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Planetary Diameters in the Surya-Siddhanta

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Abstract — This paper discusses a rule given in the Indian astronomical text Surya-siddhanta for computing the angular diameters of the planets. By combining these angular diameters with the circumferences of the planetary orbits listed in this text, it is possible to compute the diameters of the planets. When these computations are carried out, the results agree surprisingly well with modern astronomical data. Several possible explanations for this are discussed, and it is hypothesized that the angular diameter rule in the *Surya-siddhanta* may be based on advanced astronomical knowledge that was developed in ancient times but has now been largely forgotten.

Keywords: Astronomy – *Surya-Siddhanta* – Indology

Introduction

In chapter 7 of the Surya-siddhanta (Burgess, 1989), the 13th verse gives the following rule for calculating the apparent diameters of the planets Mars, Saturn, Mercury, Jupiter, and Venus:

7.13. The diameters upon the moon's orbit of Mars, Saturn, Mercury, and Jupiter, are declared to be thirty, increased successively by half the half; that of Venus is sixty.

The meaning is as follows: The diameters are measured in a unit of distance called the yojana, which in the Surya-siddhanta is about five miles. The phrase "upon the moon's orbit" means that the planets look from our vantage point as though they were globes of the indicated diameters situated at the distance of the moon. (Our vantage point is ideally the center of the earth.) Half the half of 30 is 7.5. Thus the verse says that the diameters "upon the moon's orbit" of the indicated planets are given by 30, 37.5, 45, 52.5, and 60 yojanas, respectively.

The next verse uses this information to compute the angular diameters of the planets. This computation takes into account the variable distance of the planets from the earth, but for the purposes of this paper it is enough to consider the angular diameters at mean planetary distances. The diameters upon the moon's orbit were given for the planets at these mean distances from the earth. The Surya-siddhanta says that there are 15 yojanas per minute of arc at the distance of the moon (giving 324,000 yojanas as the circumference of the moon's orbit). Thus the mean angular diameters of the planets can be

TABLE 1
Angular Diameters of Planets in Minutes of Arc.

Planet	Surva-*Siddhanta*	Ptolemy	Tycho Brahe	Modern Minimum	Modern Maximum
Mars	2.0	1.57	1.67	0.058	0.392
Saturn	2.5	1.74	1.83	0.249	0.344
Mercury	3.0	2.09	2.17	0.076	0.166
Jupiter	3.5	2.61	2.75	0.507	0.827
Venus	4.0	3.13	3.25	0.159	1.050

The modern angular diameters are for the greatest and least distances of the planets from the earth.

computed by dividing the diameters upon the moon's orbit by 15. Table 1 gives the results of this computation and lists other estimates of planetary angular diameters for comparison.

The Surya-siddhanta figures are roughly the same size as the planetary angular diameters reported by the 2nd century Alexandrian astronomer Claudius Ptolemy in his book Planetary Hypotheses. Ptolemy attributed his angular diameters to the Greek astronomer Hipparchus, but he did not say how they were measured. According to the historian of astronomy Noel Swerdlow (1968), no earlier reports of planetary angular diameters are known, and Ptolemy's angular diameters were reproduced without change by later Greco-Roman, Islamic, and European astronomers up until the rise of modem astronomy in the days of Galileo, Kepler, and Tycho Brahe.

Brahe's figures were obtained by sighting through calibrated pinholes by the naked eye. They are very similar to Ptolemy's, and they are clearly much larger than the angular diameters measured in more recent times by means of telescopes (Burgess, 1989). It is well known that a small, distant light source looks larger to the naked eye than it really is. This phenomenon makes it likely that angular diameters of planets would inevitably have been over-estimated by astronomers before the age of the telescope.

It has been argued that Indian astronomy was heavily influenced by Hellenistic astronomy between the second and fifth centuries A. D. (Pingree, 1976). This suggests that the angular diameters given in the Surya-siddhanta may have been based on Ptolemy's angular diameters. Indeed, Ptolemy's figures are very close to $94/(60 - 7.5n)$, where $n+1$ is the line number in Table 1. The corresponding Surya-siddhanta figures are given by $(30 + 7.5n)/15$.

Whether or not this indicates an Indian adaptation of Greek material, the angular diameters from Surya-siddhanta have an important property that the Ptolemaic angular diameters lack. To see this, it is first necessary to examine the sizes of the planetary orbits, as given in Surya-siddhanta.

Orbital Dimensions in the *Surya-Siddhanta*

Verses 12.85–90 of the Surya-siddhanta give the circumferences of the planetary orbits in yojanas, and these figures are reproduced in Table 2. The orbits are represented as simple circles centered on the earth, and their circumferences are proportional to the mean orbital periods of the planets. For Mercury and Venus, the mean planetary position is the same as the position of the sun, and thus the orbital circumferences in the table are the same for Mercury, Venus, and the sun. For Mars, Jupiter, and Saturn, the mean position corresponds to the average motion of the planet in its heliocentric orbit.

Verse 1.59 of the Surya-siddhanta gives the diameter of the earth as 1,600 yojanas. Several scholars have argued that the yojana in the Surya-siddhanta is about 5 miles, thereby bringing the earth's diameter to the realistic value of 5x1600 = 8,000 miles. Examples are Sarma (1956), Burgess (1989), and Dikshit (1969).

Different standards were adopted for the yojana by different medieval Indian astronomers. This was noted by the astronomer Paramesvara (1380–1450 A. D.), who said:

> What is given by ***Aryabhata*** as the measure of the earth and the distances [of the Planets from it], ***etc.***, is given as more than one and a half times by other [astronomers]; this is due to the difference in the measure of the yojana [adopted by them] (Sarma, 1956).

Verse 4.1 of the Surya-siddhanta gives the diameters of the sun and moon as 6,500 and 480 yojanas, respectively. Given 5 miles per yojana, the resulting lunar diameter of 5x480 = 2,400 miles is about 11% higher than the modem value. The corresponding earth-moon distance of about 258,000 miles (listed in Table 2) is high by 8.3%. However, the sun's diameter comes to 5x6500 = 32,500 miles, which is far too small.

It is easy to see why the diameter of the moon should be reasonably accurate. The dimensions of the moon and its orbit were well known in ancient times. For example, the lunar diameter given by Ptolemy in his Planetary Hypotheses falls within about 7% of the modern value, if we convert his earth-diameters into miles using the modem diameter of the earth (Swerdlow, 1968).

TABLE 2
Geocentric Orbital Circumferences

Planet	***Surya-siddhanta*** Orbital Circumference (yojanas)	***Surya-siddhanta*** Orbital Radius (miles)
Moon	324,000	258,000
Mercury	4,332,000	3,447,000
Venus	4,332,000	3,447,000
Sun	4,331,500	3,447,000
Mars	8,147,000	6,483,000
Jupiter	51,376,000	40,884,000
Saturn	127,668,000	101,595,000

As given in texts 12.85–90 of the ***Surya-siddhanta.*** The orbital radii are computed from these circumferences using 5 miles per yojana.

It is also easy to see why the diameter for the sun is too small. Ancient astronomers tended to greatly underestimate the earth-sun distance, and Table 2 shows that this also happened in the Surya-siddhanta, The angular diameter of the sun is easily seen to be about the same as that of the moon — about 1/2 degree. This angular diameter, combined with a small earth-sun distance, leads inevitably to a small estimate for the diameter of the sun. Ptolemy's solar diameter figure is similar to the Surya-siddhanta's.

Computing Planetary Diameters

What about the planets? Ptolemy listed wildly inaccurate diameters for Mercury, Venus, Mars, Jupiter, and Saturn in his Planetary Hypotheses. To see what the Surya-siddhanta says about the diameters of these planets, we should multiply the orbital radii in Table 2 by the angular diameters (converted to radians) in Table 1. This is done in Table **3.**

Note that even though the angular diameters are too large, and the orbital radii are too small, the calculated diameters are close to modern values for Mercury, Mars, and Saturn. For Venus and Jupiter, they are too small by about 50%. One might argue that this balancing is due to pure chance. However, since the balancing works for five distinct cases, it is worthwhile to estimate just how probable it is.

This probability can be evaluated by setting up a model in which diameters are chosen at random. One can then check to see if the observed correlation between modem and Surya-siddhanta diameters is likely to show up in this model. Of course, it is difficult to propose a realistic probabilistic model of how ancient people would have generated astronomical data. But it is possible to set up a simple model in which it is assumed that all planetary diameters, ancient and modern, are given by positive random numbers. It is easy to show that the observed correlation between modern and Surya-siddhanta diameters is highly unlikely to arise by chance, according to this model. This is discussed in the appendix.

If the observed correlation did not happen by chance, then perhaps it happened by design. One hypothesis is that at some time in the past, ancient as-

TABLE 3
Planetary Diameters in Miles

Planet	Modern Diameter	***Surya-siddhanta*** Diameter	% Error
Mercury	3032	3008	−1
Venus	7523	4011	−47
Mars	4218	3772	−11
Jupiter	88748	41624	−53
Saturn	74580	73882	−1

Computed using the ***Surya-siddhanta*** orbital radii from Table 2 and angular diameters from Table 1. The error percentages compare the *Surya-siddhanta* diameters with the corresponding modern planetary diameters.

tronomers possessed realistic values for the diameters of the planets. They might have acquired this knowledge during a forgotten period in which astronomy reached a high level of sophistication. Later on, much of this knowledge was lost, but fragmentary remnants were preserved and eventually incorporated into texts such as the Surya-siddhanta. In particular, the real diameters of the planets were later combined with erroneous orbital circumferences to compute the diameters "upon the moon" given in verse 7.13. These figures were then accepted because they gave realistic values for the angular diameters of the planets as seen by the naked eye.

This hypothesis is supported by the fact that the Surya-siddhanta diameters of Jupiter and Venus in Table **3** are almost exactly half of the corresponding modern diameters. If we multiply these Surya-siddhanta diameters by 2, we get 83248 miles for Jupiter and 8022 miles for Venus. These figures differ from the corresponding modern values by –6% and +7%. Given this correction, all five planets have an error of 11% or less. (The root-mean-square error comes to 6.3%.)

One can argue that the Surya-siddhanta diameters for Jupiter and Venus were actually the radii for these planets, and somehow they were accepted as diameters by mistake. Or radii might have been deliberately used instead of diameters in order to allow for the simple rule of $30+7.5n$ used in verse 7.13. This is consistent with the fact that such verses were intended as memory aids and brevity was considered to be a virtue.

Alternative Explanations

Of course, it could be argued that this is just number jugglery, and by juggling numbers one can create false correlations. But let us review the steps taken thus far. The angular diameters in Table 1 were given by the text of the Surya-siddhanta. The orbital radii of Table 2 were computed from Surya-siddhanta orbital circumferences using the conversion factor of 5 miles per yojana. This factor is based on the Surya-siddhanta's diameter for the earth, and it has been discussed by other authors. There is no scope for juggling numbers here.

The only proposed adjustment of the numbers is the doubling of the Surya-siddhanta diameters of Jupiter and Venus. Since the Surya-siddhanta numbers can be so easily brought into line with modem data, it may be that they have a genuine relationship with this data.

One possible explanation is that verse 7.13 may have been written recently, using modem planetary data, and falsely interpolated into the text. But this is ruled out by the fact that there is a manuscript of the Surya-siddhanta that scholars date to the year 1431 **A.** D. (Shukla, 1957). This manuscript includes a commentary by Paramesvara, who died in 1450 **A.** D., and thus it definitely dates back to the 15th century. Verse 7.13 is present in this manuscript, and it agrees with the Burgess translation quoted above. The commentary explains the verse point by point, and thus it confirms that the verse was present in the manuscript in the same form in which it appears today.

In 15th century Europe, the prevailing ideas concerning the sizes of the planets came from medieval Islamic astronomers who were following the teachings of Ptolemy. The first telescopic observations of planets were made by Galileo in 1609–10 (Drake, 1976). As late as 1631, Pierre Gassendi of Paris was shocked when his telescopic observation of a transit of Mercury across the sun revealed that its angular diameter was much smaller than he had believed possible (Van Helden, 1976). It is clear that the information on planetary diameters in the Surya-siddhanta antedates the development of modern knowledge of these diameters.

It is also clear that Hellenistic astronomers did not have accurate diameters for the planets. Ptolemy computed planetary diameters from his angular diameters and his estimates of planetary distances, and these were reproduced without significant change by European and Islamic astronomers for centuries (Swerdlow, 1968). However, his figures disagree strongly both with modem data and with the diameters computed from Surya-siddhanta in Table 3.

Deriving the *Surya-siddhanta* Rule

If we hypothesize that verse 7.13 incorporates knowledge of the actual diameters of the planets, then one natural question is this: If one started with the modem diameters of the planets and the *Surya-siddhanta* orbital circumferences, could one arrive at the rule given in this verse? We can answer this question by computing planetary diameters "upon the moon's orbit" as follows: For each planet, multiply its modem diameter, converted to yojanas, by the ratio between the orbital circumferences of the moon and the given planet, as listed in Table 2. Here we use the radius in place of the diameter for Jupiter and Venus. The resulting values are listed in the leftmost column of Table 4.

The idea behind the rule in verse 7.13 is to arrange the planets so that the diameters on the moon's orbit are in increasing order and then approximate them by a simple arithmetic progression. We can see from Table 4 that the order of the planets used in this rule does put the computed diameters "on the moon's orbit" in increasing order. One can approximate them by an arithmetic progression of the form $an+b$ either by trial and error or by using an optimization method such as least squares. I did this by least squares and got $a = 6.356$ and

TABLE 4
Deriving Verse 7.13 from Modern Data

Planet	Modern projection	Least squares fit	Angular diameter
Mars	33.6	33.1	2.2
Saturn	37.9	39.4	2.6
Mercury	45.4	45.8	3.0
Jupiter	56.0	52.2	3.5
Venus	56.3	58.5	3.9

b= 33.089. This arithmetic progression is listed in the middle column of Table 4.

In the leftmost column, modern planetary diameters are projected to the orbit of the moon, assuming the planetary orbits given in Surya-siddhanta. The projected diameters are expressed in yojanas (and radii are used in place of diameters for Jupiter and Venus). In the middle column, these projected diameters are fit to an arithmetic progression using least squares. The angular diameters in the rightmost column are obtained by dividing the figures in the middle column by 15 yojanas per minute of arc.

One could arrive at the rule in verse 7.13 by observing that 33.1 is about 30, 45.8 is about 45, and 58.5 is about 60. Or one could compute the angular diameters listed in the rightmost column of Table 4 by dividing the numbers in the arithmetic progression by 15. It is plausible that someone looking for a simple rule might round off these angular diameters to the Surya-siddhanta series of 2, 2.5, 3, 3.5, 4.

Thus it is possible to derive the rule in verse 7.13 from modern values for the diameters of the planets.

Conclusion

In summary, verses 7.13 and 12.85–90 of the Surya-siddhanta contain information regarding the true diameters of the five planets Mercury, Venus, Mars, Jupiter, and Saturn. This information enables us to compute the diameters of three of these planets with errors of 11% or less. If the computed figures for Jupiter and Venus are interpreted as their radii rather than their diameters, then these radii are in error by about 6% and 7%, respectively. This may not be due to mere coincidence. Rather, it may indicate that accurate knowledge of planetary diameters was possessed by ancient astronomers and used in the composition either of the Surya-siddhanta or of some earlier astronomical text on which it was based. It is not apparent how such knowledge may have been obtained, but we should be on the alert for other possible examples.

References

Burgess, Ebenezer, trans. (1989). *The Surya Siddhanta.* Gangooly, P., ed., Delhi: Motilal Banarsidass.

Dikshit, S. B. (1969). *English Translation of Bharatiya Jyotish Sastra.* R. V. Vaidya, trans., Delhi: Manager of Publications, Civil Lines.

Drake, Stillman (1976). Galileo's first telescopic observations. *Journal for the History of Astronomy,* 7, 153.

Pingree, David (1976). The recovery of early Greek astronomy from India. *Journal for the History of Astronomy, 7,* 109.

Sarma, K. V., trans. (1956). *The Goladipika* by *Paramesvara.* Madras: The Adyar Library and Research Center.

Shukla, K. S. (1957). *The Surya Siddhanta with the Commentary of Paramesvara.* Dept. of Mathematics and Astronomy: Lucknow University.

Swerdlow, Noel Mark (1968). *Ptolemy's Theory of the Distances and Sizes of the Planets: A Study of the Scientific Foundations of Medieval Cosmology.* Yale Univ. Ph.D. thesis.

Van Helden, Albert, (1976). The importance of the transit of Mercury of 1631. ***Journal for the History of Astronomy, 7***, *1*.

Appendix: Statistical Evaluation

In this appendix a simple probabilistic model is used to evaluate whether or not the correlation between modern and Surya-siddhanta diameters shown in Table 3 could have arisen by chance. First, randomly choose 5 numbers between 0 and B, where B is some fixed positive number. Call these numbers $X_1,\ldots,X_5$, and let them represent the diameters of Mercury, Venus, Mars, Jupiter, and Saturn, as calculated from data in the Surya-siddhanta. Then randomly choose 5 numbers $Y_1,\ldots,Y_5$ between 0 and B to represent the modem values for these diameters. What is the probability that the X's will agree with the Y's as well as do the Surya-siddhanta and modem diameters listed in Table 3?

For each (X, Y), let $P = 1 - \min(X/Y, Y/X)$. P is a measure of how close X is to Y, and $P=0$ if $X=Y$. It is easy to see that if X and Y are chosen independently in $(0,B)$ with a uniform distribution, then P is distributed uniformly on $[0,1)$. (It does not matter what value we choose for B.)

Let S be the sum of the P's for the 5 pairs (X,Y). If we compute S using the 5 pairs of diameters from Table 3, we get S = 1.121. What is the probability that S will be no greater than this for the 5 randomly chosen (X,Y) pairs?

It is easy to compute an upper bound on the probability that $S < y$, where S is the sum of n independent random variables distributed uniformly on $[0,1)$. This upper bound is $y^n/n!$. Using S = 1.121 and $n = 5$, we get .0147 for this upper bound. Therefore, the actual pairs of diameters in Table **3** exhibit a significant deviation from chance expectation.

Note that in this probability estimate, the Surya-siddhanta diameters of Jupiter and Venus have not been doubled. Thus the probability estimate of .0147 is for the unedited Surya-siddhanta diameters. If we do double the diameters of Venus and Jupiter (taking them to be radii), then the probability estimate becomes 7.7×10^{-6}.