

Appendix on Units and Dimensions

The question of units and dimensions in electricity and magnetism has exercised a great number of physicists and engineers over the years. This situation is in marked contrast with the almost universal agreement on the basic units of length (centimeter or meter), mass (gram or kilogram), and time (mean solar second). The reason perhaps is that the mechanical units were defined when the idea of "absolute" standards was a novel concept (just before 1800), and they were urged on the professional and commercial world by a group of scientific giants (Borda, Laplace, and others). By the time the problem of electromagnetic units arose there were (and still are) many experts. The purpose of this appendix is to add as little heat and as much light as possible without belaboring the issue.

1 Units and Dimensions; Basic Units and Derived Units

The *arbitrariness* in the *number* of fundamental units and in the *dimensions* of any physical quantity in terms of those units has been emphasized by Abraham, Planck, Bridgman,* Birge,[†] and others. The reader interested in units as such will do well to become familiar with the excellent series of articles by Birge.

The desirable features of a system of units in any field are convenience and clarity. For example, theoretical physicists active in relativistic quantum field theory and the theory of elementary particles find it convenient to *choose* the universal constants such as Planck's quantum of action and the velocity of light in vacuum to be *dimensionless* and of *unit magnitude*. The resulting system of units (called "natural" units) has only *one* basic unit, customarily chosen to be mass. All quantities, whether length or time or force or energy, etc., are expressed in terms of this one unit and have dimensions that are powers of its dimension. There is nothing contrived or less fundamental about such a system than one involving the meter, the kilogram, and the second as basic units. It is merely a matter of convenience.[‡]

A word needs to be said about basic units or standards, considered as independent quantities, and derived units or standards, which are defined in both magnitude and dimension through theory and experiment in terms of the basic units. Tradition requires that mass (m), length (l), and time (t) be treated as basic. But for electrical quantities there has been no compelling tradition. Consider, for example, the unit of current. The "international" ampere (for a long

*P. W. Bridgman, *Dimensional Analysis*, Yale University Press, New Haven, CT (1931).

[†]R. T. Birge, *Am. Phys. Teacher* (now *Am. J. Phys.*), **2**, 41 (1934); **3**, 102, 171 (1935).

[‡]In quantum field theory, powers of the coupling constant play the role of other basic units in doing dimensional analysis.

period the accepted practical unit of current) is defined in terms of the mass of silver deposited per unit time by electrolysis in a standard silver voltameter. Such a unit of current is properly considered a basic unit, independent of the mass, length, and time units, since the amount of current serving as the unit is found from a supposedly reproducible experiment in electrolysis.

On the other hand, the presently accepted standard of current, the “absolute” ampere “is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed one metre apart in vacuum, would produce between these conductors a force equal to $2 \cdot 10^{-7}$ newton per metre of length.” This means that the “absolute” ampere is a derived unit, since its definition is in terms of the mechanical force between two wires through equation (A.4) below.* The “absolute” ampere is, by this definition, exactly one-tenth of the em unit of current, the abampere.

Since 1948 the internationally accepted system of electromagnetic standards has been based on the meter, the kilogram, the second, and the above definition of the absolute ampere plus other derived units for resistance, voltage, etc. This seems to be a desirable state of affairs. It avoids such difficulties as arose when, in 1894, by act of Congress (based on recommendations of an international commission of engineers and scientists), independent basic units of current, voltage, and resistance were defined in terms of three independent experiments (silver voltameter, Clark standard cell, specified column of mercury).† Soon afterward, because of systematic errors in the experiments outside the claimed accuracy, Ohm’s law was no longer valid, by act of Congress!

The *Système International d’Unités* (SI) has the unit of mass defined since 1889 by a platinum-iridium *kilogram* prototype kept in Sèvres, France. In 1967 the SI *second* was defined to be “the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.” The General Conference on Weights and Measures in 1983 adopted a definition of the *meter* based on the speed of light, namely, the *meter* is “the length of the distance traveled in vacuum by light during a time $1/299\,792\,458$ of a second.” The speed of light is therefore no longer an experimental number; it is, by definition of the meter, exactly $c = 299\,792\,458$ m/s. For electricity and magnetism, the *Système International* adds the absolute ampere as an additional unit, as already noted. In practice, metrology laboratories around the world define the ampere through the units of electromotive force, the volt, and resistance, the ohm, as determined experimentally from the Josephson effect ($2e/h$) and the quantum Hall effect (h/e^2), respectively.‡

*The proportionality constant k_2 in (A.4) is thereby given the magnitude $k_2 = 10^{-7}$ in the SI system. The *dimensions* of the “absolute” ampere, as distinct from its magnitude, depend on the dimensions assigned k_2 . In the conventional SI system of electromagnetic units, electric current (I) is arbitrarily chosen as a *fourth* basic dimension. Consequently charge has dimensions It , and k_2 has dimensions of $mI^{-2}t^{-2}$. If k_2 is taken to be dimensionless, then current has the dimensions $m^{1/2}l^{1/2}t^{-1}$. The question of whether a fourth basic dimension like current is introduced or whether electromagnetic quantities have dimensions given by powers (sometimes fractional) of the three basic mechanical dimensions is a purely subjective matter and has no fundamental significance.

†See, for example, F. A. Laws, *Electrical Measurements*, McGraw-Hill, New York (1917), pp. 705–706.

‡For a general discussion of SI units in electricity and magnetism and the use of quantum phenomena to define standards, see B. W. Petley, in *Metrology at the Frontiers of Physics and Technology*, eds. L. Corvini and T. J. Quinn, Proceedings of the International School of Physics “Enrico Fermi,” Course CX, 27 June–7 July 1989, North-Holland, Amsterdam (1992), pp. 33–61.

2 Electromagnetic Units and Equations

In discussing the units and dimensions of electromagnetism we take as our starting point the traditional choice of length (l), mass (m), and time (t) as independent, basic dimensions. Furthermore, we make the commonly accepted definition of current as the time rate of change of charge ($I = dq/dt$). This means that the dimension of the ratio of charge and current is that of time.* The continuity equation for charge and current densities then takes the form:

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0 \quad (\text{A.1})$$

To simplify matters we initially consider only electromagnetic phenomena in free space, apart from the presence of charges and currents.

The basic physical law governing electrostatics is Coulomb's law on the force between two point charges q and q' , separated by a distance r . In symbols this law is

$$F_1 = k_1 \frac{qq'}{r^2} \quad (\text{A.2})$$

The constant k_1 is a proportionality constant whose magnitude and dimensions *either* are determined by the equation (if the magnitude and dimensions of the unit of charge have been specified independently) *or* are chosen arbitrarily in order to define the unit of charge. Within our present framework all that is determined at the moment is that the product ($k_1 qq'$) has the dimensions (ml^3t^{-2}).

The electric field \mathbf{E} is a derived quantity, customarily defined to be the force per unit charge. A more general definition would be that the electric field be numerically proportional to the force per unit charge, with a proportionality constant that is a universal constant perhaps having dimensions such that the electric field is dimensionally different from force per unit charge. There is, however, nothing to be gained by this extra freedom in the definition of \mathbf{E} , since \mathbf{E} is the first derived field quantity to be defined. Only when we define other field quantities may it be convenient to insert dimensional proportionality constants in the definitions in order to adjust the dimensions and magnitude of these fields relative to the electric field. Consequently, with no significant loss of generality the electric field of a point charge q may be defined from (A.2) as the force per unit charge,

$$E = k_1 \frac{q}{r^2} \quad (\text{A.3})$$

All systems of units known to the author use this definition of electric field.

For steady-state magnetic phenomena Ampère's observations form a basis for specifying the interaction and defining the magnetic induction. According to Ampère, the force per unit length between two infinitely long, parallel wires separated by a distance d and carrying currents I and I' is

$$\frac{dF_2}{dl} = 2k_2 \frac{II'}{d} \quad (\text{A.4})$$

*From the point of view of special relativity it would be more natural to give current the dimensions of charge divided by length. Then current density J and charge density ρ would have the same dimensions and would form a "natural" 4-vector. This is the choice made in a modified Gaussian system (see the footnote to Table 4, below).

The constant k_2 is a proportionality constant akin to k_1 in (A.2). The dimensionless number 2 is inserted in (A.4) for later convenience in specifying k_2 . Because of our choice of the dimensions of current and charge embodied in (A.1), the dimensions of k_2 relative to k_1 are determined. From (A.2) and (A.4) it is easily found that the ratio k_1/k_2 has the dimension of a velocity squared (l^2t^{-2}). Furthermore, by comparison of the magnitude of the two mechanical forces (A.2) and (A.4) for known charges and currents, the magnitude of the ratio k_1/k_2 in free space can be found. The numerical value is closely given by the square of the velocity of light in vacuum. Therefore in symbols we can write

$$\frac{k_1}{k_2} = c^2 \quad (\text{A.5})$$

where c stands for the velocity of light in magnitude and dimension.

The magnetic induction \mathbf{B} is derived from the force laws of Ampère as being numerically proportional to the force per unit current with a proportionality constant α that may have certain dimensions chosen for convenience. Thus for a long straight wire carrying a current I , the magnetic induction \mathbf{B} at a distance d has the magnitude (and dimensions)

$$B = 2k_2\alpha \frac{I}{d} \quad (\text{A.6})$$

The dimensions of the ratio of electric field to magnetic induction can be found from (A.1), (A.3), (A.5), and (A.6). The result is that (E/B) has the dimensions $(lt\alpha)$.

The third and final relation in the specification of electromagnetic units and dimensions is Faraday's law of induction, which connects electric and magnetic phenomena. The observed law that the electromotive force induced around a circuit is proportional to the rate of change of magnetic flux through it takes on the differential form

$$\nabla \times \mathbf{E} + k_3 \frac{\partial \mathbf{B}}{\partial t} = 0 \quad (\text{A.7})$$

where k_3 is a constant of proportionality. Since the dimensions of \mathbf{E} relative to \mathbf{B} are established, the dimensions of k_3 can be expressed in terms of previously defined quantities merely by demanding that both terms in (A.7) have the same dimensions. Then it is found that k_3 has the dimensions of α^{-1} . Actually, k_3 is equal to α^{-1} . This is established on the basis of Galilean invariance in Section 5.15. But the easiest way to prove the equality is to write all the Maxwell equations in terms of the fields defined here:

$$\left. \begin{aligned} \nabla \cdot \mathbf{E} &= 4\pi k_1 \rho \\ \nabla \times \mathbf{B} &= 4\pi k_2 \alpha \mathbf{J} + \frac{k_2 \alpha}{k_1} \frac{\partial \mathbf{E}}{\partial t} \\ \nabla \times \mathbf{E} + k_3 \frac{\partial \mathbf{B}}{\partial t} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \right\} \quad (\text{A.8})$$

Then for source-free regions the two curl equations can be combined into the wave equation,

$$\nabla^2 \mathbf{B} - k_3 \frac{k_2 \alpha}{k_1} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \quad (\text{A.9})$$

The velocity of propagation of the waves described by (A.9) is related to the combination of constants appearing there. Since this velocity is known to be that of light, we may write

$$\frac{k_1}{k_3 k_2 \alpha} = c^2 \quad (\text{A.10})$$

Combining (A.5) with (A.10), we find

$$k_3 = \frac{1}{\alpha} \quad (\text{A.11})$$

an equality holding for both magnitude and dimensions.

3 Various Systems of Electromagnetic Units

The various systems of electromagnetic units differ in their choices of the magnitudes and dimensions of the various constants above. Because of relations (A.5) and (A.11) there are only two constants (e.g., k_2 , k_3) that can (and must) be chosen arbitrarily. It is convenient, however, to tabulate all four constants (k_1 , k_2 , α , k_3) for the commoner systems of units. These are given in Table 1. We note that, apart from dimensions, the em units and SI units are very similar, differing only in various powers of 10 in their mechanical and electromagnetic units. The Gaussian and Heaviside-Lorentz systems differ only by factors of 4π .

Table 1 Magnitudes and Dimensions of the Electromagnetic Constants for Various Systems of Units

The dimensions are given after the numerical values. The symbol c stands for the velocity of light in vacuum ($c \approx 2.998 \times 10^{10}$ cm/s $\approx 2.998 \times 10^8$ m/s). The first four systems of units use the centimeter, gram, and second as their fundamental units of length, mass, and time (l , m , t). The SI system uses the meter, kilogram, and second, plus current (I) as a fourth dimension, with the ampere as unit.

System	k_1	k_2	α	k_3
Electrostatic (esu)	1	$c^{-2}(l^2 t^{-2})$	1	1
Electromagnetic (emu)	$c^2(l^2 t^{-2})$	1	1	1
Gaussian	1	$c^{-2}(l^2 t^{-2})$	$c(lt^{-1})$	$c^{-1}(tl^{-1})$
Heaviside-Lorentz	$\frac{1}{4\pi}$	$\frac{1}{4\pi c^2}(l^2 t^{-2})$	$c(lt^{-1})$	$c^{-1}(tl^{-1})$
SI	$\frac{1}{4\pi\epsilon_0} = 10^{-7}c^2$ $(ml^3 t^{-4} I^{-2})$	$\frac{\mu_0}{4\pi} \equiv 10^{-7}$ $(mlt^{-2} I^{-2})$	1	1

Only in the Gaussian (and Heaviside–Lorentz) system does k_3 have dimensions. It is evident from (A.7) that, with k_3 having dimensions of a reciprocal velocity, \mathbf{E} and \mathbf{B} have the same dimensions. Furthermore, with $k_3 = c^{-1}$, (A.7) shows that for electromagnetic waves in free space \mathbf{E} and \mathbf{B} are equal in magnitude as well.

For SI units, (A.10) reads $1/(\mu_0\epsilon_0) = c^2$. With c now defined as a nine-digit number and $k_2 \equiv \mu_0/4\pi = 10^{-7} \text{ H/m}$, also by definition, 10^7 times the constant k_1 in Coulomb's law is

$$\frac{10^7}{4\pi\epsilon_0} = c^2 = 89\,875\,517\,873\,681\,764$$

an exact 17-digit number (approximately 8.9876×10^{16}). Use of the speed of light without error to define the meter in terms of the second removes the anomaly in SI units of having one of the fundamental proportionality constants ϵ_0 with experimental errors. Note that, although the right-hand side above is the square of the speed of light, the *dimensions* of ϵ_0 (as distinct from its magnitude) are not seconds squared per meter squared because the numerical factor on the left has the dimensions of μ_0^{-1} . The dimensions of $1/\epsilon_0$ and μ_0 are given in Table 1. It is conventional to express the dimensions of ϵ_0 as farads per meter and those of μ_0 as henrys per meter. With $k_3 = 1$ and dimensionless, \mathbf{E} and $c\mathbf{B}$ have the same dimensions in SI units; for a plane wave in vacuum they are equal in magnitude.

Only electromagnetic fields in free space have been discussed so far. Consequently only the two fundamental fields \mathbf{E} and \mathbf{B} have appeared. There remains the task of defining the macroscopic field variables \mathbf{D} and \mathbf{H} . If the averaged electromagnetic properties of a material medium are described by a macroscopic polarization \mathbf{P} and a magnetization \mathbf{M} , the general form of the definitions of \mathbf{D} and \mathbf{H} are

$$\left. \begin{aligned} \mathbf{D} &= \epsilon_0\mathbf{E} + \lambda\mathbf{P} \\ \mathbf{H} &= \frac{1}{\mu_0}\mathbf{B} - \lambda'\mathbf{M} \end{aligned} \right\} \quad (\text{A.12})$$

where ϵ_0 , μ_0 , λ , λ' are proportionality constants. Nothing is gained by making \mathbf{D} and \mathbf{P} or \mathbf{H} and \mathbf{M} have different dimensions. Consequently λ and λ' are chosen as pure numbers ($\lambda = \lambda' = 1$ in rationalized systems, $\lambda = \lambda' = 4\pi$ in unrationalized systems). But there is the choice as to whether \mathbf{D} and \mathbf{P} will differ in dimensions from \mathbf{E} , and \mathbf{H} and \mathbf{M} differ from \mathbf{B} . This choice is made for convenience and simplicity, usually to make the macroscopic Maxwell equations have a relatively simple, neat form. Before tabulating the choices made for different systems, we note that for linear, isotropic media the constitutive relations are always written

$$\left. \begin{aligned} \mathbf{D} &= \epsilon\mathbf{E} \\ \mathbf{B} &= \mu\mathbf{H} \end{aligned} \right\} \quad (\text{A.13})$$

Thus in (A.12) the constants ϵ_0 and μ_0 are the vacuum values of ϵ and μ . The relative permittivity of a substance (often called the *dielectric constant*) is defined as the dimensionless ratio (ϵ/ϵ_0) , while the relative permeability (often called the *permeability*) is defined as (μ/μ_0) .

Table 2 displays the values of ϵ_0 and μ_0 , the defining equations for \mathbf{D} and \mathbf{H} , the macroscopic forms of the Maxwell equations, and the Lorentz force equation

Table 2 Definitions of ϵ_0 , μ_0 , \mathbf{D} , \mathbf{H} , Macroscopic Maxwell Equations, and Lorentz Force Equation in Various Systems of Units

System	ϵ_0	μ_0	\mathbf{D}, \mathbf{H}	Macroscopic Maxwell Equations	Lorentz Force per Unit Charge
Electrostatic (esu)	1	c^{-2} (l^2l^{-2})	$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$ $\mathbf{H} = c^2\mathbf{B} - 4\pi\mathbf{M}$	$\mathbf{V} \cdot \mathbf{D} = 4\pi\rho$ $\mathbf{V} \times \mathbf{H} = 4\pi\mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}$ $\mathbf{V} \times \mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0$ $\mathbf{V} \cdot \mathbf{B} = 0$	$\mathbf{E} + \mathbf{v} \times \mathbf{B}$
Electromagnetic (emu)	c^{-2} (l^2l^{-2})	1	$\mathbf{D} = \frac{1}{c^2}\mathbf{E} + 4\pi\mathbf{P}$ $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$	$\mathbf{V} \cdot \mathbf{D} = 4\pi\rho$ $\mathbf{V} \times \mathbf{H} = 4\pi\mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}$ $\mathbf{V} \times \mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0$ $\mathbf{V} \cdot \mathbf{B} = 0$	$\mathbf{E} + \mathbf{v} \times \mathbf{B}$
Gaussian	1	1	$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$ $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$	$\mathbf{V} \cdot \mathbf{D} = 4\pi\rho$ $\mathbf{V} \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}$ $\mathbf{V} \times \mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = 0$ $\mathbf{V} \cdot \mathbf{B} = 0$	$\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}$
Heaviside-Lorentz	1	1	$\mathbf{D} = \mathbf{E} + \mathbf{P}$ $\mathbf{H} = \mathbf{B} - \mathbf{M}$	$\mathbf{V} \cdot \mathbf{D} = \rho$ $\mathbf{V} \times \mathbf{H} = \frac{1}{c}\left(\mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}\right)$ $\mathbf{V} \times \mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = 0$ $\mathbf{V} \cdot \mathbf{B} = 0$	$\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}$
SI	$\frac{10^7}{4\pi c^2}$ ($l^2m^{-1}l^{-3}$)	$4\pi \times 10^{-7}$ ($ml^{-2}l^{-2}$)	$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$ $\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$	$\mathbf{V} \cdot \mathbf{D} = \rho$ $\mathbf{V} \times \mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}$ $\mathbf{V} \times \mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0$ $\mathbf{V} \cdot \mathbf{B} = 0$	$\mathbf{E} + \mathbf{v} \times \mathbf{B}$

 Where necessary the dimensions of quantities are given in parentheses. The symbol c stands for the velocity of light in vacuum with dimensions (lt^{-1}).

in the five common systems of units of Table 1. For each system of units the continuity equation for charge and current is given by (A.1), as can be verified from the first pair of the Maxwell equations in the table in each case.* Similarly, in all systems the statement of Ohm's law is $\mathbf{J} = \sigma\mathbf{E}$, where σ is the conductivity.

4 Conversion of Equations and Amounts Between SI Units and Gaussian Units

The two systems of electromagnetic units in most common use today are the SI and Gaussian systems. The SI system has the virtue of overall convenience in

Table 3 Conversion Table for Symbols and Formulas

The symbols for mass, length, time, force, and other not specifically electromagnetic quantities are unchanged. To convert any equation in SI variables to the corresponding equation in Gaussian quantities, on both sides of the equation replace the relevant symbols listed below under "SI" by the corresponding "Gaussian" symbols listed on the left. The reverse transformation is also allowed. Residual powers of $\mu_0\epsilon_0$ should be eliminated in favor of the speed of light ($c^2\mu_0\epsilon_0 = 1$). Since the length and time symbols are unchanged, quantities that differ dimensionally from one another only by powers of length and/or time are grouped together where possible.

Quantity	Gaussian	SI
Velocity of light	c	$(\mu_0\epsilon_0)^{-1/2}$
Electric field (potential, voltage)	$\mathbf{E}(\Phi, V)/\sqrt{4\pi\epsilon_0}$	$\mathbf{E}(\Phi, V)$
Displacement	$\sqrt{\epsilon_0/4\pi} \mathbf{D}$	\mathbf{D}
Charge density (charge, current density, current, polarization)	$\sqrt{4\pi\epsilon_0} \rho(q, \mathbf{J}, I, \mathbf{P})$	$\rho(q, \mathbf{J}, I, \mathbf{P})$
Magnetic induction	$\sqrt{\mu_0/4\pi} \mathbf{B}$	\mathbf{B}
Magnetic field	$\mathbf{H}/\sqrt{4\pi\mu_0}$	\mathbf{H}
Magnetization	$\sqrt{4\pi/\mu_0} \mathbf{M}$	\mathbf{M}
Conductivity	$4\pi\epsilon_0\sigma$	σ
Dielectric constant	$\epsilon_0\epsilon$	ϵ
Magnetic permeability	$\mu_0\mu$	μ
Resistance (impedance)	$R(Z)/4\pi\epsilon_0$	$R(Z)$
Inductance	$L/4\pi\epsilon_0$	L
Capacitance	$4\pi\epsilon_0 C$	C

$$c = 2.997\,924\,58 \times 10^8 \text{ m/s}$$

$$\epsilon_0 = 8.854\,187\,8 \dots \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 1.256\,637\,0 \dots \times 10^{-6} \text{ H/m}$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = 376.730\,3 \dots \Omega$$

*Some workers employ a modified Gaussian system of units in which current is defined by $I = (1/c)(dq/dt)$. Then the current density \mathbf{J} in Table 2 must be replaced by $c\mathbf{J}$, and the continuity equation is $\nabla \cdot \mathbf{J} + (1/c)(\partial\rho/\partial t) = 0$. See also the footnote to Table 4.

Table 4 Conversion Table for Given Amounts of a Physical Quantity

The table is arranged so that a given amount of some physical quantity, expressed as so many SI or Gaussian units of that quantity, can be expressed as an equivalent number of units in the other system. Thus the entries in each row stand for the same amount, expressed in different units. All factors of 3 (apart from exponents) should, for accurate work, be replaced by (2.997 924 58), arising from the numerical value of the velocity of light. For example, in the row for displacement (D), the entry ($12\pi \times 10^5$) is actually ($2.997\ 924\ 58 \times 4\pi \times 10^5$) and "9" is actually $10^{-16} c^2 = 8.987\ 55 \dots$. Where a name for a unit has been agreed on or is in common usage, that name is given. Otherwise, one merely reads so many Gaussian units, or SI units.

Physical Quantity	Symbol	SI		Gaussian
Length	l	1 meter (m)	10^2	centimeters (cm)
Mass	m	1 kilogram (kg)	10^3	grams (g)
Time	t	1 second (s)	1	second (s)
Frequency	ν	1 hertz (Hz)	1	hertz (Hz)
Force	F	1 newton (N)	10^5	dynes
Work	W	1 joule (J)	10^7	ergs
Energy	U			
Power	P	1 watt (W)	10^7	ergs s^{-1}
Charge	q	1 coulomb (C)	3×10^9	statcoulombs
Charge density	ρ	1 C m^{-3}	3×10^3	statcoul cm^{-3}
Current	I	1 ampere (A)	3×10^9	statamperes
Current density	J	1 A m^{-2}	3×10^5	statamp cm^{-2}
Electric field	E	1 volt m^{-1} (Vm^{-1})	$\frac{1}{3} \times 10^{-4}$	statvolt cm^{-1}
Potential	Φ, V	1 volt (V)	$\frac{1}{300}$	statvolt
Polarization	P	1 C m^{-2}	3×10^5	dipole moment cm^{-3}
Displacement	D	1 C m^{-2}	$12\pi \times 10^5$	statvolt cm^{-1} (statcoul cm^{-2})
Conductivity	σ	1 siemens m^{-1}	9×10^9	s^{-1}
Resistance	R	1 ohm (Ω)	$\frac{1}{9} \times 10^{-11}$	s cm^{-1}
Capacitance	C	1 farad (F)	9×10^{11}	cm
Magnetic flux	ϕ, F	1 weber (Wb)	10^8	gauss cm^2 or maxwells
Magnetic induction	B	1 tesla (T)	10^4	gauss (G)
Magnetic field	H	1 A m^{-1}	$4\pi \times 10^{-3}$	oersted (Oe)
Magnetization	M	1 A m^{-1}	10^{-3}	magnetic moment cm^{-3}
Inductance*	L	1 henry (H)	$\frac{1}{9} \times 10^{-11}$	

*There is some confusion about the unit of inductance in Gaussian units. This stems from the use by some authors of a modified system of Gaussian units in which current is measured in electromagnetic units, so that the connection between charge and current is $I_m = (1/c)(dq/dt)$. Since inductance is defined through the induced voltage $V = L(dI/dt)$ or the energy $U = \frac{1}{2}LI^2$, the choice of current defined in Section 2 means that our Gaussian unit of inductance is equal in magnitude and dimensions (t^2I^{-1}) to the electrostatic unit of inductance. The electromagnetic current I_m is related to our Gaussian current I by the relation $I_m = (1/c)I$. From the energy definition of inductance, we see that the electromagnetic inductance L_m is related to our Gaussian inductance L through $L_m = c^2L$. Thus L_m has the dimensions of length. The modified Gaussian system generally uses the electromagnetic unit of inductance, as well as current. Then the voltage relation reads $V = (L_m/c)(dI_m/dt)$. The numerical connection between units of inductance is

$$1 \text{ henry} = \frac{1}{9} \times 10^{-11} \text{ Gaussian (es) unit} = 10^9 \text{ emu}$$

practical, large-scale phenomena, especially in engineering applications. The Gaussian system is more suitable for microscopic problems involving the electrodynamics of individual charged particles, etc. Previous editions have used Gaussian units throughout, apart from Chapter 8, where factors in square brackets could be omitted for the reader wishing SI units. In this edition, SI units are employed exclusively in the first 10 chapters. For the relativistic electrodynamics of the latter part of the book, we retain Gaussian units as a matter of convenience. A reminder of the units being used appears at the top of every left-hand page, with the designation, **Chapter Heading—SI** or **Chapter Heading—G**. Some may feel it awkward to have two systems of units in use, but the reality is that scientists must be conversant in many languages—SI units are rarely used for electromagnetic interactions in quantum mechanics, but atomic or Hartree units are, and similarly in other fields.

Tables 3 and 4 are designed for general use in conversion from one system to the other. Table 3 is a conversion scheme for *symbols and equations* that allows the reader to convert any equation from the Gaussian system to the SI system and vice versa. Simpler schemes are available for conversion only *from* the SI system *to* the Gaussian system, and other general schemes are possible. But by keeping all mechanical quantities unchanged, the recipe in Table 3 allows the straightforward conversion of quantities that arise from an interplay of electromagnetic and mechanical forces (e.g., the fine structure constant $e^2/\hbar c$ and the plasma frequency $\omega_p^2 = 4\pi ne^2/m$) without additional considerations. Table 4 is a conversion table for units to allow the reader to express a given amount of any physical entity as a certain number of SI units or cgs-Gaussian units.