Part One: Accounting for the experiments of Trouton and Noble with classical and with relativistic mechanics

Introduction: Larmor, Lorentz, and Laue on the experiments of Trouton and Noble

In June 1903, Trouton and Noble published the negative result of an ingenious etherdrift experiment (Trouton and Noble 1903). They had a plate condenser suspended on a torsion wire, and expected to find a turning couple acting on the condenser because of its motion through the ether (see section 1.1, Figs. 1.3–1.5). Trouton and Noble, however, found no such effect. In 1901–1902, in a closely related experiment, Trouton had also obtained a negative result (Trouton 1902). Using a torsion balance with a condenser in it, Trouton had tried in vain to detect the impulse he expected to find upon charging or discharging the condenser (see section 1.1, Figs. 1.1–1.2)

I want to look at various accounts of these two experiments. In chapter one, I will discuss the ether-theoretic accounts given by Larmor (1902) and Lorentz (1904b). In chapter two, I will discuss the canonical relativistic account of the Trouton-Noble experiment due to Laue (1911a, 1912b), and, taking my inspiration from Rohrlich (1960, 1965), develop an alternative to it. I will also work out the relativistic account of the Trouton experiment. To my knowledge, no one has ever bothered to do this before.[1]

The accounts of Larmor, Lorentz, and Laue are substantially different from one another, and, at some points, flatly contradict each other. Before turning to these differences and contradictions, however, let me state what they have in common, for the similarities are at least as important as the differences. Larmor, Lorentz, and Laue agree that the equations governing the

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[1] At first sight, it may seem that there is nothing to be explained at all in the relativistic setting. After all, the negative results of the experiments of Trouton and Noble follow directly from the principle of relativity itself. The problem, as John Norton succinctly put it, "is to see how relativity theory allows one to predict [these] null result[s]" (Norton 1992a, p. 44). Or, as Laue put it in the paper in which he first published his account of the Trouton-Noble experiment: "the theory of relativity can, of course, explain this result [i.e., the negative result of the Trouton-Noble experiment] simply through the fact that the earth, with respect to which the condenser is at rest, is a justified [i.e., to a good approximation inertial] frame of reference. However, how does the theory work out if one chooses a different frame of reference? The electromagnetic turning couple is present according to the theory of relativity as well [i.e., as well as in the classical theory]. Why is it nonetheless the case that a rotation does not occur?" (Laue 1911a, p. 136). What I will be concerned with in chapter two are accounts of the experiments that provide detailed answers to such questions

electromagnetic part of the system are Maxwell's equations plus the equation for the Lorentz force applied to the case of a moving plate condenser shortened in the direction of motion by a factor $(1 - \beta^2)^{1/2}$ (where $\beta \equiv v/c$ with v the velocity of the condenser and c the velocity of light in vacuo). Against this common backdrop, however, one quickly notices some major differences between the three accounts.

First of all, Larmor and Lorentz's picture of the physical system under consideration is very different from Laue's. For Lorentz and Larmor, the problem is to find out what happens in a condenser moving with the earth through the stationary ether. Due to peculiarities of the forces holding it together, a condenser, in Larmor and Lorentz's view, undergoes the Lorentz-FitzGerald contraction when it is set in motion with respect to the ether. For Laue, on the other hand, the problem is to describe what happens in a condenser in an inertial frame in which it is in uniform motion.[2] Because of this choice of reference frame, the condenser will be contracted. This is not a dynamical but a purely kinematical effect.

A superficial look at the actual calculations of Larmor, Lorentz, and Laue also suffices to see that they do their calculations in very different ways. Larmor does his calculations in terms of energy, Lorentz in terms of momentum, more specifically in terms of a quantity called electromagnetic momentum, and Laue avails himself of the energy-momentum tensor, the quantity he put at the center of his relativistic mechanics.

In and of themselves, these differences do not amount to contradictions. In fact, Lorentz's account of the Trouton-Noble experiment carries over unproblematically to special relativity. All we have to do is replace Lorentz's ether theoretic perspective by the relativistic perspective on the equations. Lorentz's account of the original Trouton experiment, however, does not carry over to special relativity in this way. Neither do the accounts of the Trouton experiment and the Trouton-Noble experiment due to Larmor. Moreover, in the case of both experiments, Larmor's account is incompatible with Lorentz's.

The main task I set myself in studying these accounts of the experiments of Trouton and Noble was to find out exactly what is responsible for these incompatibilities.[3] The analysis in chapters one and two will reveal a simple pattern. All incompatibilities stem from a failure to (properly) take into account two closely related effects which manifest themselves in these experiments. Upon charging a moving condenser, the condenser not only gains energy, it also gains momentum and it gains mass. These effects are totally unexpected on the basis of

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[2] See the quotation from Laue 1911a in the preceding footnote.

[3] To facilitate comparison between the different accounts, I have reconstructed all calculations in the language of modern textbooks on electrodynamics and relativity such as Jackson 1975. Unlike Jackson, however, I will use the SI or MKSA system of units throughout this dissertation. For those readers who are more at home with other systems of units or who want to check my reconstructions against the primary sources, I refer to the conversion tables in the appendix of Jackson 1975, pp. 817-819.

Newtonian mechanics, but they fall into place quite naturally in the relativistic mechanics Laue developed and published in 1911, in a seminal paper (Laue 1911a) and in the first textbook on special relativity ever published (Laue 1911b). Laue showed how the effects mentioned above can be encoded very elegantly in the energy-momentum tensor and its behavior under Lorentz transformation. Minkowski (1908) had introduced the energy-momentum tensor in electrodynamics and Planck (1908) and Laue (1911a, 1911b), introduced to special relativity in late 1905 while studying with Planck (Miller 1981, p. xxvii), made it the basis for the description of all physical phenomena. Not surprisingly, given its electrodynamical credentials, Laue's relativistic mechanics is tailor-made for analyzing the experiments of Trouton and Noble. For Larmor and Lorentz, who used Newtonian mechanics and some special assumptions such as the contraction hypothesis in conjunction with electrodynamics and Lorentz's clever scheme of "corresponding states" (see section 1.2), it was much harder to analyze these experiments. To illustrate the difficulties they were facing, let me give a preview of Larmor's account of the Trouton-Noble experiment (see section 1.3) and of Lorentz's account of the Trouton experiment (see section 1.4).

The basic idea of Larmor's account of the Trouton-Noble experiment is as follows. If there is a net turning couple on a moving condenser, it has to be the case that its total energy depends on the orientation of the plates with respect to the direction of motion. The turning couple will try to rotate the condenser to the orientation minimizing the total energy. If one can show that the total energy is, in fact, independent of the orientation of the condenser, one has shown that there will be no turning couple, thereby explaining the negative result of the Trouton-Noble experiment. Larmor calculated the electromagnetic energy of a moving condenser and found that it will be independent of the orientation of the condenser with respect to the direction of motion, if one assumes that the condenser is subject to the Lorentz-FitzGerald contraction. So, the contraction hypothesis seems to account for the null result of the Trouton-Noble experiment.

There are two complications. First, in Laue's relativistic account of the Trouton-Noble experiment, the electromagnetic energy of the condenser does depend on the condenser's orientation with respect to the direction of motion. From Laue's point of view, Larmor failed to take into account the energy needed to build up the electromagnetic momentum created upon charging a moving condenser. If this energy is added to Larmor's result, Laue's expression for the electromagnetic energy of the condenser is recovered. Nothing in Newtonian mechanics could have prepared Larmor for this complication. The energy of the electromagnetic field does not have mass in Newtonian mechanics. Using the Newtonian relation momentum equals mass times velocity, it is hard to see how it could have momentum, especially in the context of a theory positing a stationary ether such as Larmor's.[4]

If Larmor's calculation is corrected for this effect, it looks as if the Trouton-Noble experiment should have given a positive result. This is not the case. In Laue's account, the non-electromagnetic energy, energy associated with stresses in the material part of the condenser that prevent the plates from collapsing on to one another under the influence of their Coulomb attraction, also depends on the condenser's orientation with respect to the direction of motion, and does so in such a way that the total energy, the sum of the electromagnetic and the non-electromagnetic energy, does not. So, in Laue's account, there are two turning couples on a moving condenser that exactly cancel one another. Again, Larmor could hardly have expected anything like this on the basis of Newtonian mechanics. The tacit assumption among ether theorists such as Larmor and Lorentz was that, as a rule, physical phenomena satisfy a Galilean principle of relativity, electromagnetic phenomena being an exception to that rule. The energy of an object at rest does not change when the object is simply being rotated, how could this be different when the object is uniform motion?[5]

Lorentz runs into a similar problem in his account of the Trouton experiment.[6] Lorentz does take into account the momentum of the electromagnetic field (unlike Larmor), but not the inertia of its energy. Lorentz reasoned as follows. When a moving condenser is charged, it gains electromagnetic momentum. Invoking momentum conservation, Lorentz argued that this gain in electromagnetic momentum should be compensated by a loss in ordinary momentum. Hence, Lorentz concluded, the condenser should recoil upon charging it. This would constitute a violation of the so-called center of mass theorem according to which an isolated system can not change its state of motion, but Lorentz's calculations showed that the effect was too small to

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[4] In his lectures in New York in 1906, we see Lorentz grappling with this problem (Lorentz 1916, sections 23–24, pp. 30–33). Lorentz clearly recognizes the importance of the quantity that Abraham defined as electromagnetic momentum (and he gives Abraham full credit for this contribution (ibid., p. 32, note 2)). Yet, he has not quite made the step of accepting electromagnetic momentum as momentum *sui generis*. To be sure, Lorentz already entertains that option, but he also suggests an alternative option of interpreting the electromagnetic momentum as an instance of ordinary Newtonian momentum, equal to the product of mass and velocity, recognizing that the immobile character of his stationary ether would require the mass to be very large so that the velocity can be very small (ibid., p. 31).

[5] Incidentally, Larmor's account of the Trouton-Noble experiment is closer to the alternative relativistic account I will present in chapter two, in which there are no turning couples whatsoever and in which neither the electromagnetic nor the non-electromagnetic energy depend on the condenser's orientation with respect to its velocity. However, this account is based on a rather counter-intuitive definition (taken from Rohrlich 1960) of the energy of the system. The rationale of this definition can only be understood in the light of the relativity of simultaneity, an insight Larmor obviously did not have in 1902. Moreover, the first objection to Larmor's account (i.e., that it fails to take into account the energy needed to build up the electromagnetic momentum created upon charging a moving condenser) obviously remains.

[6] Lorentz does not get into trouble in his account of the Trouton-Noble experiment. This is because he was more careful about the interpretation of electromagnetic momentum in the context of the Trouton-Noble experiment than in the context of the Trouton experiment, and skillfully managed to reduce the role of Newtonian mechanics to an absolute minimum in his derivation of an expression for the turning couple (see section 1.4).

be detected by apparatus with the accuracy of the apparatus used by Trouton in 1902.[7] Moreover, Lorentz had made it clear as early as 1895 that violations of the center of mass theorem—more precisely: violations of the closely related principle that action equals reaction, i.e., Newton's third law—had to be expected in his theory. Poincaré (1900b) had sharply attacked Lorentz's theory on just this point. Poincaré's criticism formed the starting point for one of Einstein's derivations of the inertia of energy. Einstein (1906) showed that the inertia of energy is necessary and sufficient to prevent violations of the center of mass theorem.[8] If the inertia of energy is factored in in Lorentz's analysis of the Trouton experiment, it is easily seen that the experiment will always give a negative result, no matter how accurate the apparatus. The important new element is that we have to take the battery supplying the energy to charge the condenser into consideration. When it charges the condenser, the battery will lose not just the energy but also the mass and the momentum gained by the condenser. The total momentum of the system is conserved without the system having to recoil (see section 2.5).

In chapter one, I will examine their arguments in more detail, but even from this brief discussion of Larmor and Lorentz's struggles with the experiments of Trouton and Noble, it will be clear that their theories can easily be modified to incorporate the relations between energy, mass, and momentum that manifest themselves in these experiments and that are usually associated with relativity theory. As we just saw, Lorentz's theory of 1904 did, in fact, already include the relativistic relation between energy and momentum, at least for the case of the electromagnetic field. Consequently, the correction of Larmor's 1902 calculation of the electromagnetic energy of a moving charged condenser made on the basis of Laue's relativistic mechanics can already be made on the basis of Lorentz's theory of 1904 (see section 1.4).

Lorentz's theory will only be empirically equivalent to special relativity, if it takes over all relations between energy, mass, momentum, and stresses encoded in the energy-momentum tensor and its transformation properties. In other words, Lorentz's theory will only be empirically equivalent to special relativity, if Newtonian mechanics is, in effect, replaced by relativistic mechanics. I will argue in part two of this dissertation that Lorentz was fully prepared to make this replacement. However, he still retained the ether, Newtonian space and time, and Newtonian kinematics.

His theory is therefore of a rather peculiar nature. In order to appreciate this, I want to make a few general remarks about what I will call, following a suggestion by John Stachel (private

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[7] Perhaps, Lorentz subliminally expressed his irritation over being stood up by these elusive jolts, when he wrote: "Trouton has not been able to observe these jerks [sic]" (Lorentz 1904b, p. 196).

[8] I do not claim originality for stressing the connection between Poincaré 1900b and Einstein 1906. Darrigol (1994b) recently emphasized and explored this connection. I am indebted to John Stachel for providing me with a preprint of this important paper.

communication), general dynamical consequences of the space-time kinematics, which are part of what Planck called “general dynamics” (*allgemeine Dynamik*) in an important talk in 1908, from which I will quote at length in sections 1.4 and 2.1. General dynamics provides the general framework for treating the dynamics of particular systems in a space-time with a certain structure. In the context of classical Newtonian mechanics, for instance, the general dynamics stipulates that we describe physical phenomena in terms of forces represented by quantities that transform as vectors under the rotations of the Euclidean group and that are invariant under Galilean transformation. This transformation behavior has nothing to do with the details of the dynamics, it simply reflects the space-time structure posited by Newtonian theory. Hence, the phrase ‘general dynamical consequences of the space-time kinematics.’ Laue’s relativistic general dynamics of 1911 stipulates that we describe physical phenomena in terms of their energy-momentum tensors, where the fact that this quantity transforms as a second-rank tensor under Lorentz transformations reflects the Minkowskian space-time structure posited by relativity theory.[9]

If Laue’s relativistic mechanics is used in conjunction with Newtonian space-time, how do we interpret those relations that, in a relativistic context, are just general dynamical consequences of the kinematics of Minkowski space-time? As long as we believe in the viability of the so-called electromagnetic view of nature, in which all physics is ultimately reduced to electrodynamics, we can interpret these relations in terms of properties of Maxwell’s equations. However, as I already mentioned in the overall introduction, the electromagnetic view of nature had ceased to be an option for Lorentz by 1906 at the latest, after Abraham and Poincaré had made it clear that his electron model needs non-electromagnetic cohesive forces for its stability (see section 2.2 and section 3.4).

Imagine the following situation. Suppose we have some physical phenomenon instantiating a relation in relativistic mechanics that, in the relativistic setting, simply reflects the space-time structure. In other words, we have a phenomenon that is purely kinematical from a relativistic point of view, in the sense that it is part of the normal spatio-temporal behavior of systems in Minkowski space-time, just as the contraction of a moving rod is part of that rod’s normal spatio-temporal behavior in Minkowski space-time.[10] Such a phenomenon will clearly not be

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[9] In quantum mechanics, both non-relativistic and relativistic, we can similarly identify such general dynamical consequences of the space-time kinematics. In the context of non-relativistic quantum mechanics, the general dynamics stipulates that observables are represented by operators on a Hilbert space, whose transformation behavior under rotations and Galilean transformations reflect the Galilean space-time structure. In the context of relativistic quantum mechanics, the general dynamics stipulates that observables are represented by operators on a Hilbert space, whose transformation behavior under Lorentz transformations reflect the Minkowskian space-time structure.

[10] In chapter two, I will give a more detailed discussion of the way in which I distinguish between kinematical and dynamical effects.

kinematical if relativistic mechanics is made part of a theory with a different space-time structure, such as the Newtonian space-time structure in Lorentz's theory. After all, the normal spatio-temporal behavior in Minkowski space-time is different from the normal spatio-temporal behavior in Newtonian space-time.

The Trouton-Noble experiment, it turns out, provides a beautiful example of the sort of phenomenon I have in mind. As I will show in chapter two, the delicately balanced turning couples in both Lorentz's and Laue's accounts of the experiment turn out to be purely kinematical effects in special relativity, in the sense indicated above. The main purpose of developing the alternative relativistic account of the Trouton-Noble experiment that I mentioned above is, in fact, to establish this claim (see section 2.3). The phenomenon that a charged condenser experiences two opposing turning couples when it is set in motion has the exact same status, I will argue, as the phenomenon that a rod contracts when set in motion or the phenomenon that a clock when set in motion starts ticking at a slower rate. I will coin the term "Laue effect" for this phenomenon manifesting itself in the Trouton-Noble experiment. The purely kinematical nature of this phenomenon has so far gone unrecognized, although Laue clearly realized that its status in relativity theory was different from its status in Lorentz's theory (see the introduction to chapter two).