Chapter 1

NUCLEAR REACTOR CONCEPTS AND THERMODYNAMIC CYCLES

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1.1 INTRODUCTION

Nuclear fission reactors share the same basic design concept in that they have a core in which the fission chain reaction proceeds.

They differ from fossil fuel power plants in that they are closed systems, rather than open systems. In open systems the products of combustion with their associated pollutants are dispersed and diluted in the atmosphere, since their amounts are so large that containing them is not possible. Nuclear power plants generate small amounts of fission products that are contained within the system for later disposal. Being radioactive, their release from the reactor core is prevented with incorporation into the design of multiple barriers and engineered safety features.

The fission energy is there initially the kinetic energy of the fission products of some fissile element fuel in the core. Often a fertile material is also present which can be bred into a fissile material through neutron transmutations. This kinetic energy is lost to the surrounding structure in the form of heat. A coolant is brought in to extract the heat.

The coolant can also act as a moderator reducing the energy of the fast fission neutrons energy from its average value of 2 MeV to the thermal equilibrium energy of 0.025 eV, where the probability of fissioning the fissile element is dramatically increased. A neutron reflector surrounds the core. The heat from the coolant is moved to a heat transfer cycle where eventually electricity or process heat for other applications such as desalination, hydrogen production, or district heat, is produced. Safety systems are incorporated into the design to provide safety to the operators and members of the public under all foreseen conditions.

1.2 MAIN REACTOR CONCEPTS

The power generated within the core is proportional to the neutron density, or the number of neutrons per unit volume and their speed. The power of the reactor can be controlled through managing the neutron density by moving rods of neutron absorbing material like cadmium or boron, into or out of the core. When these control rods are fully inserted, the chain reaction is stopped. Alternatively, a fuel rod moved into or out of the core, or a portion of a reflector displaced, would have the same control effect.

Different reactor concepts can be considered by choosing a compatible set of fuel, coolant, moderator and safety and control strategies.

There exist several fission reactor concepts that constitute the present generation of nuclear fission power plants. Table 1 shows some of these main concepts regarding their core volume and their power density defined as:

$$q''' = E_f \; \Sigma_f \; \Phi \; [\text{MWth/m}^3] \tag{1}$$

where: E_f is the energy release per fission event
= 200 [MeV/fission] = $3.2\text{x}10^{-11}$ [Watt.sec/fission],
Σ_f is the macroscopic fission cross section in [cm^{-1}],
Φ is the average neutron flux in [neutrons/(cm^2.sec)].

Higher power densities are associated with smaller core sizes and volumes. Small core volumes are favorable from a capital cost perspective, meaning that fewer materials will have to be manufactured for constructing the core. However, higher power densities require stringent heat transfer systems and higher levels of needed operational safety. The design engineers always try to achieve a compromise between the cost and the desired level of safety.

Table 1: Power densities and core volumes in fission power reactors.

Type	Description	Core Average Power Density [MWth/m³]	Core Volume [m³]
PWR	Pressurized Water Reactor, H_2O	75.0	40.0
BWR	Boiling Water Reactor, H_2O	50.0	60.0
HTGR	High temperature Gas-cooled Reactor, Graphite moderated, He cooled	7.0	428.6
GCFR	Gas Cooled Fast Reactor, He cooled, Fast neutron Breeder	280.0	10.7
LMFBR	Liquid Metal Fast Breeder Reactor, Na cooled, Fast neutron Breeder	530.0	5.7

Table 2: Power data for fission power reactors.

Type	Power Level MWe	Primary coolant	Primary coolant exit temperature and pressure °C/atm	Turbine temperature and pressure °C/atm	Overall thermal efficiency [percent]
PWR	1,300	Pressurized H_2O	H_2O 330/158	H_2O 284/68	33
BWR	1,300	Boiling H_2O	H_2O 286/71	H_2O 281/67	34
AGR	600	Advanced Gas-cooled Reactor, Graphite moderated, CO_2 cooled	CO_2 648/40	H_2O 538/163	42
HTGR	1,200	High temperature Gas-	He	H_2O	38

		cooled Reactor, Graphite moderated, He cooled	778/48	510/166	
LMFBR	1,000	Liquid Metal Fast Breeder Reactor, Na cooled, Fast neutron Breeder	Na 615/10	H_2O 538/169	42
GCFBR	1,000	Gas cooled Fast Breeder Reactor, He cooled	He 568/114	H_2O 510/180	36

Table 3: Fuel data for fission power reactors.

Type	Fuel loading [tonnes] heavy metal	Fissile enrichment [percent]	Discharge burnup [MW.day/ tonne heavy metal]	Fuel Rating [kW/kg heavy metal]	Fissile rating [MW/kg fissile rating]	Power density [kW/liter]	Conversion, Breeding ratio
PWR	102 UO_2	3.1	31,500	37	1.50	93.0	0.60
BWR	147 UO_2	2.7	27,500	25	1.10	56.0	0.70
AGR	120 UO_2	2.3	18,000	13	0.54	2.7	0.50
HTGR	39 UC_2-ThO_2	4.1	98,000	77	1.90	8.4	0.65
LMFBR	19 PuO_2-UO_2	11.5	67,000	116	1.00	380.0	1.27
GCFBR	28 PuC_2-UO_2	12.7	73,000	93	0.73	259.0	1.39

1.3 MULTIPLE BARRIERS DESIGN CONCEPT

Underlying nuclear fission reactors designs is their large inventory of fission products and the concern that an accident might expose the population around the reactor site to hazardous levels of radiation. It is not possible for a power plant to disperse these substances with the explosive force of a weapon device, since they are designed differently. The concern is that an accident might release a hazardous amount of radioactive elements to the environment.

Different reactor systems designs share the same concept for the containment of radioactive releases. To make certain that the radioactive materials, principally the fission products are retained within the reactor system; they are surrounded by a series of physical barriers as shown in Table 4. To reach the environment, the radioactive fission products would have to penetrate all these barriers in succession. These barriers are primarily part of the plant's design for normal operation.

Table 4: Multiple barriers concept to contain fission products in nuclear power plants.

Barrier or Layer	Function
1. Ceramic fuel pellets	A fraction of the gaseous and volatile fission products can be released from the porous ceramic fuel pellets.
2. Metal fuel cladding	Contains the fission products released from the fission process. Less than 0.5 percent of the tubes develop pinhole sized leaks through which fission products would escape, over the lifetime of the fuel.
3. Reactor vessel and piping.	The 8-10 inch thick steel vessel and 4-inch thick steel piping contain the reactor coolant. A portion of the coolant is continuously passed through filtering traps keeping the coolant activity at a low level.
4. Concrete shield	Plant operators and equipment are protected from core radiation by biological concrete shields 7-10 feet thick.
5. Containment structure	The entire nuclear island is enclosed to protect from the outside elements such as hurricanes or tornado winds. Any release of radioactivity in case of reactor cooling water pipe leakage or rupture is quenched by a containment spray system. High pressure and low-pressure coolant pump provide cooling in case of rector primary coolant loss.
6. Exclusion area	A designated area around each plant separates the plant from the public. Entrance is restricted.
7. Plant separation distance	Plants are sited at a distance from population centers.

1.4 ENGINEERED SAFETY FEATURES (ESFs) AND DEFENSE IN DEPTH DESIGN PRINCIPLE

Successive Engineered Safety Features (ESFs) are a part of a nuclear power plant design to protect against three types of occurrences:
1.   Equipment failures.
2.   Human Error.
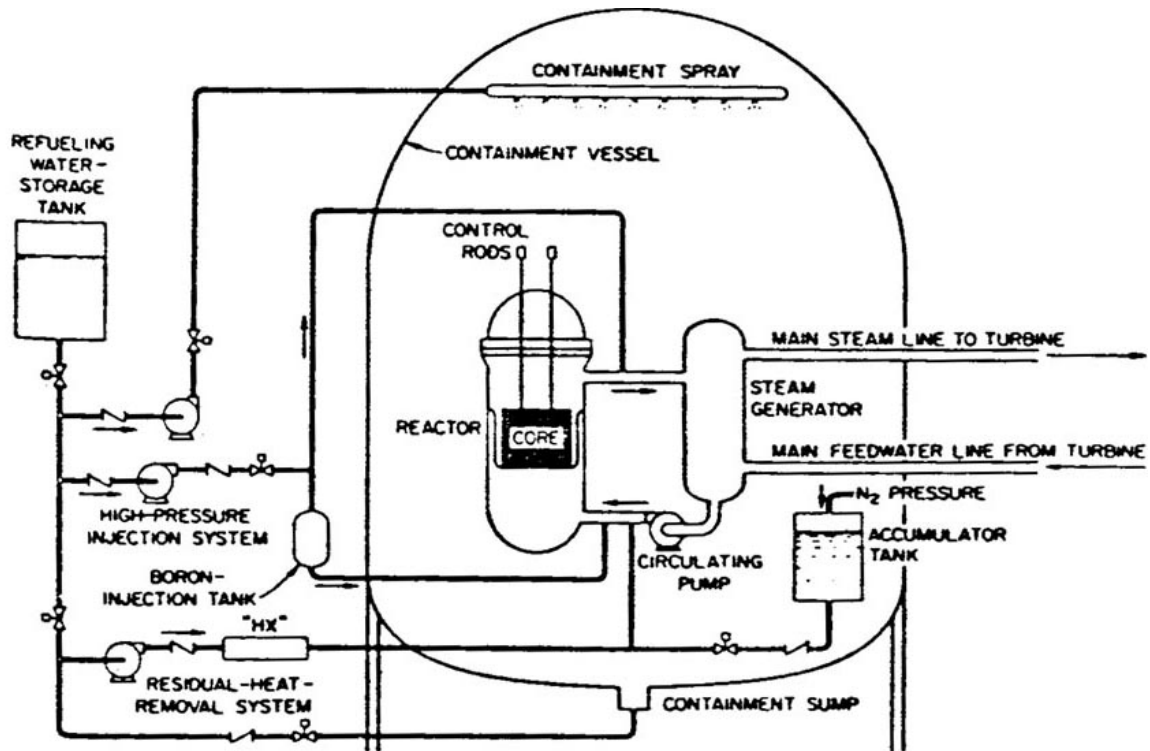3.   Severe natural events.

Fig. 1: Engineered Safety Features for the PWR Concept.

Any sudden increase in power level is countered and limited by physical self-regulating processes such as the negative temperature coefficients of reactivity.

The ESFs for the PWR concept are shown in Fig.1. These include:

1. The control rods, to shut down the chain reaction.
2. The containment vessel and its spray system, to quench any steam released into the containment.
3. The accumulator tanks containing a supply of water under nitrogen pressure for emergency cooling.
4. A residual heat removal system heat exchanger.
5. A High Pressure Coolant Injection system, HPCI.
6. A Low Pressure Coolant Injection system, LPCI.
7. A boron injection tank to shut down the chain reaction in case the control rods are not capable of being inserted into the core.
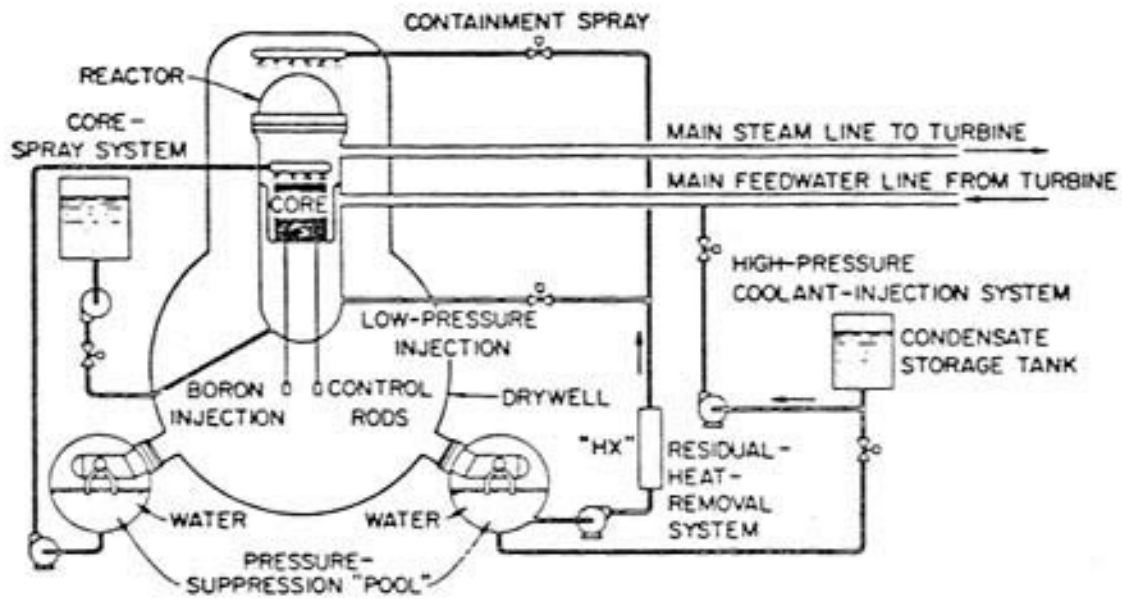8. An extra supply of cooling water in the refueling storage tank.

Fig. 2: Engineered Safety Features for the BWR Concept.

The ESFs for the Boiling Water Reactor (BWR) concept are also shown in Fig.2. It shares similar components with the PWR ESFs, and include:

1. The control rods, to shut down the chain reaction.
2. The containment spray system, to quench any steam released under abnormal conditions.
3. The pressure suppression pool to condense any steam leaking into the containment vessel.
4. A residual heat removal system heat exchanger.
5. A High Pressure Coolant Injection system, HPCI.
6. A Low Pressure Coolant Injection system, LPCI.
7. A boron injection tank to shut down the chain reaction in case the control rods are not capable of being inserted into the core.
8. An extra supply of cooling water in the condensate storage tank.
9. An internal core spray system.

The system is designed to operate with wide margins of stability, so that it will tolerate a broad spectrum of malfunctions and errors. Only tested, proven materials are used in construction, and they assembled and tested with strict quality assurance criteria.

Instruments and controls are redundant, with one system substituting for another so that operators at all times are aware of, and can regulate reactor conditions. The assumption is made that the equipment will fail and that operators will make errors. The reactor thus has built into it extensive systems to monitor temperature, pressure, water levels in the core, and other aspects of operation bearing on safety. The sensors are linked to automatic control systems that adjust or shut down the reactor if predetermined levels are exceeded.

The control mechanisms are designed to be fail safe: that is the malfunction of any component in the network activates the overall system. Mechanisms are designed to be redundant and independent: if one fails, another is available to perform the same protective

action. The system is designed for accidents mitigation, a safety measure shared with the aerospace industry.

The most severe hypothetical accidents are assumed, reflecting combinations of highly improbable failures occurring all at the same time. These design basis accidents include such remotely credible events as the sudden ejection of the most critical control rod, a break in a steam line, a Loss of Coolant Accident (LOCA) and other events. External events are assumed such as a 100 miles/hour tornado or hurricane, the most severe earthquake in the seismic region and the probable maximum flood.

To consider these events does not imply that they are likely to occur, but for each of these incidents, the designer provides diverse and redundant safeguards in the form of the Engineered Safety Features.
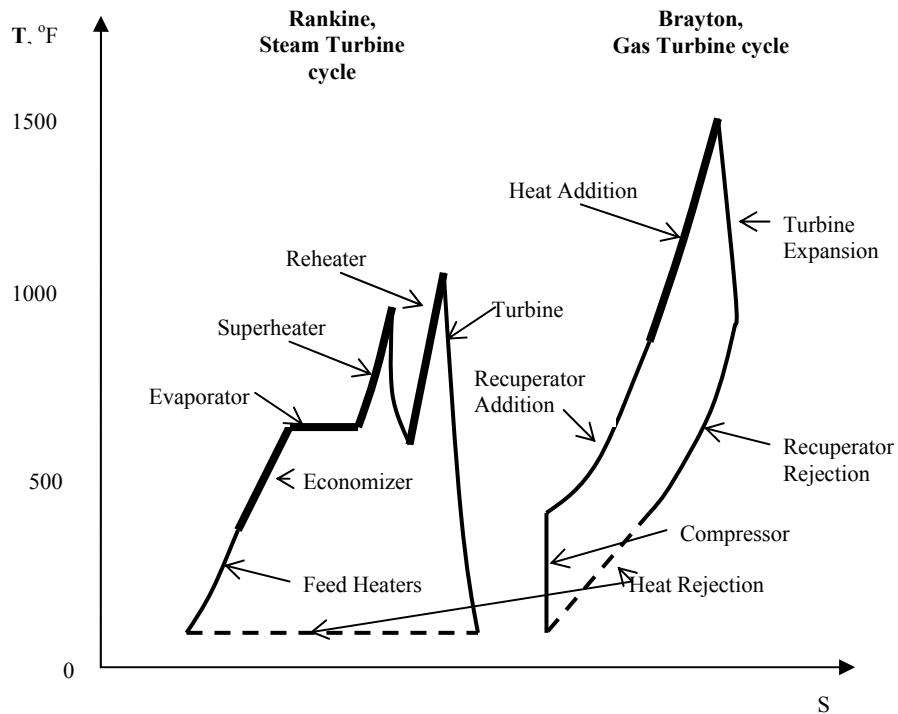
Fig. 3: Comparison of the Gas Turbine Brayton Cycle to the Rankine, Joule or Steam Cycle on a Temperature-Entropy TS Diagram.

1.5 POWER CYCLES

FLOW SYSTEMS ASSYMETRIES AND ENERGY EXTRACTION

A basic law of energy conversion engineering can be simply enunciated as:

"Energy can be extracted or converted only from a flow system."

In hydraulics, the potential energy of water blocked behind a dam cannot be extracted unless it is allowed to flow. In this case only a part of it can be extracted by a water turbine.

In a heat engine, the heat energy cannot be extracted from a totally insulated reservoir. Only when it is allowed to flow from a high temperature at which heat is added to a low temperature where it is rejected to the environment, can a fraction of this energy be extracted by a heat engine.

Totally blocking a wind stream does not allow any energy extraction. Only by allowing the wind stream to flow from a high speed region to a low speed region can energy be extracted by a wind turbine.

A second law of energy conversion can be enunciated as:

"Natural or artificial asymmetries in a hydraulic, thermodynamic or aerodynamic system allow the extraction of only a fraction of the available energy at a specified efficiency."

Ingenious devices take advantage of existing asymmetries or create configurations or situations favoring the creation of these asymmetries, to extract energy from the environment.

A corollary ensues that the existence of a flow system necessitates that only a fraction of the available energy can be extracted at an efficiency characteristic of the energy extraction process with the rest returned back to the environment to maintain the flow process.

In thermodynamics, the ideal heat cycle efficiency is expressed by the Carnot cycle efficiency. In a wind stream, the ideal aerodynamic cycle efficiency is expressed by Betz's efficiency equation.

POWER CYCLES

Most power reactor designs use the Steam, Rankine or Joule cycle as shown in Fig. 3. The newer prospective designs take advantage of new developments in turbine technology such as magnetic bearings, and use the Brayton or Gas Turbine cycle shown in Fig. 3.

With the use of the Brayton gas turbine cycle, the gaseous coolant such as helium coolant is enclosed in a single circuit moving from the compressor to the turbine. The possibility of its depressurization or leakage is minimized, and it is not reactive with graphite like steam would be. The designs can operate at higher temperatures and offer a high value of the thermal efficiency around 40 percent, compared with the 30 percent value for light water reactors. The high temperatures offer the possibility of process heat generation and use in industrial processes such as high temperature water electrolysis for the production of hydrogen for future non-fossil transportation fuel supplies.

RANKINE OR STEAM TURBINE CYCLE

The Rankine or steam cycle system is the most widely used cycle in nuclear and fossil power plants, dating back to the Watt's steam engine used in boilers and steam locomotives from the beginning of the industrial revolution. It uses a liquid that evaporates in a steam generator when heated and expands to produce work, such as rotating a turbine or piston, which when connected to the shaft of a generator, produces electricity. The exhaust vapor leaving the turbine condenses in a condenser and the liquid is pumped back to the steam generator to be evaporated

again. The working fluid most commonly used is water, though other liquids such as ammonia or mercury can also be used. The Rankine cycle design is used by most commercial electric power plants.

BRAYTON , JOULE OR GAS TURBINE CYCLE

The Brayton, Joule or gas turbine cycle is suggested for new nuclear power plants since it allows operation at higher temperatures, hence higher cycle efficiencies. It incorporates a turbine and a compressor on the same shaft connected to an electrical generator and uses a gas as the working medium. There exist open cycle and closed cycle Brayton systems. The gas turbine is a common example of the open cycle Brayton system. Air is drawn into a compressor, heated and expanded through a turbine, and exhausted into the atmosphere. Nuclear power plants use the closed cycle Brayton system where a gas, such as steam, H_2, CO_2 or He. The gas in the closed cycle system gives up some of its heat in a heat exchanger after it leaves the turbine. It then returns to the compressor to continue the cycle again.

STIRLING CYCLE

The Stirling cycle is also called an external combustion engine differs from the Rankine cycle in that it uses a gas, such as air, helium, or hydrogen, instead of a liquid, as its working fluid. The external source of energy could be from a radioisotope, heat pump from a fission reactor, concentrated sunlight, biomass, or fossil fuels. The external heat is provided to one cylinder. This causes the gas to alternately expand and contract, moving a displacer piston back and forth between a heated and an unheated cylinder.

DISSOCIATING GASES CYCLE

Dissociating gases which dissociate upon heating and recombine upon cooling can be used in nuclear power plants to considerably reduce the weight of the heat exchange and rotating machinery. Such a reaction can occur in nitrogen tetroxide:

$$N_2O_4 \rightleftharpoons 2NO_2 \tag{2}$$

The doubling of the number of molecules in the working gas from n to 2n, doubles the amount of work per unit mass in the ideal gas equation:

$$PV = 2nRT \tag{3}$$

The resulting doubling of the work done per unit mass of the working fluid allows the use of smaller size and weight turbines, compressors and heat exchangers. As proposed by Ragheb and Hardwidge, if used in the propulsion system of a nuclear submarine, it can increase its power to weight ratio and consequently its attainable speed by 30 percent for the same reactor power. The weight reduction makes it also suitable for space power applications. Other gases such as aluminum chloride and aluminum bromide can be used.

Table 5: Candidate dissociating gas systems.

Dissociating gas	Increase factor in gas constant	Thermal release from reaction [Kcal/mole]	Temperature Range ^{o}C
$N_2O_4 \rightleftharpoons 2NO_2$	2	13.7	25-170
$2NO_2 \rightleftharpoons 2NO + O_2$	1.5	27.0	140-850
$Al_2Br_6 \rightleftharpoons 2AlBr_3$	2	30.0	300-1,400
$Al_2Cl_6 \rightleftharpoons 2AlCl_3$	2	29.8	200-1,100
$Al_2I_6 \rightleftharpoons 2AlI_3$	2	26.4	230-1.200
$2NOBr \rightleftharpoons 2NO + Br_2$	1.5	-	25-500
$2NOCl \rightleftharpoons 2NO + Cl_2$	1.5	-	25-900
$Al_2Cl_6 + 4Al(liquid) \rightleftharpoons 6AlCl$	6	263.8	670-1,200
$Al_2Br_6 + 4Al(liquid) \rightleftharpoons 6AlBr$	6	282.4	670-1,400
$Al_2I_6 + 4Al(liquid) \rightleftharpoons 6AlI$	6	196.4	670-1,300
$HgCl_2 + Hg(liquid) \rightleftharpoons 2HgCl$	2	70.4	280-700
$HgBr_2 + Hg(liquid) \rightleftharpoons 2HgBr$	2	63.7	250-700
$SnCl_4 + Sn(liquid) \rightleftharpoons 2SnCl_2$	2	38.6	-
$SnBr_4 + Sn(liquid) \rightleftharpoons 2SnBr_2$	2	65.3	-
$Ga_2Cl_6 \rightleftharpoons 2GaCl_3$	2	20.0	10-1,000
$Ga_2Br_6 \rightleftharpoons 2GaBr_3$	2	18.5	150-1,200
$Ga_2I_6 \rightleftharpoons 2GaI_3$	2	11.0	250-1,300
$Ga_2Cl_6 + 4Ga(liquid) \rightleftharpoons 6GaCl$	6	58.8	100-1,000

Table 6: Charateristics of different turbines using steam and dissociating gases.

	Working Fluid			
	H_2O Steam Turbine	H_2O Steam Turbine	Al_2Cl_6 Gas Turbine	Al_2Br_6 Gas Turbine
Output, MWe	500	300	555	340
Pressure, turbine inlet, ata	240	240	80	80
Temperature, turbine inlet, ^{o}C	580	580	600	750
Pressure, turbine exhaust, ata	0.035	0.035	5	5
Mass flow rate, metric tonne/hr	1,495	880	17,900	21,900
Turbine revolutions, rpm	3,000	3,000	3,000	3,000
Number of exhausts	4	3	2	4
Total number of turbine stages	42	39	6	12
Mean diameter of last stage, m	2.550	2.480	1.338	0.915

Height of last stage blade, m	1.050	0.960	0.495	0.250
Internal efficiency				
High pressure cylinder	-	80.0	89.9	90.0
Intyermediate pressure cylinder	-	89.5	-	-
Low pressure cylinder	-	82.0	-	-
Number of turbine shafts	1	1	1	1
Turbine length, m	29.1	21.3	9.0	7.6
Weight of turbine, metric tonnes	964	690	55	90
Power to weight ratio, [MWe/Metric tonne]	0.52	0.43	10.09	3.78

The dramatic advantage of using dissociating gases is a reduced size and weight in the turbo machinery. A 500 MWe steam turbine would measure 21.3 meters in length compared with just 9 meters for a 555 MWe Al_2Cl_6 turbine. This is associated with an increase by a factor of 10.09/0.52 = 19.4 in the power to weight ratio. The reduced weight in the other associated heat transfer equipment makes dissociating gases a promising choice for space, naval propulsion, space applications, as well as central station applications.

KALINA CYCLE

The Kalina cycle can be used in nuclear power applications increasing the efficiency up to 30 percent. It is simple in design and can use readily available, off the shelf components. It is similar to the Rankine cycle except that it heats two fluids, such as a mixture of ammonia and water, instead of one. The dual component vapor consisting for instance of 70 percent ammonia and 30 percent water is directed to a distillation subsystem which creates three additional mixtures. One is a 40/60 mixture, which can be completely condensed against a normal cooling source. After condensing, it is pumped to a higher pressure, where it is mixed with a rich vapor produced during the distillation process. This recreates the 70/30 working fluid. The elevated pressure completely condenses the working fluid and returns it to the heat exchanger to complete the cycle. The mixture's composition varies throughout the cycle with the advantages of variable temperature boiling and condensation, and a high level of recuperation. Its main use has been so far in geothermal heat extraction.

1.6 ENTROPY AND THE TEMPERATURE-ENTROPY T-S DIAGRAM

For an internally reversible process the change in entropy is related to the absolute temperature and change in heat transferred as:

$$dS = \frac{dQ}{T} \tag{4}$$

or:

$$\Delta S = S_2 - S_1$$

$$= \int \frac{dQ}{T} \qquad (5)$$

where: dQ = Heat transferred in [BTU/lb$_m$] or [kJoules/kg]

T = Temperature in degrees Rankine[^oR=460+^oF] or degrees Kelvin [^oK=273+^oC]

S = Entropy in [BTU/(lb$_m$.^oR)] or [kJoules/(kg.^oK)]

From Eqn. 5, we can deduce that:

$$Q = \int_1^2 T dS \qquad (6)$$

A Temperature-Entropy or T-S diagram shown in Fig. 4 or thermodynamic tables can thus be used to calculate the heat Q.

1.7 THE MOLLIER CHART: ENTHALPY-ENTROPY DIAGRAM

A fluid possesses stored internal energy u [kJ/kg] due to the internal potential and kinetic energy of its molecules.

That fluid passing through a system's boundary possesses an energy quantity entering or leaving the system denoted as flow energy:
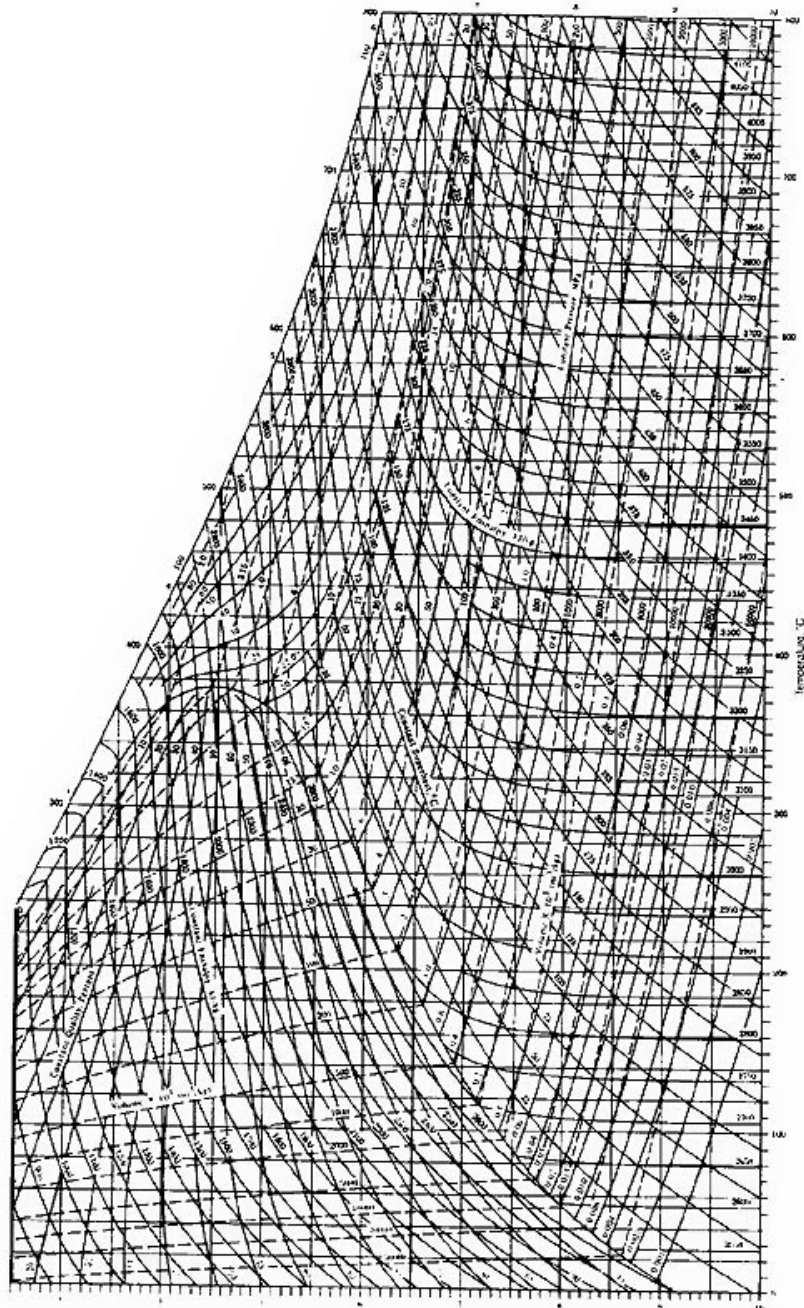
Fig. 4: Temperature-Entropy, or T-S diagram for water in SI units.
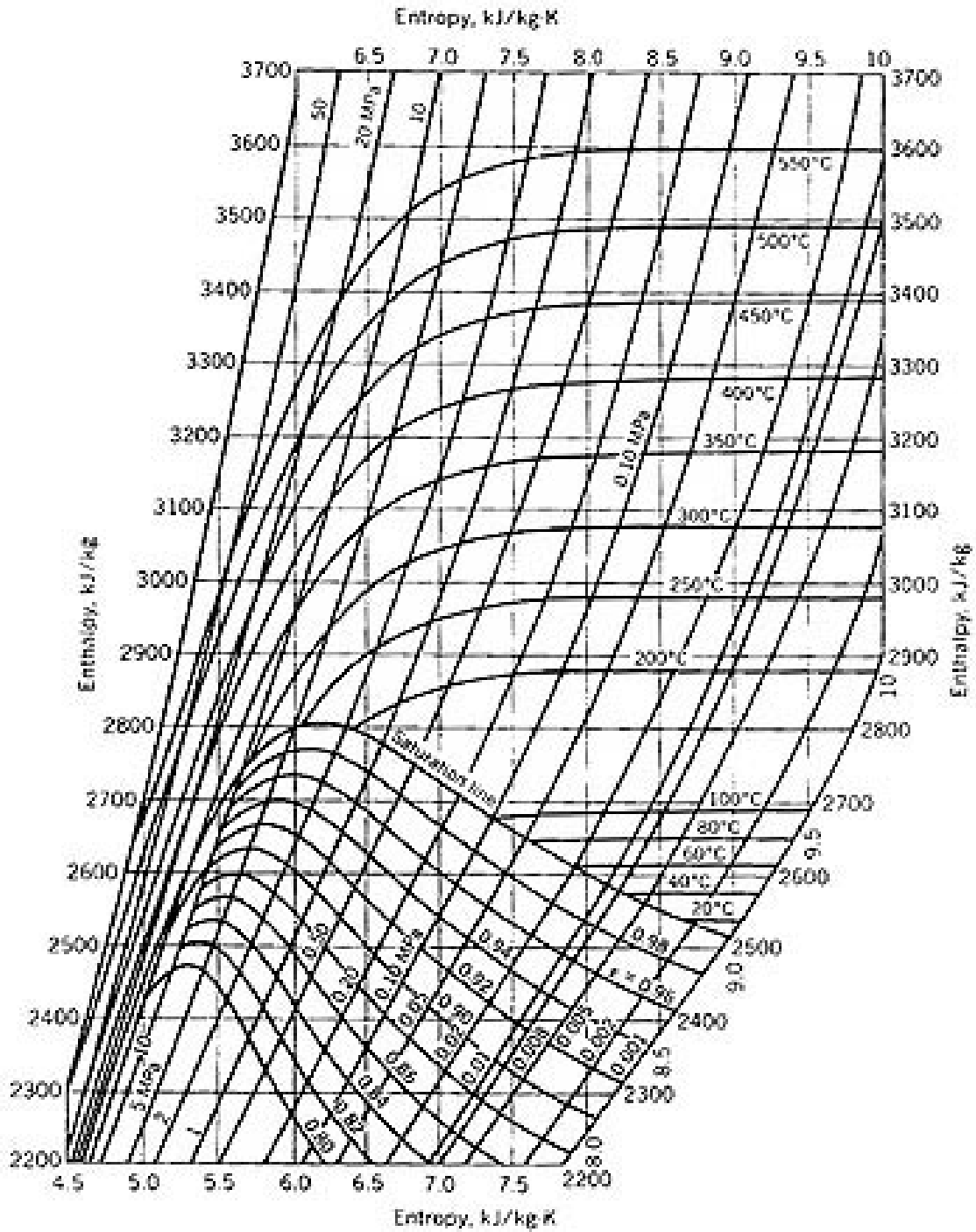
Fig. 5: Mollier, Enthalpy-Entropy, or H-S diagram for water in SI units.

$$\text{Flow Energy} = PV = \frac{P}{\rho} \qquad (7)$$

where: P is the pressure of the fluid

       V is the specific volume of the substance

       ρ is the density of the substance.

Because it often happens in flow problems, the sum of the flow energy and internal energy, using Joule's constant J, is designated as the enthalpy h:

$$h = u + \frac{PV}{J} \text{ [kJ/kg]} \tag{8}$$

To obtain the values of the enthalpy, the thermodynamic tables or the Mollier Chart or Enthalpy-Entropy diagram shown in Fig. 5 are used.

1.8 CARNOT CYCLE EFFICIENCY

A thermodynamic cycle is defined as a series of processes during which a substance starts in a certain state and returns to its initial state. Heat is added to the substance as Q_a and heat is extracted at a heat sink as Q_r. A heat engine such as a turbine generates work as $W_{turbine}$, and the fluid is circulated through the system using a pump or a compressor as W_{pump}. In power production, the overall thermal efficiency is defined as:

$$\begin{aligned}
\eta_{th} &= \frac{\text{Net Work Output}}{\text{Heat Input}} \\
&= \frac{W_{net}}{Q_a} \\
&= \frac{W_{turbine} - W_{pump}}{Q_a} \\
&= \frac{Q_a - Q_r}{Q_a}
\end{aligned} \tag{9}$$

The Carnot Cycle is an idealized power production process. It is the most efficient cycle that is conceivable and can be used as the standard of comparison for all other heat engines. It consists of four reversible processes, as shown in Fig. 6 for an idealized Boiling Water Reactor (BWR):

1. Isothermal heat addition at the absolute temperature T_a, from point 1 to point 2.
2. Isothermal heat rejection at the absolute temperature T_r, from point 3 to point 4.
3. Isentropic or constant entropy expansion in a turbine from point 4 to point 1.
4. Isentropic or constant compression in a pump or compressor from point 4 to point 1.

Considering that the heat addition and rejection can be expressed in terms of the change in entropy as:

$$Q_a = T_a . \Delta S$$
$$Q_r = T_r . \Delta S$$

(10)

In this case the overall thermal efficiency can be deduced by substituting from Eqn. 10 into Eqn. 9 as:

$$\eta_{Carnot} = \frac{T_a . \Delta S - T_r . \Delta S}{T_a \Delta S}$$

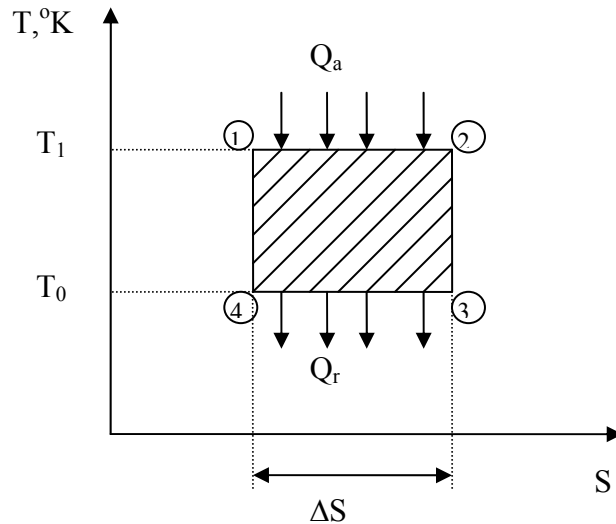$$= \frac{T_a - T_r}{T_a}$$

(11)

$$= 1 - \frac{T_r}{T_a}$$
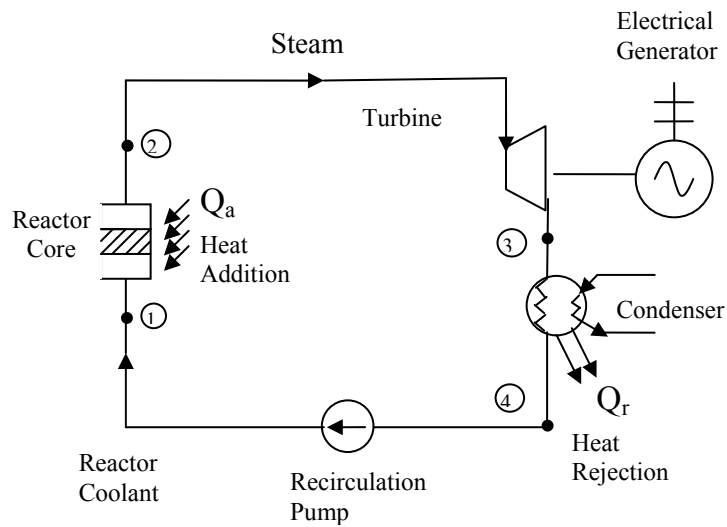
Fig. 6: Carnot Cycle efficiency for an idealized BWR system.

The Carnot Cycle efficiency is less than unity. It increases as the difference between the heat addition temperature and the heat rejection temperature is increased. Thus increasing the heat addition temperature and decreasing the heat rejection temperature becomes an objective of any heat engine design.

There is limited control on the heat rejection medium such as air in a cooling tower, or a body of water such as a pond, lake, river or the ocean. Increasing the heat addition temperature becomes the major engineering objective, justifying the use high temperature materials such as ceramics such as uranium dioxide (UO_2), or uranium carbide (UC) as nuclear reactor fuels.

The upper limit is reached according to the maximum temperature that materials can safely withstand without deterioration in their mechanical properties. It also depends on our ability to design efficient cooling systems that use materials at high temperatures without a loss of their mechanical properties or before catastrophic failure, melting or vaporization.

In nuclear applications, this suggests for instance the use of graphite as a moderator material in gas cooled reactors instead of water in light water reactors. This is associated with an increase in the overall thermal efficiency from about 33 percent in light water reactors to about 40 percent is gas-cooled reactors.

1.9 THE REVERSIBLE SATURATED STEAM RANKINE CYCLE

The internally reversible Rankine cycle for an idealized Pressurized Water Reactor (PWR) is shown on the T-S diagram in Fig. 7. The saturated steam at point 1 expands to point 2 where it is condensed to point 3 then pumped to point B. There heat is added in the feed water heater from point B to point 4, where it enters in the heat exchanger at 4. Heat is then added in he heat exchanger from point 4 to point 1. Notice that even though the heat addition in the core is at a temperature T_A, the working medium is actually receiving the heat at a temperature T_1.
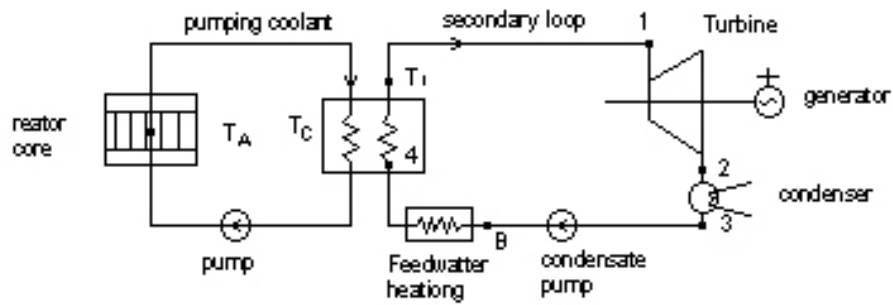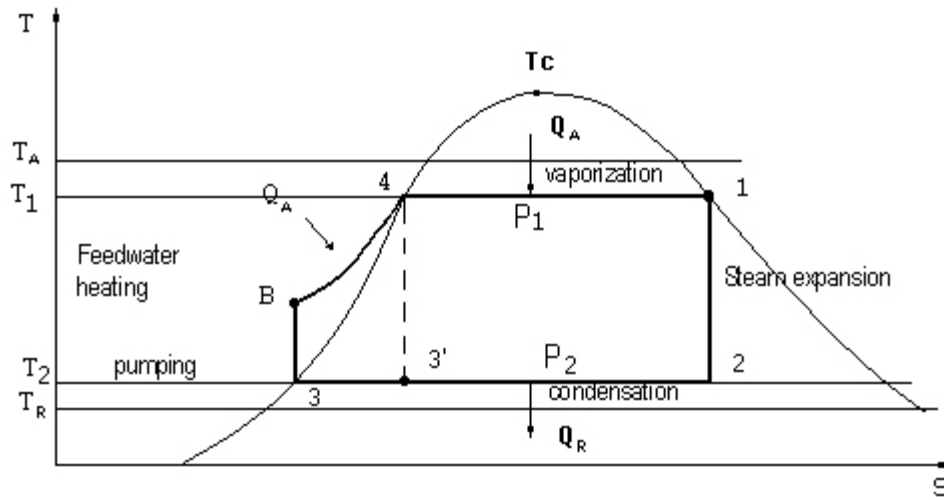
Fig. 7: Rankine reversible saturated Steam Cycle for a PWR system.

The overall thermal efficiency in terms of the enthalpy of this cycle becomes;

$$\eta_{Rankine} = \frac{W_{turbine} - W_{pump}}{Q_a}$$
$$= \frac{(h_1 - h_2) - (h_B - h_3)}{h_1 - h_B} \tag{12}$$

Normally for a liquid coolant the pumping work is negligible, thus:

$$h_B \approx h_3 \tag{13}$$

Substituting from Eqn. 11 into Eqn. 10 we get:

$$\eta_{Rankine} \simeq \frac{h_1 - h_2}{h_1 - h_3}$$ (15)

1.10 USING THE MOLLIER CHART

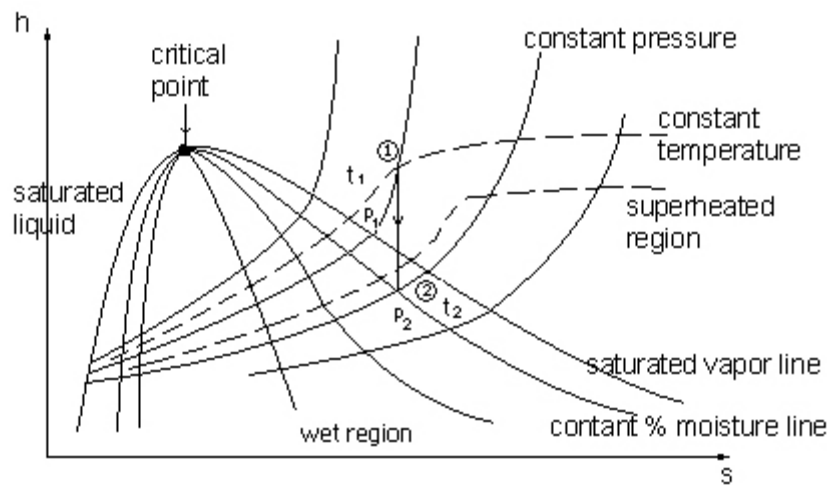To obtain values of the enthalpy using the Mollier Chart the following approach can be followed as shown in Fig. 8.

Fig. 8: Use of the Mollier Chart.

1. If the steam pressure p_1 and the steam temperature t_1 are known, this determines point 1 on the diagram.
2. If the steam quality x_2 at exhaust and the steam pressure p_2 at exhaust are known, then point 2 can be located on the chart.
3. Following the P_2 = constant line we can get point 3 on the saturated liquid line, so that we can estimate the efficiency from Eqn. 12.

1.11 INCREASING THE EFFICIENCY OF THE RANKINE CYCLE

Several approaches present themselves for increasing the thermal efficiency of the Rankine cycle. As shown in Fig. 9 the reference thermal efficiency is given by:

$$\eta_{th} = \frac{W_{net}}{Q_A} = \frac{area\ 1234}{area\ 12356}$$
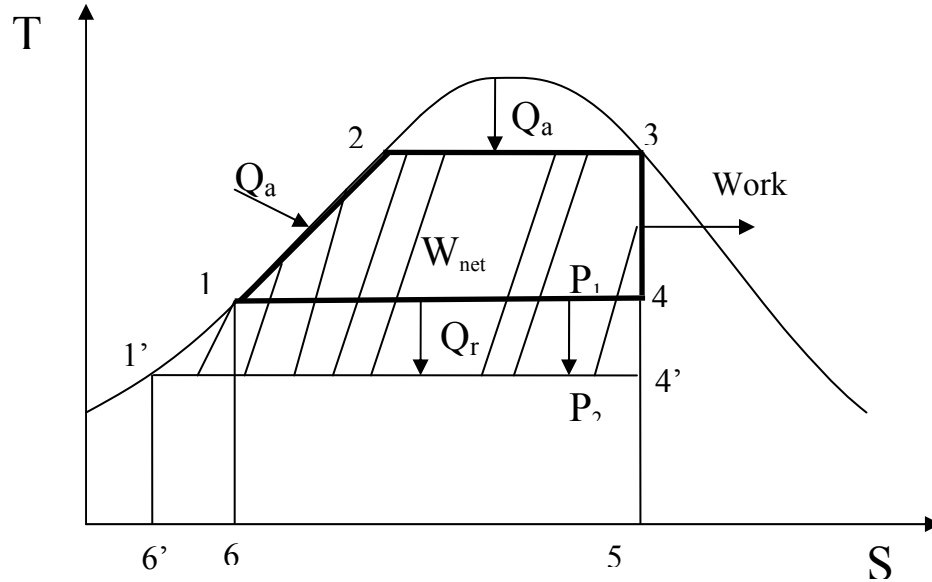

Fig. 9: Enhancing the efficiency of the Rankine cycle through lowering of the condenser pressure.

1.   Lowering the condenser pressure as shown in Fig. 9 leads to a new value of the thermal efficiency:

$$\eta_{th}^1 = \frac{area\ 1'234'}{area\ 1'2356'}$$

This clearly leads to an enhanced thermal efficiency.
2.   Raising the heat exchanger pressure as shown in Fig. 10 leads to the new value of the thermal efficiency:

$$\eta_{th}^2 = \frac{area\ 12'3'4'}{area\ 12'3'5'6}$$

In this case the new efficiency may be larger or lower than the initial efficiency.
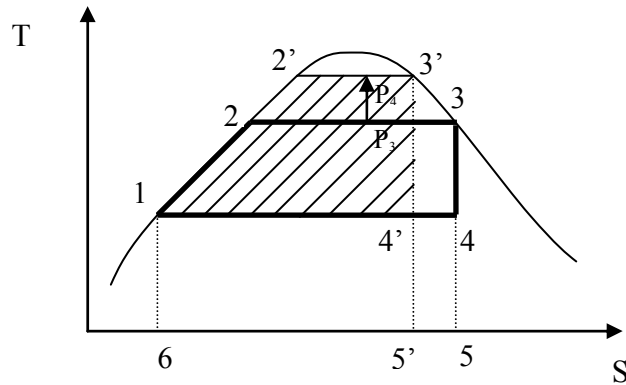
Fig. 10: Enhancing efficiency of the Rankine steam cycle through raising the heat exchanger pressure.
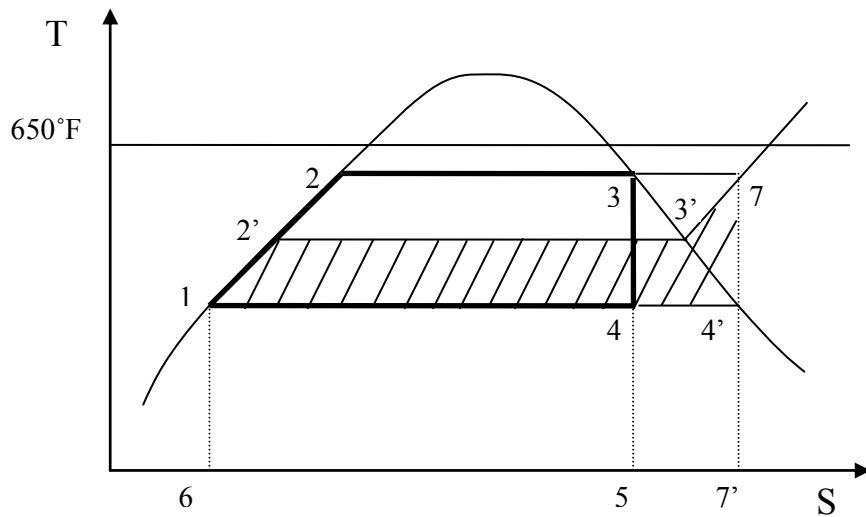

Fig. 11: Enhancing efficiency of the Rankine steam cycle using superheat.

3.   The use with superheat with a temperature limitation. According to Fig. 11 the thermal efficiency becomes:

$$\eta_{th}^3 = \frac{area\ 12'3'74'}{area\ 12'3'77'6}$$

Again in this case the new efficiency may be larger or smaller than the initial efficiency.

The advantage of the use of superheating is that we obtain a higher quality steam at the turbine at point 4' instead of point 4. Only if there is not a limitation on the heat addition temperature should we expect that;

$$\eta_{th}^3 > \eta_{th}$$

1.12 TWO STAGE REGENERATIVE RANKINE CYCLE

We consider a pressurized water reactor steam cycle where the steam exiting the steam generator enters the turbine and is partially expanded. As it expands, the steam is bled to heat the water in two feed water heaters before it reenters the steam generator.
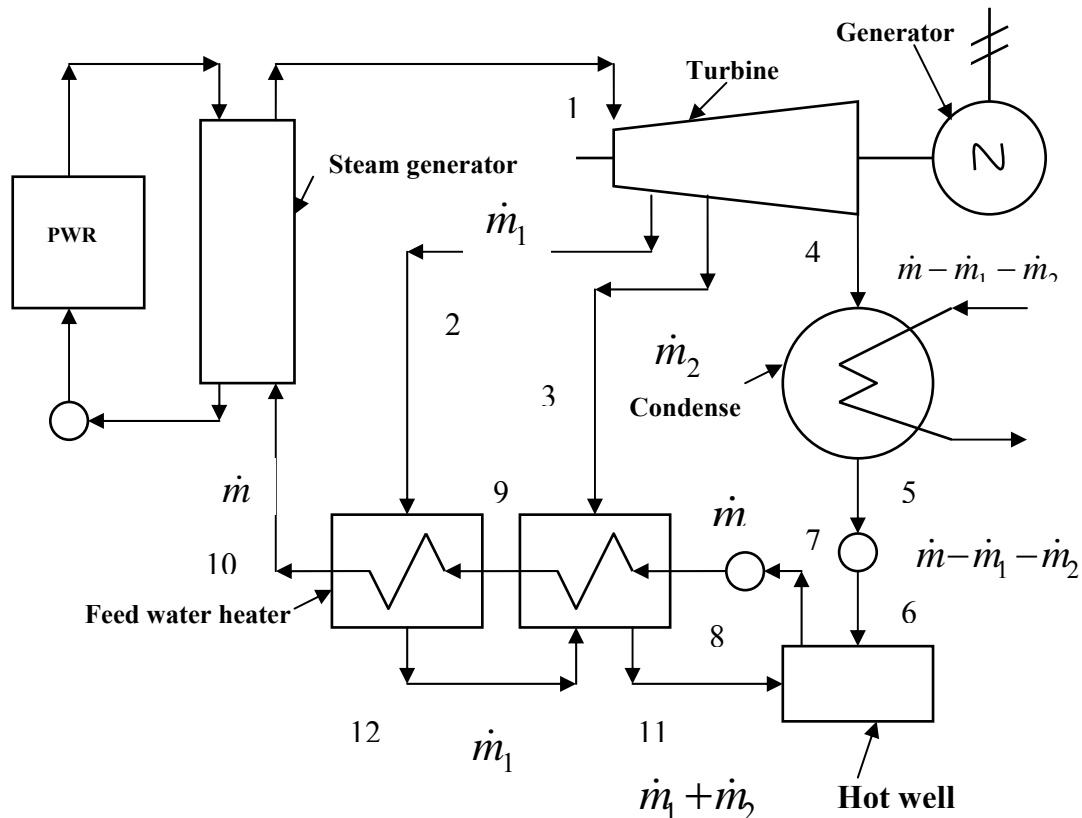

Fig. 12: Two stage Pressurized Water Reactor Rankine cycle with regeneration.

The condensate from the first heater is fed back through a steam trap to the previous heater and then to the hot well of the condenser.

The remaining steam is used to drive the electrical generator then is fed to the condenser and its hot well.

We can perform energy and mass balances on the individual components of the system. In terms of the mass flow rates and the enthalpies, the entering energy into the first feedwater heater is:

$$Q_1 = \dot{m}h_9 + \dot{m}_1 h_2$$

The exiting energy is:

$$Q_2 = \dot{m}h_{10} + \dot{m}_1 h_{12}$$

Equating those two energies we get:

$$Q_1 = Q_2$$
$$\dot{m}h_9 + \dot{m}_1 h_2 = \dot{m}h_{10} + \dot{m}_1 h_{12} \tag{16}$$

The quantities in this equation are usually known except for the bleeding mass flow rate $\dot{m}_1$ which can be calculated from:

$$\dot{m}_1 = \dot{m}\frac{h_{10} - h_9}{h_2 - h_{12}} \tag{17}$$

The energy balance on the second feedwater heater and the hot well taken together yields:

$$\dot{m}_2 h_3 + \dot{m}_1 h_{12} + (\dot{m} - \dot{m}_1 - \dot{m}_2)h_5 + W_p = \dot{m}h_9 \tag{18}$$

where Wp is the total power of the pumps in the system.

All quantities in this equation are known except for the bleeding mass flow rate $\dot{m}_2$ which can be estimated from:

$$\dot{m}_2 = \frac{\dot{m}(h_9 - h_5) - \dot{m}_1(h_{12} + h_5) - W_{pump}}{h_3 - h_5} \tag{19}$$

The power generated by the turbine can be written as:

$$W_{turbine} = \dot{m}(h_1 - h_2) + (\dot{m} - \dot{m}_1)(h_2 - h_3) + (\dot{m} - \dot{m}_1 - \dot{m}_2)(h_3 - h_4) \tag{20}$$

The net work done is:

$$W_{net} = W_{turbine} - W_{pump} \tag{21}$$

The heat added in the heat exchanger is:

$$Q_a = \dot{m}(h_1 - h_{10}) \tag{22}$$

It follows that the thermal efficiency of the regenerative cycle is:

$$\eta_{\text{Regenerative}} = \frac{W_{net}}{Q_a}$$

$$= \frac{W_{turbine} - W_{pump}}{\dot{m}(h_1 - h_{10})} \tag{23}$$

$$= \frac{\dot{m}(h_1 - h_2) + (\dot{m} - \dot{m}_1)(h_2 - h_3) + (\dot{m} - \dot{m}_1 - \dot{m}_2)(h_3 - h_4) - W_p}{\dot{m}(h_1 - h_{10})}$$

1.13 THE GAS TURBINE OR BRAYTON CYCLE

INTRODUCTION

The gas turbine or Brayton cycle is under consideration for future nuclear power plants. The higher achievable temperatures imply operation at higher thermal efficiency. In addition, high temperature process heat that they are capable of generating would be useful in the production of hydrogen as a carrier of fission energy. It has also been suggested that air cooling can be used in location where a shortage exists of water for cooling.
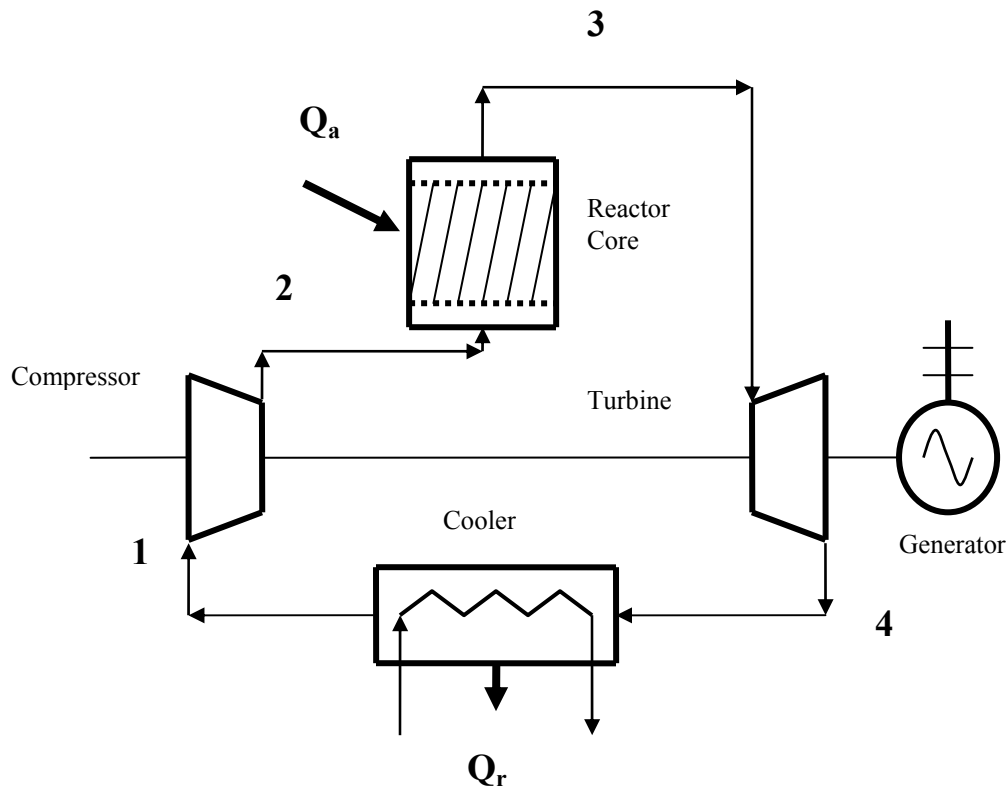
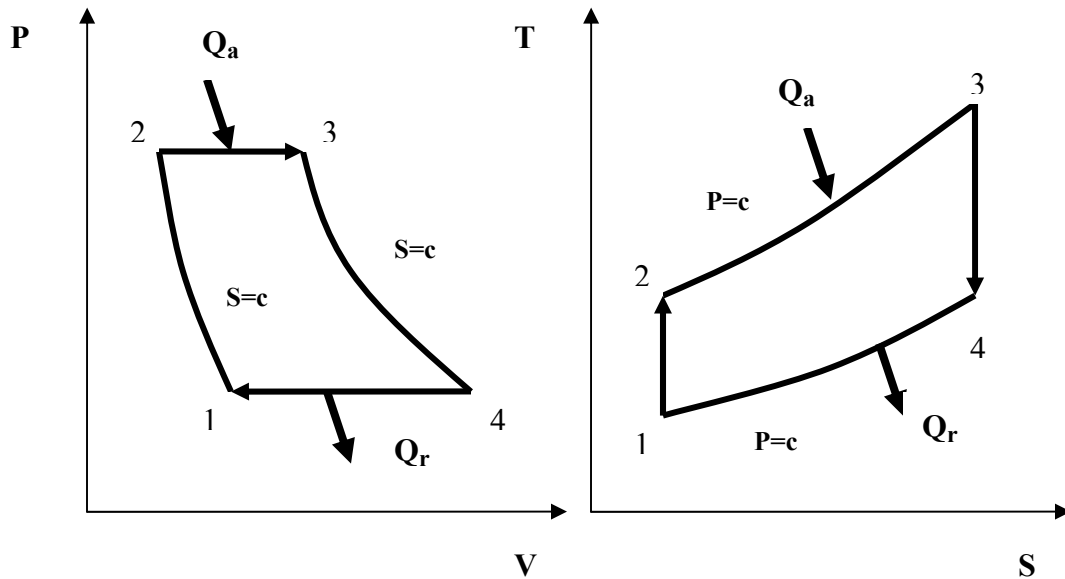**Fig. 13: The ideal direct gas turbine or Brayton cycle.**

Fig. 14: Brayton cycle PV and TS diagrams.

In the ideal Brayton cycle, a high pressure, high temperature gas such as helium undergoes an adiabatic and isentropic expansion through a turbine to a lower pressure and temperature state. After expansion, heat is removed from the gas at constant pressure in a cooler. The gas is then compressed in a compressor adiabatically and isentropically, before entering the reactor core where it receives the heat input at a constant pressure.
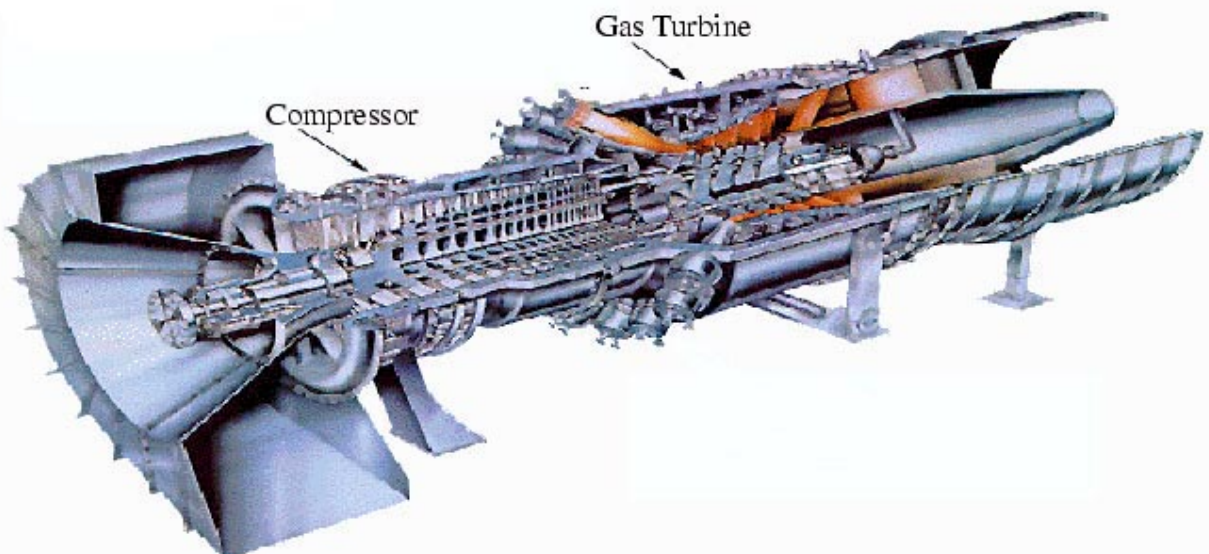
Fig. 15: Open cycle gas turbine and compressor set producing 150 MWe at 50 percent efficiency.

Assuming that the potential and kinetic energy losses are negligible, the turbine work is given by the gas enthalpy change in the turbine:

$$W_{turbine} = h_3 - h_4 \tag{24}$$

The compressor work is:

$$W_{compressor} = h_2 - h_1 \tag{25}$$

The heat added to the gas is:

$$Q_a = h_3 - h_2 \tag{26}$$

The overall thermal efficiency of the ideal Brayton cycle in terms of enthalpies becomes:

$$\eta = \frac{W_{net}}{Q_a} = \frac{W_T - W_C}{Q_a} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} \tag{27}$$

USE OF THE GAS TABLES

If one needs to use the Gas Tables, two independent thermodynamics properties at each state must be specified. The materials and design limitations are usually expressed in terms of the pressure and temperature values.

Assuming a perfect gas, the enthalpy differences can be expressed in terms of temperature differences. The work done by the turbine can the written as:

$$W_{turbine} = h_3 - h_4 = c_p(T_3 - T_4) = c_p T_3 (1 - \frac{T_4}{T_3})$$

c_p = gas specific heat at constant pressure assumed constant   (28)

T = absolute temperature in $^\circ$K.

The expansion through the turbine is assumed to be isentropic and adiabatic. For such an expansion the perfect gas law applies:

$$PV^\gamma = \text{constant}$$

$$\gamma = \frac{c_p}{c_v} \tag{29}$$

c_p = specific heat at constant pressure

c_v = specific heat at constant volume

Table 7 shows the specific heat and γ for some possible potential gaseous coolants.

Table 7: Specific heat and γ for gaseous coolants at 68 ^oF.

Gas	Specific heat, c_p [BTU/(lb$_m$.^oF)]	γ [c_p/c_v]
Hydrogen, H_2	3.420	1.405
Helium, He	1.250	1.659
Carbon dioxide, CO_2	0.202	1.290
Air	0.240	1.400
Nitrogen, N_2	0.248	1.400

From Eqn. 20, we can write:

$$P_3 V_3^\gamma = P_4 V_4^\gamma \tag{30}$$

The ideal gas law, not to be confused with the perfect gas law, is:

$$PV = nRT \tag{31}$$

For n=1 it becomes:

$$PV = RT \tag{32}$$

From the ideal gas law we can write:

$$P_3 V_3 = RT_3$$
$$P_4 V_4 = RT_4 \tag{33}$$
$$\frac{T_4}{T_3} = \frac{P_4 V_4}{P_3 V_3}$$

From Eqn. 18 we can write:

$$P_3 V_3{}^{\gamma} = P_4 V_4{}^{\gamma}$$

$$\left(\frac{V_4}{V_3}\right)^{\gamma} = \frac{P_3}{P_4} \tag{34}$$

$$\frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{\frac{1}{\gamma}} = \left(\frac{P_4}{P_3}\right)^{-\frac{1}{\gamma}}$$

Substituting from Eqn. 25 into Eqn. 24 we get:

$$\frac{T_4}{T_3} = \frac{P_4 V_4}{P_3 V_3} = \frac{P_4}{P_3}\left(\frac{P_4}{P_3}\right)^{-\frac{1}{\gamma}} = \left(\frac{P_4}{P_3}\right)^{1-\frac{1}{\gamma}} = \left(\frac{P_4}{P_3}\right)^{\frac{(\gamma-1)}{\gamma}} \tag{35}$$

The work in the turbine can be rewritten from Eqns. 18 and 26 as:

$$W_{turbine} = c_p T_3\left(1 - \frac{T_4}{T_3}\right) = c_p T_3\left[1 - \left(\frac{P_4}{P_3}\right)^{\frac{(\gamma-1)}{\gamma}}\right] = c_p T_3\left[1 - \frac{1}{\left(\frac{P_3}{P_4}\right)^{\frac{(\gamma-1)}{\gamma}}}\right] \tag{36}$$

The compressor work can similarly be written as:

$$W_{compressor} = h_2 - h_1 = c_p(T_2 - T_1) = c_p T_2\left(1 - \frac{T_1}{T_2}\right)$$

$$= c_p T_2\left(1 - \frac{T_1}{T_2}\right) = c_p T_2\left[1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{37}$$

$$= c_p T_2\left[1 - \frac{1}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}\right]$$

The compressor pressure ratio is defined as:

$$r_p = \frac{p_2}{p_1} \tag{38}$$

In actual cycles, the turbine pressure ratio is less than in the compressor because of the pressure losses in the heating and the cooling processes. However, for simplicity we can assume that those losses are negligible with the pressure ratio in the turbine equal to the pressure ratio in the compressor:

$$r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4} \qquad (39)$$

The Turbine and compressor work in the turbine and compressor in term of the pressure ratio become:

$$W_{turbine} = c_p T_3 [1 - \frac{1}{(r_p)^{\frac{(\gamma-1)}{\gamma}}}] \qquad (40)$$

$$W_{compressor} = c_p T_2 [1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}] \qquad (41)$$

The net work in the cycle becomes:

$$W_{net} = W_{turbine} - W_{compressor}$$
$$= c_p (T_3 - T_2)[1 - \frac{1}{(r_p)^{\frac{(\gamma-1)}{\gamma}}}] \qquad (42)$$

The heat addition to the cycle is:

$$Q_a = h_3 - h_2 = c_p (T_3 - T_2) \qquad (43)$$

Thus the thermal efficiency of the cycle is:

$$\eta = \frac{W_{net}}{Q_a} = \frac{c_p (T_3 - T_2)[1 - \frac{1}{(r_p)^{\frac{(\gamma-1)}{\gamma}}}]}{c_p (T_3 - T_2)} = [1 - \frac{1}{(r_p)^{\frac{(\gamma-1)}{\gamma}}}] \qquad (44)$$

The last result shows that the ideal cycle thermal efficiency depends only on the pressure ratio r_p and not on the temperature differences. In the actual cycle is, however, the efficiency is temperature dependent.

THE MAXIMUM THERMAL EFFICIENCY BRAYTON CYCLE

The material limitations impose an upper limit on the achievable upper temperature T_3. To attain maximum thermal efficiency, one must expand the working gas to the lowest possible

temperature T_1, and also compress the gas to the maximum possible temperature T_3. The pressure ratio is given in this case by Eqn. 26 as:

$$r_p = \frac{p_3}{p_1} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

(45)

Substituting in the expression for the thermal efficiency we get:

$$\eta = [1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}] = 1 - \frac{1}{\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1} \cdot \frac{\gamma-1}{\gamma}}} = 1 - \frac{T_1}{T_3}$$

(46)

This reveals that the maximum thermal efficiency Brayton cycle is nothing but a Carnot cycle. However, it is an unrealizable goal since in this case the net work per unit mass of the gas is zero, and an infinite gas flow is needed for a finite amount of work.

OPTIMAL PRESSURE RATIO

An important design consideration is to determine the pressure ratio r_p corresponding to optimal power plant performance. In this case we consider the expression for the net work from Eqn. 39 and rewrite it in term of the maximum gas temperature T_3 and minimum gas temperature T_1.

$$
\begin{aligned}
W_{net} &= c_p(T_3 - T_2)[1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}] \\
&= c_p[T_3 - T_1\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}] \cdot [1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}] \\
&= c_p\{T_3[1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}] - T_1\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}[1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}]\} \\
&= c_p\{T_3[1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}] + T_1[1 - \left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}]\} \\
&= c_p T_1\{\frac{T_3}{T_1}[1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}] + [1 - \left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}]\}
\end{aligned}
$$

(47)

Since:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}} = \left(r_p\right)^{\frac{(\gamma-1)}{\gamma}} \tag{48}$$

For maximum net work we take the derivative of the expression for the net work with respect to the pressure ratio for constant temperatures, and equate it to zero, yielding:

$$\frac{dW_{net}}{dr_p} = c_p T_1 \frac{d}{dr_p} \{\frac{T_3}{T_1}[1 - \frac{1}{\left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}}] + [1 - \left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}]\}$$

$$= c_p T_1 \frac{d}{dr_p} \{\frac{T_3}{T_1}[1 - \left(r_p\right)^{\frac{(1-\gamma)}{\gamma}}] + [1 - \left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}]\}$$

$$= -c_p T_1 [\frac{T_3}{T_1} \frac{(1-\gamma)}{\gamma} \left(r_p\right)^{\frac{(1-\gamma)}{\gamma}-1} + \frac{(\gamma-1)}{\gamma} \left(r_p\right)^{\frac{(\gamma-1)}{\gamma}-1}] \tag{49}$$

$$= -c_p T_1 [-\frac{T_3}{T_1} \frac{(\gamma-1)}{\gamma} \left(r_p\right)^{\frac{(1-2\gamma)}{\gamma}} + \frac{(\gamma-1)}{\gamma} \left(r_p\right)^{-\frac{1}{\gamma}}]$$

$$= 0$$

This results in:

$$[-\frac{T_3}{T_1} \left(r_p\right)^{\frac{(1-2\gamma)}{\gamma}} + \left(r_p\right)^{-\frac{1}{\gamma}}] = 0$$

$$\left(r_p\right)^{-\frac{1}{\gamma}} = \frac{T_3}{T_1} \left(r_p\right)^{\frac{(1-2\gamma)}{\gamma}}$$

$$\frac{\left(r_p\right)^{-\frac{1}{\gamma}}}{\left(r_p\right)^{\frac{(1-2\gamma)}{\gamma}}} = \frac{T_3}{T_1} \tag{50}$$

$$r_p^{\frac{2(\gamma-1)}{\gamma}} = \frac{T_3}{T_1}$$

Hence the optimal compression ratio for maximum net work is:

$$r_{p,opt} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} \tag{51}$$

The ratio $\frac{\gamma}{2(\gamma-1)}$ decreases with increasing γ. Consequently for fixed maximum and minimum gas temperatures, the optimal pressure ratio for monatomic gases such as helium is

lower than for polyatomic gases such carbon dioxide. This results in higher turbine exhaust pressures, and plant sizes are consequently reduced with improved economics from the perspective of the plant's capital cost.

OPTIMAL THERMAL EFFICIENCY OF BRAYTON CYCLE

The thermal efficiency corresponding to maximum net work becomes from Eqns. 44 and 51 is:

$$
\begin{aligned}
\eta_{opt} &= [1 - \frac{1}{\left(r_{p,opt}\right)^{\frac{(\gamma-1)}{\gamma}}}] \\
&= [1 - \frac{1}{\left(\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}\right)^{\frac{(\gamma-1)}{\gamma}}}] \\
&= [1 - \frac{1}{\left(\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}\right)^{\frac{(\gamma-1)}{\gamma}}}] \\
&= 1 - \left(\frac{T_1}{T_3}\right)^{\frac{1}{2}}
\end{aligned}
\tag{52}
$$

The appearance of the square root makes that expression different from the one for the Carnot cycle efficiency.

OPTIMAL GAS TEMPERATURE AFTER COMPRESSION

It is of interest to estimate the gas temperature after compression, T_2. From Eqn. 42:

$$
\left(r_{p,opt}\right)^{\frac{(\gamma-1)}{\gamma}} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2}}
\tag{53}
$$

However, from Eqn. 39:

$$
\frac{T_2}{T_1} = \left(r_p\right)^{\frac{(\gamma-1)}{\gamma}}
\tag{54}
$$

Equating the last two equations yields:

$$\left(r_{p,opt}\right)^{\frac{(\gamma-1)}{\gamma}} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2}} = \frac{T_2}{T_1} \tag{55}$$

Thus:

$$T_2 = \sqrt{T_1 T_3} \tag{56}$$

This reveals that the gas temperature after compression for optimal net work is the geometric mean of the highest gas temperature at turbine inlet and the lowest gas temperature at compressor inlet.

IMPROVING THE BRAYTON CYCLE'S EFFICIENCY

Methods similar to those used to enhance the efficiency in the steam cycle can be used in the gas turbine cycle. Reheat is used to increase the work output. Regenerative heat exchange between the turbine exhaust gas and compressor outlet reduces the heat input. Intercooling between the compressor stages reduces the compressor work.

1.14 STIRLING CYCLE

INTRODUCTION

The Stirling cycle engine, also called an external-heat engine differs from the Rankine or steam cycle in that it uses a gas, such as air, helium, or hydrogen, instead of a liquid, as its working fluid. Fission energy using a heat pipe or radioisotope heat, provide external heat to one cylinder. This causes the gas to alternately expand and contract, moving a displacer piston back and forth between a heated and an unheated cylinder.
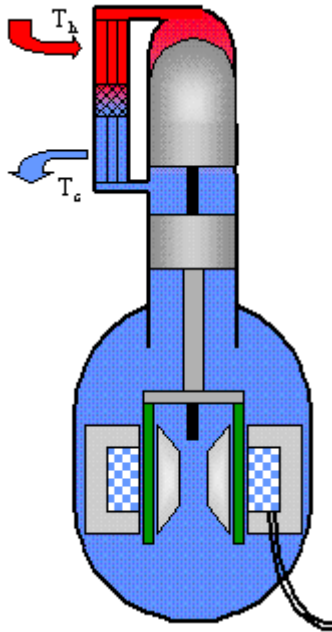
Fig. 16: Free piston Stirling cycle power converter for electrical generation.

HISTORY

In 1816, Dr. Robert Stirling (1790-1878), a preacher in Scotland obtained a patent for his "Heat Economizer." He described how to use a power piston in combination with a separate displacer, linked together on the same shaft with air as a working fluid. He explained how to use a regenerator or heat economizer, which he places between the hot and cold ends of the displacer cylinder. The cycle he described was of a closed nature, meaning that the working fluid is not exchanged during the cycle's operation. The heat required to drive the cycle is provided from the outside rather than from an internal combustion process.

The internal volumes consist of a hot area for heat addition and a cold one for heat rejection. The two areas are separated by an insulated piston. The volumes are connected with a bypass that keeps the pressure in the two volumes equal and encloses a regenerator or economizer. The regenerator stores some of the energy when the hot air is transferred from the hot volume to the cold volume. The energy is then transferred back to the cold air when it returns to the hot volume. This minimizes the heat rejected and helps maximize the thermal efficiency of the cycle.

The Ford Motor Company and Phillips developed in the 1970s an automotive Sterling engine that powered the Ford Torino model. The Sterling cycle has been used for generating electrical power in remote locations.

The ideal Stirling cycle consists of two isothermal or constant temperature and two isochoric or constant volume processes.

POWER CYCLE

The engine consists of two pistons one is a displacer piston, the other a power piston, and a regenerator. Using appropriate areas and pressures, the displacer drive rod can become unloaded and act in a self driving operation. With the addition of a linear alternator as a load, electrical power can be generated. With appropriate masses, spring rates and damping, or dynamic tuning, the converter can resonate as a free piston Stirling converter.

The ideal Stirling cycle consists of four distinct processes:

Process 1-2 is an isothermal compression process. Work W_C is done on the system, while an equal amount of heat Q_C is rejected by the system to the cooling medium at a constant temperature T_C.

Process 2-3 is a constant volume displacement process. Heat Q_R is absorbed by the working gas from the regenerator matrix.

Process 3-4 is an isothermal expansion process. Work W_H is done by the system, while an equal amount of heat Q_H is added to the system from the heat source at a constant temperature T_E.

Process 4-1 is a constant volume displacement process. Heat Q_R is rejected by the working gas to the regenerator matrix.

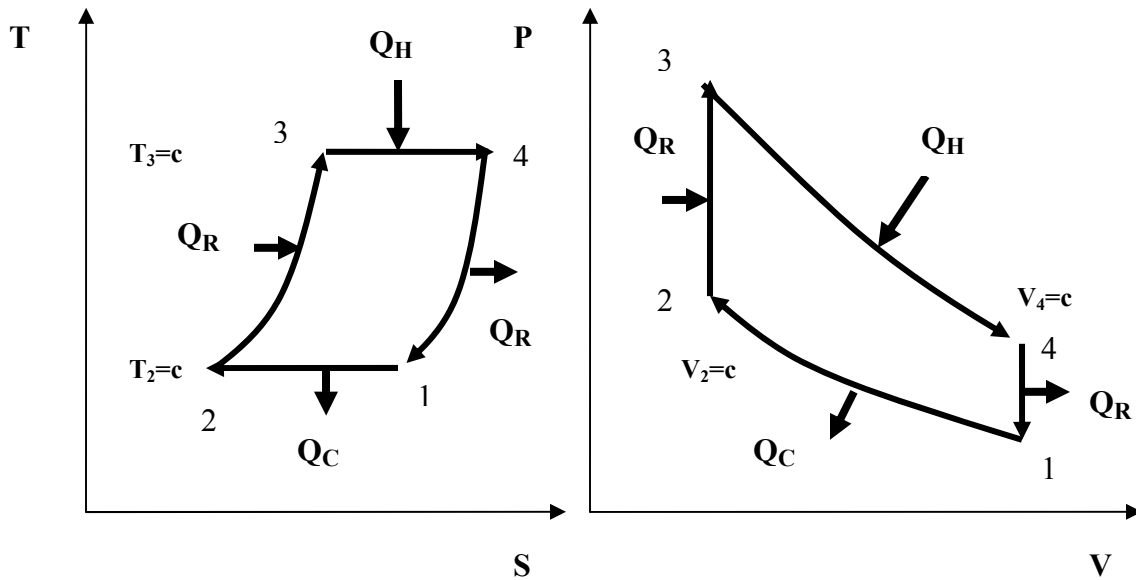**Fig. 17: PV diagram for the Stirling cycle. Compression process 1-2 and expansion process 3-4 are isothermal processes.**

Since the Stirling cycle is a closed cycle, each process can be analyzed separately. The work done for each process can be determined by integration of the area under the PV diagram.

For the isothermal compression process 1-2 the work done is:

$$Q_C = W_C = \int_1^2 PdV \qquad (57)$$

Similarly the work done and heat transfer in the expansion process 3-4 is:

$$Q_H = W_H = \int_3^4 PdV \qquad (58)$$

Since the compression work is negative, the net work done per cycle is: the area within the cycle diagram 1-2-3-4-1. Thus:

$$W_{net} = W_H - W_C \qquad (59)$$

The power becomes the net work done multiplied by the cycle frequency f (Hz or cycles/second):

$$P = fW_{net} = f(W_H - W_C) \qquad (60)$$

To get the heat input during the process 2-3 we use the energy equation with $W_{23} = 0$:

$$Q_R = Q_H - W_{23} = mC_V(T_3 - T_2) \qquad (61)$$

and finally the thermal efficiency is given by:

$$\eta_{th} = \frac{W_{net}}{Q_H} \qquad (62)$$

After noted success in the 19th century, the Stirling cycle was considered impracticable and non competitive with the internal combustion engine. Recently new interest in the cycle has arisen for use with external heat sources such as solar concentrators, radioactive isotopes, or fission reactors equipped with heat pipes.

In the theoretical cycle, the heat transfer between the working fluid and its surroundings is achieved isothermally as in the Carnot cycle, and pressure increase and decrease at constant volume are achieved by internal heat transfer between the two processes.

An ideal regenerator with a 100 percent effectiveness to achieve the heat transfer from process 1-4 to process 2-3. With the areas under the lines 1-4 and 2-3 equal on the TS diagram, the thermal efficiency then is the same as that for the Carnot cycle between the temperatures T_3 and T_2. This should be the case since the Carnot cycle and Sterling cycle are both reversible cycles receiving heat and rejecting heat at two identical temperatures, thus:

$$\eta_{Stirling} = \eta_{Carnot} = \frac{W_{net}}{Q_H} = 1 - \frac{T_2}{T_3} \qquad (63)$$

An advantage of the Stirling cycle over the Carnot cycle is that the work area is larger for the same change in specific volume.

PRACTICAL CYCLE

In real cycles, the achievable cycle efficiency is always lower than the Carnot cycle efficiency due to several reasons. Continuous rather than discontinuous volume changes, temperature losses at the hot and cold surfaces, imperfect isothermal processes with the very short expansion and compression times, friction and piston seals leakages, can reduce the real efficiency. However this particular engine design leaves a large potential for improvements and research to approach the theoretical cycle efficiency.

SPACE APPLICATIONS

An effort to replace the currently employed Radioisotope Thermoelectric Generators (RTGs) with a radioisotope heated 55 Watt(e) Stirling converter is underway to provide power for deep space probes. The higher overall thermal efficiency of the Strirling cycle in excess of 25 percent, will allow the reduction of the isotope inventory in current RTGs.

Conventional convective cooling cannot be used in space. Instead radiative cooling using radiators must be used, adding weight to prospective spacecraft. The use of dissociating gases instead of He can significantly increase the power to weight ratio of the engine.
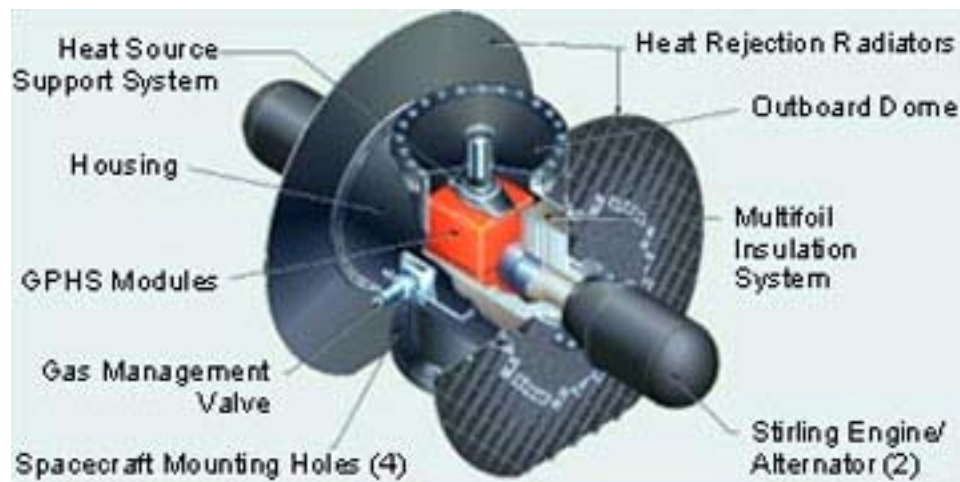

Fig. 18: Stirling cycle engine using radioisotopes as a heat source for space applications, NASA Glenn Research Center..

Fig. 19: Stirling engine powered by a radio isotope heat source, NASA Glenn Research Center..

Work was initiated by the National Aeronautics and Space Administration (NASA) in its Civil Space Technology Initiative (CSTI) to develop a nuclear powered Stirling engine to provide electric power generation in future Lunar or Mars missions.

Different designs are possible to realize a Stirling engine, some of them with rotating crankshafts. For space applications an engine producing power through linear motion is favorable. This is achieved with linear alternators or reversibly operated linear motors. The advantage here is that the engine does not produce any torque which would have to be balanced in order to prevent the whole space from rotating in space.

The number of converters is usually an even number so that the linear momentum produced is balanced. The choice of two or four converters depends on the system's redundancy and specific power needs of the application. The entire unit would be hermetically sealed with stainless steel or Inconel 718 heater heads and flexure bearings that support the pistons with non contacting clearance seals which are used to provide a maintenance free system over its expected lifetime of more than 100,000 hours.

Helium gas at charge pressure of 2.5 MPa is used as a working fluid for its good heat transfer properties. The power piston frequency is 82 Hz with a power piston amplitude of less or equal to 6 mm. The mass of a 55 Watts(e) generator is about 4 kgs. The design temperatures of the heating heads and cold end temperatures are 650 and 60-120 ^{o}C respectively. The actual cold sink temperature to which heat would be radiated in space is considered to be at –40 ^{o}C, which would vary through space.

The nominal thermal heat input is 235 Watts(th), resulting in a thermal efficiency of:

$$\eta_{th} = \frac{Watts(e)}{Watts(th)} = \frac{55\ W(e)}{235\ W(th)} = 23.4\% .$$

This value can be compared to the ideal Carnot cycle efficiency of the Stirling cycle as:

$$\eta_{Stirling} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{273 + 120}{273 + 650} = 1 - \frac{393}{923} = 1 - 0.4258 = 57.42\%$$

The maximum power output is not just dependent on the temperatures of the hot and cold reservoirs, but is also limited by the maximum allowable stroke of the power piston. For a given frequency, the stroke of the pistons determines the produced voltage. To increase the power over a load of constant resistance, the voltage and the stroke must be increased, which can only be achieved within the geometric limits.

1.15 KALINA ENGINE CYCLE

INTRODUCTION

The Kalina cycle engine, which is at least 10 percent more efficient than the other heat engines, is simple in design. This new technology is similar to the Rankine cycle except that it heats two fluids, such as an ammonia and water mixture, instead of just water. Conventional equipment such as steam turbines can be used in the Kalina cycle. The molecular weight of ammonia NH_3 and water H_2O are close at 17 and 18 respectively.

Such a cycle can be applied to low temperature nuclear power plants that would be dedicated to fresh water production.

The closed system technology uses a mixture of water and ammonia rather than water alone to supply the heat recovery system for electricity generation in a power plant. Because ammonia has a much lower boiling point than water, the Kalina Cycle is able to begin spinning the steam turbine at much lower temperatures than typically associated with the conventional steam boiler and turbine systems. The lower boiling point of ammonia allows additional energy to be obtained on the condenser side of the steam turbine.

Instead of being discarded as waste at the turbine exhaust, the dual component vapor as 70 percent ammonia and 30 percent water enters a distillation subsystem. This subsystem creates three additional mixtures. One is a 40/60 mixture, which can be completely condensed against normal cooling sources. After condensing, it is pumped to a higher pressure, where it is mixed with a rich vapor produced during the distillation process. This recreates the 70/30 working fluid. The elevated pressure completely condenses the working fluid and returns it to the heat exchanger to complete the cycle. The mixture's composition varies throughout the cycle. The advantages of this process include variable temperature boiling and condensing, and a high level of recuperation.

A Kalina cycle engine was built in 1991, at the Energy Technology Engineering Center in Canoga Park, California. The power plant may also improve heat engine efficiency through better thermodynamic matching in the boiler and distillation subsystem, and through recuperation of the heat from the turbine exhaust. Data from the operating trials confirmed the principle of the Kalina Cycle technology. The technology is currently being used in geothermal power plants.
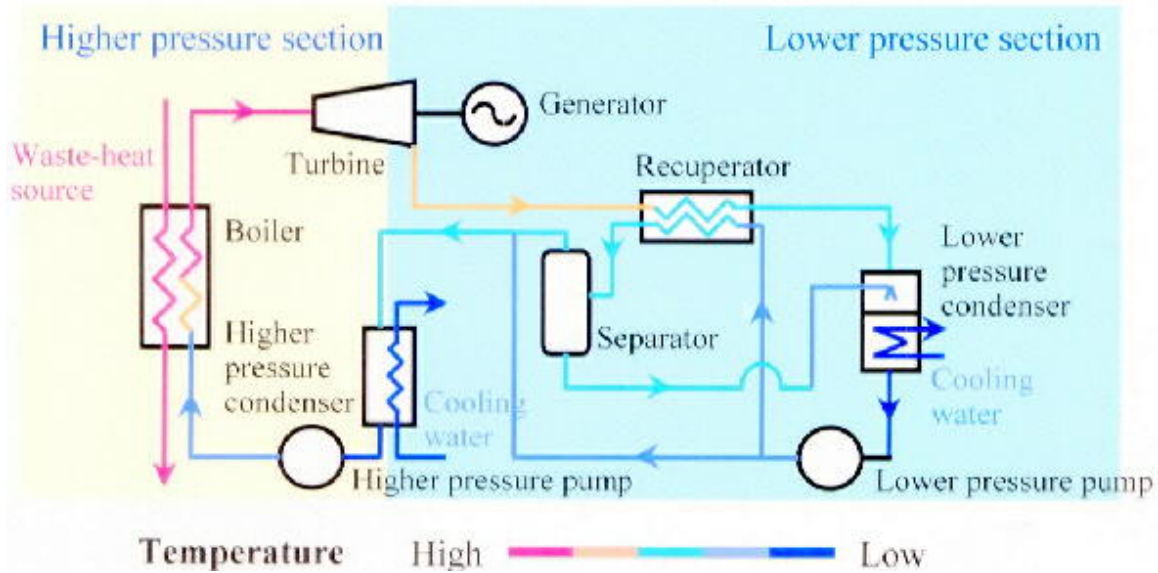
Fig. 20: Flow diagram of a Kalina cycle using a high pressure and low pressure sections and a mixture of two working fluids.

When the Kalina Cycle replaces the Rankine Cycle in any power plant it reduces entropy production significantly. In the case of liquid metal or helium cooled reactors, the mismatch between the properties of water and the reactor coolant causes very large entropy losses. The use of a mixture as the working fluid reduces these losses and can increase the thermal efficiency of the cycle by approximately 30 percent. The Kalina Cycle is the creation of Alexander Kalina, meaning "flower" in the Slavic languages, who came to the USA from Russia.
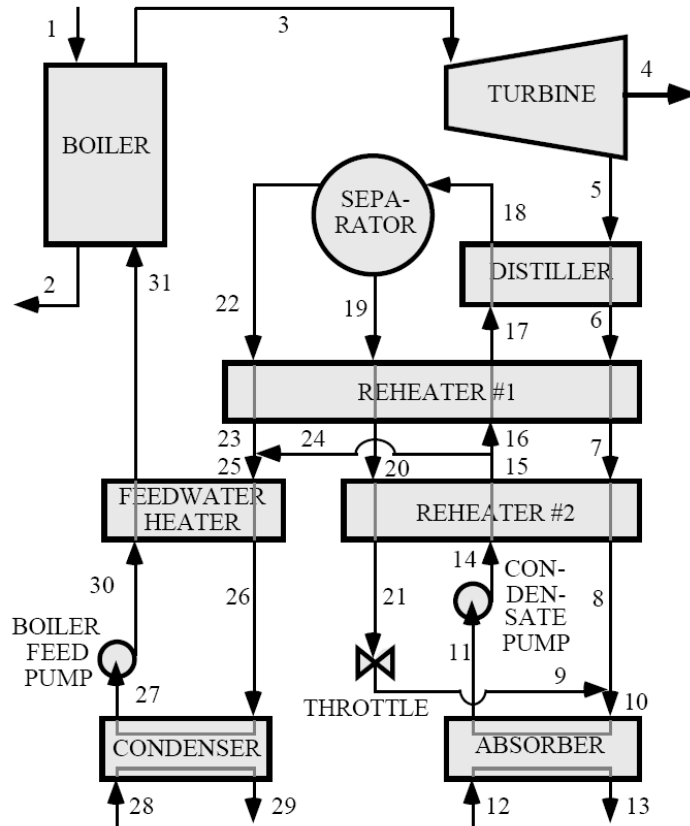
Fig. 21: Simplified Kalina cycle using exhaust gases to a boiler.

By circulating the mixture at different compositions in different parts of the cycle, condensation (absorption) can be done at slightly above atmospheric pressure with a low concentration of ammonia, while heat input is at a higher concentration for optimum cycle performance.

EXERGY FLOW IN THE KALINA CYCLE

The exhaust gases enter a boiler at point 1 and exit at point 2. The generated ammonia-water vapor mixture is evaporated in the boiler, exits at 3 then is expanded in a turbine to generate work at 4.

The turbine exhaust 5 is cooled in a distiller at 6, a first reheater at 7 and a second reheater at 8. It is then diluted with an ammonia poor liquid stream at points 9 and 10, then condensed at 11 in the absorber by the cooling water flow from 12 to 13.

The saturated liquid leaving the absorber is compressed at 14 to an intermediate pressure and heated in a reheater at 15, another reheater at 17 and the distiller at 18. The saturated mixture is separated into an ammonia poor liquid at 19 which is cooled at 20, and 21 then depressurized in a throttle. Ammonia rich vapor at 22 is cooled at 23 and some of the original condensate at 24 is added to the nearly pure ammonia vapor to obtain an ammonia concentration of about 70 percent in the working fluid at 25.

The mixture is then cooled at 26, condensed at 27 by the cooling water at 28 and 29, compressed at 30, and sent to the boiler through a regenerative feed water heater at 31.

The mass flow circulating between the separator and the absorber is about 4 times that through the turbine, thus, causing some additional condensate pump work. However, this loop makes possible the changes in composition between the initial condensation in the absorber and the heat addition in the boiler. By modifying the dew point of the mixture, the waste heat from the turbine exhaust, which is lost in a Rankine cycle, can be used to dilute the ammonia water vapor with a stream of water, thus, producing a mixture with a substantially lower concentration of ammonia which allows condensation at a much higher temperature.

The thermodynamic properties of pure fluids and information about the departure from ideal solutions are sufficient to derive the mixture properties. Stability, secondary reactions, and safety must be taken into consideration.

COMPARISON OF THE KALINA AND STEAM CYCLES

The Kalina cycle is a new concept in heat recovery and power generation, which uses a mixture of 70 percent ammonia and 30 percent water as the working fluid with the potential of significant efficiency gains over the conventional Rankine cycle. Basically this concept is suitable for medium to low gas temperature heat recovery systems with gas inlet temperatures in the range of 400 to 1000 ^{o}F, offering more gains, over the Rankine cycle, as the gas temperature decreases.

Gas turbine based combined cycles using this concept have 2-3 percent higher efficiency over multi pressure combined cycle plants using steam and water as the working fluid. In low gas temperature heat recovery systems such as diesel engine exhaust or fired heater exhaust, the energy recovered from the hot gas stream is more significant and Kalina cycle output increases by 20-30 percent. The main reason for the improvement is that the boiling of ammonia water mixture occurs over a range of temperatures, unlike steam and hence the amount of energy recovered from the gas stream is much higher.

Considering a 550 ^{o}F gas temperature source with a cold end fluid temperature of 100 ^{o}F and 70/30 NH_3/H_2O mixture at 500 psia, by virtue of its varying boiling point, is able to match or run parallel to the gas temperature line while recovering energy and hence the exit gas temperature can be as low as 200 ^{o}F.

°F

Kalina   Rankine

600

                                          Gas
500

                                                        Gas
Gas   Steam
400   500 psia

300   Water

200   70/30 HN₃/H₂O
           500 psia

100

                                          Energy Recovery
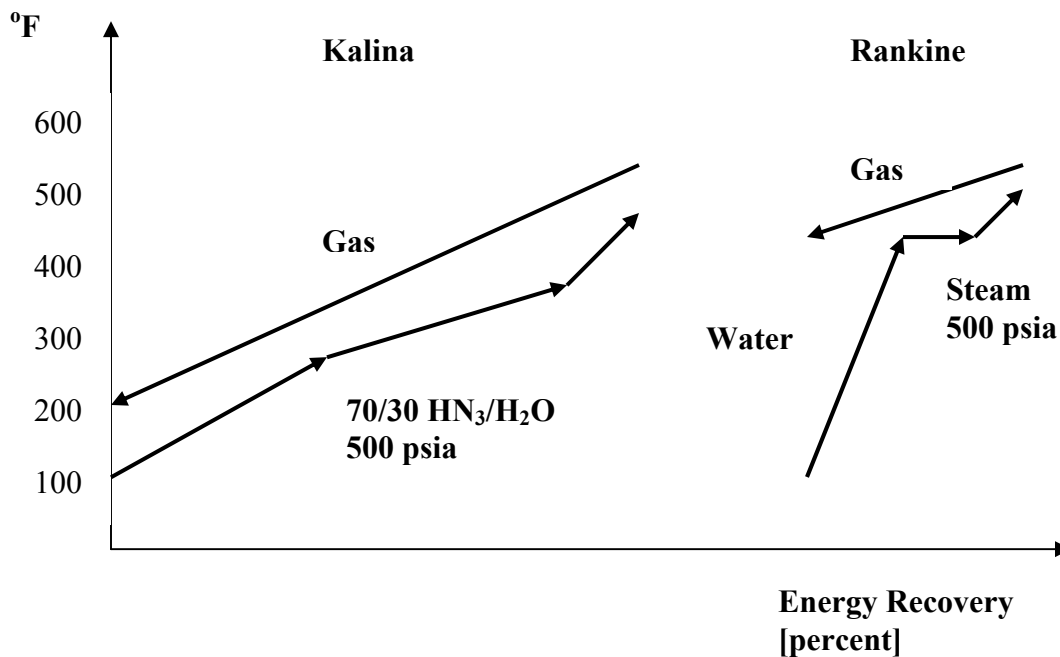   [percent]

Fig 22: Comparison of the heat recovery in the Kalina and Steam cycles.

The steam-water mixture at 500 psia, on the other hand, due to pinch and approach point limitations and a constant boiling point of 467 °F, cannot cool the gases below about 400 °F. Only about 15-20 percent of the energy is recovered, compared with 100 percent in the Kalina cycle.

The condensation of ammonia-water also occurs over a range of temperatures and hence permits additional heat recovery in the condensation system, unlike the Rankine cycle, where the low end temperature, affected by ambient conditions, limits the condenser back pressure and power output of the system. If the cooling water temperature is say 100 °F, less power is generated by the steam turbine compared to say 40 °F cooling water. The condenser pressure can be much higher in a Kalina cycle, and the cooling water temperatures does not impact the power output of the turbine as in the Rankine cycle. The thermo-physical properties of ammonia-water mixture can also be altered by changing the concentration of ammonia. This helps to recover energy in the condensation system. Modifications to the condensing system are also possible by varying the ammonia concentration and thus more energy can be recovered from the exhaust gases.

Expansion in the turbine produces a saturated vapor in the Kalina cycle compared with wet steam in Rankine cycle, which requires protection of the turbine blades in the last few stages. Also due to the higher pressure of vapor and lower specifc volume, the exhaust system size can be smaller compared to steam. For example the specific volume of a 70 percent ammonia water mixture exhausting from a turbine at its dew point of 240 F is 5.23 ft³/lb,while steam at its condensing temperature of 70 F(sat pres=0.36 psia) has 868 ft³/lb. Thus the equipment size can be smaller with a Kalina system.

EXERCISES

1. Assuming that heat rejection occurs at an ambient temperature of 20 degrees Celsius, for the average heat addition temperatures T_a given below, compare the Carnot cycle thermal efficiencies of the following reactor concepts:
    1. PWR, 168 ^{o}C.
    2. BWR, 164 ^{o}C.
    3. CANDU, 141 ^{o}C.
    4. HTGR, 205 ^{o}C.
    5. LMFBR, 215 ^{o}C.

2. A boiling water reactor produces saturated steam at 1,000 psia. The steam passes through a turbine and is exhausted at 1 psia. The steam is condensed to a subcooling of 3°F and then pumped back to the reactor pressure. Compute the following parameters:
a. Net work per pound of fluid.
b. Heat rejected per pound of fluid.
c. Heat added by the reactor per pound of fluid.
d. The turbine heat rate defined as: [(Heat rejected +Net turbine work)/Net turbine work] in units of [BTU/(kW.hr)]
e. Overall Thermal efficiency.
You may use the following data:
From the ASME Steam Tables, saturated steam at 1,000 psia has an enthalpy of h=1192.9 [BTU/lbm]
At 1 psia pressure the fluid enthalpy from an isentropic expansion is 776 [BTU/lbm]
The isentropic pumping work is 2.96 [BTU/lbm]
The enthalpy of the liquid at 1 psia subcooled to 3 ^{o}F is 66.73 [BTU/lbm]
1 [kW.hr] = 3412 [BTU]

3. In the preceding problem calculate the quantities of interest for a practical cycle with a turbine efficiency of 80 percent and a pump efficiency of 80 percent.

4. For a 100 percent regenerative effectiveness, prove that the Stirling cycle has a Carnot cycle efficiency.

5. A Stirling cycle engine using a radioactive isotope for space power applications operates at a hot end temperature of 650 ^{o}C and rejects heat through a radiator to the vacuum of space with a cold end temperature at 120 ^{o}C. Calculate its ideal Stirling cycle efficiency.

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