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High-Precision Ephemerides of Planets—EPM and Determination of Some Astronomical Constants

E. V. Pitjeva

Institute of Applied Astronomy, Russian Academy of Sciences, nab. Kutuzova 10, St. Petersburg, 191187 Russia

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Abstract—The latest version of the planetary part of the numerical ephemerides EPM (**E**phemerides of **P**lanets and the **M**oon) developed at the Institute of Applied Astronomy of the Russian Academy of Sciences is presented. The ephemerides of planets and the Moon were constructed by numerical integration in the post-Newtonian metric over a 140-year interval (from 1880 to 2020). The dynamical model of EPM2004 ephemerides includes the mutual perturbations from major planets and the Moon computed in terms of General Relativity with allowance for effects due to lunar physical libration, perturbations from 301 big asteroids, and dynamic perturbations due to the solar oblateness and the massive asteroid ring with uniform mass distribution in the plane of the ecliptic. The EPM2004 ephemerides resulted from a least-squares adjustment to more than 317000 position observations (1913–2003) of various types, including radiometric measurements of planets and spacecraft, CCD astrometric observations of the outer planets and their satellites, and meridian and photographic observations. The high-precision ephemerides constructed made it possible to determine, from modern radiometric measurements, a wide range of astrometric constants, including the astronomical unit AU = (149597870.6960 ± 0.0001) km, parameters of the rotation of Mars, the masses of the biggest asteroids, the solar quadrupole moment $J_2 = (1.9 \pm 0.3) \times 10^{-7}$, and the parameters of the PPN formalism β and γ. Also given is a brief summary of the available state-of-the-art ephemerides with the same precision: various versions of EPM and DE ephemerides from the Jet Propulsion Laboratory (JPL) (USA) and the recent versions of these ephemerides—EPM2004 and DE410—are compared. EPM2004 ephemerides are available via FTP at ftp://quasar.ipa.nw.ru/incoming/EPM2004.

HISTORICAL INTRODUCTION: MATHEMATICAL MODELING OF PLANET MOTION

Until the 1960s, the classic analytical theories of planetary motions developed by Le Verrier, Hill, Newcomb, and Clemens (Abalakin, 1979) were being refined along with the development of astronomical practice. Experiments performed in deep space and the introduction of new observational techniques (radar ranging, lunar laser ranging, VLBI measurements, etc.) required the development of precise planetary ephemerides that would be more accurate than the classical ones. On the other hand, it was modern observations that made it possible to develop a new generation of ephemerides.

The errors of the best ranging observations do not exceed several meters, making it necessary to compute delay times correctly up to the 12th decimal digit. Such high precision requires the construction of an appropriate model of the motion of celestial bodies. This is a serious problem; the easiest way to solve it is to perform computer-assisted numerical integration of the equations of motion of the planets and the Moon.

Eckert *et al.* (1951) were the first to compute the coordinates of five outer planets over a four-hundred-year time interval using numerical integration. Constructing a high-precision numerical theory of the motion of the major planets requires simultaneous numerical integration of the equations of orbital motion of the planets and the Moon and equations of rotation of the Earth and Moon. Oesterwinter and Cohen (1972) were the first to numerically integrate the equations of the orbital motion of the major planets and the Moon over the 1911–1973 time interval.

In the late 1960s, several research groups in the United States and Russia developed numerical theories to support space flights. American groups worked at the California (JPL) (Standish *et al.*, 1976; Newhall *et al.*, 1983) and the Massachusetts (Ash *et al.*, 1967) Institutes of Technology. Russian high-precision numerical ephemerides of planets (Akim *et al.*, 1986) were created as a result of research carried out at the Institute of Applied Mathematics (Akim and Stepanianz, 1977), the Institute of Radio Engineering and Electronics, the Space Flight Control Center (Kislik *et al.*, 1980), and the Institute of Theoretical Astronomy, where Glebova (1984), Eroshkin *et al.* (1992), and a group led by Krasinsky (Krasinsky *et al.* 1981, 1982) developed independent theories. This work was continued at the Institute of Applied Astronomy, where a series of EPM (**E**phemerides of **P**lanets and the **M**oon) ephemerides was produced.

In this paper, we consider two dynamical models of planetary motion that are most completed by the present time, have the same precision, and are adequate to modern radiometric observations. These are the series of EPM ephemerides and the well-known series of DE (**D**evelopment **E**phemeris, JPL) ephemerides.

The most accurate analytical ephemerides (theories of motion) are represented by the VSOP series of French ephemerides (Bretagnon and Francou, 1988) developed at the Bureau des Longitudes (BDL) and the Institut de Mecanique Celeste et de Calcul des Ephémérides (IMCCE). Recently, considerable progress has been achieved for the new ephemerides, VSOP2002b (Fienga and Simon, 2005), which include the perturbations from the Moon, 300 asteroids, solar oblateness, and relativistic effects. However, a comparison with the numerical ephemerides that the same group (Fienga and Simon, 2005) began to compute at IMCCE (their dynamical model of the motion of planets is close to the DE405 model, and the initial parameters of integration coincide with those of DE405) shows discrepancies of up to 100 m over 30 years. Moreover, the initial constants of integration of these ephemerides were obtained by fitting to DE200, DE403, and DE405, not to observations.

EPM AND DE DYNAMICAL MODELS OF THE MOTION OF PLANETS

The main common feature of EPM and DE ephemerides is that they are based on simultaneous integration of the equations of motion of the nine major planets, the Sun, the Moon, as well as the lunar physical libration in the post-Newtonian approximation described by a three-parameter metric (α, β, γ) in the harmonic coordinate system $\alpha = 0$; all variants of ephemerides were computed within General Relativity: $\beta = \gamma = 1$ (Newhall *et al.*, 1983).

Different versions of EPM and DE ephemerides differ slightly in the following:

(a) modeling of lunar libration,

(b) reference frames in which the ephemerides are computed,

(c) adopted value of the solar oblateness,

(d) modeling of perturbations from asteroids,

(e) sets of observations to which ephemerides are adjusted.

Table 1 lists some characteristics of ephemerides DE118, DE200, DE403, DE405, DE410, EPM87, EPM98, EPM2000, and EPM2004 (Standish, 1990, 1998; Standish *et al.*, 1995; Krasinsky *et al.*, 1993; Pitjeva, 2001a; Pitjeva, 2004).

The earliest ephemerides (DE118 and EPM87) were in the reference frame of the FK4 catalog and, then, in the reference frame of the dynamical equator and equinox (DE200); present-day ephemerides are referred to the International Celestial Reference Frame (ICRF) by including in the adjustment the ICRF-based VLBI measurements of spacecraft near planets.

The solar oblateness causes secular variations in the orbital elements of planets, with the exception of semimajor axes and eccentricities (Brumberg, 1972), and, therefore, starting with DE405 and EPM2000, ephemerides are integrated adopting the nonzero value of the quadrupole moment of the Sun, $J_2 = 2 \times 10^{-7}$, obtained from astrophysical estimates. The solar oblateness is now determined during the processing of high-precision ranging measurements.

A serious problem arises in the construction of modern ephemerides due to the necessity of allowing for the perturbations produced by asteroids. DE200 and EPM87 included only the perturbations from several of the biggest asteroids, which proved to be insufficient. DE403 and DE405, EPM98, and several others included the perturbations from 300 asteroids; however, the masses of most of them are either unknown or are known with insufficient accuracy, and Standish and Fienga (2002) showed that the accuracy of planetary ephemerides deteriorates substantially with time as a result of this factor.

The masses of several asteroids that produce the strongest perturbations on the orbits of Mars and Earth can be estimated by processing high-precision observations of Martian landers and spacecraft orbiting Mars. The masses of asteroids Eros (433) and Mathilde (253) were determined with high precision from trajectory perturbations of the *Near-Earth Asteroid Rendezvous* (NEAR) spacecraft. Recently, binary asteroids and asteroids with satellites have been discovered and studied, and the masses of these systems are now known rather accurately. Unfortunately, the accuracy of the dynamical determination of asteroid masses from gravitational perturbations caused by other asteroids has proved to be insufficient in many cases due to the uncertainties in the masses of perturbing asteroids, insufficient allowance for the perturbations from other asteroids, and observation errors (Krasinsky *et al.*, 2001; Hilton, 2002). Therefore, the masses of the asteroids that remain of the original 300 asteroids and those of the more than 57 additional asteroids producing the strongest perturbations upon the orbits of planets were estimated by an astrophysical method (Krasinsky *et al.*, 2001, 2002) that involved analyzing the data on the radii and classes of asteroids. To this end, the researchers used the most recent published diameters based on IRIS and MSX infrared observations (Tedesco *et al.*, 2002a, 2002b) and on the observations of stellar occultations by asteroids (Dunham *et al.*, 2002) and radar observations (Ostro *et al.*, 2002). The mean densities of asteroids of three taxonomic classes were estimated during processing of the ranging observations of planets and spacecraft.

In addition, thousands of small asteroids, many of which are too small to be ever discovered from the Earth, produce a substantial cumulative effect on the orbits of inner planets. The total additional effect produced by the asteroids for which individual perturba-

Table 1. DE and EPM ephemerides

Ephemerides	Interval of integration	Reference frame	Mathematical model	Type of observations	Number of observations	Time interval
DE118	1599 → 2169	FK4	Integration: the Sun, the Moon, nine planets + perturbations from three asteroids (two-body problem)	Optical	44755	1911–1979
(1981)				Radar	1307	1964–1977
⇓		⇓		Spacecraft and landers	1408	1971–1980
DE200		J2000.0		LLR (lunar laser ranging)	2954	1970–1980
		system		Total	50424	1911–1980
EPM87	1700 → 2020	FK4	Integration: the Sun, the Moon, nine planets + perturbations from five asteroids (two-body problem)	Optical	48709	1717–1980
(1987)				Radar	5344	1961–1986
				Spacecraft and landers	–	–
				LLR (lunar laser ranging)	1855	1972–1980
				Total	55908	1717–1986
DE403	–1410 → 3000	ICRF	Integration: the Sun, the Moon, nine planets + perturbations from 300 asteroids (mean elements)	Optical	26209	1911–1995
(1995)				Radar	1341	1964–1993
⇓	⇓			Spacecraft and landers	1935	1971–1994
DE404	–3000 → 3000			LLR (lunar laser ranging)	9555	1970–1995
				Total	39057	1911–1995
EPM98	1886 → 2006	DE403	Integration: the Sun, the Moon, nine planets, five asteroids + + perturbations from 295 asteroids (mean elements)	Optical	–	–
(1998)				Radar	55959	1961–1995
				Spacecraft and landers	1927	1971–1982
				LLR (lunar laser ranging)	10000	1970–1995
				Total	67886	1961–1995
DE405	1600 → 2200	ICRF	Integration: the Sun, the Moon, nine planets + perturbations from 300 (integrated) asteroids	Optical	28261	1911–1996
(1997)				Radar	955	1964–1993
⇓				Spacecraft and landers	1956	1971–1995
DE406	–3000 → 3000			LLR (lunar laser ranging)	11218	1969–1996
				Total	42410	1911–1996
EPM2000	1886 → 2011	DE405	Integration: the Sun, the Moon, nine planets, 300 asteroids	Optical	–	–
(2000)				Radar	58076	1961–1997
				Spacecraft and landers	24587	1971–1997
				LLR (lunar laser ranging)	13500	1970–1999
				Total	96163	1961–1999
DE410	1901 → 2019	ICRF	Integration: the Sun, the Moon, nine planets + perturbations from 300 asteroids	Optical	39159	1911–2003
(2003)				Radar	978	1964–1997
				Spacecraft and landers	154685	1971–2003
				LLR (lunar laser ranging)	9555	1970–1995
				Total	204377	1911–2003
EPM2004	1880 → 2020	ICRF	Integration: the Sun, the Moon, nine planets, 301 asteroids, and a ring	Optical	46064	1913–2003
(2004)				Radar	58116	1961–1997
				Spacecraft and landers	197271	1971–2003
				LLR (lunar laser ranging)	15590	1970–2003
				Total	317041	1913–2003

tions were not accounted for in the simultaneous numerical integration was modeled by the potential of a circular asteroid ring with a constant mass distribution in the plane of the ecliptic. The formulas for the perturbing force of the asteroid ring can be found in the paper by Krasinsky *et al.* (2002). The mass M_r and radius R_r of the ring have been included in the set of solution parameters.

Thus, the dynamical model of EPM2004 ephemerides includes the mutual perturbations of the major planets and the Moon computed in terms of General Relativity, effects due to the physical libration of the Moon, perturbations from the 301 biggest asteroids and the massive asteroid ring, and dynamic perturbations due to the solar oblateness. The equations of motion of planets used in the EPM2004 ephemerides can be found in the paper by Pitjeva (2004).

Along with the planetary ephemerides, the ephemerides of the orbital and rotational motion of the Moon were produced and improved by processing Lunar Laser Ranging (LLR) observations performed in 1970–2003. The most recent version of the lunar theory can be found in the paper by Krasinsky (2002), where a number of subtle selenodynamical effects is described.

The equations of motion were numerically integrated in the J2000.0 barycentric coordinate system over a 140-year time interval (1880–2020) using the lunar and planetary integrator of the **E**phemeris **R**esearch in **A**stronomy package (ERA-7) based on Everhart's method (Everhart, 1974). This package was developed to support the research in the field of ephemeris astronomy and celestial mechanics (Krasinsky and Vasilyev, 1997).

RADAR AND OPTICAL OBSERVATIONS AND THEIR PROCESSING

The EPM2004 ephemerides were fitted to 317041 position observations (1913–2003) of various types, including radiometric measurements of planets and spacecraft, CCD astrometric observations of outer planets and their satellites, and meridian and photographic observations. The data used for the production of the EPM ephemerides were taken from the JPL database (http:/ssd.jpl.nasa.gov/iau-comm4/), developed and maintained by Dr. Standish, and from the database of optical observations of Dr. Sveshnikov, and were extended to include Russian ranging observations of planets made in 1961–1995 (available from the site of the Institute of Applied Astronomy of the Russian Academy of Sciences, //www.ipa.nw.ru/PAGE/DEPFUND/LEA/ENG/englea.htm). All observations used to construct the ephemerides are described in Tables 2 and 3.

Ranging observations of planets started in 1961 and have become widely used in astronomical practice since then, making it possible to determine various astronomical constants with high precision. Reductions of radar observations including relativistic corrections—the delay of the radio signal near the Sun (the Shapiro effect); the transition from the coordinate time, the argument of ephemerides, to the proper time of the observer; the delay of radio signals in the Earth's troposphere and in the plasma of the solar corona—are well known, and a description of them can be found, e.g., in the paper by Standish (1990). Only the reduction for the topography of the planets may cause some problems. Topographic correction of observations of Mars and Venus was performed using modern hypsometric maps of the surfaces of these planets and a representation of the global topography with an expansion of spherical functions of 16th–18th degrees. The global topography of Mercury is unknown, and, therefore, we represented it by the second-order Legendre functions. The coefficients of the harmonics were estimated from ranging observations of Mercury (Pitjeva, 2001b).

We should point out the special importance of ranging observations of the Martian landers *Viking 1* and *2* (1976–1982), which are free of topographic errors, errors that persist in ranging observations of planets despite careful topographic reductions. These observations remained the most accurate position observations of planets for 20 years (they have an a priori accuracy of about 10 m). The data from the new *Pathfinder* lander were received during three months in 1997. The computation of the positions of the landers on the surface of Mars in the ephemeris reference frame requires a theory of Martian rotation that includes not only precession and nutation of the rotation axis of Mars but also seasonal terms in the Martian rotation (see Youder and Standish, 1997; Folkner *et al.*, 1997; Pitjeva, 1999).

Since 1998, the database has been augmented by ranging observations of the *Mars Global Surveyor* (MGS) spacecraft, and, since 2002, by the *Odyssey* spacecraft. These measurements have an accuracy of 2 m.

All observations of Mars and, as a rule, those of Mercury and Venus, performed during one day and, after introducing all the required corrections, including the reduction for the topography of the planets, were grouped into normal places. The normal places for the MGS and *Odyssey* data were obtained by combining the measurements made during the same session: it was assumed that the measurements belong to different sessions if the corresponding observation times differed by more than one hour. When combining observations, we assigned weight to all observations according to their a priori accuracy, which is usually given in the corresponding publications.

Unfortunately, unlike the observations of the *Viking* spacecraft, which were made at two frequencies and, therefore, allowed the delay in the solar corona to be taken into account, the MGS and *Odyssey* observations were carried out at one X band and the effect of the solar corona delay was considerable, especially near the superior solar conjunctions in 1998 and 2002. We reduced these observations using the following model of the solar corona:

$$N_e(r) = \frac{A}{r^6} + \frac{B + \dot{B}t}{r^2},$$

where $N_e(r)$ is the electron density. We determined the parameters B and $\dot{B}$ from observations, and these parameters differed for different conjunctions. Although this reduction for the effect of the solar

Table 2. Radiotechnical observations

Observatory or object	Type of observations	Interval o observations	Number of observations	Normal places	A priori accuracy
Mercury					
Millstone	τ	1964	5	–	7.5–75 km
Haystack	τ	1966–1971	217	–	3 km
Arecibo	τ	1964–1982	341	323	3–30 km
Goldstone	τ	1971–1997	259	138	1.5–3 km
Goldstone C	τ	1990–1997	40	–	0.15–2.5 km
Crimea	τ	1980–1995	75	23	1.2–4.8 km
Mariner-10	τ	1974–1975	–	2	0.1 km
Venus					
Millstone	τ	1961–1967	135	–	1.5–120 km
Haystack	τ	1966–1971	219	–	1.5 km
Arecibo	τ	1964–1970	319	–	3–15 km
Goldstone	τ	1964–1990	512	–	1.5–6 km
Crimea	τ	1962–1995	1139	170	0.15–22.5 km
Magellan	$\alpha\delta$	1990–1994	–	18	$0''.001$–$0''.004$
Mars					
Haystack	τ	1967–1973	3801	133	0.075–12 km
Arecibo	τ	1965–1973	1680	43	0.075–45 km
Goldstone	τ	1969–1994	48989	149	0.075–0.6 km
Crimea	τ	1971–1995	381	78	0.15–4.8 km
Mariner-9	τ	1971–1972	643	–	15–270 m
Viking-1	τ	1976–1982	1161	–	7–12 m
Viking-1	$d\tau$	1976–1978	14980	–	0.16–3.2 m
Viking-2	τ	1976–1977	80	–	7–10 m
Mars Pathfinder	τ	1997	90	–	10–22 m
Mars Pathfinder	$d\tau$	1997	7576	–	0.012 m
Phobos	τ	1989	–	1	0.2 km
MGS	τ	1998–2003	110538	4930	2–7.5 m
Odyssey	τ	2002–2003	62093	1715	2–3 m
Spacecraft	$\alpha\delta$	1984–2003	–	44	$0''.0003$–$0''.006$
Jupiter					
Spacecraft, VLA	α	1979–1995	–	4	$0''.003$–$0''.046$
Spacecraft, VLA	δ	1979–1995	–	4	$0''.005$–$0''.2$
Spacecraft	τ	1973–1995	–	6	1–6 km
Spacecraft	$\alpha\delta$	1996–1997	–	24	$0''.007$–$0''.012$
Arecibo 3, 4*	τ	1992	–	4	3–14 km

* Arecibo 3, 4 corresponds to ranging observations of Jovian satellites nos. 3 and 4 made at Arecibo.

corona substantially reduced the residuals of the observations, the remaining influence of the corona is still obvious (Fig. 1 for MGS and *Odyssey*). Moreover, the parameters of the corona correlate with other parameters and impair their determination. This fact must be taken into account in high-precision astrometric observations in future space missions.

For Jupiter, unlike other outer planets, a number of precise radiotechnical observations by spacecraft (*Pioneer 10* and *11*, *Voyager 1* and *2*, *Ulysses*, and *Galileo*) approaching the planet or orbiting it have been performed, which allow its orbit to be determined much more accurately than those of other outer planets.

Table 3. Optical and VLA observations of outer planets

Observatory or object	Planet (p) or satellite (s)	Type of observations	Time interval of observations	Number of observations	A priori accuracy
Jupiter					
USNO	p	Transit observations	1913–1994	4388	0″.5
Tokyo	p	Photoelectric transit observations	1963–1988	568	0″.5–0″.8
La Palma	s 3, 4	Photoelectric transit observations	1986–1997	1316	0″.25
Nikolaev	s 1, 2, 3, 4	Photographic observations	1962–1998	2628	0″.2
Flagstaff	s 1, 2, 3, 4	CCD observations	1998–2003	2408	0″.2
Mountain	s 1, 2, 3, 4	CCD observations	2002	16	0″.5
Saturn					
USNO	p	Transit observations	1913–1982	3054	0″.5
Tokyo	p	Photoelectric transit observations	1963–1988	506	0″.5–0″.8
Bordeaux	s 6, 8	Photoelectric transit observations	1987–1993	238	0″.25
La Palma	s 5, 6, 7, 8	Photoelectric transit observations	1987–1997	1460	0″.25
Nikolaev	s 3, 4, 5, 6, 8	Photographic observations	1973–1997	1264	0″.2
Flagstaff	s 3, 4, 5, 6, 7, 8	CCD observations	1998–2003	4014	0″.2
Mountain	s 3, 4, 5, 6, 7, 8	CCD observations	2002–2003	628	0″.15
VLA	p	Radiotechnical observations	1984	8	0″.03–0″.06
Uranus					
USNO	p	Transit observations	1913–1993	4244	0″.5
Tokyo	p	Photoelectric transit observations	1963–1988	366	0″.5–0″.8
Bordeaux	p	Photoelectric transit observations	1985–1992	330	0″.25
Bordeaux	p	CCD observations	1997	34	0″.2
La Palma	p, s 4	Photoelectric transit observations	1984–1997	2072	0″.25
Nikolaev	p	Photographic observations	1961–1998	440	0″.2
Flagstaff	p, s 3, 4	CCD observations	1995–2003	2324	0″.2
Mountain	p, s 3, 4	CCD observations	1998–2003	174	0″.15
VLA	p	Radiotechnical observations	1977–1985	16	0″.03–0″.2
Neptune					
USNO	p	Transit observations	1913–1993	3804	0″.5
Tokyo	p	Photoelectric transit observations	1963–1988	320	0″.5–0″.8
Bordeaux	p	Photoelectric transit observations	1985–1993	366	0″.25
Bordeaux	p	CCD observations	1997	28	0″.2
La Palma	p	Photoelectric transit observations	1984–1998	2212	0″.25
Nikolaev	p	Photographic observations	1961–1998	436	0″.2
Flagstaff	p, s 1	CCD observations	1995–2003	1888	0″.2
Mountain	p, s 1	CCD observations	1998–2003	120	0″.15
VLA	p	Radiotechnical observations	1981–1997	22	0″.03–0″.2
Pluto					
Various stations	p	Photographic observations	1914–1967	1164	0″.5–1″
Various stations	p	Photographic observations	1969–1988	674	0″.5–1″
Various stations	p	Photographic observations	1989–1995	82	0″.5–1″
Pulkovo	p	Photographic observations	1930–1993	416	0″.5
Tokyo	p	Photographic observations	1994	24	0″.3
Bordeaux	p	Photoelectric transit observations	1996	12	0″.3
Bordeaux	p	CCD observations	1995–1997	64	0″.2
La Palma	p	Photoelectric transit observations	1986–1998	760	0″.25
Flagstaff	p	CCD observations	1995–2003	1152	0″.2
Mountain	p	CCD observations	2000–2003	68	0″.15

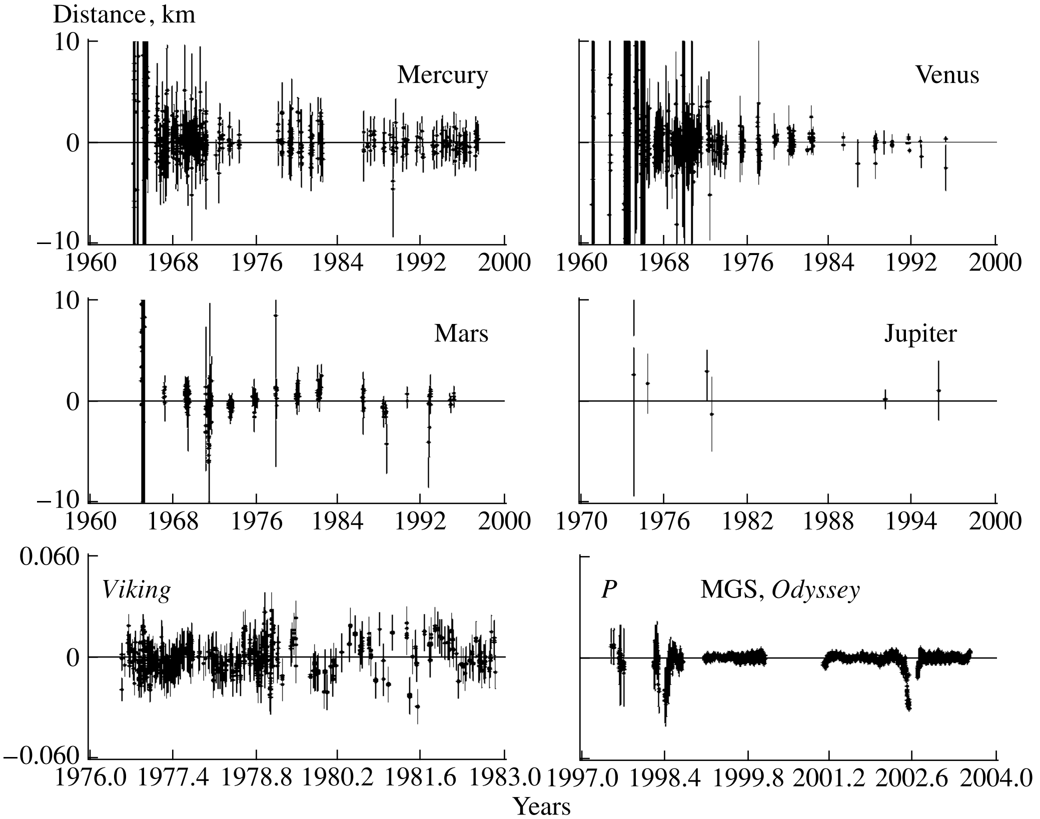

Fig. 1. Ranging residuals of Mercury, Venus, Mars, Jupiter, *Viking*, *Pathfinder P* (1997), MGS (1998–2003), and *Odyssey* (2002–2003).

Figure 1 shows the residuals of ranging observations. The root-mean-square errors of fits to observations are equal to 1.4 km for Mercury; 0.7 km for Venus and Mars; 8 and 4.4 m for the *Viking* and *Pathfinder* landers, respectively; and 1.4 m for MGS and *Odyssey.*

The orbits of other outer planets rely entirely upon optical observations (see Table 3). Observations of the satellites of outer planets are of special importance for the improvement of the orbits of these parent planets, because satellite observations are much more accurate than observations of their parent planets and are practically free of the phase effect, which is difficult to allow for. Figure 2 shows the residuals for all outer planets.

We reduced EPM2004 ephemerides to the ICRF reference frame. Most of the modern optical observations of planets and their satellites (made at Flagstaff, Mountain, Nikolaev, and La Palma) have been already referred to the ICRF frame by the observers. The remaining optical observations referenced to various catalogs and were transformed to the reference frame of the FK4 catalog by Sveshnikov (1974, 2000). We then referenced these observations to the FK5 reference frame using well-known formulas (see, e.g., Standish *et al.*, 1995) and finally referred them to the ICRF frame using three angles of rotation between the HIPPARCOS and the FK5 catalogs, with J2000 in milliarcseconds (mas) (Mignard, 2000): $\varepsilon_x = -19.9$, $\varepsilon_y = -9.1$, and $\varepsilon_z = 22.9$.

The orbital elements (except orientation) of the four inner planets are determined completely by ranging observations of planets and spacecraft. The orientation of the system of these planets was provided by using the ICRF-based VLBI measurements of spacecraft. The orientation of this system was improved substantially by the incorporation, in addition to the earlier data for the *Magellan* and *Phobos* spacecraft, of new VLBI observations of the MGS and *Odyssey* spacecraft. The angles of rotation between the EPM2004 ephemerides and the ICRF frame are (in mas): $\varepsilon_x = 1.9 \pm 0.1$, $\varepsilon_y = -0.5 \pm 0.2$, and $\varepsilon_z = -1.5 \pm 0.1$; they are close to the angles of rotation between DE405 and DE410.

DETERMINATION OF ASTRONOMICAL CONSTANTS

In this section, the parameters of EPM2004 ephemerides determined from all observations (Tables 2 and 3) made in 1913–2003 are presented. Table 4 gives the formal standard errors of the orbital elements of planets, where a is the semimajor axis, i is the orbital inclination, Ω is the longitude of the ascending node, e is the eccentricity, π is the longitude of the perihelion, and λ is the mean longitude. The accuracy of the determina-

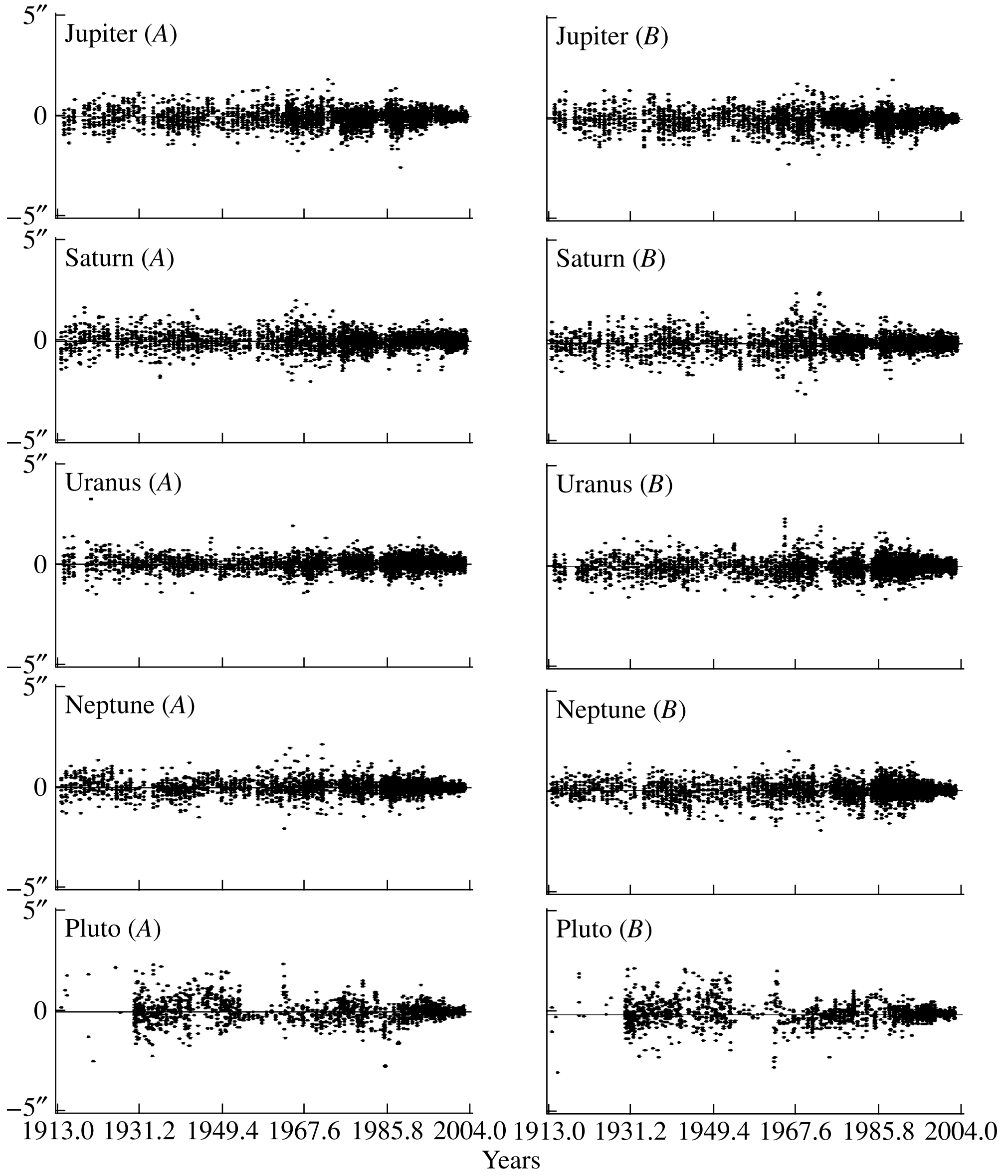

Fig. 2. Residuals of outer planets $\alpha\cos\delta$ (*A*) and δ (*B*); the scale is ±5″.

tion of the orbital elements of planets is very high; e.g., the formal standard errors of the least-squares method (LSM) amount to only a fraction of a meter for the semimajor axes; however, it should be pointed out that, as experience shows, the real errors may be larger than LSM errors by an order of magnitude.

The following value for the astronomical unit (AU) was found: AU = 149597870696.0 ± 0.1 m, which differs from the most recent estimate based on approximately the same set of observations (Standish, 2005), $\mathrm{AU}_{\mathrm{Standish}}$ = 149597870697 m, by 1 m, which is the likely real error of the determination of this value.

Observations of the *Viking 1* and *2* and *Pathfinder* landers yielded the following parameters for the rotation of Mars: $\dot{V}$, the velocity of Martian rotation; Ω_q, $\dot{\Omega}_q$, I_q, and $\dot{I}_q$, the mean longitude of the node and inclination of the Martian equator to the mean orbit of Mars and their derivatives, respectively (Table 5); the positions of the landers; and the seasonal terms in the axial rotation of Mars (Pitjeva, 1999). Our result for the precession of the Martian rotation proved to be close to the following recent estimate (Yoder *et al.*, 2003): $\dot{\Omega}_q = [-7''.597 \pm 0''.025(10\sigma)]$/yr, obtained from observations of the lander modules and from the MGS radio tracking.

High-precision radar observations, which already span a time interval of 43 years, make it possible to very accurately determine not only the orbital elements of planets but also other constants of planetary theory, e.g., the masses of the biggest asteroids and the total

Table 4. The formal standard deviations of planetary orbital elements

Planet	a, m	$\sin i\cos\Omega$ [mas]	$\sin i\sin\Omega$ [mas]	$e\cos\pi$ [mas]	$e\sin\pi$ [mas]	λ [mas]
Mercury	0.105	1.654	1.525	0.123	0.099	0.375
Venus	0.329	0.567	0.567	0.041	0.043	0.187
Earth	0.146	–	–	0.001	0.001	–
Mars	0.657	0.003	0.004	0.001	0.001	0.003
Jupiter	639	2.410	2.207	1.280	1.170	1.109
Saturn	4222	3.237	4.085	3.858	2.975	3.474
Uranus	38484	4.072	6.143	4.896	3.361	8.818
Neptune	478532	4.214	8.600	14.066	18.687	35.163
Pluto	3463309	6.899	14.940	82.888	36.700	79.089

Note: a mas (milliarcsecond) is one thousandth of an arcsecond.

Table 5. Obtained values of astronomical parameters

Parameters of Mars rotation

$\dot V$ [deg/day]	I_q [deg]	$\dot I_q$ [arcsecond/year]	Ω_q [deg]	$\dot\Omega_q$ [arcsecond/year]
350.891985294 ±0.000000012	25.1893930 ±0.0000053	−0.0002 ±0.0007	35.437685 ±0.000021	−7.5844 ±0.0015

Masses of asteroids in $(GM_i/GM_\odot) \times 10^{-10}$

(1) Ceres	(2) Pallas	(3) Juno	(4) Vesta	(7) Iris	(324) Bamberga
4.753 ±0.007	1.027 ±0.003	0.151 ±0.003	1.344 ±0.001	0.063 ±0.001	0.055 ±0.001

Quadrupole moment of the Sun; radius and mass of the asteroid ring;
the total mass of the main-belt asteroids; parameters of PPN formalism, and $\dot G/G$

J_2 10^{-7}	R_{ring} AU	M_{ring} $10^{-10} M_\odot$	M_{belt} $10^{-10} M_\odot$	$\beta - 1$ 10^{-4}	$\gamma - 1$ 10^{-4}	$\dot G/G$ 10^{-11} year^{-1}
1.9 ± 0.3	3.13 ± 0.05	3.35 ± 0.35	15.0 ± 1.0	0 ± 1	−1 ± 1	–0.002 ± 0.005

mass of asteroids in the main belt. Of special interest is the possibility of experimentally detecting the hypothetical secular variation of the gravitational constant, because it characterizes fundamental properties of our physical space–time, and of estimating the PPN parameters and the dynamical oblateness of the Sun. The results obtained (Table 5) show no substantial deviations from the values of General Relativity.

COMPARISON OF EPM2004 AND DE410 EPHEMERIDES

It may be beneficial to know the discrepancies of different ephemerides, because such discrepancies indicate real accuracies of the computed ephemerides. We compared the recent versions of EPM2004 and DE410 ephemerides over the 1970–2010 time interval. These ephemerides are based on approximately the same sets of observations and similar mathematical models of the motion of planets, but they differ in their methods of allowing for the perturbations produced by asteroids, their masses, and reductions for the topography of planetary surfaces and the solar corona.

The coordinates of Mercury and Venus were obtained from fitting ranging observations of these planets, which have errors about 1 km, and, therefore, the maximum differences of 258 and 139 m in the heliocentric distances for Mercury and Venus (Fig. 3) can be considered acceptable. The maximum differences in the heliocentric distances of the Earth and Mars in these ephemerides are much smaller—up to 12.8 and 35.7 m, respectively. This is not surprising, because the accuracy of the MGS and *Odyssey* data used to construct the ephemerides of these planets is on

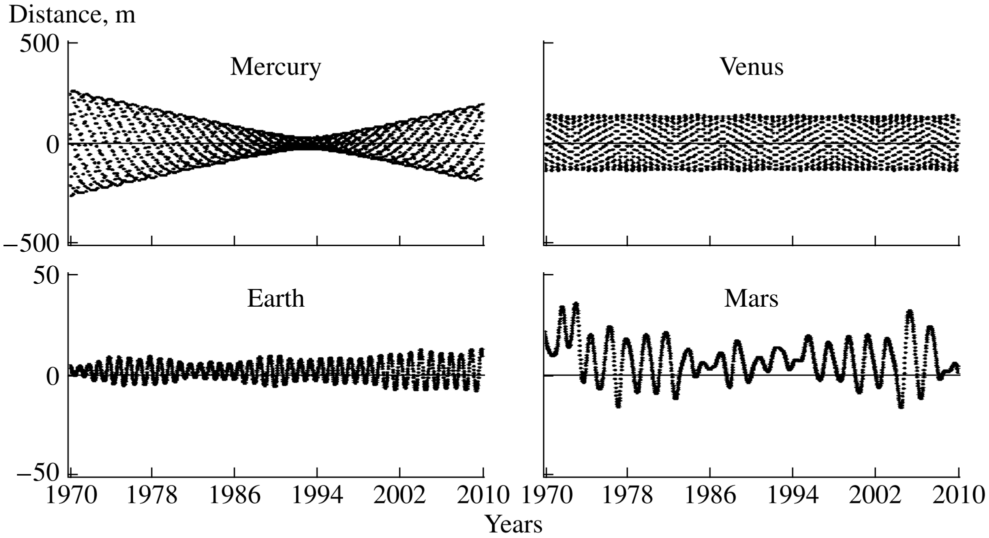

Fig. 3. EPM2004–DE410: differences in the heliocentric distances of inner planets.

the order of two meters. The differences in coordinates can be explained by the differences in the allowance for the perturbations produced by asteroids and the solar corona. As mentioned above, the availability of a number of radiotechnical observations of Jupiter allowed us to construct its orbit more accurately than those of other outer planets. The differences in the heliocentric distances for Jupiter do not exceed 10 km. The orbits of the remaining outer planets were determined by optical observations exclusively; moreover, more or less accurate observations do not cover even a single orbital period for either Neptune or Pluto. The differences amount to 180, 410, 1200, and 14000 km for Saturn, Uranus, Neptune, and Pluto, respectively, and give the current accuracy of modern ephemerides.

CONCLUSIONS

The quality of ephemerides and the accuracy of all the parameters of planetary theories depend on the following three factors: the accuracy of the procedures of reduction of observations, the dynamical models of the motion of planets, and the observations to which ephemerides are adjusted. An improvement in the quality and an increase in the number of observations are crucial factors in this process. It should be emphasized that the use of ranging observations of planets and spacecraft made it possible to achieve milliarcsecond accuracy of the ephemerides constructed and to thereby determine the astronomical parameters with high accuracy. Further increase in the accuracy of planetary parameters depends on supplementing the observational database with new ranging observations of spacecraft and landers and on making progress in the determination of accurate masses of many asteroids.

We point out in conclusion that EPM2004 numerical ephemerides of all the planets and the Moon are available via FTP at ftp://quasar.ipa.nw.ru/incoming/EPM2004 or via the website of the Institute of Applied Astronomy of the Russian Academy of Sciences at http://www.ipa.nw.ru/PAGE/DEPFUND/LEA/ENG/englea.htm.

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