EC201 Intermediate Macroeconomics

Lecture 7-8: Fiscal and Monetary Policy in the IS-LM Model

Lecture Outline:

   - the *IS-LM* model equilibrium
   - how to use the *IS-LM* model to analyze the effects of fiscal and monetary policy

Essential reading:

Mankiw: Ch. 11

IS-LM Model: the Short-Run Equilibrium

The IS-LM model is based on the assumption that the general price level is fixed.

With that we mean that the general price level will not suddenly adjust when economic conditions change. Think about a single market for a good in which the price is given by the intersection of demand and supply. If the economic conditions do not change (meaning: demand and supply do not change) the price will not change. Now suppose that there is a change, suppose an increase in the demand. Given the supply an increase in the demand should result in an increase in the price level (and in the quantity exchanged in the market). However, if the price is fixed, after the increase in the demand the price will not change. How long the price will remain at the same level after the demand has changed will denote the short-run. How long it will take to the price to finally increase after the demand has increased will denote the long-run. Here we have the same idea just applied to the general price level instead to the price of a particular good.
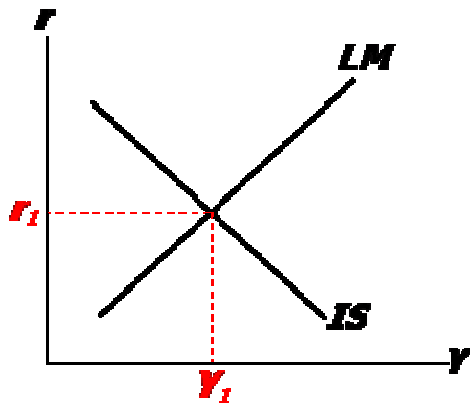
The *IS* curve represents equilibrium in the goods market.

The curve is given by: $Y = C(Y - T) + I(r) + G$

The *LM* curve represents money market equilibrium.

The curve is given by: $\dfrac{M}{P} = L(Y, r)$

The intersection of the two curves determines the unique combination of Y and r that satisfies equilibrium in both markets.

In the graph: the values Y_1 and r_1 represent the values of real income and the interest rate that ensure that the goods market and the money market are simultaneously in equilibrium. So, if we believe that an aggregate economy is well approximated by the IS-LM model then we have an idea about the mechanism that determines the equilibrium level of real income in a given closed economy. If we know the mechanism we can try to affect this mechanism through economic policy in order to affect the equilibrium level of real income. Notice the difference with the determination of real GDP we discussed in lecture 2 about national accounting. In that lecture we gave an idea about how the real GDP is calculated. Here we have an idea about the economic process that determine the real GDP, meaning we have an idea about the economic relationship between different variables and how this interaction between different variables determine the equilibrium level of the real GDP.

Now we can ask the following question:

What happens when for some reasons we are not at the equilibrium implied by the IS-LM model? For example, suppose we start from a situation where the goods market is in equilibrium, however, the money market is not.

Are there market forces that can naturally (without any external intervention) bring the system to the general equilibrium level?

Graphically, a situation like this is represented by point H in the IS-LM graph:

r

LM

r_H H

r_E

r_A B

E

IS

A

Y_H Y_E Y_B Y
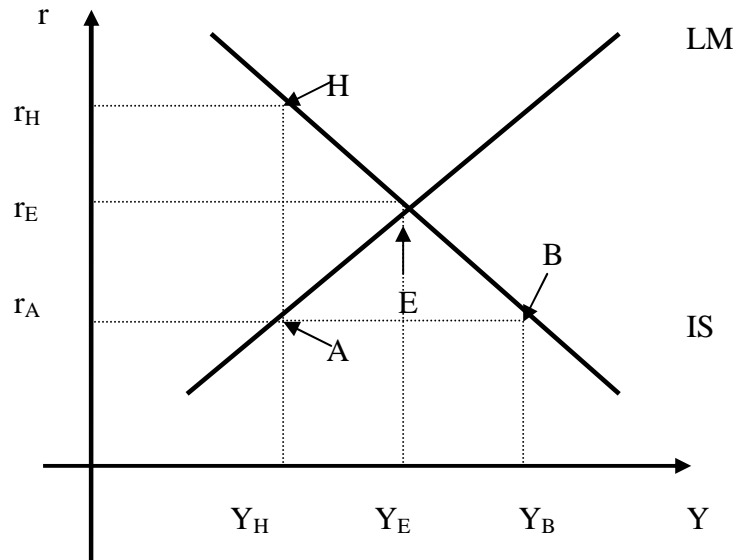
At point H the goods market is at the equilibrium (since it is a point on the IS curve) however the money market is not. In particular, at Y_H and at r_H the goods market is in equilibrium, however, at that level of income Y_H, the interest rate is too high for the equilibrium to take place also in the money market. With an income level of Y_H the interest rate that would ensure that the money market is in equilibrium should be $r_A < r_H$. If the interest rate is too high in the money market ($r_A < r_H$), we know that the demand for money will be low. Since the money market is not at the equilibrium this means that the money supply is larger than the money demand (remember that the money market equilibrium is defined as real money balances supply = demand for real money balances. A disequilibrium must occur only if that inequality is not satisfied). We know that if money supply is larger than money demand the interest rate must decrease to eliminate this excess of money supply (according to the liquidity preference theory). This means that r_H must decrease. However, if the interest rate decreases then Investment will increase, and so income will increase as well. As income increases, money demand increases and this together with the reduction of the interest rate will contribute to eliminate the excess of money supply in the money market. When this process is going to end? When we reach point E, where the interest rate is now lower and the income is larger than at point H. At point E, no agents in both markets have an incentive to change their behaviour and therefore we are at a general equilibrium. This means that there exist market forces (the movement of r and of Y) that will ensure that we will converge to the equilibrium in both market starting from any situation where one of the markets is in disequilibrium.

A similar reasoning can be made for other points like point A, where the money market is in equilibrium but the goods market is not.

At point A, given the interest rate prevailing in the market (r_A), the real income is too low to ensure the equilibrium in the goods market ($Y_H < Y_B$). The interest is low and this means that Investments will be too high (remember that the goods market equilibrium implies I = S, any disequilibrium implies that equality does not hold). If investments are high, income will increase, if income will increase, money demand will increase. In the money market, an increase in the demand of money, for a given money supply, will increase the interest rate. The increase in the interest rate will depress investments and so income will stop increasing after a while. When this process will stop? At point E again, where the interest rate will be higher and income will be higher than at point A.

You can show that this kind of adjustment towards the equilibrium E will hold for any point different from E.

So we can say that there are market forces that naturally will establish the equilibrium in the money market and in the goods market simultaneously.

Now we can ask the following question: if there are market forces that ensure that an equilibrium will be reached, why should we intervene in the market using economic policies?

The answer depends on what we think about the equilibrium that the market forces will reach. If we think that the equilibrium E is not a good equilibrium we may want to change the market result using policies. First of all we need to define what a good equilibrium is. We can define the concept of **Full-employment equilibrium**: *a situation where the number of unemployed workers is equal to the number of job vacancies available.* Full employment does not mean zero unemployment rate (we have seen that unemployment is never zero). It means that unemployment in mainly based on a voluntary basis. We will define more formally this concept when we will talk about the trade-off between inflation and unemployment by introducing the concept of Natural Rate of Employment. A full-employment equilibrium is therefore a situation where most of the labour resources are used in the production process and so the real GDP is very close to the **Potential real GDP**, that is the maximum level of output that can be produced, given the technology available (= the production function), using all the resources available in the economy. (It is also called Natural real GDP).

It is clear that an economy would preferred to be close to the potential real GDP most of the time, since this will imply that the real income is high and there are few wastes of economic resources.

The point now becomes: are the market forces (that ensure the equilibrium in the IS-LM model) capable to ensure that the equilibrium coincides with the full-employment equilibrium? If you are a neoclassical economist (or a monetarist) your answer should be yes. Those economists believed that markets work well, and that sooner or later the economy will converge to the full-employment equilibrium. This will be true especially in the long-run where all prices will be flexible and so markets should work properly. However, if you believe that the description of an economic system made by Keynes is more correct, your answer to the previous question should be no.

In particular, one of the main contributions of Keynes was the idea that equilibria lower than the full-employment equilibrium can be stable and persist for long time. The main idea is that markets may not work very well, since there are market failures (externalities), rigidities (prices and wages that are not flexible) and so on. This implies that market forces alone cannot guarantee that the economy converges towards the full-employment equilibrium, and therefore some external interventions (like by governments) would be necessary. Obviously the analysis of Keynes is based on the short-run (in the long-run we all agree that markets should work well). However, as Keynes used to say: "in the long-run we are all dead".

The analysis of Keynes is highly influenced by the Great Depression that started in 1929 (the General Theory was published in 1936) and affected many countries and lasted for some years. For Keynes this was an example of a situation where market forces alone could not improve the economic situation.

Therefore, since the ideas of Keynes had a great influence on macroeconomic theory, the IS-LM model has been used by economists to study the effects of fiscal and monetary policy on the equilibrium of an economic system. Furthermore, the IS-LM model has been also used to explain what we called "short-run fluctuations" in real GDP (or Business Cycles). This is because the IS-LM model is a model that explains the behaviour of the Aggregate Demand of an economy. Many economists that followed the ideas of Keynes believed that movements in aggregate demand are the main determinants of real output fluctuations in the short-run (this is the opposite idea of the Real Business Cycle theory)

Fiscal Policy in the IS-LM model

First, we write the IS-LM model using specific functional forms. In particular, we focus on linear functions.

Consider the following functions:

Consumption function: $C = C_0 + c(Y - T)$

Where $C_0 > 0$ is a constant and $0 < c < 1$ is the Marginal Propensity to Consume. Y is real income and T is the tax level, so (Y-T) is the disposable income.

Investment function: $I = I_0 - br$

Where $I_0 > 0$ is a constant and $b > 0$ measures the responsiveness of investments to the interest rate.

Government expenditure is exogenous and equal to G.

Tax level is exogenous and equal to T (you may assume that the tax level depends on income as we have done in one of the problem sets, however, nothing substantial will change in the analysis).

The equilibrium in the goods market is: $Y = C + I + G$

Using the functions defined above we have:

$Y = C_0 + c(Y - T) + I_0 - br + G$

Solving for Y:

$$Y = \frac{1}{1-c}\left[C_0 + I_0 + G - cT - br\right] \quad 1)$$

Equation 1) is our IS curve. However, the IS curve as we plotted so far should be written as r as a function of Y.

Therefore, we can rewrite the **IS curve** as:

$$r = \frac{1}{b}\left[C_0 + I_0 + G - cT - Y(1-c)\right] \quad\quad 2)$$

Equations 1) and 2) are obviously the same thing written in a different way.

A bit of notation about the slope of the IS curve and the elasticity: when b is particularly large (or c is very close to 1) the slope of the IS curve in 2) is a small number in absolute value. This means that a big change in income will result in a small change in r. This implies that the IS curve is relatively flat. We call this situation as an IS curve that is **ELASTIC** with respect to Y (if a big change in Y

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results in a small change in r, then reversing the causality, a small change in r will result in a big change in Y).

On the other hand, is b is particularly small, the slope of the IS curve in 2) is a large number in absolute value. This means that a small change in Y will result in a big change in r. The IS curve in 2) will be relatively steep. We say in this case that the IS curve is **INELASTIC** with respect to Y.

So: **b large (or c close to 1) = IS elastic = IS flat**

  b small = IS inelastic = IS steeper

Define the money demand function as:

$$\left(\frac{M}{P}\right)^d = kY - hr$$

where k > 0 and h > 0 are two parameters.

The money supply $M^s = M$ and prices are exogenous.

The equilibrium in the money market is given by:

$$\frac{M}{P} = kY - hr$$

The **LM curve** is therefore given by:

$$r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{P} \qquad\qquad 3)$$

A bit of notation about the slope and elasticity of the LM curve in 3): if h is particularly large (or k is particularly small) the slope of 3) is a small number. This means that a big change in income will result in a small change in r. This implies that the LM curve is quite flat. We call this case as an LM curve **ELASTIC** with respect to Y.

If h is a small number the slope of 3) is a large number. A small change in Y will result in a big change of r. This means that the LM curve defined by 3) is relatively steep. We call this case as an LM **INELASTIC** with respect to Y.

So: **h large (or k small) = LM elastic = LM flat**

  h small = LM inelastic = LM steeper

The IS-LM model is given by the following two linear equations in two variables (Y and r):

$$r = \frac{1}{b}\left[C_0 + I_0 + G - cT - Y(1-c)\right]$$

$$r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{P}$$

We can solve this system and the solution is:

$$Y^* = \frac{h}{bk + (1-c)h}A + \frac{b}{bk + (1-c)h}\frac{M}{P} \quad 4)$$

where $A = C_0 + I_0 + G - cT$

$$r^* = \frac{k}{bk + (1-c)h}A - \frac{(1-c)}{bk + (1-c)h}\frac{M}{P} \quad 5)$$

(see the Appendix of this lecture note)

Now we see the effects of fiscal policy on the equilibrium of our model. Here the two variables that define a fiscal policy are G and T.

From equation 4) we have that:

$$\frac{\partial Y^*}{\partial G} = \frac{h}{bk + (1-c)h} > 0 \quad 6)$$

$$\frac{\partial Y^*}{\partial T} = \frac{-ch}{bk + (1-c)h} < 0 \quad 7)$$

An increase in G (everything else constant) will increase the equilibrium level of income while an increase in T (everything else constant) will decrease the equilibrium level of Y.

From equation 5) we have that:

$$\frac{\partial r^*}{\partial G} = \frac{k}{bk + (1-c)h} > 0 \quad 8)$$

$$\frac{\partial r^*}{\partial T} = \frac{-ch}{bk + (1-c)h} < 0 \quad 9)$$

An increase in G (everything else constant) will increase the equilibrium level of r while an increase in T (everything else constant) will decrease the equilibrium level of r.

Notice the difference between the effect of fiscal policy here and in the Keynesian Cross framework.

In the Keynesian Cross the multiplier effect of G was given by $\frac{1}{(1-c)}$. This was because there was not an interest rate effect in the Keynesian cross model. Therefore, the result of the Keynesian cross model was a **Partial Equilibrium Effect**, based only on the goods market framework.

However, when we consider also the interaction between the goods market and the money market, the effect of *G* on *Y* becomes a **General Equilibrium Effect**, where the interaction between different markets is considered.

In particular notice that:

$$\frac{1}{1-c} > \frac{h}{bk+(1-c)h} \qquad 10)$$

if *h*, *k* and *b* are greater than zero.

Notice that the two expressions coincide when $h \to \infty$. In this case the LM curve is infinitively elastic (completely flat) and the crowding out is zero:

$$\lim_{h\to\infty} \frac{h}{bk+(1-c)h} = \frac{1}{1-c}$$

(see the Appendix of this lecture note).

Inequality 10) implies that the effect of *G* on *Y* is now lower than in the Keynesian cross case. The reason is that an increase in G increases income BUT also the interest rate (equation 8)). The increase in the interest rate will decrease investment.

Therefore, the increase in income due to the increase in G will be partly compensated by the decreasing in the Investments.

The fact that an increase in public expenditure (or a decrease in Taxes) increases the interest rate is called **CROWDING OUT**.

The increase in government expenditure crowds out some of private investments.

Higher is the crowding out effect and lower if the effectiveness of public expenditure on equilibrium income.

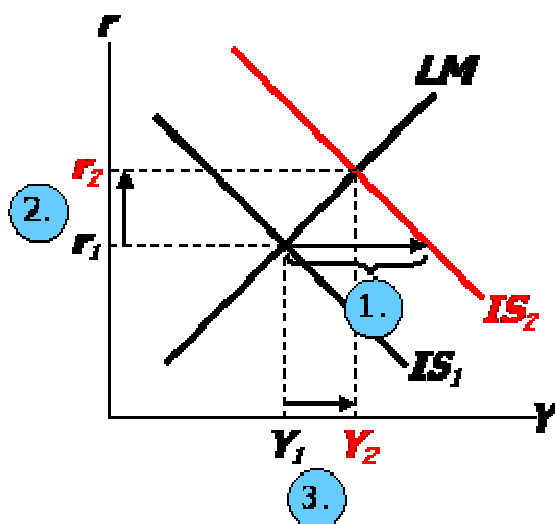The level of crowding out depends on the relative slopes of the IS and the LM curves. In particular:

1) *Given the slope of the IS curve, the Crowding out is higher the more inelastic is the LM curve*;

2) *Given the slope of the LM curve, the Crowding out is higher the more elastic is the IS curve*;

Graphically:

The effect of an increase in *G*:

1) After the change in *G*, say ΔG, the IS curve will shift up. From equation 2) we know that an increase in G will increase the vertical intercept of the IS curve and from equation 1) we know that if G increases, real income must increase by $\dfrac{\Delta G}{1-c}$.

2) After the increase in *G* and then the increase in *Y* also the interest rate increases. The reason is that an increase in income will increase money demand for a given level of money supply, creating an excess of money demand in the money market. This will increase the interest rate.

3) An increase in *r* will decrease *I* and so the initial increase in *G* will partly offset by this decreasing in *I*. Equilibrium income will increase but by less than $\dfrac{\Delta G}{1-c}$, because of the crowding out effect.

Crowding out and the relative slopes of the IS and LM curves:

a) Suppose the LM is particularly inelastic
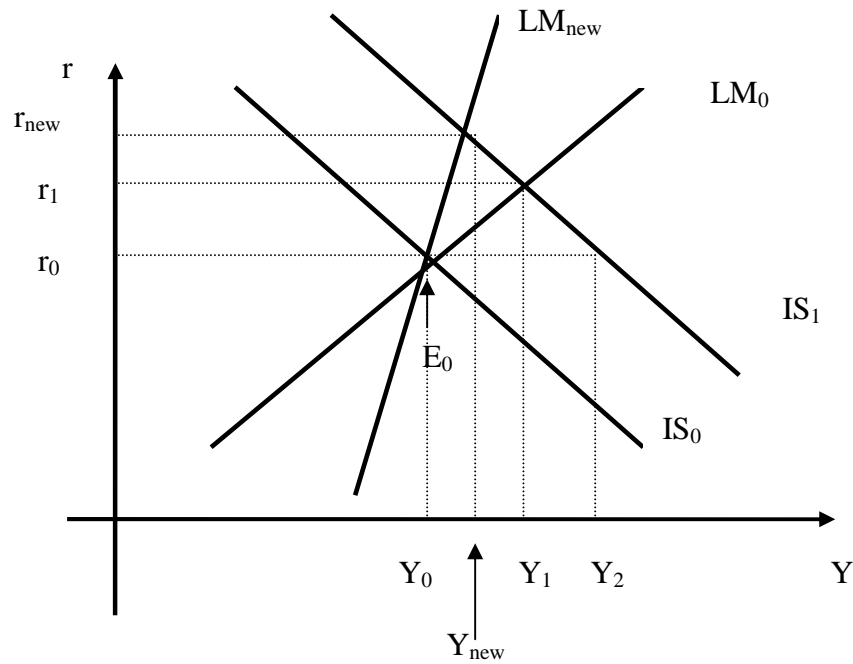
Suppose an increase in G. The initial IS curve is IS_0 and the equilibrium is given by r_0 and Y_0. The initial LM is LM_0. After the increase in G the new IS curve is IS_1 and the new equilibrium is r_1 and Y_1. Notice that the LM curve does not move since G does not enter in the LM equation. The Crowding out effect is given by the difference between Y_2 (**the level of income we could get if the increase in G would not affect the interest rate**) and Y_1. Now consider an LM curve steeper (meaning less elastic) than LM_0, call it LM_{new}. Notice that now, the same change in G as before, will have a lower effect on Y and a higher effect on r. The equilibrium with this less elastic LM curve will be r_{new} and Y_{new}. Notice that now the difference between Y_2 and Y_{new} is larger than the one between Y_2 and Y_1, meaning that the crowding out is now larger. The reason is that now an increase in G will have a bigger effect on the interest rate than before and so the reduction in investment will be higher than before.

The slope of the LM curve is given by k/h. The slope is large (curve inelastic) if h is a really small number. If h is very small it means that the demand for money is not very sensitive to the interest rate. If income increases, money demand increases and for a given level of money supply the interest rate must increase. However, if money demand is not very sensitive to r then the increase in r must be large in order to restore the equilibrium in the money market.

b) Suppose the IS curve is particularly elastic
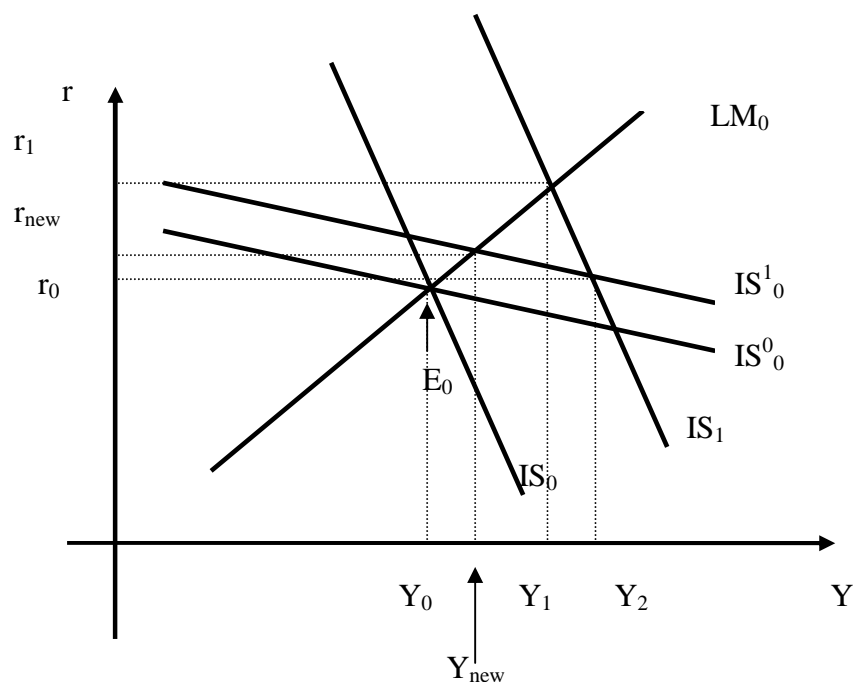
In this graph we have two IS curves quite inelastic (IS_0 and IS_1) and two more elastic (IS^0_0 and IS^1_1). As we can see the same change in G will have a much smaller effect when the IS curve is elastic. Notice that also the effect on *r* is lower when the IS curve is elastic than in the case where the IS curve is less elastic ($r_{new} < r_1$).

The slope of the IS curve is $-\dfrac{1-c}{b}$. The IS curve is elastic if the slope is a small number. This will happen if *b* is large for example. If *b* is large, a change in the interest rate will have a big impact on the level of investment.

An increase in *G* will increase *Y*. This will increase the money demand and given the level of money supply will increase the interest rate. A small increase in the interest rate will have a big negative effect on investments and therefore the initial increase in *G* will be highly compensated by the big reduction in *I*, and so the effect of *G* on *Y* will be very limited in this case.

A similar reasoning can be made for the case where Taxes are reduced, since in this case the shift in the IS curve is similar to the case where *G* is increased.

Conclusion: *an expansionary fiscal policy (increasing in G or decreasing in T) is more effective in affecting the level of real income the lower is the crowding out effect.*

The effect of an increase in the tax level is less interesting in the IS-LM model since the result is a reduction in the equilibrium income.

Monetary Policy in the IS-LM model

In the IS-LM model the only exogenous variable related to monetary policy is the level of money supply. So we consider a monetary policy as a change in the money supply.

From equation 4) and 5) we have that:

$$Y^* = \frac{h}{bk + (1-c)h} A + \frac{b}{bk + (1-c)h} \frac{M}{P}$$

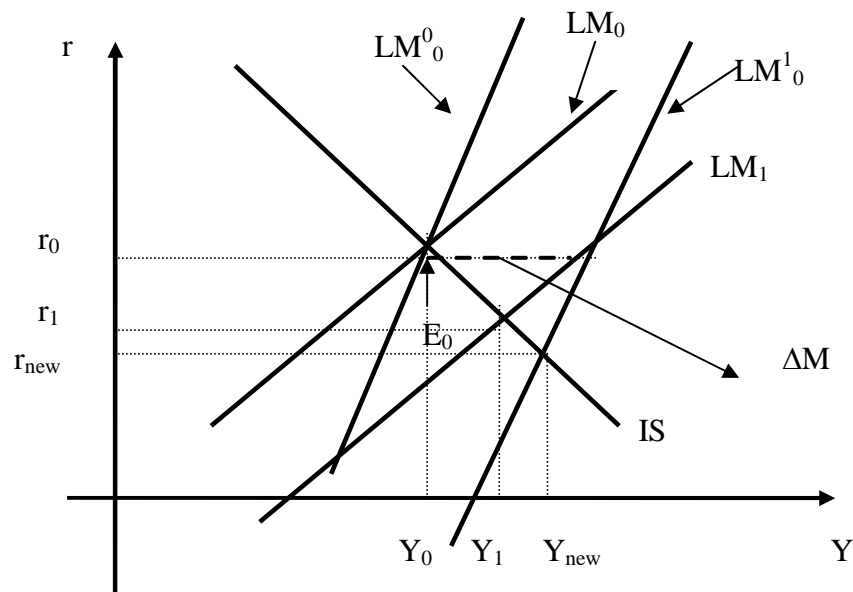$$r^* = \frac{k}{bk + (1-c)h} A - \frac{(1-c)}{bk + (1-c)h} \frac{M}{P}$$

So we have:

$$\frac{\partial r^*}{\partial M} = -\frac{(1-c)}{bk + (1-c)h} \frac{1}{P} < 0 \qquad\qquad 11)$$

$$\frac{\partial Y^*}{\partial M} = \frac{b}{bk + (1-c)h} \frac{1}{P} > 0 \qquad\qquad 12)$$

An increase in money supply (everything else constant) will increase the equilibrium level of output and will decrease the equilibrium interest rate.

An increase in *M* will shift the LM curve (*M* does not appear in the IS curve) down, by an amount given by the change in *M*.

Graphically:

In this graph we have 4 different LM curves. Two of them are relatively more elastic (LM_0 and LM_1) than the other two (LM^0_0 and LM^1_0).

Consider first the LM relatively more elastic. An increase in M by ΔM will shift down the LM curve (**remember that the intercept of the LM curve is negative**). Starting with an initial LM_0, after the increase in M, the new LM curve is LM_1. As we can see the effect is an increase in income from Y_0 to Y_1 and a reduction in r from r_0 to r_1.

The idea is that an increase in the money supply, given the money demand, will create an excess of money in the money market. To eliminate this excess of money, the theory of liquidity preference says that the interest rate must decrease. However if the interest rate decreases, Investment will increase and therefore income will increase. As Income increases money demand increases helping to restore the equilibrium in the money market. The process ends at the new equilibrium given by Y_1 and r_1.

Now consider the same change in money supply, but with a less elastic LM curve like LM^0_0. After the change in M, the LM curve shifts down to the new curve LM^1_0.

The same change in M has now a bigger effect on income and on the interest rate.

The slope of the LM curve is given by k/h. In particular, remember that the LM is inelastic if h is small.

If h is small it means that the demand for money is not very sensitive to the interest rate. If money supply increases, in order to increase the money demand and restore the equilibrium in the money market the interest rate must decrease quite a lot.

However, if the interest rate decreases a lot, Investment will increase substantially and so the level of income.

Here we have considered an **expansionary monetary policy** (*M* increases). A similar reasoning can be made for the case of a **restrictive monetary policy** (*M* decreases).
In the case of a restrictive monetary policy the LM will shift to the left and the result will be a reduction in Income and an increase in the interest rate in equilibrium. However, the effectiveness of this policy will depend on the slope of the LM as in the case of an expansionary policy.

Obviously the effectiveness of monetary policy depends also on the slope of the IS curve.
In the following graph we consider two possible IS curves, one less elastic (IS) than the other (IS^1). As we will see an increase in M will tend to have a bigger effect on *Y* the more elastic is the IS curve ($Y_{new} > Y_1$).
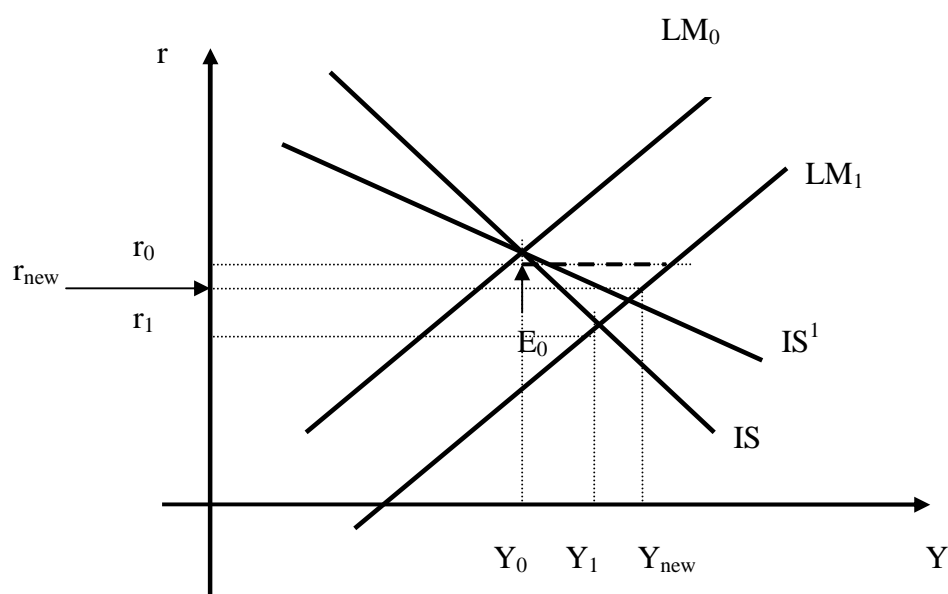
If the IS curve is elastic it means that investments are very sensitive to the interest rate (*b* is large). A small decrease in the interest rate will have a big impact on *I* and therefore on *Y*.

Conclusion: *for a given slope of the IS curve, monetary policy (expansionary or restrictive) is more effective in affecting the level of real income the less elastic is the LM curve. Given the slope of the LM curve, monetary policy (expansionary or restrictive) is more effective in affecting the level of real income the more elastic is the IS curve*

The Liquidity Trap: the Keynesian Case

The liquidity trap is also called the "Keynesian case" of the neoclassical synthesis since it represents the economic situation that is closer to the original ideas of Keynes on how an economic system should work.

A liquidity trap is a situation where the interest is very close to zero. At that interest rate people will hold any amount of money for any given level of income.

The point is: when the interest rate is close to zero, the cost opportunity of holding money is close to zero, so people will just keep all the money they have in their pocket instead of investing some of it into assets that will pay a return that is close to zero. We say that in this case money and less liquid assets are essentially the same thing.

This implies that the LM curve is infinitively elastic (completely flat) at this low interest rate.

In terms of the slope of the LM curve it means that $h \rightarrow \infty$, meaning that the demand for money is infinitely sensitive to *r*.

If there is a huge increase in Income and money demand increases, it is enough to have a small decrease in the interest rate to compensate the increase in the money demand.

So the first two basic elements of a liquidity trap are:

1) Interest rate extremely low, possibly zero;
2) The LM curve is completely flat at that low interest rate;

Now we introduce another element that makes the liquidity trap closer to Keynes' ideas. According to Keynes investments are not very sensitive to the interest rate, but they depend mainly on what Keynes called "*Animal Spirits*". You may think at the animal spirits as the subjective expectations of the investors about the future. The idea for Keynes is: an investor will undertake an investment if he expects that the investment is a good idea, independently on the level of the interest rate. You may

think at a sort of optimism or pessimism of investors when you talk about animal spirits. If an investor is pessimistic about the future, then even if the interest rate is low, he may not take the risk to invest.

The fact that investment are not very related to the interest rate in Keynes' opinion, implies that the IS curve is relatively inelastic since b is a small number.

The fact that the IS curve is inelastic accounts for another idea of Keynes: the full-employment equilibrium may be impossible to reach by the market forces alone and an external intervention should be needed to reach that equilibrium. Otherwise the economy can be stuck in a lower equilibrium for a long time.

This is shown graphically:
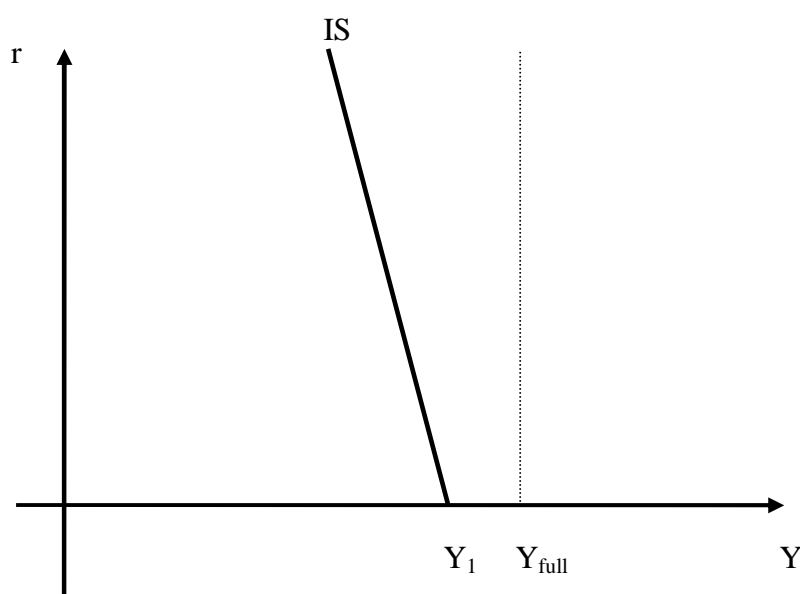
Suppose that the full employment equilibrium if Y_{full} on the graph. If the IS curve is inelastic, it may cut the horizontal axis to the left of Y_{full}, like at Y_1 in the figure.

It is clear that in such a situation there is not a positive interest rate that can be consistent with the IS curve and the full-employment equilibrium. In this situation, independently on where the LM curve will lie, the equilibrium in the market cannot be the full-employment equilibrium. However, by increasing the public expenditure or by reducing taxes we may be able to shift the IS curve to the right and therefore the full-employment equilibrium can be, in theory, achieved.

Now we add the LM curve to the previous figure to show the liquidity trap case:
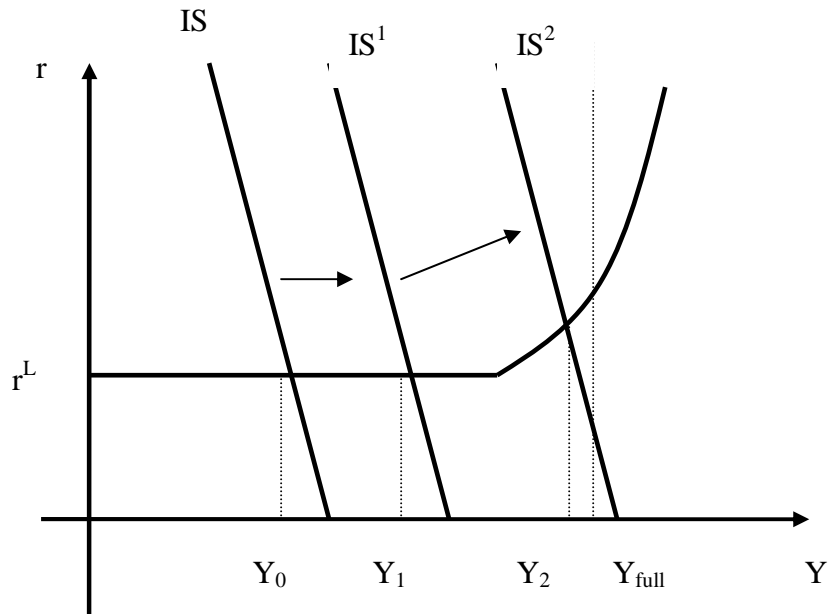
Suppose that the interest rate is very low at level r^L (notice that this interest rate should be close to zero, however, for graphical convenience, we plotted it as if it is much higher than zero). The LM curve is flat at that interest rate. This is the case of the liquidity trap.

As the income becomes large enough the LM starts to increase again showing when the system can get out for the liquidity trap. The idea according to Keynes is: as income increases, the economic situation improves, after a while investors will start to gain confidence, they will become optimistic, they will start taking investments again and the system will start working again as in the usual IS-LM model we have seen before.

Suppose we are at the equilibrium defined by r^L and Y_0. If we don't intervene in some way the system will remain at that equilibrium and we will be stuck in the liquidity trap. How do we escape from the liquidity trap?

We need to increase equilibrium income.

There are two ways to do that according the IS-LM model:

1) **Expansionary Monetary Policy**: in the liquidity trap this policy does not work. An increase in money supply would reduce the interest rate, however, in the liquidity trap the interest rate is already at the minimum level and cannot be decreased further.

2) **Expansionary Fiscal Policy**: this policy is really effective in a liquidity trap. Suppose an increase in government expenditure. We have seen that when the

LM curve is flat, the multiplier of the government expenditure is equal to the case in the Keynesian Cross case, meaning that this policy will be very effective in increasing equilibrium income. The same will hold for a reduction in Taxes. So an expansionary fiscal policy will shift the IS curve to IS^1 for example. A further increase in G (or reduction in T) will continue to shift the IS curve and after a while the system may be reach the part of the LM curve that is increasing (for example with an IS curve given by IS^2) and it will escape the liquidity trap.

Are the liquidity traps common in reality?

Not really, however, in the Great Depression of 1929 the nominal interest rate dropped quite dramatically and its level started to be very close to zero after few years.

In Japan in the 90's a similar situation happened, where the nominal interest fell almost to zero.

The recent credit crunch is another example of a situation where nominal interest rate became very close to zero.

Fiscal Policy and Monetary Policy Together

The IS-LM model can explain how to coordinate monetary and fiscal policy together in order to achieve some economic objectives.

In particular, monetary policymakers may adjust *M* in response to changes in fiscal policy, or vice versa. Such interaction may alter the impact of the original policy change.
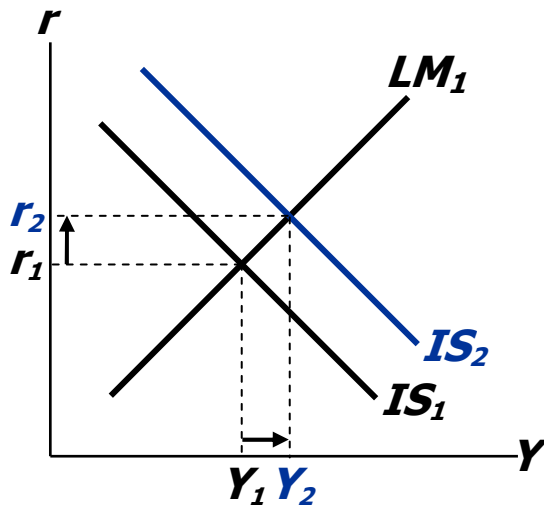
Suppose that the government increases *G*.

Possible responses by the Central Bank:

1. hold *M* constant

2. hold *r* constant

3. hold *Y* constant

In each case, the effects of the Δ*G* are different:
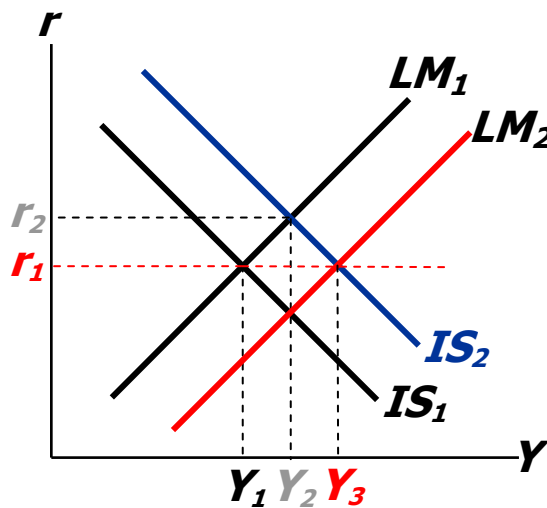
Case 1) Holding *M* constant

If Congress raises *G*, the *IS* curve shifts right. If the central bank holds *M* constant, then *LM* curve doesn't shift. The result is in the following graph

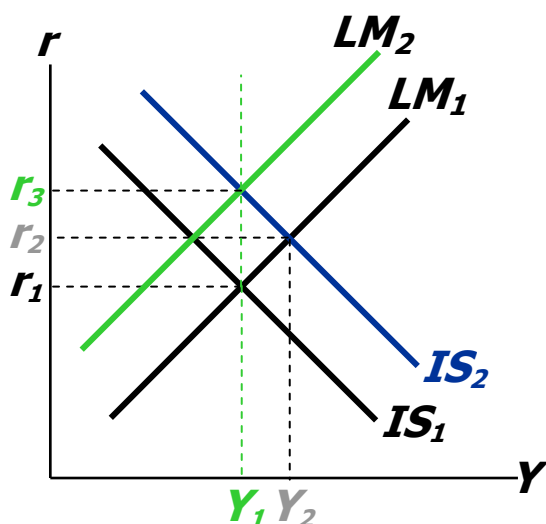Result: *income increases and interest rate increases.*

Case 2) Holding *r* constant

To keep **r** constant, the central bank increases **M** to shift the *LM* curve to the right, as in the following graph:

Result: *income increases more than in the case where the LM is fixed and the interest rate does not change.*

Case 3) Holding Y constant

In this case the central bank must decrease M to compensate the increase in G.

Result: *income does not change and the interest rate increases more than in the case where the LM remains fixed.*

Is the general price level really fixed in the short-run?

The analysis done so far is based on the assumption that the general price level is fixed. Is this an assumption that holds in reality?

To see this we should run an experiment and see how long it takes to the general price level to change after a change in economic conditions has occurred.

An empirical experiment of this sort has been done by Bernanke and Gertler (1995).[1]

They estimate a VAR (Vector Autoregression that is a system of dynamic equations to be estimated) and they check if after a positive shock in the interest rate (an increase in the interest rate) the general price level does change. The increase in the interest rate is interpreted as a change in monetary policy. In particular it is interpreted as tight monetary policy (like a reduction in money supply). The experiment is as follow: first we increase the interest rate in the first period, then nothing else changes and we see over time how long it takes for the general price level to change.

The result of their analysis is depicted in the following picture:

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[1] Bernanke and Gertler (1995), **Inside the Black Box: The Credit Channel of Monetary Policy Transmission,** *Journal of Economic Perspectives*, Vol. 9, No. 4. (Autumn, 1995), pp. 27-48.

Figure 1

Responses of Output, Prices and Federal Funds Rate to a Monetary Policy Shock
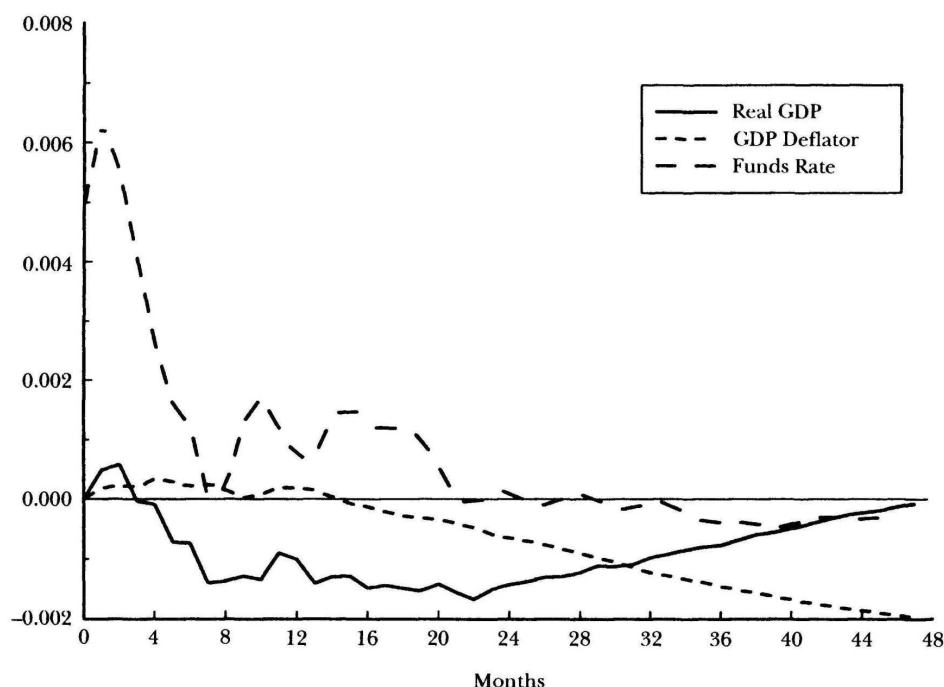
First, the interest rate increases (the Fund Rate), then they look how long it takes for the general price level to change. In the figure above, we see that the general price level (measured by the GDP deflator) remains stable for at least 12 months, then it starts to decrease. Output instead starts to decrease quite early, around 4 months after the increase in the interest rate. This graph seems to support the idea that the general price level tends to be fixed for around a year and that output is the first variable that changes in the short run after a change in the economic conditions. This is consistent with the analysis of the IS-LM model.

However, another way to look at how long prices remain fixed is to check a bundle of goods over time and see in average how long it takes for prices of those goods to change. This is different from the experiment of Bernanke and Gertler. For example, Bils and Klenow (2004)[2] checked for a sample of 350 consumption goods the frequency of price changes. They found that in average a price remains fixed for 5.5 months or less. This result seems to suggest that prices are sticky but not as sticky as we may think.

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[2] M. Bils and P. Klenow (2004), "Some Evidence on the Importance of Sticky Prices", *Journal of Political Economy* vol. 112, 947-985.

Mathematical Appendix

1) L'Hopital Theorem of Limits

Consider two functions $f(x)$ and $g(x)$, both continuous functions of x.

Suppose we want to find the limit for x that tends to infinity of the ratio of the two functions:

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

Suppose that the result of that limit is:

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$$

This means that the limit is not defined. We cannot say if that ratio is convergent (converging to some real number, including zero) or divergent (equal to plus or minus infinity).

In that situation we can apply the Hopital theorem that says that we can calculate:

$$\lim_{x \to \infty} \frac{f'(x)}{g'(x)}$$

where $f'(x) = \dfrac{df(x)}{dx}$ and $g'(x) = \dfrac{dg(x)}{dx}$ are first derivative. If the limit of the ratio of the first derivatives is convergent or divergent, then we can say that $\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = \lim_{x \to \infty} \dfrac{f'(x)}{g'(x)}$.

Suppose that also $\lim_{x \to \infty} \dfrac{f'(x)}{g'(x)} = \dfrac{\infty}{\infty}$, then we can continue in the process and calculate:

$$\lim_{x \to \infty} \frac{f''(x)}{g''(x)}$$

where $f''(x) = \dfrac{d^2 f(x)}{dx^2}$ and $g''(x) = \dfrac{d^2 g(x)}{dx^2}$.

We stop in taking derivatives until we arrive to a limit that is defined. Meaning: convergent or divergent.

The same principle can be applied to limits for which the result is $\dfrac{0}{0}$ independently to what x is tending to (x may tend to infinity as in the case above, or to zero or to any other number). The important thing is that the result of the limit is $\dfrac{\infty}{\infty}$ or $\dfrac{0}{0}$.

Examples.

Consider the following limit:

$$\lim_{x\to\infty}\frac{x-1}{x}$$

The result is: $\lim\limits_{x\to\infty}\dfrac{x-1}{x}=\dfrac{\infty}{\infty}$.

We cannot say anything about that. However, by using the Hopital theorem, and taking the limit of the ratio of the first derivatives we have that:

$$\lim_{x\to\infty}\frac{x-1}{x}\Rightarrow\lim_{x\to\infty}\frac{1}{1}=1$$

So we can say that $\dfrac{x-1}{x}$ converges to 1 when x tends to infinity.

Consider the following limit:

$$\lim_{x\to\infty}\frac{x^2}{x^3-2}$$

The result is again: $\lim\limits_{x\to\infty}\dfrac{x^2}{x^3-2}=\dfrac{\infty}{\infty}$

The first derivative of the numerator is $2x$. The derivative of the denominator is $3x^2$. Therefore:

$$\lim_{x\to\infty}\frac{x^2}{x^3-2}\Rightarrow\lim_{x\to\infty}\frac{2x}{3x^2}=\lim_{x\to\infty}\frac{2}{3x}=0$$

So we can say that $\dfrac{x^2}{x^3-2}$ converges to zero as x tends to infinity (actually in this case we don't really need to use this theorem, since we know that the denominator will tend to infinity faster than the numerator. In the denominator we have x to the power of 3 that grows faster than x to the power of 2. Therefore the denominator will tend to infinity faster than the numerator and the result is that the limit will tend to zero)

In the lecture notes we have considered the following limit:

$$\lim_{h\to\infty}\frac{h}{bk+(1-c)h}$$

The result is $\lim_{h\to\infty}\dfrac{h}{bk+(1-c)h}=\dfrac{\infty}{\infty}$

Using the Hopital theorem, takes the first derivative of the numerator with respect to *h*. That is equal to 1.

The first derivative of the denominator with respect to *h* is equal to *(1-c)*.

So we can write the limit as:

$$\lim_{h\to\infty}\frac{h}{bk+(1-c)h}\Rightarrow\lim_{h\to\infty}\frac{1}{(1-c)}=\frac{1}{(1-c)}$$

<u>2) The Algebra of the IS-LM model</u>

The IS-LM model we analysed in the lecture is given by:

$$r=\frac{1}{b}\left[C_0+I_0+G-cT-Y(1-c)\right]$$

$$r=\frac{k}{h}Y-\frac{1}{h}\frac{M}{P}$$

We solve this system by equalising the right-hand side of each equation (they are both equal to the same thing, r, so they must be equal):

$$\frac{1}{b}\left[C_0+I_0+G-cT-Y(1-c)\right]=\frac{k}{h}Y-\frac{1}{h}\frac{M}{P}\qquad\text{1)}$$

Solving equation 1) for Y:

$$\frac{1}{b}\left[C_0+I_0+G-cT\right]-\frac{(1-c)}{b}Y=\frac{k}{h}Y-\frac{1}{h}\frac{M}{P}$$

$$\Rightarrow\frac{1}{b}\left[C_0+I_0+G-cT\right]+\frac{1}{h}\frac{M}{P}=\frac{k}{h}Y+\frac{(1-c)}{b}Y$$

$$\Rightarrow\frac{1}{b}\left[C_0+I_0+G-cT\right]+\frac{1}{h}\frac{M}{P}=Y\left[\frac{k}{h}+\frac{1-c}{b}\right]$$

$$Y\left[\frac{kb + (1-c)h}{hb}\right] = \frac{1}{b}[C_0 + I_0 + G - cT] + \frac{1}{h}\frac{M}{P}$$

$$\Rightarrow Y = \frac{1}{b}[C_0 + I_0 + G - cT]\frac{hb}{kb + (1-c)h} + \frac{1}{h}\frac{hb}{kb + (1-c)h}\frac{M}{P}$$

$$\Rightarrow Y^* = \frac{h}{kb + (1-c)h}A + \frac{b}{kb + (1-c)h}\frac{M}{P}$$

where $A = C_0 + I_0 + G - cT$.

Substitute the result for Y^* into the IS or the LM curve to find the corresponding value of r. For example using the LM curve:

$$r = \frac{k}{h}\left[\frac{h}{kb + (1-c)h}A + \frac{b}{kb + (1-c)h}\frac{M}{P}\right] - \frac{1}{h}\frac{M}{P}$$

$$\Rightarrow r = \frac{k}{kb + (1-c)h}A + \frac{k}{h}\frac{b}{kb + (1-c)h}\frac{M}{P} - \frac{1}{h}\frac{M}{P}$$

$$\Rightarrow r = \frac{k}{kb + (1-c)h}A + \frac{1}{h}\frac{M}{P}\left[\frac{bk}{kb + (1-c)h} - 1\right]$$

$$\Rightarrow r = \frac{k}{kb + (1-c)h}A + \frac{1}{h}\frac{M}{P}\left[\frac{bk - bk - (1-c)h}{kb + (1-c)h}\right]$$

$$\Rightarrow r^* = \frac{k}{kb + (1-c)h}A - \frac{(1-c)}{kb + (1-c)h}\frac{M}{P}$$