Radar Imaging

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September 18, 2005

with thanks to Brett Borden and various web authors for figures

RADAR = RAdio **D**etection **A**nd **R**anging

- developed within engineering community
  - how to transmit high power (physics, engineering)
  - how to detect signals (physics, engineering, math)
  - how to interpret and use received signals (math)
- mathematically rich
  - PDE (electromagnetic theory, wave propagation)
  - harmonic analysis, group theory, microlocal analysis
  - linear algebra, sampling theory
  - statistics
  - scientific computing
  - coding theory, information theory

Why make images with radar?

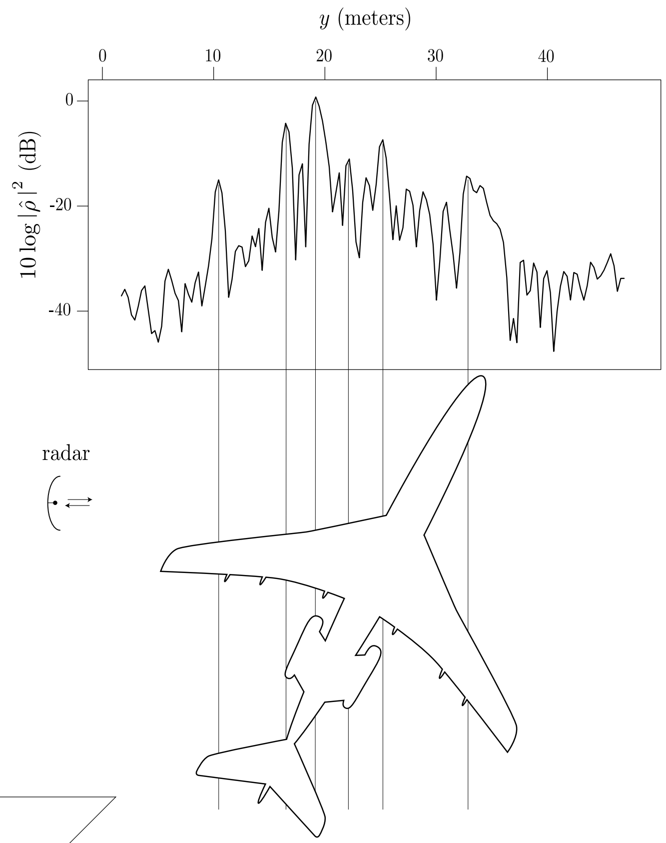
- works day or night (unlike optical imaging)
- works in all weather
  penetrates clouds, smoke
  some radars can penetrate foliage, buildings, soil, human tissue
- can provide very accurate distance measurements
- sensitive to objects whose length scales are cm to m
- can measure velocities (changes in range)

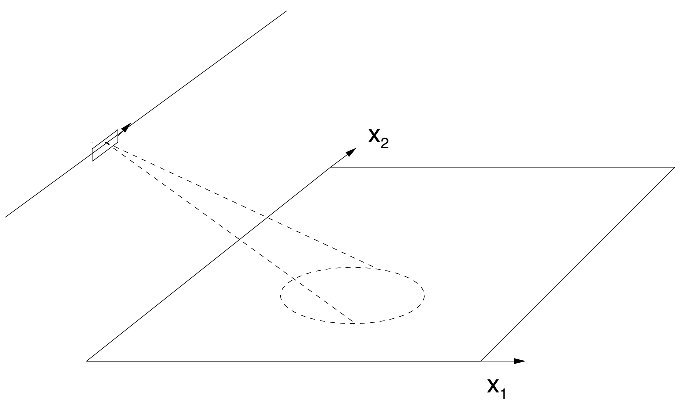
Radar history

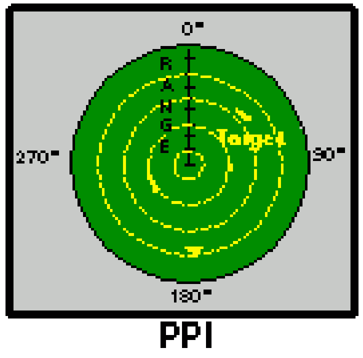
1886	Heinrich Hertz confirmed radio wave propagation
1904	Hülsmeyer patented ship collision-avoidance system
1922	ship detection methods at NRL (Taylor & Young, 700MHz)
1930	Hyland used radar to detect aircraft
	⇒ first US radar research effort, directed by NRL
1930s	England and Germany radar programs developed:
	Chain Home early warning system (22-50 MHz)
	fire control systems
	aircraft navigation systems
	cavity magnetron to transmit high-power microwaves
1940s	establishment of MIT Rad Lab (British + American)
	radar for tracking, U-boat detection

Rudimentary imaging

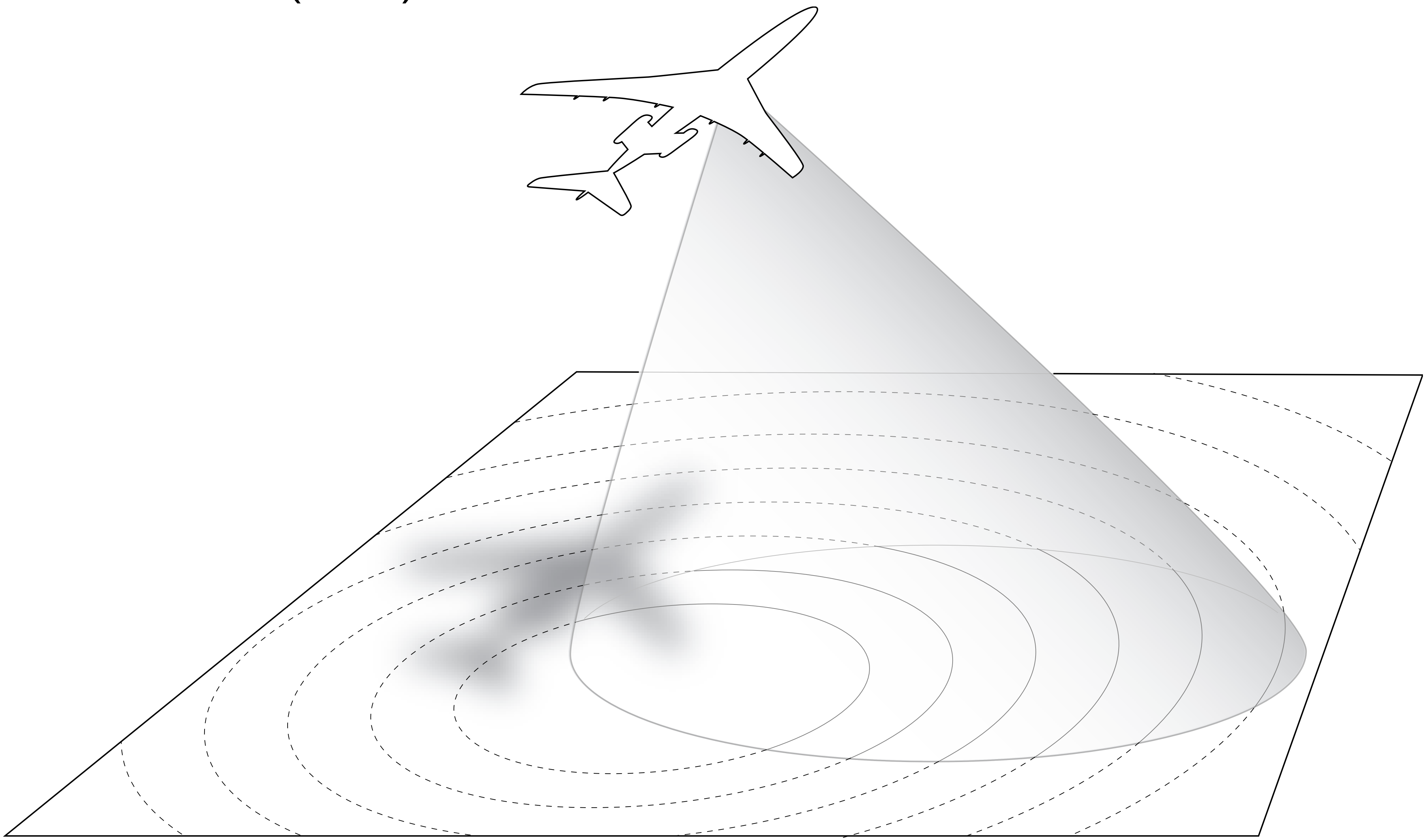
- Detection. For a target at distance r, see blip at time $2r/c$.
- High Range-Resolution (HRR) imaging
- Real-aperture imaging
- Plan position indicator





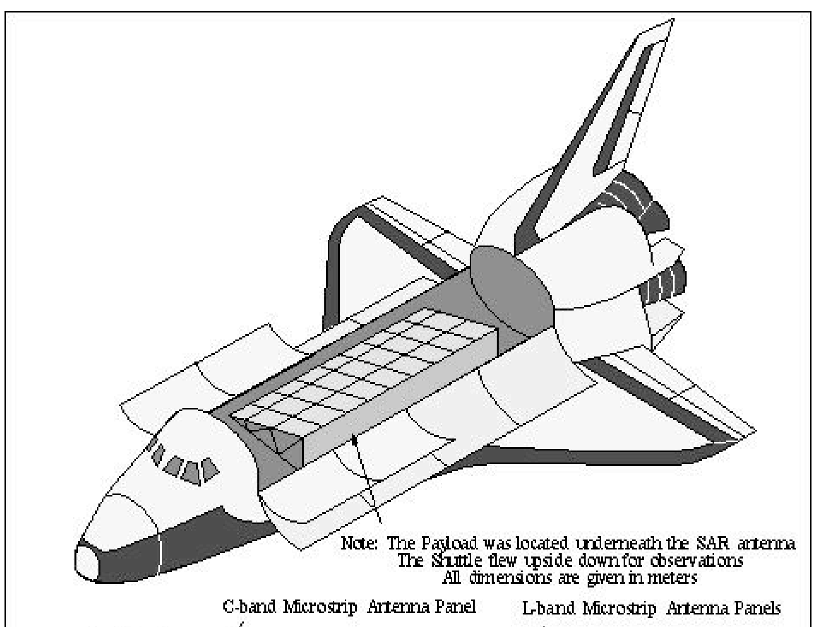


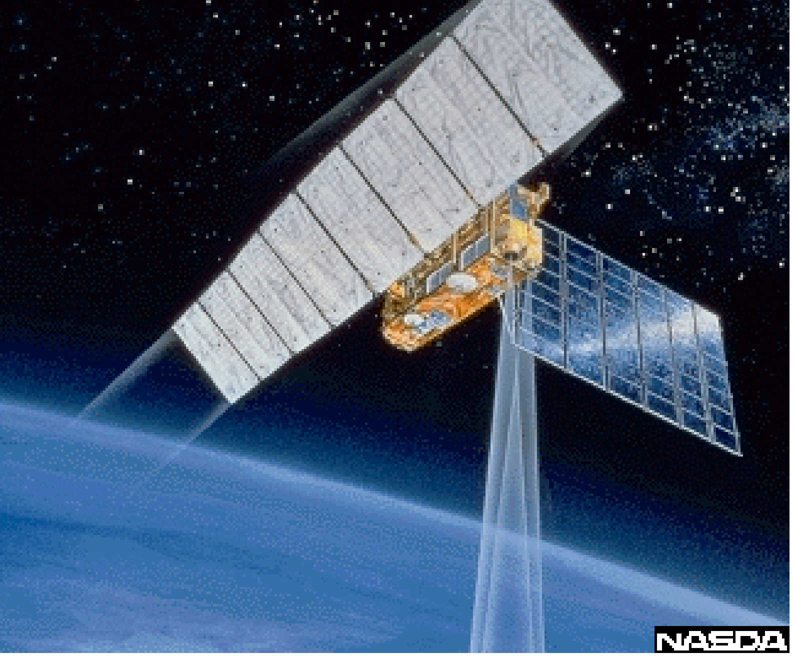
Synthetic Aperture Radar (SAR)



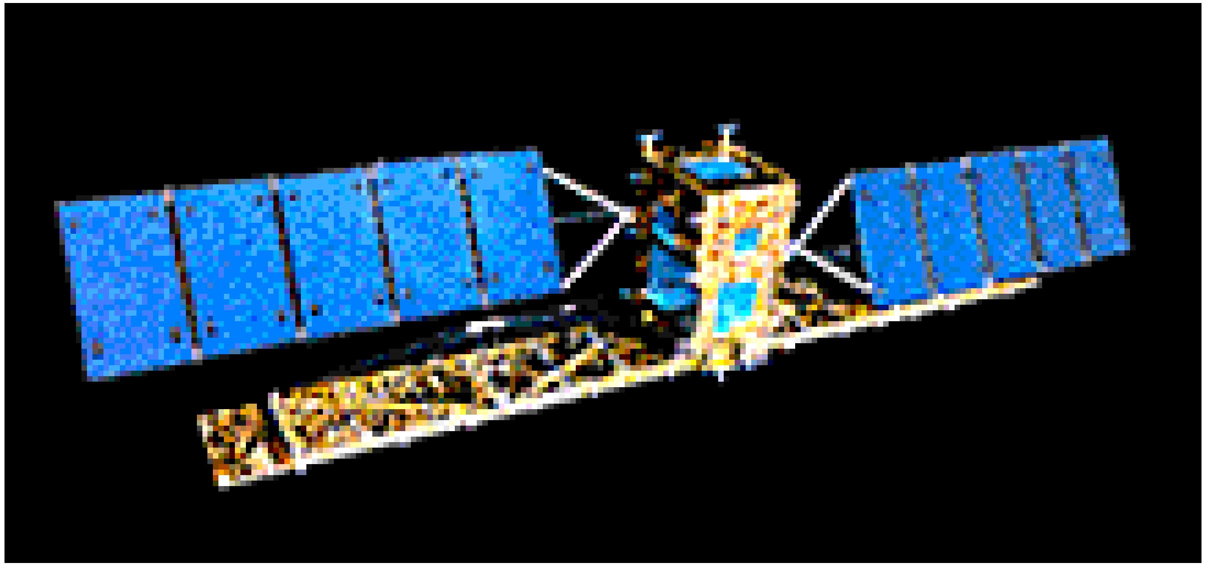
SAR History

1951	SAR invented by Carl Wiley, Goodyear Aircraft Corp.
mid-’50s	first operational systems, under DoD sponsorship:
	U. of Illinois, U. of Michigan, Goodyear Aircraft,
	General Electric, Philco, Varian
late ’60s	NASA sponsorship (unclassified!)
	first digital SAR processors
1978	SEASAT-A
1981	beginning of SIR (Shuttle Imaging Radar) series
1990s	satellites sent up by many countries
	SAR systems sent to Venus, Mars, Titan

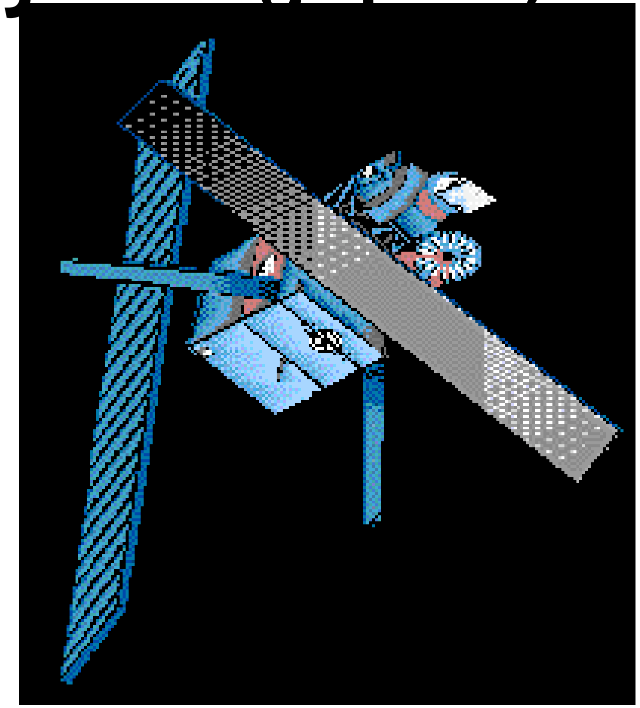




JERS (Japan)

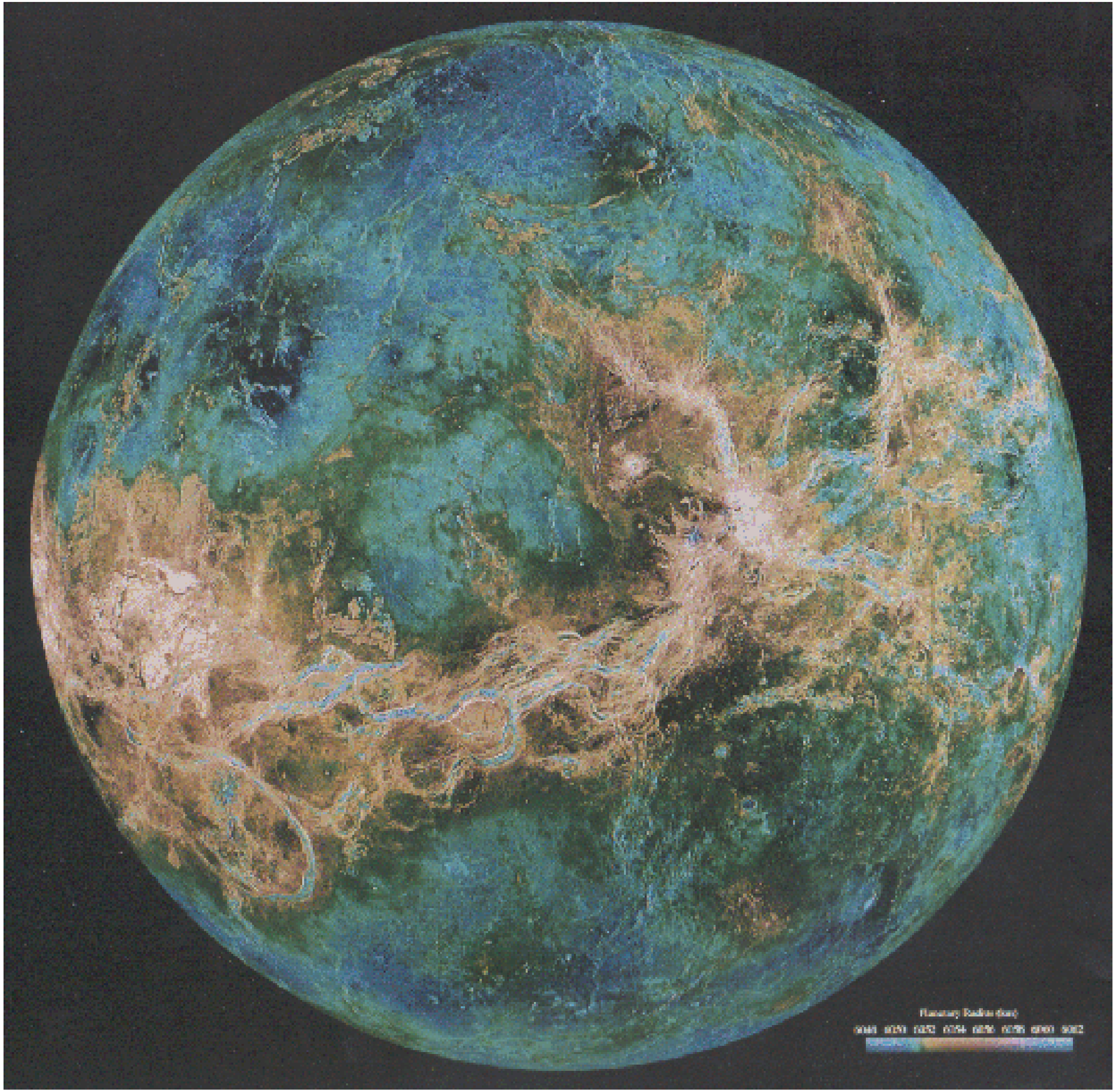


Radarsat (Canada)

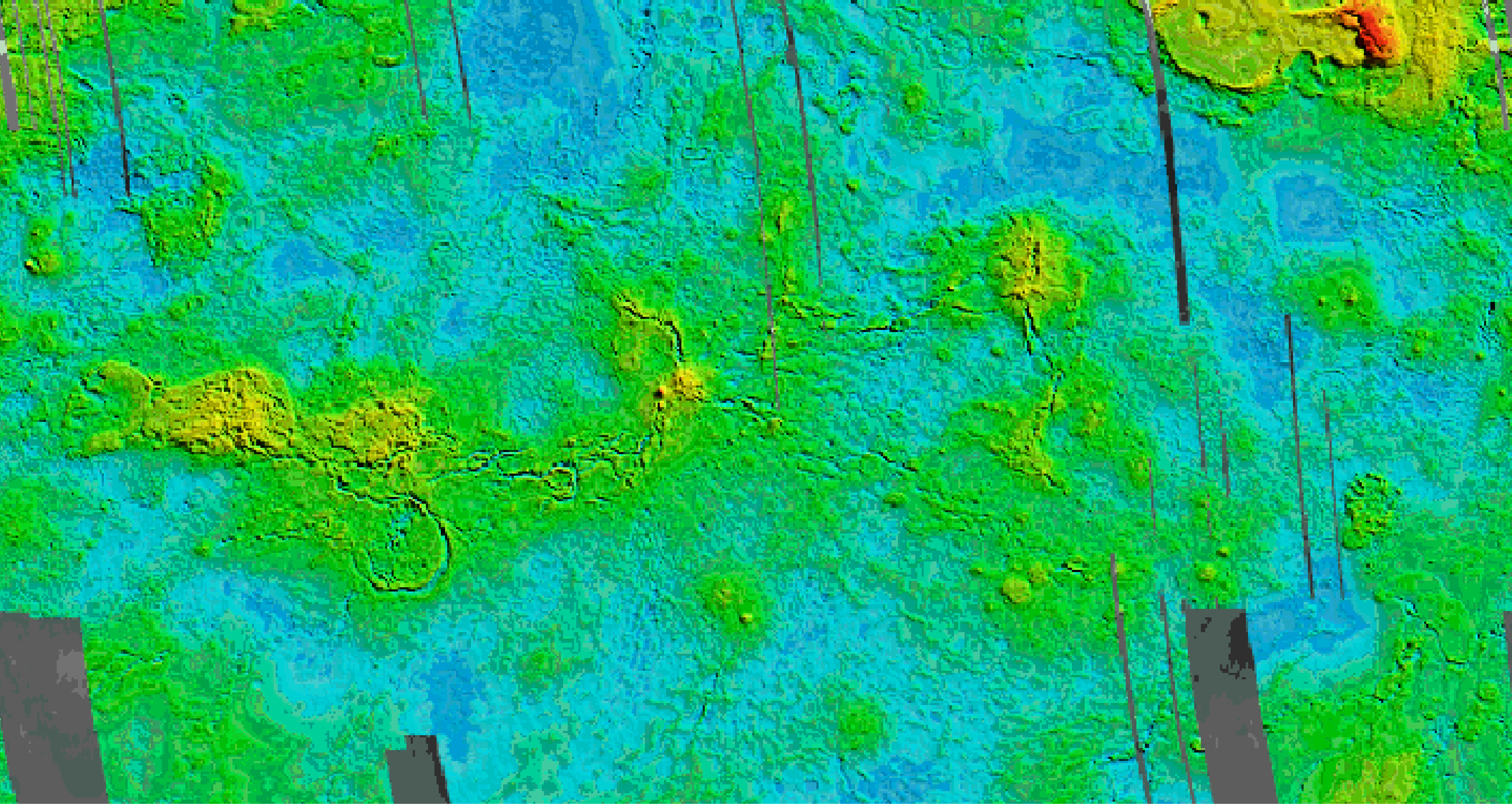


ERS-1 (Europe)

Envisat (Europe)



Venus radar penetrates cloud cover



Venus topography

AirSAR

CARABAS

UAVs

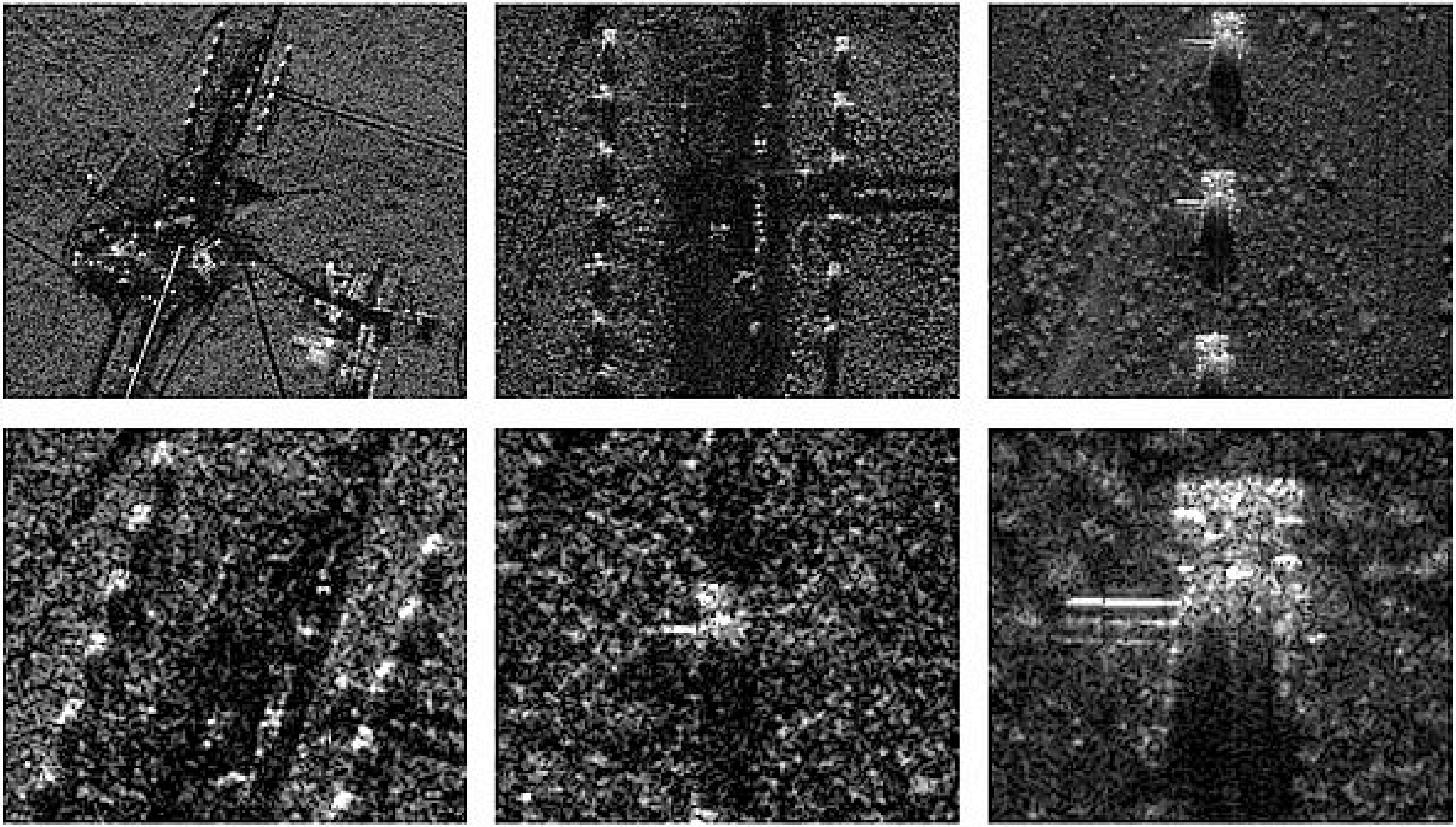
Lynx SAR

Applications

- military: early warning, tracking, targeting
- commercial aviation, navigation, collision-avoidance
- land use monitoring, agricultural monitoring, ice patrol, environmental monitoring
- surface topography, crustal change
- speed monitoring (police radar)
- weather radar: storm monitoring, wind shear warning
- search and rescue
- medical microwave tomography

M-47 Tanks On Kirtland AFB

Comparison of Resolutions At Actual and 4x Enlarged Views

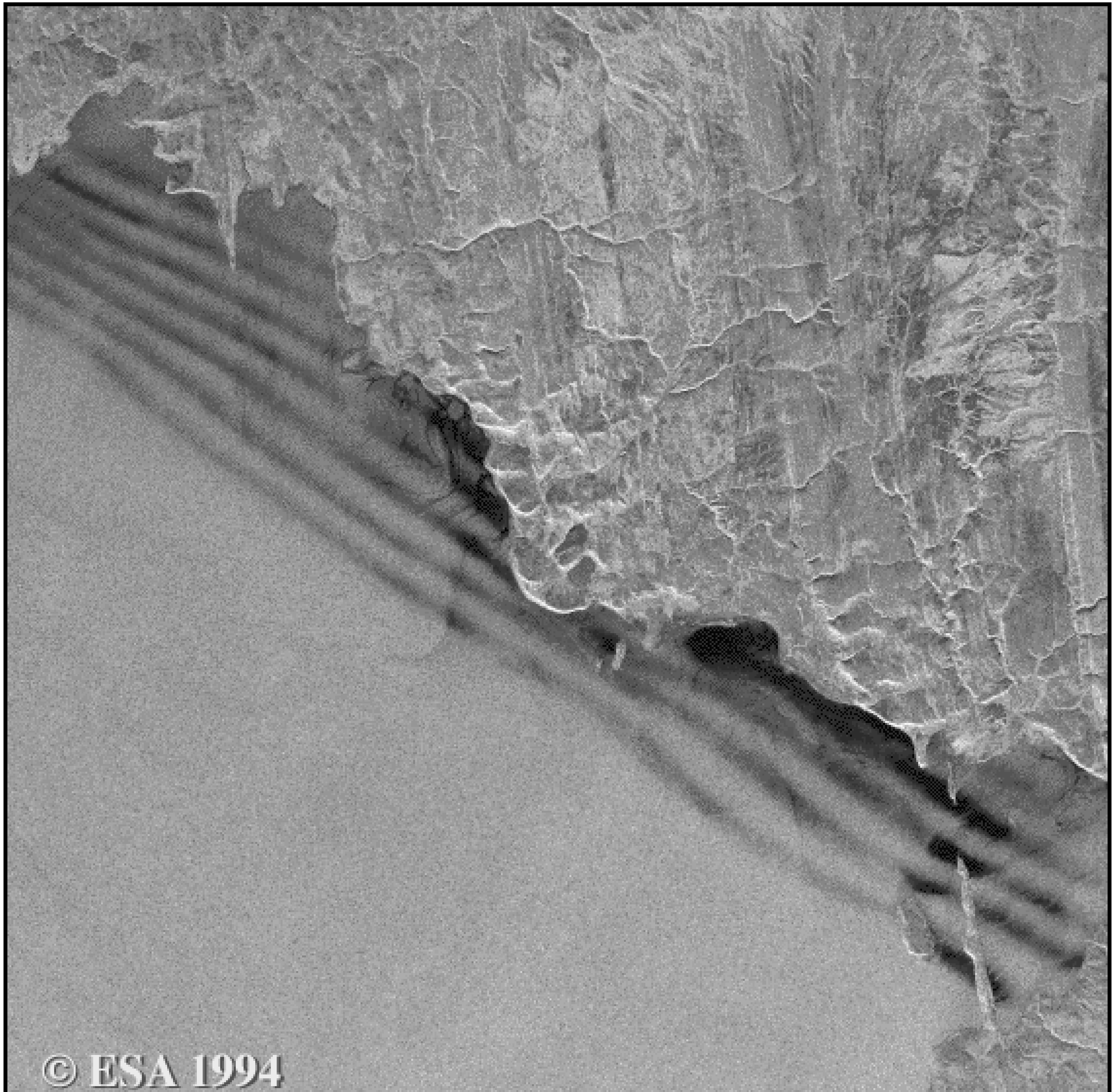


Resolution = 1 Meter | Resolution = 1 Foot | Resolution = 4 Inches

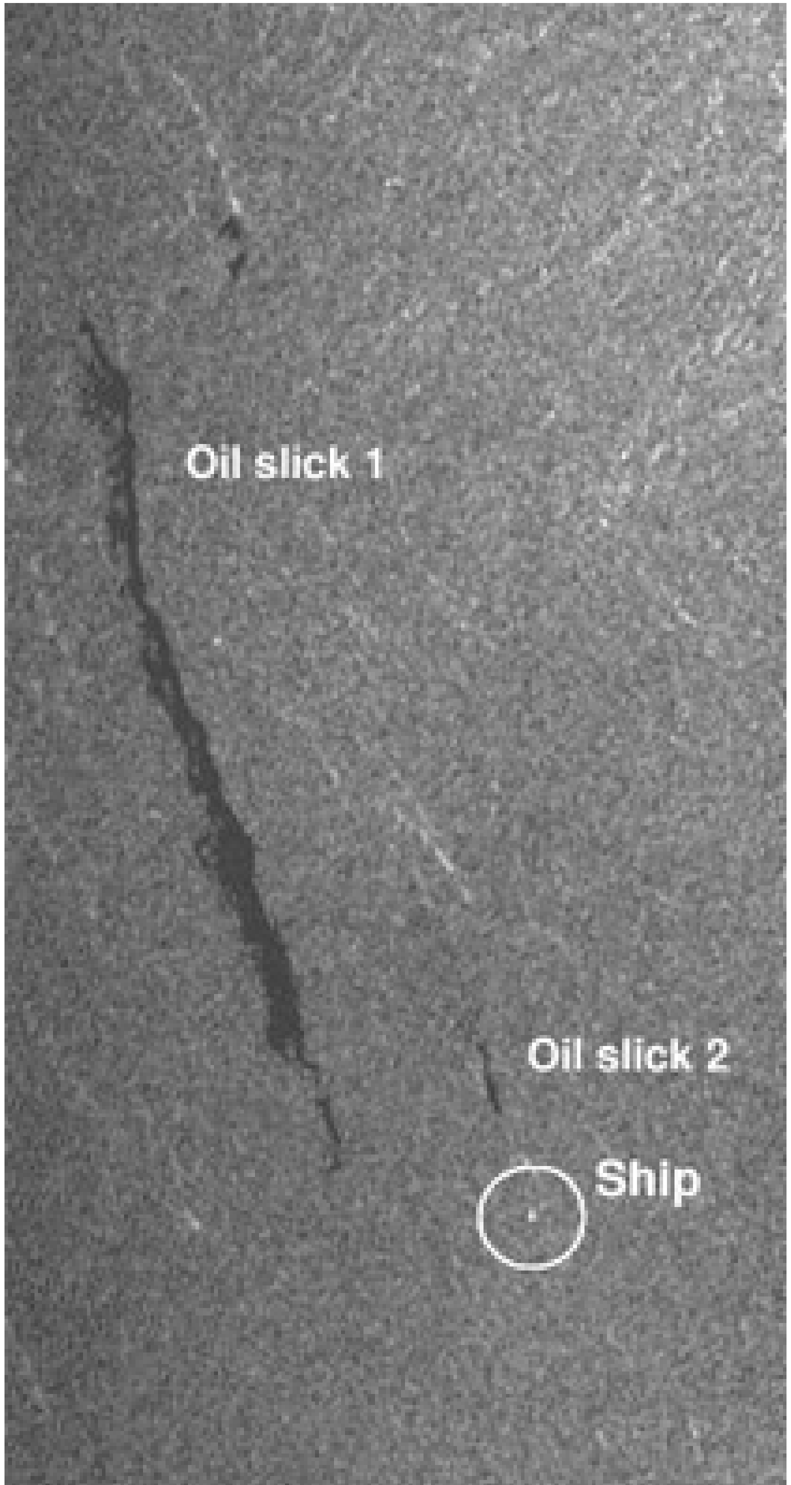
GENERAL ATOMICS   LYNX SAR

Deforestation in Brazil

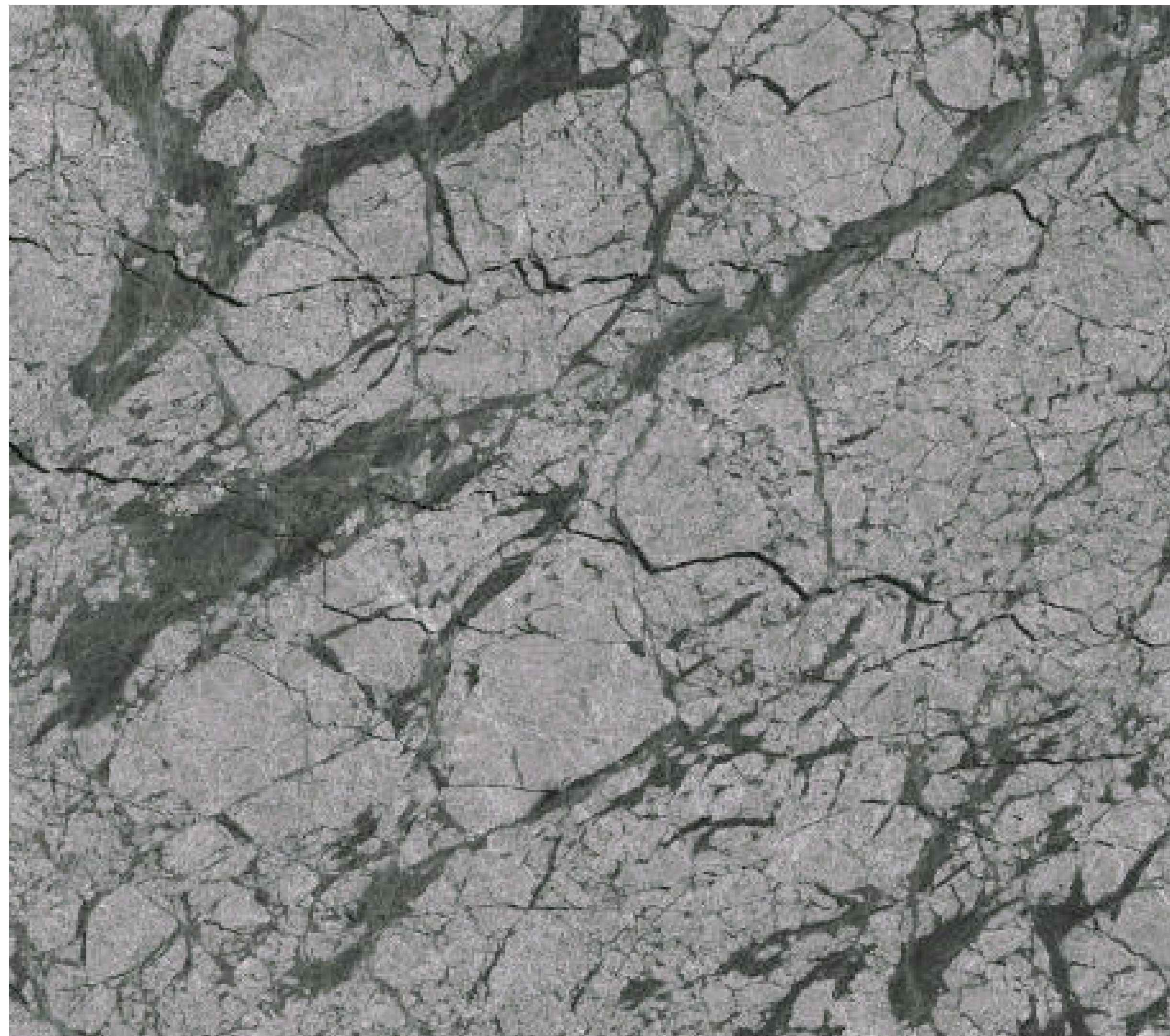
Ocean waves (texture due to wind)



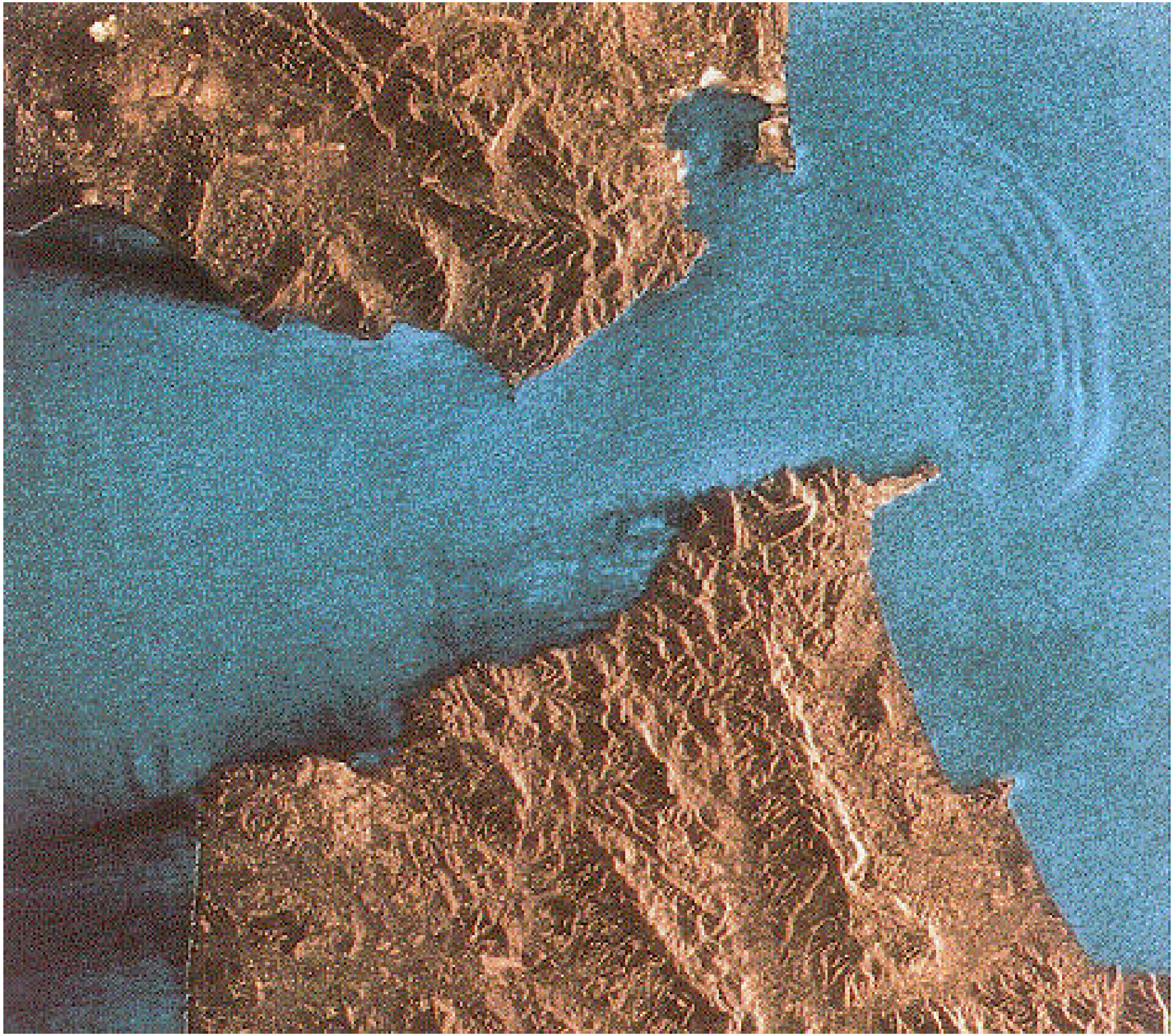
Oil slicks on the ocean

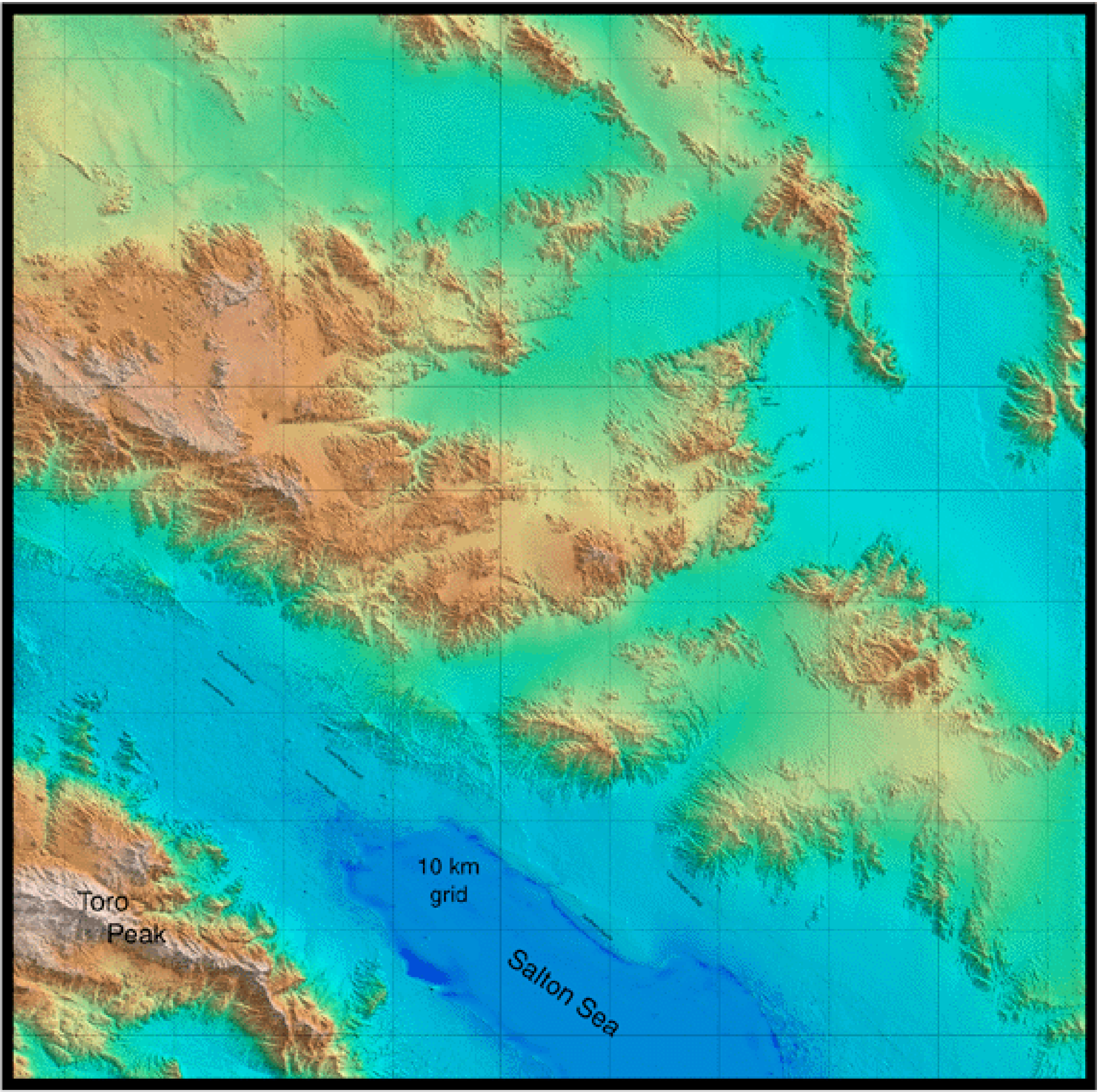


Sea ice

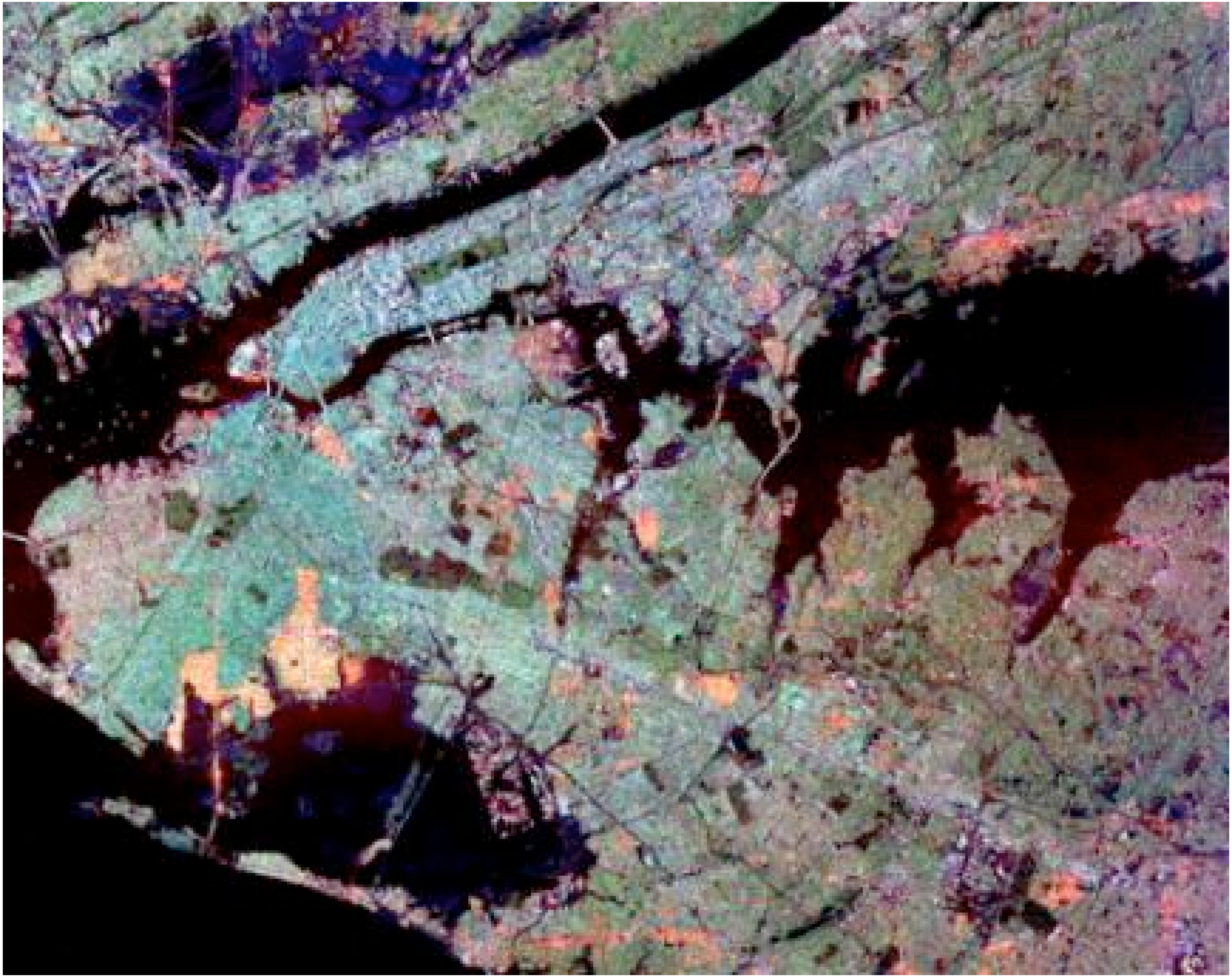


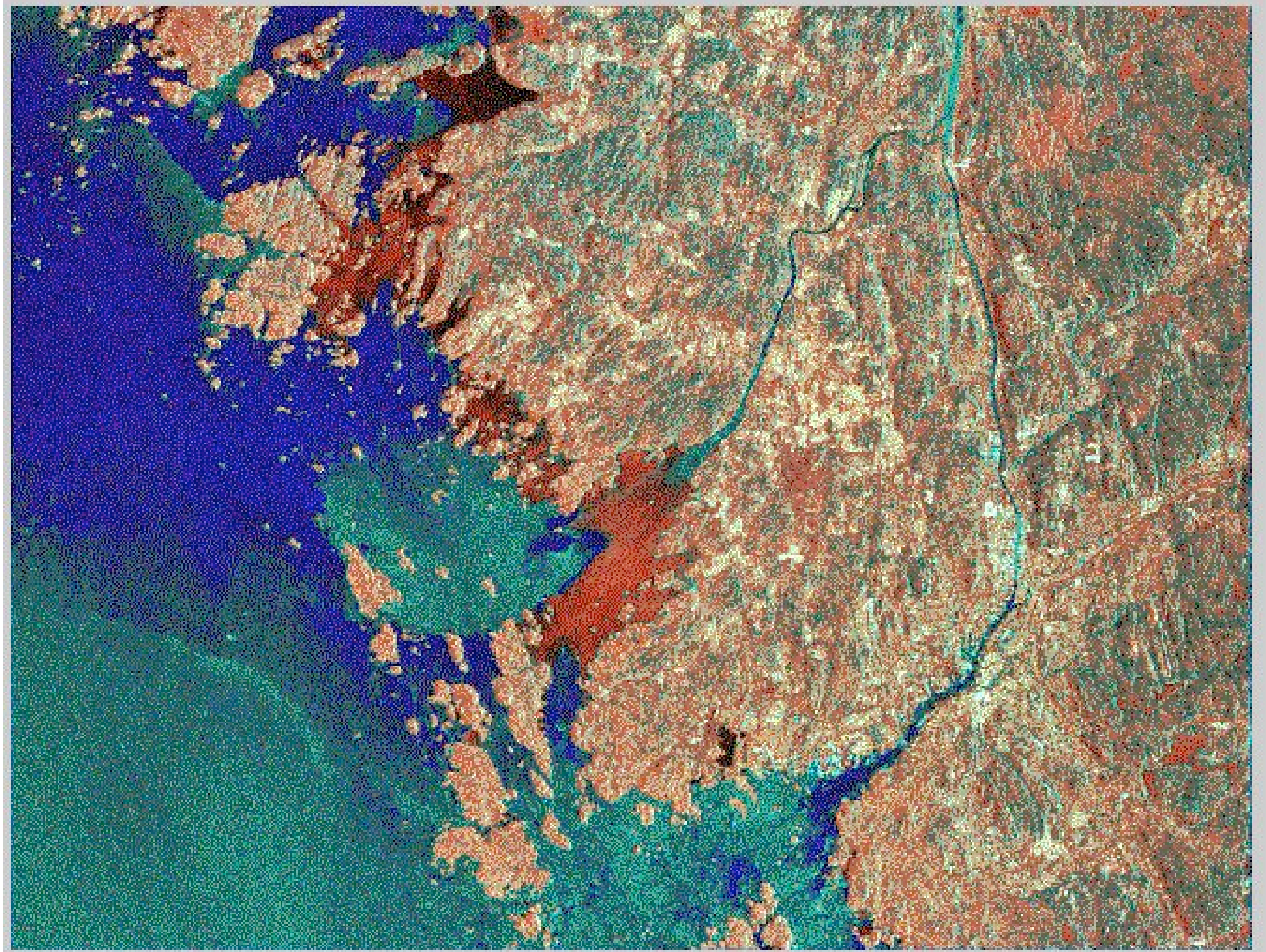
Ocean internal waves at Gibraltar



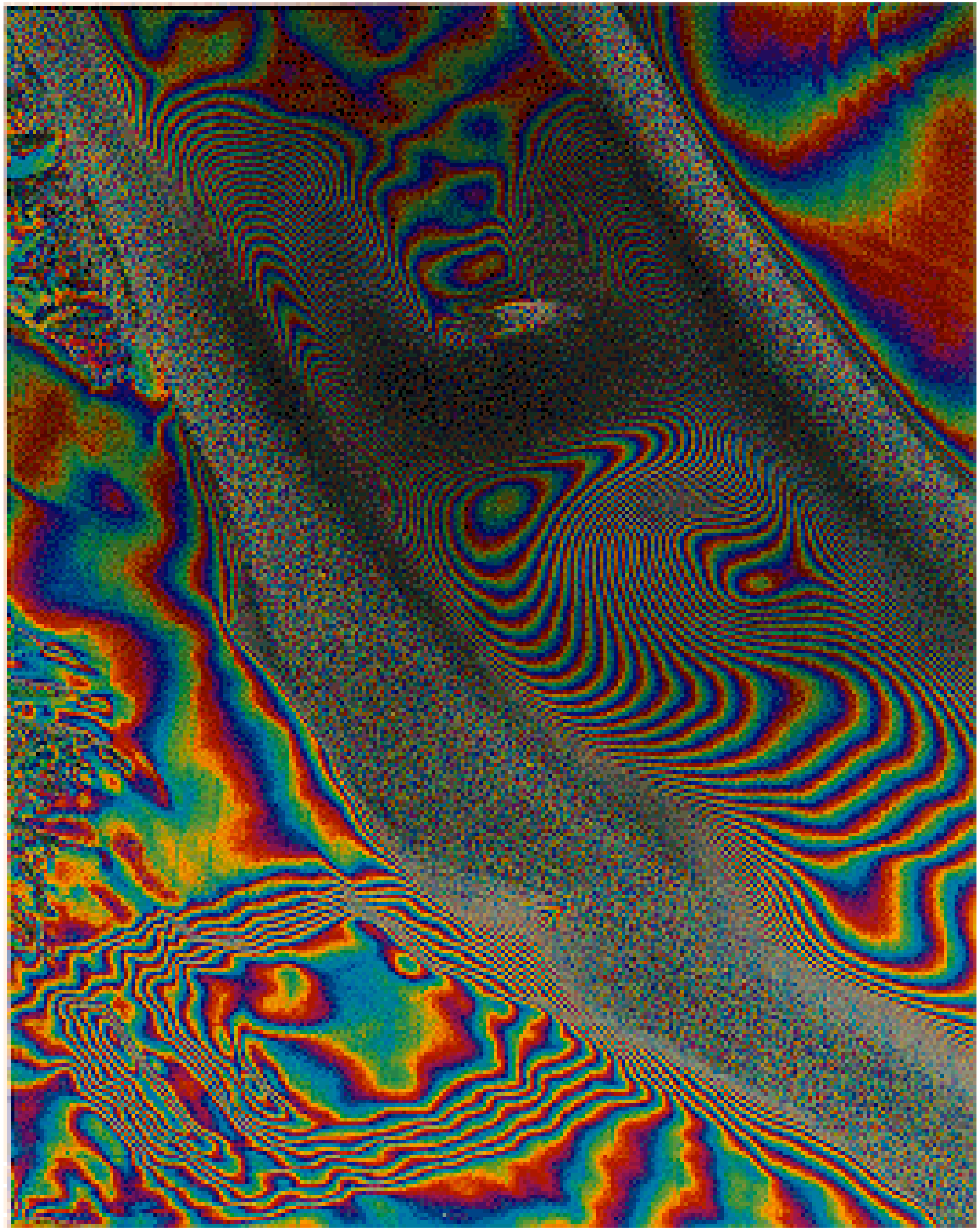


Southern California topography





Mikrovågsbild av Göteborg utnyttjande interferometrisk syntetisk aperturradar från ERS–1 och ERS–2 satelliterna den 10 och 11 mars 1996. Rött beskriver stabilitet (koherens) hos ytor, grönt besrkriver radartvärsnitt och blått beskriver färändringar i radartvästnitt. Fastisen längs kusten är röd, havet är blått med nvarier på grund av vindegenskaper, skogen är grönaktig etc.



Glacier flow via SAR interferometry

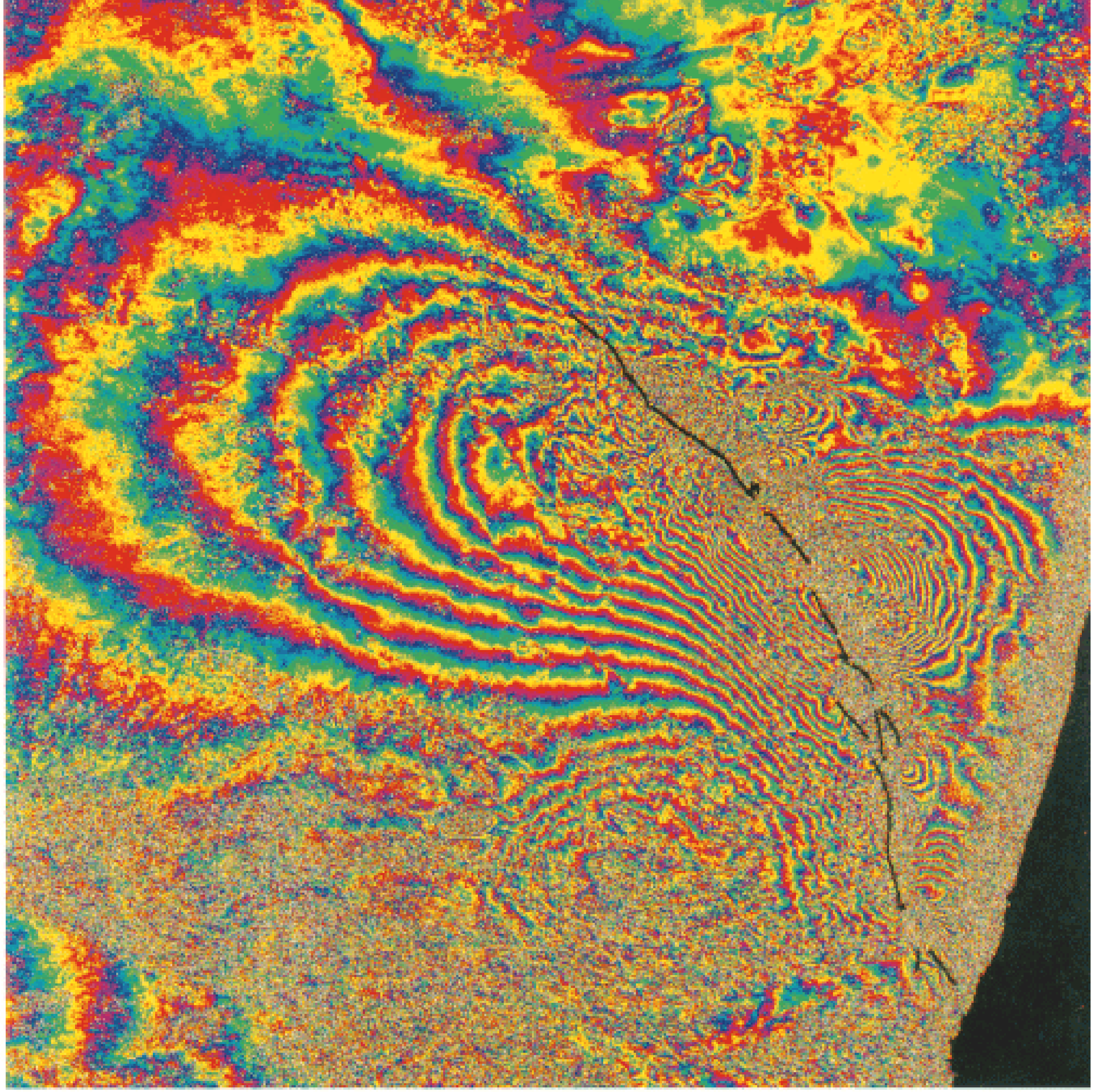


Fig. 3.3a. Compare this observed coseismic interferogram for the Landers earthquake [*Massonnet et al.*, 1993] with the synthetic interferogram in Fig. 3.3b. One cycle of color represents 28 mm of change in range. Black segments depict the fault geometry as mapped in the field. Both this image and Fig. 3.3b cover a 90-by-110-km area from April 24 to

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Assumed background

- Fourier transform
- delta function
- $(\partial_x^2 - \partial_t^2)u(t,x) = 0$ has solutions of the form $u(t,x) = f(t-x) + g(t+x)$
- Cauchy-Schwartz inequality ($\int fg^* \leq \|f\|\|g\|$)
- $f = O(g)$ means $f \leq (const.)g$
- $\nabla \cdot \boldsymbol{B} = 0 \Rightarrow \boldsymbol{B} = \nabla \times \boldsymbol{A}$ and $\nabla \times \boldsymbol{E} = \mathbf{0} \Rightarrow \boldsymbol{E} = -\nabla\phi$
- $\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2\boldsymbol{E}$

Fourier transform

$$\mathcal{F}[F](t) := f(t) = \frac{1}{2\pi}\int e^{-i\omega t}F(\omega)d\omega = \int e^{-2\pi i\nu t}\tilde{F}(\nu)d\nu$$

inverse transform: $$F(\omega) = \int e^{i\omega t}f(t)dt$$

Properties

1. If $g(t) = \int h(t-t')f(t')dt'$, then $G(\omega) = H(\omega)F(\omega)$.
2. $\partial_t f(t) = \mathcal{F}[-i\omega F](t)$
3. $\delta(t) = (2\pi)^{-1}\int e^{i\omega t}d\omega$

in n dimensions:

$$\mathcal{F}[F](\boldsymbol{x}) := f(\boldsymbol{x}) = \frac{1}{(2\pi)^n}\int e^{i\boldsymbol{\xi}\cdot\boldsymbol{x}}F(\boldsymbol{\xi})d\boldsymbol{\xi} \qquad F(\boldsymbol{\xi}) = \int e^{i\boldsymbol{\xi}\cdot\boldsymbol{x}}f(\boldsymbol{x})d\boldsymbol{x}$$

Books

- B. Borden, Radar Imaging of Airborne Targets, Institute of Physics, 1999.
- C. Elachi, Spaceborne Radar Remote Sensing: Applications and Techniques, IEEE Press, New York, 1987.
- W. C. Carrara, R. G. Goodman, R. M. Majewski, Spotlight Synthetic Aperture Radar: Signal Processing Algorithms, Artech House, Boston, 1995.
- G. Franceschetti and R. Lanari, Synthetic Aperture Radar Processing, CRC Press, New York, 1999.
- L.J. Cutrona, "Synthetic Aperture Radar", in Radar Handbook, second edition, ed. M. Skolnik, McGraw-Hill, New York, 1990.
- C.V. Jakowatz, D.E. Wahl, P.H. Eichel, D.C. Ghiglia, and P.A. Thompson, Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach, Kluwer, Boston, 1996.
- I.G. Cumming and F.H. Wong, Digital Processing of SAR Data: Algorithms and Implementation, Artech House, 2005

Maxwell's equations

$$\nabla \times \boldsymbol{\mathcal{E}} = -\partial_t \boldsymbol{\mathcal{B}} \tag{1}$$

$$\nabla \times \boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{J}} + \partial_t \boldsymbol{\mathcal{D}} \tag{2}$$

$$\nabla \cdot \boldsymbol{\mathcal{D}} = \rho \qquad \nabla \cdot \boldsymbol{\mathcal{B}} = 0 \tag{3}$$

$\boldsymbol{\mathcal{E}}$ = electric field $\quad$ $\boldsymbol{\mathcal{D}}$ = electric displacement

$\boldsymbol{\mathcal{B}}$ = magnetic field $\quad$ $\boldsymbol{\mathcal{H}}$ = magnetic induction

$\boldsymbol{\mathcal{J}}$ = current density $\quad$ ρ = charge density

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Constitutive laws in free space

$$\boldsymbol{\mathcal{D}} = \epsilon_0 \boldsymbol{\mathcal{E}} \qquad \boldsymbol{\mathcal{B}} = \mu_0 \boldsymbol{\mathcal{H}} \qquad \boldsymbol{\mathcal{J}} = 0 \qquad \rho = 0$$

$$\nabla \times (1) \quad + \quad \text{constitutive laws} \quad + \quad (2) \quad \Rightarrow$$

$$\underbrace{\nabla \times \nabla \times \boldsymbol{\mathcal{E}}}_{\underbrace{\nabla(\nabla \cdot \boldsymbol{\mathcal{E}})}_{0} - \nabla^2 \boldsymbol{\mathcal{E}}} = -\partial_t \nabla \times \boldsymbol{\mathcal{B}} = -\mu_0 \partial_t \underbrace{\nabla \times \boldsymbol{\mathcal{H}}}_{\epsilon_0 \partial_t \boldsymbol{\mathcal{E}}}$$

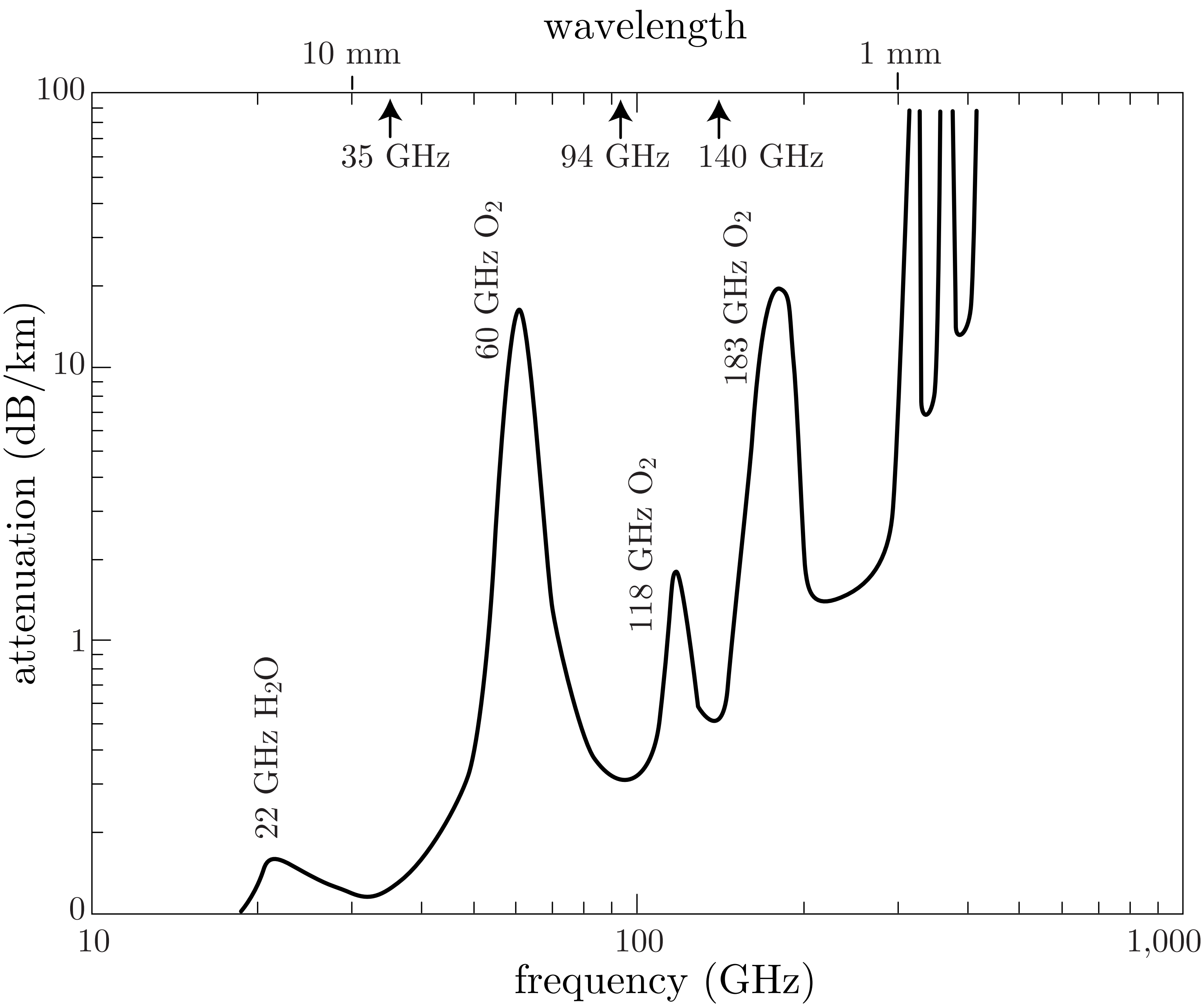
$$\Downarrow$$

$$\boxed{\nabla^2 \boldsymbol{\mathcal{E}} - \underbrace{\mu_0 \epsilon_0}_{1/c_0^2} \partial_t^2 \boldsymbol{\mathcal{E}} = \mathbf{0}}$$

Fourier transform $\quad \downarrow \quad \mathbf{E}(\omega) = \int e^{i\omega t} \boldsymbol{\mathcal{E}}(t) dt$

$$\boxed{\nabla^2 \mathbf{E} + \underbrace{\frac{\omega^2}{c^2}}_{k^2} \mathbf{E} = \mathbf{0}}$$

Atmospheric Attenuation



Radar frequency bands

Band Designation	Approximate Frequency Range
HF	3–30 MHz
VHF	30–300 MHz
UHF	300–1000 MHz
L-band	1–2 GHz
S-band	2–4 GHz
C-band	4–8 GHz
X-band	8–12 GHz
Ku-band	12–18 GHz
K-band	18–27 GHz
Ka-band	27–40 GHz
mm-wave	40–300 GHz

Decibels

$$\log_{10}\left(\frac{\text{power in}}{\text{power out}}\right) = \text{Bel} \qquad \text{too small}$$

instead use:

decibel $\boxed{\text{dB} = 10\log_{10}\frac{\text{power in}}{\text{power out}}} = 10\log_{10}\frac{V_{in}^2}{V_{out}^2} = 20\log_{10}\frac{V_{in}}{V_{out}}$

$\uparrow$

power $\propto$ (voltage)^2

dB	Power ratio
0 dB	1
10 dB	10
20 dB	100
30 dB	1000

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Radar systems

1. Stepped-frequency radars (laboratory systems)

transmit	receive	measure
$\underbrace{\cos(\omega_1 t)}_{\text{Re}(e^{-i\omega_1 t})}$	$\underbrace{R_R(\omega_1)\cos(\omega_1 t) + R_I(\omega_1)\sin(\omega_1 t)}_{\text{Re}[R(\omega_1)e^{-i\omega_1 t}]}$	$R(\omega_1)$
$\text{Re}(e^{-i\omega_2 t})$	$R(\omega_2)e^{-i\omega_2 t}$	$R(\omega_2)$
$\vdots$	$\vdots$	$\vdots$
$\text{Re}(e^{-i\omega_N t})$	$R(\omega_N)e^{-i\omega_N t}$	$R(\omega_N)$

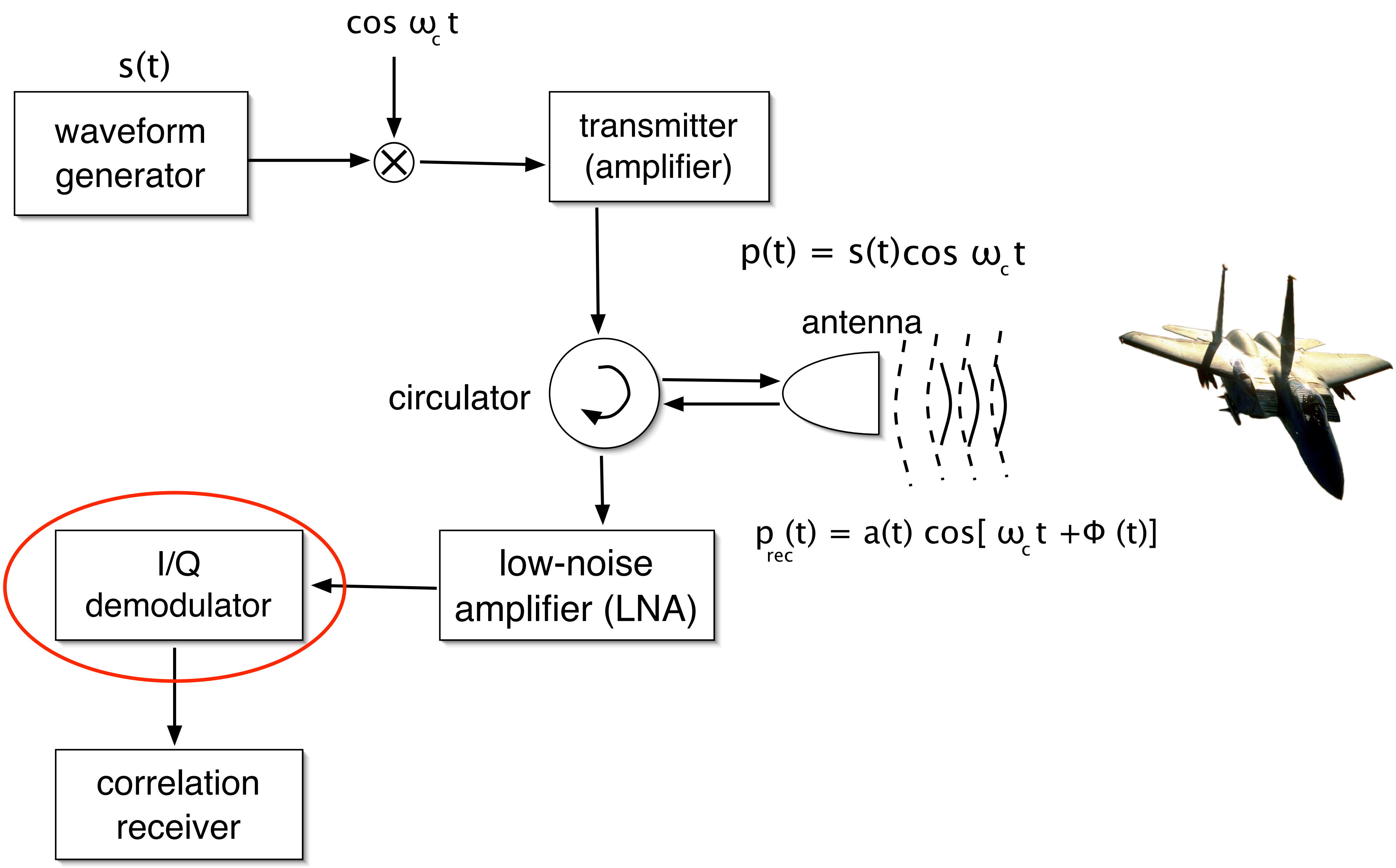
From the Rs, can synthesize response to any waveform

$$s^{in}(t) = \sum a_n(\omega_n)e^{-i\omega_n t} \approx \int_{\omega_1}^{\omega_N} a(\omega)e^{-i\omega t}d\omega$$

Response would be

$$s^{rec}(t) = \sum a_n(\omega_n)R(\omega_n)e^{-i\omega_n t} \approx \int_{\omega_1}^{\omega_N} a(\omega)R(\omega)e^{-i\omega t}d\omega$$

2. Pulsed radar systems



I/Q Demodulation

in-phase (I) channel:

$$p_{rec}(t)\cos(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\cos(\omega_c t)$$

$$= a(t)\tfrac{1}{2}\left(\underbrace{\cos(\phi(t)+2\omega_c t)}_{\text{filter out}} + \cos\phi(t)\right)$$

quadrature (Q) channel (90° out of phase):

$$p_{rec}(t)\sin(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\sin(\omega_c t)$$

$$= a(t)\tfrac{1}{2}\left(\underbrace{-\sin(\phi(t)+2\omega_c t)}_{\text{filter out}} + \sin\phi(t)\right)$$

I and Q channels together give the *analytic signal*

$$\boxed{s_{rec}(t) = a(t)e^{i\phi(t)}}$$

(approximately analytic in upper half-plane, when $a(t)$ is slowly varying, i.e., in narrowband case)

Filters

$$H(\omega)\ \textit{transfer function}$$

$$f(t) \xrightarrow{\mathcal{F}} F(\omega) \rightarrow \bigotimes \rightarrow F(\omega)H(\omega) \xrightarrow{\mathcal{F}^{-1}} (h * f)(t)$$

$$\begin{aligned} \mathcal{F}^{-1}\left[H(\omega)(\mathcal{F}f)(\omega)\right](t) &= \frac{1}{2\pi}\int e^{-i\omega t}H(\omega)\int e^{i\omega t'}f(t')dt'd\omega \\ &= \frac{1}{2\pi}\int \underbrace{\left[\int e^{-i\omega(t-t')}H(\omega)d\omega\right]}_{h(t-t')} f(t')dt' \end{aligned}$$

Example: Low-pass filter. Take $H(\omega) = \begin{cases} 1 & |\omega| < \omega_1 \\ 0 & \text{otherwise} \end{cases}$

$\Rightarrow \qquad h(t) = \frac{\omega_1}{\pi}\frac{\sin\omega_1 t}{\omega_1 t} = \frac{\omega_1}{\pi}\operatorname{sinc}\omega_1 t$

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1D Scattering by a fixed perfect conductor at range R

waveform generator $\rightarrow s_{inc}(t)$

transmitter output:

$$s_{inc}(t)\cos(\omega_c t) = Re\left[s_{inc}(t)e^{-i\omega_c t}\right] := f(t)$$

transmitted electromagnetic wave: (1D model)

$$\boldsymbol{\mathcal{E}}^{in}(\boldsymbol{r},t) = \boldsymbol{e}^{in} f(t - x/c) \text{ where } x = \hat{\boldsymbol{e}}\cdot\boldsymbol{r}$$

$\boldsymbol{\mathcal{E}}^{in}$ is a right-going solution of

$$\partial_x^2 \boldsymbol{\mathcal{E}}^{in} - \frac{1}{c^2}\partial_t^2 \boldsymbol{\mathcal{E}}^{in} = \boldsymbol{0}$$

Write total field as $\boldsymbol{\mathcal{E}}^{tot} = \boldsymbol{\mathcal{E}}^{in} + \boldsymbol{\mathcal{E}}^{sc}$ (think $f(t-x/c) + g(t+x/c)$)

$\boldsymbol{\mathcal{E}}^{tot}$ satisfies

$$\begin{aligned}
\partial_x^2 \boldsymbol{\mathcal{E}}^{tot} - \frac{1}{c^2}\partial_t^2 \boldsymbol{\mathcal{E}}^{tot} &= \boldsymbol{0} \\
\boldsymbol{\mathcal{E}}^{tot}\Big|_{x=R} &= 0 \qquad \leftarrow \text{conducting B.C.}
\end{aligned}$$

$\Rightarrow$

$$\begin{aligned} \partial_x^2 \boldsymbol{\mathcal{E}}^{sc} - \frac{1}{c^2}\partial_t^2 \boldsymbol{\mathcal{E}}^{sc} &= \mathbf{0} \\ \boldsymbol{\mathcal{E}}^{sc}\Big|_{x=R} &= -\boldsymbol{\mathcal{E}}^{in}\Big|_{x=R} \end{aligned}$$

expect $\boldsymbol{\mathcal{E}}^{sc}(\boldsymbol{r}, t) = \boldsymbol{e}^{sc} g(t + x/c)$ (left-going solution of wave equation)

B.C. $\Rightarrow \boldsymbol{e}^{sc} g(\underbrace{t + R/c}_{w}) = -\boldsymbol{e}^{in} f(t - R/c) \;\; \Rightarrow \;\; \boldsymbol{e}^{sc} = -\boldsymbol{e}^{in}$

$t = w - R/c \qquad \Rightarrow \qquad g(w) = f(w - 2R/c)$

received field at $\boldsymbol{r} = \mathbf{0}$:

$$\boldsymbol{\mathcal{E}}^{sc}(\mathbf{0}, t) = -\boldsymbol{e}^{in} f(t - 2R/c)$$

$\boxed{\text{transmit } f(t), \text{ receive } p_{rec}(t) = f(t - 2R/c)}$ (fixed target)

1D Scattering by a moving conductor at range $R(t)$

$$g(\underbrace{t + R(t)/c}_{w}) = f(t - R(t)/c)$$

solve $w = t + R(t)/c$ for t (via Implicit Function Theorem) $\rightarrow t = \tau(w)$

for pulsed systems: use Taylor series expansion $R(t) = R + vt + \cdots$

$$w = t + \overbrace{(R + vt)}^{R(t)}/c \qquad \Rightarrow \qquad t = \frac{w - R/c}{1 + v/c} := \tau(w)$$

$$\begin{aligned} g(w) &= f(t - (R + vt)/c)\Big|_{t=\tau(w)} \\ &= f\left(\underbrace{\left(\frac{1 - v/c}{1 + v/c}\right)}_{\alpha}(w - R/c) - R/c\right) \end{aligned}$$

$\uparrow$

Doppler scale factor

RF field scattered from moving target

For $f(t) = s(t)\cos(\omega_c t)$,

$$p_{rec}(t) = s(\alpha(t - R/c) - R/c)\cos[\omega_c \underbrace{(\alpha(t - R/c) - R/c)}_{\alpha t-(1+\alpha)R/c}]$$

frequency of cosine $= \omega_c \alpha$

For $\dfrac{v}{c} << 1, \quad \alpha \approx 1 - \dfrac{2v}{c} \quad \Rightarrow \quad \omega_c \alpha \approx \omega_c \underbrace{- \frac{2v}{c}\omega_c}$

$\uparrow$

Doppler shift $= \omega_D$

I/Q demodulation of signal from moving scatterer

$$p_{rec}(t)\cos(\omega_c t) = s(\alpha(t - R/c) - R/c)\cos[\omega_c(\alpha t - (1+\alpha)R/c)]\cos(\omega_c t)$$

$$= s(\alpha(t - R/c) - R/c)\frac{1}{2}\left(\overbrace{\cos[\text{sum}]}^{\text{filter out}} + \cos\left[\omega_c(\alpha t - (1+\alpha)R/c) - \omega_c t\right]\right)$$

$$I(t) = s(\alpha(t - R/c) - R/c)\cos\omega_c\left[(\alpha - 1)t - (1+\alpha)R/c\right]$$

$$Q(t) = s(\alpha(t - R/c) - R/c)\sin\omega_c\left[(\alpha - 1)t - (1+\alpha)R/c\right]$$

$$s_{rec}(t) = s(\alpha(t - R/c) - R/c)e^{i\omega_c[(\alpha-1)t-(1+\alpha)R/c)]}$$

For $\frac{v}{c} << 1$ and s slowly varying:

$$s_{rec}(t) \approx s(t - 2R/c)e^{i\omega_D(t-R/c)}e^{-2i\omega_c R/c}$$

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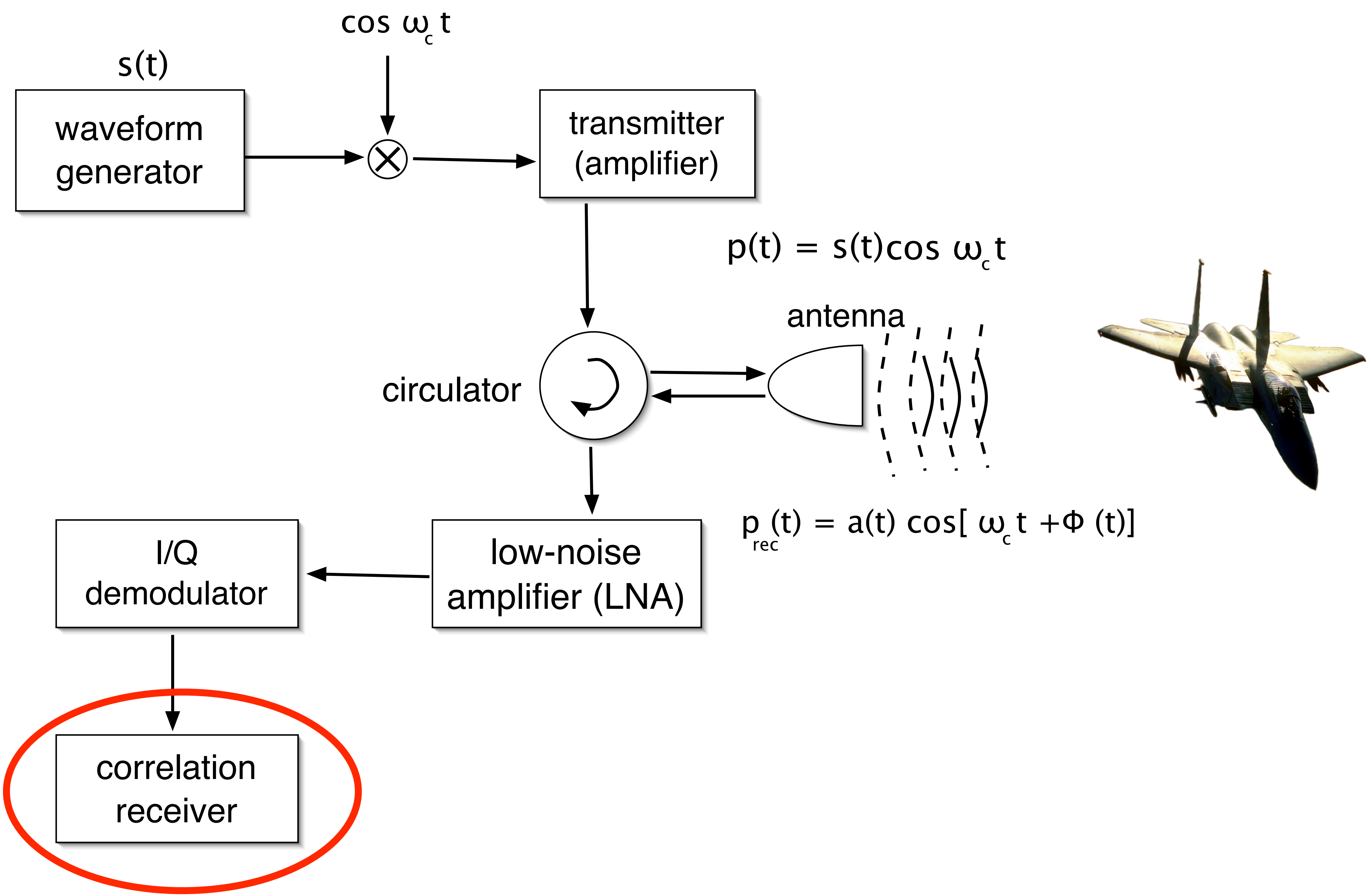


Receiver design

For good range resolution, want a short pulse

But a short pulse has little energy $\Rightarrow$ hard to detect signal in noise

energy density $\propto \frac{1}{R^4}$!

signal is swamped by thermal noise in the receiver!

target can't even be detected, much less imaged

Brilliant solution:

Use (long) *coded pulses* and mathematical processing

$\uparrow$

matched filter or correlation receiver

pulse compression

Matched filter: sketch of derivation

receiver input: $r(t) = \rho s(t-\tau) + n(t)$ (= demodulator output)

$\nearrow \quad \nearrow \quad \nwarrow$ want to find τ

$ae^{i\phi}/R^4 \quad 2R/c$ noise, assumed white, zero mean

power spectral density N

Apply filter $\eta(t) = (h * r)(t) = \int h(t-t')r(t')dt' = \eta_s(t) + \eta_n(t)$

Choose h so that $|\eta_s(\tau)/\eta_n(\tau)|$ is as large as possible.

$$SNR = \max_h \frac{|\eta_s(\tau)|^2}{E|\eta_n(\tau)|^2} = \max_h \frac{\rho^2 \left|\int h(\tau - t')s(t'-\tau)dt'\right|^2}{N \int |h(t)|^2 dt}$$

$$= \max_h \frac{\rho^2 \left|\int h(t')s(-t')dt'\right|^2}{N \int |h(t)|^2 dt}$$

Cauchy-Schwartz inequality $\Rightarrow h(t) = s^*(-t)$

$$\boxed{\eta(t) = \int s^*(t'-t)r(t')dt' = \int s^*(t'')r(t+t'')dt''}$$ correlation

Pulse compression from matched filtering

Example: the 5-bit Barker code +++-+

				+	+	+	-	+	correlator output
+	+	+	-	+					1
	+	+	+	-	+				-1+1=0
		+	+	+	-	+			1-1+1=1
			+	+	+	-	+		1+1-1-1=0
				+	+	+	-	+	1+1+1+1+1=5

Multiple fixed targets

Two fixed targets: $r(t) = \rho_1 s(t - \tau_1) + \rho_2 s(t - \tau_2) + n(t)$

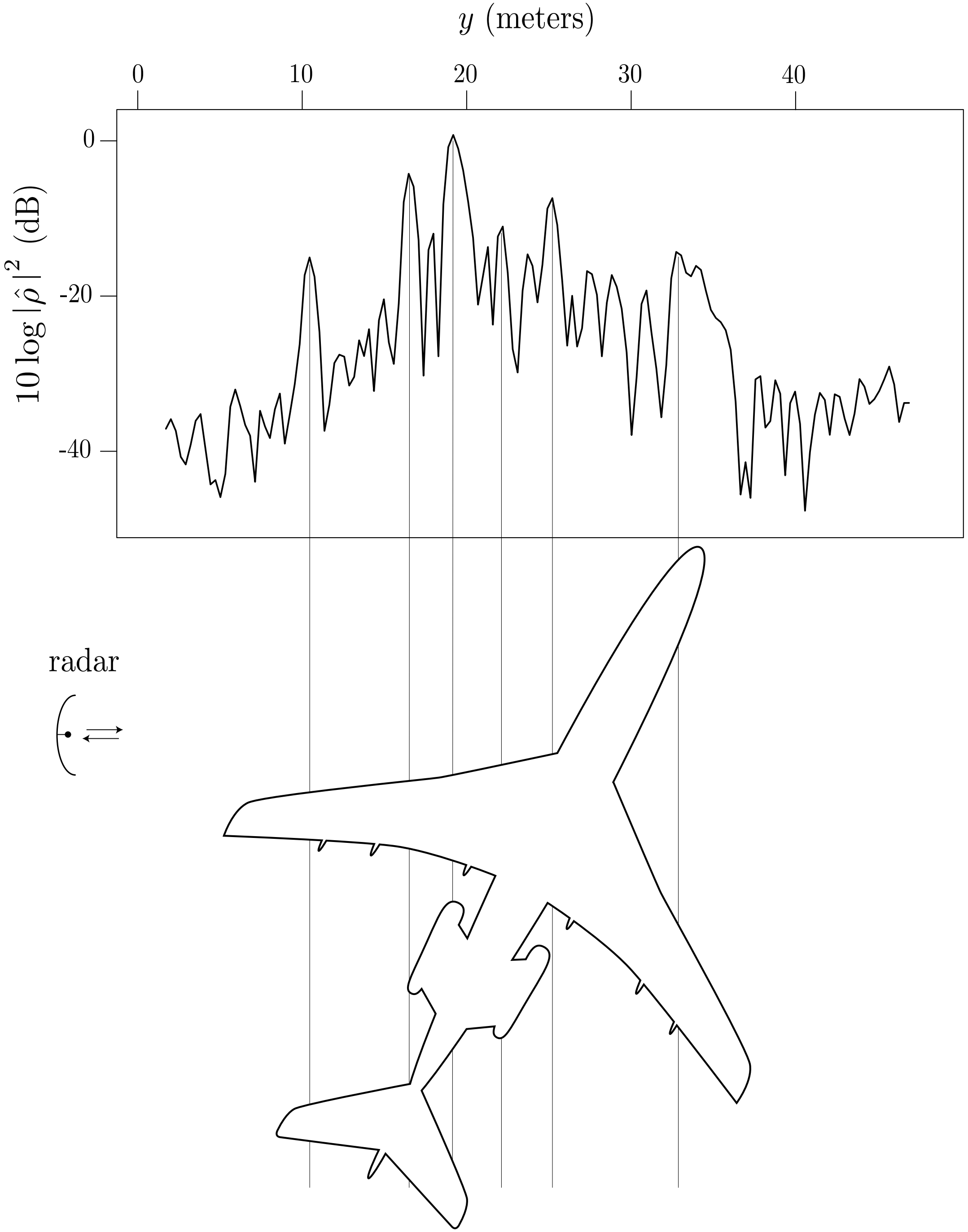
Distribution of fixed targets: $r(t) = \int \rho(\tau')s(t - \tau')d\tau' + n(t)$

Apply matched filter:

$$\begin{aligned}
\eta(t) &= \int s^*(t' - t)r(t')dt' \\
&= \int s^*(t' - t) \int \rho(\tau')s(t' - \tau')d\tau' dt' + \text{noise} \\
&= \int \underbrace{\int s^*(t' - t)s(t' - \tau')dt'}_{\chi(\tau' - t)} \rho(\tau')d\tau' + \text{noise}
\end{aligned}$$

$\chi(t) = \int s^*(t'' + t)s(t'')dt'' =$ *point spread function* for
1D "imaging system"

High Range-Resolution (HRR) Imaging

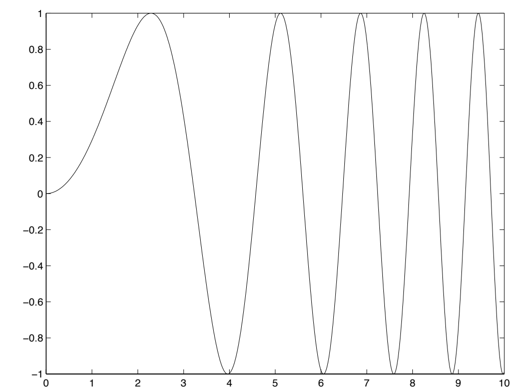


Chirp = Linearly Frequency Modulated (LFM) waveform

$s(t) = e^{i\phi(t)}\text{rect}(t/t_p)$ where $\frac{d\phi}{dt}(t) =$ instantaneous frequency

$$\text{rect}(t) = \begin{cases} 1 & -1/2 < t < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d\phi}{dt}(t) = at \qquad \Rightarrow \qquad \phi(t) = at^2$$



$$\Rightarrow \boxed{s(t) = e^{iat^2}\text{rect}(t/t_p)}$$

gives rise to a point spread function

$$\chi(t) = (1 - |t|)\text{sinc}(at(1 - |t|))$$

where sinc $x = (1/x)\sin x$.

(see p. 170 in Rihaczek *Principles of High Resolution Radar*
or work out yourself)

Matched filter for single moving target

receiver input = demodulator output $= r(t) = s(t-\tau)e^{i\omega_D(t-\tau)} + n(t)$

want to find τ and ω_D.

use a *filter bank* = set of filters that depend on a parameter ν:

$$\eta(t,\nu) = \int h_\nu(t-t')r(t')dt'$$

to maximize SNR, choose $h_\nu(t) = s^*(-t)e^{i2\pi\nu t}$

Matched filter for distribution of moving targets

demodulator output $= r(t) = \int\int \rho(\tau', \nu') s(t - \tau') e^{2\pi i \nu'(t-\tau')} d\tau' d\nu'$

output of filter bank is

$$
\begin{aligned}
\eta(t, \nu) &= \int s^*(t' - t) e^{2\pi i \nu (t-t')} r(t') dt' \\
&= \int s^*(t' - t) e^{2\pi i \nu (t-t')} s(t' - \tau') e^{2\pi i \nu'(t'-\tau')} dt' \rho(\tau', \nu') d\tau' d\nu' \\
&= \int\int \chi(\tau' - t, \nu' - \nu) e^{2\pi i \nu (t-\tau')} \rho(\tau', \nu') d\tau' d\nu'
\end{aligned}
$$

where

$$
\boxed{\chi(\tau, \nu) = \int s^*(t'' + \tau) s(t'') e^{2\pi i \nu t''} dt''}
$$

(narrowband) *radar ambiguity function*

point spread function for imaging system

Typically one considers only the *magnitude* of the ambiguity function.

Outline

1. introduction, history, frequency bands, dB, real-aperture imaging
2. radar systems: stepped-frequency systems, I/Q demodulation
3. 1D scattering by perfect conductor
4. receiver design, matched filtering
5. ambiguity function & its properties
6. range-doppler (unfocused) imaging
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10. spotlight SAR
11. stripmap SAR

Properties of the ambiguity function

1. $|\chi(\tau,\nu)| \leq |\chi(0,0)| = \int |s(t)|^2 dt =$ signal energy
   $= 1$ for a normalized signal

2. $\int\int |\chi(\tau,\nu)|^2 d\tau d\nu = 1$ (for a normalized signal)
   Radar uncertainty principle or conservation of ambiguity volume

3. $|\chi(-\tau,-\nu)| = |\chi(\tau,\nu)|$

4. If χ is the ambiguity function for s, then the ambiguity function χ_a for $e^{-i\pi a t^2} s(t)$ satisfies $|\chi_a(\tau,\nu)| = |\chi(\tau,\nu + a\tau)|$

5. The ambiguity function for $s(t)e^{ia}$ is the same as that for $s(t)$.

6. The (magnitude of the) ambiguity function for $s(t)e^{-i\omega t}$ is the same as that for $s(t)$.

Resolution and cuts through the ambiguity function

Doppler (frequency) resolution:

$$|\chi(0,\nu)| = \left| \int |s(t)|^2 e^{2\pi i \nu t} dt \right|$$

$\Rightarrow$ Frequency (Doppler) resolution is determined by amplitude.

For good Doppler resolution, want $|s(t)| \approx 1$.

Range resolution:

$$|\chi(\tau,0)| = \left| \int |S(2\pi\nu)|^2 e^{2\pi i \nu \tau} d\nu \right|$$

where $S(\omega) = \int e^{-i\omega t} s(t) dt$.

$\Rightarrow$ Range resolution (for a fixed target) is determined by bandwidth.

Example: Range resolution with a CW pulse

baseband signal is $s(t) = \mathrm{rect}(t/t_p)$ $\qquad t_p =$ time duration of pulse

ambiguity function is

$$|\chi(\tau,\nu)| = \begin{cases} \left(1 - \frac{|\tau|}{t_p}\right) \left|\mathrm{sinc}\left[\pi \nu t_p \left(1 - \frac{|\tau|}{t_p}\right)\right]\right| & \text{for } |\tau| < t_p \\ 0 & \text{otherwise} \end{cases}$$

Range resolution is obtained from

$$|\chi(\tau,0)| = \begin{cases} \left(1 - \frac{|\tau|}{t_p}\right) & \text{for } |\tau| < t_p \\ 0 & \text{otherwise} \end{cases}$$

whose first null is at $\delta\tau_{pn} = t_p$.

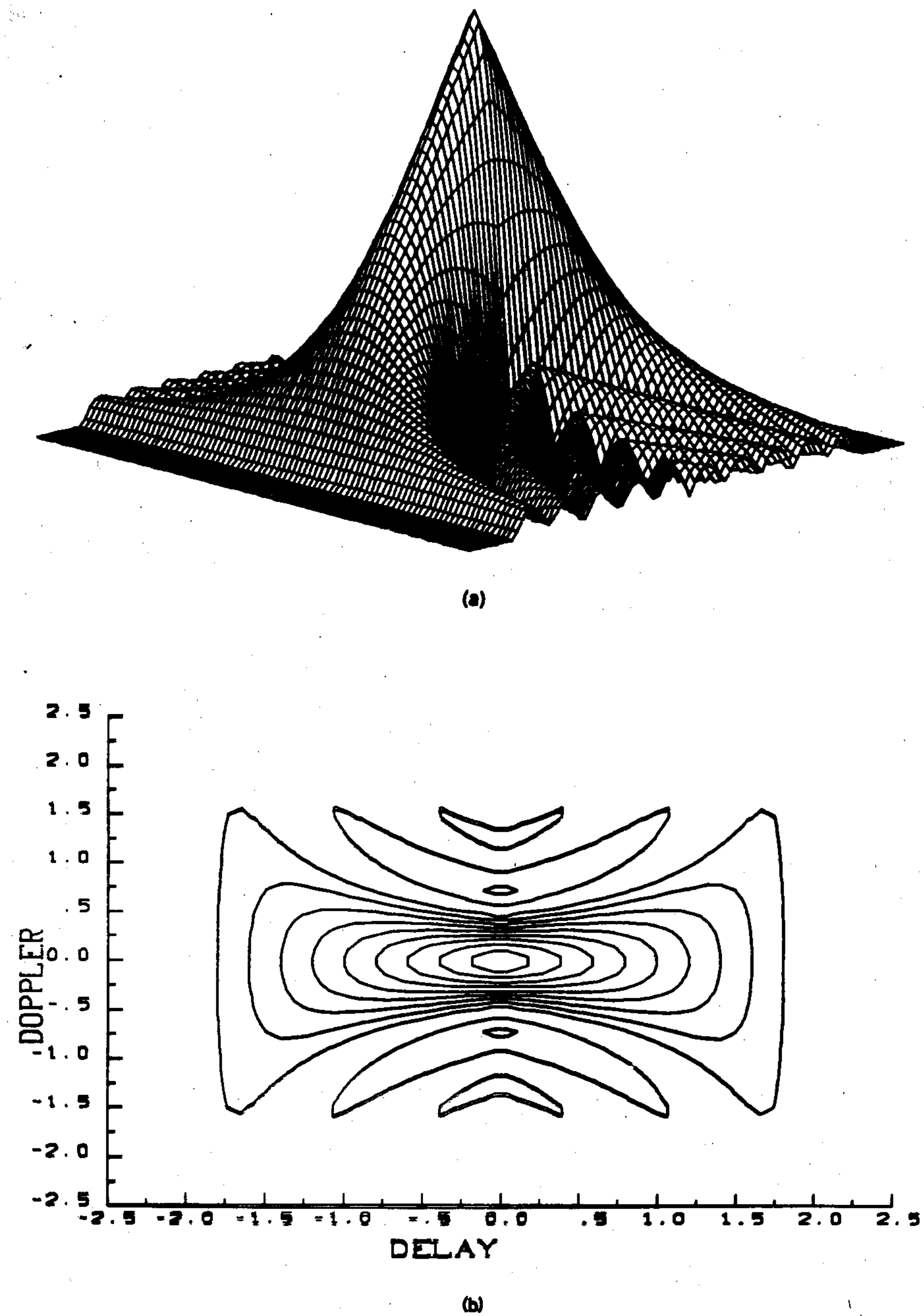


***Figure** 7.2 The ambiguity function of a single-frequency pulse: (a) 3-D view. (b) Contour plot for a pulse duration $t_p = 2$.*

ambiguity
function
for CW pulse

N. Levanon,
Radar Principles,
Wiley 1988

Example: Range resolution with a chirp

For $s(t) = \mathrm{rect}(t/t_p)e^{i\pi a t^2}$

the ambiguity function is

$$|\chi(\tau,\nu)| = \begin{cases} \left(1 - \frac{|\tau|}{t_p}\right) \left|\mathrm{sinc}\left[\pi t_p \left(1 - \frac{|\tau|}{t_p}\right)(\nu + a\tau)\right]\right| & \text{for } |\tau| < t_p \\ 0 & \text{otherwise} \end{cases}$$

Range resolution is obtained from

$$|\chi(\tau,0)| = \begin{cases} \left(1 - \frac{|\tau|}{t_p}\right) \left|\mathrm{sinc}\left[\pi t_p \left(1 - \frac{|\tau|}{t_p}\right) a\tau\right]\right| & \text{for } |\tau| < t_p \\ 0 & \text{otherwise} \end{cases}$$

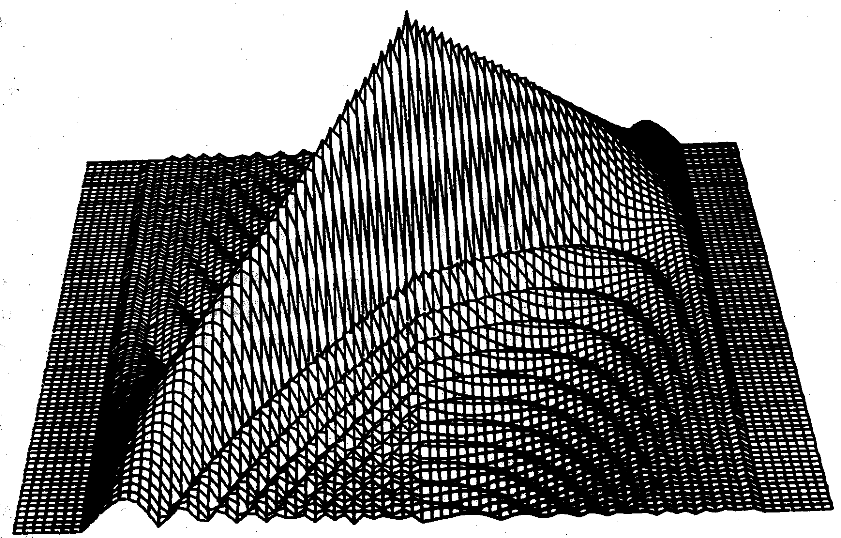
The first null is at $\delta\tau_{pn} = \frac{1}{a t_p} = \frac{1}{B}$ where $B =$ bandwidth

---

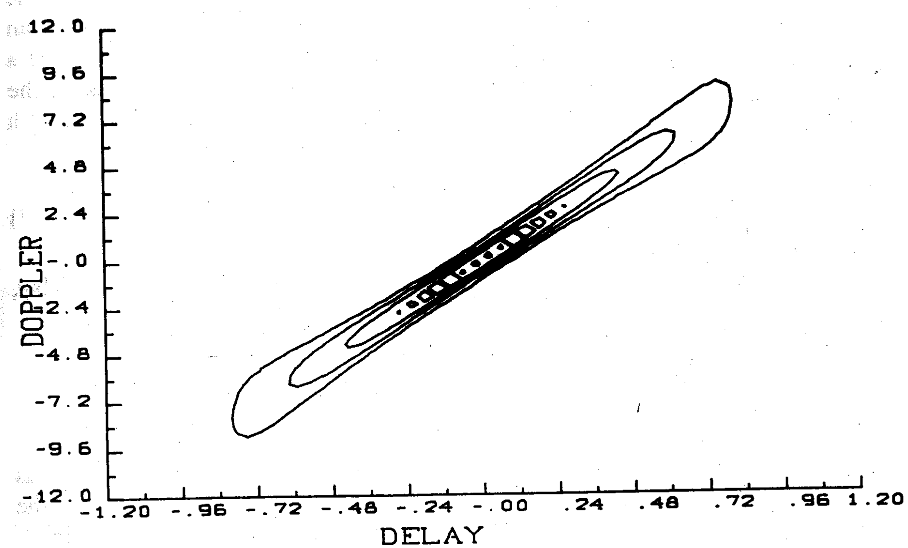
Phase modulation improves range resolution by a factor of

$$\text{pulse compression ratio} = \frac{\delta\tau_{pn,CW}}{\delta\tau_{pn,chirp}} = \frac{t_p}{(1/B)} = \underbrace{t_p B}_{\text{time-bandwidth product}}\ !$$

ambiguity function for chirp



(a)



(b)

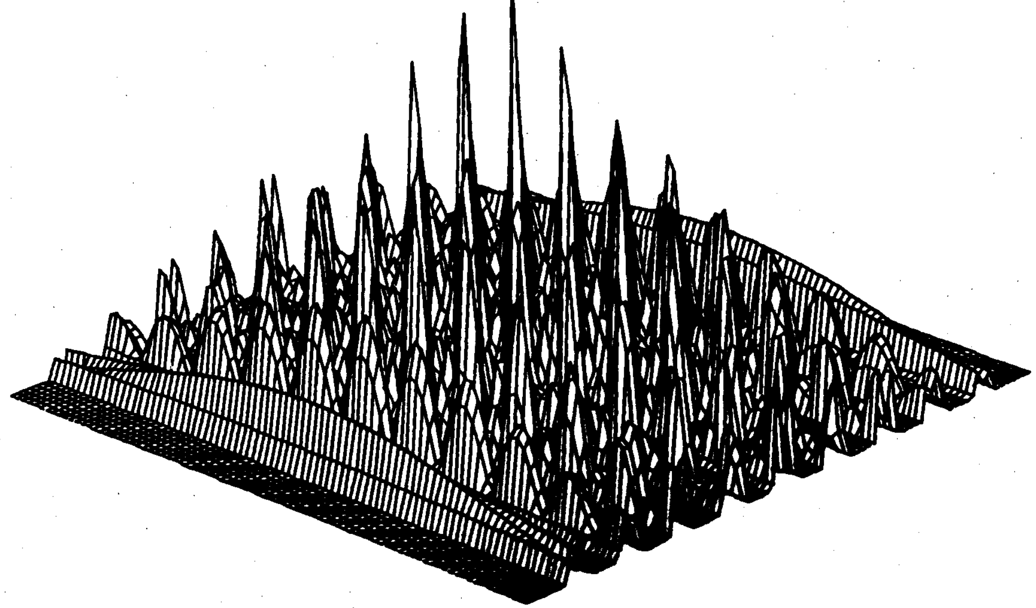
Figure 7.5 The ambiguity function of a linear FM pulse ($t_p = 1$, $k = -10$): (a) 3-D view. (b) Contour plot.

A train of high-range-resolution (HRR) pulses

Doppler shift can be found by change in phase of successive returns

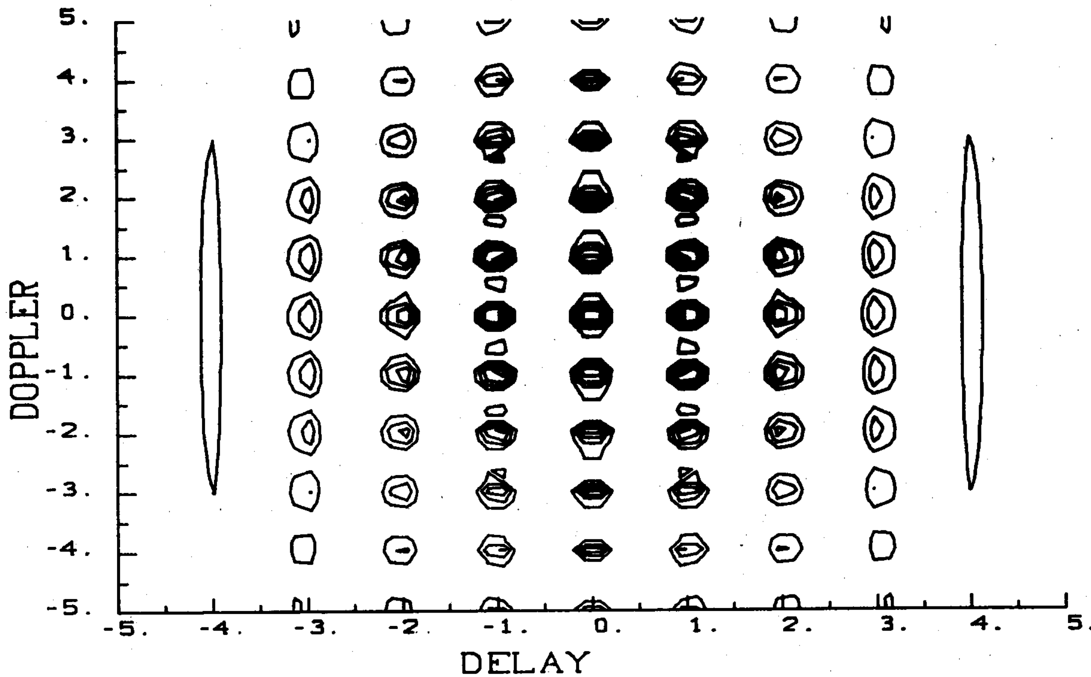
Suppose target travels as $R(t) = R_0 + vt$; write $R_n = R(nT)$

1. transmit $p_n(t) = s(t)e^{-i\omega_c t}$
   receive $r_n(t) = p_n(t - 2R_n/c)e^{i\omega_D(t-2R_n/c)}$
2. demodulate: $s_n(t) = s(t - 2R_n/c)e^{i\omega_D(t-R_n/c)}e^{-2i\omega_c R_n/c}$
3. correlate: $\eta_n(\tau) = \int s^*(t' - \tau)s_n(t')dt' =$
   $\int s^*(t' - \tau)s(t' - 2R_n/c)e^{i\omega_D(t'-R_n/c)}e^{-2i\omega_c R_n/c}dt'$
4. at peak, $\tau = 2R_n/c$:
   $\eta_n(2R_n/c) = \underbrace{\int |s(t' - 2R_n/c)|^2 e^{i\omega_D(t'-R_n/c)}dt'}_{\chi(0,\omega_D)} e^{-2i\omega_c R_n/c}$
5. phase difference between successive pulses:
   $2\omega_c[R_0 + v(n+1)T]/c - 2\omega_c[R_0 + vnT]/c = 2\omega_c v/c = -\omega_D$
6. note *blind speeds* when $2\omega_c v/c = 2\pi$(integer)



(a)

ambiguity function for a train of pulses



(b)

pulse repetition frequency gives rise to delay ambiguities

Figure 7.7 *The ambiguity function of five coherent pulses ($T_R = 1$, $t_P = 0.2$):*

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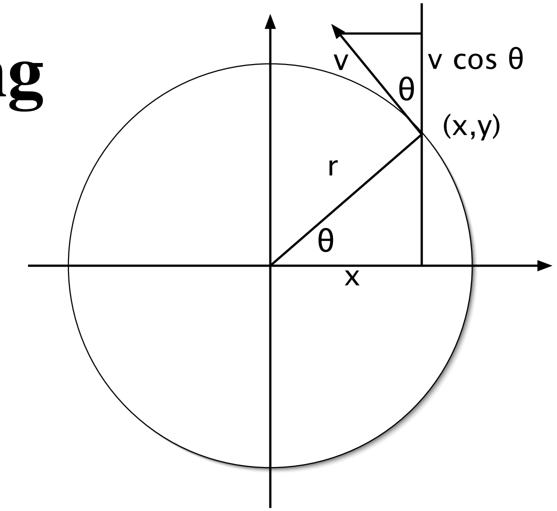
Range-Doppler Imaging



Stationary radar, rotating 2D object

If radar is at $(0, -R)$, scatterer at (x, y):

- range is $R + y$
- if rotation rate is Ω, then

  $$|v| = r\Omega \qquad \Rightarrow \qquad v_{LOS} = v_y = |v| \cos\theta = \Omega \underbrace{r \cos\theta}_{x}$$

  recall Doppler shift is $\dfrac{\omega_D}{\omega_c} = -\dfrac{2v_{LOS}}{c} = -\dfrac{2\Omega x}{c}$
- As the object rotates, x and y change ("scatterer moves out of resolution cell")

  $\Rightarrow$ blurring

Need 3D scattering model that incorporates target motion

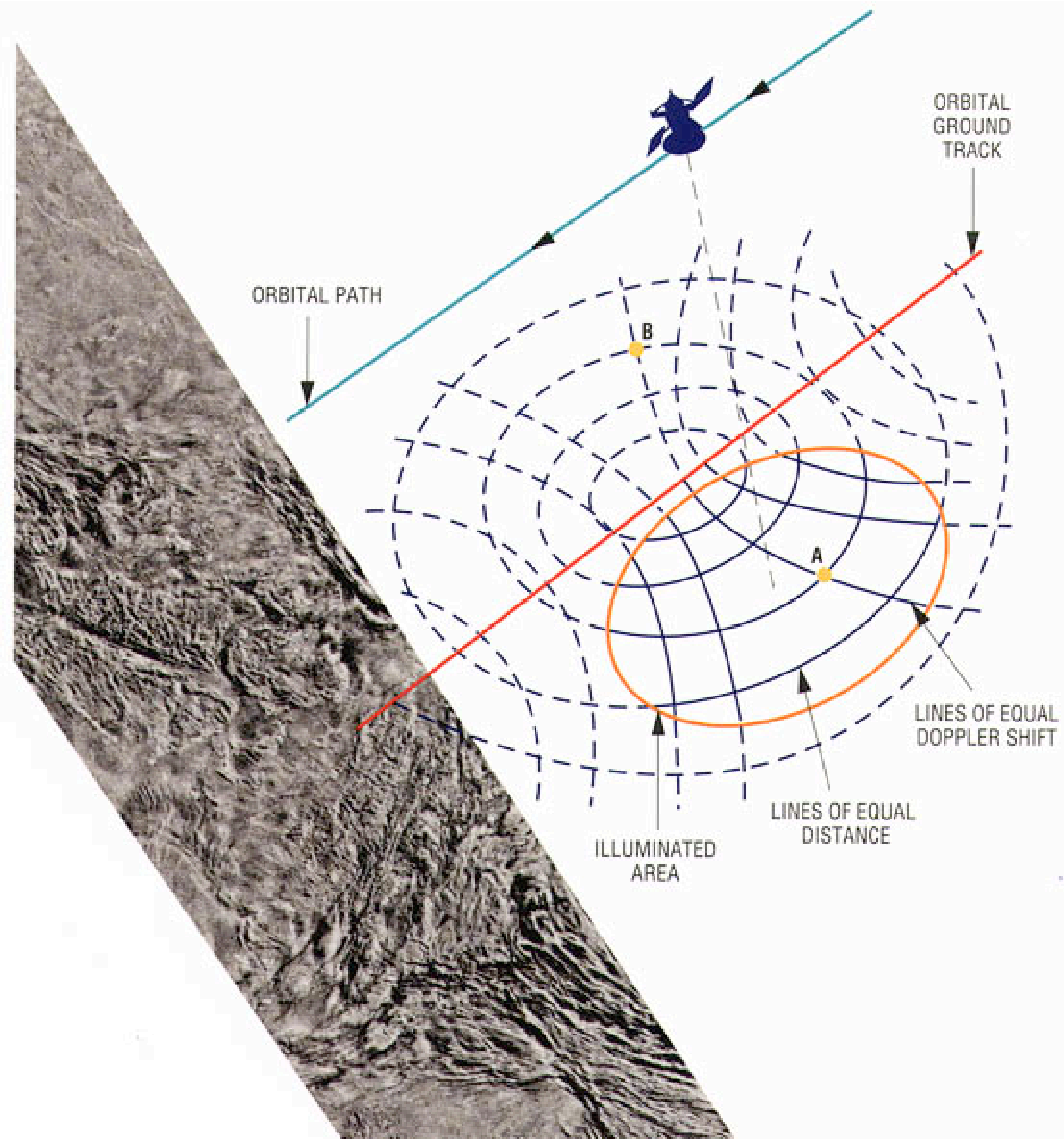
Moving radar imaging a stationary planar scene

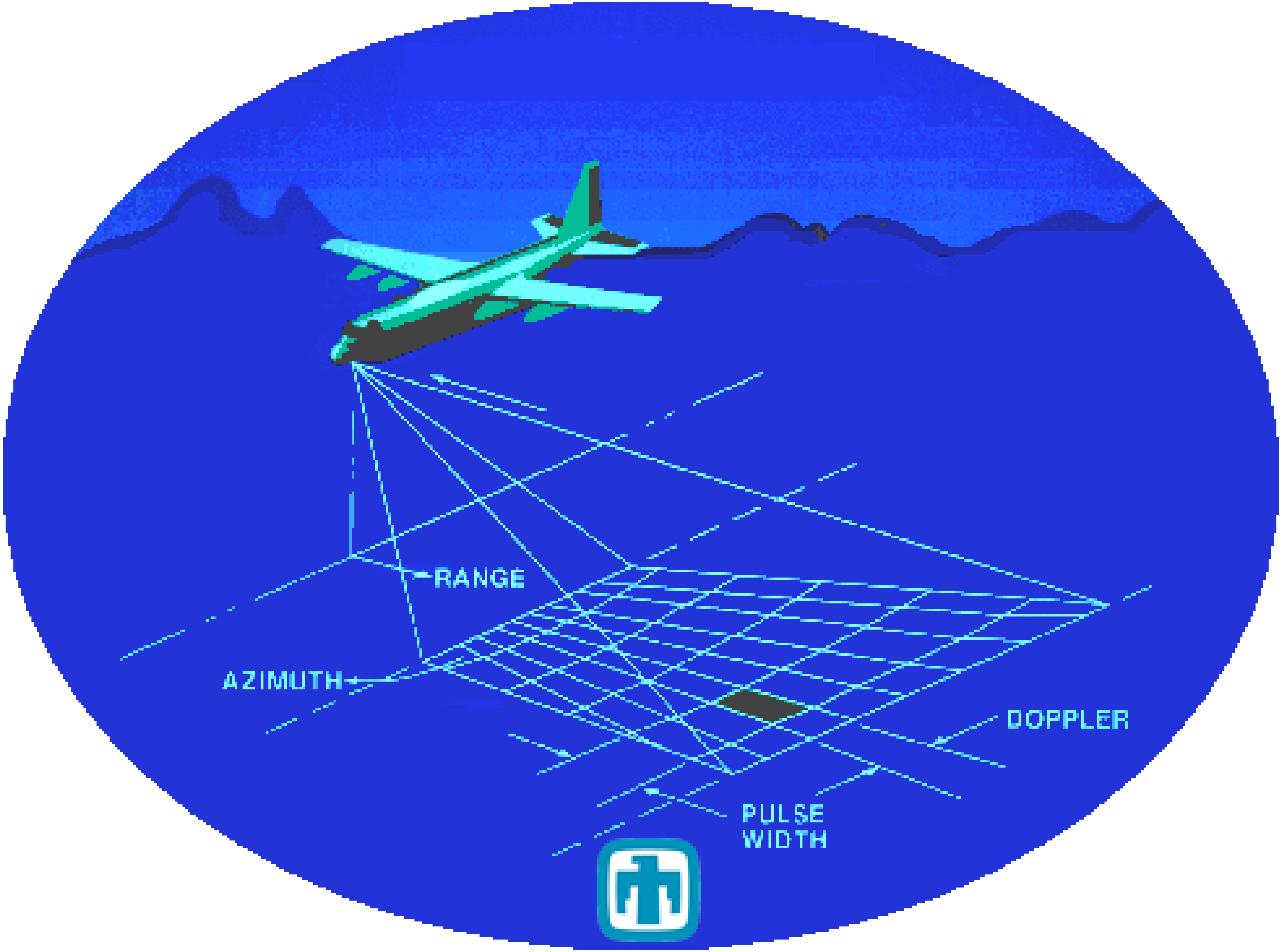
- delay $\Rightarrow$ range $\Rightarrow$ scatterer lies on a constant-range sphere
$\Rightarrow$ scatterer on plane lies on a constant-range circle

- Doppler shift $\Rightarrow$ line-of-sight relative velocity
$\Rightarrow$ scatterer lies on the iso-Doppler cone $v_{LOS} = \hat{\mathbf{R}} \cdot \mathbf{v} = const$
$\Rightarrow$ scatterer on plane lies on iso-Doppler hyperbola

- does not account for change in radar position as measurements are taken ("scatterers migrate through resolution cell")
$\Rightarrow$ get an unfocused image

Need a 3D scattering model that incorporates changes in sensor position


ORBITAL GROUND TRACK
ORBITAL PATH
B
A
LINES OF EQUAL DOPPLER SHIFT
LINES OF EQUAL DISTANCE
ILLUMINATED AREA



RANGE
AZIMUTH
DOPPLER
PULSE WIDTH