THE

PURE THEORY OF FOREIGN TRADE.

CHAPTER I.

THE PREMISES OF THE PURE THEORY OF FOREIGN TRADE. THE
METHOD OF DIAGRAMS. THE FUNDAMENTAL LAWS OF CURVES
WHICH REPRESENT INTERNATIONAL DEMAND.

THE function of a pure theory is to deduce definite conclu-
sions from definite hypothetical premises. The premises should
approximate as closely as possible to the facts with which the
corresponding applied theory has to deal. But the terms used
in the pure theory must be capable of exact interpretation, and
the hypotheses on which it is based must be simple and easily
handled.

The pure theory of foreign trade satisfies these conditions.
This theory is based upon the hypothesis that two countries, say
England and Germany, carry on trade with each other but only
with each other. It is assumed that they are not under any
obligations to make foreign payments excepting those arising
from trade, so that in equilibrium the exports of each country
exchange for her imports. It is assumed that the pure theory
of domestic values has provided the means of measuring the
value in England of all the various wares exported by England in
terms of any one of them. Suppose cloth of a definite quality to be
one of them; then the value, in England, of all the wares which
England exports may be expressed as that of a certain number
of yards of cloth. So the value in Germany of all the wares
which Germany exports, may be expressed as that of, say, a
certain number of yards of linen.

We may for brevity use the phrase "a certain number of yards of cloth," as a substitute for the complete phrase "English wares the equivalent of a certain number of yards of cloth": and so for linen. Further we may consider that the processes of producing the cloth and the linen are not completed until the cloth and the linen are delivered in Germany and England respectively. By this means we shall avoid the necessity of specially mentioning the expenses of transport; so that we shall find no occasion to follow Mill in making the assumption that the expenses of transport may be neglected.

We may apply this method of speaking to express the conditions under which trade is in equilibrium; i.e. is such that there is no tendency for the imports and exports of the countries in question to increase or to diminish. Thus:—In equilibrium a certain number, say ten million, of yards of cloth are exported annually to Germany and sold there for a price which covers the expenses of producing a certain number, say fifteen million, of yards of linen. Vice versa, fifteen million yards of linen are exported to England and sold there for a price which covers the expense of producing ten million yards of cloth.

We are now in a position to give a definite interpretation to the phrase "the rate of interchange between two countries" in place of the inexact account sometimes given. We may measure the rate of interchange between England and Germany by the amount of linen which England obtains in return for each yard of cloth which she exports.

It seems on the whole best thus to represent the value of the wares which England exports as equivalent to that of a certain number of yards of cloth. But we might measure it as equivalent to a certain number of units of English capital and labour, or as we may say as equivalent to a certain number of units of English cost of production[1]. We should then measure the rate of interchange between England and Germany by the number of units of German cost of production which England obtains in return for the produce of a given number of units of her cost of production. This latter method of measurement has several advantages, and there is no reason why it should not be adopted in the treatment of some portions of the pure theory of foreign trade. But for the general purposes of the theory the method of measurement first given will be found to be the most convenient.

The theory of foreign trade is necessarily difficult. Mill when introducing it says, "I must give notice that we are now in the

[1] For a solution of the ambiguities connected with the use of this phrase see Appendix I.

region of the most complicated questions which Political Economy affords: that the subject is one which cannot possibly be made elementary; and that a more continuous effort of attention than has [in the earlier portions of the science] been required will be necessary in order to follow the series of deduction." The unavoidable difficulties of the subject are great: but students frequently fall into errors which they may easily avoid if they will resolve that when discussing the pure theory they will not speak of the imports or exports of a country as measured in terms of money.

Suppose that the fact to be expressed is that England has increased her demand for the wares of Germany; and has thereby caused the rate of interchange to be altered to her disadvantage.

It is found by experience that students commencing the subject have a tendency to describe this fact thus:—England used to import (say) ten million pounds worth of German wares, giving for them (after allowing for carriage) ten million pounds worth of English: but her demand for German wares increases so that she purchases twelve million pounds worth; and, the rate of interchange being altered to her disadvantage she has to give in return for them (after allowing for carriage) thirteen million pounds worth of her own wares.

This statement is inaccurate because it ignores the changes that will meanwhile have occurred in the standards of prices in the two countries. After, as well as before, the change in England's demand, each million pounds worth of English goods will be exchanged (allowance being made for the cost of carriage) for a million pounds worth of German goods, prices being measured according to the new standard. But the change will have caused gold to flow from England to Germany, so as to raise prices in Germany and lower them in England. So that the above statement should have been:—England imports an amount of German wares which according to the old standard of German prices was worth twelve million pounds, but according to the new standard of prices is worth (say) twelve and a half millions. In exchange England exports an amount of her own wares which according to the old standard of her prices was worth thirteen million pounds, but according to the new standard is worth twelve and a half millions. This statement is accurate but uselessly complex. And the complexities of which this is an instance, increase till they become wholly unmanageable if the attempt is made to proceed far into the pure theory of foreign trade on the plan of measuring exports and imports in terms of money.

§ 2. We may now proceed to consider the laws which govern the demand of one country for the wares of another. The explanation of these laws is tolerably simple, so long as we are dealing only with the normal conditions of foreign trade. Under ordinary circumstances, a decrease in a country's exports will cause her to obtain her imports on terms more advantageous, but not much more advantageous than before. There will be increased competition for her wares in foreign markets, and consequently their price will tend to rise; but as some at least of her wares will be closely pressed by the rivalry of foreign producers, the rate of interchange will not be altered in her favour sufficiently to prevent a decrease in the amount of her imports. Similarly any increase in her exports will cause her to obtain an increase in her imports, though she will obtain them on somewhat less advantageous terms. So long, then, as we assume these normal conditions to exist, we may trace the changes of foreign trade by means of the ordinary processes of general reasoning, without the aid of any artificial apparatus. But such a treatment becomes very difficult, if not impossible, when we pass to consider exceptional cases in which these normal conditions fail: and it is the special task of the pure theory of foreign trade to deal with such exceptional cases. The only apparatus which Ricardo and Mill brought to bear on the problems of pure economic theory was that of arithmetical illustration. But this is inadequate to the work. The use of numerical examples will perhaps enable the investigator to ascertain some of the consequences which may arise from the causes into whose operation he is inquiring: but it affords no security that he will discover all of these consequences or even the most important of them. Moreover when he has deduced certain conclusions from a particular set of numbers which he has chosen to illustrate certain general premises, he is not unlikely to infer that these conclusions follow necessarily from the premises he has laid down: whereas these conclusions may be latent in the particular choice of numbers that he has made, and may not be capable of being deduced from every set of numbers which satisfy the conditions laid down in the general premises. Experience proves that even powerful thinkers are liable to be thus led into error in spite of their being well aware that the legitimate use of numerical examples is only to illustrate and not to prove general rules. The weakness and inefficiency of this apparatus will be demonstrated in the course of the present examination of the theory of foreign trade. For the free use of numerical examples has not enabled Ricardo and Mill to discover the conclusions which follow necessarily from their hypothesis.

The pure theory of economic science requires the aid of an apparatus which can grasp and handle the general quantitative relations on the assumption of which the theory is based. The most powerful engines for such a purpose are supplied by the various branches of mathematical calculus. But diagrams are of great service, wherever they are applicable, in interpreting to the eye the processes by which the methods of mathematical analysis obtain their results. It happens that with a few unimportant exceptions all the results which have been obtained by the application of mathematical methods to pure economic theory can be obtained independently by the method of diagrams.

Diagrams present simultaneously to the eye the chief forces which are at work, laid out, as it were, in a map; and thereby suggest results to which attention has not been directed by the use of the methods of mathematical analysis. The method of diagrams can be freely used by every one who is capable of exact reasoning, even though he have no knowledge of Mathematics. The reader, who will take the trouble to assure himself that he thoroughly understands the account of the curves given in the following paragraphs, will not find difficulty in following the reasoning to which they are afterwards applied.

§ 3. The most convenient mode of procedure will be to commence by examining the conditions of the first of the exceptional cases to which reference has been made; then to interpret the normal conditions of the problem as well as these exceptional conditions into the language of diagrams; and afterwards to treat the second exceptional case, which is of minor importance.

The first exceptional case is that of a group of problems in which it is assumed that a diminution of the total exports of a country may cause these to be in such urgent demand abroad that she obtains in return for her diminished exports an increased instead of a diminished supply of foreign wares. The results of an investigation of this exceptional case are capable of being applied in the partial and indirect solution of some practical problems connected with the trade that is carried on between existing countries; particularly in connexion with duties on exportation. But the chief importance of these results arises from the fact that they may be applied to the trade that a compact industrial group carries on with its neighbours. We shall refer to this class of problems as "Class I."; and shall give the name of "Class II." to the second exceptional case to which reference has been made; the case, namely, in which an increase in the amount of wares which a country produces for exportation effects a very great

diminution in the expenses at which she can produce them; so that the consequent fall in their value diminishes the total amount of the imports that she receives in exchange for them. When we are considering the circumstances of trade from which both these exceptional cases are excluded, we may for brevity say that we are discussing the "Normal class" of problems.

Applying this classification to the special case of the trade in cloth and linen which we have supposed to be carried on between England and Germany, we may say:—Every increase in the amount of linen which is thrown annually on the English market will necessitate a cheapening of the terms on which it is offered for sale. The effect of this cheapening will (save in problems of Class II.) cause each yard of linen to exchange for the means of producing and exporting a smaller amount of cloth than before : that is, will alter the rate of interchange in England's favour[1]. In the Normal Class this alteration will be slight, so that every increase in the amount of linen imported will occasion an increase in the amount of cloth exported. But in Class I. an increase in the amount of linen imported will depress the price at which it can be sold in England, and it will alter the rate of interchange in England's favour to so great an extent as to cause the amount of cloth exported not to increase but diminish. A precisely similar statement of course applies to Germany's demand for cloth. Class I. may be illustrated numerically thus :—Suppose the sale of 10 million yards of linen in England to afford the means of purchasing and exporting 10 million yards of cloth, the rate of interchange being thus, one yard of cloth to one yard of linen. An increase in the amount of linen to 15 million yards may perhaps cause the amount of cloth to increase to 12 million : while it is possible that a further increase in the linen to 20 million may so force down its price in the English market as to cause the rate of interchange to become two yards of linen for one of cloth; in which case the amount of cloth which Germany obtains will fall to ten million yards.

§ 4. Let us now commence to interpret the laws of international demand into the language of diagrams. Let distances measured along a fixed straight line Ox (fig. 1) represent numbers of yards of cloth. Let distances measured along a straight line Oy at right angles to Ox represent numbers of yards of linen. Let a curve OE be drawn as follows :—N being any point upon Oy, let it be determined from a knowledge of the

[1] An examination of extraordinary circumstances in which this may not be true will be found in §§ 5, 6.

circumstances of England's demand for linen, what is the number of yards of cloth, the expenses of producing and exporting which will be covered annually by the proceeds of the sale in England of an amount of linen represented by ON. From Ox measure off OM, equal to this number of yards of cloth. Draw lines through M and N at right angles to Ox and Oy respectively, meeting in P; then P is a point on the required curve, OE. If N be moved from O gradually along Oy, P will assume a series of positions, each of which corresponds to one position of N; the continuous string of points thus formed will be the curve OE. [In other words, OE will be the locus of P.] If we were applying the method of diagrams to the trade that is actually carried on between two countries, we could not indeed obtain trustworthy data for drawing more than a limited portion of the curve. For it is not possible to conjecture with any approach to certainty what would be the terms on which it would be possible to sell in a country an amount of imports, either very much greater, or very much less, than that which is actually sold there. But for the purposes of the pure theory we are at liberty to suppose that the curve is properly drawn throughout its entire length. We may call OE "England's demand curve;" and bearing in mind that PM is equal to ON, we may describe it thus :—

England's demand curve is such that any point P *being taken on it, and* PM *being drawn perpendicular to* Ox *;* OM *represents the amount of cloth which England will be willing to give annually for an amount of linen represented by* PM.

In exactly the same way we may construct a curve OG which may be called Germany's demand curve, and which may be described thus :—

Germany's demand curve is such that any point p *being taken upon it and* pm *being drawn perpendicular to* Ox *;* pm *represents the amount of linen which Germany will be willing to give annually for an amount of cloth represented by* Om.

It may not be superfluous to state explicitly that the period for which the supplies of cloth and linen are reckoned is taken as a year only, for the purposes of definiteness and brevity. If the phrase "in a given unit of time" were not cumbrous, it might be substituted throughout for the word "annually."

The terms in which the curves are described imply that there is no change in the circumstances which govern the amount of linen that England is willing to take at each particular rate of interchange : and similarly that the circumstances which govern the German demand for cloth remain constant. As a matter of fact the causes which govern the demand of a country for foreign wares do vary from time to time. They are altered by

every change that affects her power of raising on the one hand the wares which she exports, and on the other domestic rivals to the wares which she imports; by almost every invention, and almost every change of fashion. But, as has been already said, we should aim at simplicity in our first approximations, in order that they may be easily manageable. Therefore, we are to neglect for the present all consideration of the disturbances arising from such variations; leaving account to be taken of them in the applications of the results of the pure theory to practical issues.

§ 5. We may now interpret into the language of curves the laws of international demand. The first proposition to be laid down requires no proof. It is that corresponding to every statement that can be made with regard to the terms on which England may be willing to export cloth in exchange for linen, there is a similar statement with regard to the terms on which Germany may be willing to import linen in exchange for cloth. Or in other words :—

The laws of the two curves are symmetrical.

PROP. I. *Every statement as to the shape which it is possible for OE to assume, has corresponding to it a similar statement as to the shape which it is possible for OG to assume ; but wherever Ox occurs in the former statement, Oy will occur in the latter, and vice versa; whenever reference is made to a horizontal straight line in the former, there must be made reference in the latter to a vertical straight line, and vice versa.*

If the reader should be unaccustomed to such a process of substitution, he may be helped to realize its validity, if he will draw any one of the figures that belong to the pure theory of foreign trade, with a broad pen on thin paper. He should then hold the paper between him and the light, with the reverse of the paper to him, with *Oy* horizontal, and *Ox* pointing vertically upwards. He will see through the paper, the two curves *OE* and *OG* with their places interchanged. Whatever proposition the figure has been used to prove with regard to *OE*, will now apply without any change or substitution to *OG*; when he has gone through this proof, he may turn the figure back again to its old position. He will observe that this proposition does not affirm that in any particular state of the trade, the shape of *OG* will be similar to the shape of *OE* : but only that whatever be the limits within which the possible variations in the shape of *OE* are confined by the fundamental laws of foreign trade, there exist precisely similar limits for *OG*.

Laws relating to the Nor-

It will suffice therefore to examine at length the laws which relate to the shape of *OE*. We may first lay down some laws which hold in the Normal Class and Class I., but not in Class II.

Let us suppose *N* to move from *O* along *Oy*, and let us watch the corresponding changes in the magnitude of *OM* and in the ratio of *ON* to *OM*. We find :—

mal Class and Class I.

PROP. II. *For the Normal Class and Class I.: if* P *be a point moving along* OE, *and*, PM, PN *drawn perpendicular to* Ox *and* Oy *respectively, every increase in* PM *is accompanied by an increase in the ratio of* PM *to* OM.

For the greater the amount of linen that has to be disposed of annually in England, the less will be the general purchasing power over which each yard of it will give command : and therefore, the less the amount of cloth that will be given in exchange for each yard of linen. The only exception to this is in the problems of Class II., in which an increase in the amount of cloth made for exportation may conceivably so increase the economy of its production as to enable a yard of cloth to be obtained by a less amount of general purchasing power than before. From this proposition we obtain at once,

PROP. III. *In curves of the Normal Class and of Class I. if* P *be any point in* OE, *every point in that portion of* OE *which is between* O *and* P *must lie below the straight line* OP; *and every point in the remaining portion of* OE *must lie above the straight line* OP *produced. Similarly if* p *be any point in* OG *every point in that portion of* OG *which is between* O *and* P *must lie to the left of the line* Op, *and every point in the remaining portion of* OG *must lie to the right of the straight line* Op *produced.* Hence we obtain at once,

PROP. IV. *If either of the curves belongs to the Normal Class or to Class I. it cannot cut twice any straight line through* O.

This result may be expressed in another form which will be more convenient for some purposes thus :

A geometrical expression for the rate of interchange.

Let *P* be a point such that *PM* being drawn perpendicular to *Ox*, *PM* is the amount of linen which Germany is actually sending to England at any time in exchange for *OM* cloth. (We shall hereafter (see Ch. II.) call this point *P* the "Exchange-index.") Then the rate of interchange is indicated by the ratio between *PM* and *OM*. This ratio will be constant whatever position *P* may have on any given straight line through *O*. So that the rate of interchange is determined by the magnitude of the angle which the straight line joining *P* and *O* makes with *Ox*[1]: the greater this angle is, the more advantageous the

[1] It is measured mathematically by tan *POx* from the point of view of England, and by cot *POx* from the point of view of Germany. The mathematical reader will observe that in the Normal Case and in Case I. the curves may have points of contrary flexure. That is, if $y = f(x)$ be the equation to *OE*, $f''(x)$ may change sign at any point of the curve. But $\frac{d}{dy}\left(\frac{y}{x}\right)$ must remain positive :

rate of interchange is to England, and the less advantageous it is to Germany. Therefore Prop. IV. may be put in the form,

PROP. IV. COR. *If the demand curve of a country belong to the Normal Class or to Class I., the amount of foreign wares which she will import is determined when the rate of interchange is known.*

Again, in the Normal Class and in Class I. when the amount of linen offered for sale in England is very small, it will be disposed of on terms advantageous to Germany, so that the amount of cloth exported in exchange for it will be proportionally large. Thus where PM is small, the ratio of PM to OM is small: and a point moving from O along OE will keep at first close to Ox. So a point moving from O along OG will keep at first close to Oy.

It can hence be inferred, or it can be proved directly from Prop. IV., that—

PROP. V. *In the Normal Class and in Class I. that portion of OE which is adjacent to O lies below that portion of OG which is adjacent to O.*

Thus we may not invert the positions which OE and OG have in fig. (1) in the neighbourhood of O.

Under Class II. we shall have to discuss the forms which OE may assume if the production of cloth on a large scale for exportation renders possible important economies that would otherwise be impossible. But however extensive these economies may be, they cannot cause the total expenses of producing any given amount of cloth to be less than the total expenses of producing a smaller amount. Hence the general condition of the arts of production being assumed, we know definitely the expenses of producing any given amount of cloth in England for exportation.

Therefore OE cannot bend downwards towards Ox after the manner of the curve in fig. (2). For if OE could assume a shape such that a horizontal line AB could be drawn cutting it in A and B; then, AC and BD being drawn perpendicular to Ox, the shape of the curve would imply the following statement:—AC linen is just capable of being sold for the expenses of producing OC cloth: and also BD linen (which is the same as AC linen) is capable of being sold for the expenses of producing OD cloth. But this is impossible. Thus we obtain a fundamental law which is valid for the Normal Class and for

i.e. $x - y\dfrac{dx}{dy}$ must remain positive; i.e. every straight line which touches OE must cut Ox to the right of O. There is an obvious geometrical proof of this result.

Classes I. and II. and is the only law to which the curves must conform under all circumstances : viz.

PROP. VI. *OE cannot in any case be cut more than once by a horizontal line. Similarly OG cannot in any case be cut more than once by a vertical line.*

§ 6. Let us next investigate the laws which bind the curves if they belong to the Normal Class, but not if they belong to Class I. For the Normal Class, but not for Class I. it is assumed that every increase in the amount of linen offered for sale annually in England increases the total proceeds of the sale, and consequently increases the amount of cloth that is exported in exchange for it. That is to say: if from N any point in Oy, NP be drawn at right angles to Oy to meet the curve OE in P, then the greater be ON the greater also is NP. But in Class I., as N moves from O along Oy the increase in ON though it is at first accompanied by an increase in NP, yet when N arrives at a certain point (V in fig. 3) NP ceases to increase and begins to diminish, and the curve bends round towards Oy. These and corresponding results may be put in the following convenient form :

PROP. VII. *In the Normal Class OE cannot cut the same vertical line more than once : but it may in Class I. So in the Normal Class OG cannot cut the same horizontal line more than once; but it may in Class I.*

In fig. 3 the curves cut one another only in one point; but consistently with the conditions of Class I. they may cut one another several times, as represented in fig. 4. It may be well formally to prove that—

PROP. VIII. *In the Normal Class OE and OG cannot cut one another in more than one point (besides O).*

Let A be a point of intersection of the curves (see fig. 1); then AE must lie entirely above OA produced, by Prop. IV.; and AG must lie entirely to the right of OA produced: consequently AE and AG cannot cut again. Nor can AE cut the portion of OG which lies between O and A. For by Prop. VI. the portion of OG between O and A must lie entirely to the left of a vertical straight line through A; and by Prop. VII. AE must lie entirely to the right of this straight line. Similarly AG cannot cut the portion of OE which lies between O and A. Therefore OE and OG cannot meet except in O and A.

PROP. IX. *Every point in which the two curves cut one another corresponds to an equilibrium of the trade.*

Let AH, BK, CL be drawn perpendicular to Ox. Then since A is a point on OE, AH linen can be sold annually in the English market for a price which will just cover the expenses of producing (and exporting to Germany) OH cloth: and since A is a

point on OG, OH cloth can be sold annually in the German market for a price which will just cover the expenses of producing (and importing to England) AH linen. That is, when OH cloth is exchanged for AH linen, there is no force present either to increase or diminish England's exports or imports: trade is in equilibrium. A precisely similar proof shews that trade is in equilibrium when OK cloth is exchanged for BK linen. In the following chapter it will be proved that the equilibrium of the trade is *stable* in each of the positions represented by A in fig. (3) and by A and C in fig. (4): but that it is unstable in the position represented by B in fig. (4). The possibility of more than one position of equilibrium in such cases as this has been noticed by Mill. His treatment of the matter is certainly inadequate: for he has failed to discover the laws which determine whether any particular position of equilibrium is stable or unstable. It is, generally speaking, true of Mill as of Adam Smith, that much of his work which appears at first sight to contain error, proves itself on further investigation to be only incomplete or incompletely expressed. This is however one of the few instances in which careful study has failed to convince me that Mill's work is right as far as it goes. The reader who may care to inquire into this matter is referred to the Note at the end of the present chapter.

§ 7. We may proceed to the discussion of problems of Exceptional Class II. The case does not yet appear to have much direct bearing on questions relating either to the trade that is actually carried on between existing countries, or to the terms on which any compact industrial group is able to sell its wares or its services. But it claims attention on the ground that it is not logically excluded by the hypothesis on which the pure theory of foreign trade has been constructed since the time of Ricardo. Moreover history shews that the practical applications of the work of pure science have in general been discovered after, and not before, that work was done; advances in that applied knowledge which gives us direct command over nature have never been made with rapidity except when men have been willing to expend some pains on completing the solution of problems suggested to them by pure science, even although the practical purposes which the various portions of their work would subserve could not be discovered beforehand. Finally no great amount of additional trouble will be involved in working out this exceptional case.

This case has its origin in the fact that the wares which a country exports may be such that the difficulty of producing them diminishes very rapidly when their amount increases. It is indeed true, as has been said, that in general the production

of a commodity on a large scale for home consumption precedes the development of any considerable foreign trade in it. Still the extent to which division of labour in the production of it can be carried, is enlarged by every extension of the foreign markets for it. For instance, there exist in England large groups of works each of which groups is filled with expensive machinery that is adapted exclusively for making the special machinery that is required in some one class of manufactures, and the growth of such works has been very greatly promoted by foreign trade. Adam Smith mentioned as one of the chief advantages of foreign trade that "by means of it the narrowness of the home market does not hinder the division of labour in any particular branch of art or manufacture from being carried to the highest perfection." And it is certain that in the century which has followed the publication of Adam Smith's work England's export trade has exerted a quiet but constant influence in developing broad inventions and economies in manufacture. These have benefited foreign countries in the first instance by causing England to sell them her manufactured goods on cheap terms, and in the second instance by passing over to those countries and assisting them to manufacture for themselves.

Thus it is possible, to revert to our old hypothesis, that an increase in Germany's demand for English cloth may to so great an extent develope the facilities which England has for producing cloth as to cause a great and permanent fall in the value of cloth in England. It is true that in order to obtain this cloth Germany will have in general to force a sale here for an increased amount of her own products, and consequently to lower their price. But it is conceivable that under exceptional conditions the increase in the amount of English cloth required for exportation to Germany may cause an increase in the economy of producing cloth so rapid and extensive that the fall in the price of cloth in England may be greater than the fall in the price of linen. Thus it is possible that an increase in Germany's demand for English cloth may cause each yard of linen to be sold here on such terms as to give command over a larger amount of cloth than before; it is possible that an increase in Germany's demand for English cloth may cause her to obtain an import of English cloth increased *in a greater ratio* than is her export of linen to England.

The introduction of the economies which were requisite in order to render possible such cases as this on a large scale have seldom been effected within a short space of time. The lapse of generations has been required for that development of England's invention and economies in manufacture which was

above attributed in part to her export trade. And the practical importance of such cases as have occurred on a somewhat small scale is in general less than at first sight appears. Let us examine one such case. The agricultural implements which England makes for herself are not always adapted for use in countries where the population is sparse. Eastern Europe wants field steam engines in which straw can be used as fuel; she wants mowing and reaping machines that can be used on uneven ground. Special knowledge, special skill and special machinery are to a greater or less extent required for the manufacture of these implements. For some time England played a very poor part in the work; partly because she had to compete with America who had organised this manufacture for her own market. At length the steady increase in the volume of the demand for these implements is enabling Englishmen to produce them with rapidly increasing economy. But their present success arises in great measure from their having had experience in the manufacture of wares of similar kind; and the main body of the work in which this experience has been obtained, is directed to the supply of the home market.

Let us proceed to interpret problems of Class II. in the language of diagrams. Let P, Q (fig. 5) be two points on $O. E.$ such that PM and QR being drawn perpendicular to Ox, QR is greater than PM. We found that in the Normal Class and in Class I. the ratio of QR to OR must be greater than the ratio of PM to OM (Prop. II.). But in Class II. it is possible for an increase in the amount of cloth produced in England so to diminish the expenses of producing each yard, that an increase in the amount of linen imported although it will cause the value of each yard of linen in England to fall, may yet cause each yard of linen to give the means of purchasing a greater amount of cloth than before: so that the exports of cloth increase not in a less ratio, but in a greater ratio than the imports of linen. So that the ratio of QR to OR may be less than the ratio of PM to OM. Hence,

PROP. X. *In Class II. the curves do not necessarily conform to the laws which are enunciated in Prop. II. III. IV. V. and VI. as valid for the Normal Class and for Class I.*

Thus for instance OE and OG may lie as in fig. 5; and may cut each other at A, B and C. The proof given in Prop. IX. that every point of intersection of the curves corresponds to a position of equilibrium of the trade applies to this case. It will be proved in the next chapter that A and C correspond to stable, and B to unstable, equilibrium. It must be remembered that it has been proved in Prop. VI. that in no case whatever can OE cut any horizontal line twice, nor can OG cut

any vertical line twice. It is possible for OE in fig. 5, ultimately to bend back towards Oy, as does the dotted portion CD, if it happen that a very large amount of linen is incapable of being sold in England except on terms extremely advantageous to England.

The reader may exercise his fancy by drawing various forms which the curves may have consistently with the fundamental laws that have been laid down, and combining them in pairs so as to observe their possible points of intersection. After reading the next chapter he may interpret the points of intersection; of course the positions of the curves in fig. 5 are capable of being inverted. They would then represent a case in which the trade between the two countries could not grow up gradually; but could be carried on with profit to both if it were once started on a large scale by any external cause.

NOTE ON MILL'S TREATMENT OF AN EXCEPTIONAL CASE.

In § 6 of Ch. XVIII. of Book III. Mill attempts to deal with difficulties in the theory of foreign trade, of which a solution is offered in the Examination of Class I. in the present Essay. He has seen that under certain circumstances there may be several different positions of equilibrium of trade : so that the problem arises of determining at which of these several positions the trade will remain. Mill has undertaken to illustrate by an example the method in which this general problem may be solved. But it appears to me that the special example which he has chosen does not illustrate the general problem in question. For I understand him to mean that the amount of cloth which England will expend on the purchase of linen is a given quantity, independent of the rate of interchange, say OV; and that the amount of linen which Germany is willing to expend in the purchase of cloth is a given quantity ; say OW. On this hypothesis the trade has only one possible position of equilibrium; viz. that in which OV cloth is exchanged for OW linen. Mill has proved, what indeed is obvious, that the division of the total benefits of the trade between the two countries depends upon the relative magnitudes of OV and OW.

Mill's example may be represented in a diagram thus. Draw (fig. 6) VPQ and WRS at right angles to Ox and Oy respectively, cutting one another in A. Let VP be the amount of linen which England could make for herself with the expense to which she is put in order to make and export OV cloth, then PQ is a portion of England's demand curve, which in this case has "degenerated" (in mathematical phrase) into a straight line. Similarly if WR be the amount of linen which Germany could make for herself with the

expense to which she is put in order to make and export OW linen, then RS is a portion of Germany's demand curve. These two straight lines PQ and RS cannot intersect in more than one point. Mill's example therefore does not afford any aid towards the solution of the class of problems which are suggested by the intersections of the curves in figure 4. With regard to division of the benefits of the trade between the two countries it may be remarked that if A coincides with P England has to pay for her imported linen the full equivalent of what it would cost her to make it herself; and therefore she derives no benefit from the trade. So if A coincides with R, Germany derives no benefit from the trade. The further A is above P, the greater is the benefit that England derives from the trade : the further A is to the right of R, the greater is the benefit that Germany derives from the trade. Of course, by the conditions of the problem, A cannot lie below P, or to the left of R.

CHAPTER II.

STABLE AND UNSTABLE EQUILIBRIUM OF FOREIGN TRADE.

§ 1. It will be convenient to have a name for the point Some which corresponds to the actual position of the trade between technical England and Germany at any time. It generally happens in terms are fact that the exports and imports of a country are not distri-buted evenly all over the year. Allowance must be made for these irregularities before the results of the pure theory can be applied to practice. But for the purposes of the pure theory it is allowable to assume that the importation and the consump-tion of foreign wares is distributed evenly all over the year.

Thus we may say that cloth is at any time being imported The scale into Germany on the scale of OM annually (or in a given unit of import-ation. of time); meaning thereby that the scale on which it is being imported is such that if it were to continue, the amount im-ported in the year (or unit of time) would be OM.

We have then the following :—

DEFINITION. If at any time cloth be exported from England Definition on the scale of OM annually, in exchange for linen on the scale of Ex-change-index. of ON annually; and MP, NP be drawn at right angles to Ox, Oy respectively, meeting in P; then P is the exchange-index at that time.

It has been proved in Prop. IX. that the trade is in equili-brium when the exchange-index is at any point of intersection of OE and Oy. In the present chapter it will be shewn that some points of intersection correspond to stable equilibrium of the trade and others to unstable : and a fundamental law will be laid down by which the one set may be distinguished from the other. It will be convenient to commence by supposing that the exchange-index is not at A : but that some external disturbing force, as a war, or a bad harvest, has jerked the exchange-index to some position such that the trade corre-sponding to it is not in equilibrium; and to investigate the forces which will govern its motion.

We know from Prop. VI. that OE cannot cut a horizontal straight line through P more than once : and that OG cannot

cut a vertical straight line through P more than once. We may have therefore the following

Definition of the phrases, "to the right of OE," "above OG."

DEFINITION. A point P is said to be *to the right* or *to the left* of OE according as it is to the right or the left of the point in which OE is cut by the horizontal straight line through P: and the point P is said to be *above* or *below* OG according as it is above or below the point in which OG is cut by a vertical straight line through P.

§ 2. The greater part of the pure theory of foreign trade consists of a series of corollaries from the laws with regard to the shapes of OE and OG, which were laid down in the last chapter, together with the following law:—

The law of the forces which control the movement of the Exchange-index.

PROP. XI. *If the Exchange-index be at any time to the right of OE it will tend to move to the left; if it be to the left of OE it will tend to move to the right. Similarly, if the Exchange-index be at any time above OG it will tend to move downwards; if it be below OG it will tend to move upwards.*

Such interpretation as this proposition may require will be contained in the proof of it. It must be remembered it is assumed throughout that the export trade of each country is conducted by private traders competing against one another. So that when the terms on which a country's foreign trade is conducted are such as to afford a rate of profits higher than the rate current in other industries, the competition of traders to obtain these higher profits will lead to an increase in the exportation of her wares: and *vice versa* when the rate of profits in the foreign trade are exceptionally low.

Let the exchange-point P be to the left of OE, as in fig. 7, and let NP produced cut OE in Q. Then since Q is a point on OE, ON linen is capable of being disposed of annually in England in exchange for the means of producing and exporting NQ cloth. But at the time in question linen is being imported on the scale of ON annually, and cloth is being exported in exchange for it on the scale of only NP annually. Consequently the exportation of cloth in exchange for linen must be a trade which affords abnormally high profits. Consequently, since competition in the trade is supposed to be free, the exportation of cloth will increase. Therefore when the exchange-index is to the left of OE it will tend to move to the right. So if the exchange-point lay at P' in NQ produced, it would shew that cloth was being exported at the rate of NP' annually in exchange for an amount of linen ON, which could be disposed of in England only for the expenses of producing and exporting NQ cloth: consequently the exportation of cloth would tend to diminish, *i.e.* when the exchange-point is to the right of OE, it will tend to move to the left.

Similar proofs apply to the second part of the proposition which relates to OG.

In order therefore to determine the directions in which the amounts of the exports of cloth and linen are tending to change at any time, it is requisite only to determine the position of the exchange-index at that time, and through it to draw arrowheads—an arrowhead pointing towards the right if the exchange-index lies on the left of OE, towards the left if this point lies on the right of OE; and an arrowhead pointing upwards if the exchange-index lies below OG, downwards if this point lies above OG[1].

The exchange-index will in each case tend to move in some direction within the angle made by the arrowheads. Thus, if the exchange-index be at P (fig. 7), it will tend to move in some direction lying within the angle RPQ. So that, unless some external event should arise to disturb the trade relations between the two countries, the exchange-index must soon strike either OE between Q and A, or OG between R and A. But, as we cannot tell the relative magnitude of the horizontal tendency along PQ, and of the vertical tendency along PR, we cannot predict which of the two curves it will strike first. Suppose it strike OE first: when it is on OE there will be no force tending to make it move either to the right or to the left. But there will be a force attracting it upwards. It will therefore tend to oscillate along QA towards A. For we may use this brief phrase to express the fact that the exchange-index will not necessarily remain on QA during the whole of its motion to A, but may oscillate first on one side of QA and then on the other: under the action of the forces which urge it to the right whenever it is to the left of OE, and to the left whenever it is to the right of OE. It will, however, unless its movements be disturbed by some powerful cause extraneous to the ordinary circumstances of the trade, in general adhere somewhat closely to QA. It will be convenient also

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[1] Thus the motion of the exchange-index is in every respect similar to that of a material particle moving freely under the action of forces which attract it towards OE and OG. Suppose OE to be a rigid wire which exerts attractions only in a horizontal direction and always towards the right when the particle is, according to the definition in the text, on the left of OE, and *vice versa*. Similarly suppose OG to be a rigid wire which exerts attractions only in a vertical direction, and always upwards when the particle is, according to the definition in the text, below OG, and *vice versa*. Then this particle will move exactly in the same manner as does our exchange-index, so that if we chose to assign to these horizontal and vertical forces any particular laws, we should obtain a differential equation for the motion of the exchange-index. This equation when integrated would give us the path which on this particular supposition the particle would describe. Such calculations might afford considerable scope to the ingenuity of those who devise mathematical problems, but as we shall see further on (§ 6) they would afford no aid to the Economist.

to place at each of several points on the curve an arrowhead, to indicate the direction in which the exchange-index, if at that point, would be made to oscillate along the curve on which it is by the force exerted on it by the other curve. Similarly, if the exchange-index moving from P had struck the curve OG first, it would have oscillated along RA towards A.

Exactly in the same way it may be proved that if the exchange-index were at any time at P' it would be impelled by the forces acting on it to move upwards to the left: that if it struck OE first it would oscillate along QA towards A; and that if it struck OG first it would oscillate along GA towards A. And similarly for the points P'' and P'''.

Finally, if the exchange-index coming towards A shoot beside it or beyond it in any direction, or if the exchange-index be displaced by any disturbing event from A in any direction, the forces acting upon it will bring it back to OE or OG, and cause it to oscillate along that curve which it strikes first toward A.

§ 3. It will be convenient to speak of the equilibrium of the trade between England and Germany corresponding to a point of intersection of OE and OG as the equilibrium at that point. We may now give a formal

Definition of Stable and Unstable Equilibrium.

DEFINITION. The equilibrium at a point of intersection of OE and OG is *stable*, provided that when the exchange-index strikes either of the curves in the neighbourhood of that point, the forces acting on the index tend to make it oscillate along the curve *towards* that point. In other cases the equilibrium is *unstable*.

It will be seen hereafter that the equilibrium at every point in which OE and OG cut one another, if it is unstable for displacements in any direction, is unstable for displacements in every direction. But this result does not hold of points in which the curves meet but touch without cutting one another.

We may now enunciate the fundamental rule for deciding whether any particular point of intersection of the curves corresponds to a stable or to an unstable equilibrium of the trade. But, in order that this may be given in a convenient form, it is necessary to have some handy means of distinguishing the various directions in which different parts of the curves may lie.

If a point moves from O along OE in fig. 8, it at first increases its distance from Oy at the same time that it increases its distance from Ox. It continues to do so until it arrives at R when the direction of the curve is vertical. If the point continues its motion from R onwards to C and B, it will continue to recede, but it will approach towards Oy. It will be

convenient to express the difference between the portions of OE by saying that between O and R the curve is inclined *positively;* and that from R to B, and for some distance beyond B, the curve is inclined *negatively*. Or more generally:—

Whatever portion of a curve lies in such a direction that a point, which moves along it so as to recede from Ox, recedes also from Oy; that portion of the curve is said to be *inclined positively*. Conversely, whatever portion of a curve lies in such a direction that a point which moves along it so as to recede from Ox approaches Oy; that portion of the curve is said to be *inclined negatively*.

Using these terms we may enunciate

PROP. XII. *The equilibrium is stable at every point of intersection of* OE *and* OG, *excepting those at which both curves are inclined positively, but* OG *is more nearly vertical than* OE, *and excepting those at which both curves are inclined negatively, but* OG *is more nearly vertical than* OE.

In accordance with this Proposition, the equilibria at A and C in each of the figures 8 and 9 (which are repetitions of figs. 4 and 5 respectively) are stable, and the equilibrium at B in each of these figures is unstable, as has been already indicated. The most convenient mode of establishing this Proposition is perhaps to draw a number of figures representative of every position in which the curves can lie at a point of intersection. Arrowheads should then be inserted to indicate, in conformity with Prop. XI., the directions of the forces which would act upon the exchange-index at different points in the figures, so as to exhibit the motion of the exchange-index.

If through B in fig. 8 there be drawn the straight lines TBU from left to right, and VBW vertically upwards, then, if the exchange-point be displaced to a position within the quadrant TBW, it will tend to move to A. If displaced to a position within the quadrant VBU it will tend to move to C. If displaced to a position in either of the quadrants TBV, WBU, it will tend to move to A or C, according to whether the forces acting upon it bring it into the quadrant TBW, or into the quadrant VBU. In this last case it is just possible that the exchange-index may on its way back strike B. This possibility is worthy of note. But the motion of the exchange-index is not likely to be arrested at B; and if disturbed from B ever so little along either of the curves it would tend to move off to A or C. Therefore it is not inaccurate to describe the equilibrium at B as unstable. Indeed precisely analogous cases occur in Mechanics. A body displaced from equilibrium may pass through a position of unstable equilibrium on its way

towards a position of stable equilibrium. Similar remarks apply to the unstable equilibrium at B in fig. 9.

The informal proof of the proposition that has already been suggested might perhaps suffice. But it seems advisable to indicate the manner in which a formal proof of it may be given.

Let then D be any point of intersection of OE and OG. Let horizontal and vertical straight lines TDU, VDW be drawn as in fig. 10.

Firstly let England's curve be inclined positively at D: let it point at D in the direction of the straight line eDE. Then will the equilibrium be stable provided that at D Germany's curve either (i) be inclined positively but make a greater angle with the vertical than eDE does, pointing at D for instance in the direction of gDG; or (ii) be inclined negatively, and pointing at D for instance in the direction of $g'DG'$: or in other words provided that Germany's curve lie within the angles eDW, EDV.

For suppose the exchange-index to strike OE just below D, then it must be below OG, whether OG lie in the direction gD or $g'D$; because eD lies below both gD and $g'D$: therefore it must be attracted upwards. Therefore the arrowhead on eD must point towards D. So it may be proved that the arrowhead on DE points towards D: and that the arrowheads on gD and DG, and on $g'D$ and DG' all point towards D. Which proves that under the stated condition D is a point of stable equilibrium.

In exactly the same manner it may be proved that the equilibrium at D will be unstable if while England's curve lies at D in the direction eDE Germany's curve is positively inclined and makes a smaller angle with the vertical than eDE does, and lies therefore in the angles eDV, WDE.

In the same manner also it may be proved that if OE is inclined negatively at D, the equilibrium at D is stable unless OG be inclined negatively at D and be more nearly vertical than OE is, which completes the proof of the Proposition.

§ 4. It may promote clear conceptions with regard to the drift of the above reasoning if some portion of it be expressed directly in terms of the motives which govern the exportation of cloth and of linen. Let us take for this purpose the case in which the exchange-index has been jerked by some disturbance from OC to the point P in fig. 8 within the loop BC. This may mean that some abnormal event such for instance as a passing difficulty in the English cloth producing trade, has checked the supply of cloth, so that cloth is imported into Germany on the scale of OM yards annually instead of OL annually. Although Germany would be willing permanently to purchase this amount

only by giving linen in return for it on the scale of QM yards annually; yet being taken by surprise, and unprovided with a substitute for cloth, or for some other transitional cause she pays for it on the scale of PM yards of linen annually. Let us then inquire what tendencies there will be, as soon as the disturbance is past, to increase or diminish the scales on which cloth and linen are sent from one country to the other.

Let us look first at Germany's side of the case. As soon as the disturbing causes have ceased to operate, cloth imported on the scale of OM yards annually will be capable of being disposed of in Germany only on terms so disadvantageous to England as not to enable linen to be exported in exchange for it on a scale as great as that of PM annually. Consequently those who export linen from Germany will find it unprofitable to carry on an extensive trade until they are able to obtain cloth on more favourable terms of interchange. Therefore there will be a diminution in the scale on which linen is exported from Germany.

England's side of the case is the reverse of this. Linen imported on the scale of PM annually will be capable of being disposed of in England on terms which will enable cloth to be exported in exchange for it on the scale of more than OM annually. Consequently the exporters of cloth from England will find that their trade affords at the present rates of interchange abnormally high profits. These traders are supposed to act not in combination, but in free competition with one another; so that each of them will strive to obtain for himself as large a share as possible of this profitable trade and will push the sale of his cloth to Germany even if in order to do so he should be compelled to submit to a slight reduction of the price on which he disposes of it. Therefore there will be an increase in the scale on which cloth is exported from England. That is to say the exchange-index will move from P downwards to the right until it strikes OE or OG. Suppose it to strike OG first in the point F. At this time cloth is being imported from England on the scale of OZ annually, and linen exported in exchange for it on the scale of FZ annually: and with this state of the trade Germany is just satisfied. The terms on which cloth can be sold in Germany are just sufficient to sustain the trade in this position. But linen imported into England on the scale of FZ annually can be disposed of there on terms more than sufficiently advantageous to cover the expenses of exporting cloth on the scale of OZ annually: consequently the exportation of cloth will continue to increase. So long as the exchange-index remains on OG the only force tending to change its motion will be a horizontal force to the

right. But if the index falls below OG the exporters of linen from Germany will have an inducement to extend their sales to England; and *vice versa* if the index rises above OG they will at once contract their sales to England. Whereby the index will be compelled to oscillate along OG towards C. So if the index had struck OE first, it would have been compelled to oscillate along OE towards C. Therefore the equilibrium at C is stable.

In the course of the proof of Prop. XII. it was proved implicitly that if at a point of intersection of the two curves, the equilibrium was stable for displacements in any one direction it was stable for displacements in all directions: and similarly for unstable equilibrium. Of course these results are capable of an easy independent proof.

But if the curves touch without cutting one another, those arrowheads on the curves which are on one side of the point of contact will be directed towards that point and those which are on the other will be directed away from it as in figure 11. In fact the position of OE in this figure is obtained from the position which it has in fig. 8 by pressing it downwards so that the two points of intersection B and C in fig. 8 run together to make the point of contact D in fig. 11. So that D is really two coincident points of intersection one of which corresponds to stable and the other to unstable equilibrium.

Of course, since disturbances of equilibrium occur in every direction, a point at which equilibrium is unstable for displacements in any direction is a point at which trade cannot rest and therefore has no practical importance. An investigation of the many various conditions under which the curves may touch one another will afford to the reader some curious amusement; but so far as at present appears, it is devoid of any practical utility.

§ 5. PROP. XIII. *If from a point of intersection of* OE *and* OG *at which the equilibrium is stable we proceed along either of the curves in either direction until we arrive at another point of intersection, this second point must be one of unstable equilibrium, and vice versa.*

This proposition is obviously true. For if we proceed from a point of intersection along that portion of OE which lies above OG; and place arrowheads on OE on our way, these must all point downwards until we come up to the next point of intersection, therefore that point of intersection is unstable. And a precisely similar proof applies, mutatis mutandis, to every other case. It may be an interesting exercise to attempt to draw diagrams in which one of the curves shall be represented as passing through two points of stable equilibrium consecutively, or through two points of unstable equilibrium; and to

notice how each attempt is foiled by the necessity of conforming to the fundamental laws of the curves. Of course the Proposition is capable of a direct geometrical proof.

It was proved in Prop. V. that if OE belong to the Normal Class or Class I., that portion of OE which is adjacent to O lies below that portion of OG which is adjacent to O. Therefore arrowheads placed on OE in the neighbourhood of O must point upwards and those placed on OG must point to the right. Therefore the first point of intersection at which we arrive if we proceed along either of the curves from O must be a point of stable equilibrium. In other words, O is a point of unstable equilibrium if both the curves belong to the Normal Class or to Class I. But if either of them belong to Class II. O may be a point of stable equilibrium, and the first point of intersection at which we arrive when we pass along either of the curves from O may be a point of unstable equilibrium.

In this last case the total number of points of intersection (O not being included,) will be two or some other even number. But in every other case the total number must be one or three, or some other odd number. For it is obvious that if we proceed from O along either of the curves, the last point of intersection that we arrive at must be one of stable equilibrium.

§ 6. We have seen how the position of the exchange-index relatively to OE and OG determines the directions of the horizontal and of the vertical force which act on it: but there are no general laws by which the magnitude of each of these forces can be determined. Therefore even if we knew exactly the shapes which the curves assumed in any particular problem, we should not have data on which to base a calculation of the precise path which the exchange point would describe[1].

The task of discovering laws by which the shapes of the curves may in any case be approximately determined does not appear to transcend the resources which the science of statistics at present affords us. It will indeed, be a long time before this task is achieved: when it is achieved, it may be worth while to

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[1] For the mathematical functions introduced into the original differential equation could not, in the present condition of our knowledge, be chosen so as to represent even approximately the economic forces that actually operate in the world. And by integrating them we should move further away from, instead of approaching nearer to the actual facts of life. For this reason, among others, the method of diagrams seems to me to be generally speaking of greater use to the Economist, than the methods of mathematical analysis. For when using the former method we have continually before us those assumptions which are justified by economic facts, and no others. Whereas the use of mathematical analysis has been found to tempt men to expend their energy on the elaboration of minute and complex hypotheses, which have indeed some distant analogy to economic conditions, but which cannot properly be said to represent in any way economic laws.

hand over the curves to be manipulated by the processes of analytical mathematics: but until then, the mathematical treatment of the curve cannot lead us to any results which cannot be at once obtained from inspection of the diagrams. Even then the methods of mathematical analysis will not be able to afford any considerable assistance in the task of determining the motion of the exchange-index. For a large amount of additional work will have to be done before we can obtain approximate laws for representing the magnitude of the horizontal and vertical forces which will act upon the exchange-index in any position.

Finally, even when this is done there will yet remain a further difficulty in the way of the mathematical treatment of the problem. It is necessary to inquire with considerable care into this difficulty; because it extends so far as even to impair to some extent the efficiency of the treatment of the problem by the method of diagrams.

§ 7. It has been remarked, that in economics every event causes permanent alterations in the conditions under which future events can occur. This is to some extent, the case in the physical world, but not to nearly so great an extent. The forces that act on a pendulum in any position are not to any appreciable extent dependent on the oscillations that the pendulum has already made. And there are many other classes of movement in the physical world, which are exact copies of movements that have gone before. But every movement that takes place in the moral world alters the magnitude if not the character of the forces that govern succeeding movements. And economic forces belong to the moral world in so far as they depend upon human habits and affections, upon man's knowledge and industrial skill. Where, for instance, any casual disturbance increases the amount of English wares of any kind that are consumed in Germany it leaves behind it a permanent effect in an increased familiarity on the part of German consumers with English wares; and in this and other ways occasions permanent alterations in the circumstances of demand. An alteration of the shape of Germany's demand curve is rendered necessary by any change which alters the amount of German wares that can be exported annually with the proceeds of the sale in Germany of any given amount of English wares. Consequently, every movement of the exchange-index entails some alteration in the shapes of the curves, and therefore in the forces which determine its succeeding movements. If the curves belong to the Normal Class, or to Class I., the alterations thus required are not likely to be extensive. At all events, the general character of the curves will seldom be changed: and though the positions

of equilibrium may be slightly shifted; the general tenor of the reasonings that have been based on the assumption that the shapes of the curves remain rigid and unchanged, will not be thereby invalidated.

But these reasonings may be frequently invalidated if either of the curves belongs to Class II. For suppose that an increase in the amount of cloth produced for exportation leads to the introduction of extensive economies. Such economies when they have once been obtained are not readily lost. Developments of mechanical appliances, of division of labour, and of organisation of transport, when they have once been effected are not readily abandoned. Capital and skilled labour which have once been devoted to any particular industry, may indeed become depreciated in value when there is a falling off in the demand for the wares which they produce; but they cannot quickly be converted to other occupations. So that for a time their competition will prevent a diminished demand from causing an increased price of the wares.

Thus for instance the shape of OE in fig. 12 implies that if cloth were produced for exportation on the scale of OU annually, the economies introduced into its production would be so extensive as to enable it to be produced and exported for a total price which would be covered by the sale in England of linen on the scale of TU annually. If these economies were once effected the shape of the curve would probably cease to represent accurately the circumstances of England's demand. The expenses of production, for instance, of OV cloth would no longer be much greater proportionately than those of OU cloth: so that cloth on the scale of OV annually could be produced and exported by means of the proceeds of the sale of linen imported on a scale considerably less than that of RV. Thus in order that the curve might again represent the circumstances of England's demand it would be necessary to draw it lower down; possibly so much lower as to make it fall in the position of the dotted curve in the figure, so as to have only one point of intersection with OG. And generally if the circumstances of the production of cloth are such that an increased production of it for exportation, within certain limits, cause greatly increased economies in its production; then the curve between these limits will require some special treatment. For it can be taken to represent the conditions of England's demand only before and up to the occurrence of any event which renders it profitable to produce cloth on a large scale for a time sufficiently long for the introduction of these economies. After the occurrence of such an event, the curve must be, partially at least, re-drawn. Thus if at a point just to the right of this portion

of the curve there be drawn, in accordance with the rules laid
down, an arrowhead pointing to the left; this arrowhead will
indicate a resistance that must be overcome before the ex-
change-index can move to this point.   But if by any means the
exchange-index is brought to this point, the existence of the
arrowhead will not justify us in assuming without investigation
that in the corresponding practical problem there will be in
operation a force tending to make the exchange-index move
toward the left.   Conclusions based upon the assumption of the
rigidity of the curves may be applied to practical problems
coming under Class II. in so far as the conclusions relate to the
resistances which must be overcome before there can be effected
an increase in the scale on which cloth or linen is exported:
but not in so far as they relate to the forces which may operate
to diminish this scale.   Therefore the account of positions of
unstable equilibrium which has been deduced from an exami-
nation of the curves in Class II. may not be applied to practical
problems generally until a careful inquiry has been instituted
in each particular case as to the probability that economies
which had once been introduced, would be quickly lost.   It is
chiefly for this reason that as has already been said the results
obtained from the curves in Class II. are of less importance than
those obtained in the Normal Class and in Class I.   But though
they cannot so far as at present appears be largely used for the
immediate deduction of conclusions in matters of practice, there
seems to be large scope for the use of them in the suggestion of
new practical problems.

fig. 8

fig. 10

fig. 20

fig. 21

fig 9

fig. 12

fig. 22

fig 23

fig. 11

fig. 13

fig 22 A

fig. 24
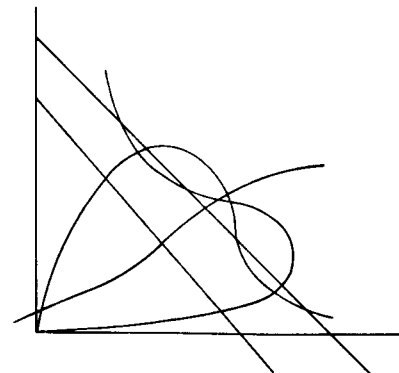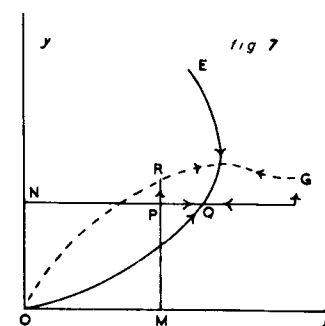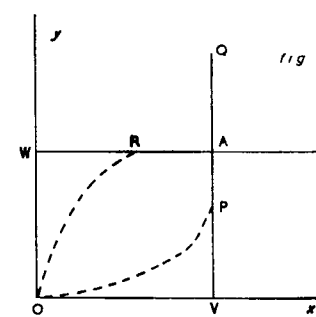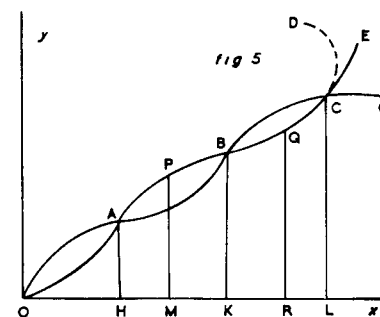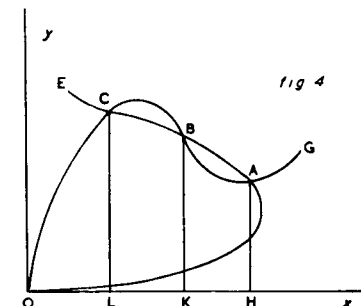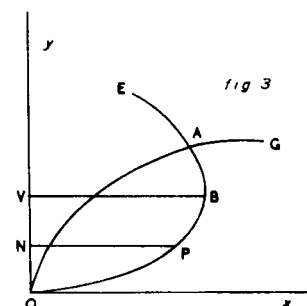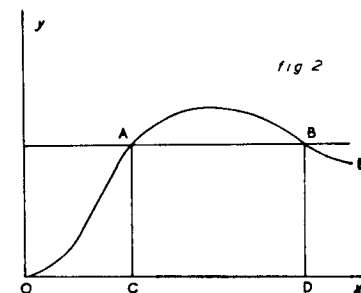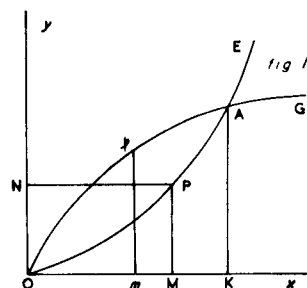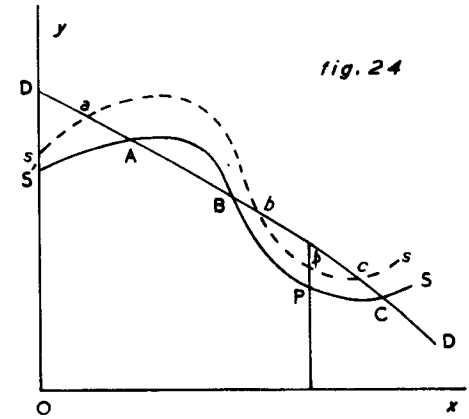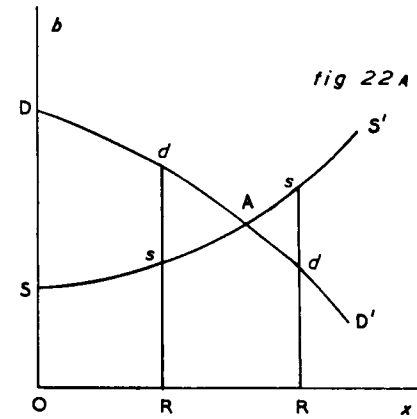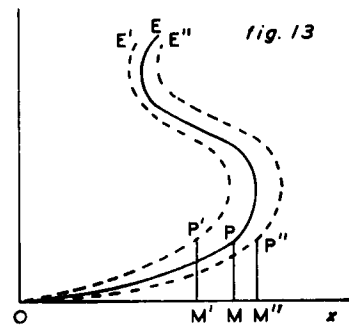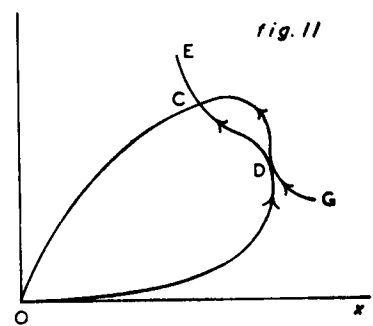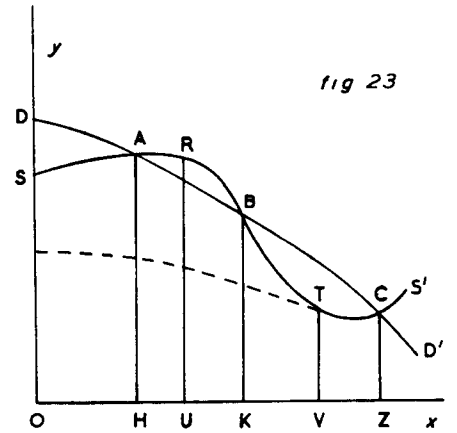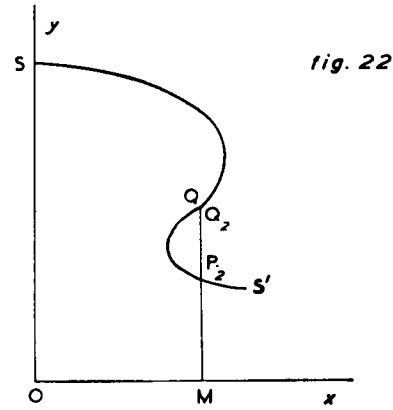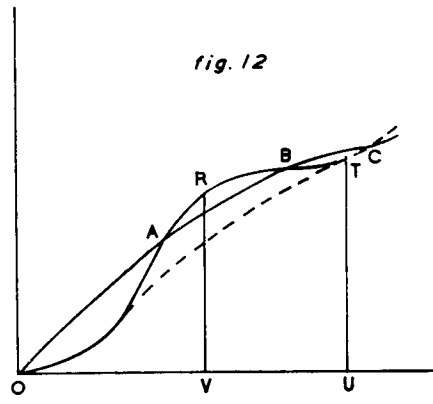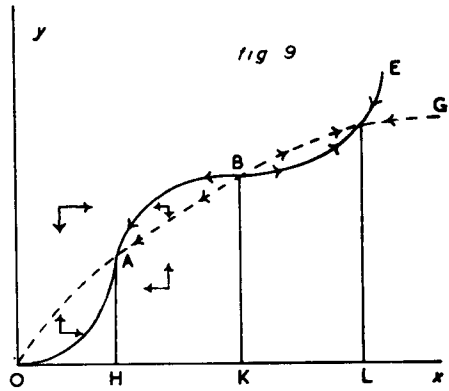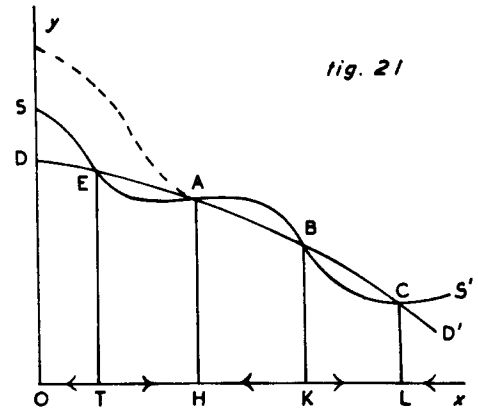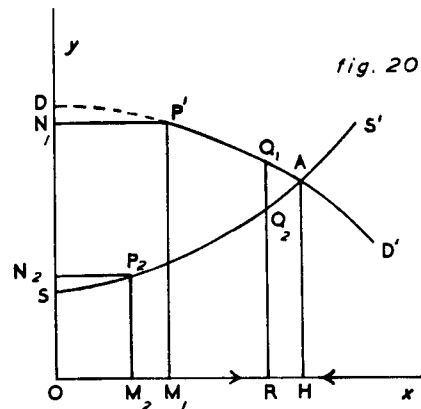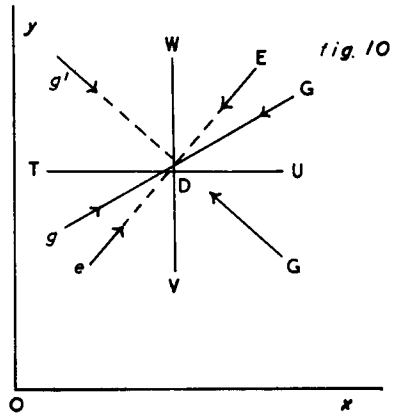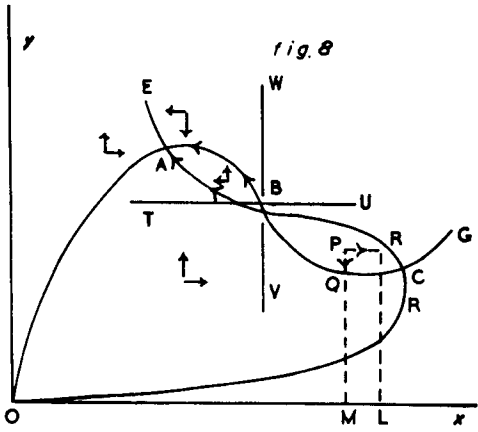
COMMENTARY

Pure Theory of Foreign Trade

Page 2. The footnote reference to Appendix I probably refers to part of the original document not printed by Sidgwick (see Editorial Note). In all probability, the material contained in this Appendix had already been published in Marshall's essay "On Mr. Mill's Theory of Value".[1] (Reprinted in *Memorials of Alfred Marshall*, p. 119.) This conjecture is reinforced by the reference to an Appendix on Mill's Theory of Value contained in *Pure Theory of Domestic Values*, p. 7.

Page 11, line 32. "Prop. IV" should read "Prop. VI".

Page 20, lines 8 and 12. No indication has been found of the positions of P', P'', or P''' on Fig. 7.

Fig. 13. This figure is not referred to in the text but it has been reproduced in Appendix I of *Money Credit and Commerce* (p. 356). This indicates that the missing portions of the work on international trade (Props. XIV–XVI, see p. 24 of this essay and p. 4 of *The Pure Theory of Domestic Values*) probably covered the theory of import taxes and other influences changing the shape of the curves contained in sections 5–7 of that Appendix. However, it must be remembered that the concept of elasticity was first adopted by Marshall in "The Graphic Method of Statistics" read to the International Statistical Conference in 1885 (reprinted in *Memorials*, p. 175).

Pure Theory of (Domestic) Values
It is interesting to note that, in one copy at least, Marshall substituted in the title the word "Inland" for "Domestic".

Page 5, line 26. "either . . . or" should read "neither . . . nor".

Page 10, footnote "§ 9" should be "§ 5".

Page 11, line 20. "equilibrium" should be inserted after "unstable".

Page 11, line 25. "Prop. XIV" should be "Prop. XIX".

Page 12, line 18. "Part II, Ch. III, § 7" obviously refers to Ch. II, § 7 of *Pure Theory of International Trade*.

Page 13, line 12. "OG" should be "DD' ".

Page 22, footnote. The material contained in this Appendix was probably reproduced in Notes VI, VII, VIII and (perhaps) IX of the mathematical Appendix to *Principles of Economics*. It should be noted that this Appendix is numbered "III" and the only other Appendix number referred to is "I". (See *Pure Theory of Foreign Trade*, p. 2.) This would indicate that an Appendix originally intended for publication is missing, although it may be the Appendix on Rent referred to first on p. 23.

(1) *Fortnightly Review*, Vol. XIX, C.XII, Apr. 1876, pp. 591-602.

Page 23, line 35. No clear indication of the contents of this Appendix can be gathered from the text. (See also p. 31, line 3.)

The references to rectangular hyperbolæ (in the complete footnote to the Preface of the first edition of *Principles of Economics*, referred to in the Editorial Note to this edition) and the "apparatus of curves" (see p. 31) might indicate that Marshall used equal outlay curves in this Appendix. This view is strengthened by his use of these curves in his outline of the theory of consumers' surplus in *Principles* (Figs. 36 and 37, pp. 488 and 490).

Page 26, line 9. "C" should be "Cc".

Page 26, line 16. "CAa" should be "CAac".

Page 27, line 40. This construction was not contained in the diagrams included in the original edition of these papers. However, the required additions are quite clear from the text.

G. S. Dorrance