What are the difficult choices?

The most common points of difficulty and subjects for discussion at our meetings were as follows.

(i) How do you define a unit by fixing the value of a fundamental constant?

(ii) Would it not be better to use the mass of an atom, such as $m(^{12}\mathrm{C})$, rather than the Planck constant h as a reference to define the kilogram?

(iii) How are the proposed changes going to affect our ability to realise and use the new definitions?

(iv) How can you fix exactly the value of a fundamental constant? What if you choose the wrong value?

(v) Is the Avogadro constant truly a fundamental constant, and the quantity ‘amount of substance’ anything more than a shorthand for counting entities ?

Using a reference constant to define a unit

If Q is a fundamental constant, then the *value* of Q is not for us to choose; we call it a “constant of nature”.

But the *value* of Q is the product of a *numerical value* $\{Q\}$, and a *unit* $[Q]$:

$Q = \{Q\}\,[Q]$ (example: $c = \{c\}[c] =$ 299 792 458 m/s)

We may choose the two factors in different ways. If we define the unit $[Q]$ independently, we must determine the numerical value $\{Q\}$ by experiment.

But if we choose to define the numerical value $\{Q\}$ to suit our convenience, then that **fixes** (or **defines**) the unit $[Q]$.

(example: If we choose the numerical value $\{c\}$ to be exactly 299 792 458, then in the process we define the unit m/s. This is how the metre was re-defined in 1983)

Thus by choosing the ***numerical value*** of a fundamental constant we may define its ***unit***. We choose the numerical value to maintain continuity with the previous value of the unit.

Choosing the reference constant to define the kilogram: *h* or $m(^{12}\mathrm{C})$?

Some people argue that the reference constant for the kilogram should be a mass, just as the reference constant for the second is a time (the period of the caesium transition). But note:

1. The reference constant for the metre is the speed of light *c*, which is a speed – and is not a length. Fixing {*c*} actually defines the unit m/s, not the m.
2. Thus the reference constants do not ***have*** to be the same dimension as the unit. The reference constants are best regarded as a set of constants, which together define the entire SI.
3. The quantities *h* and $m(^{12}\mathrm{C})$ are related by the equation : in which the value of the RHS is at present known with a relative standard uncertainty of 3.2×10^{-10}. Thus we can calculate *h* from $m(^{12}\mathrm{C})$, or vice versa, to a very close approximation.

$$\frac{h}{m(^{12}\mathrm{C})} = \frac{\alpha^2 c}{2R_\infty}\left(\frac{m_\mathrm{e}}{m(^{12}\mathrm{C})}\right)$$

5. Fixing {*h*}, combined with fixing {*e*} to define the ampere, has the clear advantage that we would have exactly known values of K_J and R_K , thus bringing electromagnetic units within the SI. Thus fixing {*h*} is strongly preferred by the electromagnetic community.
6. Also *h* and *e* are amongst the most fundamental constants of physics.
7. Either of the two alternative definitions of the kg can be realised by using both the watt balance to measure *h* and the XRCD experiment to measure the $m(^{12}\mathrm{C})$.

Choosing the reference constant to define the ampere: $\boldsymbol{e}$ or $\boldsymbol{\mu_0}$?

This choice defines all the electrical units

1. Both are fundamental constants.
2. The elementary charge $\boldsymbol{e}$ is a simpler concept to most people than the magnetic constant $\boldsymbol{\mu_0}$. The magnetic constant calls for a deeper understanding of electromagnetic theory than is common among bio-scientists, for example.
3. Fixing $\boldsymbol{e}$ and $\boldsymbol{h}$ has the clear advantage that we would have exactly known values of $K_J = 2e/h$, and $R_K = h/e^2$, thus bringing electromagnetic units within the SI. For this reason fixing $\boldsymbol{e}$ is strongly preferred to fixing $\boldsymbol{\mu_0}$ by the electromagnetic community.

Base units and derived units, or reference constants ?

The traditional presentation of the SI is made in terms of ***base units*** and ***derived units***. The choice of the seven base units is somewhat arbitrary; it could be made in many different ways without changing the system.

The particular choice in the current SI, **m, kg, s, A, K, mol,** and **cd,** is based on history rather than logic.

The Draft Resolution A recommends a more fundamental presentation of the SI by specifying the values of the ***seven reference constants*** used to define the units, without explicitly connecting the individual constants to individual base units.

We also recommend retaining the equivalent presentation in terms of base and derived units, in order to preserve the historical connection with the traditional language. We also recommend changing the order used to present the base units to **s, m, kg, A, K, mol, cd** so that no definition of a unit involves one of the other units later in the list.

Is the Avogadro constant N_A really a fundamental constant, analogous to the speed of light c or the Planck constant h ?

It is sometimes argued that N_A is a man-made constant chosen for our convenience to establish the magnitude of the unit mole. But that is a misunderstanding: it is the *numerical value* of N_A, not the *value* of N_A, that defines the mole.

The value of N_A is given by the product of a *numerical value* × *unit*,

$$N_A = \{N_A\}\,[N_A] = 6.022\ldots\times 10^{23}\ \mathrm{mol}^{-1}$$

The ***value*** of N_A is a constant of nature, just like the Planck constant h. It is an invariant. It is the conversion factor between number of entities N and amount of substance n, just as h is the conversion factor between frequency ν and energy E.

It is the ***numerical value*** of N_A that determines the ***unit*** mole,

just as the ***numerical value*** of h determines the ***unit*** kg in the New SI.

Relative standard uncertainties for some fundamental constants in the current SI and the New SI, multiplied by 10^8 (in parts per hundred million)

constant	***current SI***	***New SI***
$m(\mathrm{K})$	0	4.4
h	4.4	0
e	2.2	0
k	91	0
N_A	4.4	0
R	91	0
F	2.2	0
σ	360	0
m_e	4.4	0.064
m_u	4.4	0.070
$m(^{12}\mathrm{C})$	4.4	0.070
$M(^{12}\mathrm{C})$	0	0.070

constant	***current SI***	***New SI***
α	0.032	0.032
K_J	2.2	0
R_K	0.032	0
μ_0	0	0.032
ε_0	0	0.032
Z_0	0	0.032
$N_\mathrm{A}\,h$	0.070	0
J↔kg	0	0
J↔m^{-1}	4.4	0
J↔Hz	4.4	0
J↔K	91	0
J↔eV	2.2	0