

Numerical model for the fall speed of raindrops in a rainfall simulator

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Abstract

The impact velocity or kinetic energy of raindrops is an important factor when studying processes such as splash erosion and crusting. Often rainfall simulators are used to study these processes, but most rainfall simulators are not high enough for large raindrops accelerate to velocities close to their terminal velocity. A numerical model is developed to quantify the difference between the terminal velocity and the impact velocity of raindrops falling from a rainfall simulator of limited height. The model accounts for the development of turbulence in the flow around the drop by making friction a function of the particle Reynolds number. Distortion of the raindrops is expressed in terms of the dimensionless Weber number, which relates the hydrodynamic pressure perturbation (tends to distort the drop) to surface tension (tends to keep the drop spherical). The terminal velocities calculated with the model compare well with measured terminal velocities over a very wide range (diameters of 0.01 to 5.8 mm). The model results show that only drops smaller than 1 mm in diameter will attain 95% of their terminal velocity within 2 m falling distance. For drops larger than 1 mm in diameter the required distance increases rapidly (5.6 m distance for a 2 mm drop). Most harm is usually done by drops larger than 1 mm. Most rainfall simulators are not high enough for these drops to obtain their terminal velocity. The presented model can be used to evaluate the difference between the terminal velocity and the impact velocity of drops from a rainfall simulator.

1. Introduction

The impact of raindrops can detach soil particles, thus causing splash erosion. Strong winds will increase the kinetic energy or momentum raindrops, thereby increasing splash erosion. Also the kinetic energy or momentum of falling raindrops is an important factor in crusting. Crusting will decrease the infiltration capacity of the soil. If the rainfall intensity exceeds the infiltration capacity of the soil overland flow will occur, which can induce water erosion. Falling raindrops can further dissociate soil aggregates, resulting in a decrease of the soil structure. Often these processes are studied using rainfall simulators.

It is generally assumed that the raindrops hit the surface with the terminal velocity of fall. This terminal velocity has been studied experimentally (e.g. Gunn & Kinzer 1949, Beard & Prupacher 1969). Several equations have been published that describe the terminal velocity of fall as a function of the drop diameter. The power law relations ($V_t = a d^b$), as proposed by Spilhaus (1948), Kessler & Wilks (1968), Liu & Orville (1969), Sekhon & Srivista (1971), Atlas & Ulbich (1977), Grosh (1996), are mostly inaccurate. The relations proposed by Sekhon & Srivista (1971) and by Grosh (1996) do have a

reasonable accuracy for drops in the range 1-3 mm diameter, but overestimate the velocity of fall for both larger and smaller drops. Best (1950) proposed $V_t = 10.30 - 9.65 \exp(-0.6 d)$, which gives a good fit for diameters larger than 0.4 mm (see Figure 1), but predicts negative terminal velocities for very small drops. Gossard (1992) changed this to $V_t = 9.65 (1.0 - \exp(-0.53 d))$, which does not produce negative values, but is less accurate (Figure 1). Uplingers (1977) relation ($V_t = 4.85 d \exp(-0.195 d)$) overestimates fall velocities for drops smaller than 0.5 mm and predicts a strong decreases in terminal velocity when drops diameters exceed 5 mm (Figure 1). Some equations are restricted to solid hydrometeors (e.g. Bohm 1989).

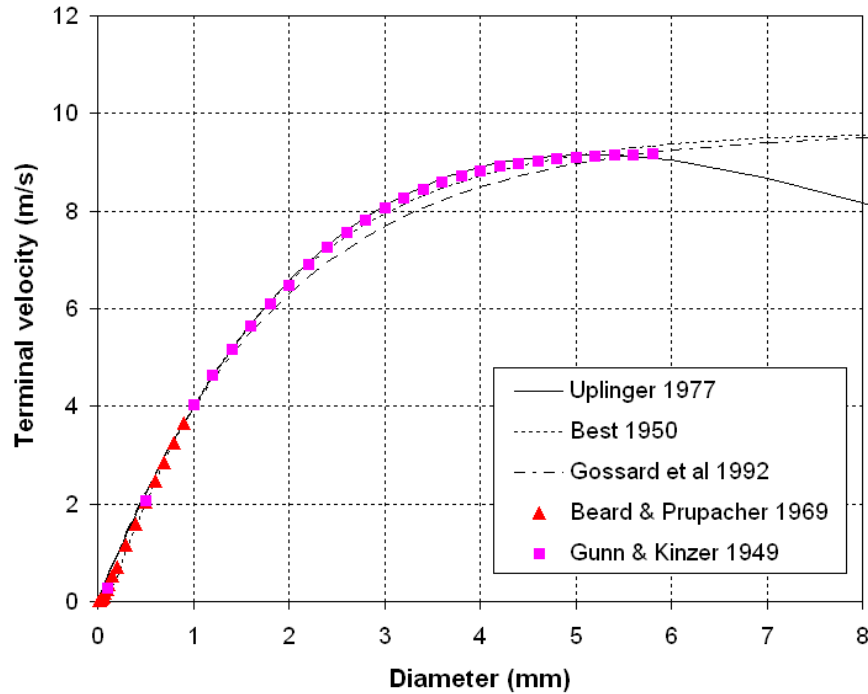


Figure 1: Terminal velocity of fall for raindrops. Measurements by Gunn & Kinzer (1949) and Beard & Prupacher (1969) and equations from Best (1950), Uplinger (1977) and Gossard et al (1992).

During windy conditions the velocity of the raindrop hitting the surface will be higher and the impact will be at an angle. This might be very important for erosion processes like splash erosion. The equations for the terminal velocity however are not suitable for quantifying the effect of the wind speed and the wind shear near the surface on the impact velocity, or the kinetic energy of the raindrop.

Often erosion processes are studied using rainfall simulators. Most rainfall simulators are not higher than 2 m, many of them are even lower. The question is how the impact velocity or kinetic energy of a water drop from a rainfall simulator will relate to the impact velocity or kinetic energy of a raindrop with the same size under natural conditions.

In order to address some of these questions a numerical model, based on physical principles is developed that calculates the fall path and velocity of a raindrop starting with zero velocity. Measurements of terminal velocities (Gunn & Kinzer 1949, Beard & Prupacher 1969) have been used to parameterize the model.

2. Theory

Small cloud droplets (< 0.05 mm) are spherical and the flow around the drops can be considered to be laminar. For these droplets the terminal velocity of fall can be found by equating the force of gravity ($F_g = g \rho_w \pi d^3/6$) to friction ($F_d = 3 \pi d \mu V$) yielding Stokes law:

$$V_t = \frac{\rho_w g d^2}{18 \mu} \quad (1)$$

Where V_t is the terminal velocity of fall, ρ_w the density of water, g the acceleration of gravity, d the diameter of the drop and μ the dynamic viscosity of air ($\mu = 18 \cdot 10^{-6}$ Pa s at 20 °C).

Larger drops fall faster and the flow around the drop becomes turbulent. Therefore Stokes law will fail for water drops larger than 0.1 mm. The transition to a turbulent flow regime is characterized by the particle Reynolds number, which expresses the ratio of turbulent and laminar forces. Turbulence will increase friction by a factor C_t :

$$C_t = 1 + 0.16 \text{Re}_p^{2/3} \quad \text{with} \quad \text{Re}_p = \frac{\rho_a V d}{\mu} \quad (2)$$

Where ρ_a is the density of air.

This factor can be added to Stokes law (eq. 1), but since the fall speed of the drop is in the Reynolds number this will not yield an explicit equation for the terminal velocity. An iterative calculation is needed to obtain correct values for the terminal velocity.

For drops smaller than 1 mm the surface tension is strong enough to keep the drop shape close to spherical. For larger drops aerodynamic pressure differences around the drop will distort the drop, causing a decrease in the vertical axis and an increase in the horizontal axis (McDonald 1954, Beard et al 1991). This distortion is illustrated in figure 2. For larger drops the base is more distorted than the top because of the reduced pressure in the separating wake (Tokay & Beard 1996). For drops larger than 2 mm, the effect of distortion is that the increase in fall velocity with diameter is much less than it would be without distortion (Figure 3). Hailstones are not distorted and will have a larger terminal

velocity than raindrops of the same size. Drops larger than approximately 9 mm tend to become unstable and break up.

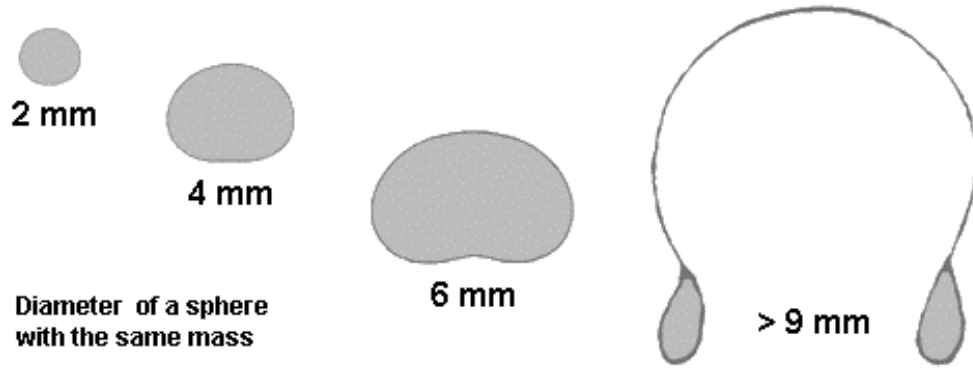


Figure 2: Schematic presentation of the distortion of raindrops due to aerodynamic pressure (From Fraser 1997).

The ratio of surface tension ($\sigma = 0.073 \text{ N/m at } 20^\circ \text{C}$) and hydrodynamic pressure perturbation is expressed in the dimensionless Weber number. In order to incorporate the effect of deformation on the friction a second correction factor was introduced which is a function of the Weber number:

$$C_d = 1 + a(We + b)^c - ab^c \quad \text{with} \quad We = \frac{\rho_a V^2 d}{\sigma} \quad (3)$$

Fitting the model results to the fall velocities measured by Gunn & Kinzer (1949) yielded $a=0.013$, $b=2.28$ and $c=2.12$.

The oscillations that occur when the drop shape deviates from spherical (Beard & Kubesh 1991, Tokay & Beard 1996) will probably also influence friction. Since (3) is obtained by fitting the model to measured fall velocities, these oscillations are implicitly incorporated in the equation.

The equations discussed above can be applied in an iterative procedure to obtain the terminal velocity. Also the terminal velocity can be obtained with a reasonable accuracy from Uplingers equation. This does however not provide information about the height of a rainfall simulator that is required for the drops to hit the surface with the terminal velocity. In order to obtain more insight into the change of velocity with fall distance we will examine the response time of the drops.

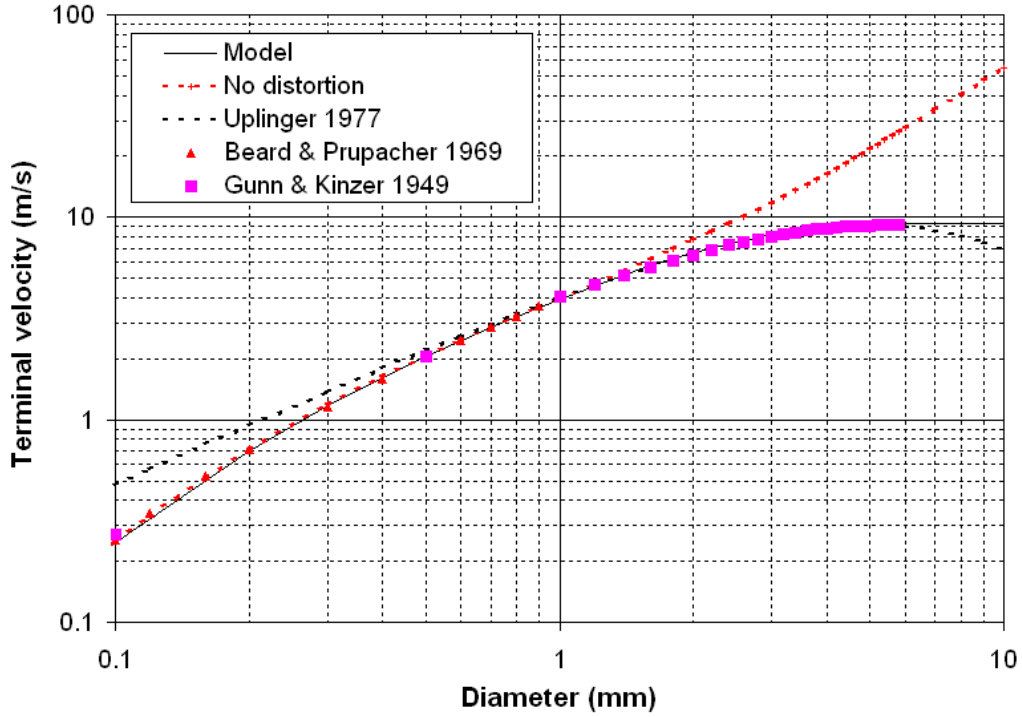


Figure 3: Modeled terminal velocities compared with measurements by Gunn & Kinzer (1949) and Beard & Prupacher (1969). Also shown are terminal velocities calculated with Uplinger's (1977) equation.

When no deformation occurs (diameter < 1 mm) the speed of the drop as a function of time can be approximated by a first order response function:

$$V(t) = V_t [1 - \exp(-t / \tau)] \quad \text{with} \quad \tau = C_\tau \frac{V_t}{g} \quad (4)$$

When turbulence can be neglected ($d < 0.05 \text{ mm}$) it can be shown theoretically that $C_\tau = 1.0$. When the flow around the drop is fully turbulent ($d > 0.3 \text{ mm}$) $C_\tau \approx 0.75$. After $t = \tau$ the drop reaches 63% of its terminal velocity and after $t = 3\tau$ it is within 95% of its terminal velocity. So a raindrop of 1.0 mm diameter with a terminal velocity of 3.9 m/s will have a response time of 0.3 second and it will take 0.9 s before the drop reaches 95% of its terminal velocity.

In order to establish the distance (D) that has to be traveled before reaching 95% of the terminal velocity (4) will be integrated with respect to time, yielding:

$$D(t) = V_t [t - \tau + \tau \exp(-t / \tau)] \quad (5)$$

For the 1.0 mm drop this means that a fall path about of 2.4 m is needed before the drop attains 95% of its terminal velocity (90% of its kinetic energy at terminal velocity). Most rainfall simulators are not this high.

For drops larger than 1.0 mm diameter the velocity cannot longer be approximated by a simple first order response, since as the speed increases different processes start influencing the velocity. For these drop sizes a model is developed.

3. The model

From the theory discussed above we find the two principle forces that determine the fall velocity of raindrops, gravity (F_g) and friction, or drag force (F_d), can be expressed as:

$$F_g = \frac{g \rho_w \pi d^3}{6} \quad \text{and} \quad F_d = 3 \pi d \mu V C_t C_d \quad (6)$$

The coefficients C_t and C_d represent the effect of turbulence and shape distortion on the drag force and are expressed by (2) and (3).

In the model the drop starts with a certain velocity (so far we have always used 0.0 m/s as initial velocity). The force of gravity and the drag force are calculated. The acceleration of the drop is calculated by dividing the net force by the mass of the drop. A simple forward integration with small time steps yields the course of the velocity during the fall over a preset distance. The time step is taken 0.05 times the response time estimated with (4), where C_t is taken 0.75 for drops smaller than 0.3 mm diameter and C_t increases with diameter for larger drops.

So far the effect of the wind speed has not been included, but it is just a minor extension of the model to include the wind speed and its variation with height.

4. Results

First of all we tested the ability of the model to predict the terminal velocity of fall for a wide range of diameters, by letting the drops fall over a long distance. Figure 3 compares the results from this run with measured terminal velocities. The model fits the measured terminal velocities over a wide range of diameters. For drops smaller than 1 mm the fit is very good. The largest discrepancies between the model and the measurements are found near diameters of 2 mm, where the model overestimates the measured terminal velocity by about 4%. For larger diameters the error becomes less. Possibly the form of (3), which describes the effect of distortion, can still be improved. For drops larger than 5.8 mm there are no measurements available, probably because of the tendency of large water drops to become unstable. Figure 3 also shows that the terminal velocity of hailstones (no deformation) of 5 mm diameter is more than twice that of raindrops of comparable size.

Figure 4 shows that drops of 0.1 to 0.4 mm diameter attain their terminal velocity within 1.0 m, but drops of 0.7 mm and larger need a fall path of more than 2.0 m to obtain their terminal fall speed. From figure 4 it is clear that raindrops of 1.0 mm diameter and larger, after falling over 2.0 m, are still far from their terminal velocity. Therefore, if the effect of raindrops larger than 2.0 mm in diameter on splash erosion or crusting is studied using a rainfall simulator less than 2 m in height, one needs to compare the fall speed or kinetic energy of the drops in the simulator to their speed under natural conditions.

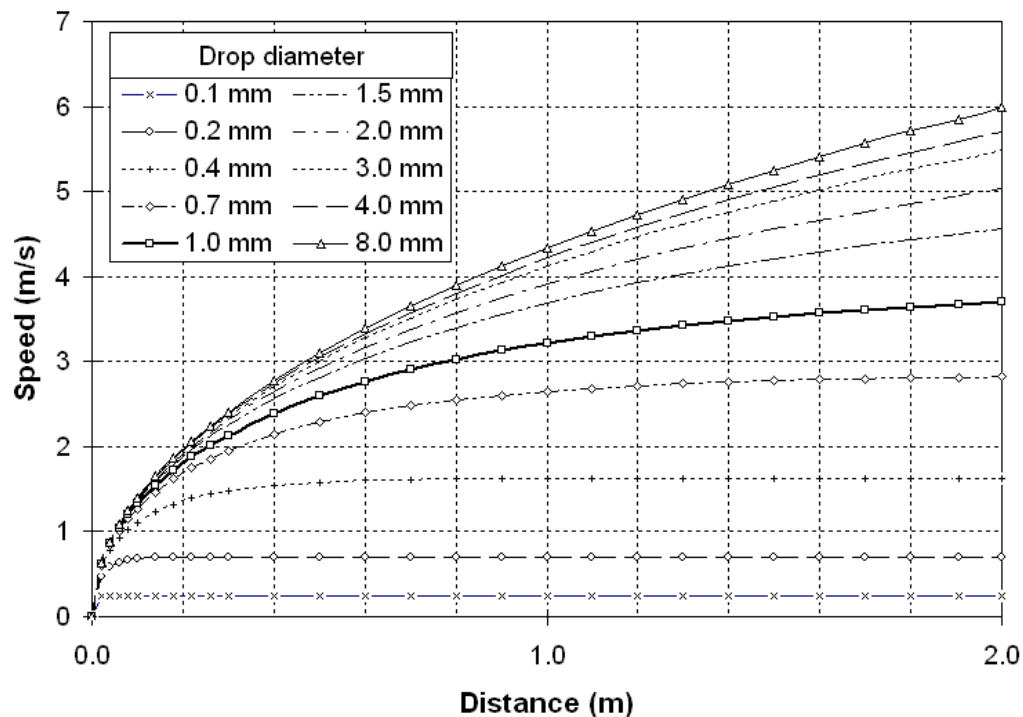


Figure 4: Velocity of raindrops as a function of fall distance for drops with diameters of 0.1, 0.2, 0.4, 0.7, 1.0, 1.5, 2.0, 3.0, 4.0 and 8.0 mm.

Figure 5 shows the fall distance that is needed for a raindrop to obtain 50, 75, 90, 95 or 99% of its terminal velocity as a function of the drop diameter. Only drops with a diameter less than 1.0 mm will reach 95% of their terminal velocity within 2.0 m. A 1.0 mm drop needs about 2.2 m in order to accelerate to 95% of its terminal velocity. This 2.2 m is slightly less than the estimate of 2.4 m that was obtained by using equation (4), because initially friction is low and the drop can accelerate relatively fast. When the speed increases, turbulence starts to develop, resulting in a higher friction. Drops larger than 1.0 mm diameter require substantially more than 2 m to obtain 95% of their terminal velocity. A 2.0 mm drop will attain 95% of its terminal velocity after falling about 5.6 m. The model, presented here, can be used to evaluate the difference between the terminal velocity and the impact velocity of drops from a rainfall simulator.

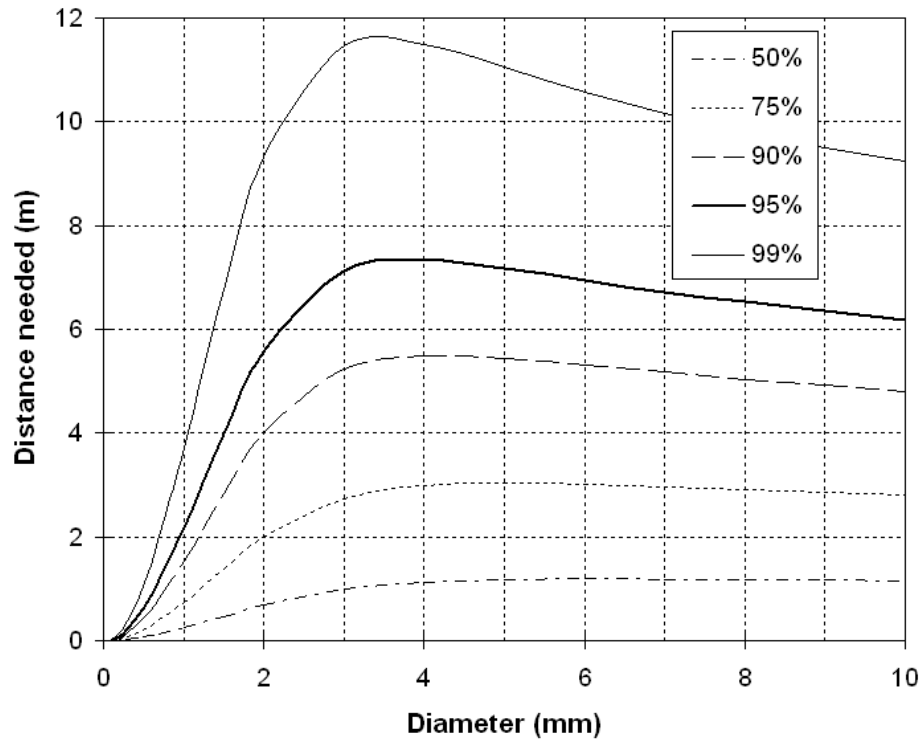


Figure 5: Falling distance needed by raindrops to obtain 50%, 75%, 90%, 95% or 99% of their terminal velocity as a function of drop diameter.

5. Conclusions

A model was developed that calculates the fall speed of raindrops that start to fall (e.g. from a rainfall simulator). The model predicts the terminal velocity of raindrops with a high accuracy over a very wide range. Model results show that drops with a diameter larger than 0.5 mm will not obtain their terminal velocity within 1.0 m. Raindrops with diameters over 2 mm require a falling distance of more than 5 m to get to 95% of their terminal velocity (90 % of their kinetic energy at terminal velocity), which is much more than the height of most rainfall simulators.

6. References

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