3.4 Capacitance - introduction

When electric charge is added to an isolated conductor the potential of the conductor is raised to V volts.

We say the isolated conductor *stores* charge. A conductor with the capacity to store charge is called a **capacitor**.

When a capacitor is 'charged' work is done to assemble the charges together (in the one place: on the conductor).

Such an isolated charged conductor holds a collection of charges (of the same sign) stored in proximity, thus there is an increase in potential energy upon charging a capacitor. This energy comes from the work done in moving the charges together.

A capacitor stores charge and ***energy***

(Imagine this: we take a group of positive (or negative) charges and squeeze them together (we *do* work). This raises the energy of the group of charges because they'd like to repel one another.)

3.5 Defining equation $C = Q/V$

The greater amount of charge a conductor can store the greater the capacitance. If a given amount of charge is stored at a lower potential, V volts, this also means greater capacitance, thus,

$$C \propto Q \quad \text{and} \quad C \propto \frac{1}{V}$$

or,

$$\boxed{C = \frac{Q}{V}} \qquad \text{definition of capacitance}$$

The units are coulombs per volt (C/V) or farads (F)

SI unit of capacitance,
named for Michael Faraday

One farad (1F) is a pretty big capacitance! We usually encounter practical components (capacitors) with values in the ranges

μF (microfarad), 10^{-6} F

nF (nanofarad), 10^{-9} F

pF (picofarad), 10^{-12} F

With a capacitance bridge (meter) it is not too difficult to measure directly capacitance values in the ranges 10^{-15} F (femtofarads) to 10^{-18} F (attofarads).

3.6 Example capacitor : isolated hollow metal sphere

The potential of a metal sphere is given by $V = \frac{k_0 Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

(see p 670 of Hecht) R = radius of sphere

therefore,

$$C = \frac{Q}{V} = \frac{Q(4\pi\varepsilon_0)R}{Q} = 4\pi\varepsilon_0 R$$

$$\boxed{C = 4\pi\varepsilon_0 R}$$

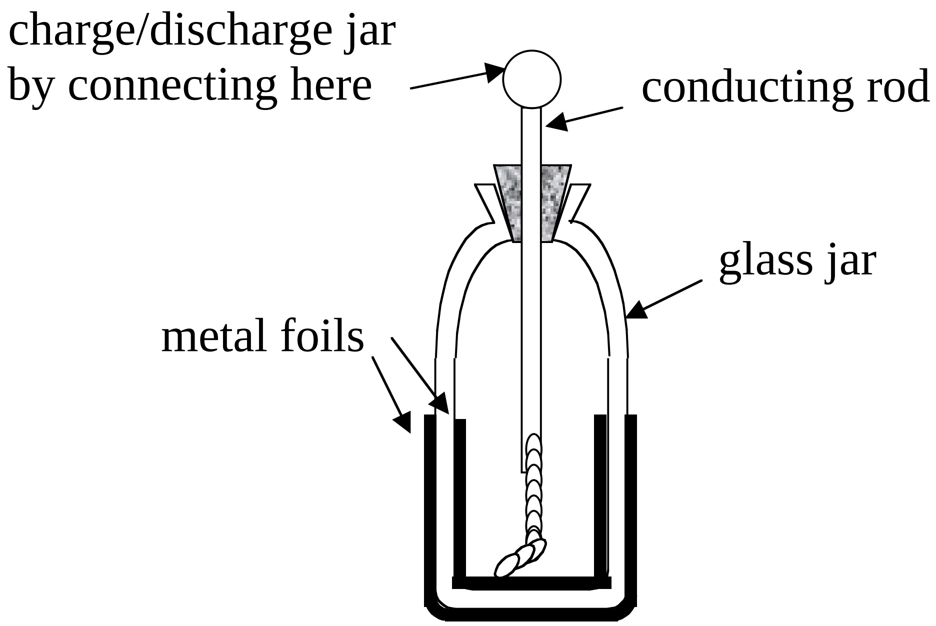
capacitance of sphere of radius R

e.g. remember the (approximately) metal sphere on the van der Graaf generator lecture demonstration, $R \sim 20\text{cm}$

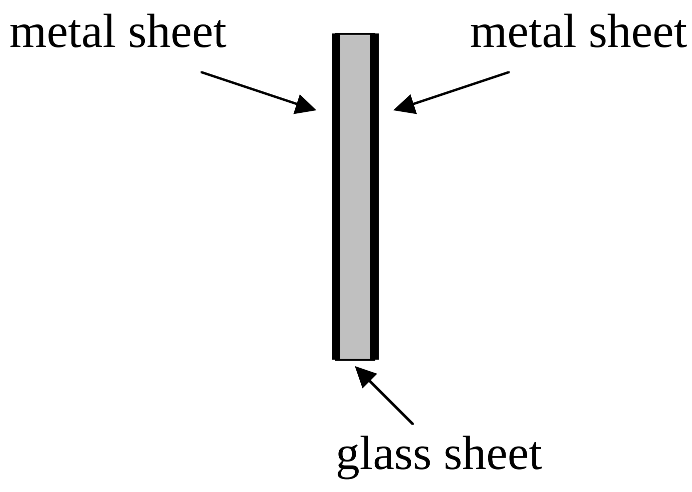
$C = 4\pi\varepsilon_0 R = 4\pi(8.85\text{x}10^{-12})(20\text{x}10^{-2}) \approx 2\text{x}10^{-11}$ F, this is about 20 picofarads (20pF).

3.7 Leyden jar and parallel plate capacitor

Amongst the earliest experimental capacitors, the **Leyden jar** consisted of a glass jar having a pair of electrically isolated metal foils:

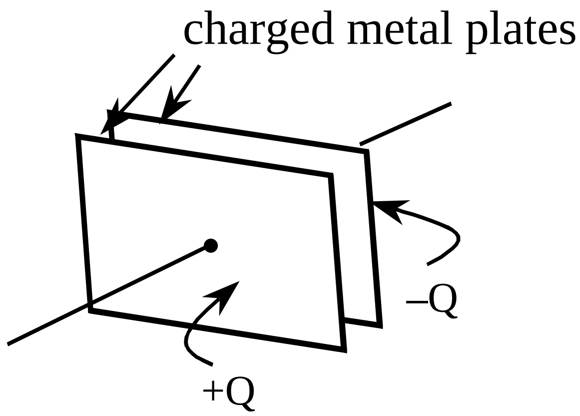


Later experimenters (notably Benjamin Franklin (in 1750s), although many people were interested in ‘electricity’ at this time) made the first **parallel plate capacitors** using metal sheets separated by glass sheet (window glass!):

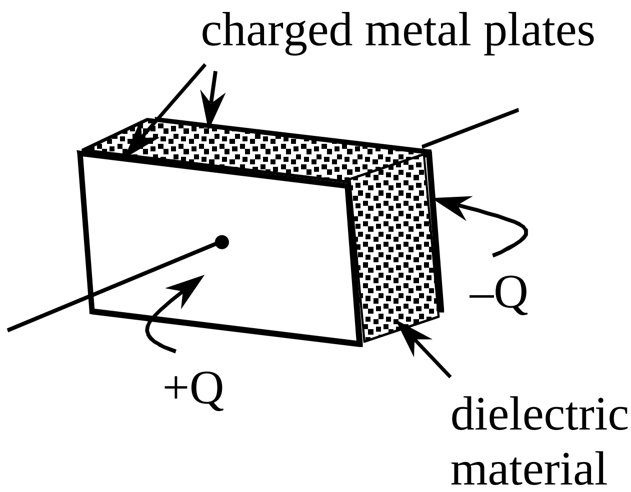


The **parallel plate capacitor** is *the* prototypical capacitor. We consider two versions

(i) vacuum (≈air) in the gap between the charged plates:

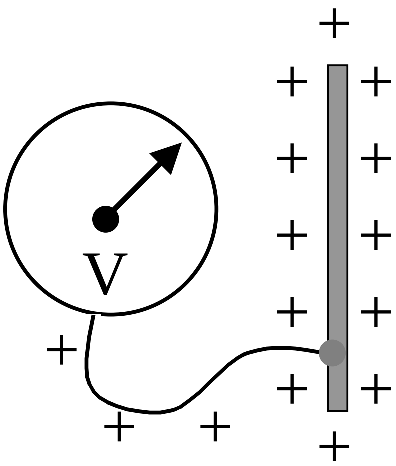


(ii) dielectric material (insulator) between the charged plates:



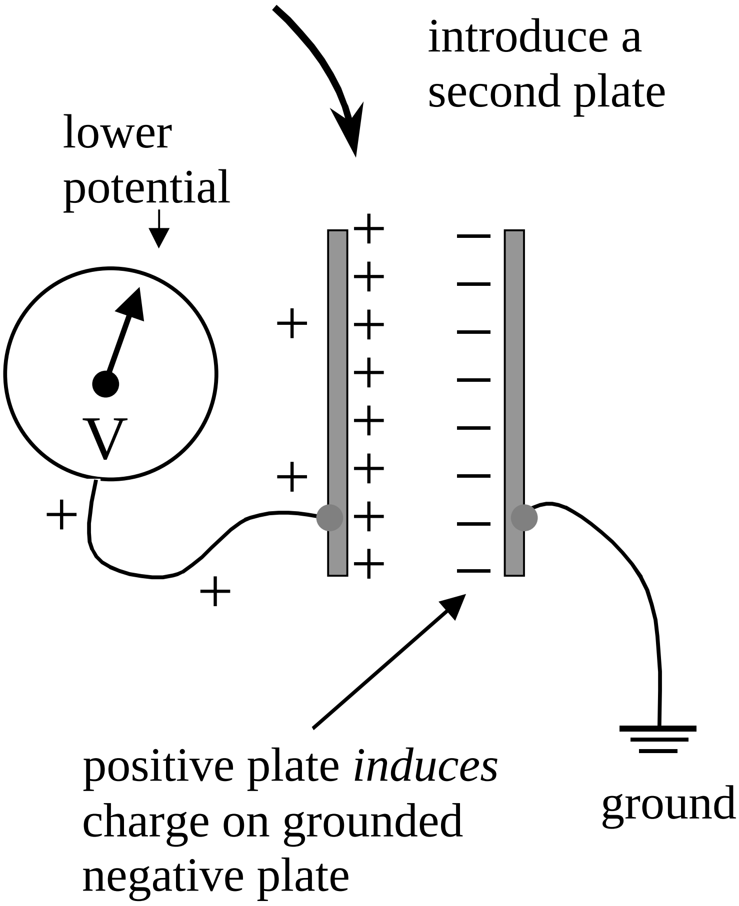
(i) Parallel plate capacitor – air between plates

Any single, isolated conductor can have capacitance – e.g. a single metal plate. In the parallel plate capacitor the effect of the second plate is to increase the charge storing capacity – the capacitance!



Single charged plate. Charges +Q lead to a potential V. The juxtaposition of like charges – their position relative to one another – brings the plate to potential V, indicated on meter

Positive and negative charges now reside on inside faces of plates. The proximity of the negative charge drops the potential of the positive plate. We can increase the amount of charge to bring V up to its original (one plate) value – hence C has been increased.



The capcitance of the parallel plate capacitor is easily calculated:

Recall (from 3.1 example), the difference in the potential between the +ve and –ve plates is

$$\Delta V = Ed$$

or

$$E = \frac{\Delta V}{d}$$

d is the plate separation

where E is the electric field between plates. The electric field E between the plates is (see end of Section 2.7 (a), this result comes from using Gauss' law)

$$E = \frac{\sigma}{\varepsilon_0}$$

σ is surface charge density (C/m^2)

ε_0 is permittivity of free space

and since

$$\sigma = \frac{Q}{A}$$

we have

$$\Delta V = Ed = \frac{Q}{\varepsilon_0 A} d$$

The definition of C is

$$C = \frac{Q}{\Delta V}$$

So that

$$C = \frac{Q}{V} = \frac{Q}{Qd/A\varepsilon_0}$$

or

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Capacitance of parallel plate capacitor of plate area A and plate separation d

By inspecting this formula for C we find that large values of C are achieved by making the plates large and putting them close together.

The other factor is the ε_0 - permittivity of free space, giving the electric field strength between the plates. If we put an insulating material with an ε value bigger than that of vacuum ($\approx$ air) we can increase C by reducing E between the plates ($E = \frac{\sigma}{\varepsilon}$).

This means inserting a dielectric layer.

(ii) Parallel plate capacitor – dielectric between plates

Common dielectrics for capacitors include polystyrene ($\varepsilon_r = 2.55$), mica($\varepsilon_r = 6.0$), waxed paper (3-4), mylar (polyimide) film ($\varepsilon_r = 3.4$).

The number ε_r is the **relative permittivity** or **dielectric constant**

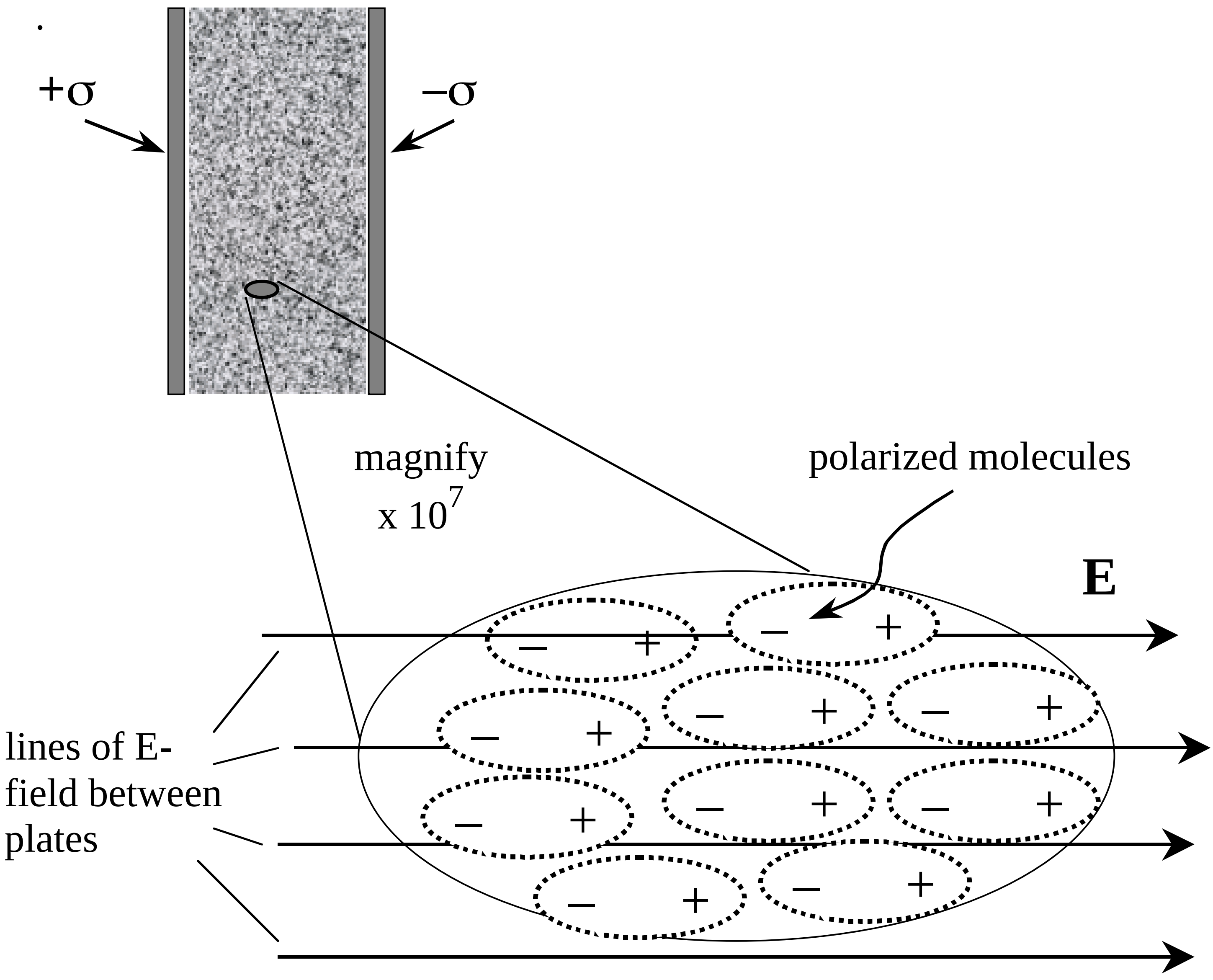
given by

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

(**N.B.** dielectric constant ε_r is also given the symbol κ)

When a dielectric material is inserted between the plates of a capacitor the material becomes polarized: the electric charge dintribution on the molecular structure is ‘stretched’ by the E field between the plates

.



The polarized dielectric has its own 'internal' E-field – in a direction opposite to the 'external field' set up by the capacitor plates. The internal field of the dielectric is that due to the external field.

The effect is to reduce the total E-field between the plates leading to capacity for more charge storage because of the lowered potential between the plates:

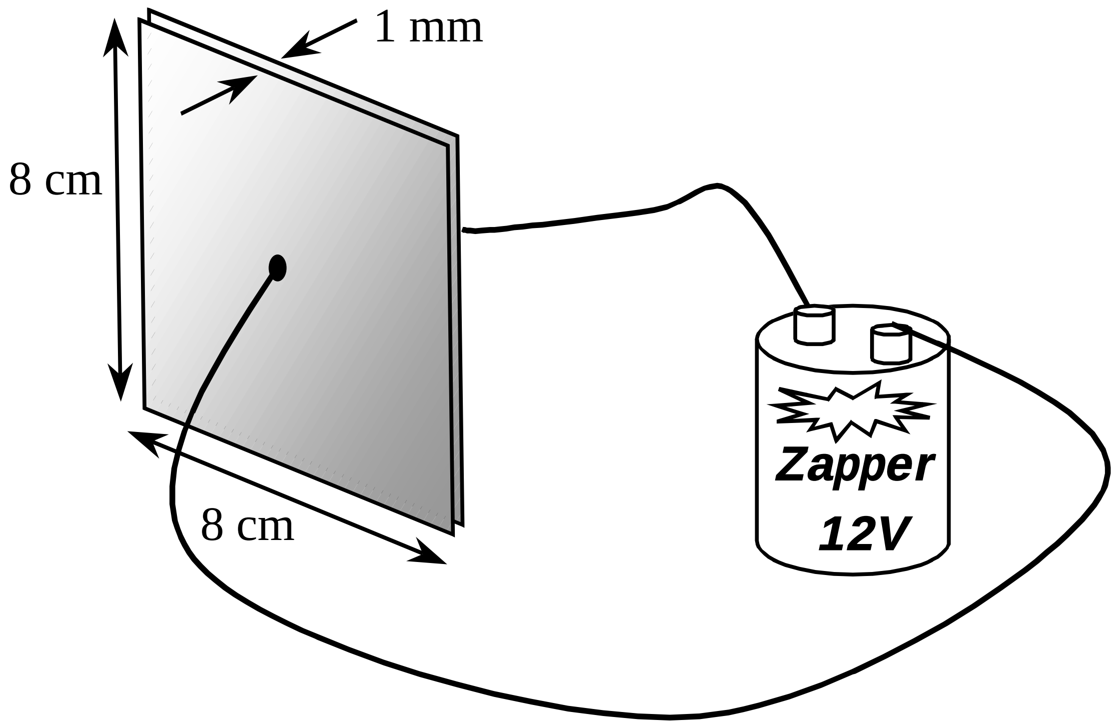
$$\Delta V = Ed$$

lowered lowered fixed

3.8 Example: Parallel plate capacitance

A parallel plate capacitor consists of two square plates, side $l = 8$ cm separated by a distance $d = 1$ mm. The capacitor has either (a) air between the plates, (b) polystyrene ($\varepsilon_r = 2.55$) between the plates. Determine (i) the capacitance of each, (ii) the charge on the capacitor in each case when connected to a 12 V battery.

(a)



(i) The capacitance is

$$C = \frac{\varepsilon_0 A}{d}$$

$A = (8\text{x}10^{-2})^2 = 6.4\text{x}10^{-3}\ \text{m}^2$,

$d = 1\ \text{mm} = 1\text{x}10^{-3}\ \text{m}$

$\varepsilon_0 = 8.85\text{x}10^{-12}\ \text{C}^2/\text{Nm}^2$

So

$$C = \frac{(8.85x10^{-12})(6.4x10^{-3})}{1x10^{-3}} = 5.7x10^{-11}\ F = 57\ pF$$

(ii) The charge stored Q is (note this is the charge stored on each plate)

$$Q = CV = (57x10^{-12})(12\ V) = 6.8x10^{-10}\ C = 0.68\ nC$$

(b) With the polystyrene dielectric $\varepsilon_r = 2.55$.

(i) The capacitance is now

$$C = \frac{\varepsilon A}{d}$$

where

$$\varepsilon = 2.55\varepsilon_0$$

and

$$C = \frac{(2.55)(8.85x10^{-12})(6.4x10^{-3})}{1x10^{-3}}$$

$$= 14.5x10^{-11}\ F = 145\ pF$$

(capacitance has increased by x 2.55)

(ii) the charge stored is

$$Q = CV = (145x10^{-12})(12\ V) = 1.74x10^{-9}\ C = 1.74\ nC$$

(and the stored charge has increased by a factor x2.55)

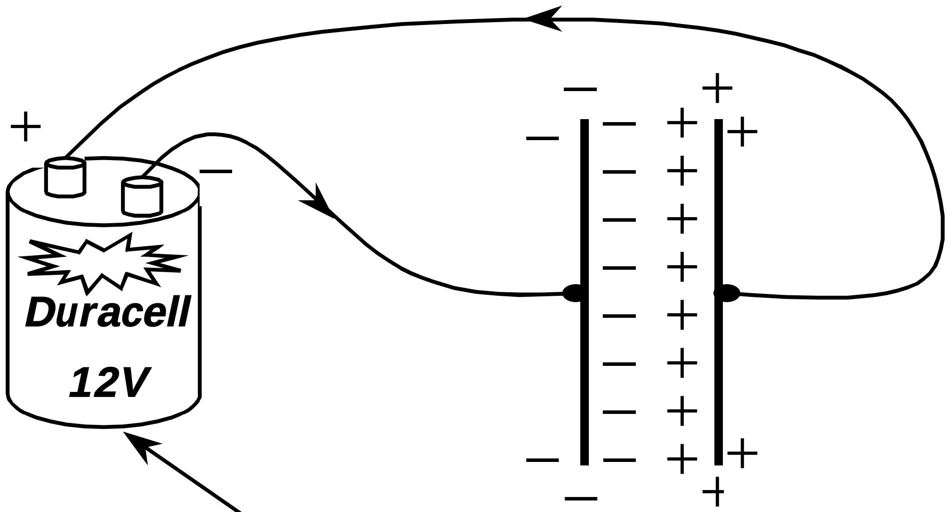
3.9 Charging a capacitor with a battery

Consider what happen when we connect a capacitor to a battery. We can use a regular dry cell (a ‘Duracell’ or an ‘Ever Ready’):

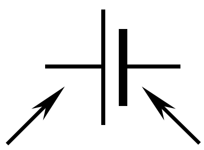
–ve charge flows out of battery to the left hand plate.

This charge induces a +ve charge on the right hand plate by depleting the plate of –ve charge.

This (‘excess’) –ve charge flows back to the –ve terminal of the battery.



The circuit symbol for this is:

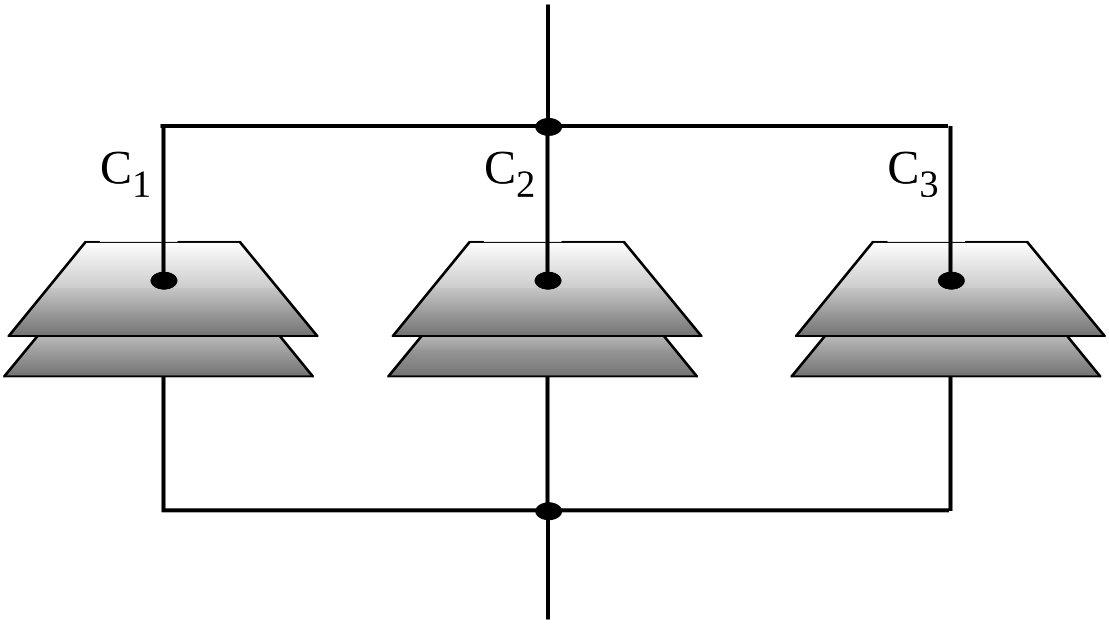


positive terminal

negative terminal

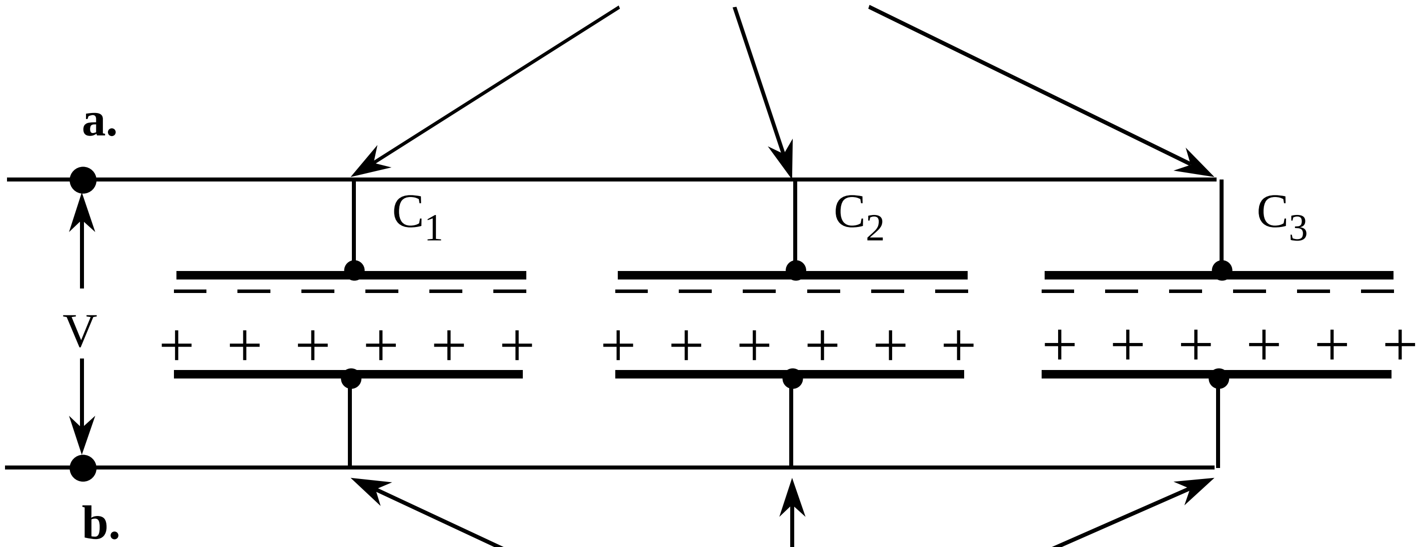
4.0 Parallel Connection of Capacitors

In parallel the capacitors are connected thus:



and the circuit symbol representation is:

top plates all at same potential V (same potential as point **a.**) since they are all connected to the same wire



bottom plates all at same potential –V (same potential as point **b.**) since they are all connected to the same wire

When points in a circuit are connected together with a conducting wire these points are brought to the same potential. (This is always assumed to be the case - at least in introductory physics.)

$$C_1 = \frac{Q_1}{V} \qquad C_2 = \frac{Q_2}{V} \qquad C_3 = \frac{Q_3}{V}$$

and

$$Q_{total} = Q_1 + Q_2 + Q_3$$

total charge on a single capacitor C_{total} that is equivalent to the combination of the three capacitors C_1, C_2, C_3

so that

$$C_{total}V = C_1V + C_2V + C_3V$$

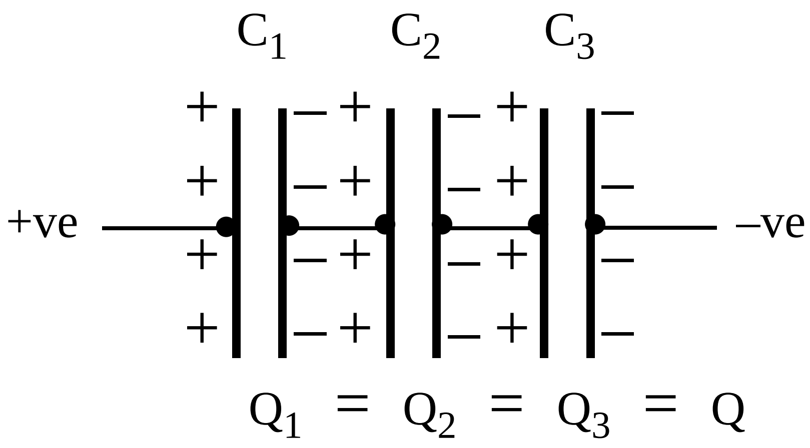
and

$$C_{total} = C_1 + C_2 + C_3$$

capacitors in parallel

4.1 Series Connection of Capacitors

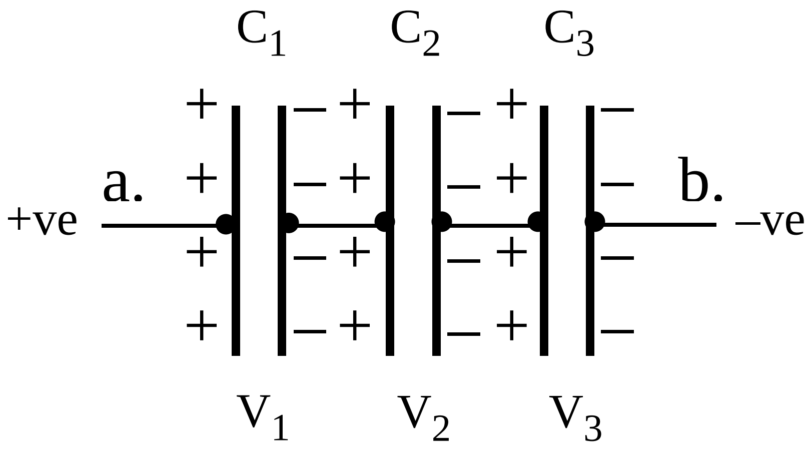
In series the capacitors are connected thus (we choose three arbitrarily for purpose of illustration):



On each of the capacitors C_1, C_2, C_3, the left had plate induces an equal and opposite charge Q on the right hand plate. Thus,

$$Q_1 = Q_2 = Q_3 = Q$$

The potentials across each of C_1, C_2, C_3 are V_1, V_2, V_3 respectively:



and the potential difference across the combination of the three capacitors, between points **a.** and **b.** is

$$V = V_1 + V_2 + V_3$$

thus,

$$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

and since

$$Q_1 = Q_2 = Q_3 = Q$$

we have

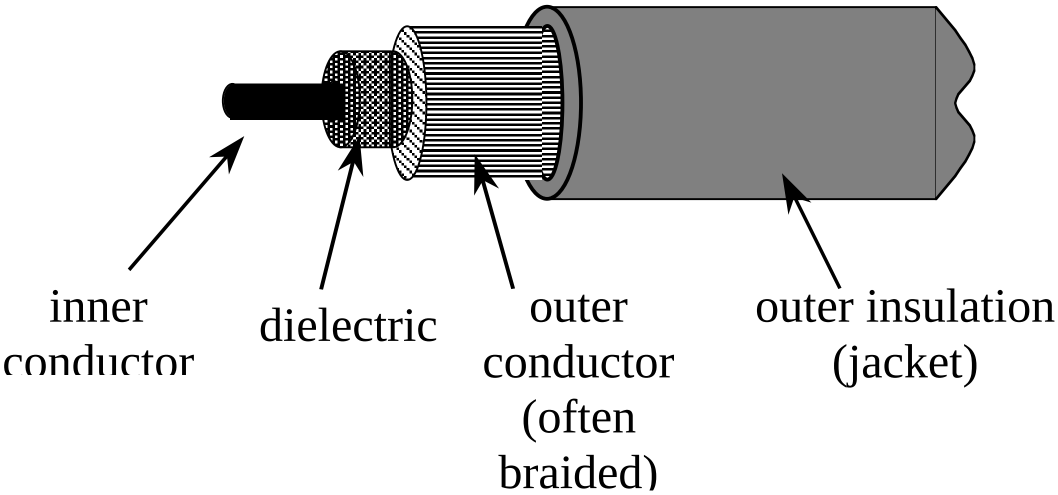
$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

capacitors in series

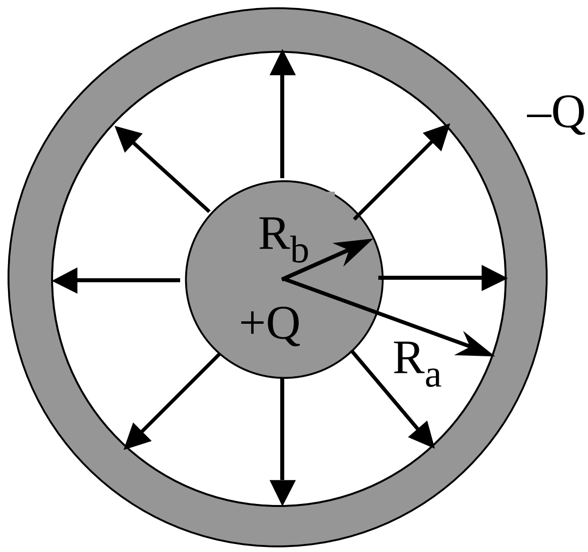
Clearly this is the general result for series connection – not just for 3 capacitors!

4.2 Cylindrical geometry: coaxial cable

Coaxial cables are used widely in electronics, data and communications applications:



Viewed schematically and in cross-section:



Let' s say the coaxial conductors have length L.

We want to find $$C = \frac{Q}{V_{ba}}$$

where V_{ba} is the potential difference between the inner and outer conductor.

Considering the inner conductor as a long straight wire, the electric field is (given by Gauss' law page 642 Hecht)

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

at distance r away from the wire due to charge per unit length λ; ($\lambda = \frac{Q}{L}$ coulombs per metre).

So the electric field between the conductors is

$$E = \frac{Q}{L(2\pi\varepsilon_0 r)}$$

and

$$V_{ba} = -\int_a^b \mathbf{E}.d\boldsymbol{l} = -\frac{Q}{2\pi\varepsilon_0 L}\int_{r=a}^{r=b}\frac{dr}{r}$$

$$= -\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{r_b}{r_a}\right)$$

$$= +\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{r_a}{r_b}\right)$$

and the capacitance C is

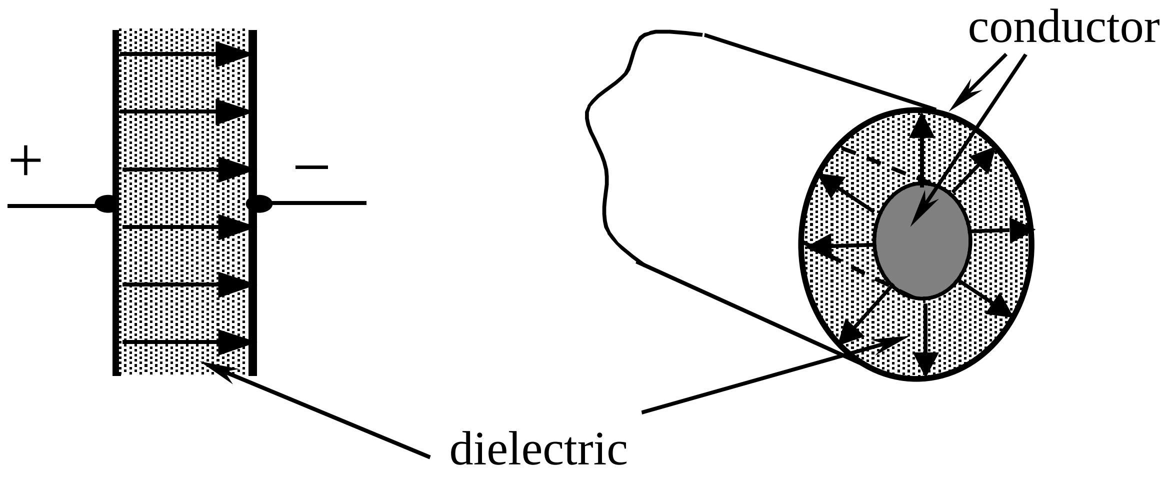
$$C = \frac{Q}{V_{ba}}$$

$$\boxed{C = \frac{2\pi\varepsilon_0 L}{\ln(r_a / r_b)}}$$

capacitance of cylindrical co-axial conductors

4.3 Dielectric strength and dielectric breakdown

In a dielectric filled capacitor or in a coaxial cable, for example, there is an electric field across the dielectric/insulator:



If this electric field is sufficiently strong electrons in the dielectric material are ‘ripped’ free of their host atoms/molecules. These freed electrons can collide with other electrons in the dielectric material causing them to be released also (a sort of ‘chain reaction’) and a current can then flow in the dielectric (which was acting as an insulator).

This process occurs when the **dielectric strength** of the material is exceeded. The dielectric strength is the electric field value (E volts per metre (V/m)) at which this **dielectric breakdown** occurs.

Examples of dielectric breakdown: spark plug, lightning, oil in high voltage transformers, failure of ultra-thin silicon dioxide layers in MOSFETS (a limiting problem for very small transistors?): http://www.eeel.nist.gov/lab_office/nano_tech_eeel_2002/researchers_help_determine_mechanism_for_ultrathin_dielectric_breakdown.htm

Breakdown paths are characterized as multifractals – see http://math.nist.gov/mcsd/savg/vis/dielectric/

4.4 Electric stored energy

A capacitor stores electrical energy. The energy is stored in the electric field **E**.

The use of capacitors as energy storage devices is ubiquitous. Some examples are: camera flash, electronics power supplies (as current reserve), large pulsed magnetic fields and pulsed lasers (Lawrence Livermore Labs Nova laser – 16 trillion watts in 2.5 ns pulses).

We consider the work done in charging a capacitor to find an expression for the charged energy. (Hecht p 682-684)

(Clearly work is done in charging a capacitor; in a superficial way we see this when capacitors are charged from a battery, for example, the battery's energy is used and it will run down!)

Following Hecht (p 682-684) we find,

$$\boxed{PE_E = \frac{1}{2}QV}$$

energy stored on capacitor

or

$$PE_E = \frac{1}{2}CV^2$$

energy stored on capacitor

or

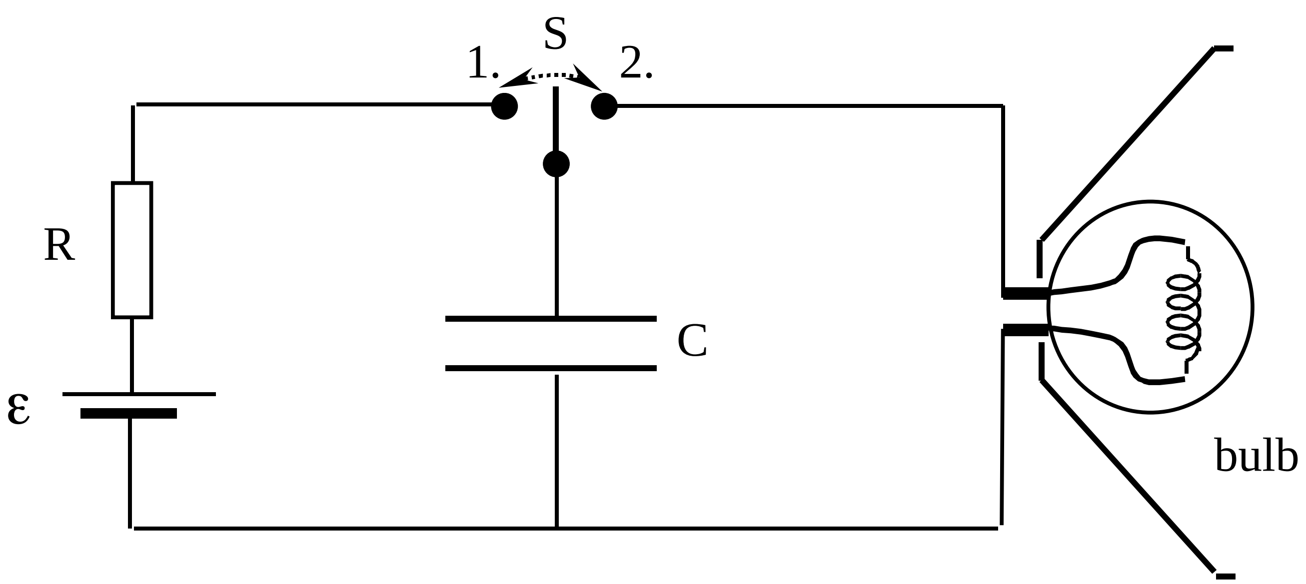
$$PE_E = \frac{1}{2}\frac{Q^2}{C}$$

energy stored on capacitor

where Q is charge on capacitor in coulombs (C), C is the capacitance in farads (F) and V is the potential across the capacitor in volts (V).

Three very handy expressions for stored energy!

Example: camera flash



S is a switch. In position 1. it is connected to the battery only and charges the capacitor from the battery which is represented by an emf ε in series with the battery's own internal resistance r.

In position 2. The switch is disconnected from the battery and connects the charged capacitor to the bulb (where the filament is shown as a coil in a glass envelope mounted in a reflector).

The capacitor discharges stored energy into the bulb's filament which is released as a brief, intense burst of light (as we know!)

• Camera flash sets carbon nanotubes alight!
http://www.sciam.com/article.cfm?articleID=0006C256-22D2-1CD0-B4A8809EC588EEDF

• Dismantle you own diposable camera flash (TAKE NOTE OF THE SAFETY INSTRUCTIONS ON THIS WEBSITE….and only do this to a cheap (~$10) disposable camera!!):
http://isaac.exploratorium.edu/~pauld/activities/camera_electronics.html

Example: Heart defibrillator

In many movies and TV medic shows you'll see poor people having their heart restarted by a doctor who places a couple of paddles on the patient's chest and dumps some electric charge into them. The principle is to stop the heart momentarily allowing it to start beating again in regular rhythm.
http://www.nitcentral.com/oddsends/defibril.htm

The amount of energy involved – which is stored on capacitors – is several hundred J.