Askar'yan, et al., 1994, and at the plasma-vacuum interface in the overdense plasma, Daido, et al., 1986, Kuznetsov, et al., 2001. In the latter case plasma quasi-neutrality requires that the fast-electron current be canceled by a cold electron current of opposite sign. These oppositely directed currents repel each other. This repulsion and the increase in the magnetic field value are the manifestation of the current filamentation, Weibel, 1959, Bychenkov, Silin and Tikhonchuk, 1990, Pegoraro et al., 1996, Califano, et al., 2001, Honda, et al., 2000, Sakai, et al., 2002. Due to symmetry of the laser pulses, the quasi-static magnetic field reverses its sign at the laser beam axis. As a result it can focus charged particles, e.g., fast particles in a Laser Particle Accelerator Tajima and Dawson, 1979, and Bingham, 1994. In addition, in the Fast Ignitor concept of ICF (Tabak et al., 1994) the quasi-static magnetic field is expected to collimate superthermal electrons and to provide the energy transfer from the relatively low plasma density region where these electrons are produced by the laser pulse to the overdense plasma in the high-density core where they ignite the fuel.

In the relativistic regime of the laser radiation self-focusing the magnetic field generation becomes dynamically important. As a result we see the magnetic interaction of the self-focusing channels. Magnetic interaction appears due to the fact that the electrons accelerated inside a self focused laser pulse produce electric currents in the plasma and a quasi-static magnetic field associated with them. The attraction of the electric currents leads to the redistribution of the fast electrons. This in turn changes the refractive index because, due to the relativistic increase of the electron mass, the effective plasma frequency is smallest in the regions with the highest concentration of fast electrons. This process causes the high intensity laser radiation filaments to merge and provides a mechanism for transporting the laser energy over long distances. In order to estimate the strength of the magnetic field, we note that the velocity of the current-carrying electrons is limited by the speed of light c and write the channel radius as $R = \sqrt{a_0} d_e$, where $d_e = c / \omega_{pe}$. We obtain

$$B = \sqrt{a_0} m_e c \omega_{pe} / e, \tag{57}$$

which gives the field of order 1 gigagauss for typical values of the laser plasma parameters. The huge magnetic field – over 340 megagauss, in the interaction of the linearly polarized $I=9\cdot 10^{19}$W/cm^2 laser pulse with thin solid target has been measured by Tatarakis et al., 2002. In the case of the circularly polarized laser pulse – plasma interaction, the 7 megagauss magnetic field has been observed in the experiments by Najmudin,et al., 2001, where its generation has been attributed to the inverse Faraday effect.

Regarding the magnetic interaction of self-focused channels we observe that the merging of the self-focusing channels and the associated self generated magnetic field were already seen in the 2D PIC simulations presented by Forslund et al., 1985. The physical mechanism of the merging due to the attraction of the electric currents inside the filaments, and the subsequent change of the refraction index due to relativistic electron redistribution, was formulated by Askar'yan et al., 1984. This mechanism was later called "magnetic lensing" or "electron pinching" and discussed in many papers including Pukhov and Meyer-ter-Vehn, 1996, Borghesi et al., 1998, and by Ruhl, Sentoku, Mima, et al., 1999.

The self-generated magnetic field in laser plasma evolves into the structures, which are associated with the electron vortices as a consequence of the equation $\nabla \times \mathbf{B} = -4\pi e n \mathbf{v} / c$, Bulanov, Esirkepov, Lontano, et al., 1996. In this case the electron fluid vorticity is $\nabla \times \mathbf{v} = c \Delta \mathbf{B} / 4\pi e n$. The vortex row is shown in **Fig. 17**. Near the laser pulse this vortex row is symmetrical, but it is unstable against bending and is transformed into an antisymmetric configuration. The distance between the vortices is comparable to, or in their final stage even larger than, the collisionless skin depth. The vortex row moves as a whole in the direction of the laser pulse propagation with a velocity much smaller than the pulse group velocity. The velocity of the vortex row decreases with increasing distance between the vortex chains that form the row, Bulanov, Esirkepov, Lontano, et al., 1997.

Inside a stationary vortex, the radial component of the force due to the magnetic pressure and the centrifugal force of the electron rotation is balanced by the force due to the charge-separation electric field, Gordeev and Losseva, 1999. The electric current curried by the fast electrons forms the electron vortex chain in the electron time scale. During this period of time the ions can be assumed to be at rest. The vortices nevertheless can interact with their neighbor vortices, resulting in a redistribution of the quasistatic magnetic field. A typical timescale this regime corresponds to the whistler wave scaling.

As we have seen above the fast electron electric current is localized inside the self-focusing filaments. Since the net electric current of the filament is zero the electric current inside the filament core and the electric current in the filament shell have opposite signs. Oppositely directed electric currents repel each other. However, inside the core a dominant force corresponds to the self-pinching. These repelling and pinching forces act on the electron component of the plasma. The electrons shift radially producing the electric field due to electric charge separation. In turn the force which appears due to the charge separation field balances the repelling-pinching force. As a result on the ion component act the forces, which compress the ions in the inner region and push them away in the outer region of the filament. We use formula (57) to estimate the magnetic field value inside the filament. The magnetic pressure is balanced by the electric charge separation field if $B^2/8\pi = e\,\delta n\,\varphi$. Here $B^2/8\pi = e\,\delta n\,\varphi$. Here $e\,\delta n$ is the separation electric charge and φ is the electrostatic potential. The electrostatic potential is equal to $\varphi = 2\pi neR^2$ for $\delta n \approx n$. These estimations were done within the framework of the approximation of unmovable ions. Ions can be assumed to be at rest during a time approximately equal to $1/\omega_{pi}$, where $\omega_{pi} = \sqrt{4\pi ne^2/m_i}$. For longer times the ions start to move and as a first effect we see fast ions accelerated outwards by the electric field of the charge separation. They maximum energy equals $E_{\max} = e\varphi = 2\pi ne^2R^2 = m_ec^2(R/d_e)^2$, i. e. it is of the order of $m_ec^2a_0$, Sakai et al., 2002. Koga (2001) has worked on self-organized criticality phenomenon of the self-focusing and defocusing of intense laser beam propagation to explain the experiment (Nakajima, 2001)

C. Relativistic Transparency

The dependence of the relativistically strong electromagnetic wave frequency on its amplitude results in the relativistic transparency of overdense plasmas. A low frequency wave can propagate through the plasma if the plasma electrons do not screen the electric field of the wave. The condition for wave propagation implies that the convection electric current density $-env$ is smaller than the displacement current $\partial_t E / 4\pi$ in the wave, i.e. $en_0 v \leq \omega E / 4\pi$. In the nonrelativistic limit the electron quiver velocity is proportional to the wave electric field $v \sim eE / m_e \omega$, and the condition of transparency is equivalent to $\omega > \omega_{pe}$. In the ultrarelativistic limit the electron velocity can not exceed the speed of light $v \approx c$ and we can write that the plasma becomes transparent if $\omega > \omega_{pe} / \sqrt{a_0}$. This corresponds to the cutoff frequency $\omega_{pe} / (1 + a_0{}^2)^{1/4}$ of transverse electromagnetic wave described by equation (39) in the limit $a_0 \gg 1$.

A high-power laser pulse interacting with a very thin foil, modeled as a thin slab of overdense plasma, exhibits features that are not encountered either in underdense or in overdense plasmas, Vshivkov et al., 1998, and offers special experimental conditions for investigating the basic properties of the laser-plasma interaction (some of these features were discussed by Denavit, 1992). This problem has been the subject of experimental and computer studies, Giulietti et al., 1997, Miyamoto et al., 1997. These novel features become important when the foil thickness is shorter than, or of the order of, both the laser wavelength and the plasma collisionless skin depth. This interaction can be exploited in order to change the shape of the laser pulse. Shaping a laser pulse provides a method for exciting regular wake fields in a plasma leading to effective acceleration of charged particles. The present method is based on the relativistic dependence of the electron mass on the quiver energy. The leading and the rear parts of the pulse are reflected by the foil, which is relativistically transparent for the central part of the pulse where the intensity is higher. This process cuts out the outer part of the laser pulse and produces a sharp leading (and rear) edge, as is needed in order to generate a good quality wake field. The conditions for the foil to be transparent depend on the polarization and incidence angle of the pulse.

To study the interaction of relativistically intense electromagnetic radiation with a thin foil in the 1-D case we reduce the problem to the solution of the Cauchy problem for the wave equation with a nonlinear source, i.e., finally, to a system of ordinary differential equations for the electric field inside the foil. This approach is valid for an arbitrary incidence angle of the laser pulse, since, as discussed above, a Lorentz transformation to a reference frame moving along the foil can be used to reduce the problem of oblique incidence to that of normal incidence, Bourdier, 1983. In the moving frame all variables are assumed to depend only on time and on the coordinate perpendicular to the foil.

Here we apply this analytical model to study the relativistic transparency of the foil and to investigate how the form of the laser pulse changes depending on the foil thickness, on the plasma density inside the foil and on the amplitude of the pulse. Within this model the foil transparency is found to depend on the relative magnitude of the pulse dimensionless amplitude a and of the dimensionless foil parameter $\varepsilon_0 = 2\pi n e^2 l / m_e \omega c$ besides the pulse incidence angle and polarization. Here l is the foil width and n is the plasma density inside the foil plasma.

In the analytical model the foil is assumed to be infinitely thin and the wave equation in dimensionless units for the dimensionless vector potential $\mathbf{a}(x,t)$ is written in the form

$$\partial_{tt}\mathbf{a} - \partial_{xx}\mathbf{a} = \delta(x)\mathbf{j}(\mathbf{a}). \qquad (58)$$

We consider here the case of normal incidence of the wave. The term on the right hand side of equation (58) describes the electric current of the 1D electric charge and the delta function, $\delta(x)$ models its sharp localization at $x = 0$. The dimensionless rationalized electric current $\mathbf{j}(\mathbf{a})$ is a nonlinear functional of the vector potential $\mathbf{a}(x = 0, t)$ at the charge position $x = 0$. Using Green's functional method the dimensionless electric field $\mathbf{E}(x,t)$ and the magnetic field $\mathbf{B}(x,t)$ on the two sides of the foil can be written as

$$\mathbf{E}(x,\tau) = \mathbf{E}_0(x,\tau) - \mathbf{j}(\mathbf{a}(0,\tau)), \qquad (59)$$

$$\mathbf{B}(x,\tau) = \mathbf{B}_0(x,\tau) - \mathrm{sign}(x)\,\mathbf{e}_x \times \mathbf{j}(\mathbf{a}(0,\tau)), \qquad (60)$$

Here $\mathbf{E}_0(x,\tau)$ and $\mathbf{B}_0(x,\tau)$ are the electric and magnetic field of the incident pulse, $\mathbf{e}_x$ is the unit vector along x and $\mathbf{a}(0,\tau)$ is the vector potential at the foil at the retarded time, $\tau = t - |x| / c$. Equations (59) and (60) play the role of a nonlinear boundary condition for the electromagnetic waves at $x=0$.

The vector potential at the foil satisfies the ordinary differential equation

$$\dot{\mathbf{a}}(0,\tau) - \mathbf{j}(\mathbf{a}(0,\tau)) / 2 = \dot{\mathbf{a}}_0(0,\tau). \qquad (61)$$

This equation is equivalent to equation (59). The second term in the left hand side describes the effect of the radiation force in the 1D case. We see that this equation does not have high order derivatives with respect to time, contrary to the case of a point three-dimensional charge, where the equations of motion with the radiation force have unphysical "self accelerated solutions" (see discussion in the textbooks by Landau and Lifshits, 1971, by Ginzburg, 1979, and by Barut, 1980).

The dependence of $\mathbf{j}$ on $\mathbf{a}$ follows from the model that we adopt in order to describe the foil. We assume that the ions do not move under the action of the electromagnetic wave and take the electrons to be collisionless. Since we disregard charge separation effects, electrons are allowed to move in the (x, y) plane only and their density is taken to be constant. Using the conservation of the y, z components of the canonical electron momentum in the moving frame, we find that the electric current $\mathbf{j}$ takes the form

$$\mathbf{j}(\mathbf{a}) = -2\varepsilon_0 \mathbf{a} / \sqrt{1 + |a|^2}. \tag{62}$$

This expression can be used to obtain an approximate form of the transmitted and reflected fields through the foil, of the harmonic generation, including the generation of a quasi-steady DC current in the case of oblique incidence, and of the polarization change due to the relativistic nonlinearities.

1. Relativistic Foil Transparency and Pulse Shaping

Equations (59-62) indicate that the transmission through the foil depends on the pulse amplitude, polarization and on the dimensionless parameter ε_0. In the simple case of a circularly polarized pulse, $\mathbf{a}(x,t) = \mathbf{a}_0(t)\exp[i(x-t)]$, we can solve equations (61,62) by looking for solutions of the form $\mathbf{a}(0,t) = \mathbf{a}(t)\exp[-it]$, where we represent the two dimensional vector $\mathbf{a}(t)$ as a complex valued function $a_y + ia_z = \mathcal{A}(t)\exp[i\Psi(t)]$, with amplitude $\mathcal{A}(t)$ and phase $\Psi(t)$. If we assume that $\mathcal{A}(t)$ and $\Psi(t)$ are slowly varying functions of time and neglect the time derivatives, we find

$$\mathcal{A}(t) = \mathcal{A}(\varepsilon_0, a_0) = \frac{1}{\sqrt{2}} \sqrt{\sqrt{(1+\varepsilon_0^2 - a_0^2)^2 + 4a_0^2} - (1+\varepsilon_0^2 - a_0^2)} \tag{63}$$

and

$$\Psi = \Psi(\varepsilon_0, a_0) = -\arccos(\mathcal{A}/a_0). \tag{64}$$

We obtain the amplitude and the shape of the transmitted and of the reflected pulse. We find that the condition for the foil to be transparent to the electromagnetic radiation in the limit of moderate intensity $a_0 \ll 1$ is $\varepsilon_0 \ll 1$. It can be rewritten as $\omega \gg \omega_{pe}(l/2d_e)$ which differs from the transparency condition for uniform plasmas by the factor $l/2d_e = l\omega_{pe}/2c$. For relativistically strong waves with $a_0 \gg 1$, a foil with $\varepsilon_0 \gg 1$ is transparent if $a_0 \gg \varepsilon_0$. This condition can be written as $\omega \gg \omega_{pe}(l/2d_e a_0)$, while according to Akhiezer and Polovin, 1956, and to Kaw and Dawson, 1970, a uniform plasma is transparent to relativistically strong radiation if $\omega \gg \omega_{pe} / \sqrt{a_0}$ as we discussed above. The relativistic transparency of the overdense plasma slab has been studied also experimentally by Fuchs et al., 1997.

Let us now consider a laser pulse with amplitude varying along x . The amplitude is zero at the beginning of the pulse, increases up to its maximum value a_m and then decreases to zero. If $a_m > \varepsilon_0$ the portion if the pulse where $a < \varepsilon_0$ is reflected by the foil, while the portion with $a > \varepsilon_0$ propagates through the foil. The model for the foil response used above can also be used to study the dependence of the pulse transmission on its incidence angle and polarization. However this model is based on a number of approximations and their validity must be checked in the framework of a more detailed description such as Particle in cell (PIC) simulations. In **Fig. 18** we present the results of 3D PIC simulations of a laser-foil interaction Vshivkov et al., 1998. A circularly polarized pulse, of initial width $l_\perp = 10\lambda$, is shown before (left column), during (central column) and after (right column) its interaction with the foil. Row (a) gives the x,y dependence of the pulse electromagnetic energy density and shows that the pulse looses its outer part, where the amplitude is smaller than ε_0 , due to its interaction with the foil. This “peeling” of the pulse provides an example of the nonlinear relativistic transparency of the plasma foil. As a result of this peeling a pulse with a sharp edge is formed as shown in row (b). The energy absorbed by the particles in the foil is only a few percent of the total pulse energy. The pulse curves the foil and makes it concave. The modification of the foil shape (row c) acts as a concave mirror and focuses the reflected radiation into a narrow beam with a width much smaller than that of the incident pulse (d).

D. Relatisvistic self-induced transparency of short e.m. packets

The relativistic transparency of the overdense plasma can be considered as a self-induced nonlinear change of the refractive index of the plasma. In the limit of relatively low intensity radiation, McCall and Hahn, 1969, have first discussed the self-induced transparency of optical beams. They found a regime for the laser pulse to propagate with anomalously low energy loss while at resonance with a two-quantum-level system of absorbers, when the initial pulse has evolved into a symmetric hyperbolic-secant pulse function of time and distance, and has the area characteristic of a "2π pulse." Ideal transparency then persists when coherent induced absorption of pulse energy during the first half of the pulse is followed by coherent induced emission of the same amount of energy back into the beam direction during the second half of the pulse. A relativistic version of this in intense laser-matter interaction has been discussed by Mima et al. (1986) and Tajima (1987) and found a condition to form a triple soliton structure that allows no trace of the laser wake behind the pulse. (A similar idea was formulated by Kaw, et al., 1992). The idea is to provide two different colored lasers with specific profiles (one is peaking at the pulse center and the other lowering at the same point) in such a way to induce the beat at the front of the rising peak of the first laser, while the beat wave returns its energy back to the laser in its back. Furthermore, through such an arrangement, it was found that the group velocity of photons (and the velocity of the triple soliton) can be increased from that less than the speed of light c to beyond it (superluminous propagation). This idea may be extended by adopting an active lasing medium that is pumped up in advance of a short pulse laser. If the laser pulse length is taken in such a way to match the Rabbi period of the transition between the lasing electron levels, the laser can absorb energy from the active medium in its front portion, while the back loses it energy back to the medium. In a judicious choice of parameters (Fisher and Tajima, 1993; Schaechter, 1999) we can make the laser propagation speed from less than c in the medium to that equal or greater than c; proposals of superluminous laser propagation have been made in the atomic physicists' community(Raymond Chiao,1993))

E. Relativistic Solitons

In general, in the interaction between intense short-pulsed laser and matter the nonlinear interaction between these tends to enforce (or reinforce) the self-binding forces between these two, be it the longitudinal force (the Forward Raman instability, for example) or the transverse force (the self-focusing instability, for example) (see e.g. Bulanov et al., 2001). This is because for ultra-short laser pulses ions have too large inertia to respond to the laser and thus the interaction is void of ionic motion. Thus nearly all deleterious instabilities in a plasma need to involve ions and their simultaneous motion with electrons to set in. When only electrons move in a plasma, there remains a strong electrostatic restoring force from inertial ions. For example, in the self-focusing, the intense laser creates a density cavity because light accumulates near the axis and evacuate electrons radially outward. However, since ions remain in the central region where electrons are evacuated, this forms an ionic channel. In solitons whose phase velocity is close to the speed of light, this comment nearly always applies (see Kozlov et al., 1979, Kaw, et al., 1992). On the other hand, there is a class of solitons that have slow phase velocity (Marburger and Tooper, 1975, Esirkepov et al., 1998, Farina and Bulanov, 2001, Naumova, et al., 2001) that are coupled with ions. In such a structure, the above general stability argument is not applicable and we have to consider the problem more carefully. Nonetheless, in general the binding forces that constitute a stable soliton structure are the ponderomotive force of the radiation and the space charge force set up by electron charge separation.

The ponderomotive force displaces electrons away from the center of the soliton, while the electrostatic force retains electrons to it. On the other hand, ions are driven away from the soliton from the center due to this electrostatic force (Naumova, et al., 2001).

Since a long time the solitons have attracted a great attention because of their resilient, robust behavior, Whitham, 1974. The relativistic solitons have been seen in the multi-dimensional particle in cell PIC simulations of the laser pulse interaction with the underdense and the plasmas by Bulanov, Inovenkov, Kirsanov, et al., 1992, Mima, et al., 2001, Mourou, et al., 2002. The solitons are generated in the wake left behind the laser pulse and they propagate with the velocity well below the speed of light toward the plasma-vacuum interface. Here they disappear suddenly radiating away their energy in the form of low frequency electromagnetic bursts, Sentoku, Esirkepov, Mima, et al., 1999. The soliton can also be considered as coherent structures which form the electromagnetic turbulence and they can be observed via the modification of the plasma density behind the laser pulse and the low frequency, broad spectrum back scattered radiation. The analytical theory of the relativistic electromagnetic solitons has been developed by Gersten and Tsoar, 1975, Tsintsadze and Tskhakaya, 1977, Kozlov et al., 1979, Shukla, et al., 1986., Kaw, et al., 1992, Esirkepov, et al., 1998, Farina and Bulanov, 2001, Poornakala, Das, Sen, et al., 2002. In the case of the relativistic but relatively low amplitude of the soliton (compared to $a_c = \sqrt{m_i / m_e}$) the ions can be assumed to be at rest during approximately $\sqrt{m_i / m_e}$ periods of oscillations of the electromagnetic field inside the soliton. The time $2\pi / \omega_{pi}$, when the analytical solution for the low frequency, zero velocity soliton, obtained in Esirkepov, et al., 1998, provides a rather good description, is substantially longer than the period of the electromagnetic field oscillations inside the soliton, and in the underdense plasma it is much longer than the laser period. However, for a time interval longer than $2\pi / \omega_{pi}$ the ponderomotive pressure of the electromagnetic field inside the soliton starts to dig a hole in the ion density and the parameters of the soliton change, Naumova, et al., 2001. In the ion time scale, therefore, ions move outward and get accelerated to the energy level of $m_e c^2 a_m$. As a result, bubbles of ion density depletion are formed (Borghesi, et al., 2002).

The post soliton development is shown in **Figs. 19** and **20** [Naumova et al., 2001 and Naumova et al, 2002]

In **Figs. 21-23** we present the results of a three dimensional simulation of laser induced sub-cycle relativistic electromagnetic soliton by Esirkepov, et al., 2002.

In **Fig. 21** we see one isolated soliton and a soliton train behind the laser pulse. A substantial part of the laser pulse energy (up to 30%) is transformed into these coherent entities. The soliton consists of oscillating electrostatic and electromagnetic fields confined in a prolate cavity of the electron density. The cavity size is of the order of few laser wavelengths. The cavity is generated by the ponderomotive force and the resulting charge separation induces a dipole electrostatic field. **Fig 22** presents the soliton structure.

In **Fig. 23** we show the ion phase plane. We see the ion acceleration predominantly in radial direction.

We can describe the scenario of the post-soliton formation as follows. Since the soliton formation time is much shorter than time of the ion response $2\pi / \omega_{pi}$, ions can be assumed to be at rest during the soliton formation. Inside the nonpropagating soliton (half-cycle soliton by Esirkepov, et al., 1998) the maximum of the e.m. field a_m and the soliton frequency Ω_S are connected as $a_m = 2\sqrt{\omega_{pe}{}^2 - \Omega_S{}^2} / \Omega_S$ and the soliton width is equal to $c / \sqrt{\omega_{pe}{}^2 - \Omega_S{}^2}$. The ponderomotive pressure of the e.m. field inside the soliton is balanced by the force due to the electric field which appears due to charge separation. The amplitude of the resulting electrostatic potential is given by $\phi_m = \sqrt{1 + a_m{}^2}$. The ponderomotive pressure displaces the electrons outward and the Coulomb repulsion in the electrically non neutral ion core pushes the ions away. The typical ion kinetic energy corresponds to the electrostatic potential energy which is of the order of $m_e c^2 a_m$. This process is similar to the so-called "Coulomb explosion" inside of self-focusing channels (see Sarkisov, et al., 1999, Esirkepov, et al., 1999, Bulanov, et al., 2000, Sentoku, et al., 2000, Krushelnik, et al., 2000) and in the case of the cluster targets irradiated by the high-intensity laser light (Nishihara, et al., 2001, Kumarappan, Krishnamurthy, Mathur, 2001, Kishimoto, Masaki, Tajima, 2002). In Fig. 3 we show the ion phase plane. We see the ion expansion in the radial direction leads to the digging of a hole in the ion density. The cavity formation in the distribution of the electron and ion densities is shown in **Figs. 19 and 20**. The plasma cavity forms a resonator for the trapped e.m. field. During the cavity expansion, the amplitude and the frequency of the e.m. field decrease. Since the radius of the cavity increases very slowly compared to the period of the e.m. field oscillations we can use the adiabatic approximation to find their dependence on time as explained in Landau and Lifshits, 1984. The adiabatic invariant in this case is the ratio between the energy and the frequency of the e.m. field:

$$\int E^2 dV / \Omega_S = const. \qquad (65)$$

As a simple analytical model to describe the e.m. field inside the post-soliton, we can use the well known electric- or magnetic-dipole oscillations inside a spherical resonator (see Landau and Lifshits, 1984, Jackson, 1984) where the lowest frequency depends on the cavity radius as $\Omega_S = 2.74\, c / R$, for the electric-dipole mode, and as $\Omega_S = 4.49\, c / R$ for the magnetic-dipole mode. From Eq. (65) we obtain that $E^2 \propto R^{-4}$. Under the action of the e.m. pressure the wall of the cavity moves, piling up plasma as a snow plough. In the "snow plough" approximation, Zel'dovich and Raizer, 1967, all the mass of the plasma pushed by the e.m. pressure $E^2 / 8\pi$ is located inside a thin shell. The mass inside the shell is equal to the mass initially contained inside a sphere of the radius R: $M(R) = 4\pi n m_i R^3 / 3$. Using the second Newton's law for the motion of the mass *M*, we find the timescale of the cavity expansion $\tau = \sqrt{6\pi n_0 m_i R_0^2 / E_0^2}$. Asymptotically, when $t \to \infty$, the post-soliton radius increases as $R \approx R_0 (t/\tau)^{1/3}$, the amplitude of the e.m. field and its frequency decrease as $E \propto t^{-2/3}$ and $\Omega_S \propto t^{-1/3}$.

Analytically the relativistic electromagnetic solitons with non-zero propagation velocity in the 1D approximation are described by a set of equations (38) and (39). As it has been shown by Farina and Bulanov, 2001, within the framework of the approximation corresponding to equations (38) and (39) there are at least three types of nonlinear waves: the bright solitons, the dark solitons and the collisionless shock waves.

If we consider fast solitons with the propagation velocity $\beta_g > \sqrt{\rho}$, in this case we have the bright solitons with the amplitude which has a maximum and vanishes at infinity. This solution to equations (38) and (39) is consistent with the boundary conditions when $a_0 = 0$. The bright soliton is described by the well known expression $a = a_m / \cosh(\kappa \xi)$ or

$$A_y + iA_z = \frac{a_m}{\cosh(\kappa(x - \beta_g t))} \exp(-i\omega(t - \beta_g x)). \tag{66}$$

Here $\xi = x - \beta_g t$, the inverse soliton width is $\kappa = (a_m / 2\beta_g)\sqrt{(\beta_g^2 - \rho)/(1 - \beta_g^2)}$ and the frequency $\omega = \sqrt{(1+\rho)/(1-\beta_g^2) - (a_m^2 / 4\beta_g^2)(\beta_g^2 - \rho)/(1-\beta_g^2)}$. As we see, when the soliton propagation velocity approaches $\beta_{g,c} = \sqrt{\rho}$ the soliton width κ^{-1} tends to infinity for fixed soliton amplitude a_m. On the other hand, if we assume the soliton width to be fixed, its amplitude becomes infinite as $\beta_g \to \beta_{g,c}$. In this case, we expect the soliton breaking and appearance of the charged particle trajectory self-intersection.

Fig. 24 shows the structure of one node in the profile of the vector potential soliton for the propagation velocity close to the velocity of breaking $\beta_{g,br} \approx 0.32$. At $\beta_g = \beta_{g,br}$, and $\omega \approx 0.224$, the solution branch ends since the soliton breaks and the singularity appears in the soliton solution, with the ion density n_i going to infinity at $\xi = 0$, i.e., $R_i = 0$. From this last condition, we obtain the peak value of the potential, $\phi_{br} = (1-\sqrt{1-\beta_{g,br}^2})/\rho$. After the break a portion of the ions will be injected into the acceleration phase. This shows that the soliton breaking can provide an additional mechanism for the generation of fast ions in laser irradiated plasmas.

If the velocity β_g is smaller than $\sqrt{\rho}$, then equations (38) and (39) have a solution which describes the dark soliton. The solution requires the frequency to be equal to $\omega = \sqrt{(1+\rho)/(1-\beta_g^2) - (a_m^2/2)/(1-\beta_g^2)}$. The dark soliton is given by

$$A_y + iA_z = a_m \tanh(\kappa(x-\beta_g t))\exp(-i\omega(t-\beta_g x)), \tag{67}$$

where the soliton inverse width is given by $\kappa = (a_m/2)\sqrt{(\rho-\beta_g^2)/(1-\beta_g^2)}$. These expressions describe the dark soliton (the kink state) of small amplitude: the wave amplitude changes monotonously from $-a_m$ at $x=-\infty$ to a_m at $x=+\infty$. In the dark soliton, we have a minimum of the electromagnetic energy density and a minimum of the plasma density, which propagate with the velocity β_g without change of their form. The dark solitons are known in optical systems, Kivshar and Luther-Davies, 1998, and Kivshar and Pelinovsky, 2000. Recently, they have been observed in the Bose-Einstein condensate Burger et al., 1999. We see that in the limit of low propagation velocity the electron - ion plasma exhibits properties similar to those in the Bose-Einstein condensate with the positive scattering length Burnett, et al., 1999. In the electron-positron plasma the dark solitons are a natural nonlinear mode, Tajima and Taniuti, 1990, Farina and Bulanov, 2001.

Effects the finite temperature of the plasma on the soliton properties have been studied by Lontano et al., 2001, and Lontano et al., 2002, as well as the modification of the soliton structure due to the quasistatic magnetic field effects has been analyzed by Farina et al., 2000.

Shocks are another type of structure formation in the laser-matter interaction. Collisionless relativistic electromagnetic shock waves are described by equations (38), (39) in the case $\beta_g \approx \beta_{g,c} = \sqrt{\rho}$. Their form is given by

$$A_y + iA_z = \frac{a_w \exp(-i\omega(t-\beta_g x))}{\sqrt{1+(a_w^2/\sqrt{1-\beta_g^2})\exp(\kappa(x-\beta_g t))}}, \tag{68}$$

where the shock wave amplitude equals $a_w = \sqrt{\rho}$. The shock wave is compressional with the carrying frequency of the electromagnetic wave equal to $\omega = \sqrt{[8(1+\rho)-a_w^2]/[8(1-\beta_g^2)]}$ and its width $\kappa^{-1} = (2/a_w^2)\sqrt{1-\beta_g^2}$. We see that as larger is the shock wave amplitude as steeper is the shock wave.

The collisionless shock wave corresponds to a nonlinear regime of the penetration of relativistically strong electromagnetic wave into the overdense plasma. Above, we have discussed the regimes of the relativistic transparency, when the electromagnetic wave could propagate through the overdense plasma due to relativistic correction of the electron mass (see Akhiezer and Polovin, 1956, Kaw and Dawson, 1970, Marburger and Tooper, 1975, Goloviznin and Schep, 1999, Cattani et al., 2000). In our case the effective Lagmuir frequency changes due to both the relativistic correction of the electron mass and the change of the plasma density. The formation of the collisionless shock wave with a stationary and monotonous profile contrary to that discovered by Sagdeev, 1966, does not require any dissipative process.

When the laser pulse is longer than the resonant length of wakefield (Tajima and Dawson, 1979) such as in the experiments by Nakajima et al., 1995, and Modena et al., 1995, the laser pulse is subject to plasma instabilities of electronic time scales. The most effective one is the stimulated Raman scattering instabilities. The forward Raman process is the one that modulate the laser pulse inn such a way to reinforce the wakefield resonance as a part of the self-organization of the laser pulse in the plasma. On the other hand, the stimulated backward Raman instability (SBRS) has a greater growth rate than the forward process, though the latter has a longer interaction time because of the co-propagation nature of the forward scattering. The SBRS leads to the erosion of the leading edge of the laser pulse over the time scale of $\omega_{pe}^{-1} (\omega_0/\omega_{pe})^2$ (Bulanov et al., 1992). (see **Fig. 25**). Such a shock front facilitates to generate wakefields in a very sharp and crisp way Bulanov, Esirkepov, Naumova, 1996. This mechanism may be employed for electron acceleration as well as ion acceleration (Esirkepov, et al., 1999, Koga et al. 2001) See later in ion acceleration section.

In addition perhaps as a combination of self-focusing and other nonlinearity, the formation of jets may be observed. Some of the spectacular jet observations included: Ruhl et al. 1999, Kodama et al., 2000, Kando et al., 1997.

F. High Harmonic Generation

Generation of high order harmonics of the electromagnetic radiation during interaction of high-intensity laser pulses with underdense and overdense plasmas presents a manifestation of one of the most basic nonlinear processes in physics. High order optical harmonics have been observed in the laser plasma interaction with plasmas for the radiation intensity ranging from moderate level up to relativistic intensities. High order harmonics attract great attention because of the key role they play in the theory of nonlinear waves and due to a wide range of their possible applications for the diagnostics, the UV and x-ray sources of coherent radiation, lithograpy etc. (see Bloembergen, 1965, Shen, 1984, Boyd, 1992, Zhou, et al., 1996, Altucci, et al., 1999, Villoresi, et al., 2000). The physical mechanisms of the generation of high order harmonics have much in common because they lie upon a property of nonlinear systems to react in an anharmonical manner on the action of periodic driving force. On the other hand, specific realization of this property depends on the circumstances of the laser-matter interaction and mainly on the laser intensity. At the moderate intensity generation of high order harmonics occurs due to anharmonicity of the atom response on the finite amplitude oscillating electric field. When the laser radiation intensity becomes above the level when the electron quiver energy is higher than the rest mass energy and the Relativistic Nonlinear Optics comes into play, Mourou, et al., 2002, Tajima and Mourou, 2002, the generation of high order harmonics is due to nonlinear dependence of the particle mass on the momentum and modulations of the electron density in the field of the electromagnetic wave. The first relativistic harmonics was observed with the large scale CO_2 laser, Antares in the early 1980's.

In the underdense plasmas the high harmonics are produced by the mechanism of the parametric excitation by the laser light of the electromagnetic and electrostatic waves with different frequencies. As we mentioned above, the linearly polarized electromagnetic wave in the underdense plasma has the transverse component which spectrum contains odd harmonics

$$E_y = -\omega a_0 \cos(\omega t - kx) - \omega a_0{}^3 \frac{3(8\omega^2 + 3\omega_{pe}{}^2)}{8(4\omega^2 - \omega_{pe}{}^2)} \cos(3\omega t - 3kx) + \ldots, \tag{69}$$

and the longitudinal component with even harmonics

$$E_x = -k a_0{}^2 \frac{1}{4\omega^2 - \omega_{pe}{}^2} \sin(2\omega t - 2kx) + \ldots, \tag{70}$$

where the wave frequency depends on the wavenumber as $\omega = \sqrt{k^2c^2 + \omega_{pe}{}^2}$.

When the laser radiation interacts with the overdense plasmas it reflects back at the plasma-vacuum interface in the case of sharp plasma boundary or at the surface of critical density in the case of gradual density profile. The reflection layer of the plasma dragged by the electromagnetic wave back and forth as well as in the plane of the surface of the plasma-vacuum interface (in the plane of the critical surface) forms the oscillating mirror (see Bulanov, et al., 1994, Gibbon, 1996, Lichters et al., 1996, Von der Linde, 1997, Vshivkov, et al., 1998, Zepf, et al., 1998, Il'in, et al., 1999, Tarasevich, et al., 2000). The spectrum of the reflected at the oscillating mirror light contains odd and even harmonics, which polarization and amplitude depend on the incidence angle of the pulse, its intensity and the pulse polarization.

In the relativistic regime on a solid target one of the interpretations is that the large ponderomotive force will drive the critical surface at twice the laser frequency at relativistic velocities and provides a new mechanism of harmonic generations. This elegant explanation has been for the first time proposed by Bulanov, et al., 1994 and further studied by Lichters et al., 1996. Harmonics up to the 60^{th} has been observed. Harmonic generation by the Rutherford and Von der Linde's group, Zepf, et al., 1998, Tarasevich, et al., 2000.

A basic phenomenon responsible for the high harmonic generation is connected to the change of the frequency and amplitude of the electromagnetic wave during its reflection at the moving mirror. The electromagnetic wave reflection at the mirror moving with constant velocity, $V = c\beta$, has been described by Einstein, (see also Pauli, 1981). If the electric field in the incident wave is given by a function $f[\omega_0(t - x/c)]$, in the reflected wave we have $J(\beta)f[\omega_0 J(\beta)(t - x/c)]$, where $J(\beta) = (1-\beta)/(1+\beta)$. The problem of the electromagnetic wave reflection at the uniformly accelerating mirror has been solved using the Rindler transformation to the accelerating reference frame by Van Meter, Carlip, and Hartemann, 2001. They noticed that to find the reflected wave at the accelerating mirror one can use the subsequent Lorentz transforms into the reference frame moving with the instantaneous velocity calculated at the proper time of the reflection. Using this recipe we write approximate expression for the reflected wave

$$a(x,t) = -J(\beta(t'))a_0\left(\int^{t'} J(\beta(s))ds\right), \qquad (71)$$

where the retarded time is $t' = t - x/c + \int^{t'} \beta(s)ds$.

When the plasma is exposed to the linearly (elliptically) polarized light the surface of critical density oscillates with the double frequency, $\beta(t) = \kappa a_0 \cos 2\omega t$, where a coefficient κ is of the order of unity. In the limit of small but finite amplitude we can find that the reflected wave is given by

$$a(t) \approx -a_0(1 + \kappa a_0 + 3\kappa^2 a_0^2)\sin(\omega t) - 3\kappa^2 a_0^3 \sin(3\omega t) + 5\kappa^4 a_0^5 \sin(5\omega t)\dots \qquad (72)$$

We see that at the normal incidence the reflected light spectrum contains odd harmonics. In the case of oblique incidence the reflected light spectrum has both the odd and the even harmonics with different polarization which depend on the polarization of the incident pulse. According to the selection rules of the harmonic generation at the solid target surface (see Bulanov, et al., 1994; Lichters, et al., 1996; Vshivkov et al., 1998), the *s*-polarized incident pulse generates *s*-polarized odd harmonics and *p*-polarized even harmonics. The *p*-polarized incident pulse generates only *p*-polarized odd and even harmonics.

Macchi et al., 2001 and 2002 have shown that the parametric instability development at the vacuum - plasma interface results in the nonlinear distortion of the oscillating mirror in the transverse direction and provides additional mechanism for high order harmonic generation.

As we see, low density, but over dense plasma can produce large amplitude of the critical surface yielding to an efficient harmonic generation. Relativistic harmonic generation can also be the source of sub-femtosecond pulses. An interesting approach has been recently demonstrated by the Michigan group with their λ^3 laser. In order to reach relativistic intensities this group uses single millijoule pulses delivered in few optical periods i.e. 6-20fs at kHz repetition rates. A f# 1 paraboloid combined with a deformable mirror is able to focus the beam on a spot size of a single wavelength dimension ~1µm. Intensities in the 5 10^{18}W/cm^2 or a_0^2 ~ 2 has been demonstrated at 1kHz repetition rate. We believe that these truly compact relativistic lasers will make relativistic studies accessible to a much larger community. The progress in this relativistic field can be appreciated when we contrast the building size few shots a day CO_2 laser used in the first relativistic intensity experiments and the relativistic kHz laser (**Figures 26-27**).

In addition to the above harmonic radiation, there is radiation called Larmor radiation. The Larmor radiation is the classic radiation due to the acceleration of electrons by the laser electric and magnetic fields, which turns electron orbits around. Ueshima et al., 1999 evaluated this radiation. The radiation intensity increases in proportion to a_0^2, while the peak frequency increases as a_0^3:

$$\hbar\omega_{\max} \approx a_0^3 \hbar\omega_0 \tag{73}$$

Laser driven Larmor radiation has been observed by Chen et al. (1998).

In the next subsection we discuss in details the effect of the radiation on the charged particle (electron) dynamics.

V. Interaction of charged particle with the electromagnetic wave in the radiation dominant regime

The investigations of free electron radiation during its interaction with the electromagnetic wave have always, starting from the J. J. Thomson works, been of great significance. The literature devoted to studies of the electromagnetic wave – particle interaction is vast (see for example, Jackson, 1975, Landau and Lifshits (1980), Nikishov and Ritus (1964), Sharachik and Schappert (1970), Zel'dovich (1975), Waltz and Manley, 1978).

Below we shall consider the relativistic electron interaction with the circularly polarized electromagnetic wave. In the case of the circularly polarized electromagnetic wave the charged particle moves along a circle trajectory, and one may borrow the expressions for the properties of the radiation emitted by the particle from the theory of synchrotron radiation

(Ginzburg and Syrovatskii, 1965, Sokolov and Ternov, 1968, Ginzburg, 1989). Taking into account the effects of the radiation damping force, equations of the electron motion can be written as

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B} + \mathbf{f}, \qquad \frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{m\gamma}. \tag{74}$$

Here the particle momentum $\mathbf{p}$, velocity $\mathbf{v}$, and Lorentz factor γ are $\mathbf{v} = d\mathbf{x}/dt$, $\gamma = \sqrt{1 + |\mathbf{p}|^2 / m^2c^2}$ and the radiation force $\mathbf{f}$ is equal to (Landau and Lifshits, 1980),

$$\mathbf{f} = -\frac{2r_e^2}{3}\frac{\mathbf{p}}{m^3c^5\gamma}\left[\left(mc^2\gamma\mathbf{E} + \mathbf{p}\times\mathbf{B}\right)^2 - (\mathbf{p}\cdot\mathbf{E})^2\right]. \tag{75}$$

We retained the leading in the ultrarelativistic limit terms in the radiation force.

The electromagnetic wave is assumed to propagate in plasma with the velocity v_{ph} along the *x*-direction. It is given by the vector potential

$$\mathbf{A} = a_0(mc^2/e)\left(\mathbf{e}_y \sin\psi + \mathbf{e}_z \cos\psi\right), \tag{76}$$

where $\psi = \omega(t - x/v_{ph})$. The electric and magnetic fields are $\mathbf{E} = -c^{-1}\partial\mathbf{A}/\partial t$ and $\mathbf{B} = \nabla\times\mathbf{A}$.

When the radiation damping force is taken into account the longitudinal component of the force (it is the radiation pressure) does not vanish. We assume that in this case the particle does not move along the *x*-axis because the radiation pressure force is balanced by the force due to the charge separation electric field in a plasma. The *x*-component of a total force on the particle vanishes: $eE_x + e(v_yB_z - v_zB_y)/c = 0$, and the particle coordinate along the *x*-axis is equal to $x=0$. Here the *x*-component of the electric field E_x that occurs due to the electric charge separation in a plasma. In the transverse direction the particle rotates along a circle. From equation (75) we obtain for the transverse components of the particle momentum

$$\dot{p}_y = a_0 mc\omega\cos\omega t - \dot{y}\Phi(a_0,\gamma), \tag{77}$$

$$\dot{p}_z = a_0 mc\omega\sin\omega t - \dot{z}\Phi(a_0,\gamma) \tag{78}$$

with $\Phi(a_0,\gamma) = (2/3)\left(e\omega a_0/c\right)^2(\gamma^2\cos^2\varphi - \sin^2\varphi)$, where we introduced a phase φ between the particle rotation and the wave field, i.e. $y = r\cos(\omega t + \varphi)$, $z = r\sin(\omega t + \varphi)$, $p_y = -p\sin(\omega t + \varphi)$ and $p_z = p\cos(\omega t + \varphi)$. From equations (77,78) we find (Zel'dovich and Illarionov, 1972, Zel'dovich, 1975)

$$p = a_0 mc\cos\varphi, \tag{79}$$

$$mc\sin\varphi = \varepsilon_{rad}a_0 p(\gamma^2\cos^2\varphi - \sin^2\varphi). \tag{80}$$

Here a dimensionless parameter ε is

$$\varepsilon_{rad} = (4\pi r_e / 3\lambda). \tag{81}$$

Eliminating $\sin\varphi$ and $\cos\varphi$ from equations (11) and (10) we find equation for the particle momentum

$$\varepsilon_{rad}{}^2 (p/mc)^8 + (p/mc)^2 = a_0{}^2. \tag{82}$$

We see that in the limit of relatively low amplitude of the laser pulse, when $1 \ll a_0 \ll a_{rad} = \varepsilon_{rad}{}^{-1/3}$, the particle momentum depends on the laser pulse amplitude as $p = mca_0$, and in the limit $a_0 \gg a_{rad}$ the momentum dependence on a_0 is given by $p = mc(a_0/\varepsilon_{rad})^{1/4}$.

The quantum physics effects become important when the photon, generated due to the Compton scattering, has the energy of the order of the electron energy, i.e. $\hbar\omega_m \approx \mathcal{E}_e$. (We do not discuss here the quantum fluctuations of the electron orbit similar to the quantum fluctuations of the trajectory of the moving in a magnetic field, Sokolov, Ternov, Loskutnov, 1962). The electron with the energy $\mathcal{E}_e = \gamma mc^2$ rotates with the frequency ω in the circularly polarized wave propagating in a plasma and it emits the photons with the frequency $\omega_m = \gamma^3\omega$ (see Landau and Lifshits, 1975). We obtain that the quantum effects come into play when

$$\gamma \geq \gamma_q = \sqrt{mc^2/\hbar\omega}. \tag{83}$$

For the electron interacting with one micron laser light we find $\gamma_q \approx 600$. From the previous analysis of the radiation effects we have for the electron gamma factor $\gamma = (a_0/\varepsilon_{rad})^{1/4}$. That is why the quantum limit is

$$a_q = \frac{2e^2 mc}{3\hbar^2\omega} = \frac{1}{3\pi}\frac{r_e\lambda}{l_c{}^2}. \tag{84}$$

For the equivalent electric field of the electromagnetic wave it yields

$$E_q = \frac{2em^2c^2}{3\hbar^2} = \frac{2}{3}\frac{l_c}{r_e}E_{Schw}. \tag{85}$$

Here

$$E_{Schw} = m^2c^3/e\hbar \tag{86}$$

is the Schwinger electric field (Schwinger, 1951). The quantum limit electric field E_q is in a factor $3/2\alpha$, i. e. approximately 200 times, smaller than the Schwinger electric field.

In the radiation dominant regime in the quantum limit we have instead equation (14)

$$a_0{}^2 - (p/mc)^2 = \varepsilon_{rad}{}^2 (p/mc)^8 \, \mathrm{I}(\Upsilon). \tag{87}$$

Here the dimensionless variable Υ is

$$\Upsilon = \frac{\hbar\omega}{mc^2}\left(\frac{p}{mc}\right)^2 \quad . \tag{88}$$

In the quantum limit, when $\Upsilon \gg 1$, the function $I(\Upsilon)$ is given by the expression (see Ritus, 1979, Beresteskii, Lifshitz, Pitaevskii, 1982)

$$I(\Upsilon) \approx \frac{32\Gamma(2/3)}{243}\frac{e^2 m^2}{\hbar^2}(3\Upsilon)^{2/3}. \tag{89}$$

Substituting this expression into equation (87) we find the electron momentum as a function of the electromagnetic wave amplitude in the limit $a_0 > a_q$:

$$p = \frac{mc}{\sqrt{3}}\left[\frac{243}{48\Gamma(2/3)}\right]^{3/4}\left(\frac{\hbar\omega}{mc^2}\right)\left(\frac{a_0}{\varepsilon_{rad}}\right)^{3/4} . \tag{89}$$

When the electromagnetic wave packet interacts with the charged particle in a vacuum, and the particle is at rest before the interaction, the particle momentum and the Lorentz factor are given by (Landau and Lifshits, 1975, Lai, 1980) $p_x = mc\,a_0^2/2,\quad p_\perp = mc\,a_0,\quad \gamma = 1 + a_0^2/2$. In the ultrarelativistic limit, when $a_0 \gg 1$ the longitudinal component of the particle momentum is much larger than the transversal component. The particle drift velocity along the x-direction is equal to $v_D = p_x/2m\gamma = ca_0^2/(2+a_0^2)$. We perform the Lorentz transformation into the reference frame moving with the particle drift velocity v_D . We find that in the moving reference frame the dimensionless amplitude value of the laser pulse is the same as its value in the laboratory reference frame: $\bar{a}_0 = a_0$. This is a consequence of the fact that the transverse component of a four vector does dot change during the Lorentz transformation. Instead, the parameter ε_{rad}, given by (81), is not a Lorentz invariant. We can find that it is

$$\bar{\varepsilon}_{rad} = \frac{4\pi}{3}\frac{r_e}{\bar{\lambda}} = \frac{\varepsilon_{rad}}{\sqrt{1+a_0^2}}, \tag{90}$$

where we have used a fact that the wavelength of the laser pulse in the moving reference frame is equal to $\bar{\lambda} = \sqrt{(c+v_D)/(c-v_D)}\lambda = \sqrt{1+a_0^2}\lambda$. The limit of the radiation dominant regime now reads as $a_0^3 \gg \bar{\varepsilon}_{rad}^{-1}$ or $a_0 \gg \varepsilon_{rad}^{-1/2}$. It is easy to show that the quantum effects, in the case of the charged particle interaction with the electromagnetic wave in a vacuum, become important when the wave electric field reaches the Schwinger limit.

For one micron laser pulse interaction with plasmas, as it is well known, the relativistic effects become important for $a_0 \geq 1$, that corresponds to the radiation intensity above $I_{rel} = 1.38 \cdot 10^{18} W/cm^2$. The radiation dominant regime begins at $a_0 \approx a_{rad}$

with $a_{rad} \approx 400$, i.e. for the laser light intensity of the order of $I_{rad} = 3 \cdot 10^{23} W / cm^2$. Quantum physics effects come into play at $a_0 \approx a_q = 2500$, which gives $I_q = 1.38 \cdot 10^{26} W / cm^2$. We reach a limit when the nonlinear quantum electrodynamics effects with the electron-positron pair creation in the vacuum come into play, when the laser pulse electric field becomes equal to the Schwinger electric field $E_{Schw} = m^2 c^3 / e\hbar$, which corresponds to $a_{Schw} = mc^2 / \hbar\omega = 5 \cdot 10^5$ and $I_{Schw} = 3 \cdot 10^{29} W / cm^2$.

For the freely accelerated by the electromagnetic wave in a vacuum, the radiation dominant regime is reached at the one micron laser light intensity of the order of $I_{Schw} = 10^{26} W / cm^2$. The quantum effects become important at the laser pulse electric field equal to the Schwinger electric field, i.e. at the intensity $I_{Schw} = 3 \cdot 10^{29} W / cm^2$.

In the radiation dominant regime a substantial part of the laser energy is transformed into the hard (X-ray) radiation (see Zhidkov et al., 2002).

An other approach to study the radiation dominant regimes for the laser – plasma interaction is connected with usage of the cluster targets. The laser – cluster interaction is accompanied by the efficient transformation of the laser light energy into the energy of the scattered electromagnetic wave (Kishimoto and Tajima, 1999, Kishimoto et al., 2002), and by the ion acceleration, Nishihara et al., 2001, and Kishimoto et al., 2002. In typical situations the cluster size is smaller than the wavelength of the laser light. In this case the scattering occurs in the collective regime and the scattering cross section increases in N^2 times. Here N is a number of electrons involved into the scattering process. Typical value of the electron number in the cluster can be estimated to be equal to $N = 10^8$. We see that the parameter $a_{rad} = (4\pi N r_e / \lambda)^{-1/3}$ becomes ≈ 500 times larger. It corresponds to the laser intensity of the order of $I_q = 10^{18} W / cm^2$. Thus in this regime we can model the radiation dominant laser plasma interaction using the moderate power lasers and to provide a source of powerful ultra-short electromagnetic bursts in a process similar to that discussed by Kaplan and Shkolnikov, 2002.

VI. Relativistic Engineering

By taking these processes described above systematically and consciously, we are now at the verge of witnessing the emergence and maturation of the utilization techniques of intense lasers (and other tools such as relativistic electrons) to affect the dynamics of matter so drastic that the relativistic effects of the dynamics are of paramount importance. We may be able to call such endeavor as **relativistic engineering**. In laser section we shall see applications of lasers and high energy electron beams as a ‘marriage’ between the laser technology and the accelerator technology in such examples as the γ-γ collider via the inverse Compton scattering process of laser. Here, let us show some spectacular implications of what we call relativistic engineering through another example. This involves at least three elements of ‘relativistic engineering’ of intense laser pulses. They have shown that three elements of (1) the longitudinal pulse length compression (or pulse compression, for short), (2) the frequency upshift, and (3) angular focusing in combination may lead to ‘manufacture’ laser pulses of unprecedented parameter regimes. Imagine a first laser pulse induces a laser wakefield. The wafefield has the phase velocity v_{ph} and its associated Lorentz factor γ_{ph}. The nonlinearity of strong wakefield amounts to nonlinear wave profile, including the steepening of the wave and what is called the cusp formation in its density. It can be shown that because of this steep cusp effect, substantial optical effects emerge. For example, this cusp acts as a relativistic mirror. By properly designing the wakefield and thus these relativistic mirrors, we should be able to modify the properties of the second laser that are now injected toward this relativistic mirror(s).

With the ideal realization of the above dynamics, we should be able to compress the pulse length by $\gamma_{ph}^{\,2}$. At the same time, the frequency of the laser goes up by the same factor. Because the wavelength compactifies by this factor, it is possible to focus (if it can be focused to the diffraction limited size) down to the spot that is smaller by the same factor in of the two transverse dimensions. This amounts to the compactification of the original laser pulse in three dimensions to new higher energy photons by a factor of $\gamma_{ph}^{\,6}$ in the most optimistic scenario. This can be immense. Take an example of the wakefield excitation in a gas of density at $10^{19}\,\mathrm{cm}^{-3}$. This means the Lorentz factor associated with the phase velocity of the wakefield is related to ω/ω_p, which is on the order of 10. Thus the laser compactification of the order of 10^6 may be realized. If one has a laser of 1PW and focuses it down to the intensity of $10^{22}\,\mathrm{W/cm}^2$, the relativistic engineering of this compactification may lead to the intensity of $10^{28}\,\mathrm{W/cm}^2$. Evidently, this is astounding energy density. How well such relativistic engineering may be accomplished remains to be investigated in the coming future. But it surely offers an immense promise and challenge ahead of us.

A. Flying mirrors

We notice that the laser frequency upshifting and the pulse compression also can be achieved using a broad variety of the other configurations. In particular, the wave amplification reflected at the moving relativistic electron slab has been discussed by Landecker, 1952 and Ostrovskii, 1976, the Backward Thompson Scattering at relativistic electron bunch was considered by Arutyunian and Tumanian, 1963, Li, et al., 2002, the reflection at the moving ionization fronts has been studied by Semenova, 1967, Mori, 1991, Savage et al., 1992, Mori et al., 1995, Bakunov et al., 2001, Dias et al, 2002, and with the use of various schemes of the counter-propagating laser pulses discussed by Shvets et al., 1998, Shvets et al., 1999, and by Ping et al., 2000.

As it has been said, ultra high intensity electromagnetic radiation limit can be reached as a result of subsequent laser radiation frequency up-shifting and focusing into the one-wavelength focus spot. Within the framework of this scheme we use the properties of the wake field generated in underdense plasmas by the ultra-short relativistically strong laser pulse – driver. The electron density modulation within nonlinear wake plasma waves can be regarded as the high density plasma shells moving with the velocity v_{ph} close to the speed of light in a vacuum. The second laser pulse, which counter propagates with respect to the driver pulse, may now be reflected back at these relativistic electron shells with the frequency upshifting and with the compression of the reflected pulse (See. **Fig. 28**). We may say that in a wake behind the laser pulse – driver we see "flying relativistic mirrors". As a result the wavelength of the reflected wave becomes in a factor $4\gamma_{ph}{}^2 = 4/(1 - v_{ph}{}^2/c^2) \gg 1$ shorter as it is well known.

Within the framework of the scheme under consideration it is important that the relativistic dependence of the Langmuir frequency on the wave amplitude results in the formation of the wake waves with the curved fronts that have a form close to the paraboloid as it has been discussed above. The electromagnetic wave reflection at the paraboloid flying mirror leads to the to the electromagnetic wave focusing. In the reference frame moving with the mirror velocity the reflected light has the wavelength equal to $\lambda_0 / 2\gamma_{ph}$. It can be focused into the spot with the transverse size $\lambda_0 / 2\gamma_{ph}$, which can result in the light intensity increase in a factor $4\gamma_{ph}{}^2(R_0/\lambda_0)^2$, where R_0 is the radius of the incident laser beam. The resulting intensity in the laboratory frame increases in a factor $16\gamma_{ph}{}^4(R_0/\lambda)^2$. This value must be multiplied on the reflection coefficient which is smaller than one.

This scheme of the laser pulse intensification is illustrated in **Fig. 28**. Upper row corresponds to the laboratory frame (**L**) before reflection of the laser pulse from the "flying mirror": The laser pulse propgates from right to left; middle row corresponds to the co-moving reference frame (**K**): Laser pulse reflection and focusing occurs into the focus spot with the size $\lambda' \approx \lambda_0 / 2\gamma_{ph}$; lower row corresponds to the laboratory frame (**L**): The reflected e.m. radiation has the wavelength $\lambda_f \approx \lambda_0 / 4\gamma_{ph}{}^2$, and it propagates in a narrow angle $\theta \approx 1/\gamma_{ph}$. It can be shown that because of this steep cusp effect, substantial optical effects emerge. For example, this cusp acts as a relativistic mirror. The interaction of a probe laser pulse with a counter-propagating wake field corresponds to the reflection of light at a mirror moving with a relativistic velocity v_{ph}. As is well known the frequency of the reflected light is

$$\omega_R = \omega_0 \frac{1+\beta_{ph}}{1-\beta_{ph}} \approx 4\gamma_{ph}{}^2 , \tag{86}$$

where $\beta_{ph} = v_{ph}/c$, ω_0 is the frequency of the incident electromagnetic wave, and ω_R is the frequency of the reflected wave.

This relativistic "effective mirror" can be formed during the breaking of the Langmuir wake wave that propagates in plasma with phase velocity close to the speed of light in vacuum. In a nonlinear Langmuir wave near the breaking threshold, when the electron quiver velocity v_E approaches the phase velocity of the wave, the dependence of the electron density on the coordinate $\xi = x - v_{ph}t$ is given by Expression (15). The distribution of the electron density (15) corresponds to an integrable singularity ($\int_{-\infty}^{+\infty} n(\xi) d\xi \neq \infty$). However, it breaks the geometrical optics approximation and leads to the reflection of a portion of the laser pulse in the backward direction and to the upshifting of the frequency of the reflected pulse.

In order to calculate the reflected radiation, we consider the interaction of an electromagnetic wave with a spike of the electron density formed in a breaking Langmuir wave (15). The electromagnetic wave, given by the z - component of the vector potential $A_z(x,t)$, is described by the wave equation

$$\partial_{tt} A_z - \frac{1}{c^2}\left(\partial_{xx} A_z + \partial_{yy} A_z\right) + \frac{4\pi e^2 n(x - v_{ph}t)}{m_e \gamma_e} A_z = 0, \tag{87}$$

where the electron Lorentz factor γ_e near the maximum of the density $n(\xi)$ is equal to γ_{ph}.

In the reference frame moving with the phase velocity of the wake wave we write the vector potential in the form

$$A_z = \left(A_0 \exp(-ik_x' x') + A_R(x')\right) \exp\left(-i\left(\omega' t' - k_y' y'\right)\right), \tag{88}$$

where A_0 and A_R correspond to the incident and reflected waves, and $\mathbf{x}'$, t' and $\mathbf{k}'$, ω' are the coordinates and time and the wave vector and frequency in the boosted frame.

From equation (88) we obtain for the reflected wave

$$\frac{d^2 A_R}{dx'^2} + q^2 A_R = -\frac{g}{(x')^{2/3}} \left(A_0 \exp(-ik_x' x') + A_R(x') \right), \tag{89}$$

where $q^2 = k_y{}^2 - \omega'^2 / c^2$ and $g = (2/9)^{1/3} k_p{}^{4/3} \gamma_{ph}{}^{2/3}$.

Assuming $\omega \gg \omega_{pe}$, which is equivalent to $k_p \ll k_x$, and considering the first term in the brackets in the right hand side of equation (89) to be much smaller than the second term, we find the reflected wave:

$$A_R = \frac{i^{4/3} g}{q(q + k_x')^{1/3}} \Gamma\left(\frac{2}{3}\right) A_0 \exp(-iq_x x'), \tag{90}$$

where $\Gamma(x)$ is the gamma function.

Performing the inverse Lorentz transformation, we obtain that in the case of normal incidence ($k_y = 0$) the frequency of the reflected wave is equal to the frequency given by Eq. (86), in agreement with the expression for the frequency change after a reflection at a relativistic counter-moving mirror. The wave amplitude (the electric field) is increased by the factor $2\left(k_p{}^2 \gamma_{ph} / k_x{}^2\right)^{2/3} \Gamma(2/3) / 9^{1/3}$. The length of the reflected pulse is $\approx 4\gamma_{ph}{}^2$ times shorter than the length of the incident pulse. If the Langmuir wave is generated by a laser pulse with carrier frequency $\omega = \omega_0$, the ratio of the intensity of the reflected and incident wave is

$$\frac{I_R}{I_0} \approx \left(\frac{\omega \omega_{pe}}{\omega_0{}^2} \right)^{4/3}. \tag{91}$$

As we have discussed above the electron density modulations in the wake plasma wave have a form of paraboloids (see Fig. 28)

By properly designing the wakefield and thus these relativistic mirrors, we should be able to modify the properties of the second laser that are now injected toward this relativistic mirror(s). With the ideal realization of the above dynamics, we should be able to compress the pulse length by $\gamma_{ph}{}^2$. At the same time, the frequency of the laser goes up by the same factor. Because the wavelength compactifies by this factor, it is possible to focus (if it can be focused to the diffraction limited size) down to the spot that is by the factor γ_{ph} smaller in of the transverse dimension. This amounts to the intensification of the original laser radiation in three dimensions to new higher energy photons by a factor of $\gamma_{ph}{}^{4-4/3}$ in the most optimistic scenario. Take an example of the wakefield excitation in a gas of density at $10^{19} cm^{-3}$. This means the Lorentz factor associated with the phase velocity of the wakefield is related to ω / ω_{pe}, which is on the order of 10. Thus the laser pulse intensification of the order of 465 may be realized. For the plasma density equal to $10^{17} cm^{-3}$ the Lorentz factor associated with the wake field phase velocity is equal to 100, and the laser pulse intensification may be reached up to 2×10^5. In this case one finds if one has a laser of 1PW and focuses it down to the intensity of $10^{22} W / cm^2$, the relativistic engineering of this intensification may lead to the intensity of $2 \times 10^{27} W / cm^2$. We see that the reflected radiation intensity can approach the Schwinger limit. In this range of the electromagnetic field intensity it becomes possible to investigate the fundamental problems of the nowadays physics using the available at present laser devices.

Now we present the results of 3D PIC simulations of the laser pulse reflection at the “flying mirrors”. The results of the simulations are presented in Figs. **29** and **30**. Fig. 29. shows the paraboloidal modulations of the electron density in the wake behind the driver laser pulse at t=16. Their transverse size is larger than the reflecting (incident from the right hand side along the x – direction) laser pulse wavelength.

In **Fig. 30** we present the projections of the electric field components. The x-component of the electric field in the wake wave is shown in the projection onto the x,y – plane. The Projection of the y - component of the electric field onto the x,z -. plane shows the electric field of the reflected laser pulse. The driver laser pulse is shown by the contours in the right hand side of the computation box.

In Fig. 30 we see that the reflected laser light has its wavelength substantially shorter than in the incident wave as well as its focusing in the region with the size also much smaller than the wavelength of the incident pulse. For the parameters of the simulations the phase velocity of the wake wave corresponds to $\beta_{ph} = 0.87$, i. e. to the Lorentz gamma factor equal to. $\gamma_{ph} = 2$. The reflected light has the frequency in a factor 14 higher than the incident radiation in perfect agreement with expression (16), because in this case $(1+\beta_{ph})/(1-\beta_{ph}) \approx 1.44$. The electric field of the reflected radiation is at about 16 times higher than in the incident pulse, i. e. the intensity increases in 256 times.

These results provide us a proof of principle of the electromagnetic field intensification during reflection of the laser radiation at the flying paraboloidal relativistic mirrors in the wake plasma waves.

VII. Nuclear Physics

A. Rutherford, Livermore , Michigan, Osaka, LULI experiments

In intense laser regimes beyond $10^{xx} Z^{2\ (or\ 3)}$ W/cm^2, electrons are stripped from atoms with charge number Z. For a certain element the removal of inner shell electrons changes the nuclear bound state so much that it destabilizes the nucleus itself. An example of this is ^{163}Dy (Jung et al., 1992), where the removal of inner shell electrons destabilizes the nucleus.

B. Tridents

The trident process is a process in which a nucleus plays an additional 'photon' in the interaction among electrons and photons. This may be written in terms of the Feynman diagram as compared with the gamma photon initiating the electron-positron pair creation.

This process allows us to access the positron production by intense laser almost witin the currently available technology. If it were just for the vacuum pair creation, the necessary electric field needs to reach the Schwinger field (see later section). With the presence of the trident process, this condition is greatly relaxed (Shearer, et al., 1973; Liang et al., 1978; Mima et al. 1999) and may be amenable under the laser intensity at around 10^{22}W/cm^2.

In the experiments conducted by the intense laser irradiation on a solid target (Cowan et al.) high energy electrons are generated and these electrons in sold lead to the creation of high energy gamma photons by the bremsstrahlung. These gamma rays are the likely culprits that induce the nuclear transmutation (Cowan et al., 2000) The energetic electron production as well as other particle acceleration may provide an avenue to create substantional amount of isotopes of ions in the laser irradiated plasma/matter. Isotope productions by laser acceleration (Leemans et al.2001; Yamagiwa, 1999; Nemoto 2001; Ledingham 2001, Spencer, et al., 2001). For example, the minor actinide transmutation may be carried out by using a new fission decay mechanism through populating the vibrational levels created by the hyperdeformation of nuclei(suvh as the formation of isomers) (Shizuma, et al., 2002). Such a process may be accessed by gamma rays generated by the inverse Compton scattering of laser off high energy electron beam. Such gamma rays induce various (γ,n) nuclear processes, as opposed to the more common (n,γ) processes in nature and in the past nuclear experiments.

C. Superhot plasma and cluster interaction, Coulomb explosion, cluster fusion, neutron sources

Nanoclusters and microclusters have attracted strong interest over the years (Gotts,62; Doremus 64; Kawabata 66; Purnell.94). In particular their interaction with intense laser (Ditmire et al. 1996; Shao et al., 1996; Ditmire et al., 1997, Ditmire et al., 1998, 1999, 2000; Zweiback et al, 2000, Zweiback and Ditmire, 2001) has sparked recent interest. The interaction of clusters with laser has many salient features. One of them is that this interaction as compared with plainly prepared materials such as gas or solid is much enhanced. This results in superhot matter and much higher absorption of laser.

The cluster targets irradiated by the laser light show the properties of both underdense and overdense plasmas as well as novel optical properties Tajima, Kishimoto, Downer, 1999. Very efficient absorption of laser energy has been demonstrated by Ditmire et al. 1996, Lezius, 1998, Ditmire et al., 1999 with the formation of underdense plasmas with very high temperature and X-ray emission. Such high temperature plasma makes table top fusion experiments (Ditmire et al., 1999; Zweiback et al, 2000; Parks et al., 2001; Last and Jortner, 2001) possible and provides a mechanism for ion injection into accelerators.

The regimes of laser-cluster interaction, with the generation of fast ions, investigated by Ditmire et al. 1996, Krainov, V. P. and Smirnov 2000 refer to conditions dominated by collisional absorption and by heating of the cluster plasma. In this case the hot cluster plasma expansion occurs in the ablation regime. With the increase of the laser pulse intensity up to the range of 10^{21} *to* 10^{22} W/cm^2, the laser light can rip electrons away from atoms almost instantaneously, instead of going through secondary processes of heating and collisions. In the petawatt range of parameters the laser radiation has such a high intensity that it can blow off all the electrons and prepare a cloud made of an electrically non-neutral plasma. Provided the cluster has large enough size and the density of a solid, ions are accelerated up to high energy during the Coulomb explosion of the cloud, Last, Schek and Jortner, 1997; Nishihara et al., 2001; Eloy et al., 2001; Kumarappan et al., 2001, 2002, Kishimoto, et al. 2002.

An electrostatic potential appears in the plasma formed by a cluster irradiated by a laser pulse. The value of this electrostatic potential, which is due to the separation of the electric charges, can be at most equal to the value of the potential at the surface of a charged sphere with a radius R and density n: $\varphi_{\max} = 4\pi n e^2 R^2 / 3$.

Now we consider the motion of the ion component under the Coulomb repulsion in this second phase. Assuming the ions to be cold and to move radially, we obtain the energy integral $K_i - \Pi(r_0, t) = const$, where the ion kinetic energy is $K_i = \sqrt{m_i^2 c^4 + p_r^2 c^2} - m_i c^2$ and the potential energy is $\Pi(r_0, t) = 4\pi e^2 Q(r_0)(1/(r_0 + \xi(r_0, t)) - 1/r_0)$, where r_0 is the initial ion position, $\xi(r_0, t)$ is the ion displacement at time t and $Q(r_0) = \int_0^{r_0} n_i(r_0) r_0^2 dr_0$. During the expansion of the cloud the ion kinetic energy increases, for $\xi \to \infty$, up to the value $4\pi e^2 Q(r_0)/r_0$ which depends on the initial position of the ion inside the cloud. Assuming a homogeneous distribution of the ion density inside the cloud, n_i, we obtain that an ion acquires a final energy $K_i = 2\pi e^2 r_0^2 / 3$, which is limited by $K_{\max} = 2\pi e^2 R^2 / 3$.

Since the ion energy is proportional to r_0^2 we can calculate the ion energy spectrum df/dK_i which, due to the flux continuity in phase space, is proportional to $4\pi r_0^2 dr_0 / dK_i$. We obtain (Nishihara et al., 2001)

$$\frac{df}{dK_i} = \frac{3R}{e^2}\theta(K_{\max} - K_i)\sqrt{\frac{K_i}{K_{\max}}}, \tag{91}$$

where the unit step function is $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$. This form of the fast ion energy spectrum has been observed in the 3D PIC simulations of the Coulomb explosion of the cluster exposed to the high intensity laser radiation, which were presented by Nishihara et al., 2001.

When the ion energy is smaller than $m_i c^2$, we can use a nonrelativistic description of the Coulomb explosion. In this approximation we write the following system of equations of motion

$$\ddot{\xi} = (\omega_{pi}^2 / 3) r_0^3 / (r_0 + \xi)^2. \tag{92}$$

Here $\omega_{pi} = \sqrt{4\pi n e^2 / m_i}$ is the ion plasma frequency. Integrating equation (92) with the initial conditions $\xi(0) = 0$ and $\dot{\xi}(0) = 0$ yields

$$\frac{1}{2}\ln\left(\frac{2\xi + r_0 + 2\sqrt{\xi^2 + r_0\xi}}{r_0}\right) + \frac{\sqrt{\xi^2 + r_0\xi}}{r_0} = \sqrt{\frac{2}{3}}\omega_{pi} t. \tag{93}$$

When the displacement is small, $\xi \ll r_0$, ions move with constant acceleration $\xi \approx r_0(\omega_{pi} t)^2 / 6$, while, for $\xi \to \infty$, we have $\xi \approx \sqrt{2/3} r_0 \omega_{pi} t$. In the latter case ions move with constant velocity. The typical time of the ion cloud expansion is of the order of ω_{pi}^{-1}. Above we assumed that the Coulomb explosion of the cluster is spherically symmetric. The effects of the cluster asymmetry were discussed by Askar'yan and Bulanov, 1983, by Nishihara et al., 2001 and by Kumarappan et al., 2001, 2002.

For the case of deuteron clusters, because of the superhigh temperatures of matter, copious neutrons of fusion origin have been observed (Ditmire, et al., 1999). Kishimoto and Tajima have shown that the enhanced interaction of the laser-cluster arises from the nonlinearity of electron orbits from clusters (Kishimoto, et al., 2000). When the cluster size is sufficiently small or the laser intensity is sufficiently strong, electrons in the cluster execute spatial oscillation whose excursion length ξ is greater than the size of the cluster a. The polarization of the cluster set up by the oscillating electrons induced on the surface of the cluster becomes nonlinear. Electrons see strong polarization fields set up by themselves and can no longer come back to its original spot. The electron orbits exhibit remarkable chaos within a matter of single optical cycle (Kishimoto, et al., 2000). This strong orbital nonlinearity is responsible in absorbing much of laser energy within ultrashort time of less 10fs. Some or many of electrons meander out of their original cluster. When this happens, the cluster is void of much of electrons. This leads to Coulomb explosion. The energy of exploding ions is high as well as takes an almost shell distribution with predominant population in high energy side. The energy of these ions approximately scales as $E_i \propto a_0^{\,2}$ and it reaches about 1MeV at $a_0 = 10$ (Kishimoto, et al. 2002).

Fast ions accelerated during the cluster explosions have also been observed in the experiments by Springate et al., 2000. These results open up a way for construction of the table top neutron sources as well as for the table top scale of nuclear fusion devices (Parks, et al., 2001; Last and Jortner, 2001, Kishimoto, et al. 2002).

D. Fast ignition

The conventional approach of the laser fusion is to compress and heat the target to the thermonuclear conditions simultaneously by one set of lasers. In this the thermonuclear burn is given by

$$\phi = \rho r/(\rho r + \xi(T)), \tag{94}$$

where $\xi(T) = 8\, m_I c_s / <\sigma v>$ and m_I is the ion mass. At the value of ρr = 3 g/cm^3 we obtain 1/3 of burnup. The confinement time (or more precisely the disassembly time of the fuel capsule) τ and the density of the fuel n are related to the value of ρr, to yield a Lawson criterion like condition

$$n\tau = \rho r/\, 4 c_s\, m_{I,}, \tag{95}$$

yielding an approximate criterion for ignition as 2 x 10^{15} s/cm^3 (Lindl, 1995). In order to achieve this energetically most favorably (i.e. with the least amount of compression energy), it is to take the route along (or near) the Fermi degenerate state. The laser pulse needs to be smoothly rising in order to make the shock minimize the entropy increase upon the compression. In addition to the adiabatic compression. one wants to make sure that toward the end of the compression phase (i.e. the slowdown phase), the Rayleigh-Taylor instability not to cause detrimental effects on the fuel (Lindl, 1995). These considerations lead to the well-known strategy of the smooth and slow rise of the laser pulse over some 20nsec with a sharp rise toward the end of the pulse that rise in about 2nsec with 10 times the pulse height. In this standard approach it is clear that the fuel compression is tied to the temperature rise through the adiabat. By incorporating the driver energy requirement and the fusion energy gain, one arrives at a scaling law of energy gain as a function of the driver energy E_{dr} in the high gain area (Kozaki, 1998) as

$$G = 100\,(E_{dr} / E_0)^{1/3}, \tag{96}$$

Where E_0 (in MJ) is the normalizing driver energy that achieves G =100 in 'direct drive', which is about 4MJ according to Kozaki, 1998.

In 1994 Tabak et al. have proposed to decouple the condition for compression and that for heating of the fuel to the thermonuclear temperature. In this scheme, they propose to first compress the fuel in a smooth adiabatic fashion without achieving the thermonuclear temperature at the fuel core first, which allows a far smaller energy in the driver laser, because the energy of the laser is directly tie to the final pressure in compression. One can choose a lot lower adiabat in entropy in this case. The moment we achieve the density dictated by Eq.(WW), a short intense laser is injected to heat the core. In Tabak's proposed scheme (a fllowup by Key et al. 1998; Mima et al. 2000), this short pulse laser (of the length on the order of psec) interacts with the atmosphere of the compressed target at the resonant surface (of the density about 10^{21} cm^{-3}). Here according to the Tabak et al., 1994 scheme electrons are heated and turn into an energetic beam with approximate energy of MeV. With a judicious choice of the electron beam energy (i.e. the absorption process of the laser by the target matter) and the linear density of the fuel, we can deposit this electron energy in the fuel core. This condition for the sufficient electron range not to exceed the target size may be written as

$$\rho r \approx 0.5 \text{ in g/cm}^2 . \tag{97}$$

The laser pulse length is given as

$$\tau_l = 40\,(100 \text{ g/cm}^2 / \rho) \text{ ps}. \tag{98}$$

This yields the pulse length between 10-20 ps for compressed fuel density of 200-300g/cm^2 (Key et al. 95). According to Atzeni, 1999, Atzeni et al., 2002, Temporal, et al., 2002, the required laser energy for the fast ignition drive is

$$E_{las} = 80\ (100gcm^{-3}/\rho)^{1.8}\ \text{kJ}. \tag{99}$$

This sets the laser energy for fast ignition about 50kJ, while 10-20kJ of electron energy needs to be delivered at the spot. In terms of the gain with the assist of fast ignition, the gain is scaled as a function of the compression driver energy in the same fashion as in the standard 'direct drive' scaling (Kozaki, 1998) as

$$G^{fi} = G^{fi0}\,(E_{dr}\,/\,E_{fi0}\,)^{1/3}\,, \tag{100}$$

where G^{fi0} is between 100 and 300 and E_{fi0} = 0.5MJ (Kozaki, 1998). Thus, given compression driver energy (even with a modest increase of fast ignition driver energy and added complexity), the fusion gain is greatly enhanced over the standard method.

The crucial question is how the laser energy is transferred to electrons and how this electron beam is transported to the fuel core and deposits most of its energy in the core (Key et al. 98; Mima, 2000). It is expected that a straightforward propagation of the electron beam that is created at the resonant surface to reach the small core spot. There are several expected plasma and beam instabilities along the way, which consist of dense hot plasma with density ranging from 10^{21} to 10^{26}cm^{3}. These include the hose instability, the sausage instability, and the filamentation instability. The expected current far exceeds the Alfven current, above which the induced magnetic field of the electron beam itself bends the electron orbits severely. Thus usually a strong return current is expected nearly cancelling this original electron current. On the other hand this electron stream of the return current can give rise to the secondary plasma instabilities.

The recent success in the design of the target with a coned access may allow to alleviate some or major difficulties of the above (Kodama et al., 2001).

To overcome this difficulty of electron beam transport over a long distance, Mourou and Tajima have proposed to use lasers with an even shorter pulse (of the order of 10fs) with much higher intensity of 10^{25}W/cm^{2}. Such an intense laser pulse penetrates the dense plasma beyond the ordinary critical density because of the relativistic effect (see above sec.). It remains to be seen if the resultant energy of electrons on MeV can be the main constituent of the electron energies from such intense laser interaction.

An other modification of the fast ignition concept is related to the proposed by Roth, et al., 2001 and Bychenkov, et al., 2001 scheme, when instead of fast electrons the beam of the laser accelerated ions ignites the precompressed target.

IIX. High Energy Physics

First we need to ask the question when we can consider that optics is in high energy physics. To determine if it is the case we will use the following ratio.

$$R = \frac{\Delta}{Mc^2} > 1 \qquad (101)$$

That expresses, for a given reaction the ratio between the binding energy (Δ) and the rest

$$R = \frac{\Delta}{Mc^2} > 1 \qquad (102)$$

mass energy (Mc^2) of the constituents. For a chemical reaction where Δ ~1eV, Mc^2~10GeV,the ratio R ~10^{-9}. For nuclear reaction where Δ~10MeV and Mc^2~10GeV, R~10^{-3}. In high energy physics R is of the order or greater than 1. The production of positron for instance from the scattering of a relativistic electron with an energy of few mc^2 from the nucleus by the trident process leads to R ~1. The observation of the positron by Anderson in early 1932, predicted by Dirac is considered as the birth of the field of high energy physics. Similarly, we could argue that the laser-produced positrons that has been demonstrated few years ago by the Garching and Livermore groups and could be considered as the entry of optics in high energy physics.

Since the first electron acceleration experiments demonstrating the high field gradients **(**Clayton, 1993; Modena, et al., 1995; Nakajima et al, 1995; Umstadter 1996), we see an increasing number of novel potential applications of ultra -high -intensity lasers in high energy physics. They could be grouped in three categories:

8) Large field gradient applications: low emittance injector of stable particle, lepton, hadron and unstable particle pions, muons, etc.
9) Particle production, positron, pions, neutrinos, polarized positrons and electrons. (include Habs' scheme)
10) Efficient γ production for $\gamma\gamma$ or γe colliders.
11) Non-luminosity paradigm with extreme high energies.

A. Large field gradient applications

1. Electron injector

Already a large body of work has demonstrated the generation of gargantuan electrostatic field gradients**.** Large numbers of electrons (nC) have been accelerated over only few tens of μm to energy reaching 150 MeV and corresponding to gradients in the range of 200GeV/m (**Figure 3**) It is worth noting that these large gradients confer to the beam a low transverse emittance (high quality). The transverse emittance expresses the quality of a beam. It is the product of the beam waist area and the beam solid angle in the far field. It needs to be as low as possible that with a minimum given of λ^2. Laser accelerator beams have shown already to have a better transverse emittance than conventional accelerator. Various methods to induce small emittance electron beam sources driven by laser have been introduced by utilizing the large electric fields of laser plasma interaction to kick electrons from the plasma into the beam. These include the methods of the self-modulated laser wakefield acceleration (SMLWFA), LILAC, beatwave, and subcyclic injectors. Possible extractionby applying the rf acceleration of these beams has been considered (Chao, 2003) They have discussed the space charge effects that play a role in emittance growth and control . A series of recent experiments using the SMLWFA generated quite remarkable results (Nakajima, et al., 1995; Modena, et al., 1995; Chen, et al., 1999; Assamagan, et al., 1999; Kodama 2000; Leemans, et al., 2002; Malka, Faure, and Amiranoff, 2001). When the plasma density is sufficiently high, the laser pulse is longer than the resonant length of Tajima-Dawson. However, the self-modulating instability of the plasma electrons via the forward Raman instability (e.g. Kruer, 1988) can give rise to the undulated laser profile with the plasma period with induced plasma waves. The phase velocity of the plasma wave is equal to the group velocity of the laser (Tajima and Dawson, 1979)

$$v_p = c(1-\omega_p^2/\omega^2)^{1/2} \approx c(1-\omega_p^2/2\omega^2). \qquad (103)$$

Because of the large amplitude and relatively slow phase velocity due to the high plasma density in these experiments, electrons in the plasma can be easily picked up and trapped into the plasma wave (Esarey and Piloff, 1995). The general features of these experiments are as follow.

First, a large amount of electrons (in the order of 1nC) are trapped and accelerated. Secondly, the transverse emittance is surprisingly small, though it is far from clear how accurately the emittance may have been measured so far, amounting to the order of 0.1 mmmrad, at least an order of magnitude smaller than the rf-based electron injector's emittance. Third, the longitudinal energy spread is rather large (up to 100%), because of the pickup from the bulk electrons.

The product of the bunch length and the energy spread is the longitudinal emittance and it is about comparable to the conventional rf sources. The tiny transverse spot size of the bunch corresponds to the laser spot size and therefore small transverse emittance. For example in Assamagan et al., 1999 at least 5 x 10^8 electrons are accelerated to the average energy of 7 MeV with transverse emittance as low as 10^{-7} mrad. (It should be noted that though this energy spread is substantial, the relative energy spread $\Delta E / E$ for high energy applications is certainly tolerable, as E gets larger. Meanwhile, there have been many theoretical proposals to reduce the energy spread and thus the longitudinal emittance in general [Umstadter, et al.; Rau et al.; Esarey et al.].) Because these experiments were a first generation experiments without particular sophistication of the beam handling and dynamics, this low transverse emittance has been a pleasant surprise as well as a puzzle. Because of this preliminary nature of experiments, it is highly desirable to measure the beam properties more precisely. It is understandable that the laser driven electron source has low emittance to begin with, as the laser is focused to a small (such as < 10μ) spot and electrons are promptly accelerated to relativistic energies. It is still puzzling, however, during the beam transport after the electron bunches emerge from the plasma, space charge effects can blow up the emittance, but the experiments appear to indicate quite low values of emittance. However, it has been pointed out (Chao et al., 2003) that this coupling can be important. This is because in these experiments (a) the longitudinal bunch length is much smaller than that of the conventional beams; (b) the longitudinal energy spread is much larger than that of the conventional ones. The longitudinal emittance (the product of the bunch length and the energy spread) is in a similar ballpark in the laser as conventional sources, i.e. MeV psec =keV nsec. These two characteristics of laser driven sources make the bunch length change rapidly as soon as the beam emerges out of the plasma wave. This bunch lengthening has a sensitive influence on the transverse space charge effects. The bunch lengthening gives rise to the dilution of space charge. On the other hand, the transverse beam spread can also mitigate the longitudinal bunch lengthening, as it too reduces the space charge effects. It is, therefore, crucial to incorporate the coupling between the longitudinal and transverse dynamics in order to evaluate the property of the laser driven bunches and how to control and utilize this potentially important new technology in high energy accelerators. The incorporation of this coupling has been shown to explain the experimentally measured (apparent) emittance being in fact quite small. The emaittance at the plasma source is estimated as small as 10^{-8} m rad (Chao, et al.2003). A good way to balance the desire to have small emittance and beam size and the wish to have a large number of electrons is to have a fairly long pulse (up to 1psec) and to extract it into a traditional rf (such as the X-band) cavity to accelerate electrons to higher energies (beyond 100MeV) before their space charge exert influence on the emittance. If and when such a beam may be extracted, the injection of such a beam (with emittance 10^{-7} m rad) into an X-ray free electron laser (FEL), the undulator length of the FEL may be greatly reduced (from 100m to 30m in the example of the LCLS, the SLAC's proposed X-ray FEL, according to Chao, et al., 2003).

In terms of the application of the laser accelerated beams to an injector to an rf accelerator, it is important to understand if the rapid dynamical changes still allow us to properly insert the bunches into the rf accelerator structure and how to do so. Since the longitudinal bunch lengthening happens quickly, we have to capture the beam before it becomes too long for that. Since the transverse beam spread takes place rapidly as well, we may need to focus the beam with magnetic field.

This is a direct benefit of abrupt acceleration. In a particle beam the emittance grows mostly at the front end of the beam, where the particles are not relativistic yet and can be easily subject to coulomb interaction. The coulombic interaction scaling as $1/\gamma^2$ it is important for the particles to reach the relativistic regime as fast as possible and therefore makes abrupt acceleration highly desirable.

This foil physics of electron bunch production.such as electron energy vs. a_0 is discussed. When the laser intensity is modest (a_0 less than unity), the main acceleration of electrons is in the direction opposite to the incident laser. As the laser intensity increases, more and more electrons are accelerated forward direction through the foil. The ionizational process is likely a combination of the Coulomb barrier suppression, the above threshold ionization, and multi-photon ionization. The level of ionization of high Z (here Z is greater than several) has been qualitatively studied (Richie et al 1998; Zhidkov et al 1998). When the peak intensity of laser pulse enters the target, electrons that have been stripped from atoms of the target material will be accelerated to high energies. The electron acceleration process in relatively low Z matter with ultrashort pulse laser is related to the wakefield generation and its associated processes. Here the electron energy spectrum tends to exhibit a power law (Nakajima, et al., 1995, Modena, et al., 1995; Cowan, et al., 1999) with a spectrum index between 0 and 2. In the nonrelativistic regime, the wakefield amplitude is proportional to the intensity, while the acceleration length is multiplied yo it to get the energy gain. Maximum energies of the electrons are proportional to the intensity of laser in the relativistic regime, if it is based on ponderomotive acceleration.

When the pulse length is sufficiently short and the surface of the metallic foil is sharp enough to cause the electron to execute an orbit out of the foil surface leading to the removal of electrons out of the uniform medium, this nonlinear electron orbital effect gives rise to rapid loss of electron memory and thus heating of electrons. This is the mechanism of the so-

called Brunel heating or the vacuum heating of electrons by short pulsed laser (Brunel, 1987). On the other hand, if the pulse is long enough to cause the surface to ablate to form a gentle density gradient or the density gradient is small to start with, electron orbits are buried in the density. In this case, the primary absorption mechanism is due to the resonant absorption. The criterion of which regime it belongs to is the competition between the electron excursion length in the laser $\xi = eE/m\omega^2$ and the density gradient scale length L_n. A clear experimental demonstration of this has been carried out by Grimes et al., 1999. In the nonrelativistic regime, a rapid rise of the electron energy from the irradiated foil has been observed. The energy of electrons continuously rises after it becomes relativistic. This is due primarily by the ponderomotive acceleration at the front of the laser when the foil is thin enough for the laser to burn though the solid electrons. (Gibbon, 1996, Denavit, 1992, Zhidkov et al. 1999) When the laser is longer and the surface of the foil is ablated, the plasma is heated by resonant absorption, leading to some two-temperature distribution (Kishimoto, et al., 1983). A recent work by Nakamura and Kawata (2003) implies that if the pulse is long and the foil is thick enough, the laser front become filamentarily fragmented to result in stochastic acceleration. This leads to heating. Such heating may have taken place in a thick target-large energy experiment as the LLNL Petawatt experiment (Cowan, et al. 2000). When the laser is irradiated obliquely with the p-polarization, the electrons are directly driven into the foil matter, yielding excitation of a large-amplitude longitudinal oscillation of plasma waves in a solid-state density, which can result in ultra-short pulses of high energy electrons (Ueshima, et al., 1998). Similarly, Downer et al. (2002) have considered extraction of electrons in high concentration in low emmitance from the surface of the laser incident side. Sometimes, the prepulse induced electron heating can be beneficial to accelerate electrons. Using these hot electrons, one can make a large space charge separation (Ueshima, et al., 2000). There is a possibility of extracting and accelerating polarized electrons. Polarized electron sources have been studied, including the GaAs laser irradiation (JJAP, 46, L555, 2001). In addition to this method, we can think of a new approach based on the intense laser irradiation of a thin target that is magnetized. The relatively small angular spread of the picked up electrons (combined with the small spot size of them) provides the basis for the small source emmitance, just as in the case of the gas target laser acceleration considered earlier. Moreover, if we magnetize the metallic target (such as Fe), the outershell electrons should get their spins polarized. As the spin depolarization is smaller by the factor of g ($\ll 1$) over the orbital divergence, such a beam should preserve the spin as well as the (orbital) emmitance (Chao et al, 2003).
This can lead to the next subsection of proton acceleration.

2. Laser Accelerated Ions

Laser accelerators of ions are based on the high efficiency of converting laser energy into the energy of fast ions in the interaction of petawatt laser pulses with plasma. Collimated beams of fast ions were recorded in experiments on the interaction of laser pulses with solid targets (Maksimchuk et al., 2000; Clark, et al., 2000; Snavely , et al., 2000; Mackinnon, et al., 2001) as well as the isotropic component of fast ions was observed during interaction of laser radiation with gas targets by Fritzler et al., 2002. The ion acceleration processes are also investigated numerically (Denavit, 1992; Esirkepov et al., 1999; Bulanov, et al., 2000; Sentoku et al., 2000; Ueshima, et al., 2000; Ruhl et al., 2001; Kuznetsov, et al., 2001; Pukhov, 2001; Sentoku et al., 2002; Mackinnon, et al., 2002) by means of two- and three-dimensional particle-in-cell (PIC) computer simulations. In the experiments mentioned above, electrons were accelerated to energies of about several hundred MeV while the proton energy was about tens of MeV, the number of fast protons ranged from 10^{12} to 10^{13} per pulse and with a 12% efficiency of transformation of the laser energy into fast ion energy. The generation of fast ions becomes highly effective when the laser radiation reaches the petawatt power limit as it was shown by Bulanov et al., 2001. Particle-in-cell (PIC) computer simulations show that by optimizing the laser-target parameters it becomes possible to accelerate protons up to energies of several hundreds of MeV,

The mechanism of laser acceleration of ions (protons and other ions) is by the electric field set up by the space charge separation of hot or energetic electrons and the ions. Thus the temperature or the energy of the electrons that are driven by the laser determines the energy of ions (Snavely et al. 2000; Clark et al. 2001; Tajima, 2002 theory in AAC book). Particular mechanism of the fast electron energy transformation into the ion energy depends on the specific conditions of the laser-target interaction. Koga, et al., (2002) have shown that a strong solitary density pileup and associated density cavity provide some 500TeV/m acceleration gradient. This can happen at a ‘modest’ intensity of 10^{21} W/cm^2.

Bulanov et al., 2001 have shown the intensity of 10^{22} W/cm^2 can accelerate ions to 1GeV. Before these experiments that showed laser-driven ion acceleration , Rau et al (1998) have suggested a graded density for Alfven shocks to gradually pick up the phase velocity so that ions pickup and acceleration can be accomplished at a modest laser intensity 10^{18}W/cm^2 that can reach energies beyond 100 MeV, and its application to medicine (Tajima, 1998).

3. High energy proton beam

It has been shown that the interaction of laser with a thin target can in turn produce a copious MeV protons beam with superior transverse emittance (Roth, 2002). The proton generation is a direct consequence of the electron acceleration. The electrons that are violently accelerated in the laser field can attract behind them the protons that are either on the front or back surface of the target. Highly energetic proton beams have been demonstrated at Livermore, LULI, CUOS, Rutherford with intensity of 10^{18}-10^{20}W/cm^2. They could lead to important applications such as Fast Ignition for Inertial Confinement Fusion (ICF) as it has been pointed by Roth et al., 2001, proton therapy Bulanov and Khoroshkov, 2002, Fourkal et al., 2002, fast ion beam injection to conventional accelerators (see Krushelnick, K., Clark, E., Allot, et al., 2000), and the proton imaging, Borghesi, Campbell, Schiavi , et al., 2002.

The proton use in the radiotherapy in the oncology provides several advantages. First of all, the proton beam scattering on the atom electrons is weak and it results in low irradiation of healthy tissues aside the tumor. Second, the slowing down length for the proton with given energy is fixed, and it avoids irradiation of the healthy tissues at the rare side of the tumor. Third, the Bragg peak of the energy losses provides substantial energy deposition in the vicinity of the proton stopping point (see for example Khoroshkov and Minakova, 1998). By now, the proton beams with necessary parameters produced with classic accelerators of charged particles: synchrotron, cyclotron, and linear accelerator (Scharf, 1994). The use of the laser accelerator is very attractive because its compactness and additional possibilities to control the proton beam parameters. The typical energy spectrum of laser accelerated particles observed both in the experiments and in the computer simulations can be approximated by a quasi-thermal distribution with a cut-off at a maximum energy. The effective temperature that may be attributed to the fast ion beams is only within a factor several of the maximum value of the particle energy. On the other hand, almost all above mentioned applications require high quality proton beams, i.e. beams with sufficiently small energy spread $\Delta E / E \ll 1$. For example, for hadron therapy it is highly desirable to have a proton beam with $\Delta E / E \leq 2\%$ in order to provide the conditions for a high irradiation dose being delivered to the tumor while sparing neighboring tissues. In the concept of Fast Ignition with laser accelerated ions presented Roth et al., 2001, the proton beam was assumed to be quasi-monoergetic. An analysis carried out by Atzeni, et al., 2002 and by Temporal, et al., 2002, has shown that the ignition of the thermonuclear target with the quasi-thermal beam of fast protons requires several times larger laser energy. Similarly, in the case of the ion injector, a high-quality beam is needed in order to be able to inject the charged particles into the optimal accelerating phase. Bulanov and Khoroshkov, 2002, and Esirkepov, et al., 2002 have shown that such a required beam of laser accelerated ions can be obtained using a double layer target. Multi layer targets have been used for a long time in order to increase the efficiency of the laser energy conversion into plasma and fast particle kinetic energy (see for example Badziak et al., 2001). In contrast to the previously discussed configurations, it was proposed the usage double layer target to produce fast proton beams with controlled quality.

In this scheme the target is made of two layers with ions of different electric charge and mass. The first (front) layer consists of heavy ions with electric charge eZ_i and mass m_i. This is followed by a second (rear) thin proton layer. The transverse size of the proton layer must be smaller than the size of the pulse waist since an inhomogeneity in the laser pulse causes the inhomogeneity of the accelerating electric field and thus a degradation of beam quality, as seen in experiments where the exposed targets to the laser light had a thin proton layer on their surface.

When an ultra short laser pulse irradiates the target, heavy atoms are partly ionized and the ionized electrons abandon the foil, generating an electric field due to charge separation. Because of the large value of the ratio μ / Z_i, where $\mu = m_i / m_p$, heavy ions remain at rest, while lighter protons are accelerated. In order to achieve 10^{10} fast protons per pulse from the two-layer target required for the applications, it is enough to have a proton layer approximately $0.02 \mu m$ thick and a laser pulse focused onto a spot with diameter equal to two laser wavelengths. The first layer is made of heavy ions and the target is sufficiently thick so produce a large enough electric field due to charge separation. This electric field has opposite sign on the two different sides of the target, has a zero inside the target and vanishes at a finite distance from it. The number of protons is assumed to be sufficiently small not to produce any significant effect on the electric field. The most important requirement is that the transverse size of the proton layer be smaller than the pulse waist so as to decrease the influence of the laser pulse inhomogeneity in the direction perpendicular to its direction of propagation. The pulse inhomogeneity causes an inhomogeneity of the accelerating electric field, which results in an additional energy spread of the ion beam seen in experiments. The effect of the finite waist of the laser pulse leads also to an undesirable defocusing of the fast ion beam. In order to compensate for this effect and to focus the ion beam, we can use properly deformed targets, as suggested by Bulanov, et al., 2000, Ruhl, et al., 2001, and by Wilks et al., 2001.

In order to estimate the typical energy gain of fast ions, we assume that the main portion of the free electrons produced by ionization in the irradiated region of the foil is expelled. In this case the electric field near the positively charged layer is equal to $E_0 \approx 2\pi n_0 Z_i e l$. Here l is the foil thickness. The region of strong electric field has a transverse size of the order of the diameter $2R_\perp$ of the focal spot. Thus the longitudinal size of this region where the electric field remains essentially one-dimensional is also of order $2R_\perp$ and the typical energy of the ions accelerated by the electric field due to charge separation can be estimated as $\Delta \mathrm{E}_{max} \approx 4\pi n_0 Z_a e^2 l R_\perp$.

The energy spectrum of protons can be found by taking the electric field in the vicinity of the target to be of the form of the electric field near an electrically charged prolate ellipsoid (see Landau and Lifshits, 1984). On the axis the *x* component of the electric field is given by $E_x(x) = (4E_0 / 3)R_\perp{}^2 / (R_\perp{}^2 - l^2 + x^2)$. The distribution function of the fast protons $f(x,v,t)$ obeys the kinetic equation, which gives $f(x,v,t) = f_0(x_0,v_0)$, where $f_0(x_0,v_0)$ is the distribution function at the initial time *t=0*. The number of particles per unit volume in phase space $dxdv$ is equal to $dn = fdxdv = fvdvdt = fd\mathcal{E}dt / m_p$. We assume that at t=0 all particles are at rest, i.e. their spatial distribution is given by $f_0(x_0,v_0) = n_0(x_0)\delta(v_0)$, with $\delta(v_0)$ the Dirac delta function. Time integration of the distribution $fvdvdt$ gives the energy spectrum of the beam

$$N(\mathcal{E})d\mathcal{E} = \left(\frac{n_0(x_0)}{m_p}\right)\left|\frac{dt}{dv}\right|_{v=v_0} d\mathcal{E}. \qquad (104)$$

Here the Lagrange coordinate of the particle x_0 and the Jacobian $\left|dt / dv\right|_{v=v_0}$ are functions of the particle energy $\mathcal{E}$. The Lagrange coordinate dependence on the energy $x_0(\mathcal{E})$ is given implicitly by the integral of the particle motion: $\mathcal{E}(x,x_0) = \mathcal{E}_0 + e[\varphi(x) - \varphi(x_0)]$, with $\varphi(x)$ the electrostatic potential. In the case under consideration, we have $\mathcal{E}_0 = 0$ and $x = \infty$. The Jacobian $\left|dt / dv\right|_{v=v_0}$ is equal to the inverse of the particle acceleration at *t=0,* i.e. $\left|dt / dv\right|_{v=v_0} = 1 / eE_x(x_0)$, and it is equal to $\left|dx_0 / d\mathcal{E}\right|$. Hence, we obtain the expression for the energy spectrum in the form

$$N(\mathcal{E})d\mathcal{E} = \left[n_0(x_0)\left|\frac{dx_0}{d\mathcal{E}}\right|\right]_{x_0 = x_0(\mathcal{E})} d\mathcal{E}. \qquad (105)$$

We notice that the expression for the energy spectrum follows from the general condition of particle flux continuity in the phase space.

As we can see, in the vicinity of the target on the axis the electric field is homogeneous. Therefore, the form of the energy spectrum (104) is determined by the distribution of the proton density $n_0(\varphi^{-1}(\mathcal{E})/e)$. We see that in general a highly monoenergetic proton beam can be obtained when the function $n_0(x_0)$ is a strongly localized function, i.e. when the thickness of the proton layer Δx_0 is sufficiently small.

In order to take into account the numerous nonlinear and kinetic effects as well as to extend our consideration to multidimensional geometry, Esirkepov et al., 2002; Bulanov et al., 2002, performed numerical simulations of the proton acceleration during the interaction of a short, high power laser pulse with the two-layer target. In **Figs. 31-33** we present the results of these simulations for a linearly polarized laser pulse with dimensionless laser amplitude $a = 30$ interaction with a double layer target. The first layer of the target (gold) has the form of a disk with diameter 10λ and thickness 0.5λ. The second layer (proton) also has the form of a disk with diameter 5λ and thickness 0.03λ, and is placed at the rear of the first layer. The electron density in the heavy ion layer corresponds to the ratio $\omega_{pe} / \omega = 3$ between the plasma and the laser frequencies, for the proton layer it corresponds to $\omega_{pe} / \omega = 0.53$. The number of electrons in the first layer is 180 times larger than in the proton layer.

In **Fig. 31** we present the spectra of the proton energy and the energy per nucleon of the heavy ions. In **Fig. 32** we present the distributions of the electric field components inside the computation box, to show the shape of the transmitted laser pulse and the accelerating longitudinal electric field. The accelerating field is shown as a 3D vector field (a); it is localized in the vicinity of the first layer (the heavy ion layer) of the target and can be described as an electrostatic field from the charged disk. The transmitted laser pulse is presented by the isosurfaces of constant value of the *z* component of the electric field (b). In **Fig. 33** we show the densities of plasma species inside the computation box.

We see that the proton layer moves along the *x* axis and that the distance between the two layers increases. The heavy ion layer expands due to Coulomb explosion and tends to become rounded. Part of the electrons is blown off by the laser pulse, while the rest forms a hot cloud around the target. We notice that for the simulation parameters the electrons do not abandon the region irradiated by the laser light completely. Even if only a portion of the electrons is accelerated and heated by the laser pulse, the induced quasi-static electric field appears to be strong enough to accelerate the protons up 65*MeV*. The energy per nucleon acquired by the heavy ions is approximately 100 times smaller than the proton energy. As seen in **Fig. 31**, the heavy ions have a wide energy spectrum while the protons form a quasi-mono-energetic bunch with $\Delta\mathcal{E} / \mathcal{E} < 3\%$. The proton beam remains localized in space for a long time due to the bunching effect of the decreasing dependence of the electric field on the coordinate in the acceleration direction.

4. Laser-produced pions and muons

At much higher intensities, 10^{23}W/cm^2, 15fs duration, PIC simulation performed by Pukhov et al, 2003, shows that the interaction with a 50μm, solid target with an electron density of n= 10^{22}cm^3, leads to an electron beam of 5GeV followed by a proton beam of 5GeV. Lets note that the electrostatic field gradients involved are of the order of the laser transverse field gradients of 500 TeV. At 10^{23} W/cm2, the transverse field is given by ((3) is 500 TV/m.

Bychenkov et al., 2001, have carried out two-dimensional “particle-in-cell” modeling to determine the laser intensity threshold for pion production by protons accelerated by the relativistically strong short laser pulses acting on a solid target. The pion production yield was determined as a function of laser intensity. It was shown that the threshold corresponds to the laser intensity above 10^{21} W/cm^2.

The pion has a rest mass of ~140MeV and a lifetime at rest of only 20ns. This short lifetime prevents the acceleration of the low energy pions since

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \qquad (106)$$

at 10MeV/m accelerating rate, the pions will have disintegrated before it had the time to get accelerated significantly. Prompt acceleration offers a completely new paradigm for high energy physics. Over only a distance of the order of a mm the pions can be accelerated to many times their mass, say 100 times (Bychenkov, 2001). At 15 GeV the pions will have a lifetime of 2μs and can then be injected and accelerated to much higher energies using conventional means. At these energies let’s note that in the laboratory frame the desintegration product, muons and neutrinos will be emitted in a narrow cone angle of $1/\gamma$half-angle. This represents an attractive new paradigm for μμ collider or the generation of neutrino beams that would avoid muon cooling. Pakhomov, 2002, considered laser generation of controlled, high-flux pulses of neutrinos. The source will yield nanosecond-range pulses of muon–neutrinos, with fluxes of $\sim 10^{19}$ νμ s^{-1} sr^{-1} and energies of ∼20 MeV or higher. The process assumes a driving laser with pulse energy ∼8 kJ, providing an irradiance of $\sim 9 \times 10^{22}$ W/cm^2. The study of neutrino oscillations would be the possible applications of the source.

5 Colliders

The next frontier in high energy physics is the interaction at TeV center of mass energy. In this regime the electroweak symmetry is broken and is expected to reveal themicrophysical meaning of mass. and reach the limit of the standard model. To reach this regime, large hadron collider (LHC) (proton-proton) at CERN, are being built. Parallel to this effort, there is also a strong motivation to build lepton collider (e-e or muon-muon) or photon collider (γγ). Leptons, i.e. electrons and muons have no structure unlike hadron (proton, neutron, …). Therefore their interaction is clean, predictable delivering particle that can be unambiguously determined.

In this new high energy physics adventure ultrahigh intensity laser may play an important role. They have the potential

1) to provide large field gradients,

2) to provide an efficient way to increase unstable particle lifetime necessary to make muon-muon collider or neutrino beam,

3) to provide an efficient source of high energy γ making possible $\gamma\gamma$ collider.

1) The e-e collider can not exceed the TeV regime because of radiative effects known as beamsstrahlung. This effect scales inversely as the fourth power of the lepton mass and seriously impairs e-e collider luminosity beyond the TeV level. The lightest lepton is the electron so one way to circumvent this limit is to choose the next lighest lepton, the muon.with a rest mass energy of 104 MeV or 200 times the electron mass. In a muon collider the beamsstrahlung would therefore be attenuated by almost ten orders of magnitude and completely eliminated. As seen earlier, see (104), the muons as well as neutrinos are produced by the decay of pions into muons and a neutrinos. The pions can be produced by the interaction of high energy protons beam with a metallic target. As mentioned earlier, laser acceleration can accelerate the pions to many times their mass in a fraction of a millimeter. This mass increase will be accompanied by a lifetime dilatation making possible to inject the pions into a conventional accelerator. Let's mention an additional expected benefit. As observed in laser accelerated electron beam, a prompt acceleration will produce low beam emittance (high beam quality) beam)**.**

6. Increasing the τ-lepton lifetime

It is interesting to see that the next lepton would be the tau with a mass of 1784 Mev and a lifetime of 300fs. Lets note that 300fs correspond to a 100μm a very short distance for conventional acceleration. This distance would be in principle sufficient for prompt acceleration to accelerate a τ lepton to several times its mass and increase its lifetime accordingly.

7. Photon-photon collider or γ–γ collider

Photon-photon collider is very complementary to lepton collider. It is considered as the best instrument to address and discover new physics, Higgs, boson physics, extradimensions, supersymmetry, top quark.

In photon collision any charged particles can be produced

$$\gamma\gamma \rightarrow Higgs, WW, ZZ, t\bar{t} \qquad (106)$$

The cross sections for pairs are significantly higher than in e^+e^- collision as indicated in the **figure 6** showing interaction cross sections. Photon colliders are seen as the best instrument to discover new physics, The $\gamma\gamma$ collider relies on the scattering of photons from a high intensity laser by a super relativistic electron beams. After scattering, the photons have an energy closed to the electron energy as shown in the expression (107) below. The photon beams after focusing correspond approximately to the electron beam size.

The maximum energy of the scattered photons is:

$$\hbar\omega_m = \frac{x}{x+1}\mathcal{E}_0 \quad \text{with} \quad x \approx \frac{\mathcal{E}_0 \hbar\omega_0}{m^2c^4} \quad \text{or } 19\left[\frac{\mathcal{E}_0}{TeV}\right]\frac{\mu m}{\lambda}, \qquad (107)$$

where $\mathcal{E}_0$ is the electron beam energy, and ω_0 the laser frequency.

These are additional crossroads (or marriages) of laser and high energy charged particles. In some of these applications one can probe nonlinear QED (see later section) while some other can yield large amount of high energy $\gamma-$ gamma photons through the inverse Compton scattering process useful for high energy and nuclear physics (Fujiwara, 2002). Tajima (2002) has suggested to use such for a possible nuclear transmutation (in combination of efficient lasers such as the free electron laser (Minehara, 2002)).

IX. Astrophysics

The immenseness of the accelerating gradient (and, therefore, the compactness of the accelerating length) to reach ultrahigh energies is the unique feature of the acceleration mechanism associated with laser. Because of this feature, it has been recently recognized that this mechanism (the wakefield excitation) is pivotal in the generation of ultra-high energy cosmic rays (UHECR) (Chen et al. 2002). The recent observation of UHECR indicates that cosmic rays exist beyond 10^{20} eV and certainly beyond 10^{19} eV [energies greater than the GZK cutoff (Greisen, 1966; Zatsepin and Kuzmin, 1966) due to the pionization loss of protons that decay by the collision with cosmic microwave background photons]. This observation puts severe requirements on the acceleration mechanisms that have been proposed.

Ultra high energy cosmic ray (UHECR) events exceeding the Greisen-Zatsepin-Kuzmin (GZK) cutoff (Greisen, 1966; Zatsepin and Kuzmin, 1966) (5 x 10^{19} eV for protons originated from a distance larger than ~50 Mps) have been found in recent years (Bird, et al., 1993; Takeda, et al., 1998; Abu-Zayyad, et al., 1999). Observations also indicate a change of the power-law index in the UHECR spectrum (events/energy/area/time), $f(E) \propto E^{-\alpha}$, from α ~ 3 to a smaller value at energy around 10^{18} –10^{19} eV. These present an acute theoretical challenge regarding their composition as well as their origin (Olinto, 2000).

So far the theories that attempt to explain the UHECR can be largely categorized into the “top-down” and the “bottom-up” scenarios. In addition to relying on exotic particle physics beyond the standard model, the main challenges of top-down scenarios are their difficulty in compliance with the observed event rates and the energy spectrum, and the fine-tuning of particle lifetimes. The main challenges of the bottom-up scenarios, on the other hand, are the GZK cutoff, as well as the lack of an efficient acceleration mechanism (Olinto, 2000). To circumvent the GZK limit, several authors propose the “Z-burst” scenario (Weiler, 1999) where neutrinos, instead of protons, are the actual messenger across the cosmos. For such a scenario to work, it requires that the original particle, say a proton, be several orders of magnitude more energetic than the one eventually reaches the Earth.

Even if the GZK-limit can be circumvented through the Z-burst scenario, the challenge for a viable acceleration mechanism remains. This is mainly because the existing paradigm for cosmic acceleration, namely the Fermi mechanism (Fermi, 1949), and its variants, such as the diffusive shock acceleration (Krymsky, 1977; Axford, et al., 1978; Bell, 1978; Blanford and Osriker, 1978; Berezinskii et al., 1990), are not effective in reaching ultra high energies (Achterberg, 1999). These acceleration mechanisms rely on the random collisions of the high energy particle against magnetic field domains or the shock media, which necessarily induce increasingly more severe energy losses at higher particle energies.

According to the conversion theory of protons → neutrinos →protons via the Z-bursts (Weiler, 1999), high energy particles propagate through the cosmological distance as neutrinos and thus avoid the pionization decay by photon collisions, which reach our Galacfic Cluster and see gravitationally bound cosmic relic neutrinos. Even though this theory allows sources of UHECR at cosmological distances, a much likelier possibility than sources in our Galacfic Cluster , this puts the necessary engines of UHECR at source beyond 50 Mpc.

Nearly all astrophysical acceleration mechanisms for the bottom-up scenario have been bared on the Fermi mechanism (Fermi, 1949) or its variants. Regardless of their details, all acceleration mechanisms bared on the Fermi or its variants resort to the successive momentum scattering by “collisions” with magnetic fields or other particles or fields. In ultra high energies such momentum scattering causes severe radiative energy losses even if the scattered particles are protons in the regime beyond 10^{19} eV. Chen et al, (2002) proposed that the immense magnetic shocks created in the atmosphere of GRB can give rise to the excitation of large wakefields. These wakefields in the relativitically flowing plasma have properties that are convenient for UHECR generation. The wakefield, being the predominantly longitudinal field, is Lorentz invariant. Thus even extreme high energy particles (such as protons) see the same accelerating gradient unlike the transverse fields, which decay as $1/\gamma^2$, where $\gamma \geq O(10^{11})$. The wakefields in the GRB atmosphere amounts to 10^{16} eV/cm. The immenseness of the gamma flux in the GRB atmosphere causes the collisional acceleration [the Eddington acceleration], which amounts to the value of Schwinger field. This is a part of the mechanisms that constitute the GRB spectrum of gamma rays (Takahaihi et al, 2002). Another important feature of wakefield acceleration in the GRB atmosphere is their parallel directionality in successive acceleration. Even though the phase encounter of particles and the wakefields is random and the deceleration and acceleration are both possible, there are no overall momentum collisions as required in the Fermi mechanism. Thus the accumulation of stochastic momentum gain is possible for the wakefields (Chen et al, 2002).

The laboratory laser acceleration, much more moderate it may be in comparison with the GRB, will demonstrate the fundamental properties of wakefield acceleration in UHECR. In addition this mechanism may be responsible for the electron acceleration in the jets of Blazars. From Blazars (Punch, 1992) we observe very high energy gamma rays with a double-humped energy spectrum, in which the higher energy is believed to be from the Bremsstrahlung of high energy electrons, while the lower one to be from the synchrotron radiation from those electrons in the magnetic field in the jet. The typical energy of gamma rays and thus that of high energy electrons are on the order of TeV. If the central engine of the blazer, a massive galactic black hole, emits highly collimated high energy electrons (and positrons), it is likely that the eruption of these jet particles accompany disruptions (or modulations) of the electron (and positron) beam. Thus the lumpy electron beam carries large amplitude plasma wakes, wakefields driven by the electron beam, which can accelerate electrons in the jet plasma to high energies if they are trapped on such plasma waves. The energy gain of trapped electron is typically $\Gamma_p mc^2$, where Γ_p is the Lorentz factor of the jet flow. Often the jet is seen to have highly relativistic flows with Γ_p as large as 10^3. This amounts to the energy gain of ～TeV over the wakefield.

An Alfven wave propagating in a stationary magnetized plasma has a velocity $v_A = eB_0 /(4\pi m_i n_p)^{1/2}$, which is typically much less than the speed of light. Here B_0 is the magnetic field and n_p is the density of the plasma. The relative strength between the transverse field of the Alfven wave is $E_A/B_A = v_A/c$. Although these two field components are unequal, being mutually perpendicular to the direction of propagation they jointly generate a non-vanishing ponderomotive force that can excite a wakefield in the plasma, with phase velocity $v_{ph} = v_A << c$. Preliminary results from simulations indicae that such Alfven waves can indeed excite plasma wakefieds (Chen et al., 2003). For the purpose of ultra high energy acceleration, such a slow wave would not be too useful, as the accelerating particle can quickly slip out of phase against the wakefield. In the frame where the plasma has a relativistic bulk flow, however, the dephasing length (thus the energy gain) can be much enhanced. Furthermore, in this relativistic flow the excited wakefields are essentially unidirectional.

With our applications to astrophysical problems in mind, the Alfven-wave-plasma interaction relevant to us is in the nonlinear regime. The nonlinearity of the plasma wakefield is governed by the Lorentz-invariant normalized vector potential $a_0 = eE/mc\omega$ of the driving EM wave. When this parameter exceeds unity, nonlinearity is strong so that additional important physics incurs. In the frame of a stationary plasma, the maximum field amplitude that the plasma wakefield can support is

$$E_{max} \approx a_0 E_{wb} = mc\omega_p a_0 /c,$$

which is enhanced by a factor a_0 beyond the cold wavebreaking limit, the Tajima-Dawson field E_{wb},of the linear regime. Transform this to a frame of relativistic plasma flow, the cold wavebreaking field is reduced by a factor $\Gamma_p^{1/2}$, while a_0 remains unchanged. The maximum "acceleration gradient" G experienced by a singly-charge particle on this plasma wakefield is then

$$G = eE'_{max} \approx a_0 mc^2 (4\pi r_e n_p / \Gamma_p)^{1/2}, \tag{108}$$

where r_e is the classical electron radius.

We now apply our acceleration mechanism to the problem of UHECR. GRBs are by far the most violent release of energy in the universe, second only to the big bang itself. Within seconds (for short bursts) about erg of energy is released through gamma rays with a spectrum that peaks around several hundred keV. Existing models for GRB, such as the relativistic fireball model by Reece 1987, typically assume either neutron-star-neutron-star (NS-NS) coalescence or super-massive star collapse as the progenitor. The latter has been identified as the origin for the long burst GRBs (with time duration~10-100 sec.) by recent observations (Price , 2002). The origin of the short burst GRBs, however, is still uncertain, and NS-NS coalescence remains a viable candidate. While both candidate progenitors can in principle accommodate the plasma wakefield acceleration mechanism, the former is taken as an example. Neutron stars are known to be compact (R_{ns} ~ O(10)km) and carrying intense surface magnetic fields (B_{ns} ~ 10^{12} G). Several generic properties are assumed when such compact objects collide. First, the collision creates sequence of strong magneto-shocks (Alfven shocks). Second, the tremendous release of energy creates a highly relativistic out-bursting fireball, most likely in the form of a plasma.

The fact that the GRB prompt (photon) signals arrive within a brief time-window implies that there must exist a threshold condition in the GRB atmosphere where the plasma becomes optically transparent beyond some radius R_0 from the NS-NS epicenter. Applying the collision-free threshold condition to the case of out-bursting GRB photons, the optical transparency implies that $\sigma_C \leq \Gamma_p / n_{p0} R_0$, where $\sigma_C \sim 2 \times 10^{-25}\,\mathrm{cm}^2$ is the Compton scattering cross section for $\omega_{grb} \sim mc^2/\hbar$. Since $\sigma_{pp} < \sigma_C$, the UHWCRs are also collision-free in the same environment.

The magneto-shocks are believed to constitute a substantial fraction, say $\eta_a \sim 10^{-2}$, of the total energy released from the GRB progenitor. The energy Alfven shocks carry is therefore $\varepsilon_A \sim 10^{50}$ erg. Due to the pressure gradient along the radial direction, the magnetic fields in Alfven shocks that propagate outward from the epicenter will develop sharp discontinuities and be compactified. The estimated shock thickness is ~ O(1)m at R_0 km. From this and one can deduce the magnetic field strength in the Alfven shocks at R_0,which gives $B_A \sim 10^{10}$ G. This leads to $a_0 = eE_A / m\omega_A c$. Under these assumptions, the acceleration gradient G is as large as

$$G \sim 10^{16} (a_0/10^9)(10^9\,cm/R_9)^{1/2}\ \mathrm{eV/cm}. \qquad (109)$$

The wakefield acceleration, as considered above, provides an alternative mechanism to the Fermi acceleration (see Bell, 1978). Thus laboratory laser experiments may serve as a fascinating glimpse into cosmological processes of high energy acceleration.

X. Ultra High Intensity and General Relativity

The main postulate of General Relativity is the Einstein principle of equivalence that state that the effect of an homogeneous gravitational field is equivalent to that of a uniform accelerated reference frame. In the past there have been experiments to test the equivalence principle in its weak limit in the laboratory using neutron beams with a spinning mirror (Bonse and Wroblewski, 1983). With adoption of strong laser, we may perhaps be able to test the equivalence principle in its strong limit.

The electrons subjected to the ultrahigh electric field can become relativistic in a time corresponding to a fraction of a femtosecond. The accelerations experienced by the electrons are huge and is given by

$$a_e = a_0 \cdot \omega \cdot c, \tag{110}$$

where a_e is the electron acceleration, ω the laser frequency. For a_0 =1, a_e=10^{25}g, and for a_0=10^5, a_e=10^{30}g.

This type of acceleration is found near the Schwartzschild radius of a black hole and is given by

$$a_e = \frac{GM}{R_s^{\,2}}, \tag{111}$$

Using the gravitational red shift expression at the Swartzschild radius where

$$\frac{2GM}{R_s c^2} = 1 \tag{112}$$

An expression for the Swartzschild radius R_s and circumference C_{bh} of the equivalent black hole can be found

$$R_s = \frac{1}{a_0}\frac{\lambda_{laser}}{2\pi}, \tag{113}$$

$$C_{BH} = \frac{\lambda_{laser}}{a_0}, \tag{114}$$

For a_0=1, R_s=λ_{laser}=1μm and a mass $M \sim M_{earth.}$ For a_0=10^5, R_s =.1$\AA$ The black hole being very small will have a temperature very high. The Hawking temperature is given by the celebrated Hawking expression

$$T = \frac{\hbar c^3}{8\pi kGM}, \tag{115}$$

where we can find easily that the black hole temperature corresponds to

$$kT = \frac{h\nu}{a_0 8\pi}. \tag{116}$$

This temperature for a_0~1 the black hole temperature is of the order of 1eV or 10^4 degree C. The black hole temperature needs to be compared to the 2.7 K cosmic background temperature.

The important point of the equivalence principle is that the effect of gravity is only felt by the particle which is in the frame of being accelerated. The inertial observer does not see the effect. The Unruh radiation (1976) may be the one which breaks this bind (Chen and Tajima 1999). The signature of the Unruh radiation may be buried under the noise of the conventional radiation due to the particle acceleration, i.e. the Larmor radiation. The ratio of the two is calculated (Chen and Tajima 1999) as:

$$P_U \;/\; P_L \;=\; \hbar\omega a_0 \;/\, mc^2 \;. \tag{117}$$

It is at about one out of million at the intensity of 10^{18} W/cm^2 . This ratio increases as a function of the square root of the laser intensity. Because of the pattern in radiation and the frequency band difference, it may become possible to observe this signal according to Chen et al. in sufficiently intense laser regimes.

We notice also here the violently accelerating proton decay predicted by Ginzburg and Syrovatskii (1965). This process has been studied in details by Vanzella, and Matsas (2001).

Another important implication of violent acceleration is that the distance to the horizon is shrunken from infinite to some finite distance

$$d \;=\; c^2 / a_e \;=\; \lambda / 2\pi a_0, \tag{118}$$

where λ is the wavelength of the laser. This distance may be substantially small for huge a_0. Recently, theory of quantum gravity has been advanced (Arkani-Hamed et al. 2000, Rubakov, 2003), in which the possibility of the gravitational interaction having extra-dimensions may be manifested over a rather macrodistance has been introduced. This has caused quiet an excitement. In fact for our ultra-intense lasers this also provides a new opportunity. It is possible that for a sufficiently intense laser field the distance of the electron to its horizon is on the order or smaller than the distance r_n over which the effects of extra-dimensions manifest according to the quantum gravity theory by Arkani-Hamed et al. (1998, 2002):

$$r_n \;\sim\; 10^{32/n\;-17} \;\; cm. \tag{119}$$

Here n is the extra-dimension beyond 4. If this is the case, we expect that the wave function of the electron may begin to feel the different gravitational Gauss Law and subsequent consequences. Exactly what these consequences may be, we need to explore in the future.

XI. Nonlinear QED

In a strong electromagnetic field the vacuum behaves similarly to a birefracting, i.e. anisotropic medium. This fact is known for about 70 years since papers published by Halpern (1933), Euler (1936), and by Heisenberg and Euler (1936). After discovering the pulsars and with the emerging of the lasers able to generate relativistically strong electromagnetic fields, it becomes clear that the effects of the vacuum polarization can be observed in the cosmos and under the laboratory conditions (see for example, Ginzburg, 1989). A measure of the electromagnetic field strength in quantum electrodynamics is given by the field

$$E_{Schw} \;= m_e^2 c^3 \,/\, e\hbar = 1.3\times 10^{16} V \;/\, cm, \tag{120}$$

which is known as the Schwinger critical field. This field is to accelerate electrons to gain the electron rest mass energy $m_e c^2$ over the Compton length $l_c = \hbar / m_e c$.

Heisenberg and Euler (1936) have obtained the Lagrangian valid for arbitrarily strong free electromagnetic field. The Heisenberg-Euler Lagrangian contains corrections that come due to photon-photon scattering mediated by through exchange of virtual electron positron pairs. The quantum effects become to be of the order of $\alpha = e^2 / \hbar c = 1/137$, when the field strength approaches E_{Schw} . This Lagrangian has both the real and imaginary parts which describe the vacuum polarization and exponentially small in the limit $E / E_{Schw} \ll 1$ probability of the e^-, e^+ pair creation (see Ritus, 1979; Berestetskii, Lifshitz, and Pitaevskii, 1982; Itzykson and Zubar, 1980). In the limit $E / E_{Schw} \ll 1$ the electron-positron pair creation can occur just as a result of the quantum tunneling, and its rate is exponentially small, $W \propto \exp(-\pi E_{Schw} / E)$, as it follows from the results by Klein, 1929 and Sauter, 1931. Bunkin et al., 1969 have attracted attention for the first time to the question whether the high power lasers might provide a condition to approach a critical value of the electric field to observe the pair creation in a vacuum. Zel’dovich and Popov, 1972 studied the problem of the pair creation in the Coulomb field of colliding heavy ions with $Z_1 + Z_2 > 170$. The X-ray lasers were considered as a candidate to generate the electric field much higher than it could be met in the optic range because of focusing into a much smaller focus spot with the size of the order of 0.1 nm (see Mellissinos, 1999; Chen and Pelligrini, 1999; Chen and Tajima, 1989; Ringwald, 2001; Tajima, 2002; Roberts, et al., 2002).

Spontaneous particle creation from vacuum is one of the most important problems in quantum field theory both in regard with the development of the theoretical aspects and with the experimental verification of contemporary physics basis. The mechanism of the particle-antiparticle pair creation has been applied to various problems that range from the black hole evaporation (Hawking, 1975) to nuclear physics (Fradkin et al., 1991) and particle creation in the Universe (Parker, 1969, Zel’dovich, 1974).

Theoretically, the process of the e^-, e^+ pair creation resembles the tunneling ionization of the atom. The atom ionization by alternating electric field has been considered by Keldysh, 1965, and the electron-positron pair creation by Brezin and Itzykson (1970). In both cases we can say about breakdown either of initially neutral gas or of a vacuum in the alternating electric field. The formalism used to calculate the probability of the e^-, e^+ pair creation in a vacuum by the alternating electric field is similar to the formalism developed for the description of the ionization by Perelomov, Popov and Terentiev, 1966 (see also Popov, 1971; Popov and Marinov, 1972; Narozhny and Nikishov, 1973; Popov, 2001; 2002).

In strong laser fields, as we see, vacuum is no longer inert. The vacuum nonlinear susceptibilities appear due to the interaction between two photons via production of virtual e^-, e^+ pairs. An effective Euler-Heisenberg Lagrangian for light-light scattering has been discussed for the process $\gamma + \gamma \rightarrow \gamma + \gamma$ (**Fig. 35**) in the limit of relatively weak electric and magnetic field ($E / E_{Schw} \ll 1$ and $B / B_{Schw} \ll 1$) is given by $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$. Here $\mathcal{L}_o$ is the Lagrangian of free electromagnetic field. It describes the linear electrodynamics of vacuum.

The nonlinear quantum electrodynamics correction is described by $\mathcal{L}'$ and $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$ has a form

$$\mathcal{L} = \frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} - \frac{\kappa}{64\pi} \left[5\left(F_{\alpha\beta} F^{\alpha\beta} \right)^2 - 14 F_{\alpha\beta} F^{\beta\gamma} F_{\gamma\delta} F^{\delta\mu} \right]. \qquad (121)$$

(see Berestetskii, Lifshitz, and Pitaevskii, 1982; Itzykson and Zubar, 1980). Here $\kappa = e^4 \hbar / 45\pi m_e^4 c^7$ and $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the four-tensor of the electromagnetic field. A ratio of nonlinear terms to the linear part of the Lagrangian is of the order of $\mathcal{L}' / \mathcal{L}_0 \approx 10^{-4} (E / E_{Schw})^2$. In the case of the petawatt power laser radiation focused into a spot with the size equal to the laser light wavelength ($\lambda = 1\,\mu m$) the electric field is equal to $\approx 4.5 \cdot 10^{12}$ V/cm, and $\mathcal{L}' / \mathcal{L}_0 \approx 10^{-14}$.

Extremizing the Hamilton principal function with respect to the four potential A_α, one obtains the usual set of Maxwell's equations with the following material equations:

$$D_i = \varepsilon_{ij} E_j = (\delta_{ij} + \varepsilon_{ij}') E_j, \qquad (122)$$

$$H_i = \mu_{ij} B_j = (\delta_{ij} + \mu_{ij}') B_j, \qquad (123)$$

where

$$\varepsilon_{ij}' = \frac{\kappa}{4\pi} \left[2\left(E^2 - B^2 \right) \delta_{ij} + 7 B_i B_j \right], \qquad (124)$$

and

$$\mu_{ij}' = \frac{\kappa}{4\pi} \left[2\left(E^2 - B^2 \right) \delta_{ij} - 7 E_i E_j \right]. \qquad (125)$$

The nonlinear dependence of the vacuum susceptibilities on the electromagnetic field amplitude results in the birefringence of the vacuum, Klein and Nigam, 1964, in the scattering of light by light (McKenna and Platzman, 1963), in the parametric four-wave processes, Rozanov, 1993, and to the soliton formations as it has been shown by Soljacjic and Segev (2000). Klein and Nigam, 1964, estimated the Kerr constant of the vacuum as to be

$$\frac{7}{90\pi}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{\hbar}{m_e c}\right)^3\frac{1}{m_e c^2\lambda}. \qquad (126)$$

Here λ is the wavelength of the electromagnetic wave. The Kerr constant in the vacuum for $\lambda = 1\,\mu m$ is of the order of $10^{-27} cm^2 / erg$, which is a factor $\approx 10^{-20}$ smaller than for water.

As it is known (see Berestetskii, Lifshitz, and Pitaevskii, 1982) the Lagrangian the $\mathcal{L}'$ has an exponentially small imaginary part, which corresponds to the electron-positron pair creation in vacuum.

In 1951 Julian Schwinger calculated in detail the probability of the process when a static electric field breaks down vacuum to produce e^-, e^+ pairs:

$$W = \frac{c}{4\pi^3 l_c^4}\left(\frac{E}{E_{Schw}}\right)^2\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\pi n E_{Schw}}{E}\right). \qquad (127)$$

It reaches its optimal value at $E / E_{Schw} \approx 1$ approximately equal to $c / l_c^4 \approx 10^{53} cm^{-3} s^{-1}$.

According to Brezin and Itzykson (1970) the transition probability per unit time to spontaneously produce pairs is given by expression

$$W \approx \frac{c}{4\pi^3 l_c^4}\begin{cases}\dfrac{\pi a_0}{2\ln(4/a_0)}\left(\dfrac{\hbar\omega}{m_e c^2}\right)^2\left(\dfrac{e a_0}{4}\right)^{4 m_e c^2/\hbar\omega}, & a_0 \ll 1\\[2ex] \left(\dfrac{E}{E_{Schw}}\right)^2\exp\left(-\dfrac{\pi E_{Schw}}{E}\right), & a_0 \gg 1\end{cases}. \qquad (128)$$

The nonlinear corrections to the Maxwell equations (122-125) depend on two scalar Lorentz invariants of the field: $E^2 - B^2 = inv$ and $\mathbf{E}\cdot\mathbf{B} = inv$. It means that no pairs are produced in a field of a plane wave. The counter-propagating waves, indeed have non-zero Lorentz invariants and the pairs can be generated. In the field produced by focusing of laser beams there are also regions near the focus where $E^2 \neq B^2$ (see Bunkin, et al., 1970; Melissinos, 1998; Ringwald, 2001).

We see that the presence of the high energy electron acts as a catalyst for spontaneous pair creation by the laser, while also providing the necessary energy-momentum balance. A standing wave field, for which $E \neq 0$ but $B = 0$, can lead to pair creation without need for a catalyst, provided $E \geq E_{Schw}$ (Melissinos, 2002). The probability for $a_0 \geq 1$ is given by Eq. (128) within a factor of order unity. When the field is weak ($a_0 < 1$), the probability increases rapidly as the field intensity increases toward the critical field, as shown in Eq.(128). When it exceeds much beyond the critical field, however, the quantum expression effect sets in and the probability is now exponentially suppressed. When we consider radiation (synchrotron radiation) by a high energy electron beam, it is customary to introduce a dimensionless parameter ϒ, the beamsstrahlung parameter, to describe the pair creation due to the collision between electron (with Lorentz factor γ) and field (often created by the other beam) as

$$\Upsilon = \gamma E / E_{Schw}. \tag{129}$$

Here if the electron has a large energy (γ>> 1), the necessary threshold (ϒ > 1) to create pairs becomes much lowered:

$$E = E_{Schw} / \gamma. \tag{130}$$

In a collider application the beamsstrahlung is related to the beam parameters as

$$\Upsilon = \frac{5 r_e^2 \gamma N}{6\alpha\sigma\sigma_z(\sigma_x + \sigma_y)}. \tag{131}$$

Because of the above behavior as to the gamma pair generation and its suppression (beamsstrahlung), the collider can be corrupted by copious pair generation as ϒ approaches unity. On the other hand, if ϒ exceeds much beyond unity, there might be a room for good operating parameters (Xie et al. 1996). This because the number of photons generated at $e^- - e^+$ collision in the large ϒ regime scales as

$$n_\gamma \propto \Upsilon^{-1/3}. \tag{132}$$

However, in a real collision, there is an overlap of the tails that makes the value of ϒ at that portion of the beams of order unity, which makes substantial emission of photons. In the case for a hard photon turning into an $e^- e^+$ pair in an external field, the rate of such pair production is

$$\frac{dn}{dt} = \frac{\alpha m^2}{\omega} \begin{cases} 0.23 \exp(-8/3\Omega), & \Omega \ll 1 \\ 0.38\Omega^{2/3}, & \Omega \gg 1 \end{cases}, \tag{133}$$

where $\Omega = \Upsilon\hbar\omega / mc^2\gamma$. In this case the total energy of the produced pair is equal to that of the initial photon. This process has been called the ‘stimulated’ process by Chen and Pellegrini (1998).

The e^-, e^+ pair creation has been already observed in an experiment of scattering of high energy electrons by intense laser (Bula et al., 1996; Burke et al., 1997). In these References it was reported on measurements of quantum electrodynamic processes in an intense electromagnetic wave, where nonlinear effects (both multiphoton and vacuum polarization) are prominent. Nonlinear Compton scattering and electron-positron pair production have been observed in collisions of 46.6 GeV and 49.1 GeV electrons of the Final Focus Test Beam at SLAC with terawatt pulses of 1053 nm and 527 nm wavelengths from a Nd:glass laser. Peak laser intensities of $\approx 5 \cdot 10^{18} W / cm^2$ have been achieved, corresponding to a value of 0.4 for the parameter a_0 and to a value of 0.25 for the parameter $\Upsilon = \gamma E / E_{Schw}$. It was presented data on the scattered electron spectra arising from nonlinear Compton scattering with up to four photons absorbed from the field. The observed positron production rate depends on the fifth power of the laser intensity, as expected for a process where five photons are absorbed from the field. The positrons are interpreted as arising from the collision of a high-energy Compton scattered photon with the laser beam. The results are found to be in agreement with theoretical predictions.

Tajima, 2002, has suggested the use of a high energy electron ring (such as Spring-8 accelerator) and a high intensity laser to provide the conditions appropriate for nonlinear QED experiments. In this case the parameter Υ becomes greater than unity, while obtaining a large event number based on a high repetition rate of such a laser and ring electron bunches. See **Fig. 36 (a).** This is an example of multiplying the two technologies, the laser and the (conventional) accelerator, as mentioned earlier. In this scheme, if one replaces the high intensity laser (such as the solid state (Ti:sapph) Petawatt laser at APRC-JAERI (Yamakawa, et al, 2002) by a high fluence free electron laser (in a ring or in a supercavity), one can also obtain a high fluence γ ray generator. As an example, a 100 μm FEL turns up γ rays of 10 MeV if scattered of a Spring-8 ring electron (8 GeV), **Fig. 36 (b).** The scattering of the electron momentum (≤10 MeV/c) barely changes its ring orbit, continuing its circulation. On the other hand such γ rays may be of use for photonuclear physics. For example, such photon interaction with nuclear matter may open a new field of investigation of the coupling between the weak and strong interactions (Fujiwara, 2003). Polarized gamma photons may be used to create large flux of polarized positrons, which may be an important ingredient in a future collider beam source to enhance the signal-to-noise ratio of the desired events(Omori, 2001). Further creative combinations of lasers and electron rings (see **Fig. 36 (c))** may lead to a brand new generation of light source, such as femtosecond synchrotron X-rays and coherent soft X-rays.

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FIGURE CAPTIONS

Figure 1. Laser Intensity vs. years

Note the very steep slope in intensities that occurs during the 60's. This period corresponded to the discovery of most nonlinear optics effects. We are experiencing today a similar rapid increase in intensity opening a new regime in optics dominated by the relativistic character of the electrons.

Figure 2. Pulse duration vs. years

The laser pulse duration has also rapidly changed from microsecond (free running), nanosecond (Q-switched), picosecond mode-locking. Here we show the pulse duration evolution since the 1990 after the invention of Ti:sappire (Spence, 1991). (Courtesy of F. Krausz).

Figure 3. Amplifier efficiency.

This graph illustrates the importance for the input pulse energy to be at several time the saturation fluence to obtain a good extraction efficiency.

Figure 4. Chirped Pulse Amplification concept.

The pulse is stretched thousand times lowering the intensity accordingly without changing the input fluence. Once has been amplified, it is recompressed to its initial value. Let's stress the amount of manipulations. Stretching 10^4 times, amplification ~10^{10}, and compression 10^4 times.

Figure 5. Treacy and Martinez Gratings

The Martinez grating pair used as stretcher and Treacy grating pair used as compressor are matched. That is the pulse can be stretched and recompressed arbitrarily keeping the initial pulse un-changed.

Figure 6. Matching between the Martinez and Treacy grating pair arrangements.

The imput grating is imaged by a telescope of magnification of 1, to form a "virtual" grating parallel to the second grating. The distance between the two gratings real and virtual can be continuously changed from positive to negative.

Figure 7. OPCPA concept
In the OPCPA the pulse is amplified by Optical Parametric Amplification instead of regular optical amplification.
(Courtesy of I. Ross).

Figure 8. Pulse contrast. The optical pulse needs before the main interaction to stay below a certain intensity level (orange line) to avoid the creation of a preformed plasma.

Figure 9. Third order auto-correlation of a 27 fs, FWHM of the HERCULES laser at the University of Michigan. Note the very large dynamic range. One ns before the main pulse we can see the contribution of the ASE. The two prepulses at -100ps are due to measurement artifacts. They are not real. The slow pedestal is due to incomplete compression.

Figure 10.

a) Polarization rotation is used in a single mode optical fiber to clean the prepulse energy. Efficient temporal cleaning can be obtained without sacrificing beam quality.

b) The second order auto-correlation shows the pulse before and after the pulse cleaner. The outpulse possesses a larger spectrum and can be slightly recompressed.

Figure 11. The use of a deformable mirror (DM) in conjunction with a large numerical aperture NA=1 ellipsoid mirror can eliminate unwanted aberrations and produce single wavelength focused spot. The two pictures are with DM "off" and DM "on".

Figure 12. Comparison between Ruled and Holographic gratings

This figure illustrates the difference in spot size obtained with holographic gratings and ruled gratings. In the case of ruled gratings the structure comes from the non-sinusoidal profile and ghosts produced by the imperfect/ broken periodicity.

Figure 13. Theoretical peak power per cm^2 of beam for various amplifying media.

Figure 14. Plasma compression by Raman backscattering. This scheme is not sensitive to damage and works with a plasma a medium that is already broken down. (Courtesy of N.J. Fisch).

Figure 15. Average power
This graph illustrates the progression in average power as a function of repetition rates of CPA systems. We also show the progresses obtained in average over non-CPA systems using dyes and excimers. Note also that much progresses needs to be made to get average power greater that 1kW.

Figure 16. Relativistic self-focusing.

Figure 17. Vortex row behind the laser pulse.

Figure 18. Laser pulse shaping.

Figure 19. Interaction of an s-polarized laser pulse with the plasma: the z component of the electric field (first column), the electron density (second column) and the ion density (third column) in the x,y plane at t=30 (row a), t=70 (row b), t=120 (row c).

Figure 20. 3D plot of the z component of the electric field (a) and of the ion density (b) inside the post-soliton at t=120.

Figure 21. Isolated soliton and a soliton train behind the laser pulse.

Figure 22. Structure of the isolated soliton.

Figure 23. The post soliton phase plane at different time.

Figure 24. Electrostatic φ, and vector potential a (a), velocities of electrons $v_{\|e}$ and ions $v_{\|i}$ (b), the electron and ion density (c) versus ξ inside the one-node soliton.

Figure 25. Shock like front formation during laser pulse propagation in underdense plasma.

Figure 26. Progress in laser development (a).

Figure 27. Progress in laser development (b).

Figure 28. Upper row corresponds to the laboratory frame (L) before reflection of the laser pulse from the "flying mirror": The laser pulse propagates from right to left; middle row corresponds to the co-moving reference frame (K): Laser pulse reflection and focusing occurs into the focus spot with the size $\lambda' \approx \lambda_0 / 2\gamma_{ph}$; lower row corresponds to the laboratory frame (L): The reflected e.m. radiation has the $\lambda_f \approx \lambda_0 / 4\gamma_{ph}^2$, and it propagates in a narrow angle $\theta \approx 1/\gamma_{ph}$.

Figure 29. Paraboloidal modulations of the electron density in the wake behind the driver laser pulse.

Figure 30. Projections of the electric field components in the x,y – plane (the x-component of the wake wave) and in the x,z – plane of the y – component of the reflected pulse, at t=20. The laser pulse driver is shown by the contours in the right hand side of the computation box.

Figure 31. The proton and the heavy ion energy spectrum at t=80.

Figure 32. Distribution of the electric field near the target (a) and in the region where the laser pulse is (b) at t=40, and at t=80.

Figure 33. Distribution of the electric charge inside the computation region at t=40 (a), and at t=80 (b) (red corresponds to heavy [thick shell] and blue to light ions [thin shell] whereas green corresponds to electrons).

Figure 34. Finite horizon and leakage of wavefunction.

Figure 35. Four quanta interaction for the light-light scattering.

Figure 36. Use of a high energy electron ring and a high intensity laser to provide the conditions appropriate for nonlinear QED experiments.