

# Money Illusion and the Aggregate Consumption Function

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A standard result of the theory of rational consumer behavior in a static monetary economy is that a consumer's demand functions for commodities are homogeneous of degree zero in prices, money income, and money wealth.<sup>1</sup> Don Patinkin has defined this condition as the absence of money illusion. People whose demands for commodities would be altered by an equiproportionate change in all prices, money income, and money wealth are said to suffer from money illusion.

Aggregating over all commodities purchased by the consumer, this standard theorem leads to the conclusion that an individual's total real consumption demand is homogeneous of degree zero in prices, money income, and money wealth. Finally, aggregating over all consumers, this result would imply that the economy's aggregate real consumption should be a function of aggregate real income and aggregate real wealth, but not the price level.

Most empirical studies of aggregate con-

sumer behavior assume this absence of money illusion when specifying their consumption functions. But the world in which consumers make their decisions and take their actions is quite different from the static model of traditional consumer theory where rationality and perfect information always prevail. First, the world we observe is a dynamic one. It is also one in which irrationality may exist in the short run and in which there are difficulties associated with the collection and interpretation of reliable information. Hence, while in the long run we might expect to find people free of money illusion, it is not so clear that in the short run we should expect to find consumers' total real consumption demand homogeneous of degree zero in prices, money income, and money wealth.

When one takes into account the lags that necessarily exist in processing price information in the real world, the case for price-level misperception by consumers becomes strong and the a priori case for the existence of money illusion in consumers' short-run demand functions becomes more convincing.<sup>2</sup> If one were, for example, to estimate an equation which explained current real consumption as a function of

<sup>2</sup> On the other hand, consider two economies which are structurally identical except that at time,  $t=0$ , the price level in one economy is  $k$  times the price level in the other. Allow both systems to run for a long time, and observe the parts of their respective time paths very far out in the future. One might be surprised if after a period long enough for the transient effects of the initial conditions to be removed from the two systems, one found the two systems behaving very differently. But it should be clear, at this point, that we are analyzing only the short-run relationship between consumption and income, wealth, and prices.

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<sup>1</sup> See Patinkin [18, pp. 17-23, 403-05, and *passim*] and Samuelson [20, Ch. V].

only current money income, money wealth, and prices, one would not be very surprised to find an inhomogeneity with respect to the current price variable. In fact, a crude test of the model presented in Section I, using only current values of all variables, yielded just such an inhomogeneity.<sup>3</sup>

We must then address ourselves to a basic question. If one estimates a short-run consumption function carefully, taking account of distributed-lag adjustments, simultaneous-equation relationships and the like, will the resulting short-run relationship show that money illusion is present? And if the price level does appear in the estimated equation in the form of a money-illusion effect, exactly how much of this money illusion is there?

The usual assumption that consumers are free of money illusion has not been subjected to systematic testing.<sup>4</sup> As Patinkin concluded at the end of his note on "Empirical Investigations of the Real-

Balance Effect," "There are other basic questions which have not been dealt with here. Thus it would be desirable to carry out a direct test of the hypothesis that consumers are free of money illusion. . . ." [18, p. 664]

Only careful specification and examination of a per capita real consumption function can provide further insight and enable a more reasoned judgment to be made about the effect of the overall price level on consumer decisions and, in particular, about money illusion's presence. In Section I, we formulate a consumption function, in the framework of the Ando-Modigliani-Brumberg "life-cycle" hypothesis, that allows the price level to play an independent role in determining the level of per capita real consumption, and we discuss the alternative interpretations of this role. Fitting this function to *U.S.* quarterly data from 1955 I-1965 IV in Section II, we find that the general price level does, indeed, play a significant role in determining the level of per capita real consumption. Our results are shown to be consistent with a model which embodies a money-illusion effect via a distributed-lag adjustment to the price level or with a model which has the money-illusion effect combined with a price-expectations mechanism. But the results are not consistent with a model which hypothesizes the complete absence of money illusion. We conclude, then, that consumers do suffer from some degree of money illusion.

Section III investigates the degree to which our conclusions themselves might be illusory because of statistical problems. Our consideration of these possible statistical pitfalls leads to the conclusion that a significant and substantial degree of money illusion does exist in the *U.S.* consumption function. The paper concludes with a brief indication of the further questions our study's results would suggest should be examined.

<sup>3</sup> We estimated the consumption functions

$$\left(\frac{C}{P}\right)_t = \beta_0 + \beta_1 \left(\frac{Y}{P^\alpha}\right)_t + \beta_2 \left(\frac{W}{P^\alpha}\right)_t,$$

and

$$\begin{aligned} \left(\frac{C}{P}\right)_t = & \beta_0 + \beta_1 \left(\frac{Y}{P^\alpha}\right)_t + \beta_2 \left(\frac{W}{P^\alpha}\right)_t \\ & + \beta_3 \left(\frac{Y}{P^\alpha}\right)_t \left(\frac{W}{P^\alpha}\right)_t, \end{aligned}$$

where  $C$  is per capita consumption,  $Y$  is per capita labor income,  $W$  is per capita consumer net worth,  $P$  is the price level of consumer goods, and  $\alpha$  is a money-illusion parameter. (Clearly,  $\alpha=1$  implies the absence of money illusion, while  $\alpha=0$  implies extreme money illusion in the Patinkin sense. See [10].) Our results indicated that the maxima of the likelihood functions for the two equations were quite far from the point  $\alpha=1$ , being in fact much closer to  $\alpha=0.00$  than to  $\alpha=1$ .

<sup>4</sup> For example, consumption functions have generally been estimated using *either* money *or* real values of the variables. Rarely is any attempt even made to compare the results obtained using the same specification except for the substitution of real for money values or vice versa. Two exceptions are studies by R. Ferber [8] and J. J. Arena [3], [4]. Unfortunately they did not yield any significant conclusions about money illusion's existence.

### I. *The Specification of a Money-Illusion Consumption Function*

The life-cycle analysis of rational consumption behavior begins with a consumer who maximizes his utility function subject to the constraint imposed by his resources.<sup>5</sup> "As a result of this maximization, the current consumption of the individual can be expressed as a function of his resources and the rate of return on capital with parameters depending on age." [2, p. 56] Aggregating across all consumers and then dividing by population, one arrives at an Ando-Modigliani-Brumberg life-cycle per capita consumption function:

$$(1) \quad \frac{C}{P} = f\left(\frac{Y}{P}, \frac{W}{P}\right),$$

where  $C$  is per capita consumption,  $Y$  is per capita nonproperty income,  $W$  is per capita consumer net worth, and  $P$  is the price level of consumer goods.

Without more detailed assumptions about individual utility functions and distributional effects of changes in  $Y$  and  $W$ , the only meaningful restriction we can impose on (1) is that it is homogeneous of degree zero in  $Y$ ,  $W$ , and  $P$ .<sup>6</sup> Therefore we can specify the aggregate consumption function (1) in multiplicative form as:

$$(2) \quad \left(\frac{C}{P}\right)_t = b_0 \left(\frac{Y}{P}\right)_t^{b_1} \left(\frac{W}{P}\right)_t^{b_2}$$

If money illusion exists, however, the price level should have an independent effect on the level of  $C/P$ . Letting  $c$  denote real consumption,  $y$  denote real net labor income, and  $w$  denote real consumer net worth, all on a per capita basis, the consumption function in the presence of

<sup>5</sup> See Modigliani and Brumberg [15], [16] and Ando and Modigliani [2] for the development of the life cycle hypothesis of saving.

<sup>6</sup> [2, p. 58]. Ando and Modigliani present a much more careful derivation, based upon carefully enunciated assumptions, of their linear "life-cycle" hypothesis model. See pp. 56-62 therein.

money illusion then takes the form

$$(3) \quad c_t = b_0' (y_t)^{b_1} (w_t)^{b_2} (P_t)^{b_3}$$

This log-linear specification, (3), can be justified by noting that beyond requiring that the form chosen possess a certain set of characteristics, the choice of a particular form for a consumption function is an arbitrary process. One should, for example, ensure the proper signs and magnitudes of relevant partial derivatives—marginal propensities to consume out of labor income and out of wealth that are positive and less than unity. The signs and magnitudes of these marginal propensities can only be checked *ex post*, although we can assure the reader that the estimates we obtain do meet these prior specifications.

In logarithmic form, the consumption function in (3) can be rewritten as

$$(4) \quad \ln c_t = b_0 + b_1 \ln y_t + b_2 \ln w_t + b_3 \ln P_t$$

With the consumption function written in this form, if there is no money illusion in the Patinkin sense, we have  $b_3=0$ , and real consumption depends only on real net labor income and real consumer net worth. On the other hand, if there is money illusion in the Patinkin sense—a proportional increase in money income, money wealth, and the price level leads to an increase in the level of real consumption—we have  $b_3>0$ .

There is, however, no reason to expect consumers to react instantaneously, that is, within one quarter, to changes in real income, real wealth, or the price level. It is much more plausible to suppose that consumers react with a lag to changes in these independent variables. Alternatively, one might think it plausible that in making their real consumption decisions in a particular quarter, consumers consider an average of recent experience with regard to the consumption determining variables. Thus, we will rewrite (4) to allow for the possibility of distributed-lag adjustments

to income, wealth, and prices. Using this distributed-lag model, the basic equation to be estimated is the following:

$$(5) \quad \ln c_t = \beta_0 + \sum_{i=0}^I \gamma_i \ln y_{t-i} + \sum_{j=0}^J \delta_j \ln w_{t-j} + \sum_{k=0}^K \eta_k \ln P_{t-k} + \epsilon_t.$$

Before going on to estimate the money-illusion consumption function (5), it will be useful to discuss the interpretation of the price term,

$$\sum_{k=0}^K \eta_k \ln P_{t-k},$$

and to make clear its role in testing the no money-illusion hypothesis. There are at least three ways in which a price-level effect might make its appearance in our basic equation. Since money illusion represents only one such effect and since these three possibilities are not mutually exclusive, it is important to show how one can determine whether money illusion is present in consumer behavior. It will be helpful in examining these three effects to write the basic equation in multiplicative form, equation (6). We see that the first type of price effect possible is the case of pure money illusion in the Patinkin sense. Instead of basing their consumption decisions on real income and real wealth, consumers modify the deflating factors of income,

$$\prod_{i=0}^I P_{t-i}^{-\gamma_i} \quad \text{and wealth,} \quad \prod_{j=0}^J P_{t-j}^{-\delta_j}$$

by multiplying their product by

$$\prod_{k=0}^K P_{t-k}^{\eta_k} \neq 1.$$

When prices, money income, and money wealth all increase proportionately, consumers notice the income and wealth increases more than they do the price level rise, and increase their real consumption. Hence, in the case of *pure money illusion we would have*

$$\sum_{k=0}^K \eta_k > 0:$$

consumers exhibit money illusion via a distributed-lag adjustment to the price level.

Suppose, in contrast, that consumers do not suffer from money illusion, but that there is a price-expectations mechanism at work. That is, the real consumption function takes the form

$$(7) \quad c_t = g(y_{t-i}, w_{t-j}, P_t^e),$$

where  $y_{t-i}$  is a vector of recent real income experience,  $w_{t-j}$  is a vector of recent real wealth experience, and  $P_t^e$  is a vector of the consumers' expectation of future price levels. The hypothesized behavior lying behind such a function is that if consumers expect prices to rise in the future, they will restructure the time pattern of their consumption by moving consumption from the future toward the present. Then, if their expectations are realized, they will reduce their consumption in the future.<sup>7</sup>

Price expectations, the vector  $P_t^e$ , could be formed in several different ways. There are two formation processes on which we will focus here in order to see how a money-illusion effect can be identified. They are expectations derived on the basis of recent price level experience,

<sup>7</sup> See Power [19] for a further discussion of the role of the intertemporal substitution effect, price expectations, and the real-balance effect.

$$(6) \quad c_t = e^{\beta_0} \left[ \prod_{i=0}^I \left( \frac{Y}{P} \right)_{t-i}^{\gamma_i} \right] \left[ \prod_{j=0}^J \left( \frac{W}{P} \right)_{t-j}^{\delta_j} \right] \left[ \prod_{k=0}^K P_{t-k}^{\eta_k} \right] e^{\epsilon_t}.$$

$$(8) \quad P_t^e = h_1(P_{t-k}),$$

where  $P_{t-k}$  is the vector of recent price experience, and expectations based on recent observations of the *rate* of change of the price level,

$$(9) \quad P_t^e = h_2(\Delta P_{t-k}),$$

where  $\Delta P_{t-k}$  is the vector of recent price change experience.

Considering the level-based expectations mechanism first, in terms of our log-linear consumption function  $P_t^e$  would have to enter into the consumption decision in the form of a product of the  $P_{t-k}$ 's,

$$\prod_{k=0}^K P_{t-k}^{\eta_k},$$

with the weights summing to zero. The weights must sum to zero because of the purely allocative role of the price-anticipations mechanism and because in a steady state with  $P_{t-1} = P_t$  for all  $t$ ,  $P_t^e$  must have no effect on the consumption decision as represented in (7): prices will not be changing. In the case of a *pure level-based price-expectations mechanism and no money illusion, we would have*

$$\sum_{k=0}^K \eta_k = 0.$$

On the other hand, if consumers' price expectations were formed on the basis of recent price-level changes as in (9) (referred to here as change-based expectations), a log-linear consumption function would take the form of equation (10), where the price ratios represent the rates of change indicated in equation (9). In this case, the purely allocative role of expectations about future prices implies that the sum of the  $\theta_k$  parameters should be zero.

Note, moreover, that the consumption function in equation (10) is consistent with the behavior we should expect in a steady state. If there exists a steady state, so that prices do not change from one period to the next, the  $P_t^e$  argument should disappear from the function in (7). This is precisely what equation (10) indicates would happen, since with  $P_{t-1} = P_t$  for all  $t$ , each  $P_{t-k}/P_{t-k-1}$  ratio would equal unity and we would have

$$c_t = e^{\beta_0} \left[ \prod_{i=0}^I y_{t-i}^{\gamma_i} \right] \left[ \prod_{j=0}^J w_{t-j}^{\delta_j} \right] e^{\epsilon_t}.$$

The question remains as to how we should expect the estimated price coefficients in equation (5) to appear if there is no money illusion but the change-based price-expectations mechanism just described exists. The answer is really quite simple. Writing (10) in logarithmic form, we have

$$(11) \quad \begin{aligned} \ln c_t = & \beta_0 + \sum_{i=0}^I \gamma_i \ln y_{t-i} \\ & + \sum_{j=0}^J \delta_j \ln w_{t-j} \\ & + \sum_{k=0}^{K-1} \theta_k (\ln P_{t-k} - \ln P_{t-k-1}) \\ & + \epsilon_t. \end{aligned}$$

But the price term can be written as

$$(12) \quad \begin{aligned} & \sum_{k=0}^{K-1} \theta_k (\ln P_{t-k} - \ln P_{t-k-1}) \\ & = \theta_0 \ln P_t + \sum_{k=1}^{K-1} (\theta_k - \theta_{k-1}) \ln P_{t-k} \\ & \quad - \theta_{K-1} \ln P_{t-K}. \end{aligned}$$

Therefore, if we estimate (5) and the true

$$(10) \quad c_t = e^{\beta_0} \left[ \prod_{i=0}^I y_{t-i}^{\gamma_i} \right] \left[ \prod_{j=0}^J w_{t-j}^{\delta_j} \right] \left[ \prod_{k=0}^{K-1} \left( \frac{P_{t-k}}{P_{t-k-1}} \right)^{\theta_k} \right] e^{\epsilon_t}.$$

model is one of pure change-based price-expectations and absence of money illusion, that is (10), we would have

$$(13) \quad \sum_{k=0}^K \eta_k = \theta_0 + \sum_{k=1}^{K-1} (\theta_k - \theta_{k-1}) \\ - \theta_{K-1} = 0.$$

Thus, in the case of a *pure change-based price-expectations model and no money illusion, we would have*

$$\sum_{k=0}^K \eta_k = 0.$$

This discussion shows that estimation of the basic equation (5) will yield an unambiguous test of the no money-illusion hypothesis. If money illusion is present to some degree, we should find

$$\sum_{k=0}^K \eta_k$$

positive and significantly different from zero. If, on the other hand, the true model is one in which money illusion is absent we should find that

$$\sum_{k=0}^K \eta_k$$

is not significantly different from zero. It should be stressed that we cannot distinguish between (a) a model in which money illusion is present but there is neither a level-based nor a change-based price-expectations mechanism at work, and (b) a model in which money illusion is present and such price-expectations mechanisms are also operative. Ideally, one would like to distinguish among the following alternative deviations from the standard static model of consumer behavior: (1) the existence of only pure expectations mechanisms, (2) the presence of only short-run money illusion, and (3) the existence of a combination of some expectations mechanism and money illusion.

The statistical estimation and tests to which we now turn enable us to distinguish alternative (1) from alternatives (2) and (3), but we are unable to distinguish between the latter two possibilities.

## II. *Estimation of the Money-Illusion Consumption Function*

The money-illusion consumption function, equation (5), will now be estimated. Section III will then consider various potential problems involved in the estimation procedure, for example, common trends, simultaneity, and so on.

The data used are *U.S.* quarterly series on real consumption per capita,  $c$ , real net labor income per capita,  $y$ , real consumer net worth per capita,  $w$ , and the price level,  $P$ , for the period 1955-I to 1965-IV. These data are described in more detail in the Appendix.

The Consumer Price Index (*CPI*) (1958 = 100) was chosen as the price variable,  $P$ , since it represents the set of prices most relevant to the consumer's buying decision. Use of the principal alternative price variable, the consumption deflator, would create a statistical difficulty since the current value of the price term would be the deflator of the dependent variable, consumption. The denominator of the left-hand side of the regression equation would then appear in the numerator of the right-hand side of that equation. This would cause the coefficient of the current price variable to be negative, and, since the price series is serially correlated, it would also reduce the coefficients of other recent values of the price variable, all due to a statistical aberration. The *CPI*, of course, has its drawbacks as well.<sup>8</sup>

<sup>8</sup> For example, being a Laspeyres price index, it is subject to the customary catalogue of criticisms that can be levelled at such a fixed-weight-base index. In particular, it does not take accurate account of changes in the market basket that result from changes in the relative prices of commodities, and it does not provide an adequate means for coping with the introduction of

Given the different shortcomings of the alternative price indices, what is crucial is that our results concerning the significance or insignificance of the price term in the consumption function should be independent of the choice of index. The *CPI* will be used as the price variable on the right-hand side of the basic equation (5), but we will also show the result of substituting the consumption deflator and the deflator for personal consumption expenditures—indexes in which price relatives receive shifting weights in proportion to the expenditures incurred each year for the goods and services they represent. We can assure the reader that this substitution does not affect the qualitative results of our study.

The data on consumption, income, wealth, and prices will be used in estimating the basic distributed-lag consumption function equation (5), shown previously. An  $I$  quarter lag distribution assigns non-zero values to the coefficients of the variable lagged 0, 1, . . .  $I-1$  quarters and a zero value to the coefficients of the variable lagged  $I, I+1, \dots$  quarters. Since the basic equation is linear in natural logarithms the estimated coefficients are, of course, estimates of the elasticity of real consumption demand with respect to changes in  $y, w,$  and  $P$ .

Each of the independent variables is entered in the form of a distributed lag with current real consumption per capita dependent on current and past values of the independent variables. The distributions of the coefficients of these lagged independent variables, which show the time shape of response of  $c$  to changes in  $y, w,$  and  $P,$  are estimated using the flexible Almon interpolation technique.<sup>9</sup> This

new commodities or quality changes. For an excellent discussion of the problems associated with the *CPI* as a price index, see National Bureau of Economic Research [22, especially Ch. IV].

<sup>9</sup> See Almon [1] for the basic theory. Almon, Bishchoff [5] and Modigliani and Sutch [17] have all used the Almon technique extensively.

method takes the lagged values of each of the independent variables as a set and estimates a separate smooth distribution of coefficients for each variable, subject only to the constraint that the coefficients be interpolated from Lagrange polynomials of a given degree.

Since two critical values might be needed to capture the lag distribution on the price terms if the purely allocative level-based expectations hypothesis discussed in Section I were correct, third-degree polynomials are used in estimating the coefficients of the price terms in (5). We will also use third-degree polynomials in estimating the distributed lags of income and wealth in (5). The freedom this accords to the shape of the distributions of the income, wealth, and price coefficients will ensure that our results do not come from the imposition of monotonic lag distributions on the coefficients.<sup>10</sup>

Since changing the lag length on one variable in (5) will usually affect the coefficients of all terms, searching for the optimal lag length on all three variables is a fairly complex procedure. In addition, there can sometimes be a conflict between criteria for determining the best lag length: overall goodness of fit, significance of the last coefficient, shape of the lag distribu-

<sup>10</sup> The Almon technique permits the user to constrain the coefficient of the value of the independent variable one period forward, for example,  $\ln y_{t+1}$ , to equal zero, and/or to constrain the distribution of coefficients to taper off gradually to zero at the far end, e.g., as  $i$  approaches  $I$ , by setting the last coefficient equal to zero. Since we do not want to exclude the possibility of convex monotonically decreasing lag distributions, we will not apply the zero constraint to the coefficients of the one-period ahead values of the independent variables. But since we expect the coefficient distributions, whatever their shapes, to approach zero gradually rather than abruptly, as the relevant variable values recede into the past, we will constrain the distributions to taper off gradually. This constraint will smooth the distribution somewhat and it should yield the same length "best" lag distribution as would unconstrained estimation. Our estimated equation is virtually the same in both the constrained and unconstrained versions, as a glance at fn. 12 will confirm.

tion, and so on. We began by setting lag lengths  $I$ ,  $J$ , and  $K$  in (5) all at four quarters, and then experimented with changes in those lengths.

With  $I=J=K=4$  in the initial estimate of (5), the price coefficients were all positive with a significant sum. Only the current wealth coefficient was at all significant, and the income lag was obviously too short—the coefficient of  $y_{t-3}$  was significantly positive. We therefore lengthened the lag distribution on income and shortened that on wealth until we reached the first equation shown in Table 1, in which  $I=7$ ;  $J=1$ ;  $K=4$ . Table 1 lists the coefficients of each equation horizontally, with the first number for each variable giving the lag length and the second the sum of the coefficients in the lag distribution for that variable, with the standard error of the sum in parentheses. Figure 1 shows the lag distribution of the coefficients of  $\ln y$  and  $\ln P$  in Table 1 equations 1-1 through 1-4.

The regressions that led to equation 1-1 from the initial 4-4-4 specification showed

that while the current wealth term was highly significant in all cases, lagged wealth terms were uniformly insignificant, and always quite near zero, whenever the wealth lag was extended beyond one quarter. This might be expected because (a) the wealth series is highly autocorrelated, and (b) one might expect a weighted average of past labor incomes to be collinear with the wealth of very recent periods.

Equation 1-1 with the income lag at seven quarters and the price lag at four quarters shows that the coefficients of all three explanatory variables are highly significant. The income lag, in Figure 1, is positive and monotonically declining, while the price lag is positive in the shape of an inverted U. Extending the length of the price lag gave us equations 1-2 to 1-4 of Table 1. The effect of lengthening the price lag, as can be seen in Figure 1, was to change its shape from a significant inverted U to a significant monotonically declining distribution. But, as is shown in Table 1, the sum of the price coefficients rises only slightly, from 0.411 to 0.418,

TABLE 1—ESTIMATION OF THE BASIC MONEY-ILLUSION CONSUMPTION FUNCTION  
TEXT EQUATION (5)

Equation	Independent Variables								$R^2$	$SE \times 10^2$	$D.W.$
	Constant	$I$	$\ln y$ $\sum \gamma_i$	$J$	$\ln w$ $\delta_0$	$K$	$\ln P$ $\sum \eta_k$				
1-1	-1.937 (0.115)	7	0.647 (0.042)	1	0.136 (0.036)	4	0.411 (0.036)	.9982	.3006	1.727	
1-2	-1.945 (0.114)	7	0.658 (0.044)	1	0.129 (0.036)	5	0.415 (0.036)	.9982	.2980	1.756	
1-3	-1.952 (0.113)	7	0.662 (0.044)	1	0.126 (0.033)	6	0.418 (0.036)	.9983	.2964	1.755	
1-4	-1.953 (0.114)	7	0.661 (0.043)	1	0.127 (0.036)	7	0.418 (0.036)	.998 $\frac{3}{4}$	.2964	1.757	
1-5	-1.941 (0.117)	8	0.659 (0.046)	1	0.132 (0.035)	7	0.413 (0.036)	.9982	.2990	1.746	
1-6	-1.943 (0.118)	8	0.658 (0.046)	1	0.133 (0.035)	8	0.413 (0.036)	.9982	.2988	1.744	



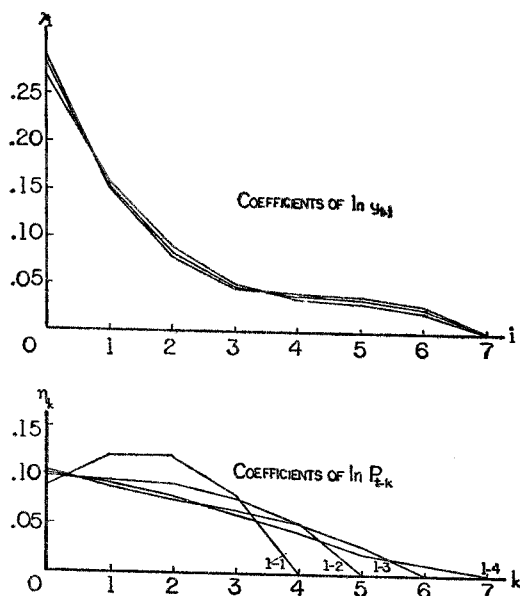


FIGURE 1. DISTRIBUTION OF COEFFICIENTS OF  $\ln y_{t-i}$  AND  $\ln P_{t-k}$  IN TABLE 1 EQUATIONS 1-1 TO 1-4

while the entire distribution becomes more significant. Lengthening the price lag from four to seven quarters leaves the income and wealth coefficients substantially unchanged. It does, however, reduce the standard error of the estimate of real consumption per capita from \$5.03 in 1-1 to \$4.99 in 1-4, compared with an average per capita consumption of \$1,837, and it raises the Durbin-Watson statistic from 1.73 to 1.76, both well above the range normally encountered in this type of time-series estimation when no lagged dependent variables are explicitly included.

When the lag on the wealth variable was extended in any of the Table 1 equations, the coefficients of all wealth terms but the current one were completely insignificant. The ratio of their sum to its standard error was less than the *t*-ratio of  $w_t$  alone in the unlagged version while the income and price coefficients and the equation statistics in these regressions were not significantly different from those of equation 1-4. This evidence led us to include only

the current value of wealth in our equations.

Equation 1-4 of Table 1 has been chosen as our best estimate of the money-illusion consumption function. While it is only marginally superior to equations 1-1 to 1-3 in a statistical sense, it does have more significant coefficients in both the income and price lags, a lower standard error than that of 1-1 and 1-2, and a higher Durbin-Watson statistic. Also, if one believes that real consumption depends on present and lagged values of money income with each value deflated by a corresponding misperceived price level, then one would think that the price lag and income lag should be roughly the same length since the variable really moving consumption is incorrectly deflated income.<sup>11</sup>

Our choice of 1-4 as the estimate of the money-illusion consumption function is buttressed by the fact that when we extend the lags on income and price beyond seven quarters, the standard error rises again, the Durbin-Watson decreases, and the significance of both the income and price lags falls. Two examples of the effect of extending these lags appear in equations 1-5 and 1-6. Furthermore, equation 1-6 with an eight-period lag on both price and income is better than equation 1-5 which has a seven-period price lag and an eight-period income lag. This tends to support the belief that the price and income lag should be the same length.

The final equation for the money-illusion consumption function is, then,

$$(14) \quad \ln c_t = -1.953 + \sum_0^7 \gamma_i \ln y_{t-i} + (0.114)$$

<sup>11</sup> It is interesting to note that with monotonically declining lag distributions of equal length on both price and income, the money-illusion consumption function could reflect an adaptive adjustment process with real consumption following incorrectly deflated income in the manner suggested by Koyck [11]. See Griliches [9] for a review of the Koyck and similar models.

$$+ 0.127 \ln w_t + \sum_0^7 \eta_k \ln P_{t-k} \\ (0.036)$$

$R^2 = .9984$ ;  $S.E. = .002964$ ;  $D.W. = 1.757$ .

The standard error of predicted  $c$  implied by (14) is only \$4.99 compared with a mean  $c$  of \$1,837.

The coefficients of lagged per capita income,  $\gamma_i$ , and lagged price,  $\eta_k$ , are plotted in Figure 1 and listed in Table 2.<sup>12</sup> The coefficients of real net labor income lagged zero to six quarters sum to 0.661 in our final equation. This is the elasticity of per capita real consumption with respect to changes in per capita real net labor income. The implied marginal propensity to consume is .71 at 1965-IV levels of \$2,118 for per capita consumption and \$1,964 for per capita income. Similarly, the per capita real wealth coefficient and elasticity is 0.127, giving a marginal propensity to consume out of real net wealth of .024 at 1965-IV per capita wealth of \$11,160. These marginal coefficients can be compared with the Ando-Modigliani values of .70 and .06 on income and on wealth.

The more interesting coefficients are those of the current and lagged values of the *CPI*. While the exact shape of the price lag is not completely clear, as a glance at Figure 1 can show, in all the versions of the money-illusion consumption function shown in Table 1, the sum of the price coefficients is between 0.411 and 0.418—the difference of .007 is completely insignificant. Furthermore, the sum of the

<sup>12</sup> Reestimation of text equation (14) without the constraint that the distributions of the coefficients of  $\ln y_{t-i}$  and  $\ln P_{t-k}$  taper off to zero as  $i$  and  $k$  approach 7 does not significantly change the equation. The income lag in the unconstrained equation has a coefficient sum of 0.662 with a standard error of 0.045; the coefficient of  $\ln y_{t-6}$  is  $-0.003$  with a standard error of 0.058. The price lag has a coefficient sum of 0.431 with a standard error of 0.039; the coefficient of  $\ln P_{t-6}$  is  $-0.045$  with a standard error of 0.094. The coefficient of  $\ln w_t$  is 0.115 with a standard error of 0.038.  $R^2 = .9983$ ;  $S.E. = .003001$ ;  $D.W. = 1.783$ .

TABLE 2—COEFFICIENTS OF  $\ln y_{t-i}$  AND  $\ln P_{t-k}$  IN EQUATION 1-4 OF TABLE 1, TEXT EQUATION (14)

Lag ( $i, k$ )	Coefficient of	
	$\ln y_{t-i}$	$\ln P_{t-k}$
0	0.274 (0.046)*	0.100 (0.082)
1	0.151 (0.015)	0.093 (0.029)
2	0.081 (0.021)	0.080 (0.047)
3	0.048 (0.018)	0.063 (0.039)
4	0.039 (0.013)	0.044 (0.023)
5	0.038 (0.019)	0.026 (0.029)
6	0.030 (0.019)	0.010 (0.032)
7	0	0
Sum	0.661	0.418
Standard error of Sum	(0.043)	(0.036)

\* The numbers in parentheses are the standard errors of the coefficients.

price coefficients is highly significant in all versions of the equation;  $\sum \eta_k$  is never less than eleven times its standard error. Thus real consumption rises when the *CPI* rises, real income and wealth being held constant, with an elasticity of 0.418 in our final equation.<sup>13</sup> If the *CPI* rises by 1 percent (not percentage point) consumption rises by 0.418 percent, or, at 1965-IV levels, \$8.85 (in 1958 dollars).

This sensitivity of real consumption to the price level in our log-linear specification of the consumption function will, of course, lead to the conclusion that real consumption will exceed Gross National Product (*GNP*) if prices rise relative to real income

<sup>13</sup> This positive value for  $\partial c/\partial P$  represents the presence of money illusion in the sense of the traditional Patinkin experiment—double all prices, money income and money wealth and see if real consumption rises.

and real wealth for a long enough period of time. It should be recalled at this point, though, that the relationship (14) presents is a *short-run* consumption function, without any necessary long-run implications for rationality or money illusion.<sup>14</sup>

To ensure that our results concerning the significance of the price variable in determining real consumption per capita are not sensitive to the use of the *CPI* as the price variable, we reestimated equation (14) using first the consumption deflator and then the deflator for personal consumption expenditures as the price variable. The use of these deflators leads one to observe the statistical artifact mentioned at the beginning of this section. In both equations the distribution of the price coefficients had the shape of an inverted U with the coefficient of the current value of the price variable insignificantly negative and that of  $\ln P_{t-1}$  insignificantly positive. Our qualitative conclusions derived from (14) were not, however, affected by either of these reestimated equations.<sup>15</sup>

Finally, to test the sensitivity of our results to the particular income series used, we also reestimated (14) using disposable personal income per capita deflated by the consumption deflator in place of real net labor income per capita. As was the case with the price variable, our qualitative conclusions, particularly those concerning the role of prices in determining real con-

sumption, were not sensitive to the choice of income series.<sup>16</sup>

The results presented in this section show that the price level has a significant, independent, positive effect on the level of per capita real consumption. When prices, money income, and money wealth rise in the same proportion, real consumption rises. We interpret this result as evidence that there exists a significant degree of money illusion in the economy in the short run.

As the discussion in Section I showed, the price level might also affect the level of real consumption through a price-expectations mechanism. But it was also shown in that section that the conclusive test for the presence or absence of money illusion in our specification of the consumption function was whether or not  $\sum \eta_k$ , the sum of the price coefficients, was positive and significantly different from zero. Since the estimates in this section lead to the conclusion that the sum of the price coefficients in (5) is significantly positive, our results are inconsistent with either a pure level-based or a pure change-based price-expectations hypothesis.

Further evidence supporting this conclusion can be obtained by looking at the effect of explicitly imposing the constraint that the sum of the price coefficients in (5) is zero. As the discussion in Section I showed, this restriction is imposed if one estimates equation (15) instead of (5):

$$\ln c_t = \beta_0 + \sum_0^I \gamma_i \ln y_{t-i} + \sum_0^J \delta_j \ln w_{t-j} + \sum_0^{K-1} \theta_k \ln \frac{P_{t-k}}{P_{t-k-1}} + \epsilon_t \quad (15)$$

<sup>14</sup> As it stands, the objection that raising prices relative to income and wealth for a long enough time leads to absurd results simply says that if we extrapolate our results sufficiently far beyond the data the results become invalid. This is not a surprising result.

<sup>15</sup> The coefficient sum of the consumption deflator was 0.332 with a standard error of 0.042; the coefficient sum using the deflator for personal consumption expenditures was 0.366 with a standard error of 0.043. The income and wealth coefficients in both equations were not significantly different from those of (14). In both cases the standard error of estimate was slightly higher than that of (14) and the Durbin-Watson statistic was lower, suggesting that the *CPI* may well be, in fact, the correct price series to use.

<sup>16</sup> As expected, substituting real disposable personal income per capita for real net labor income per capita slightly raised the sum of the income coefficients to 0.678, and reduced the wealth coefficient to 0.082. It also reduced the sum of the *CPI* price coefficients to 0.338 with a standard error of 0.029.

Setting  $I=7$ ,  $J=1$ , and  $K-1=6$ , the appropriate lag lengths determined by our earlier work, we obtained the following estimates:

$\beta_0$	-0.678 (0.066)	$K-1$	6
$I$	7	$\Sigma\theta_k$	-0.004 (0.643)
$\Sigma\gamma_i$	0.533 (0.092)	$R^2$	0.9918
$J$	1	$S.E.\times 10^2$	.6416
$\delta_0$	0.453 (0.049)	$D.W.$	0.817

The standard error of this estimate of (15) is more than twice that of our final equation (14), and the Durbin-Watson statistic is only 0.82, strongly suggesting that the equation has been misspecified.

That this estimate of (15) is significantly inferior to the money-illusion consumption function, equation (14), can also be seen from the following analysis of variance. The sum of squared residuals in (14) is  $.3163\times 10^{-3}$ ; that in the estimate of (15) is  $.1482\times 10^{-2}$ . With forty-four observations and eight regression variables in each equation, we have

$$F(1, 36) = \frac{1.4820 - .3163}{.3163/36} = 132.7$$

to test the significance of the effect of the added restriction that  $\sum \eta_k$  is zero. Since  $F(1, 36) = 7.39$  at the 1 percent level, it is clear that constraining  $\sum \eta_k$  to equal zero significantly worsens the explanatory power of the equation.<sup>17</sup>

This estimate of (15) and the corresponding  $F$ -test provide additional evidence that we are not observing simply a pure price-expectations mechanism. There

<sup>17</sup> A search for the "best fitting" estimate of (15) yielded an equation with an eleven-quarter income lag and a seven-quarter price lag. The estimate of (15) with  $I=11$ ,  $J=1$  and  $K-1=7$  fits better than the one presented in the text but still has a standard error nearly twice that of any of the equations in Table 1, and a Durbin-Watson statistic less than unity. Its  $F$ -test performance was correspondingly poor as  $F(1, 36)$  was 104.9.

may be a price-expectations mechanism at work in the determination of real consumption, but if there is, it is operating in conjunction with the existence of money illusion.

### III. *Statistical Problems of Trend, Cycle, and Simultaneity*

This section reports briefly on several further tests of the money-illusion consumption function, equation (14), which were conducted to ensure that our results are not seriously affected by problems of time and timing: trend interrelationships among variables, cyclical factors in the economy, and simultaneous equations bias.

#### *Trend Relationships Among Variables*

In any time-series regression analysis there exists the possibility that a spurious fit may be obtained due to the fortuitous presence of trends in both dependent and independent variables. While the Durbin-Watson statistic of equation (14), 1.76, suggests that we have captured more than a trend relationship, we performed a direct test of the role of time trend in our results by regressing  $\ln c/ct$  the natural log of consumption deviations from trend on similar transformations of the income, wealth and price variables.<sup>18</sup> This is, of course, equiva-

<sup>18</sup> We should note that if the *CPI* has a spurious upward trend due to its inadequacy in observing quality changes and the introduction of new products, and if this effect somehow underlies the results in Section II, then the sum of the coefficients of the price term in the deviations-from-trend equation should be insignificant. Also, if the changing age distribution of the population was increasing the aggregate propensity to consume, and if this movement was being picked up by the trend in our price variable, the sum of the price coefficients in the deviations-from-trend version should be insignificant. In fact, the percentage of all families with head aged 15-25 has been growing, but the average income of these families is only about 75 percent of the national average. Hence the effect of this population drift on the propensity to consume is ambiguous in any event. Branson and Thurow in an unpublished paper [7] have concluded that changing population distribution, in the period 1950-1970, has little effect on the aggregate propensity to consume.

lent to simply adding time to equation (14) as an independent variable.

Equation 3-1, Table 3, shows the result of reestimating the money-illusion consumption equation (14) using deviations from trend. The format of Table 3 is the same as that of Table 1 except that since lag lengths are fixed at  $I=7$ ,  $J=1$ ,  $K=7$ , they are not shown. This deviations-from-trend version of the consumption function explains 95 percent of the variance of the deviations of consumption from its logarithmic trend. All the independent variables are significant. In particular, the sum of the price coefficients is still significantly positive, 2.7 times its standard error, indicating that real consumption per capita is positively related to the price level both along trend and in deviations from trend.

#### *Price Movements and the GNP Gap*

Another potential role of the price level in determining consumption, besides the existence of money illusion or the existence of price-expectations mechanisms, could be the presumed correlation of price movements with employment and distributional factors in the business cycle. It might be possible, for example, that as aggregate demand rises relative to potential output and unemployment falls, prices rise. The falling unemployment rate could increase the income of low-income families with higher-than-average consumption propensities, shifting the per capita consumption function (of income and wealth alone) up. If this movement were generally associated with rising prices, and vice versa, we might find prices significant in the consumption function due only to this distributional effect associated with a diminishing *GNP* gap.

Two points can be made to counter this hypothesis. First, balancing the increased income at the lower end of the income distribution is the well-known tendency for the profit share to rise in a cyclical up-

swing, shifting income to families with presumably lower-than-average consumption propensities.<sup>19</sup> Second, the correlation between price movements and the ratio of actual to potential real *GNP* is not all that clear in the period over which our consumption function was estimated.<sup>20</sup>

To test directly the hypothesis that our price terms only reflect cyclical effects, we reestimated the money-illusion consumption function (14) adding the natural logarithm of the ratio of actual real *GNP* to potential real *GNP* as a variable.<sup>21</sup> If the hypothesis is correct, the price term is merely a proxy for the closing of the *GNP* gap and inserting this new variable into the equation should greatly reduce the sum of the price coefficients and the significance of that sum, while assigning a significantly positive coefficient to the actual real *GNP*/potential real *GNP* variable.

Equation 3-2 of Table 3 gives the result of this test. The variable  $y/y^p$  is the ratio of actual real *GNP* to potential real *GNP*. The rejection of the hypothesis that the price term is merely reflecting cyclical movements through a Phillips' curve mechanism is clear. While the cyclical *GNP* variable has a positive and nearly significant coefficient, the sum of the price coefficients is raised, not reduced, by the introduction of this cyclical variable.

#### *Simultaneity Among Consumption, Income, and the CPI*

Our consumption function is, of course, in reality part of a simultaneous system explaining aggregate consumption, income,

<sup>19</sup> See Kuh [12] for evidence on the cyclical behavior of the profit share.

<sup>20</sup> The period 1955 IV to 1958 II saw actual/potential *GNP* fall from 1.05 to 0.94 while the *CPI* rose from 93.5 to 100.0, a 7 percent increase. In the period 1961 I to 1965 IV, however, while actual/potential *GNP* rose from .94 to 1.03, the *CPI* rose from 103.9 to 111.1, still only 7 percent. See Kuh [13] for a recent criticism of the Phillips' curve explanation of price level determination.

<sup>21</sup> Potential *GNP* was computed following the Council of Economic Advisers' formulation. See [21, pp. 60-63].

TABLE 3—TESTS OF STATISTICAL PROBLEMS OF TREND, CYCLE, AND SIMULTANEITY

Equation	Dependent Variable	Constant	Independent Variables				Statistics			
			$\ln y/yt$	$\ln w/wt$	$\ln P/Pt$	$R^2$	$S.E. \times 10^2$	$D.W.$		
3-1	$\ln (c/ct)$	0.001 (0.001)	0.559 (0.055)	0.135 (0.040)	0.300 (0.110)	0.9496	.3337	1.430		
3-2	$\ln c$	-2.204 (0.191)	0.602 (0.056)	0.139 (0.036)	0.473 (0.049)	0.078 (0.048)	0.9984	.2900	1.887	
3-3	$\ln c$	-1.800 (0.137)	0.122 (0.102)	0.510 (0.102)	0.182 (0.041)	-0.204 (0.139)	0.566 (0.127)	0.9980	.3301	1.794
3-4	$\ln c$	-1.892 (0.131)	0.223 (0.092)	0.407 (0.089)	0.159 (0.041)		0.393 (0.042)	0.9977	.3434	1.751

and price determination. Because of the simultaneous relationships truly at work in this system, the error term in our consumption function (5) may be positively correlated with the contemporaneous values of  $y$  and  $P$  in that equation, biasing upward the estimates of the coefficients of these contemporaneous terms and downward the estimates of all the other coefficients.

We suspect an upward bias in the coefficient of  $\ln y_t$  in the consumption function (14) because of the close connection of  $c_t$  and  $y_t$  through the usual national income accounts identity which appears in such simultaneous models. The coefficient of  $\ln P_t$  is, however, less suspect because it is likely that prices in this quarter are mainly determined nonsimultaneously by events in previous quarters, through mark-up pricing procedures and the like. This leads us to consider  $\ln P_t$  a predetermined, rather than a simultaneously determined, variable. Furthermore, there is no reason to expect past income, past prices, or current wealth to be determined simultaneously with current consumption.

To test the extent of such simultaneous

equations bias, we employed the instrumental-variable technique.<sup>22</sup> Total labor income,  $ny$ , and the price level,  $P$ , were each regressed on a set of instruments, and then the resulting instrumental-variable estimates  $\hat{y}_t = (\hat{ny}/n)_t$  and  $\hat{P}_t$  were used to replace  $y_t$  and  $P_t$  in estimating the basic equation (5).<sup>23</sup> First, we reestimated (5) replacing  $\ln y_t, \dots, \ln y_{t-6}$  by  $\ln \hat{y}_t, \ln \hat{y}_{t-1}, \dots, \ln \hat{y}_{t-6}$ , and similarly replacing  $\ln P_t, \dots, \ln P_{t-6}$  by  $\ln \hat{P}_t, \ln \hat{P}_{t-1}, \dots, \ln \hat{P}_{t-6}$ . The coefficients of income and price lagged  $t-1$  to  $t-6$  were estimated using the Almon technique with a third-degree polynomial lag distribution while  $\ln \hat{y}_t$  and  $\ln \hat{P}_t$  were entered separately into the regression. The resulting estimate is equation 3-3 in Table 3. The coefficients listed under  $\ln y_{t-i}$  and  $\ln P_{t-k}$  in 3-3 are the sums of the coefficients of the variables lagged 1 to 6 quarters.

In equation 3-3 the sum of the coef-

<sup>22</sup> See Malinvaud [14, pp. 604-08] for a discussion of the instrumental-variable technique.

<sup>23</sup> The instrumental-variable equations relating  $ny$  to current and lagged money supply, government expenditure, investment, and net exports, and relating  $P$  to current and lagged wage and profit rates are shown in the Appendix.

ficients of  $\ln y$  is 0.632, slightly less than the sum 0.661 in the consumption function estimate (14). The  $\ln \hat{y}_t$  coefficient alone is 0.122 as opposed to 0.274 in (14), although this comparison is not strictly legitimate since the  $\ln y_t$  coefficient in (14) is constrained by the Almon estimation procedure while the coefficient of  $\ln \hat{y}_t$  in 3-3 is not. The sum of the price coefficients in 3-3 is 0.362, again slightly lower than the sum of 0.418 of equation (14). The coefficient of  $\ln \hat{P}_t$  is insignificant and negative, while that of  $\ln P_t$  in (14) was insignificant and positive. The fit of 3-3 is worse than that of equation (14) although the Durbin-Watson statistic is slightly higher.<sup>24</sup>

Equation 3-4 shows the result of a second test of simultaneous equations bias. In this case, we tested only for income simultaneity, as equation (5) was reestimated replacing  $\ln y_t, \dots, \ln y_{t-6}$  by  $\ln \hat{y}_t, \ln y_{t-1}, \dots, \ln y_{t-6}$ , but including the original price lag  $\ln P_t, \dots, \ln P_{t-6}$ . The resulting equation is similar to that of 3-3. The sum of the income coefficients is now 0.630, again insignificantly less than the 0.661 of the original (14), while the coefficient of  $\ln \hat{y}_t$  is 0.223, somewhat less than the coefficient 0.274 of  $\ln y_t$  in (14) but much higher than the coefficient of  $\ln y_t$  in 3-3. The sum of the price coefficients is insignificantly smaller in 3-4 than it was in (14).

These tests of simultaneous equations bias indicate that there may be a slight, but statistically insignificant, upward bias in our estimates of the coefficients of concurrent income and price in the money-illusion consumption function (14). Since the bias appears to be so small as to be insignificant it can probably be ignored quan-

<sup>24</sup> Of course, the  $R^2$  and  $D. W.$  statistics and all the standard errors of 3-3 are equal to those of the true equation only asymptotically, since we are using  $y$  and  $P$  estimates which are only asymptotically equal to the true  $y$  and  $P$  values. But with  $R^2$  values of .99+ on the instrumental variable equations, our standard errors are probably quite close to correct.

tatively and certainly does not affect our principal qualitative conclusion: in the short run the price level has an independent effect on real consumption due to what is commonly called money illusion.

#### IV. *Concluding Comments*

The principal result of this paper—that a significant degree of money illusion appeared in the aggregate consumption function for the United States over the sample period—has a number of interesting implications for macroeconomic theory and policy. Two issues upon which it has an important bearing are the degree of stability of the economy and the nature of the aggregate labor supply function. These questions are discussed in Branson and Klevorick [6].

In closing, there are a number of lines of further investigation that our results suggest it might be fruitful to pursue. First, it would be interesting to disaggregate consumption expenditure and investigate such subaggregates as real personal consumer expenditures on durables and real personal consumer expenditures on nondurables and services using a money-illusion specification of the respective demand functions. Second, it would be most useful to introduce the money-illusion consumption function into a complete simultaneous equation model, observe its performance in such a model, and observe the implications for stability as viewed through simulation experiments. The results presented in this paper suggest that in constructing such macro-models, greater attention should be paid to the link between the price-wage sector and the expenditure sector.

#### APPENDIX

##### *Instrumental-Variable Equations and Data Description*

In Section III the money-illusion consumption function was reestimated with instrumental variable estimates of the  $CPI$ ,

$P_t$ , and per capita real net labor income,  $y_t$ , substituted for the actual series in the unlagged terms. While in principle the form of the instrumental equations used to construct the estimates should be irrelevant since the sole purpose of the technique is to break the simultaneity among  $c_t$ ,  $y_t$ , and  $P_t$  (and clearly not to estimate a behavior function or structural equation for  $y_t$  or  $P_t$ ), we will show here the estimated equations for  $(ny)_t$  and  $P_t$  that were used to construct  $(\hat{ny})_t$  and  $\hat{P}_t$ . Then we will conclude with a description of the basic data series used in the study.

### I. Instrumental-Variable Equations for $(ny)$ and $P$

The instrumental variable equation for aggregate real net labor income,  $(ny)_t$ , has  $(ny)_t$  as a function of current and lagged values of the money supply, real government expenditure, real gross private domestic investment, and real net exports. In linear estimating form the equation is

$$(A-1) \quad (ny)_t = \alpha + \sum_{i=0}^I \beta_i G_{t-i} + \sum_{j=0}^J \gamma_j I_{t-j} + \sum_{k=0}^K \delta_k X_{t-k} + \sum_{m=0}^M \eta_m M_{t-m} + \epsilon_t,$$

where  $G$  is government expenditure,  $I$  is gross private domestic investment, and  $X$  is net exports, all in billions of 1958 dollars from the *Survey of Current Business*, and  $M$  is the money supply, currency plus demand deposits in billions of dollars, from the *Federal Reserve Bulletin*.

As it turns out, the estimated version of (A-1) used to construct  $(\hat{ny})_t$ , includes only current values of  $G$  and  $X$ ,  $I$  lagged 0, 1, and 2 quarters, and  $M$  lagged 0-11 quarters with the coefficients estimated using a third degree Almon lag. The estimated equation is

$$(A-2) \quad (ny)_t = -238.53 + 0.423G_t + \sum_{j=0}^3 \gamma_j I_{t-j} + 0.838X_t \quad (0.120) \quad (0.109) \quad (0.160)$$

$$+ \sum_{m=0}^{12} \eta_m M_{t-m}.$$

$R^2 = .9976$ ;  $S.E. = 1.78$ ;

Mean = \$309.27 billion;  $D.W. = 1.13$ . Period of fit: 1955 I-1965 IV. The numbers in parentheses are standard errors.

The  $\gamma_i$  coefficients of lagged investment and the  $\eta_m$  coefficients of lagged money supply are shown in Table A-1. The coefficients of equation (A-2) were used to compute  $(\hat{ny})_t$ .

The instrumental variable equation for  $P_t$ , the CPI, has  $P_t$  as a function of lagged wage rates,  $W$ , the average hourly earnings of manufacturing workers in dollars from the *Monthly Labor Review*, and the profit rate,  $R$ , the average rate of profit on stockholders' equity from the *Federal Trade Commission Quarterly Financial Reports on Manufacturing Corporations*. The estimated instrumental variable equation is

$$(A-3) \quad P_t = 52.56 + 3.006W_{t-1} + 9.860W_{t-2} + 7.973W_{t-3} + 2.851W_{t-4} - 0.138R_{t-1} \quad (0.90) \quad (4.963) \quad (4.271) \quad (2.455) \quad (3.711) \quad (0.046)$$

$R^2 = .9929$ ;  $S.E. = 0.47$ ; Mean = 101.95;  $D.W. = 0.83$ . Period of fit: 1955 I-1965 IV. The numbers in parentheses are standard errors.

The coefficients of equation (A-3) were used to compute the  $\hat{P}_t$  series used in the text.

### II. Data Description

The data used for real consumption per capita,  $c$ , are essentially aggregate real consumption expenditures on nondurables and services plus depreciation and imputed interest on durables, divided by population,  $n$ . The imputed interest on durables represents consumers' use of durables' services. Real net labor income per capita,  $y$ , is aggregate employees' compensation plus an imputed proportion of proprietors' income plus trans-



TABLE A-1—COEFFICIENTS OF  $I_{t-j}$  AND  $M_{t-m}$  IN  
INSTRUMENTAL VARIABLE EQUATION  
(A-2) FOR  $ny$

Lag ( $j, m$ )	Coefficient of	
	$I_{t-j}$	$M_{t-m}$
0	0.423 (0.092)*	0.582 (0.145)
1	0.155 (0.110)	0.290 (0.067)
2	0.204 (0.094)	0.123 (0.056)
3	0 .	0.054 (0.068)
4	. .	0.062 (0.068)
5	. .	0.121 (0.056)
6	. .	0.209 (0.041)
7	. .	0.302 (0.038)
8	. .	0.375 (0.052)
9	. .	0.405 (0.065)
10	. .	0.368 (0.066)
11	. .	0.241 (0.047)
12	0	0

\* The numbers in parentheses are the standard errors of the coefficients.

fer receipts less employees' social insurance contributions and state, local, and federal tax liabilities on labor income, deflated by the consumption deflator and divided by  $n$ . Real wealth per capita,  $w$ , is the aggregate net worth of households, including liquid assets, consumer durables, and housing, deflated by the consumption deflator and divided by  $n$ . These three series of wealth components, all in billions of 1958 dollars,

quarterly at annual rates, are updated versions of the annual series used by Ando and Modigliani [2]. Population,  $n$ , is total U.S. population in millions. The consumption and income data and the consumption deflator were provided to us by Harold Shapiro, and the wealth and population data were provided by Albert Ando. The CPI data are quarterly averages of the monthly figures published in the *Survey of Current Business*.

The authors will be happy to make the entire set of data available upon written request.

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