APS, DAMOP, Lincoln, Nebraska, May 17–21, 2005:

Emergence and Interpretation of Lorentz Invariance

Michel Janssen
Program in History of Science and Technology
University of Minnesota

In the course of his work on optics and electrodynamics in systems moving through the ether, the 19th-century medium for light waves and electric and magnetic fields, Lorentz discovered and exploited the invariance of the free-field Maxwell equations under what Poincaré later proposed to call Lorentz transformations. To account for the negative results of optical experiments aimed at detecting the earth's motion through the ether, Lorentz, in effect, assumed that the laws governing matter interacting with light waves are Lorentz invariant as well. Like Lorentz, Einstein first encountered the Lorentz transformations in electrodynamics. Unlike Lorentz, for whom the transformation merely provided convenient mathematical substitutions, but like Poincaré, Einstein recognized that the Lorentz-transformed quantities are the measured quantities for the moving observer. More importantly, Einstein recognized that the Lorentz invariance of all physical laws had nothing to do with electrodynamics per se, but reflected the kinematics in a new relativistic space-time, to be named after Minkowski who worked out its geometry a few years later.

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Outline:

1. Electromagnetism from Maxwell to Lorentz

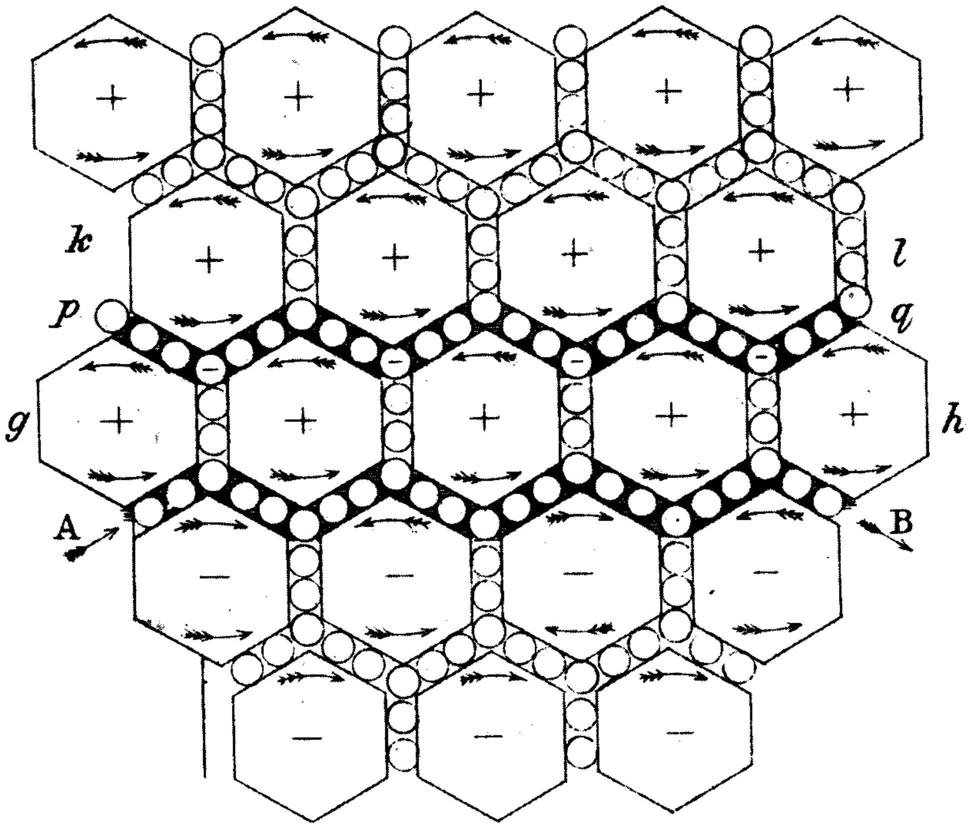
2. Lorentz's theorems of corresponding states

3. Lorentz's mature theory of electrodynamics in moving bodies

4. Einstein's electrodynamic route to special relativity

5. Minkowski's contribution

James Clerk Maxwell (1831–1879), "On the Physical Lines of Force" (1861–1862).



"honeycomb" ether model

Maxwell's equations describe macroscopic behavior of microscopic mechanical system. Equations turn out to have wave solutions.

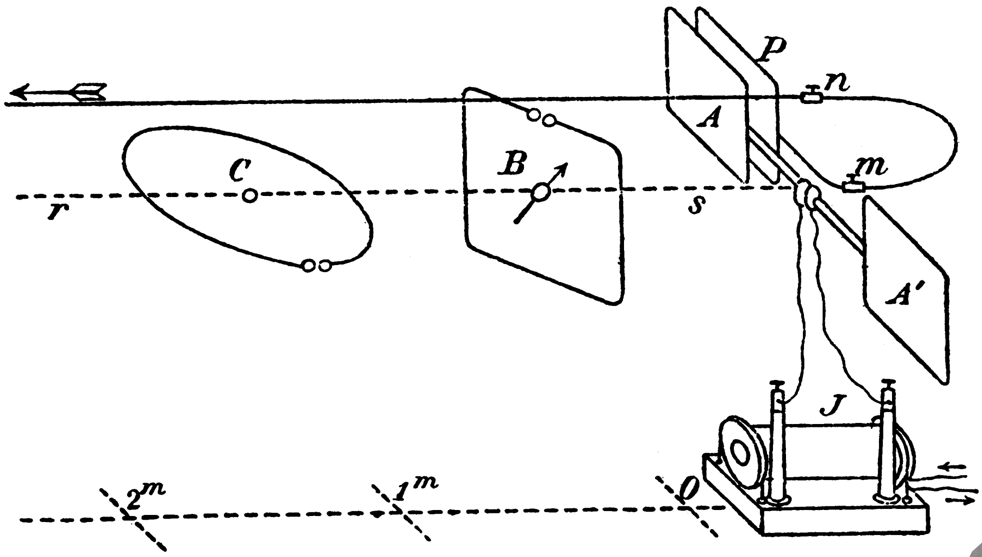
"The velocity of transverse undulations in our hypothetical medium, calculated from the electro-magnetic experiments of MM. Kohlrausch and Weber [$1/\sqrt{\varepsilon_0\mu_0}$], agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau that we can scarcely avoid the inference that *light consists of the transverse undulations of the same medium which is the cause of electric and magnetic phenomena*."

Maxwell, “A Dynamical Theory of the Electromagnetic Field” (1864).

Comparing 1861-2 and 1864 papers in letter to Tait, December 23, 1867:

“The former is built up to show that the phenomena are such as can be explained by mechanism. The nature of the mechanism is to the true mechanism what an orrery is to the solar system. The latter is built on Lagrange’s dynamical equations and is not wise about vortices.”

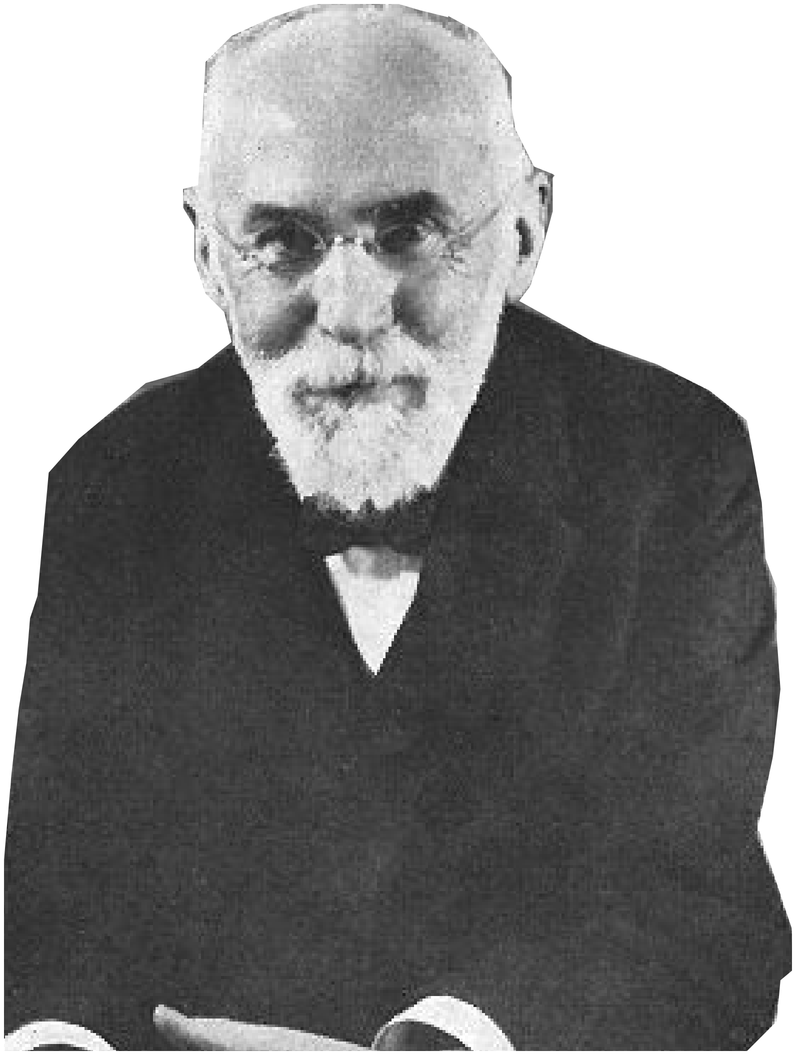
Heinrich Hertz (1857–1894) (Only the good die young: Maxwell at 48, Hertz at 37).



1888: Generates and detects electromagnetic waves

"Many a man … has … been compelled to abandon the hope of forming for himself an altogether consistent conception of Maxwell's ideas. I have fared no better myself ... What is Maxwell's theory? I cannot give any clearer or briefer answer than the following: Maxwell's theory is the system of Maxwell's equations."

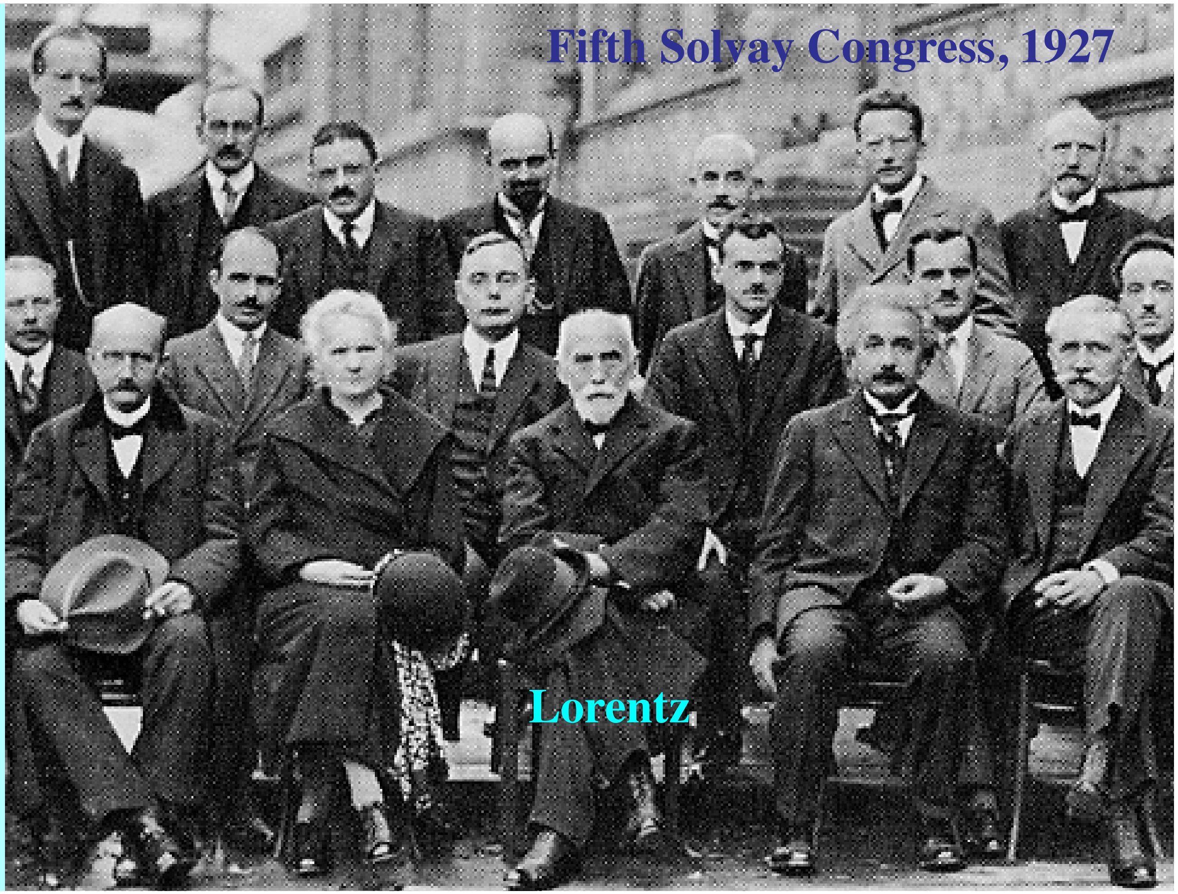
Hendrik Antoon Lorentz (1853–1928). From ether with electric and magnetic fields (in principle) reducible to mechanics to ether/matter or electrodynamics/mechanics dualism



1890s: **Electrodynamics of Moving Bodies**

- **Strict separation of ether and matter** (described by Einstein as a "liberating act"): **immobile ether** fully undisturbed by matter moving through it.
- Newtonian mechanics: space-time arena and laws governing **matter**.
- Maxwellian electrodynamics: governing electric and magnetic **fields** conceived of as states of a non-mechanical ether (Newton's third law, "action = reaction," does not apply to the ether).
- Tiny **charged particles** (ions, electrons) generate electric and magnetic fields in the ether; fields in turn exert forces (the Lorentz force) on material bodies via the charged particles they contain ("blueberry muffin model").

Albert Einstein (1879–1955) in 1953, the centenary of Lorentz's birth: "The physicists of our time are mostly not fully aware of the decisive part which H. A. Lorentz played in shaping the fundamental ideas in theoretical physics. The reason for this curious fact is that they have absorbed Lorentz's basic ideas so completely that they are virtually incapable of comprehending the boldness of these ideas and the simplification they brought about."



Problem for Lorentz's theory: Maxwell's equations do not hold in frames of reference on earth since the earth is moving through the ether with a velocity in the order of 30 km/s.

- Free field equations in ether frame (x_0, t_0) (Maxwell's equations):

$$\text{div}\,\mathbf{E} = 0, \qquad \text{curl}\,\mathbf{E} = -\partial_t \mathbf{B}$$

$$\text{div}\,\mathbf{B} = 0, \qquad \text{curl}\,\mathbf{B} = \frac{1}{c^2}\partial_t \mathbf{E}$$

- Free field equations in terrestrial frame $(x = x_0 - v t_0,\ t = t_0)$: $\partial_{t_0} \to \partial_t - v\partial_x$

$$\text{div}\,\mathbf{E} = 0, \qquad \text{curl}\,\mathbf{E} = -\partial_t \mathbf{B} + v\partial_x \mathbf{B}$$

$$\text{div}\,\mathbf{B} = 0, \qquad \text{curl}\,\mathbf{B} = \frac{1}{c^2}\partial_t \mathbf{E} - \frac{v}{c^2}\partial_x \mathbf{E}$$

Problem for Lorentz's theory: Maxwell's equations do not hold in frames of reference on earth since the earth is moving through the ether.

- Focus on the x-component of curl $\mathbf{E} = -\partial_t \mathbf{B} + v\partial_x \mathbf{B}$:

$$\begin{aligned} \partial_y E_z - \partial_z E_y &= -\partial_t B_x + v\partial_x B_x \\ \partial_z E_x - \partial_x E_z &= -\partial_t B_y + v\partial_x B_y \\ \partial_x E_y - \partial_y E_z &= -\partial_t B_x + v\partial_x B_z \end{aligned}$$

- Use div $\mathbf{B} = 0$ (i.e., $\partial_x B_x = -\partial_y B_y - \partial_z B_z$) and move $\partial_i B_i$-terms to LHS:

$$\begin{aligned} \partial_y(E_z + vB_y) - \partial_z(E_y - vB_z) &= -\partial_t B_x \\ \partial_z(\quad E_x \quad) - \partial_x(E_z + vB_y) &= -\partial_t B_y \\ \partial_x(E_y - vB_z) - \partial_y(\quad E_x \quad) &= -\partial_t B_x \end{aligned}$$

- Introduce auxiliary field $\mathbf{E}' \equiv (E_x, E_y - vB_z, E_z + vB_y) = \mathbf{E} + \mathbf{v} \times \mathbf{B}$

Problem for Lorentz's theory: Maxwell's equations do not hold in frames of reference on earth since the earth is moving through the ether.

Results so far:

- curl $\mathbf{E} = -\partial_t \mathbf{B} + v\partial_x \mathbf{B}$ can be rewritten as curl $\mathbf{E}' = -\partial_t \mathbf{B}$ with $\mathbf{E}' \equiv \mathbf{E} + \mathbf{v} \times \mathbf{B}$.
- Likewise: curl $\mathbf{B} = \frac{1}{c^2}\partial_t \mathbf{E} - \frac{v}{c^2}\partial_x \mathbf{E}$ can be rewritten as curl $\mathbf{B}' = \frac{1}{c^2}\partial_t \mathbf{E}$ with $\mathbf{B}' \equiv \mathbf{B} - \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$.

Remaining questions?

- How do we combine $\mathbf{E}'$ and $\mathbf{B}'$? curl $\mathbf{B}' \neq \frac{1}{c^2}\partial_t \mathbf{E}'$ and curl $\mathbf{E}' \neq -\partial_t \mathbf{B}'$.
- What about the other two equations? div $\mathbf{E}' \neq 0$ and div $\mathbf{B}' \neq 0$.

Problem for Lorentz's theory: Maxwell's equations do not hold in frames of reference on earth since the earth is moving through the ether.

Answer to both remaining questions: further coordinate transformation
$(x, t) \to (x' = x, t' = t - (v/c^2)x)$ **(Lorentz called t' the 'local time')**.
As a result, $\partial_x \to \partial_{x'} - (v/c^2)\partial_{t'}$.

- Consider $\text{div}'\ \mathbf{E}'$, using (*i*) $E' \equiv (E_x, E_y - vB_z, E_z + vB_y)$; (*ii*) $\partial_{x'} \to \partial_x + (v/c^2)\partial_t$; and (*iii*) $\text{div}\ \mathbf{E} = 0$:

$$\text{div}'\ \mathbf{E}' = \cancel{\text{div}\ \mathbf{E}} + (v/c^2)\partial_t E_x - v(\partial_y B_z - \partial_z B_y) = v\left[\frac{1}{c^2}\partial_t \mathbf{E} - \text{curl}\ \mathbf{B}\right]_x.$$

- Use that (*iv*) $\text{curl}\ \mathbf{B} = \frac{1}{c^2}\partial_t \mathbf{E} - \frac{v}{c^2}\partial_x \mathbf{E}$ and (*v*) $(v^2/c^2)\partial_x \mathbf{E}$ is small ($v/c \approx 10^{-4}$):

- Likewise:

$\text{div}'\ \mathbf{E}' = 0$ to first order in v/c

$\text{div}'\ \mathbf{B}' = 0$ to first order in v/c

Problem for Lorentz's theory: Maxwell's equations do not hold in frames of reference on earth since the earth is moving through the ether.

Answer to both remaining questions: local time (hence $\partial_x \to \partial_{x'} - (v/c^2)\partial_{t'}$)

- Consider the x- and the y-components of $\mathrm{curl}'\,\mathbf{E}'$ using (*i*) $\mathrm{curl}\,\mathbf{E}' = -\partial_t \mathbf{B}$; (*ii*) $\mathbf{B}' \equiv (B_x, B_y + (v/c^2)E_z, B_z - (v/c^2)E_y)$; and (*iii*) $\mathbf{E}' \equiv (E_x, E_y - vB_z, E_z + vB_y)$

$$[\mathrm{curl}'\,\mathbf{E}']_x = [\mathrm{curl}\,\mathbf{E}']_x = -\partial_t B_x = -\partial_{t'} B'_x$$

$$\begin{aligned} [\mathrm{curl}'\,\mathbf{E}']_y &= \partial_{z'} E'_x - \partial_{x'} E'_z \\ &= (\partial_z E'_x - \partial_x E'_z) - (v/c^2)\partial_t E'_z \\ &= -\partial_t [B_y + (v/c^2)E_z] - (v^2/c^2)\partial_t B_y \\ &= -\partial_{t'} B'_y \text{ to first order in } v/c \end{aligned}$$

- Likewise: $\mathrm{curl}'\,\mathbf{E}' = -\partial_{t'}\mathbf{B}'$ and $\mathrm{curl}'\,\mathbf{B}' = \dfrac{1}{c^2}\partial_{t'}\mathbf{E}'$ to first order in v/c.

Problem for Lorentz's theory: Maxwell's equations do not hold in frames of reference on earth since the earth is moving through the ether.

- Free field equations in terrestrial frame (moving through the ether):

$$\text{div}\,\mathbf{E} = 0, \qquad \text{curl}\,\mathbf{E} = -\partial_t \mathbf{B} + v\partial_x \mathbf{B}$$

$$\text{div}\,\mathbf{B} = 0, \qquad \text{curl}\,\mathbf{B} = \frac{1}{c^2}\partial_t \mathbf{E} - \frac{v}{c^2}\partial_x \mathbf{E}$$

- Free field equations in terrestrial frame in terms of auxiliary variables are Maxwell's equations to first order in v/c:

$$\text{div}'\,\mathbf{E}' = 0, \qquad \text{curl}'\,\mathbf{E}' = -\partial_{t'} \mathbf{B}'$$

$$\text{div}'\,\mathbf{B}' = 0, \qquad \text{curl}'\,\mathbf{B}' = \frac{1}{c^2}\partial_{t'} \mathbf{E}'$$

With $\mathbf{E}' \equiv \mathbf{E} + \mathbf{v} \times \mathbf{B}$, $\mathbf{B}' \equiv \mathbf{B} - \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$, and 'local time' $t' = t - (v/c^2)x$.

Lorentz's first-order "theorem of corresponding states" (1895) [essentially: first-order Lorentz invariance of Maxwell's equations]

The theorem (valid to first order in v/c): If in a **frame at rest** in the ether some **configuration of the real fields E and B as functions of the real coordinates** (x_0, t_0) is allowed, then in a **frame in motion** through the ether at some constant velocity v that **same configuration** is allowed but **in terms of the auxiliary fields E′ and B′ as functions of the auxiliary coordinates** (x', t').

The field configuration (of real fields) in the moving frame and the field configuration in the frame at rest in the ether that are related to one another in this way are called **corresponding states.**

Application of the theorem: On the assumption that the field configuration in some optical experiment in a moving frame* is the corresponding state of the field configuration in that same experiment in a frame at rest in the ether, **the pattern of light and shadow observed in the experiment in motion through the ether is the same as the pattern observed in the experiment at rest**.

*i.e., the field configuration produced by some matter configuration of sources, lenses, mirrors, screens, etc.

Lorentz's first-order theorem of corresponding states thus explains in one fell swoop why no first-order experiment had ever detected any ether drift!

Lorentz's exact "theorem of corresponding states" (1899/1904):

- Free field equations in terrestrial frame in terms of auxiliary variables are exactly Maxwell's equations:

$$\mathrm{div}'\,\mathbf{E}' = 0 \qquad \mathbf{E}' \equiv \mathrm{diag}(1,\gamma,\gamma)(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

$$\mathrm{div}'\,\mathbf{B}' = 0 \qquad \mathbf{B}' \equiv \mathrm{diag}(1,\gamma,\gamma)\left(\mathbf{B}-\frac{1}{c^2}(\mathbf{v}\times\mathbf{E})\right)$$

$$\mathrm{curl}'\,\mathbf{E}' = -\partial_{t'}\mathbf{B}'$$

$$x' = \gamma x$$

$$\mathrm{curl}'\,\mathbf{B}' = \frac{1}{c^2}\partial_{t'}\mathbf{E}'$$

$$t' = \frac{t}{\gamma} - \gamma\left(\frac{v}{c^2}\right)x \qquad \text{with } \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

- **The theorem** (same as before but now holding exactly).

- **Application of the theorem:** the pattern of light and shadow observed in the experiment in motion through the ether is **contracted by a factor $1/\gamma$ in the direction of motion** compared to the pattern observed in the experiment at rest. The complication comes from the relation $x' = \gamma x$.

Lorentz's mature theory of electrodynamics in moving bodies (1899/1904)

Theorem of corresponding states (mathematical result)
+ Generalized contraction hypothesis (physical assumption)

- **Theorem of corresponding states** says how motion through the ether affects the **laws governing fields** in systems in motion (i.e., Lorentz invariance of Maxwell's equations).

  Application of this theorem: field configurations (and thus: patterns of light and darkness) in moving systems contract by $1/\gamma$ in direction of motion.

  Without further assumptions, the theory predicts that ether drift should be detectable (patterns of light and shadow contract but optical components do not). But ether drift is not found in any experiment.

- **Generalized contraction hypothesis** says that **motion through the ether affects the laws governing matter the same way it affects the laws governing the fields** (i.e., assume that the laws effectively governing matter are Lorentz invariant as well)

  With this new assumption Lorentz's theory correctly predicts that there will be no signs of ether drift (matter and field subject to the same contraction).

George Francis FitzGerald (1851–1901)

Why Lorentz's mature theory is vastly superior to the version routine denounced as ad hoc

The Lorentz-FitzGerald contraction hypothesis of 1889/1892 was originally proposed to account for the failure of **one** experiment to detect ether drift (the 1887 Michelson-Morley experiment)

Lorentz's "Generalized Contraction Hypothesis" of 1899/1904 can account for the failure of **any** optical ether drift experiment that boils down to the observation of patterns of light and shadow

Lorentz's theory is **empirically adequate** (it gives the right predictions) but **explanatorily deficient** (it doesn't provide convincing answers to a number of eminently reasonable 'why'-questions:

Embarrassing 'why'-question #1: Why does motion through the ether affect the laws for field and matter in the same way? (Why are both the laws governing the fields and the laws governing matter Lorentz invariant)

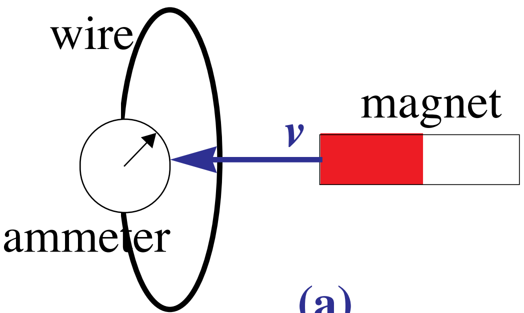
Lorentzian answer: Just a coincidence.

Relativistic answer: coincidence traced to common origin: it reflects a new relativistic space-time structure (Minkowski space-time).

Lorentzian picture: space-time is still Newtonian, but physical systems don't behave the way you'd expect them to behave in Newtonian space-time.

Relativistic picture: Newtonian space-time replaced by Minkowski space-time and physical systems behave just as you'd expect in this new space-time.

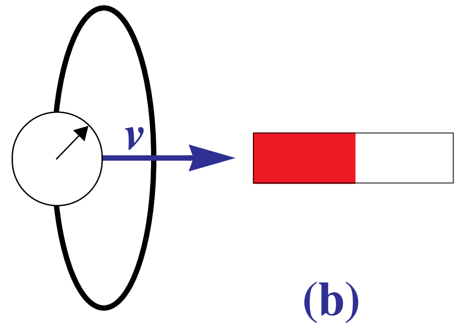
Einstein, "On the Electrodynamics of Moving Bodies" (1905). Example on first page.



(a)
magnet moving

moving magnet induces
electric field in the wire
(Faraday's induction law)

electric field makes
electrons go round the wire



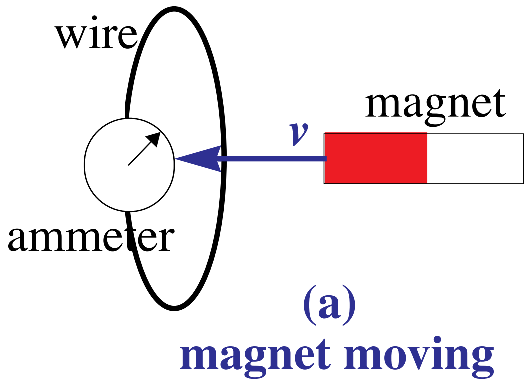
(b)
conductor moving

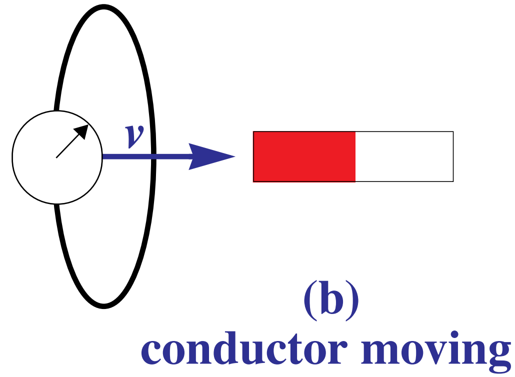
electrons in the wire move in magnetic field

Lorentz force coming from magnetic field makes electrons go round the wire

Current in the wire the same in both cases, but theoretical explanation very different.
Embarrassing 'why'-question #2 for Lorentz: "Why do these two experiments that according to your theory are completely different, give the exact same result?"

Einstein, "On the Electrodynamics of Moving Bodies" (1905).





Current in the wire the same in both cases, but theoretical explanation very different. **Embarrassing 'why'-question #2 for Lorentz:** "Why do these two experiments that according to your theory are completely different, give the exact same result?"

Einstein's answer:

Same situation looked at by two different observers

→ There is no ether

→ There's no separate electric field and magnetic field but only an electromagnetic field that splits differently into electric and magnetic components for different observers. There's no preferred split (the Dave Mason 'we just disagree'-principle)

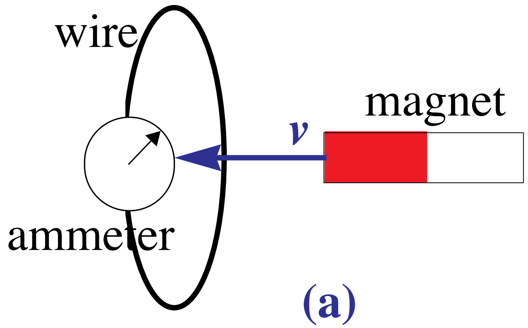
Einstein on his magnet-conductor example in 1919:

"In the formulation of the special theory of relativity, a consideration—not mentioned so far—concerning Faraday's electromagnetic induction played a leading role

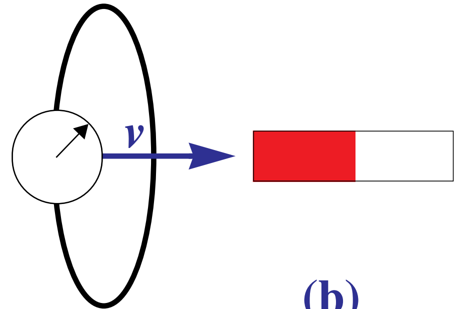
[*magnet-conductor example*]

The idea that we would be dealing here with two fundamentally different situations was unbearable to me [...] The existence of the electric field was therefore a relative one, dependent on the coordinate system used, and only the electric and magnetic field taken *together* could be ascribed some kind of objective reality."

The mathematics needed to describe the magnet-conductor experiment was readily available through Lorentz's work:



(a)
magnet moving



(b)
conductor moving

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{v} \times \mathbf{B} \neq 0$$

$$\mathbf{B}' = \mathbf{B} - \frac{1}{c^2}(\mathbf{v} \times \mathbf{E}) = \mathbf{B} \neq 0$$

$$\mathbf{E} = 0$$

$$\mathbf{B} \neq 0$$

Conjecture: Einstein originally found the Lorentz transformations essentially the way Lorentz did. He first found the transformation equations for **E** and **B** and only then found the one for time. The crucial breakthrough occurred when Einstein saw the connection between local time and relativity of simultaneity.

The crucial insight: **relativity of simultaneity**.

Einstein on the crucial breakthrough six weeks prior to publishing his paper (lecture in Kyoto, Japan, December 14, 1922):

"By chance, a friend of mine [Michele Besso] living in Bern (Switzerland) helped me. It was a beautiful day. I visited him and I said to him something like: 'I am struggling with a problem these days that I cannot solve no matter what I try. Today I bring this battle of mine to you.' I had various discussions with him. Through them it suddenly dawned on me. The very next day I visited him again and told him without further ado: 'Thank you. I have already solved my problem completely.' "

Einstein based his entire theory on two postulates:

1st Postulate (**Principle of Relativity**): "same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good"

2nd Postulate (**Light Postulate**): "light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body"

Where is the light postulate coming from?

- **Myth of the Michelson-Morley experiment:** straightforward generalization of the negative result of Michelson and Morley: light has the same velocity in all directions independently of the velocity *of the observer*. BUT: the light postulate *in Einstein's formulation* says that the velocity of light being independent of the velocity *of the source*! The result of Michelson and Morley is not even a consequence of light postulate by itself but only of the conjunction of the two postulates.
- **Einstein's later recollections (consistently):** Maxwell's equations, the accepted laws of electrodynamics, predict electromagnetic waves propagating at velocity *c* independent of the velocity of its source (as with any wave phenomenon). Rather than assuming Maxwell's equations as the definitive laws of electrodynamics, Einstein just assumed this one key prediction of the equations. He worried that Maxwell's equations would have to be changed in view of quantum phenomena.

Einstein on why he chose to formulate special relativity this way ("Autobiographical Notes," 1949):

"By and by I despaired of the possibility of discovering the true laws by means of **constructive efforts** based on known facts. The longer and more despairingly I tried, the more I came to the conviction that only the discovery of a **universal formal principle** could lead us to assured results. The example I saw before me was thermodynamics"

Cf. "What is the Theory of Relativity?" (1919)

Constructive theory: start with speculative model of salient features of the physical world (e.g, kinetic theory of gases).
Theory of principle: start with empirically well-confirmed regularities (e.g., thermodynamics).

As in thermodynamics, Einstein can derive consequences from his postulates without committing to a wave or a particle view of light

Einstein preferred constructive theories:

The London Times, 1919: "When we say that we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the processes in question."

Einstein to Arnold Sommerfeld, January 14, 1908: "The theory of relativity is ultimately as little satisfactory as […] thermodynamics was before Boltzmann had interpreted the entropy as probability."

The constructive theory version of special relativity: space-time has the geometry of Minkowski space-time.

Minkowski gave special relativity its now standard form at the annual congress of *German Natural Scientists and Physicians* in Cologne in 1908:

"The word relativity-postulate […] seems to me very feeble. Since the postulate comes to mean that only the four-dimensional world in [space-time] is given […], but that the projection in space and in time may still be undertaken with a certain degree of freedom, I prefer to call it the postulate of the absolute world (or briefly, the world-postulate)"

Application of Dave Mason principle to space and time: All that exists is space-time. **Space-Time** distances will break down differently into their **Space** and **Time** components for different observers.