

**PUTTING TO REST
MASS MISCONCEPTIONS**

Reprinted from: *Physics Today, Letters*, May 1990, pp. 13 – 14, 115, 117.

Contents

Letter by W. Rindler	143
Letter by M. A. Vandyck	144
Letter by P. Murugesan	145
Letter by S. Ruschin	146
Letter by C. Sauter	146
Okun replies	147

I am disturbed by the harm that Lev Okun's earnest tirade (June 1989, page 31) against the use of the concept of relativistic mass ("It is our duty . . . to stop this process") might do to the teaching of relativity. It might suggest to some who have not thought these matters through that there are unresolved logical difficulties in elementary relativity or that if they use the quantity $m = \gamma m_0$ they commit some physical blunder, whereas in fact this entire ado is about terminology. There are perhaps 40% of us who find it useful occasionally to write m for E/c^2 and 60% who don't. But why the latter should try to coerce the former beats me.

One can perhaps understand a desire that everyone should use the term "mass" in the same sense. I myself have never found this a stumbling block, since the context tells me which mass the author means. Nevertheless, if I am told by the particle physicists — and they *are* the largest user group of special relativity these days — that henceforth I must use the symbol m for rest mass and call it mass, so be it. But I refuse to stop using the concept of relativistic mass, which I would then denote by m_r .

I know a man who can drive a shift car without ever using the clutch — it's a question of timing. The ingenious Ernest Vincent Wright in 1939 wrote a 50 000-word novel, called *Gadsby*, in all of which the letter *e* never occurs. Its sentences look like this: "A busy day's traffic had had its noisy run." Sure, such feats can be performed. But to what end? I like using my clutch, the letter *e* and the relativistic mass.

To me, $m = \gamma m_0$ is a useful heuristic concept. It gives me a feeling for the magnitude of the momentum $\mathbf{p} = m\mathbf{v}$ at various speeds. The formula $E = mc^2$ reminds me that energy has mass-like properties such as inertia and gravity, and it tells me how energy varies with speed. I will confess to even occasionally using the heuristic concepts of longitudinal mass $\gamma^3 m_0$ and

transverse mass γm_0 to predict how a particle will move in a given field of force.

9/89

Wolfgang Rindler
University of Texas at Dallas

I read with great interest the article "The Concept of Mass" by Lev B. Okun. As the author points out, the notion of "relativistic mass" is both unnecessary and confusing, and should therefore be avoided.

In this note I would like to go even one step further and suggest that one should also avoid equations such as $d\mathbf{p}/dt = \mathbf{f}$ and $\mathbf{p} = m\gamma\mathbf{v}$, which obscure the Lorentz covariance of the theory because of the appearance of the operator d/dt . Their use is, however, common practice, even in the best textbooks.¹

The well-known manifestly Lorentz-covariant treatment employs the equation $d\mathbf{P}/d\tau = \mathbf{F}$, where $\mathbf{P} \equiv m d\mathbf{X}/d\tau$, τ denoting the proper time and the capitals representing four vectors. Not only is the Lorentz-covariant formulation conceptually clearer; it is also simpler to use in practice due to its geometrical significance.

These points are made clearly in a book by John L. Synge,² to which the reader is referred for more details, in particular for an enlightening discussion of the concept of "rest mass."

REFERENCES

1. See, for example, R. Feynman, R. Leighton, M. Sands, *The Feynman Lectures on Physics*, vol. I. Addison-Wesley, Reading, Mass. (1966), ch. 15.8.
2. J. Synge, *Relativity: The Special Theory*, North-Holland, Amsterdam (1965), ch. VI.

Michael A. Vandyck
Cork Regional Technical College
Bishopstown, County Cork, Ireland
and Dublin Institute
for Advanced Studies
Dublin, Ireland

7/89

Lev Okun's article on the concept of mass, despite the apologetic tone, is an important one. Many introductory physics and physical science texts include the erroneous concept implied in the equation

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Students are taught that as the velocity of an object of "rest mass" m_0 gets close to c , the velocity of light, its mass becomes extremely large. The argument is extended to explain why an object of "rest mass" m_0 cannot reach the velocity of light: If $v = c$, then the mass becomes infinite! The same students are also told that particles in cosmic rays could have velocities greater than $0.9c$ without any change in their masses. It is about time we eliminate such contradictory and confusing statements from our textbooks.

Okun's definition, $E_0 = mc^2$, may not eliminate the confusion, as m could still be interpreted as "rest mass" since E_0 is rest energy. To avoid that confusion, I recommend that the mass-energy relation be stated as $E = \gamma mc^2$, where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

For the special case of a reference frame at rest, $v = 0$, $\gamma = 1$ and $E_{\text{rest}} = mc^2$. Let us eliminate the subscript "0" altogether.

This definition may not gain popularity among the public. On the other hand, physics students will learn the correct concepts of special relativity.

Poovan Murugesan
Stetson University
DeLand, Florida

6/89

The article "The Concept of Mass" contains, in my opinion, a curious mistranslation of Einstein's letter of 19 June 1948 to Lincoln Barnett.

The pertinent part in the caption on page 32 reads:

It is not good to introduce the concept of the mass $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the "rest mass" m . Instead of

introducing M it is better to mention the expression for the momentum and energy of a body in motion.

The German word *daneben* does not mean "instead of," but rather "besides," "in addition to" or "moreover." I would therefore translate the passage:

It is not proper to speak of the mass $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body, because no clear definition can be given for M . It is preferable to restrict oneself to the "rest mass" m . Besides, one may well use the expression for momentum and energy when referring to the inertial behavior of rapidly moving bodies.

It should be noted that according to Einstein's letter, the expression for momentum and energy may be used to describe the inertial behavior of rapidly moving bodies, not the motion as such.

I am unqualified to evaluate the article. But it seems to me that the inaccurate translation may not be irrelevant to the argument.

Siegfried Ruschin
Linda Hall Library
Kansas City, Missouri

8/89

I found many of the remarks in the article "The Concept of Mass" thought provoking and interesting. This past semester I took a course where we discussed some of Einstein's theory of special relativity. My professor wished for us to use m_0 as the rest mass and $m = \gamma m_0$ as the relativistic mass in our equations. Our text,¹ however, did not show this notation. This conflict between my professor and the text was a source of great confusion for me. One of my main concerns was that my professor had probably been taught about relativistic mass and then passed the idea along to his students like so many other teachers. Why is it that the academic system has been able to pass along information like this for so long?

Everyone should now fully realize what terminology one should use in explaining mass, and the reasons behind it. The article was a source of greater understanding for me and I hope, for other readers. There are still concepts in physics that confuse me: Does mass truly depend on velocity?

REFERENCE

1. R. Serway, *Physics for Scientists and Engineers*, 2nd ed., Saunders, Philadelphia (1987).

Catherine Sauter
Baltimore, Maryland

7/89

OKUN REPLIES: I fully agree with Siegfried Ruschin that the English text of Einstein's letter does not correspond exactly to the German original. Moreover, I was aware of this circumstance when I published my article. However, both the German manuscript and a copy of the English typescript that Einstein sent to Lincoln Barnett are in the same file of the Einstein archives, and I did not and do not think that an inaccurate translation that did not bother Einstein was worth remarking on in the article.

I agree with Michael A. Vandyck that the most adequate way of presenting special relativity is by writing manifestly Lorentz-covariant equations. However, the connection with Newtonian mechanics is also essential. Nonrelativistic physics is an important (and correct within its realm) part of physics. And it would be a kind of "relativistic extremism" always to start from four-dimensional coordinates and momenta when considering everyday phenomena.

It is not quite clear to me why Poovan Murugesan believes that the equation $E_0 = mc^2$, where E_0 is the rest energy and m is the mass, would provoke a student to call m the rest mass. But I don't think we have a real disagreement here.

In connection with Catherine Sauter's letter I would like to point out that the textbook by Raymond Serway to which she refers contains a modern and concise essay on relativity written by George O. Abel. Curiously, the chapter on relativity in a more recent book cowritten by Serway¹ is based entirely on $m = \gamma m_0$.

Sauter wonders why "the academic system has been able to pass along information like [the idea of relativistic mass] for so long." A partial answer is given by Wolfgang Rindler, who has written an eloquent letter defending the notion of velocity-dependent mass.

My article cannot suggest that there are unresolved logical difficulties in elementary relativity, as Rindler worries, because I stressed that the matter is absolutely clear to all specialists. What is unresolved is the way the subject is treated in many textbooks. I do insist that by using $m = \gamma m_0$ the authors of these textbooks commit a "physical blunder" because they use misleading and confusing terminology.

Rindler writes, "One can perhaps understand a desire that everyone should use the term 'mass' in the same sense." I appreciate his understanding that terms in physics should mean the same thing to everyone and his readiness to "use the symbol m for rest mass and call it mass." However, the equations $m = \gamma m_0$, $\mathbf{p} = m\mathbf{v}$ and $E = mc^2$ — the heuristic value of which Rindler praises — preclude uniform usage of the word "mass." (By the way, it seems to me that $E = \gamma mc^2$ tells how energy varies with speed much better than the "heuristic" $E = mc^2$.)

As for Rindler's statement that understanding the term "mass" in various senses never was a stumbling block for him, it would be extremely strange if it were otherwise. Rindler is not a student, but an expert in relativity. I have seen three editions of his book *Essential Relativity: Special, General and Cosmological* (Springer-Verlag, New York, 1969, 1977, 1979). It is a nice introduction to the subject, except for the sections where the so-called relativistic mass and the whole bunch of other masses are introduced at length. I believe that these sections do present a "stumbling block" to students. If Rindler uses the term "mass" in only one sense in the fourth edition of his book, it will not contain such misleading statements as one that accompanies the equation $m = \gamma(v)m_0$ in section 5.3 of the last edition — "This conclusion is inevitable if momentum is conserved" — or the whole paragraph about "mass-energy equivalence according to the formula $E = mc^2$ " in the same section.

The relativistic mass is dear to Rindler's heart. For him not to use it is like driving a car without using the clutch, or writing a novel without using the letter e . But the fact is that most leading physics journals, such as *Physical Review*, *Physical Review Letters* and *Physics Letters*, don't use relativistic mass. You will not find it in professional books on particle physics. Are all these cars without clutches? To me, texts using the letter m in many senses look like a novel where m stands for $e, m, n \dots$

It is obvious that Rindler is simply accustomed to the archaic language. But is that enough to justify the promotion of this language to new generations of students?

I would like to use this opportunity to make some remarks concerning

equation 16 of my article for gravitational force:

$$\mathbf{F}_g = - \frac{G_N M (E/c^2) [\mathbf{r} (1 + \beta^2) - \beta(\beta \cdot \mathbf{r})]}{r^3}$$

According to this equation the force acting on a horizontally moving photon is twice as large as that on a vertically moving one. This factor of 2 explains the famous extra factor of 2 in Einstein's expression for the angle of light bending by the Sun. Equation 16 needs some comments, which unfortunately were missing from my article.

It is common knowledge that in a locally inertial frame the gravitational force is equal to zero. That means that equation 16 is valid only for locally non-inertial frames, such as the usual laboratory frame. Moreover, it is important to stress that we are considering the static, spherically symmetric gravitational field of the Sun or the Earth (neglecting rotation) and that the metric chosen to obtain equation 16 is isotropic: $g_{\alpha\beta} = (g_{00}, g_{ik})$, where $g_{ik} = \eta_{ik} f$, so that $ds^2 = g_{00}x^0x^0 - f(x^1x^1 + x^2x^2 + x^3x^3)$. Using this metric, equation 16 is easily derived as a first-order approximation in the gravitational potential $G_N M/r$ from equation 87.3 of *Field Theory* by Lev D. Landau and Evgenii M. Lifshitz. As the final step of this derivation one has to change to the frame with the usual local rulers and clocks. The choice of an isotropic metric does not permit us to get rid of our force by using equivalence-principle elevators, and therefore one can say that the result for the light bending angle does arise from the global geometry of the central field — the point that is usually stressed in textbooks on gravity.

Engelbert Schucking from New York University has informed me that he has obtained a generalized exact formula for what he calls "the relativistic apple," valid in all approximations with respect to the parameter $G_N M/rc^2$:

$$\mathbf{F}_g = - \frac{G_N M (E/c^2)}{r^3 \left(1 + \frac{G_N M}{2rc^2}\right)^3} \times \left[\mathbf{r} \left(1 + \beta^2 + \frac{G_N M}{2rc^2 - G_N M}\right) - \beta(\beta \cdot \mathbf{r}) \right]$$

(I'm grateful to Schucking for this communication. I am also grateful to him and to Mikhail Voloshin and Alexander Dolgov for very enlightening discussions.) The first-order approximation in gravitational coupling implicit in equation 16 is very good for the cases of the Sun and the Earth.

I have found equation 16 in only one book.² Unfortunately the formula is constructed there semiempirically, and the book itself is full of $E = mc^2$ and all that.

The lack of space and time didn't allow me to discuss in my article such important questions as the mass of a system of particles. I consider this and some other problems in more detail in an extended version of the article.³

I don't think we should try to banish $E = mc^2$ from T-shirts, badges and stamps. But in the textbooks it should appear only as an example of a historical artifact, with an explanation of its archaic origin.

REFERENCES

1. R. S. Serway, J. S. Faughn, *Physics*, Saunders, Philadelphia (1989).
2. M. G. Bowler, *Gravitation and Relativity*, Pergamon, Elmsford, N. Y. (1976).
3. L. B. Okun, *Usp. Fiz. Nauk.* **158**, 511 (1989) [*Sov. Phys. Usp.* **32**, 629 (1989)].

Lev B. Okun
*Institute of Theoretical and
Experimental Physics
Moscow, USSR*