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Liberty, Unanimity and Rights

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Two of the more widely used principles in evaluating social states are:

(a) The Pareto principle: if everyone in the society prefers a certain social state to another, then the choice of the former must be taken to be better for for the society as a whole.

(b) Acceptance of personal liberty: there are certain personal matters in which each person should be free to decide what should happen, and in choices over these things whatever he or she thinks is better must be taken to be better for the society as a whole, no matter what others think.

It was argued in Sen (1970a, b) that these two principles conflict with each other in a significant sense—a sense that was precisely described (see (T1) and (T7) in the Appendix). The Pareto principle implies that if more than one person is given the guarantee of having his preference reflected in social preference even over one pair each (no matter how "personal" to him the choice over that pair is), then contradictory cycles may result (e.g., x social preferred to y, y to z and z to x) for some set of individual preferences.

This thesis of the "impossibility of the Paretian liberal" has received searching examinations in a number of recent contributions, e.g. Batra and Pattanaik (1972), Bernholz (1974, 1975), Blau (1975), Blau and Deb (1976), Campbell (1975), Deb (1974), Farrell (1976), Fine (1975), Gibbard (1974), Hammond (1974), Hillinger and Lapham (1971), Karni (1974a, b), Kelly (1976a, b), Ng (1971), Nozick (1973, 1974), Peacock and Rowley (1972), Ramachandra (1972), Rowley and Peacock (1975), Seidl (1975) and Suzumura (1976). While some authors have disputed the existence of the conflict, others have investigated ways of escaping the difficulty, while still others have been concerned with extending this impossibility result to a wider class of social choice problems. Part of the object of this note is to reappraise the question in the light of these contributions, but partly it is also aimed at presenting some additional results which may help to clarify the nature of the conflict and its implications for social choice theory.

The formal statements of conditions and theorems and the proofs have been banished to the Appendix, permitting the text of the paper to be almost completely informal. The underlying issues are, I believe, general enough to be of interest to a wide class of readership in addition to social choice enthusiasts. Also, the discussion is self-contained and a familiarity with the earlier literature on the subject has not been presupposed.

I. RECALL

Some choices between alternative social states may involve differences that are personal to someone; e.g., *with everything else the same* Jack sleeps on his back (x) or on his belly (y). Choices of this type—though formally between alternative social states—may be taken to be the "concern" only of the relevant person. Even if persons other than Jack entertain preferences as to how Jack

should sleep, it seems reasonable to argue that the choice between x and y should be settled by Jack's preference only. Choices of this kind lie in what is sometimes referred to as a person's "protected sphere" (see Hayek, 1960). Such spheres may be taken to be very wide or rather narrow depending on, among other things, our political philosophy, but the existence of some personal protected sphere seems to be widely acknowledged; see, for example, Mill (1859), Hayek (1960) and Gramsci (1971), whose conceptions of liberty differ sharply in other respects.

The existence of such "protected spheres" for all persons was demanded in a weak condition (Condition L), which we may call "weak libertarianism"—an expression that I prefer to my earlier use (Sen, 1970a) of the more ambiguous term "liberalism". Condition L demands that for each person there is *at least* one pair of social states, say, x and y, such that his preference over that pair must be decisive for social judgment; i.e., if he prefers x to y, then x must be acknowledged to be socially better than y and correspondingly if he prefers y to x. The acceptability of Condition L will "depend on the nature of the alternatives that are offered for choice", and "if the choices are all non-personal, e.g., to outlaw untouchability or not, to declare war against another country or not", this condition "should not have much appeal" (Sen, 1970a, p. 83). But in choices involving personal variations of the kind discussed earlier, L would seem to be appealing. Indeed, the terms "liberalism" and "libertarianism" may make the condition look too narrow, since "support for L" may also "come from people who are not 'liberals' in the usual sense" (p. 83). Indeed, such mild endorsement of libertarian values is a common feature of most modern cultures, and of many ancient ones.

The "impossibility of the Paretian liberal" asserts that this Condition L conflicts with the Pareto principle if contradictory cycles of social preference must not arise for any set of individual preferences. Formally, the theorem establishes a conflict between three conditions for social choice, viz. Condition L, Condition P (the Pareto principle) and Condition U ("unrestricted domain", which essentially requires that for no set of individual preferences should the social strict preference involve a cycle). For a rigorous statement of the result and proof of it, the reader should turn to the Appendix (see T1), but, the nature of the conflict can be brought out by an example used earlier (in Sen, 1970a). There is a book (e.g., *Lady Chatterley's Lover*) which may be read by Mr A ("the prude") or Mr B ("the lascivious") or by neither. Given other things, these three alternatives define three social states, a, b and o respectively. Consider now the following possibility. The prude A most prefers o (no one reading it), then a ("I'll take the hurt on myself"), and lastly b ("imagine that lascivious lapping it up"). The lascivious prefers most a ("it will give that lilywhite baby a nice shock"), then b ("it will be fun"), and last o ("what a waste of a good book"). On grounds of individual freedom, since B wants to read the book rather than no one reading it, b is socially better than o; note that *in either case* A does not read the book here. Similarly, since A does not want to read it, o is socially better than a. But a is Pareto superior to b, yielding a preference cycle.

In establishing the "impossibility of the Paretian liberal", condition L can be further weakened, demanding only that at least two persons (not necessarily everyone) should have a protected sphere, i.e. an assigned pair each, over which the person's preference will be reflected in social judgment. This condition was

called Condition L* in Sen (1970a), in which it was given the name "minimal liberalism", but is possibly better called "minimal libertarianism".

In the context of those social choice problems in which there are personal issues, for which Condition L (or L*) makes sense, this impossibility result points towards a serious questioning of the Pareto principle. This was one of the main lessons drawn from (T1) in Sen (1970a), suggesting that the "sense of ethical invulnerability" of the Pareto principle in traditional welfare economics "does not seem to survive a close scrutiny" (p. 200). This may appear puzzling since "unanimity" is a powerful enough reason for a policy to be pursued, but the argument was that it is important "not merely . . . to know who prefers what, but also *why* he has this preference" (p. 83). People may agree on a particular ranking for quite different reasons (as in preferring a to b in the "Lady Chatterley" illustration), and a mechanical use of the Pareto rule irrespective of context seems questionable. One of the issues that will be examined in this note in the light of subsequent contributions is whether this questioning of the Pareto principle stands, since many of these contributions (e.g. Blau, 1975; Gibbard, 1974; Hillinger and Lapham, 1971; Karni, 1974a; Kelly, 1976b) have revealed a preference for resolving the conflict by weakening Condition L rather than by weakening the Pareto principle.

A more general issue in social choice theory is also related to this question. In traditional theory, social choice has been taken to be a function of individual preferences (e.g. formalized as an Arrow-type "social welfare function"). But the argument questioning the Pareto principle can be also used to establish the need to consider other information (e.g. the *motivation* behind those preferences). In this approach, judgments on social choices "would then no longer be a function of individual preferences only" (Sen, 1970a, p. 83). This line of argument is quite distinct from the important radical critique of basing social judgments on individual preferences because of the individual's "alienation from himself" (see particularly Gintis, 1972) and also from the argument for subjecting individual preferences to "rational assessment" (Broome, 1974), but it shares with these approaches a rejection of the refusal to look beyond the set of individual preferences in making a critical assessment of social choices.

Finally, one other aspect of (T1) relates to the possible argument that "the eventual guarantee for individual freedom cannot be found in mechanisms of collective choice, but in developing values and preferences that respect each other's privacy and personal choices" (Sen, 1970a, p. 85). Formally, this points the finger at Condition U (unrestricted domain) on the weakening of which one may base the possibility of realizing libertarian rights without running afoul of the Pareto principle, but more substantially it points to difficulties of taking individual preferences as *given* in pursuing demanding social objectives.

The theorem of "the impossibility of the Paretian liberal" was presented with the intention of raising these questions, and in this note the recent contributions will be examined and some new results presented with a focus on these issues.

II. The Paretian Epidemic

The Pareto principle is sometimes referred to as the "unanimity rule", requiring that preferences unanimously held must be fully reflected in social judgment. This description is somewhat misleading, since the unanimity in

question is not of the whole preference but only over a pair. A less demanding formulation which may be called "unanimity rule" is the following: if everyone has the same preference over the entire set of social states, then social judgment should reflect this preference fully. This unanimity rule—let us call it UR—will yield the Pareto principle if something is required additionally; viz., social preference over any pair must depend only on individual preferences over that pair. This last condition is called, following Arrow (1951), "the independence of irrelevant alternatives" (Condition I). The Pareto principle has this "independence" property, and this takes us beyond the unanimity rule UR.

This "independence" property of the Pareto principle can be used to obtain a rather peculiar result, which will be called the "Paretian epidemic". Define a person as being "decisive" over a pair x and y if and only if, whenever he prefers x to y, x is judged to be socially better than y. He is "decisive both ways" if in addition it is the case that, whenever he prefers y to x, y is judged to be socially better than x. Note that the condition of "weak libertarianism", L, makes each person decisive both ways over one pair of alternatives each, and that of "minimal libertarianism", L*, makes at least two persons thus decisive.

A weaker form of decisiveness is "semidecisiveness". This requires that, if this person prefers x to y, then x is judged to be socially *at least as good* as y. A still weaker requirement is "potential semidecisiveness", which requires that, given certain configurations of individual preferences over *other* pairs, if this person prefers x to y, then x is judged socially at least as good as y, no matter how the others rank x *vis-à-vis* y. Now, it can be proved—see (T2) in the Appendix—that, if social preference cycles are to be avoided no matter what the individual preferences are, then the Pareto principle, even in its weakest form, implies that a person who is decisive both ways over *any* pair of social states whatsoever must be potentially semidecisive both ways for *all* pairs of social states. The Pareto principle corrects any limited outbreak of decisiveness into a veritable "epidemic" of decisiveness, even though in the weaker form of potential semidecisiveness. In the presence of the Pareto principle, no one, it seems, can be given an inch without being given an ell.

The impossibility of the Paretian liberal is an immediate corollary of this more general result of the "Paretian epidemic". If one person is made decisive over one pair, representing a tiny protected sphere of personal choice, then by virtue of the "Paretian epidemic" he is potentially semidecisive over *every* pair of social states. So no one else can be decisive over any pair whatsoever, thereby ruling out the possibility of guaranteeing the libertarian rights of anyone else.

This epidemic property takes stronger shapes if additional conditions such as Arrow's "independence of irrelevant alternatives" and transitivity of strict preference (and not merely the absence of strict preference cycles) are imposed (see (T3), (T4), (T5) in the Appendix). But the pure Paretian epidemic (T2) gets a remarkable amount of mileage from the Pareto principle itself. The limited element of "independence" implicit in the Pareto principle, combined with the inter-pair consistency requirement of always avoiding preference cycles, is sufficient to spread decisiveness of a person from one pair to every pair, albeit in a weakened form.

III. Preference Intensity and Dependent Liberalism

The impossibility of the Paretian liberal (T1) is based on the assumption that

social preference be dependent on individual *preference orderings only* without bringing in *intensity* of preference. In this respect (T1) is similar to Arrow's impossibility theorem. Ng (1971) has proposed getting round the problem through admitting intensity of preference as part of the informational setup; a person's right over his "protected sphere" may be thought to be dependent on the strength of his preference. I am, however, inclined to argue that the notion of a person's "protected sphere" is somewhat at variance with the notion that his right depends on his preferences being sharp. Even if Jack prefers *mildly* that he sleeps on his belly rather than on his back, and busybodies feel *strongly* that he should do the opposite, one can defend on libertarian grounds Jack's right to sleep as he pleases. Ng's way out—logically perfectly feasible—seems to contain the danger of throwing the baby out with the bath water.

Blau (1975) explores an avenue that is also based on bringing in "intensity of preference" but in a way that involves nothing more than individual *orderings*, and it works by comparing the intensity of a person's preference for choices on his own assigned pair *vis-à-vis* the same person's intensity of preferences for choices on someone else's assigned pair:

> *Ordinal intensity:* If a person prefers x to a, a to b and b to y, then his preference for x over y is stronger than his preference for a over b. Furthermore, this is so even if he is indifferent between x and a, *or* between b and y, but not both.

A person is "meddlesome" according to Blau if and only if his preference over the two alternatives in his own assigned pair is weaker than his opposition to someone else's preference over that person's assigned pair.

Blau shows that, if at least one person is not meddlesome in each configuration of individual preferences. then in a two-member community the Pareto principle and Condition L (or L*) cannot lead to a social preference cycle. (Note that in the *Lady Chatterley's Lover* illustration both persons are meddlesome.)

Is this a way out of the problem of the Paretian liberal? Blau notes that it isn't so for an n-person community, since cycles can still arise based on Paretian and libertarian rankings if there are three or more persons; but for a two-person community it works. Blau proposes a "modified liberalism", which makes the libertarian rights conditional on preferences being non-meddlesome. Under various alternative versions of his "modified liberalism", the liberal privileges are withheld (a) for all if everyone's preference is meddlesome (SL'), (b) for all if someone's preference is meddlesome (WL'), and (c) for those whose preferences are meddlesome (L'). In the two-person case conflicts occur—as noted before—only if both are meddlesome, and thus all the versions of modified liberalism lead to the same conclusion, viz. withhold libertarian privileges from both. We are left with only the Pareto principle and there is no consistency problem for social decision.

But is this an acceptable solution? Since libertarian values come into their own in defending personal liberty against meddling, one can argue that the presence of meddling makes libertarian values more (*not* less) important. If everyone meddles in the sense of Blau, surely libertarian values should demand that the meddling part of each person's preferences be ignored but the non-meddling parts dealing with one's own affairs be defended against other people's meddling. Indeed, the ingenious consideration of meddlesomeness, so well discussed by Blau, seems to lead naturally to a critique of the part of the individ-

ual preferences incorporated in the Pareto relation rather than of the part incorporated in the personal rights, i.e. precisely the opposite of what Blau proposes.

Consider four alternative social states $\{x_1, y_1, x_2, y_2\}$ with $\{x_1, y_1\}$ being person 1's assigned pair and $\{x_2, y_2\}$ being person 2's pair. Let person 1 prefer x_1 to y_1, and person 2 be meddlesome by ordering the four alternatives in the strict descending order: y_1, x_2, y_2, x_1. No one denies 2's right to rank x_2 and y_2 as he likes. If we are upset about his being meddlesome, surely the object of our wrath should be his preference for y_1 over x_2, or for y_2 over x_1, or both. And these are precisely the pairs over which 2's preference gives muscle to the Pareto relation. The same applies to 1; he can be meddlesome by ordering: y_2, x_1, y_1, x_2. No one denies his right to prefer x_1 to y_1, but since he also prefers y_2 to x_1 and y_1 to x_2, he is meddlesome. Once again the finger points towards precisely the same two pairs on which person 1's preference—like person 2's—gives the Pareto relations their content.

We can divide person 2's rankings in the ordering $\{y_1, x_2, y_2, x_1\}$ into three ordered pairs, viz. a "self-regarding" ordered pair $\{x_2, y_2\}$ and two "non-self-regarding" ordered pairs $\{y_1, x_2\}$ and $\{y_2, x_1\}$. If person 1 happens to prefer x_1 to y_1 then this overall order of person 2 is meddlesome. Given that, we might decide to follow one of the following three alternative ways of discounting meddlesome 2's preference ordering:

(a) ignore his entire ordering;
(b) ignore his ordering of non-self-regarding pairs;
(c) ignore his ordering of the self-regarding pair.

It would seem rather natural to follow (a) or (b), whereas Blau follows (c), whereby the preference is ignored precisely over the pair on which the person in question can be hardly accused of being meddlesome.

The dispute can be illustrated in terms of the following example, which will be called the *work choice case*. Let persons 1 and 2 each have a part-time job, and suppose the possibility arises of a full-time job being available. Each prefers more of a job to less (i.e. 1 to $\frac{1}{2}$ to 0) given the job situation of the other, but prefers that the other should be jobless (i.e. 0 to $\frac{1}{2}$ to 1 for the other), spoilt as they are by the competitive society in which they live. In fact, they are each "meddlesome" enough to attach greater importance to the other being jobless than to their own job situation. Consider now four alternative possibilities with the first number standing for person 1's job state, while the second for person 2's: $(1, \frac{1}{2})$, $(0, \frac{1}{2})$, $(\frac{1}{2}, 1)$ and $(\frac{1}{2}, 0)$. On grounds of one having the right to work if one wishes to, no matter what others want, the choice over $(1, \frac{1}{2})$ and $(0, \frac{1}{2})$ may be assigned to person 1 and that over $(\frac{1}{2}, 1)$ and $(\frac{1}{2}, 0)$ to person 2, since the job of the other person in each case is unaffected. This will lead to either $(1, \frac{1}{2})$ or $(\frac{1}{2}, 1)$ as the solution, eliminating $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ on libertarian grounds.

Let the two persons have the following strict orders, for reasons mentioned above:

Person 1	*Person* 2
$(\frac{1}{2}, 0)$	$(0, \frac{1}{2})$
$(1, \frac{1}{2})$	$(\frac{1}{2}, 1)$
$(0, \frac{1}{2})$	$(\frac{1}{2}, 0)$
$(\frac{1}{2}, 1)$	$(1, \frac{1}{2})$

Both are "meddlesome" in the sense of Blau. On grounds of each of the alternative versions of Blau's "modified liberalism" (WL′, SL′ and L′), the liberal privilege will be withheld from each. On grounds of the Pareto principle, the choice of $(1, \frac{1}{2})$ or $(\frac{1}{2}, 1)$ should be avoided since both are Pareto-inferior, and the choice should be confined to $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$. But this amounts precisely to permitting the meddlesome parts of the two persons' preferences to hold sway. Left to himself, person 1 will prefer to work more, i.e. will choose $(1, \frac{1}{2})$ over $(0, \frac{1}{2})$, and left to himself person 2 will prefer to work more also, i.e. will choose $(\frac{1}{2}, 1)$ over $(\frac{1}{2}, 0)$, despite meddling by the other in each case, and Blau's solution amounts to eliminating the non-meddlesome part of one's preference and retaining the influence of meddling.

A more appropriate solution would seem to be to respect the self-regarding or the non-meddling parts, viz. 1's preference for $(1, \frac{1}{2})$ over $(0, \frac{1}{2})$ and 2's for $(\frac{1}{2}, 1)$ over $(\frac{1}{2}, 0)$, and to ignore the non-self-regarding parts and the Pareto relations based on them.

Blau is, of course, both right and illuminating in asserting the role of "independence" in these impossibility theorems, which is "the main thesis" of his paper. He has also taken me to task for claiming that in (T1) I had not "imposed Arrow's condition of 'the independence of irrelevant alternatives'" (Blau, 1975, p. 395). He points out that the impossibility result makes substantial use of independence properties. This is indeed so, but the fact is that these independence properties were used only *in so far as* they were already incorporated in the Pareto condition and in the liberal condition in determining social choices over a pair on the basis of individual preferences over that pair under special circumstances. There was no need to impose Arrow's condition of "independence of irrelevant alternatives" as such—a much more demanding condition, about the pervasive implications of which Blau (1957, 1971) himself has made us so aware. Incidentally, the Paretian epidemic discussed in the last section also exploits the *implicit* independence element in the Pareto principle.

Our difference does not lie in our respective recognitions of the role of independence, but on precisely *how* to bring in "non-independent" considerations into the decision. Blau would like to sacrifice personal right (based on independence), retaining Pareto (also based on independence), whereas it appears that in many circumstances (as illustrated above) being prepared to go against the Pareto principle is at least as reasonable. An alternative will be to follow approach (a) above, which will remove the sanctity of both the Pareto principle as well as of liberal privileges when preferences are "meddlesome" in the sense of Blau.

IV. Alienability of Rights

Blau's "modified liberalism" gives each person a right that is conditional on non-meddlesome preferences. This is an example of libertarian rights being treated as alienable. Gibbard (1974) has developed a rights system that makes libertarian rights alienable if it conflicts with other people's libertarian rights or the Pareto principle. A rights system is a set of ordered triples $\{x, y, i\}$ where $\{x, y\}$ is assigned to person i. *Under ordinary circumstances* person i has his way over $\{x, y\}$ in the sense that, if he prefers x to y, then x is judged socially better than y. But this right can be waived if others beside i "claim their rights" to z over x, and person i himself regards y at least as good as z. Others can claim their

rights to z over x if and only if there is a sequence of strict preferences: z over a_1, a_1 over $a_2, \ldots, a_{m-1}$ over a_m and a_m over x, such that the ranking over each pair is derived either from Pareto preference or from the preference of someone other than i to whom the relevant pair has been assigned under the rights system. This weakening of the libertarian requirements makes it consistent with the Pareto principle for a social decision function with unrestricted domain. Gibbard proceeds to show that these alienable rights are rights in a very stringent sense—waived only under very special circumstances. (For formal statements, see Gibbard, 1974. He has also provided a deep and penetrating analysis of formulations of the libertarian principle identifying some formulations which may turn out to be self-contradictory; on this see Section X below.)

There are, it seems to me, two ways of viewing Gibbard's system of alienable rights. One takes off from his observation that under certain circumstances a right may be "useless" to a person (Gibbard, 1974, p. 398) when by exercising it the person may end up at no better a position than by not exercising it, and this leads to a dichotomy between the *existence* of rights and the *exercise* of these rights. This I shall call the "pragmatic interpretation". The other starts off from Gibbard's concern with "how the conflict *ought* to be resolved" (p. 398, italics added) and basing a system of rights on that consideration. This may be called the "ethical interpretation".

The pragmatic interpretation has been penetratingly explored recently by Kelly (1976b), who seems to take that to be the only interpretation. He identifies a number of difficulties with Gibbard's system, on this interpretation, essentially arising from problems in deciding *when* a right is useful for a person. (Kelly also discusses the "very heavy demands on the information structure", viz., "not only must each individual know all his rights as well as his preference ordering, he must know the preference orderings of all other individuals and must know all right assignments" (Kelly, 1976b, pp. 5–6 of manuscript). Some of these difficulties are eliminated by modifications of the Gibbard system proposed by Kelly, but a basic problem of "correctable miscalculation" remains. The difficulty arises from each person deciding what right is "useless" for him on the basis of some presumption as to what rights the others would exercise, but one person's decision not to exercise his right (on the supposed ground of its being "useless" when others exercise their rights) renders erroneous another person's conviction that his right is "useless" (based on that person's assumption that others will exercise their rights). This problem of interdependence and of "correctable miscalculation" proves to be a deep one for the "pragmatic interpretation" of the Gibbard system (see especially Kelly, 1976b, Theorem 3). I shall not go into this interpretation anymore, since it is the "ethical interpretation" of the Gibbard system that seems to me to be the relevant one for analysing libertarian ethics in general and "the Pareto-consistent libertarian claim" in particular.

The ethical interpretation of the Gibbard system of alienable rights appears to be open to the same type of criticism as Blau's "modified liberalism". When meddling in each other's affairs causes a cycle involving the Pareto principle and personal rights, the axe in the Gibbard system falls invariably on personal rights (based on the "self-regarding" part of a person's preference), leaving intact the effectiveness of the Pareto rule (based on the "non-self-regarding" parts of people's preferences). Consider, for example, the "work choice case" discussed

in the last section. Each person prefers full-time work to being unemployed (given the other person's half-time job), and let this be the assigned pair of each over which the Gibbard system gives him an alienable right. Does person 1, in fact, end up getting the society to respect his rights of choosing $(1, \frac{1}{2})$ over $(0, \frac{1}{2})$? The answer is no, since this right is "waived" in the Gibbard system because of the fact that person 1 prefers $(0, \frac{1}{2})$ to $(\frac{1}{2}, 1)$, person 2 has an alienable right to choose $(\frac{1}{2}, 1)$ over $(\frac{1}{2}, 0)$, and $(\frac{1}{2}, 0)$ is Pareto-preferred to $(1, \frac{1}{2})$ thanks to their jealousy of each other. Similarly, person 2's right to full-time work is "waived". Given the Pareto rule—respected in the Gibbard system—social choice is confined to $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$, and the conflict with individual rights is eliminated by "waiving" these rights.

It is not my intention to argue that, even if people have meddlesome preferences, as both these people seem to have, their preferences should continue to be fully respected, but to suggest that in some cases the "waiving" should deal not with the unmeddlesome "self-regarding" part of the preference, but with the "non-self-regarding" parts. The right to be counted in to give the Pareto relation its muscle is a right too, and in situations such as this at least as strong a case can be made for "waiving" that right as for the Gibbard solution of waiving the right to choose over one's "assigned pair" unaffected by the other person's meddling. In this alternative system of "alienation", the mechanical use of the Pareto principle must go, leading in this case to the choice of $(1, \frac{1}{2})$ or $(\frac{1}{2}, 1)$, rather than $(0, \frac{1}{2})$ or $(\frac{1}{2}, 0)$.

Gibbard gives relatively little space to the justification of his ingenious system of rights, though he does establish, in my judgment convincingly, that the "extreme fears" that "a person's right on an issue would usually be waived" are "groundless" (Gibbard, 1974, p. 403). But that still leaves us with cases of waiving that are not easy to justify. Motivationally, Gibbard seems to proceed from a particular case (his "Angelina–Edwin case") in which to him "it is plain . . . how the conflict ought to be resolved", and then in his system "to generalize the moral of the example" (p. 398). But what is "plain" in one case need not be in another, as is apparent from the "work choice case", which also arises—as does the "Angelina–Edwin case"—from "one person's taking a perverse interest in the affairs of another" (p. 398).

Consider, now, Gibbard's own case. "Angelina wants to marry Edwin but will settle for the judge, who wants whatever she wants. Edwin wants to remain single, but would rather wed Angelina than see her wed the judge." Denoting "Edwin weds Angelina" as e, "the judge weds Angelina and Edwin remains single" as j and "both Edwin and Angelina remain single" as o, we have the following preference orders (in strict descending order):

Angelina: e, j, o
Edwin: o, e, j.

Gibbard takes (j, o) to be Angelina's assigned pair ("Angelina has a right to marry the willing judge instead of remaining single") and (o, e) to be Edwin's assigned pair ("Edwin has the right to remain single rather than wed Angelina"). Edwin's right is "waived" in the Gibbard system of alienable rights since Edwin prefers e to j (he is ready to marry Angelina to prevent her from marrying the judge), and Angelina "claims her right" to marry the judge rather than remaining single, preferring j to o. With Edwin's right to remain single "waived",

Edwin and Angelina would seem to be heading towards a conjugal life since both prefer *e* to *j*. "Left freely to bargain away their rights . . . Edwin and Angelina would agree to the outcome: wedding each other" (p. 398).

The appeal of this solution in this particular example lies, I would argue, not merely in the preferences specified, but also on what we presume to be the motivation underlying the preferences as described by Gibbard. Let us consider a different interpretation of the orderings. Angelina loves the judge—truly—and would have preferred most to marry him but for her fury at being scorned by the unwillingness of Edwin ("oh, I hate him!") to marry her ("I *will*, Edwin, just you see!"), and hence her strict order: *e*, *j*, *o*. Edwin hates Angelina's guts ("in so far as she has any"), and knowing that she will be very happy married to the judge, he would do anything to stop her, even—if need be—himself marrying her ("that will teach her all right"), and hence his strict order: *o*, *e*, *j*. While Gibbard makes arrangements for the wedding of Edwin and Angelina, Edwin can do worse than recite: "I don't want to wed Angelina and have a right not to—I won't let Gibbard 'waive' it; and to stop Angelina from getting happiness married to the judge is none of my bloody business, and my perverse preference on this should not really affect whether they marry or not." One can, indeed, in such a situation make a case for respecting Edwin's right not to wed Angelina, but not attach great social importance to his views on whether Angelina should marry the judge.

My point is not that the above reflects a more natural interpretation of the preferences of Angelina and Edwin, but that these preference orderings are consistent with quite different interpretations, and without going into the motivations it may *not* be "plain in this case how the conflict ought to be resolved". (Fine (1975) has pointed out that, judged purely as orderings, the configuration of preferences assumed in proving (T1) is very similar to that of the "Prisoners' Dilemma", though the lessons drawn are quite different.)

The fundamental issue really is whether individual preference orderings alone provide enough of a basis for a social judgment without going into the causation of and the motivation behind these preferences (see Section I). While there undoubtedly do *exist* cases where the optimal solution may involve waiving a person's libertarian rights—Gibbard is convincing on this existential proposition though not, in my judgment, in its generalization in the form of his system of rights—there also exist cases where the optimal solution involves waiving the Pareto principle. Principles that take account of nothing other than what individual preferences happen to be, however superficially appealing (and the Pareto principle is certainly appealing) are essentially "non-basic" (see Sen, 1967; 1970a, Chapter 5). To axe invariably personal rights over assigned pairs and never the Pareto principle, when they conflict, as Gibbard's system does, seems to me to be hard to justify.

V. Other Weakenings of Condition L

While Blau's and Gibbard's methods consist of accepting the basic idea behind Condition L but qualifying its scope in terms of preferences over other pairs, some other authors have proposed ways of avoiding the dilemma by declaring Condition L to be essentially inappropriate. Hillinger and Lapham (1971) argue that Condition L has got very little to do with liberalism as they see it, and claim that "whenever the choices of one individual impinge on the

welfare of others there is no general presumption in favour of freedom of individual choice" (pp. 1403–1404). This makes "liberalism" immediately consistent with the Pareto principle, since person *i*'s right to having his way over any pair depends on non-opposition by others over that pair itself. Everyone's right to do anything whatsoever is made conditional on non-opposition by others, and one does not see much trace of liberal values in Hillinger and Lapham's modified liberalism, consistent though it is with the Pareto principle.

Hillinger and Lapham (1971, p. 1405) argue: "In conditions of interdependence, we cannot conceive of any 'principle of liberalism' which would govern what actions are to be left to individuals independently of the majority preference of the individuals concerned". Rowley and Peacock (1975), writing on the same subject, "cannot endorse such a judgment", but agree that "Sen has grossly misinterpreted the liberalist philosophy in his condition L* as was noted by Hillinger and Lapham" (pp. 82–83). They view liberalism as "concerned essentially though not exclusively, with the maintenance and extension of individual freedom, defined as that condition of mankind in which coercion of some individuals by others is reduced to the minimum possible degree" (p. 78). Despite lengthy (and otherwise interesting) discussion, they never seem to take note of the fact that, in the type of situations considered, a denial of Condition L* would involve precisely the loss of the guarantee of even a *minimal* element of "individual freedom".

It is worth examining Karni's (1974a, b) interesting reformulation of liberalism. Essentially, he proposes that the libertarian right be weakened to being semi-decisive over a pair rather than decisive; i.e., if the pair containing x and y is assigned to a person, then whenever he strictly prefers x to y, x is judged to be socially at least as good as y. This makes person i semi-decisive over this pair x, y. This weakened libertarianism is consistent with the Pareto principle in the sense of not producing strict preference cycles, even though the social preference thus generated will involve intransitive indifference. Social *strict* preference may be transitive, but if so, all Pareto non-comparable states may have to be declared socially indifferent (a direct corollary of Lemma 5*f in Sen, 1970), which will be a very peculiar system indeed.

An even more serious difficulty with Karni's method of resolution lies in the fact that i being semi-decisive for x against y guarantees only that *x is chosen from that pair* (though not necessarily uniquely), but it does not prevent y from being chosen with x rejected from *larger* sets containing both x and y. In the *Lady Chatterley's Lover* illustration, under Karni's rule Mr B will be able to read the book if the choice is between his reading it and no one reading it (both alternatives will be judged equally acceptable), but not if the possibility of Mr A reading it is also included. Indeed, thanks to the Pareto principle, kept intact by Karni, Mr B will definitely not be able to read the book when the choice is over the three alternatives. Thus semi-decisiveness over a pair is a very limited right indeed, and it will be a very tame libertarian who will settle for it.

Kelly (1976a) has explored the possibility of introducing information on interpersonal comparisons of welfare into the framework and has used Suppes' (1966) "grading principle of justice" in restricting the scope of Condition L. Under his "weak just liberalism", at least two persons have the right to be decisive both ways over one pair each *provided* no one else believes that the opposite preference over that pair will reflect justice in the sense of Suppes.

Kelly demonstrates that, even with this restriction, Condition L conflicts with the Pareto principle (and with Suppes' grading principle of justice, applied over other pairs) for a possible set of extended orderings incorporating views on interpersonal comparisons (see Theorem 3). Kelly's "impossibility of a just liberal" is an important extension of the problem of the Paretian liberal, and as he points out the result demonstrates that, for this class of impossibility results, introducing interpersonal comparisons is not much of a cure (in contrast with the impossibility results of the Arrow type).

Condition L has been subjected to serious scrutiny also by Seidl (1975). He criticizes the existential form in which Condition L is defined, and points out that "it is perfectly consistent with Sen liberalism, if individual j is socially decisive on the regime of the society (say, whether monarchy or republic should obtain), whereas individual k is socially decisive with respect to his sleeping on back or belly" (p. 279). This is certainly so, but the result of defining liberalism in the existential form is that, if it is denied, then j loses his right to decide singlehanded on the "regime" (this loss will not be regretted) *as well as* his right to sleep as he likes (this loss will be regretted). Wanting decisiveness over "some" pair is, of course, weaker than demanding it over a *specified* pair, and since the object was to show an inconsistency, there was something to be said for choosing as weak a condition as possible. But Seidl (1975) is clearly right that a constructive study of liberalism requires us to go more into the nature of the alternatives involved, and here his investigation of "technological separability" clarifies what kind of choices can be reasonably put under a person's "protected sphere". Bernholz (1974, 1975) discusses a similar issue. However, Bernholz's (1974) assertion that "the rule of liberalism generally gives only the right to decide among certain alternative measures or actions belonging to certain issues" and not "among social states" (p. 101) would seem to be based on a misunderstanding of the type of space on which these preferences are formulated. Given the rest of the world Ω, Jack's choice over the "measure" of sleeping on his back and that of sleeping on his belly *is* a choice over two "social states". Seidl's and Bernholz's discussions do not affect the impossibility result, but are helpful in clarifying the types of choices over which such a conflict can occur.

Osborne (1975) also objects to the existential form and argues that Condition L permits "a person to govern on a pair belonging to another person's protected sphere" (p. 1286). "Sen's Condition L is as consistent with universal busy-bodiedness as with liberalism." True enough, in the sense that both imply Condition L. The consequence of this, however, is not that the impossibility of the Paretian liberal does not hold, but that, for a social decision function with unrestricted domain, more than one person cannot be permitted to determine the choices irrespective of others' preferences *even* in their *own* "protected spheres" (and not on others—a consequence that also holds but is hardly disturbing). "The impossibility of the Paretian busy-body" to which Osborne refers does not in any way reduce "the impossibility of the Paretian liberal", given the *existential* form of Condition L.

Osborne's rebuttal of the inconsistency result is based on an unadulterated piece of logical error:

> The weak Pareto principle operates only in case of unanimity; . . . in that case the liberal principle is empty. On the other hand the liberal principle is forceful only

> in cases of certain kinds of disagreement; and in those cases the Pareto principle is silent. Thus when the one is binding, the other is empty or silent. *If that is true they cannot possibly be inconsistent.* (Osborne, 1975, p. 1286, italics added.)

The Pareto principle can operate over one or more pairs (without conflicting with the liberal principle over *those* pairs) and the liberal principle can operate over two or more other pairs (without conflicting with the Pareto principle over *those* pairs), and these choices *together* can be inconsistent. The theorem of the impossibility of the Paretian liberal is based on such interpair inconsistency (as indeed are other "impossibility" theorems in this field, including Arrow's famous one).

VI. Rights as Constraints

In contrast with the weakenings of Condition L discussed in the preceding sections, Nozick (1973, 1974) suggests a way out of the dilemma that gives liberal rights priority by making social choice constrained by the exercise of these rights.

> The trouble stems from treating an individual's right to choose among alternatives as the right to determine the relative ordering of these alternatives within a social ordering. . . . A more appropriate view of individual rights is as follows. Individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. Within the constraints of these fixed features, a choice can be made by a social choice mechanism based upon a social ordering, if there are any choices left to make! Rights do not determine a social ordering but instead set the constraints within which a social choice is to be made, by excluding certain alternatives, fixing others, and so on. . . . If entitlements to holdings are rights to dispose of them, then social choice must take place within the constraints of how people choose to exercise these rights. If any patterning is legitimate, it falls *within* the domain of social choice, and hence is constrained by people's rights. *How else can one cope with Sen's result?*" [Nozick, 1973, pp. 60–61; 1974, pp. 165–166]

This neat solution of the problem is indeed attractive, since the conflict between the Pareto rule and the liberal principle is resolved by giving them two quite different roles; viz., the former determines a strict partial ordering with which the social ordering has to be consistent, and the latter restricts the choice situations over which the social ordering is to be applied. In the "work choice case" discussed in Section III, Nozick's solution—unlike Blau's, Gibbard's, Hillinger and Lapham's, Karni's, etc.—would amount to social choice of $(1, \frac{1}{2})$ or $(\frac{1}{2}, 1)$, and our criticism will not apply. The same would be true in the *Lady Chatterley's Lover* case.

There is, however, a problem of interpretation of a social ordering. It can be taken either to be purely a mechanism for choice, or as reflecting a view of social welfare. In the latter interpretation to say xPy (x is "socially preferred" to y) is to assert that in one's judgment society is better off with x than with y, and vice versa. In the "work choice case", if someone—an outsider *or* even one of the persons involved taking an "ethical" view—tries to decide what is the best solution, does he eliminate $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ by saying that, while these are socially better than $(\frac{1}{2}, 1)$ and $(1, \frac{1}{2})$ respectively (thanks to Pareto), they *cannot* be chosen since 1 and 2 have exercised their rights by eliminating them from social choice (as one would do under Nozick's solution)? Or should he assert that the choice of $(\frac{1}{2}, 1)$ or $(1, \frac{1}{2})$ would in fact be socially better, despite the

Pareto preference to the contrary, since the Pareto preference is based on preference rankings that deserve to be ignored (on some grounds, e.g. meddlesomeness)? Like the other authors, Nozick does not seem to dispute the acceptability of the Pareto ranking as a sufficient condition for higher social welfare, but eliminates its impact by excluding the Pareto-superior alternatives from social choice on grounds of rights.

The difference can be brought out in terms of Nozick's example: "If I have a right to choose to live in New York or in Massachusetts, and I choose Massachusetts, then alternatives involving my living in New York are not appropriate objects to be entered in a social ordering" (Nozick, 1973, p. 62). But one can also argue that, if I believe that it is a better society which—given other things—lets Nozick decide where he wishes to live, then I must *assert* that it is socially *better* that Nozick should be permitted to live in Massachusetts as desired by him. If Nozick is forced out of Massachusetts, then one would wish to say not only that Nozick's rights have been violated, but that society is worse off—given other things—by stopping Nozick from living where he wishes.

I do not wish to enter here into a more general discussion of Nozick's ingenious theory of rights as such, or of his analysis of the role of the state. Nozick's (1974) system of rights does, of course, have an exceptionally wide coverage, while conditions L and L* demand very little; but in either case acknowledging certain rights would seem to have consequences on our judgments on what is socially good. It is not a matter of unconcern, in making pronouncements on what is socially good and what not, to examine whether the rights acknowledged can be exercised freely. There is a clear sense in which Nozick's ability to choose to live where he pleases *is* in "the domain of social choice".

Thus the dichotomy with the help of which Nozick solves the conflict, permitting support for personal rights without conflict with the Pareto principle, can lead to fresh problems. It is certainly possible to follow Nozick in defining a "social ordering" without taking note of acknowledged rights, but if a social ordering is supposed to reflect a judgment of social welfare taking everything into account, then the ability to exercise these rights must enter the "social ordering" after all. One can use the approach of having a "ranking of rankings", as discussed in Sen (1974) (see also Nozick, 1968, and Jeffrey, 1974), in which a "higher" ranking incorporates these rights while a "lower" ranking does not. But at some stage, i.e. with a "higher" ranking, the conflict with the Pareto principle will surface again, and in the latter ordering we shall have to go against some Pareto relations in the cases discussed if it is accepted that it is a better social state which incorporates the exercise of these personal rights.

In order to avoid an ambiguity, I should make it clear that it is not my contention that Condition L, or some strengthening of it in that general line, can catch whatever there is to catch in our conception of rights in general, and of liberty in particular. Condition L does focus on the end-state, and it may be important from a libertarian point of view to ensure not merely that the consequences corresponding to the desires of the persons in question take place, but also that these consequences are brought about in the right way. For example, from a libertarian point of view it may not be sufficient to distinguish only between Nozick's remaining in Massachusetts (x) and his going to New York (y). Even though it may be known by everyone that Nozick would prefer

to live in Massachusetts, it can be argued that an order served on him to stay in Massachusetts will be an infringement of his liberty. A distinction may be made between there being such an order and Nozick living in Massachusetts (x_1), and there being no such order and Nozick living in Massachusetts (x_2). A libertarian may well prefer x_2 to x_1, even though the order does not have any consequence on where Nozick lives, and the libertarian position would seem to include "non-consequentialist" features (on the structure of "consequentialism", see Williams, 1973, pp. 82–93). This does not, however, contradict that a libertarian would find y, i.e. Nozick's living in New York (and in the case described this can happen only if he is forced) to be worse than *both* x_1 and x_2. If y is the outcome, then it is sufficient ground for concluding that the libertarian principle has been violated, and that is all that is needed for the "impossibility of the Paretian liberal". There is no need to deny that libertarian ethics might also involve other elements as long as it incorporates, *inter alia*, Condition L or L*. (This is so independently of the fact that in this case x_1 and x_2 can also be treated as two separate states, and there is no obligation to treat them as socially equivalent in our formulation.)

VII. Preference Amendment and the Liberal Partition

Farrell (1976) has explored two rather different avenues of ensuring consistency in social choices involving the exercise of liberal rights. The first takes the form of "amending" a person's preference so that "he is deemed indifferent between any pair of states for which some other individual is to be decisive" (p. 12), which amounts "in spirit" to ignoring a person's strict preference on choices that are "none of his business". This way of avoiding a conflict raises interesting problems of consistency, which have been thoroughly investigated by Farrell, and he outlines a method of moving from true preferences $\{R_i\}$ of the individuals to "amended" preferences $\{R_i'\}$ such that each R_i' is also an ordering, and each individual can be assigned (many) decisive pairs without running into social preference cycles (taking the Pareto relation as defined by the *amended* preferences).

Farrell points out several objections to this procedure, including the basic objections to "deeming" a person's preference to be different from what it is, and the particular result that the Pareto relation on the "amended preferences" may be the exact opposite of the Pareto relation on the true preferences. While Farrell finds these objections "overwhelming", I am not so sure that they are. "Amending" preferences in the Farrell system is essentially like ignoring a person's "meddlesome" preferences (defined more widely than Blau's) and spelling out the consequences of this for the rest of the preference ranking to preserve its ordering character. As we argued in Sections III and IV, there exist situations in which the case for violating the Pareto principle is strong. Indeed, we criticized Blau's "modified liberalism" and Gibbard's system of "alienable rights" for making the axe fall invariably on personal rights, ignoring a person's non-meddlesome "self-regarding" parts of the preference ranking and keeping intact "non-self-regarding" parts (reflected in the Pareto preferences). In effect, Farrell does the exact opposite, and for the type of cases we discussed in Sections III and IV, e.g. the "work choice case", the Farrell system will lead to "amended" preferences such that the pernicious consequences of the mechanical use of the

Pareto rule will be eliminated, keeping the personal rights untouched. While I shall argue in Section XI below that there is a better way of achieving this than "amending" preferences, Farrell's approach seems to have much merit in it.

One problem with Farrell's ingenious system of "amendments" lies in the fact that it too—like Blau's "modified liberalism" and Gibbard's "alienable rights"—tries to make social judgments based on what individual preferences happen to be, without going into the motivation that lies behind these preferences. And as we saw, there are situations in which, with appropriate individual motivation, the Gibbard system or the Blau system will be appealing (e.g. in Gibbard's version of the "Angelina–Edwin case"), and here Farrell's solution would be unattractive, while there are other cases with the *same* individual rankings but different motivations underlying them in which the Gibbard solution or the Blau solution will be unattractive (e.g. in the perverse interpretation of the "Angelina–Edwin case" discussed in Section IV, or in the "work choice case", under the given interpretation), and here Farrell's solution seems perfectly legitimate. While Farrell's approach is in some ways the exact opposite of those of Gibbard and Blau, and is immune to the difficulties discussed in Sections III and IV, it shares with those approaches the attempt to make do with rather inadequate information.

Farrell's second approach involves a significant departure from the usual format of social choice theory. The set X of social states is first partitioned into "socially equivalent" subsets, the motivation for which arises from his observation that a liberal is likely to hold that "there is no social choice to be made between x and y [when] they differ in a matter private to individual j" (Farrell, 1976, p. 9). Social choice is then seen as a choice over elements of a partition $\mathscr{P}$ of X. The choice among alternatives within such a "socially equivalent" subset "will be determined by private decisions" (p. 9).

Farrell notes that this radical solution "generates a number of questions". One that worries me is similar to my difficulty with Nozick's approach. If the choice among "socially equivalent" partitions are "removed from social choice theory", are we asserting that from the social point of view it *does not matter* which element of such a subset is chosen? Does it then make no difference to our idea of social welfare whether the choice between x and y differing "in a matter private to individual j" (and thus declared "socially equivalent" and left to be "determined by private decisions") is, in fact, decided the way j wants it, rather than in some other way, e.g. as strong-armed k wants it? If it does not make a difference, then in what sense is this a "liberal" approach? If it does, then in a significant sense x and y are *not* "socially equivalent"; if j prefers x to y, then in a non-trivial sense there is a social preference in favour of x against y, and the choice between x and y is not "removed from social choice theory" after all. A "liberal partition" is indeed a useful way of looking at the issue of personal rights, but in so far as such a partition is combined with recommending social support for ensuring that person j decides over alternatives that "concern" j only, the approach is not altogether different from a social choice system incorporating Condition L.

VIII. Domain Restriction

As mentioned in Section I, one way of resolving the conflict is the weakening of Condition U, the "unrestricted domain" (see Sen, 1970a, pp. 85–86; Fine,

1975, and Blau, 1975). However, the interpretation of a relaxation of Condition U may not be obvious.

If a particular configuration of individual preferences is "outside the domain" of a social decision procedure, then nothing can be deduced from that procedure if such a configuration were to arise. When we "rule out" a preference configuration, that is only a refusal to open our mouth in that case, and obviously has no bearing on whether that configuration will, *in fact*, arise or not. If such a preference configuration does, in fact, occur, then to say that it is outside the domain of a procedure is merely an admission of defeat as far as that procedure is concerned. The relevance of the investigation of "domain restriction" lies in the light it throws on the type of configurations that would have to be absent. The investigation comes into its own when we move away from the assumption of *given* individual preferences, and consider the changes that will help to eliminate the conflict. It is in *this* context that one can remark that "the eventual guarantee for individual freedom" may have to be found "in developing values and preferences that respect each other's privacy and personal choices" (Sen, 1970a, p. 85). This is a "way out" of the dilemma only in this rather limited sense.

The belief that "unrestricted domain" is the condition to axe is not uncommon, though the argument on this is rarely spelt out clearly. Perhaps the most persuasive comments on this come from Blau (1975), who does not, however, himself base his solution of the problem on this—at any rate not on this exclusively. He points out that, in the two-person case, the conflict between the Pareto principle and Condition L arises only if *both* persons are meddlesome in the sense of having stronger preferences *against* the other on the other's assigned pair than on his own assigned pair. "That one of them might exhibit such a preference is remarkable enough, but that both should do so seems to border on the socially pathological" (Blau, 1975, p. 396). Whether pathological is an appropriate description of this type of occurrence I find difficult to decide, but as we saw with several examples (e.g. the *Lady Chatterley's Lover* case, the "work choice case", the "Angelina–Edwin case"), such preference configurations may not be unplausible even over pairs the choices over which are regarded as "purely personal" from the common libertarian point of view. If meddlesomeness is a disease, it is certainly not a rare disease.

Blau shows that the conflict between the Pareto principle and Condition L will not arise in the case of two individuals and four alternatives, if only four of the possible 75^2 configurations of individual preferences were to be ruled out (Blau, 1975, p. 398). If any preference pattern were as likely as any other, this would give it a very low probability of occurrence, even though for a large community there will be a fair number of cases of conflict even under this assumption. Equi-probability is, however, not a very good assumption (on which see Sen, 1970a, pp. 163–166), so that the interpretation of Blau's striking result remains a little problematic. Blau is, therefore, quite right in not basing his "way out" of the inconsistency on the relaxation of the condition of "unrestricted domain", but on weakening the other conditions, specifying what "should" be done if such a meddlesome preference configuration were to arise. It was argued earlier that Blau's solution is not quite adequate (see Section III, above), but our differences there do not lie in the role of the condition of unrestricted domain.

IX. Non-Binary Frameworks of Choice

The conflict between the Pareto principle and libertarian requirements has been discussed so far in terms of cycles of strict preference. While a preference cycle makes it impossible to choose an alternative that is not judged strictly worse than any other, it can be argued that choice from a set of alternatives need not be based on such pair-wise non-defeat. Is there much hope of avoiding the conflicts of Paretian liberalism by moving away from this "binary" framework? This question is particularly important, since escape from Arrow's Impossibility Theorem has recently been sought by many authors in the eschewing of the binary structure and reformulation of the problem in terms of "choice functions" (see Schwartz, 1970; Campbell, 1972; Fishburn, 1973; Plott, 1973; Deb, 1974; Bordes, 1976; and Blair *et al.* 1976).

I don't believe there is much mileage in this. As was noted in Sen (1970a, pp. 81–82), the Pareto principle and conditions L and L* can be easily reformulated in non-binary terms, without losing their rationale. The Pareto principle can then be read as: if everyone prefers x to y, then y should not be socially chosen from any set if x is available in that set. Similarly, Condition L will be transformed into requiring that if any person i prefers x to y when $\{x, y\}$ is his assigned pair, then y should not be socially chosen from any set that contains x. For some configuration of individual preferences, these two conditions will leave nothing to choose (see (T7) in the Appendix). Alternatively, Condition L can be made less demanding; viz., if any person i prefers x to y when $\{x, y\}$ is his assigned pair, then y should not be socially chosen from *that* pair. Combined with a relatively mild requirement of consistency of choice proposed by Batra and Pattanaik (1972), this too makes social choice impossible from some sets for particular individual preferences when combined with the non-binary Pareto principle (see (T8) in Appendix).

X. Motivation and the Internal Consistency of Rights

The problem of consistency of libertarian and Paretian principles has recently been supplemented by Gibbard's (1974) pointer to *internal* inconsistency of the libertarian principle itself "under one natural interpretation" (see also Farrell, 1976). If more than one person is given the right to make a class of personal decisions without outside interference, this itself can give rise to a strict preference cycle.

An example may bring out the nature of the difficulty envisaged. Let Zubeida be keen on dressing in the same colour as Rehana, while Rehana wishes to differentiate from Zubeida. Consider four alternatives with R standing for red, G for green, with the first letter denoting the colour of Zubeida's dress and the second Rehana's: RR, GG, RG and GR. If it is accepted that the way one dresses is a person's own business and whatever she decides about her own dressing must be judged to be socially better, then there is now a problem of consistency. Zubeida's preference for matching Rehana leads to RR being superior to GR and GG being superior to RG (in these decisions only Zubeida's dress varies), while Rehana's preference for differentiating leads to RG being superior to RR, and GR being superior to GG (in these choices only Rehana's dress varies). There is now a cycle of strict preference order: RR, GR, GG, RG, RR.

Cycles of this kind cannot be caused by Condition L or L* alone, since for this we need at least two assigned pairs per person related to each other in this "closed circle" way. But Gibbard argues, with some force, that the rationale for Condition L or L* should extend to giving each person rights over *all* choices that differ only in a feature of the world that is exclusively his or her "business". And then the problem of internal consistency can arise, as illustrated above.

But can we decide that some choice is a person's concern alone totally independently of the *motivation* that lies behind his or her preferences? Zubeida's desire to match Rehana, and Rehana's desire to differentiate from Zubeida, are not particularly inward-looking, and what either does is clearly of real consequence to the other if their respective desires to match or differentiate are taken seriously. So while the personal right to choose the colour of one's dress may well be conceded in general, the presence of the types of motivations discussed would tend to question the coverage of this right. We may be upset about an order prohibiting the wearing of red dresses (because this *is* a private business in general), without being prepared to weep too much about Zubeida's failure to secure the right to match Rehana, should Rehana be cunning enough to frustrate Zubeida's matching programme. The problem of *internal* consistency of the kind with which Gibbard is concerned arises only with preference configurations requiring rather other-oriented motivations, and the weakening of these "rights" in the presence of other-oriented motivations would not seem to involve any great violations of libertarianism. This is so irrespective of whether or not problems of internal consistency do *in fact* arise. But, of course, the issue has an additional interest when problems of internal consistency are present, as in Gibbard's example; and more generally when the condition of "consistent rights assignment" discussed in the Appendix (section A5) is violated.

The question of motivation seems central to the force of *both* the Pareto principle (as discussed earlier) and the libertarian principles (especially in Gibbard's extension). Both types of judgments seem to be "non-basic" (in the sense discussed in Sen, 1967), and it seems difficult to assert or deny either type of judgment *irrespective* of the motivations underlying the personal preferences.

XI. Restraining the Pareto Principle

One of the main preoccupations of this paper has been the unacceptability of the Pareto principle as a universal rule. The "Paretian epidemic", that is (T1), shows how powerful the Pareto principle is in spreading decisiveness from one pair to all. The "impossibility of the Paretian liberal" (T2) is just a corollary of the "Paretian epidemic". The power of a principle is not itself an argument against it—in fact, it may even be treated as an argument in favour—but some of the consequences that follow from this power seem to be unacceptable, thereby indicating the unacceptability of the Pareto principle. The fact that even a minimal guarantee of individual liberty may have to be revoked is viewed as one such consequence.

The suggestion that the Pareto principle be rejected meets with resistance, which is perfectly understandable, since there is something very central in the idea that preferences unanimously held by members of a community cannot be rejected by that community. As Blau (1975, p. 401) puts it, "I can see no case

for an outside observer denying a unanimous choice. This leads inevitably to modifying [Condition] L."

But is an outsider necessarily involved in denying a unanimous preference? An important distinction exists, it seems to me, between person *i preferring* x to y, and person i wanting his preference for x over y to *count* in determining social choice. I can easily take the view that, while I would prefer you to read what I consider to be good literature as opposed to what appears to me to be muck, I do not want my preference to count in the social evaluation as to whether it is better that you read good literature or bury yourself in muck. I might accept taste differences as legitimate and accept the greater relevance of your taste in matters that I agree are essentially your "concern".

Extending this reasoning, I may decide, for the sake of consistency, not to insist that my preferences be taken into account even in choices over some pairs that are not exclusively your concern. Let me be the "prude" (Mr A) in the example of *Lady Chatterley's Lover* (see Section I), while you are "lascivious" (Mr B). I would rather not read the stuff myself (i.e., I prefer o to a), and I would rather you would not (i.e., I prefer o to b), but I decide to "respect" your tastes on what I agree is your benighted business (while wondering whether "respect" is quite the word), conceding that my preference for o over b be ignored. My dislike of your gloating over "muck" was so strong that I would have preferred to read the work myself to stop you from falling into this (i.e., I preferred a to b), but being a consistent kind of man, I notice that, if I insist that my preference for o over a should count as well as my preference for a over b, then there is not much point in my "renouncing" my preference for o over b. So I may decide not to want my preference for a over b to count, even though the choice over the pair $\{a, b\}$ is not exclusively your business.

On a similar ground, you might not want your preference for a over b to count, since you do wish your preference for b over o to count and decide not to want that your preference for a over o should count (since it is my business). But the Pareto preference for a over b is built on counting my preference and yours for a over b, and if neither of us wants our respective preferences over this pair to count, there can hardly be much force in the Pareto ranking in this case. If on these grounds the Pareto preference is overridden, this is not done by virtue of any "outside observer denying a unanimous choice", but on the basis of our own denial that our preferences for a over b should "count" in deciding what is socially better.

This notion of counting suggests a *conditional* version of the Pareto principle. If everyone in a community prefers x to y and wants that preference to count, then x must be socially preferred to y (conditional weak Pareto principle, denoted PC). A person can be described as respecting the rights of others if and only if he wants only a part of his total preference to count such that it can be combined with everyone's preferences over *their* respective "protected spheres". (There may be many more than one pair in any person's protected sphere, but the rights of different people are required to be consistent with each other, avoiding Gibbard's problem.) The conditions involved are stated more rigorously in the Appendix, and it is shown there—see (T9)—that, if *at least one* person respects the rights of others, then there can be no conflict between the conditional Pareto principle (even in a strengthened form) and the weak libertarian principle (even after considerable strengthening), no matter what the

individual preferences are. But it does mean that some Pareto preferences in the traditional sense may have to be violated.

In this procedure, no one pretends that his preference is, in fact, different from what it actually is, and there is no question of "amending" preferences (contrast Farrell, 1976). A person wants a part of his preference to count, thereby asserting "the truth" and "nothing but the truth", but does not demand that "the whole truth" of his preference be brought to bear on every choice for the society.

Note also that, while the conditional Pareto principle as defined here (and its strengthened version in the Appendix) do not bring in any outsiders, it is possible to consider "arbitration problems" in which the job of restricting is not left only to the persons involved. Some outsider may try to judge what should be done and may decide that certain parts of a person's preferences *should not* count in the choice in question. For example, a person judging what should be done in the *Lady Chatterley's Lover* case may decide that, while the prude Mr A prefers o to a, and a to b, and wants every ounce of it to count, there is nevertheless a case for discounting his preference for o over b since B should himself decide whether to read the book or not, and therefore A's preference for a over b should not count either since his preference for o over a must. Thus arguments may be constructed that suggest the violation of not merely the Pareto principles, but also of the *conditional* versions of these principles proposed here.

It is difficult to decide where exactly to draw the line when such judgments are concerned, and indeed, as I have tried to argue earlier in this paper, the set of individual orderings in general provides too little information for deciding what to do. To discuss whether a person's preference should count or not we may need to know more than what the preferences happen to be, e.g. the *reasons* for holding these preferences. As argued before, the same set of individual orderings under one interpretation of the motivations underlying the preferences might suggest the dropping of the Pareto principle, while under another interpretation of the motivations it might point the finger at Condition L.

XII. Concluding Remarks

No attempt will be made here to summarize the discussion in the earlier sections, but a few general remarks will be made.

(1) While most of the attempts at avoiding the conflict that may arise between the Pareto principle and Condition L (or L*) has taken the form of weakening the latter, a strong case can be made for weakening the former as well. This may appear superficially puzzling since "unanimity" is a strong argument, but the "unanimity" used in the Pareto principle is of a rather limited kind (*contrast* the "unanimity rule" UR in Section II), and it leaves many issues open.

(2) To reject the Pareto principle as a universal rule does not amount to an outsider overriding the wishes of members of the community. The difference between "preferring x to y" and wanting one's preference to "count in favour of x against y" is relevant to this. Indeed, the guarantee of a minimal amount of personal liberty may require that certain parts of individual rankings should

not count in some specific social choices, and in some cases even the persons in question may agree with this.

(3) The Pareto principle (as opposed to the unanimity rule UR) has some "epidemic" properties for a social decision function with unrestricted domain. If a person is decisive over any pair, he turns out to have a weak form of decisiveness ("potentially semidecisive") over *every* ordered pair. It is this result, viz. (T2), that does not permit even two persons to decide on certain personal things on their own, because if one is decisive over a single pair, he is potentially semidecisive over every ordered pair, and thus the other cannot be decisive over any pair at all, no matter how "personal" the choice might be to him. This leads to the impossibility of the Paretian liberal (T1).

(4) The same reason applies to two or more groups of people with no members in common, who cannot be decisive over one pair each (see (T6) in the Appendix), ruling out even the most minimal forms of local autonomy, or a federal structure. The Paretian epidemic threatens not merely individual liberty but also group autonomy.

(5) Relaxing the "binary" framework of social choice is no way out of these conflicts, since weak consistency conditions on the choice function regenerate the "impossibility", as is shown in the Appendix (see also Batra and Pattanaik, 1972).

(6) Condition L reflects only a very small part of what a "liberal", or a "libertarian", is typically concerned with. Support for Condition L in specific classes of choices will come even from those who would not be described as "liberals". (Even Joseph Stalin (1913), not especially known for his libertarian sympathies, wrote eloquently about group autonomy which involves similar problems of consistency *vis-à-vis* the Pareto principle.) While there are enormous differences in the conception of freedom in different philosophies, most social philosophies accept certain personal or group rights. The fact that unqualified use of the Pareto principle potentially threatens all such rights gives the conflict an extraordinarily wide scope. This applies also to the so-called positive (as opposed to negative) freedoms, e.g. freedom from hunger, or right to work.

(7) While Blau's partial solution of the problem through weakening Condition L to versions of "modified liberalism" in the presence of "meddlesome" preferences is illuminating, it seems to have the remarkable property of responding to meddlesomeness by ignoring the self-regarding, non-meddlesome part of the preference and making effective the non-self-regarding parts. The problem arises from Blau's determination not to weaken the Pareto principle. The same difficulty applies to Gibbard's interesting solution through weakening Condition L by making the libertarian rights "alienable", while keeping the Pareto principle unaffected. If everyone other than myself prefers x to y, then the Pareto principle gives me the right to ensure the social choice of x over y by my preferring x to y, and there exist situations in which the case for the "alienation" of this right is at least as strong as that of the libertarian right. And this does require a violation of the Pareto principle.

(8) Nozick's ingenious solution of the conflict in terms of the domain of social choice being constrained by considerations of individual rights (including Condition L), while the choice over that domain is made through a social ordering incorporating the Pareto principle, avoids the Paretian epidemic by systematic domain restrictions. There remains, however, the problem of making

judgments about changes that lie outside that constraint, e.g. when a person's recognized rights are violated. These judgments re-introduce the conflict. The difficulty seems to arise, once again, from the reluctance to go against the Pareto relation in the social ordering itself.

(9) Farrell's first approach involves "amendment" of individual preferences, so that if person i prefers x to y over his assigned pair, then everyone else is "deemed" to regard x to be at least as good as y, and other changes are made that follow from consistency requirements. With the "amended" preferences, there is no conflict between the Pareto principle and Condition L. Farrell is critical of this first approach, arising partly from his disapproval of the result that an actual Pareto preference for x over y may be changed to the opposite Pareto preference with the amended preferences. (The procedure explored in Section XI and in Appendix A5 avoids this and involves no *amendment* of the actual preferences, only taking a sub-relation of it, and I prefer it to Farrell's, but it must be pointed out that the two approaches are motivationally similar.) Farrell's second approach partitions the set of social states into "socially equivalent" subsets on liberal grounds and then confines social choice theory to choosing between these subsets. If violation of personal rights (involving two points in a "socially equivalent" subset) is, thereby, not to be described as socially inferior, this limits the scope of social choice theory rather arbitrarily.

(10) The attempted solutions seems to have been severely constrained by the unwillingness to drop the Pareto principle. I have tried to argue here that the Pareto principle, in the form in which it is used, is a prime candidate for rejection. A more general question concerns whether the set of individual preferences irrespective of the motivation underlying them is an adequate basis for social judgment involving issues such as liberty. I have tried to argue that it is not. I have argued elsewhere that, for social judgments involving issues of equity and justice, the informational framework of concentrating only on preference orderings is also inadequate (Sen, 1970, Chapter 9; 1973). While those issues indicate the need for *interpersonal comparisons of welfare*, the discussion of liberty puts us in the direction of *motivations* that underlie the preferences. This question of the relevance of motivation underlying preferences has implications for the analytical framework used in the current social choice theory, and needless to say for welfare economics.

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APPENDIX

Some Formal Statements and Proofs

A1. *Introduction*

X is the set of alternative social states, and each person i has a complete preference ordering (reflexive, complete and transitive) R_i over X, with $i=1, \ldots, n$. It is assumed

that there are at least two persons and at least three distinct social states in X. A collective choice rule f specifies a reflexive ("weak") social preference relation R for every n-tuple of individual orderings $\{R_i\}$—one ordering for each person in the community:

(A1) $R=f(\{R_i\})$.

If R must be an ordering, then f is a "social welfare function", SWF, in the sense of Arrow (1951). A weaker requirement is that R must be reflexive, complete and *acyclic* (not necessarily transitive). This is necessary and sufficient for R to generate a choice function defined over the class of all finite subsets of the set X, i.e. in each finite subset S of X, the set $C(S)$ of R-greatest elements is non-empty (see Sen, 1970a, p. 16). A collective choice rule satisfying this requirement is called a "social decision function", SDF, which is a slight broadening of SDF as defined in Sen (1970a, p. 52). Since the set of SWFs is a proper subset of the set of SDFs, the non-existence of any SDF satisfying a set of conditions will imply the non-existence of any SWF satisfying those conditions. (The converse is, of course, not true; e.g., there are SDFs satisfying the Arrow (1951) conditions but no such SWF.)

P and I stand respectively for the asymmetric and symmetric factors of R, i.e. for social "strict preference" and "indifference" respectively. Similarly, P_i and I_i are the asymmetric and symmetric factors of R_i.

The "impossibility of the Paretian liberal" uses the following conditions.

Condition U (unrestricted domain). The domain of f includes all logically possible n-tuples $\{R_i\}$ of individual preference orderings over X.

Condition P (weak Pareto principle). For any x, y from X, if xP_iy for all i, then xPy.

Condition L (minimal libertarianism).* There is at least one pair of persons decisive both ways over at least one pair of alternatives each; i.e., for each of them i, there is a pair of alternatives in X, which we may rechristen (x_i, y_i), such that $x_iP_iy_i$ implies x_iPy_i, and $y_iP_ix_i$ implies y_iPx_i.

(T1) There is no SDF satisfying Conditions U, P and L*.
For proof, see Sen (1970a, pp. 87–88).

Condition L (weak libertarianism). Everyone is decisive both ways over at least one pair of alternatives each.

(T1.1) There is no SDF satisfying Conditions U, P and L.
An immediate corollary of (T1).

A2. *The Paretian Epidemic and Related Results*

Semidecisiveness. Person J is semidecisive over an ordered pair $\{x, y\}$ if and only if for any $\{R_i\}$ in the domain of f, if xP_Jy, then xRy.

Decisiveness and semidecisiveness can be respectively weakened by making the effectiveness of person J over $\{x, y\}$ conditional on some specified individual rankings over pairs *other than* $\{x, y\}$ without however restricting the ranking of anyone over $\{x, y\}$.

Potential decisiveness and semidecisiveness. Person J is potentially decisive (resp. potentially semidecisive) over an ordered pair $\{x, y\}$ if and only if for any $\{R_i\}$ in the domain of f satisfying some specified restrictions on the rankings of pairs other than $\{x, y\}$, which leaves the rankings of $\{x, y\}$ by all $i \neq J$ completely free, if xP_Jy, then xPy (resp. xRy).

Note that in the above definition xPy (resp. xRy) is required to follow from xP_Jy irrespective of the nature of the ranking of anyone else over $\{x, y\}$.

The "Paretian epidemic" shows that, for an SDF with an unrestricted domain, the weak Pareto principle is sufficient to spread decisiveness from one pair of alternatives to all in the weaker form of potential semidecisiveness.

(T2) For any social decision function satisfying U, the weak Pareto principle P implies that if any person J is decisive both ways over any one pair in X, then J is potentially semidecisive over every ordered pair in X.

Proof. Let J be decisive both ways over $\{x, y\}$. Take any other pair $\{z, w\}$. There are three possibilities: (I) x, y, z and w are all distinct; (II) $\{z, w\}$ and $\{x, y\}$ have one element in common, and (III) $\{x, y\}=\{z, w\}$. The three cases are considered in turn.

CASE I. Let J rank the four distinct states in the strict descending order: z, x, y, w. Let everyone else strictly prefer z to x and y to w, leaving the ranking of all other pairs completely free. By the weak Pareto principle, zPx and yPw. By J's decisiveness over $\{x, y\}$, xPy. If now wPz, then there is a preference cycle, which is impossible since f is an SDF with an unrestricted domain. Hence zRw, since R must be complete. But only J's preference over $\{z, w\}$ has been specified here, and since zRw follows no matter how everyone else ranks z *vis-à-vis* w, clearly J is potentially semidecisive over $\{z, w\}$.

CASE II. There are four sub-cases. Consider first the case in which $x=z$. Let J have a strict descending order, x, y, w, and let everyone else strictly prefer y to w, leaving the ranking of y and w *vis-à-vis* x open. Then, by the decisiveness of J over $\{x, y\}$, xPy, and by the weak Pareto principle, yPw. To prevent a preference cycle, xRw, i.e. zRw. This makes J potentially semidecisive over $\{z, w\}$. The sub-case of $y=z$ is identically covered since J's decisiveness applies to both the ordered pairs $\{x, y\}$ and $\{y, x\}$. Next, let $y=w$. Consider now J's strict descending order, z, x, y, and let everyone else prefer z to x. By the decisiveness of J over $\{x, y\}$, xPy, and by the weak Pareto principle, zPx. To prevent a preference cycle, zRy, i.e. zRw. Hence J is potentially semidecisive over $\{z, w\}$. The remaining sub-case of $x=w$ is covered similarly.

CASE III. Finally, if $\{x, y\}$ and $\{z, w\}$ are the same pairs, then the implication is trivial since decisiveness over $\{x, y\}$ both ways must imply potential semidecisiveness over the ordered pair $\{z, w\}$, which completes the proof.

A corollary of the Paretian epidemic (T2) is immediate.

(T2.1) For any social decision function satisfying U, the weak Pareto principle implies that if anyone J is decisive both ways over a pair of alternatives, then no subset of individuals which does not include J can be decisive over any ordered pair whatsoever.

Note that (T2.1) rules out, *inter alia*, any person other than J being decisive anywhere at all. Thus (T1) follows from (T2.1) and, therefore, from (T2).

If quasi-transitivity of social preference (i.e. transitivity of strict P) is required, the spread of decisiveness of J is more exacting.

(T3) For any social decision function satisfying U and yielding quasi-transitive P, the weak Pareto principle P implies that, if any person J is decisive both ways over any one pair in X, then J is potentially decisive over every ordered pair of alternatives from X.

The proof is omitted here, since it is very similar to that of (T2); with quasi-transitivity zPx, xPy and yPw together imply zPw, and zPy and yPw together imply zPw.

None of the results so far invokes Arrow's much-debated condition of "independence of irrelevant alternatives".

Condition I (*independence of irrelevant alternatives*). The restriction $R^{x,y}$ of social preference R over any pair $\{x, y\}$ is a function only of the n-tuple of restrictions $R_i^{x,y}$ of R_i over $\{x, y\}$: $R^{x,y}=f^{x,y}(\{R_i^{x,y}\})$.

If this condition is imposed additionally, (T2) and (T3) transform into the following theorems (cf. Arrow, 1963, Theorem 2, pp. 97–100; Sen, 1970a, Lemma 3*a; Blau and Deb, 1976, Section VII). Decisiveness over an *ordered* pair is sufficient for (T5) but not for (T4).

(T4) For any social decision function satisfying U, the weak Pareto principle P and the independence of irrelevant alternatives I together imply that if any person J is decisive both ways over any pair in X, then J is semidecisive over every ordered pair of alternatives from X.

(T5) For any social decision function satisfying U and yielding quasi-transitive R, the weak Pareto principle P and the independence of irrelevant alternatives I together imply that if any person J is decisive over any ordered pair of alternatives from X, then J is decisive over every ordered pair from X.

To summarize, for a social decision function with unrestricted domain, if J is decisive (both ways in the first three cases) over some pair, then:

(T2) P (weak Pareto principle) $\Rightarrow$ J potentially semidecisive everywhere
(T3) $P+$ social quasi-transitivity $\Rightarrow$ J potentially decisive everywhere
(T4) $P+$ independence of irrelevant alternatives $\Rightarrow$ J semidecisive everywhere
(T5) $P+$ both $\Rightarrow$ J a dictator.

(T2) is our *pure* "Paretian epidemic", while (T5) is the central lemma for Arrow's General Possibility Theorem (see Lemma 3*a in Sen, 1970a), and the others are intermediate cases.

A3. *Libertarianism and Federalism*

While (T3), (T4) and (T5) involve strengthening of the conditions imposed in the pure "Paretian epidemic" (T2), the "impossibility of the Paretian liberal" (T1) follows immediately from (T2) itself. Another direct consequence of (T2) is the impossibility of combining Paretianism with what Batra and Pattanaik (1972) have called "minimal federalism".

*Condition F** (*minimal federalism*). There are at least two disjoint subsets of the community which are each decisive both ways over at least one pair of distinct alternatives each; i.e., if everyone in such a subset J prefers x to y (resp. y to x) when $\{x, y\}$ is the assigned pair of J, then xPy (resp. yPx).

(T6) There is no social decision function satisfying Conditions U, P and F^*.

This follows from (T2) by reinterpreting xP_Jy to mean that everyone in the subset J strictly prefers x to y. (It may be worth remarking that not only (T2) but also (T3), (T4) and (T5) hold under this reinterpretation of J.)

A4. *Non-Binary Social Choice*

The focus is now shifted from SDFs (with social choices being expressed in the form of a binary relation of social preference R) to "functional collective choice rules" FCCR (see Sen, 1976). An FCCR specifies for each n-tuple of individual preference orderings $\{R_i\}$ a choice function $C(\cdot)$ for the society yielding a non-empty subset $C(S)$ of S for any non-empty subset S of X:

(A2) $C(\cdot)=\phi(\{R_i\})$.

Conditions U, P, L, L^* and F^* may now be reformulated for an FCCR. The wording of U may be kept unchanged replacing f by ϕ and denoting the condition thus reformulated as $\hat{U}$. The other conditions may be redefined by interpreting xPy as y is not socially chosen when x is available, i.e., for any S, if $x \in S$, then $y \notin C(S)$. These "non-binary" conditions are respectively denoted as $\hat{P}$, $\hat{L}$, $\hat{L}^*$ and $\hat{F}^*$. The "impossibility of the Paretian liberal", (T1), now readily translates to the non-binary framework (see Sen, 1970a, pp. 81–82), and so does the impossibility of the Paretian federalism, (T6) (see Batra and Pattanaik, 1972).

(T7) There is no functional collective choice rule satisfying Conditions $\hat{U}$, $\hat{P}$ and $\hat{L}^*$ (or $\hat{F}^*$).

Proof. Let 1 and 2 be two persons (resp. two disjoint subsets of individuals) with assigned pairs $\{x, y\}$ and $\{z, w\}$ respectively. If the four alternatives are all distinct, consider

the following preferences of persons 1 and 2 (resp. unanimous preferences of groups 1 and 2) respectively in strict descending order: (1) w, x, y, z; and (2) y, z, w, x. Let everyone else prefer w to x and also y to z. By L^*, neither y nor w should belong to $C(\{x, y, z, w\})$. By P, neither x nor z should belong to it. Since $(C\{x, y, z, w\})$ must then be empty, the FCCR violates U. The cases of one alternative in common between $\{x, y\}$ and $\{z, w\}$ are handled similarly.

The proof makes it clear that the impossibility results will hold even if the definition of an FCCR were weakened to demand non-empty choice sets for only 3-element and 4-element subsets of X, without demanding anything about choices from subsets of other cardinality.

Batra and Pattanaik (1972) have pointed out an alternative way of deriving these impossibility results with weaker requirements of libertarianism and federalism, but with an additional condition of consistency of social choice, viz.:

BP (Batra–Pattanaik condition of choice consistency). If $\{x\} = C(\{x, y\})$, then for any S, if $x \in S$, then $y \in C(S)$ implies that $x \in C(S)$.

With this weak consistency condition, $\hat{L}^*$ and $\hat{F}^*$ can be weakened by redefining L^* and F^* through interpreting xPy as x being uniquely chosen over the *pair* $\{x, y\}$. These conditions may be called $\hat{\hat{L}}^*$ and $\hat{\hat{F}}^*$ respectively.

(T8) There is no functional collective choice rule satisfying Condition $\hat{U}$, $\hat{P}$, BP and $\hat{\hat{L}}^*$ (or $\hat{\hat{F}}^*$).

The proof of (T7) translates readily for (T8). With the postulated preferences x and z do not belong to $C(\{x, y, z, w\})$ in view of $\hat{P}$. By $\hat{\hat{L}}^*$ (or $\hat{\hat{F}}^*$), $\{x\} = C(\{x, y\})$ and $\{z\} = C(\{x, w\})$. By BP, y or w can belong to $C(\{x, y, z, w\})$ only if x or z respectively does. But neither does. Hence $C(\{x, y, z, w\})$ is empty.

A5. *Conditional Pareto Principles*

A weakening of the weak Pareto principle is now examined. Let $\bar{R}_i$ be a sub-relation of individual preference R_i reflecting the parts of his preference ordering R_i that person i wants to count in social choice. $\bar{P}_i$ and $\bar{I}_i$ are the asymmetric and symmetric factors of $\bar{R}_i$ respectively.

Condition PC (conditional weak Pareto principle). For any x, y in X, if $x\bar{P}_i y$ for all i, then xPy.

Condition PC (conditional strong Pareto principle)*. For any x, y in X, if $x\bar{R}_i y$ for all i, then xRy, and if furthermore, for some person i, $x\bar{P}_i y$, then xPy.

Next the problem of a consistent assignment of rights to avoid problems of internal consistency discussed by Gibbard (1974).

Consistent rights assignment. Each person i is assigned a non-empty set D_i of pairs such that no matter how they order them, there is an ordering T of X of which each i's preference over each $\{x, y\}$ in his D_i is a sub-relation.

Condition L+. For any consistent assignment of rights, if $\{x, y\}$ is in D_i, then $xP_i y$ implies xPy.

The concept of "respecting" other people's rights is defined in the framework of a consistent assignment.

Respecting rights. For any consistent rights assignment a person j respects the rights of others if and only if for each n-tuple of individual preference orderings $\{R_i\}$, person j wants a sub-relation $\bar{R}_j$ of his preference ordering R_j to count such that there exists an ordering T_j of which $\bar{R}_j$ is a sub-relation and so is each i's preference over each $\{x, y\}$ in D_i.

(T9) There exists a social decision function satisfying Conditions U, $L+$ and PC^* if at least one person in the community respects the rights of others.

Proof. Suppose to the contrary person j respects the rights of others but still the SDF generates a strict social preference cycle over some subset S with $xP\mu(x)$ for all x in S for some one-to-one correspondence from S to S. Let the subset M of S represent those x for which $xP\mu(x)$ follows from the conditional strong Pareto principle PC^*. Given consistent rights assignment implied in $L+$, M is non-empty. Since the Pareto relation is acyclic, M must be a *proper* subset of S. And $xP\mu(x)$ for all x *in* $(S-M)$ must be due to $L+$. Consider now $\bar{R}_j$. By the definition of PC^*, $x\bar{R}_j\mu(x)$ for all x in M. Since j respects the rights of others, there is an ordering T_j which incorporates $\bar{R}_j$ as well as $xP\mu(x)$ for all x in $(S-M)$. This is a contradiction; the strict cyclicity of P over S must imply the intransitivity of such a T_j.

Note that (T9) can be proved also constructively by showing that, from any ordering T_j (incorporating $\bar{R}_j$ and the strict preference P_i for each $\{x, y\}$ in D_i for each i), an acyclic, complete and reflexive relation P can be constructed by strengthening to P those $\bar{I}_j$ which go with a strict Pareto preference according to PC^*.

Note also that this method of accommodating libertarian rights by restricting the Pareto principle can also be used for the non-binary framework discussed in Section A4.

REFERENCES

ARROW, K. J. (1951). *Social Choice and Individual Values* (2nd edn 1963). New York: Wiley.

BATRA, R. N. and PATTANAIK, P. K. (1972). On some suggestions for having non-binary social choice functions. *Theory and Decision*, **3**, 1–11.

BERNHOLZ, P. (1974). Is a Paretian liberal really impossible? *Public Choice*, **19**, 99–107.

—— (1975). Is a Paretian liberal really impossible: a rejoinder. *Public Choice*, **23**, 69–73.

BINMORE, K. (1974). Social choice and parties (mimeographed). London School of Economics.

BLAIR, D. H., BORDES, G., KELLY, J. S. and SUZUMURA, K. (1976). Impossibility theorems without collective rationality. *Journal of Economic Theory*, forthcoming.

BLAU, J. H. (1957). The existence of a social welfare function. *Econometrica*, **25**, 302–313.

—— (1971). Arrow's theorem with weak independence. *Economica*, **38**, 413–420.

—— (1975). Liberal values and independence. *Review of Economic Studies*, **42**, 395–402.

BLAU, J. H. and DEB, R. (1976). Social decision functions and the veto. *Econometrica*, forthcoming.

BORDES, G. (1976). Consistency, rationality and collective choice. *Review of Economic Studies*, forthcoming.

BROOME, J. (1974). Consumer sovereignty. Birkbeck College Discussion Paper No. 32.

CAMPBELL, D. E. (1972). A collective choice rule satisfying Arrow's five conditions in practice. In *Theory and Application of Collective Choice Rule*, Institute for Quantitative Analysis of Social and Economic Policy, Working Paper No. 7206, University of Toronto.

—— (1975). Freedom of choice and social choice (mimeographed).

DEB, R. (1974). *Rational Choice and Cyclical Preferences*. Ph.D. dissertation, London University.

FARRELL, M. J. (1976). Liberalism in the theory of social choice. *Review of Economic Studies*, **43**, 3–10.

FINE, B. J. (1975). Individual liberalism in a Paretian society. *Journal of Political Economy*, **83**, 1277–1282.

FISHBURN, P. C. (1973). *The Theory of Social Choice*. Princeton: University Press.

GIBBARD, A. (1974). A Pareto-consistent libertarian claim. *Journal of Economic Theory*, **7**, 388–410.

GINTIS, H. (1972). Alienation and power. *Review of Radical Political Economics*, **4**, 1–34.

GRAMSCI, A. (1971). *Selections from the Prison Notebooks*. London: Lawrence and Wishart.

HAMMOND, P. (1974). On dynamic liberalism (mimeographed). Australian National University and the University of Essex.

HANSSON, B. (1969). Group preferences. *Econometrica*, **37**, 50–54.

HAYEK, F. A. (1960). *The Constitution of Liberty*. London: Routledge and Kegan Paul.

HICKS, J. R. (1956). *A Revision of Demand Theory*. Oxford: Clarendon Press.

HILLINGER, C. and LAPHAM, V. (1971). The impossibility of a Paretian liberal: comment by two who are unreconstructed. *Journal of Political Economy*, **79**, 1403–5.

JEFFREY, R. C. (1974). Preferences among preferences. *Journal of Philosophy*, **71**, 377–391.

KARNI, E. (1974a). Individual liberty, the Pareto principle and the possibility of social choice function. Working Paper No. 2, The Foerder Institute for Economic Research, Tel-Aviv University.

—— (1974b). Collective rationality, unanimity and liberal ethics (mimeographed), Tel-Aviv University.

KELLY, J. S. (1976a). The impossibility of a just liberal. *Economica*, **43**, 67–76.

—— (1976b). Rights exercising and a Pareto-consistent libertarian claim. *Journal of Economic Theory*, forthcoming.

MILL, J. S. (1859). *On Liberty*. Reprinted in M. Warnock (ed.) (1973). *Utilitarianism*. London: Fontana.

NG, Y. K. (1971). The possibility of a Paretian liberal: impossibility theorems and cardinal utility. *Journal of Political Economy*, **79**, 1397–1402.

NOZICK, R. (1968). Moral complications and moral structures. *Natural Law Forum*, **13**, 1–50.

—— (1973). Distributive justice. *Philosophy and Public Affairs*, **3**, 45–126.

—— (1974). *Anarchy, State and Utopia*. Oxford: Blackwell.

OSBORNE. D. K. (1975). On liberalism and the Pareto principle. *Journal of Political Economy*, **83**, 1283–1288.

PEACOCK, A. T. and ROWLEY, C. K. (1972). Welfare economics and the public regulation of natural monopoly. *Journal of Political Economy*, **80**, 476–490.

PLOTT, C. R. (1973). Path independence, rationality, and social choice. *Econometrica*, **41**, 1075–1091.

RAMACHANDRA, V. S. (1972). Liberalism, non-binary choice and Pareto principle. *Theory and Decision*, **3**, 49–54.

ROWLEY, C. K. and PEACOCK, A. T. (1975). *Welfare Economics: A Liberal Restatement*. London: Martin Robertson.

SCHWARTZ, T. (1970). On the possibility of rational policy evaluation. *Theory and Decision*, **1**, 89–106.

SEIDL, C. (1975). On liberal values. *Zeitschrift für Nationalökonomie*, **35**, 257–292.

SEN, A. K. (1967). The nature and classes of prescriptive judgments. *Philosphical Quarterly*, **17**, 46–62.

—— (1970a). *Collective Choice and Social Welfare*. San Francisco: Holden-Day, and Edinburgh: Oliver and Boyd.

—— (1970b). The impossibility of a Paretian liberal. *Journal of Political Economy*, **78**, 152–157.

—— (1973). *On Economic Inequality*. Oxford: Clarendon Press, and New York: Norton.

—— (1974). Choice, orderings and morality. In *Practical Reason* (Proceedings of the Bristol Conference on Practical Reason, 1972) (S. Körner, ed.). Oxford: Blackwell.

—— (1975). Is a Paretian liberal really impossible: a reply. *Public Choice*, **21**, 111–113.

—— (1976). Social choice theory: a re-examination. *Econometrica*, forthcoming.

STALIN, J. (1913). Marxism and the national question. English translation in *The Essential Stalin* (1973) (B. Franklin, ed.). London: Croom Helm.

SUPPES, P. (1966). Some formal models of grading principles, *Synthèse*, **6**, 284–306. Reprinted in his *Studies in the Methodology and Foundations of Science* (1969). Dordrecht: Reidel.

SUZUMURA, K. (1976). Remarks on the theory of collective choice. *Economica*, forthcoming.

WILLIAMS, B. (1973). A critique of utilitarianism. In *Utilitarianism: For and Against* (J. J. C. Smart and B. Williams, eds.), pp. 77–150. Cambridge: University Press.