KARAṆASĀRA OF VAṬEŚVARA

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Al-bīrūnī in his book on India has referred to a large number of books by Indian astronomers and mathematicians. Amongst these is a mention of *Karaṇasāra* of Vaṭeśvara. Copies of this book are not available in India. But another book of Vaṭeśvara has been recently discovered and some chapters published. This is *Vaṭeśvara Siddhānta*. From this it is clear that Vaṭeśvara was a follower of Āryabhaṭa although some of the constants given by him are slightly different from those of Āryabhaṭa. Al-bīrūnī quotes from *Karaṇasāra* the method of calculating *adhimāsa* which can be explained only with the help of the main book of Vaṭeśvara now published. Al-bīrūnī also gives the revolutions of the Great Bear according to Vaṭeśvara as given in the *Vaṭeśvara Siddhānta*. He also discusses the method of obtaining the angle subtended at the earth by the shadow of the earth at the orbit of the moon. This topic is not included in the portion of the main book of the Vaṭeśvara now published but is similar to the method first given by Brahmagupta. The birth-place of Vaṭeśvara is not known though in his main book he gives *Śaka* year 802 as the year of his birth. There are some indications that he belonged to Kashmir.

Abu'l Raihan Al-bīrūnī* has a very high opinion of Hindu philosophy, mathematics and astronomy and in his book on India has given accounts of a large number of Indian astronomers and their books on mathematics and astronomy. One of these is *Karaṇasāra* of Vaṭeśvara. Manuscripts of this book have not been so far reported from anywhere in India. On page 156 (Vol. 1) of his book Al-bīrūnī writes:

'Another one by Vittesvara, the son of Bhadatta (? Mihadatta) of the city of Nāgarapura, called *Karaṇasāra*, i.e. that which has been derived from the Karaṇa.'

*Abu'l Raihan Muhammad ibn Ahmad al-Biruni (or Bairuni). Born in Khwarizm (Khiva) in 973; sojourned a considerable time in India, died in 1048, probably at Ghazna in Sijistan (Afghanistan). He was by birth a Persian and a Shiite; his religion was tempered with agnostic tendencies, but his national, anti-Arabic feelings remained very strong until the end. Traveller, philosopher, mathematician, astronomer, geographer, encyclopaedist, one of the greatest scientists of Islam, and, all considered, one of the greatest of all times. His critical spirit, toleration, love of truth and intellectual courage were almost without parallel in mediaeval times. His main works were: (1) the *Chronology of Ancient Nations*, written in 1000, (2) an account of India (*Tarikh al-Hind*), composed in Ghazna *c.* 1030, (3) an astronomical encyclopaedia, (4) a summary of mathematics, astronomy and astrology. He translated several works from Sanskrit into Arabic and, on the other hand, transmitted Muslim knowledge to the Hindus.

It had not been so far possible to know anything regarding this Vittesvara as no book by him had been published or noticed anywhere. M.M.Pt. Sudhākara Dvivedī wrote in A.D. 1892 in his *Gaṇaka Tarañgiṇī* as follows:

'*Yathā brahmaguptenāryabhaṭādināṃ khaṇḍanaṃ kṛtaṃ tathaiva vaṭeśvareṇa svasiddhānte bahutra brahmaguptakhaṇḍanaṃ kṛtamasti asyaiva "kajanmano'ṣṭau sadalāḥ samā yayuri" tyādinā vartamānasya brahmaṇa āyuḥ sārddhavarṣāṣṭakaṃ gatamiti mataṃ asya siddhāntagrantho mayā sampūrṇo na dṛṣṭaḥ gvāliyaramahārājasyāśritasya śribālajyotirvido gehe'yamastīti śrutvā tatrāsakṛtpatraṃ preṣitaṃ parantvadyāvadhi kimapyuttaraṃ na prāptam*'

'As Brahmagupta refuted Āryabhaṭa and others, in the same way Vaṭeśvara in his siddhānta has refuted Brahmagupta in many ways. It is his opinion that eight years and a half of the age of the current Brahmā have already passed. I have not seen the whole of his Siddhānta book. Hearing that there was a copy of it in the house of the astronomer Śrī Bāla attached with the Maharaja of Gwalior, many letters were sent but till now no reply has been received.'

Vaṭeśvara Siddhānta

Some sections of *Vaṭeśvara Siddhānta* have been now published by the Indian Institute of Astronomical and Sanskrit Research, Delhi, and it is possible, as will be shown presently, to identify the author of *Vaṭeśvara Siddhānta* as the author of the book *Karaṇasāra* mentioned by Al-bīrūnī. But it is unfortunate that Pt. Sudhākara Dvivedī did not indicate where he had seen even fragmentary copies of the book as the present edition is based on only one copy which was prepared from one manuscript in the library of the Lahore University and the editing leaves much to be desired.

According to Al-bīrūnī, Vitteśvara (or Vaṭeśvara) was the son of Bhadatta (? Mihadatta). Now in the Persian script Mahadatta can be also read as Mihadatta in the absence of proper diacritical marks or even as Bhadatta if *ma* is taken to be *ba* which is very probable. Fortunately Vaṭeśvara has given his parentage in the very first page of his Siddhānta. He says:

'*Brahmāvanīndubudhaśukradivākarāra - jivārkasūnubhagurūn pitarau canatvā*
Brāhmaṃ graharkṣagaṇitaṃ mahadattasūnurvakṣye'khilaṃ sphuṭamatīva vaṭeśvaro'ham'

'I, Vaṭeśvara, the son of Mahadatta, after having bowed to the God, earth, moon, Mercury, Venus, sun, Mars, Jupiter, Saturn, the constellations, the teachers and my parents, give a clear account of the whole of the Brāhma mathematics of planets and constellations.'

The commentators have explained Brāhma mathematics as the mathematics of Brahmagupta, although it was known to Pt. Sudhākara Dvivedī from what little he had been able to know about Vaṭeśvara's book that he was a great critic of Brahmagupta and had devoted the whole of the tenth chapter of Madhyamādhikār to point out the faults of Brahmagupta. Actually Vaṭeśvara was a follower of Āryabhaṭa and by Brāhma mathematics he alludes to the mathematics propounded by Brahmā, one of the three gods of Hinduism, as has been done by Āryabhaṭa in the last stanza of his book *Āryabhaṭīyam*.[1] But the editors have tried their best to make the constants given by Vaṭeśvara the same as those given by Brahmagupta and his followers as will be clear from the meaning of the following stanza:[2]

'Yadyugotthamiha paryayādikaṃ tadgajābhragaganendu tāḍitaṃ
kalpajaṃ khakhanakhagrahāhataṃ tadbhavetkamalaviṣṭarāyuṣi'

'In this book the revolutions, etc., in a *yuga* (*mahāyuga*) multiplied by one thousand and eight become those to be obtained in a *kalpa* and that (i.e. *kalpa* numbers) multiplied by seventy-two thousand become the numbers in the life of Brahmā.'

First the editors changed *gaja* (8) into *bhuja* (2) and then made it zero and performed all their calculations on this basis. Other examples of this from Madhyamādhikāra have been pointed out by T. S. Kuppanna Shastri.[3]

The Kṣepa Year of Karaṇasāra

More light is thrown on the identity of Vaṭeśvara as the author of *Karaṇasāra* when we consider the astronomical calculations adopted in this book as reported by Al-bīrūnī. Al-bīrūnī complains about the bad translation of the book in his possession and then gives the following quotation:[4]

'He subtracts 821 from the years of the *Śakakāla*. The remainder is the *basis*. This would be the year 132 for our gauge year. He writes down the number in three different places. He multiplies the first number by 132 degrees. The product gives the number 17424 for our gauge-date. He multiplies the second number by 46 minutes, and gets the product 6072. He multiplies the third number by 34 and gets the number 4488. He divides it by 50 and the quotient represents minutes, seconds, etc., viz. 89′ 46″. Then he adds to the sum of degrees in the upper place 112, changing the seconds to minutes, the minutes to degrees, the degrees to circles. Thus he gets 48 circles 358° 41′ 46″. This is the mean place of the moon when the sun enters Aries.

'Further, he divides the degrees of the mean place of the moon by 12. The quotient represents the days. The remainder of the division he multiplies by 60, and adds thereto the minutes of the mean places of the moon.

He divides the sum by 12, and the quotient represents *ghaṭis* and minor portions of time. Thus we get 27° 23′ 29″, i.e. *adhimāsa* days. No doubt this number represents the past portion of the *adhimāsa* month, which is at present in the course of formation.

'The author, in regard to the manner in which the measure of the *adhimāsa* is found, makes the following remarks:

'He divides the lunar number which we have mentioned, viz. 132° 44′ 34″ by 12. Thereby he gets as the *portio anui* 11° 3′ 52″ 50‴ and the *portio mensis* 0° 55′ 19″ 24‴ 10^{iv}. By means of the latter *portio* he computes the duration of the time in which 30 days sum up as 2 years, 8 months, 16 days, 4 *ghaṭis*, 45 *chashaka*. Then he multiplies the *basis* by 29 and gets 3828. He adds thereto 20 and divides the sum by 36. The quotient represents the *ūnarātra* days, viz. $106\frac{8}{9}$.

'However, as I have not been able to find the proper explanation of this method, I simply give it as I find it, but I must remark that the amount of *ūnarātra* days which corresponds to a single *adhimāsa* month is $15\frac{7887'}{10622}$.

It is clear that the first paragraph of the above quotation gives a method of calculating the mean longitude of the moon when the sun just enters the Aries. The second gives the number of *adhimāsa* days in the current year. The third gives the number of *adhimāsa* days per annum and per month and then gives the time after which an *adhimāsa* on the average ought to occur. It also gives a method of calculating the *ūnarātra* days. It will now be shown how the numerical figures given above can be derived from the astronomical constants given in the astronomical books of Āryabhaṭa and his followers. But before doing that it is necessary to point out that due to the copyists the number of *adhimāsa* days is given as 27^d 23 *ghaṭis* and 29 *palas*. It should actually be 29^d 53 *ghaṭis* and 29 *palas*.

In a *karaṇagrantha*, a certain year is taken as the starting year and the positions of planets, etc., are given for that year and then methods are given for calculating these in the succeeding years. Here 821 *Śaka* year has been taken as the starting year and the position of the moon at the end of this year has been given as 112°. Also the number of years after 821 is to be multiplied by certain numbers which can be obtained by the constants given by Āryabhaṭa and astronomers of his school. According to them the moon makes 57753336 sidereal revolutions in 43,20,000 years. It then follows that in one year the moon must have made 13 complete revolutions and be at the point $132° 46\frac{17'}{25}$ in her path. According to Brahmagupta and his school, the moon makes 57753300 revolutions in the same number of years. Hence according to them it will have made 13 complete revolutions and be at the point 132° 46½′ in her path.

Also the *kṣepa* or the additive constant of 112° can only be obtained from the number of revolutions given by Āryabhaṭa and his school. Taking 821 *Śaka* year as the starting point means that 4,000 years of the *Kali* era have elapsed and at the beginning of *Kali* both the sun and the moon had just entered Aries. In 4,000 years, the moon has made 53475 complete revolutions and is at the point 112° in her path. According to Brahmagupta and his school, the moon should have made 53475 complete revolutions and be at the point 100° in her path.

For calculating the number of *adhimāsa* days per annum, one should divide 132° 46′ 40″ 48‴ and not 132° 46′ 34″ by 12 as $46\frac{17'}{25}$ will be equal to 46′ 40″ 48‴. The rest is straightforward calculation.

Regarding the *ūnarātra* days, Al-bīrūnī seems to have missed the point that the *Karaṇa* gives direction only for calculating the number of *ūnarātra* days which are the fractions of a day per year. In 43,20,000 years, the number of omitted days according to Vaṭeśvara are 25082520. Hence in one year the number of omitted days is $\frac{25082520}{43,20,000} = 5\frac{29021}{36,000}$ which differs from $5\frac{29}{36}$ by $\frac{7}{12,000}$. If we neglect the latter the difference in the actual and calculated number of days will be less than one even in one thousand years. Hence Vaṭeśvara recommends that one should multiply the number of years by 5 in one place and in another multiply it by 29 and divide it by 36 and then add the two.

Al-bīrūnī seems to have missed the first part of calculation from his badly translated copy of *Karaṇasāra*. In 4,000 years the additive constant will be $\frac{20}{36}$.

Revolutions of the Great Bear

Another proof of this identity is obtained when we consider the revolutions of the Great Bear. Vaṭeśvara says in his book:[5]

'Kamalaviṣṭaravaktrasaroruhasphuṭagirābhihitā muniparyayāḥ
Ya iha tānapi vacmi yugodbhavān dyucaralabdhavaro bhujago'ṣṭayaḥ'

'I, Vaṭeśvara, who have been given a boon by the planets, state the revolutions of the *munis*, as declared clearly by the lotus-like mouth of Brahmā, to be one thousand six hundred and ninety-two.'

Unfortunately the commentators have dismissed the phrase meaning one thousand six hundred and ninety-two as useless and connected this stanza with the next where the author speaks of the number of revolutions of the apogee of the sun, Mars and Jupiter.

Considering the calculation of the position of Great Bear, as given in *Karaṇasāra*, Al-bīrūnī says, 'Subtract 821 from the *Śaka Kāla*. The remainder is the *basis*, i.e. the number of years above 4000 which have elapsed since the beginning of *Kaliyuga*.

'Multiply the basis by 47 and add 68000 to the product. Divide the sum by 10000. The quotient represents the zodiacal signs and fractions of them, i.e. the position of the Great Bear which was sought.

'The addition of 68000 prescribed in this rule must be the original position of the Great Bear at the beginning of the *basis* multiplied by 10000. If we divide 68000 by 10000, we get the quotient $6\frac{4}{5}$, i.e. six zodiacal signs and twenty-four degrees of a seventh sign.

'It is evident that if we divide the 10000 by 47, the Great Bear has wandered through one zodiacal sign in 212 years, 9 months and 6 days, according to solar time. Accordingly it wanders through one degree of a sign in 7 years, 1 month and 3 days, and through one lunar station in 94 years, 6 months and 20 days.'[6]

Only two other Indian astronomers have spoken of the revolution of the Great Bear. According to Varāhamihira, the Great Bear makes 1600 revolutions in *mahāyuga*. According to Āryabhaṭa II, it makes 15,99,998 revolutions in 1000 *mahāyugas*, i.e. 1599·998 revolutions in a *mahāyuga*. As has been observed by Pt. Sudhākara Dvivedī[7] in his commentary on *Mahāsiddhānta*, the calculation of the position of Great Bear at the time of the reign of Yudhiṣṭhira as given by Varāhamihira in his *Vṛhatsaṃhitā*[8] leads to the result that the Great Bear was in the seventh lunar mansion. According to the figures of Vaṭeśvara also it will lead to the result that the Great Bear will be in the seventh lunar mansion. But according to Varāhamihira the Great Bear was in the tenth lunar mansion. Hence these statements cannot be reconciled with each other. However, the figure 1692 leads to the figures as stated by Albīrūnī.

In 4320000 years, the Great Bear makes according to Vaṭeśvara 1,692 revolutions. Hence in 10,000 years it will pass through $\frac{1692 \times 12}{432} = 47$ zodiacal signs. Also in 4,000 years up to *Śaka* year 821, it will have passed through $\frac{47 \times 4}{10} = \frac{94}{5}$ zodiacal signs, i.e. one revolution and $6\frac{4}{5}$ zodiacal signs which is the additive constant according to Albīrūnī.

Angular Size of the Earth's Shadow

Finally Al-bīrūnī has referred to the determination of the angle subtended at the earth by the shadow of the earth at the orbit of the moon. The method is:[9]

'Multiply the *bhukti* of the moon by 4 and the *bhukti* of the sun by 13. Divide the difference between the two products by 30, and the quotient is the diameter of the shadow.'

Most astronomers, including the *Sūryasiddhānta*, give this angle in terms of the daily angular speeds of the sun and moon. Only Bhāskara I has given it in terms of the angular speed of the moon only. But the others, including *Sūryasiddhānta*, give only different variations of a formula first due to Brahmagupta.[10] Even Brahmadeva in his book *Karaṇaprakāśa*[11] gives the same method although he otherwise follows Āryabhaṭa and Lalla. Bhāskara II in his commentary on *Siddhāntaśiromaṇi* has given a justification for that formula.[12]

It is difficult to say what formula Vaṭeśvara has given in his *Vaṭeśvara-siddhānta*. But we can easily derive his formula from the diameter of the earth and the sun and the distance between the centres of the sun and earth and of the earth and the moon. The diameter of the shadow at the moon is given by the formula:

$$\text{Diameter of shadow} = \text{Diameter of the earth} - \frac{(\text{diameter of sun} - \text{diameter of the earth})}{\text{distance of the sun/distance of the moon}}.$$

Putting down the values, according to Āryabhaṭa, we get the diameter of the

$$\text{shadow} = 1050\ \textit{yojana} - \frac{4410\ \textit{yojana} - 1050\ \textit{yojana}}{5,77,53,336/43,20,000}.$$

To get the angle subtended we have to divide the above by the distance of the moon from the earth which is $2,16,000/2\pi$ *yojanas* and then convert it into minutes. It will then be found that the first term is slightly less than 4/30 of moon's angular speed in minutes and the second term is slightly less than 13/30 of the angular speed of the sun. The angle then can be approximately taken to be that given by Vaṭeśvara, the difference between this value and the actual value being only of the order of 1/3200 of the angular speed of the moon.

Year of Composition of Karaṇasāra

One point which requires consideration is the time when Vaṭeśvara wrote this book *Karaṇasāra*. According to his own statement, he was born in the *Śaka* year 802 and wrote *Vaṭeśvara Siddhānta* at the age of 24, i.e. in the *Śaka* year 826.[13] If he actually wrote *Karaṇasāra* in the year 821 then it would have been written before his bigger and more authentic book. He would have then based the constants entirely on the older books of Āryabhaṭa school and not introduced his own idea about the revolution of the Great Bear and the angle subtended by the shadow of the earth. Also the number of omitted days according to Vaṭeśvara is different from that of Āryabhaṭa. It appears, therefore, that he wrote *Karaṇasāra* later than 826 *Śaka* year and

took 821 as the starting point for the sake of convenience of calculation. But it is difficult to say in which year it was composed unless we can unearth a copy of it either in India or abroad.

Birth-place of Vaṭeśvara

Finally, we have to consider the place which Vaṭeśvara belonged to. At the end of each *adhikāra* of the printed book we are told that Vaṭeśvara belonged to Ānandapura. In the introduction the editors say that Ānandapura is inside the Punjab but then throw some doubt by saying that it is not certain that Vaṭeśvara belonged to this Ānandapura or some other Ānandapura.

Vaṭeśvara cannot definitely belong to this Ānandapura as this place was known as Mākhovāl before Guru Teghabahadur bought it from the hill states in A.D. 1664 and established a *Gurudwara* there and named it Ānandapura. The evidence of *Karaṇasāra* points to the fact that he belonged to Kashmir as he gives the latitude of Kashmir as 34° 9′ which is very nearly the latitude of Śrinagar.[14] Also the name Vaṭeśvara is not very common in the rest of India and we have on the evidence of *Rājataraṅgiṇī* that there was a *Śivaliṅga* of the name of Vaṭeśvara near Srinagar which one of the kings of Kashmir used to worship daily.[15] Also Al-bīrūnī says that he belonged to the city of Nāgarapura. Now names are liable to change a little during the course of one thousand years. But there is a village between Srinagar and Punch of the name of Nāgarapathari, of which the latitude is 33° 55′. This latitude is so very close to 34° 9′ that I am tempted to believe that this was the native place of Vaṭeśvara.

Some Astronomical Constants according to Āryabhaṭa, Brahmagupta, Vaṭeśvara and Sūryasiddhānta

Constants	Value according to Āryabhaṭa	Value according to Brahmagupta	Value according to Vaṭeśvara	Value according to Sūrya-siddhānta
Revolutions of the constellations ..	1582237500	1582236450	1582237560	1582237828
Revolutions of the sun	4320000	4320000	4320000	4320000
Revolutions of the moon	57753336	57753300	57753336	57753336
Number of intercalary months ..	1593336	1593300	1593336	1593336
Number of omitted days	25082580	25082550	25082520	25082252
Angular size of earth's shadow	..	8 lunar motion −25 solar motion / 60	4 lunar motion −13 solar motion / 30	8 lunar motion −25 solar motion / 60

References

1 *Āryabhaṭiyam*, IV, 50.

2 *Vaṭeśvarasiddhānta*, ed. R. S. Sharma and his team. Institute of Astronomical and Sanskrit Research. New Delhi, 1962. Madhyamādhikāra, II, 7.

3 T. S. Kuppanna Shastri, 'The System of Vaṭeśvarasiddhānta', *Indian Journal of History of Science*, Vol. IV (1969), p. 135.

4 *Albīrūnī's India*, Translated and edited by Edward C. Sachau, Vol. II, p. 55.

5 *Vaṭeśvarasiddhānta*, Madhyamādhikāra, I, 15.

6 *Albīrūnī's India*, Vol. I, p. 392.

7 Sudhākara Dvivedī's commentary on *Mahāsiddhānta*, Table of Contents, p. 3.

8 Varāhamihira, *Vṛhatsaṃhitā*, XIII, 3–4.

9 *Albīrūnī's India*, Vol. II, p. 79.

10 Brahmagupta, *Brāhmasphuṭasiddhānta*, ed. R. S. Sharma and his team. Institute of Astronomical and Sanskrit Research, New Delhi, 1962. IV, 6 (ii).

11 Brahmadeva, *Karaṇaprakāśa*, Edited with a commentary by Sudhākara Dvivedi, Benares, 1899. V 2.

12 Bhāskara, *Siddhāntaśiromaṇi*, V 9.

13 *Vaṭeśvarasiddhānta*, Madhyamādhikāra, I, 21.

14 *Albīrūnī's India*, Vol. I, p. 317.

15 Kalhaṇa's *Rājataraṅgiṇī*, I, 194.