The Man of Numbers

In search of Leonardo Fibonacci

a pictorial tour with
Keith Devlin

Contents

Who was Leonardo Fibonacci?

Leonardo Fibonacci was an Italian mathematician who lived from around 1170 to around 1250. He spent most of his life in Pisa. When he was still a teenager, he traveled to Bugia, in North Africa, to join his father who had gone there to handle international trade for Pisa, then one of the major Mediterranean commercial hubs. While there, he observed Muslim traders using a remarkable method for performing calculations. Instead of using finger-counting (a sophisticated and popular, but complicated, method that could be used for numbers up to 10,000) or a physical counting-table, they wrote numbers on parchment using just ten symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and calculated by manipulating the symbols according to certain rules. In short, they were performing arithmetic the way we are all taught today in school. That system had been developed in India in the first seven centuries of the first Millennium, and had been learned, used, and carried northwards by Arabic speaking traders who traveled back and forth along the Silk Road. On his return to Pisa, Leonardo wrote a mammoth book describing this remarkable new way to write numbers and calculate with them, called *Liber abbaci* (spelt with two *b*'s), which translates from the Latin as "Book of Calculation". (Not the "book of the abacus", spelt with one b.) Completed in 1202, this book is generally credited with bringing the Hindu-Arabic number system and its arithmetic (as the system is now known) to Europe, and launching the modern, Western-led, commercial world.

What did Leonardo look like?

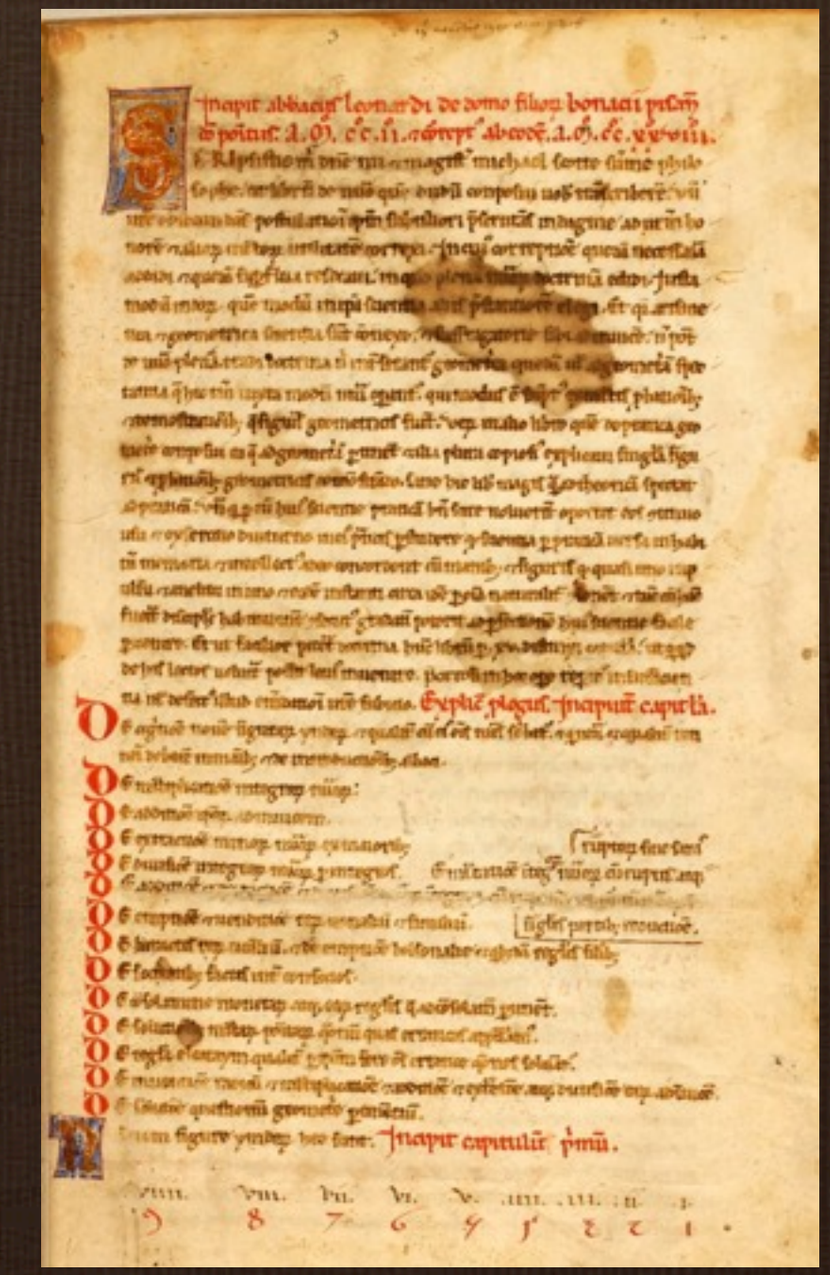
We do not know. The portrait often used to accompany references to him is from an engraving of relatively recent but unknown origin, and there is no reason to assume it is anything other than a work of fiction.

The Leonardo statue you can visit in the Camposanto in Pisa is the work of Giovanni Paganucci, a sculptor in Florence, who completed it in 1863. There is sufficient similarity to the face in the engraving that the sculptor may have used that as his source, merely aging his long deceased subject to portray the successful older man he became. These are the only two images we have. (A photograph of a rather dirty statue you may come across are of the Paganucci statue when, for a few years, it was displayed outside in a small public garden. It was cleaned and returned to its original home in the Camposanto around 1990.)

What else did Leonardo write?

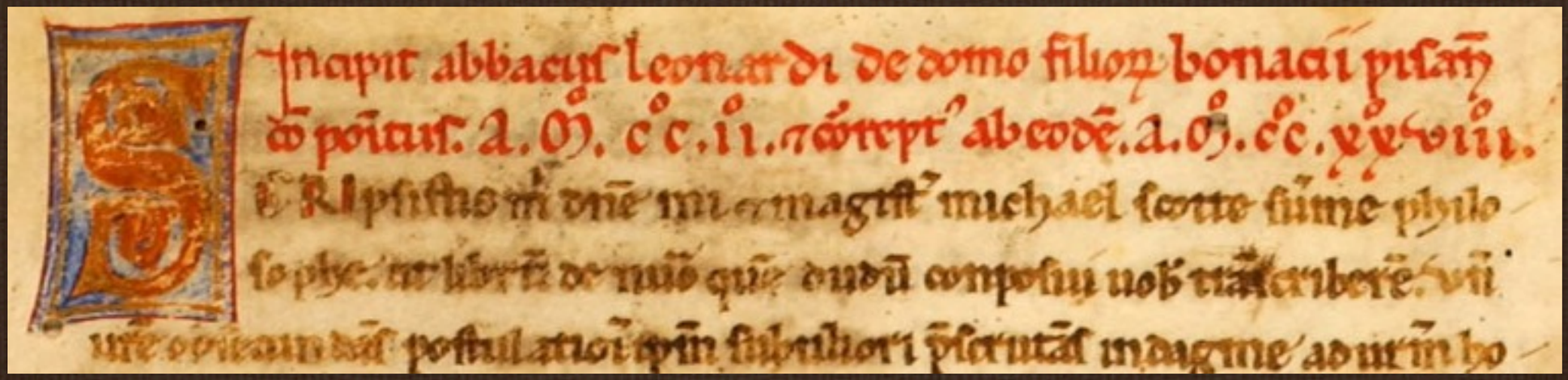
The first edition of *Liber abbaci* has not survived. Leonardo completed a second edition in 1228, and several copies exist dating from the late 13th or early 14th centuries. He wrote a shorter, simpler account of Hindu-Arabic arithmetic, *Libro di minor guisa* ("Book in a smaller manner") aimed at merchants, which modern historians think was the work most responsible for popularizing the new methods in Europe, but no copies have been found.

Three other books survive: *Practica geometriae* (1220) is a geometry textbook based on Euclid's writings, *Flos* (1225) presents Leonardo's solutions to a series of challenge problems posed to him in a public display by a mathematician in the Court of the Emperor Frederick II, and *Liber quadratorum* ("Book of Squares", 1225) is an impressive number theory book which, among other things, examines methods to find Pythagorean triples.



The first page of a copy of the 1228 edition of *Liber abbaci* kept in the Siena Public Library, believed to date from the late 13th Century.

What was his real name?



"Here begins the Book of Calculation composed by Leonardo Pisano, Family Bonaci, in the year 1202. [...]"

For sure it was not Fibonacci. That name was coined by the historian Guillaume Libri in 1838. According to the tradition of the time, he would have been known as "Leonardo Pisano" ("Leonardo of Pisa"). In his opening statement in *Liber abbaci* (shown above) he refers to himself as *filius Bonacci*, a Latin phrase that translates literally as "son of Bonacci." But Bonacci was not his father's name — as Leonardo would have known full well — so we should perhaps translate the phrase as "of the Bonacci family". Bonacci may have been the name of Leonardo's grandfather, with Bonacci then being, in colloquial terms, the "father of the family." "*Filius Bonacci*" is the origin of Libri's "Fibonacci" nickname. Leonardo occasionally referred to himself as *Bigollo*, which was a Tuscan dialect term that can sometimes mean a traveler, but there is considerable uncertainty as to the origin or meaning of the term in this connection. (In some dialects the word also meant "blockhead," but since Leonardo used the term himself, that surely was not his intended meaning.)

What do we know about Leonardo's family?

Leonardo's father was called Guilichmus, which translates as "William". (The variant Guilelmo is also found in references.) He had a brother, whose name was Bonaccinghus. And that's all we know. We know nothing of his mother, and have no idea if Leonardo married or had any children.

Guilichmus was a trader in Pisa. When Leonardo was born, Italy was a center of the vastly important, and still rapidly growing, international trade between the countries that fanned out from the Mediterranean Sea. Pisa, along with Italy's other maritime cities, Genoa to the north and Venice on the northeastern coast of Italy, dominated the trade, and their ships sailed constantly from one Mediterranean port to another. The merchants in those three cities were the key figures who were shaping the development of a new, more cosmopolitan world. When Leonardo was still a child, Guilichmus was posted to the North African port of Bugia (now Bejaïa, in Algeria) to act as a trade representative and customs official.

The Latin inscription on the plinth reads: *A Leonardo Fibonacci Insigne Matematico Pisano del Secolo XII* ("To Leonardo Fibonacci, noted mathematician of Pisa of the 12th Century"). A bit inaccurate since, though born around 1170, Leonardo wrote all his books in the 13th Century.

The Fibonacci sequence

The name “Fibonacci” is most widely known today not because Leonardo was the man who introduced modern arithmetic into Europe, rather in connection with the “Fibonacci sequence”, an unending sequence of natural numbers that begins

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...

The rule for generating new numbers in the sequence is that each number is the sum of the two preceding numbers, so 1+1=2, 1+2=3, 2+3=5, etc. This sequence arises when you solve a particular problem Leonardo gave in *Liber abbaci* (p.404 of Laurence Sigler’s English translation):

How Many Pairs of Rabbits Are Created by One Pair in One Year.

A certain man had one pair of rabbits together in a certain enclosed place, and one wishes to know how many are created from the pair in one year when it is the nature of them in a single month to bear another pair, and in the second month those born to bear also.

The numbers of pairs each month are the Fibonacci numbers. You can read off the answer to Leonardo’s problem: 377 pairs. The problem and its solution dates back well before Leonardo. The first known reference is in the *Chandahshastra* (“The Art of Prosody”) written by the Sanskrit grammarian Pingala some time between 450 and 200 BCE. Prosody was important in ancient Indian ritual. In the sixth century, the Indian mathematician Virahanka showed how the sequence arises in the analysis of metres with long and short syllables. Around 1150, the Jain philosopher Hemachandra composed a text on them.

Cont...

The Fibonacci sequence – 2

In other words, Leonardo did not discover the sequence that bears his name, and for which he is now most famous. The sequence was given that name by the French mathematician Edouard Lucas in the 1870s, after Guillaume Libri gave Leonardo the nickname Fibonacci. The rabbit problem was just one of many hundreds of worked examples Leonardo gave in *Liber abbaci* to help people master the new arithmetic.

The Fibonacci numbers in nature

For reasons that are now fairly well understood, the Fibonacci numbers arise frequently in plant growth. The number of petals on a flower is often a Fibonacci number: 3 for an iris; 5 for primroses, buttercups, wild roses, larkspur, and columbine; 8 for delphiniums, 13 for ragwort, corn marigold, and cineria; 21 for asters, black-eyed Susan; 13, 21, or 34 for daisies; and 55 or 89 for Michaelmas daisies.

For another example, if you examine a sunflower head or the base of a pine cone you will notice spirals going in opposite directions. If you count them you will find that a sunflower has 21, 34,55, 89, or 144 clockwise paired respectively with 34, 55, 89, 144, or 233 counterclockwise, and a pine cone has 8 clockwise spirals and 13 counterclockwise. Check out pineapples as well.

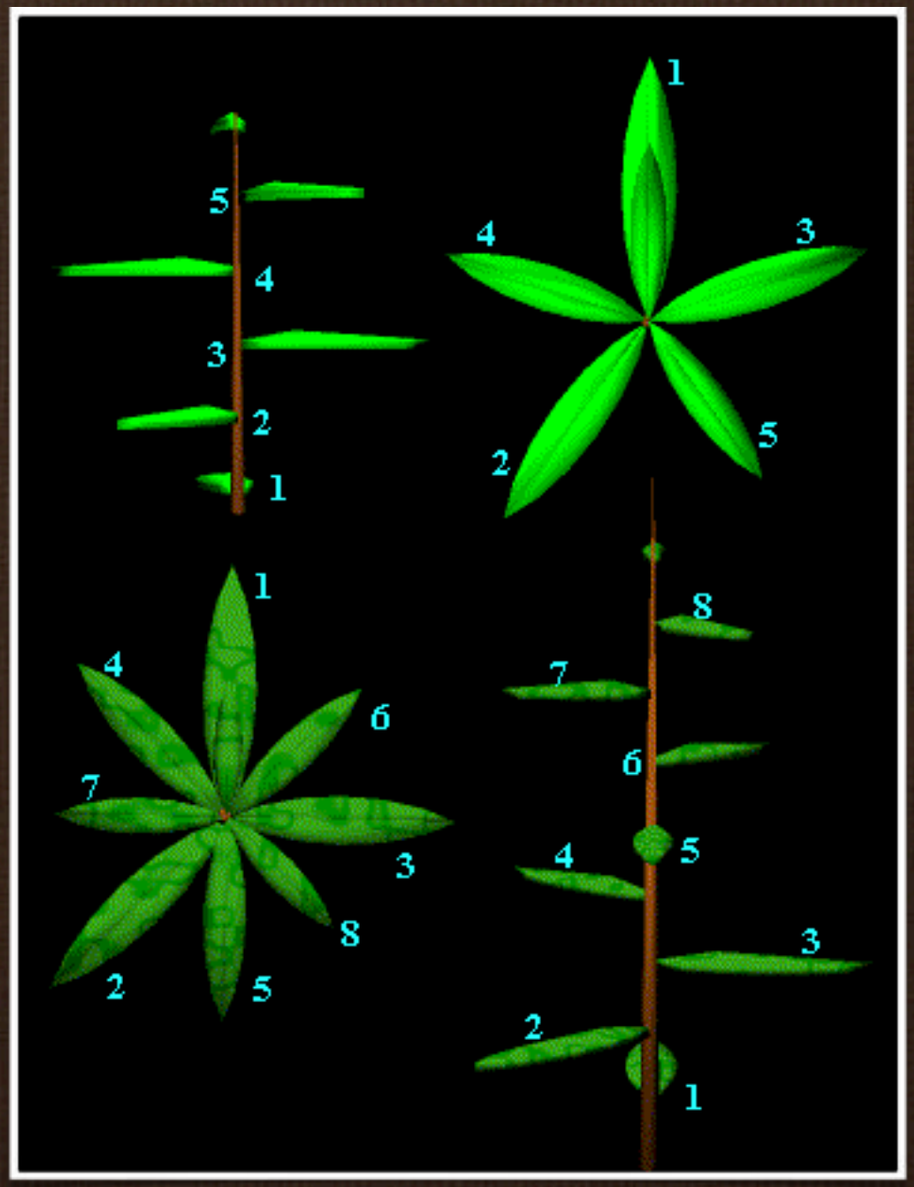
Cont...

The Fibonacci sequence – 3

A third example arises in phyllotaxis, the study of the arrangement of leaves on plant stems. As you proceed up the stem, the leaves spiral round. If you start at one leaf, there will be a fixed number p (depending on the species of plant) of complete turns of the spiral before a second leaf lies directly above the first. Let q be the number (also constant for each species) of leaves you encounter in going from the first leaf to the last in this process (excluding the last one). The (unreduced) ratio p/q is called the divergence of the plant.

Common divergences are: elm, linden, lime, and some common grasses 1/2; beech, hazel, blackberry, sedges, and some grasses 1/3; oak, cherry, apple, holly, plum, and common groundsel 2/5; poplar, rose, pear, and willow 3/8; almonds, pussy willow, and leeks 5/13. All pairs of Fibonacci numbers.

All these appearances of the Fibonacci numbers in nature are a result of the way plants grow. Claims about the relevance of the Fibonacci sequence to the behavior of the stock market and other human activities should be taken with a large grain of salt.



Plant divergences

Pisa

Pisa: The Piazza dei Miracoli

The Leaning Tower

The Baptistry, the Cathedral, and the Leaning Tower

The Camposanto

Cont...

Pisa: The Piazza dei Miracoli

Clockwise from top-left: the Baptistry and the Camposanto with the Jewish Cemetery in the background; the Jewish Cemetery; an arched entrance to the Piazza; the Camposanto.

Pisa: The River Arno

In Leonardo's time, the River Arno was one of the busiest shipping ports in the world.

Cont...

Pisa: The River Arno

One of the two medieval customs houses.

Pisa: Medieval towers

Modern Pisa contains many medieval towers
of the kind Leonardo probably grew up in.

Livorno

Pisa's Mediterranean port in medieval times

Livorno – the medieval *Porto Pisano*

The medieval Porto Pisano has become the busy modern Mediterranean port of Livorno. The ancient harbor that was the focus of activity in Leonardo's time is now a leisure marina filled with pleasure craft. The modern docks are just a few kilometers to the north.

Cont…

Livorno – the medieval *Porto Pisano*

A wall carving in Livorno depicts the medieval customs house where Leonardo's father worked prior to being posted to Bugia.

In the 13th Century, a dirt road across open marshes connected Pisa to its Mediterranean port. Today, a modern road follow the same path.

Cont...

Livorno – the modern port

Cont…

The Statue

The statue is not here

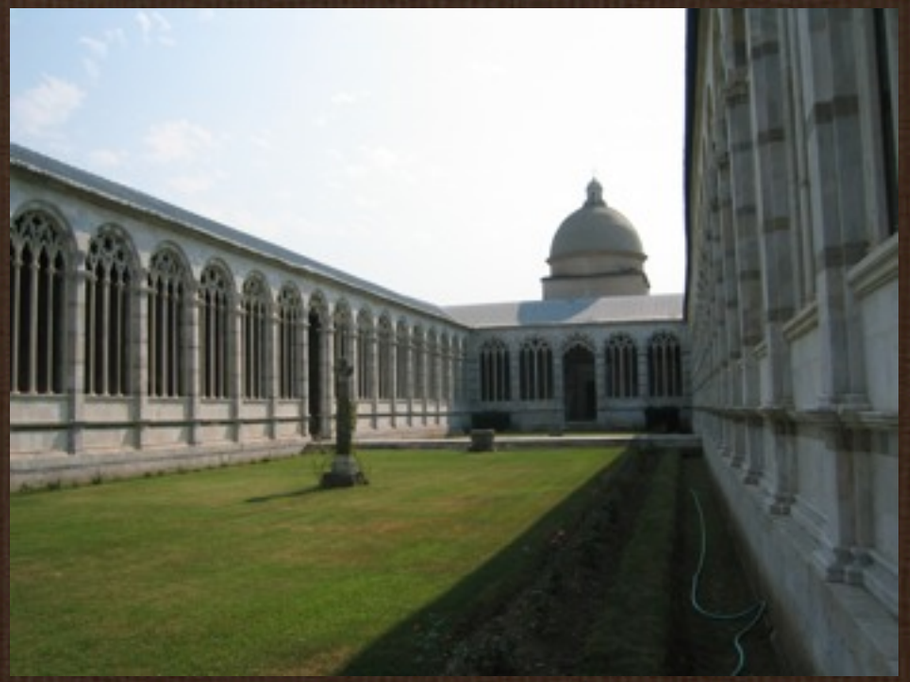
I looked for Leonardo's statue on my first research trip to Pisa in 2003. I went first to the Giardino Scotto, where I had seen a photograph of the statue, but it was not there. Other sources told me that it had been relocated in a cemetery in the Piazza dei Miracoli. (I learned subsequently that the statue had been removed from the garden around 1990 for cleaning and restoration, and then relocated.)

The only cemetery I could find was on the north west corner of the piazza, across from the Baptistry. But it was a Jewish cemetery, surely not where the city would locate a monument to the presumably Catholic Leonardo. The gate was locked with a chain, but by peering through I was able to tell that Leonardo's statue was indeed not there.

A short while later I discovered that the adjacent building, the Camposanto, translates as "Monumental Cemetery," and is the current (and original) home of the statue.

The Camposanto

The Exterior from the Piazza dei Miracoli



Inner courtyard

Inner cloister looking toward Leonardo statue at far end

Inner courtyard

The Leonardo statue in the Camposanto

Since the early 1990s, the Leonardo statue has been prominently displayed in the Camposanto

From 1926 to 1946 the statue was here

When Leonardo's statue was first unveiled on 17 June, 1863, it was in a prominent location in the Camposanto. In 1926, the Fascist authorities moved it to an outside location in a square in front of the *Logge di Bianchi*, looking out across the *Ponte di Mezzo* (Central Bridge), a main bridge across the Arno. The square was renamed *Piazza XX Settembre*, an important date in Italian history. It was still there at the end of the Second World War, when American and German troops fought an intense month long battle from opposite sides of the river. Remarkably, the statue survived, with just minor damage to the fingers. You can make it out clearly in the photograph on the left. The photograph on the right shows the same location in 2003, when I visited the site for the first time. After the war was ended, the statue was taken into storage while the bridge and the surrounding buildings were rebuilt. It remained in storage until 1966, when it was put on display in a small public garden, the Giardino Scotto.

In 1966, the statue was moved here

A postcard showing the Leonardo statue in a small public garden along the Lung'arno Leonardo Fibonacci

… where it got very dirty

Two photographs taken by Laurence and Judith Sigler in the 1980s, and one by Frank Johnson that can be found on many websites

Paying homage

Since the early 1990s, the Leonardo statue has been prominently displayed in the Camposanto, where I first saw it in 2003. The close-up photograph shows Leonardo's missing fingers, the sole damage caused in the carnage that destroyed much of the surrounding Piazza during a fierce battle at the end of the Second World War.

The Latin inscription on the plinth reads: *A Leonardo Fibonacci Insigne Matematico Pisano del Secolo XII* ("To Leonardo Fibonacci, noted mathematician of Pisa of the 12th Century").

The Declaration

The Commemorative Tablet

About 500 meters along the Arno from the *Ponte di Mezzo*, at 30 Lung'arno Mediceo, is an imposing, three-story building with attractively decorated grey stone walls. This is the *Palazzo Toscanelli*. It originally belonged to the Lanfranchi family, but then passed on to Alessandro della Gherardesca, who made a number of architectural modifications. In 1821-22, it was the home of Lord Byron and his circle of friends, during the period in which Pisa was called the "Paradise of Exiles". From here Lord Byron departed for Greece, where he met his death. Today the palace hosts the *Archivio di Stato* (State Archives) of Pisa.

On a wall in the entrance hall is a stone tablet commemorating Leonardo. It was dedicated on 16th June, 1867. Following an introductory declaration written in 1865, it reproduces the text of a document of 1241, by which the Comune of Pisa (the town government — at that time Pisa was an independent state or republic) decreed that an amount of money should be given annually to Leonardo for his merits.

This proclamation is the only indication we have that Leonardo was still alive in 1241.

Cont...

The Commemorative Tablet

DECLARATION (1865): The Rulers and People of Pisa in the year 1865 after ignoring old crushing falsehoods and where the will of the Elders was to study what was better known and proven about Leonard Fibonacci ordered the city archives to file a copy of the decree by the same Most Eminent Republic of Pisa that one monument equal to so great a man survive.

DECREE (1241): In consideration of the honor brought to the city and its citizens and their betterment by the teaching and zealous cooperation of that discreet and wise man, Master Leonardo Bigolli, as well as by his regular patriotic efforts in civic and patriotic affairs, the Pisan Commune and its Officials in certain right and conscious of our prerogative to make recompense for work that he performed in heeding and consolidating the efforts and affairs already mentioned confer upon this same Leonardo so meritorious of our love and appreciation an annual salary or reward from the Commune of 20 free denarii and the usual accompaniments. This we affirm with the present statement.

Cont…

ORDO · POPVLVSQ · PISANVS
QVO · VETERI · CONTEMPTVS · CALVMNIA · OBTRITA
MAIORVM · VOLVNTAS · STVDIVMQVE

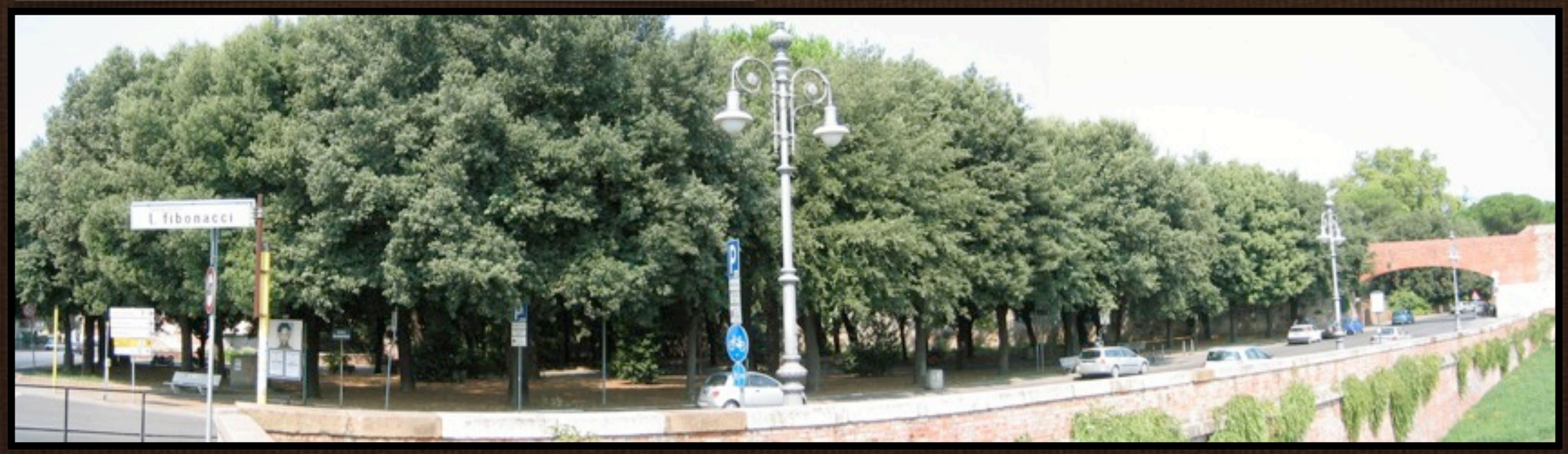
IN · LEONARDVM · FIBONACCIVM

NOTIORA · TESTATIORAQ · ESSENT
AMPLISSIMI · PISANAE · REIPVB · DE · EODEM · DECRETI · EXEMPLAR
QVOD · VNVM · SVPEREST · TANTO · VIRO · AEQVALE · MONVMENTVM
IN · PATRIO · TABVLARIO · AFFIGI · IVSSIT · A · $\overline{\text{MDCCCLXV}}$ ·

CONSIDERANTES · NOSTRE · CIVITATIS · ET · CIVIVM HONOREM · ATQVE · PROFECTVM · QVI · EIS · TAM · PER DOCTRINAM · QVAM · PER · SEDVLA · OBSEQVIA · DISCRETI ET · SAPIENTIS · VIRI · MAGISTRI · LEONARDI · BIGOLLI IN · ABBACANDIS · ESTIMATIONIBVS · ET · RATIONIBVS CIVITATIS · EIVSQVE · OFFICIALIVM · ET · ALIIS · QVOTIES EXPEDIT · CONFERVNTVR · VT · EIDEM · LEONARDO MERITO · DILECTIONIS · ET · GRATIE · ATQ · SCIENTIE · SVE PRERROGATIVA · IN · RECOMPENSATIONE · LABORIS · SVI QVEM · SVBSTINET · IN · AVDIENDIS · ET · CONSOLIDANDIS ESTIMATIONIBVS · ET · RATIONIBVS · SVPRADICTIS · A COMVNI · ET · CAMERARIIS · PVBLICIS · DE · COMVNI ET · PRO · COMVNI · MERCEDE · SIVE · SALARIO · SVO ANNIS · SINGVLIS · LIBRE · $\overline{\text{XX}}$ · DENARIORVM · ET AMISCERIA · CONSVETA · DARI · DEBEANT · IPSEQVE PISANO · COMVNI · ET · EIVS · OFFICIALIBVS · IN ABBACATIONE · DE · CETERO · MORE · SOLITO SERVAT · PRESENTI · CONSTITVTIONE · FIRMAMVS ·

Street signs

Lung'arno Leonardo Fibonacci in Pisa



The Giardino Scotto is behind the trees. The marble street sign is on the brick wall on the far side of the bridge to the right.

Via Leonardo Fibonacci in Florence

This single short street boasts no fewer than eleven “Via Fibonacci” street signs. Here are seven.

Cont...

Via Leonardo Fibonacci in Florence

The other four, and one of a number of house plates, this one for the Italian Computer Society.

The Siena manuscript

L.IV.20

The Siena manuscript

The manuscript, which is believed to date from the late 13th or early 14th Century, is bound into book form. It is large and heavy, measuring over 20 cm wide by 30 cm high and over 5 cm thick. The thick cover is a discolored brown, with gold lettering. The spine bears the inscription

LION. PISANI
DE ABACO

near the top, and the Siena Public Library reference number

L.IV.20

near the bottom. (Yes, it is just another reference book in the Public Library. Anyone can view it.)

The manuscript, which is missing much of the final chapter, comprises 224 sheets, each one written on both sides. The page numbers — running from 1 to 224, appropriately in Hindu-Arabic numerals — have been added later, one on the top center of the front side of each sheet. The parchment is thick and stiff but not brittle. Each sheet measures 20.5 cm wide by 30 cm high. Discounting the cover, the manuscript is 5 cm thick. Apart from the front page, which has partly disintegrated and been attached to a backing sheet, each page is in remarkably good condition. An occasional page has a hole in it, and the outside edges of some pages have worn away under the cumulative influence of eight hundred years worth of page-turning hands.

Many pages are a whitish-cream color, others a light brown, and many are discolored. Each page had been carefully ruled with a grid to guide the lettering, and the text is written in brown ink, with every numeral in red. At the start of some paragraphs, the initial letter is enlarged and also in red. In addition, the scribe had embellished some chapter openings with large stylized letters in blue and gold.

Despite the damage, the first page still shows the scribe's original title near the top:

Aritmetica Leonardi Bigholli de Pisa

The Siena Public Library

The Biblioteca Communale di Siena occupies a stone building dating from the mid 18th Century on Via della Sapienze ("Street of Knowledge"), a short walk from the Piazza del Campo, the main square of Siena. When I visited in 2003, and took these photographs, the rather ornate entrance was at number 5 (bottom left photo).

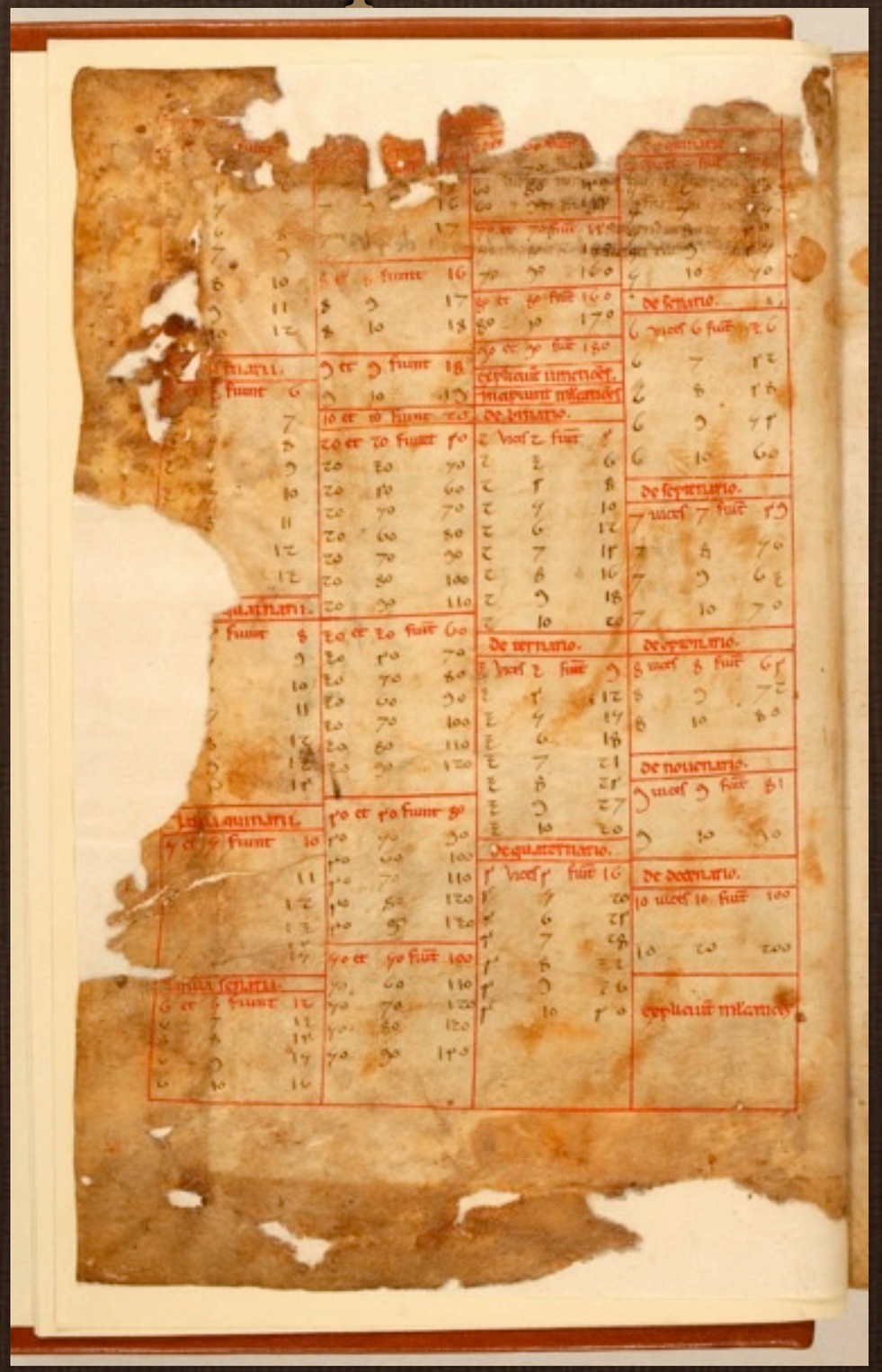
Cont...

The Siena Public Library

On my return to the library in 2009, I discovered that the ornate entrance at number 5 had been closed off (center and right pictures) and access was now through a newly installed security entrance at number 3 (left picture, nearest door). It was late in the year, and early in the morning when I took these photos, hence the grey sky and lack of crowds and colorful flags.

The Siena Manuscript

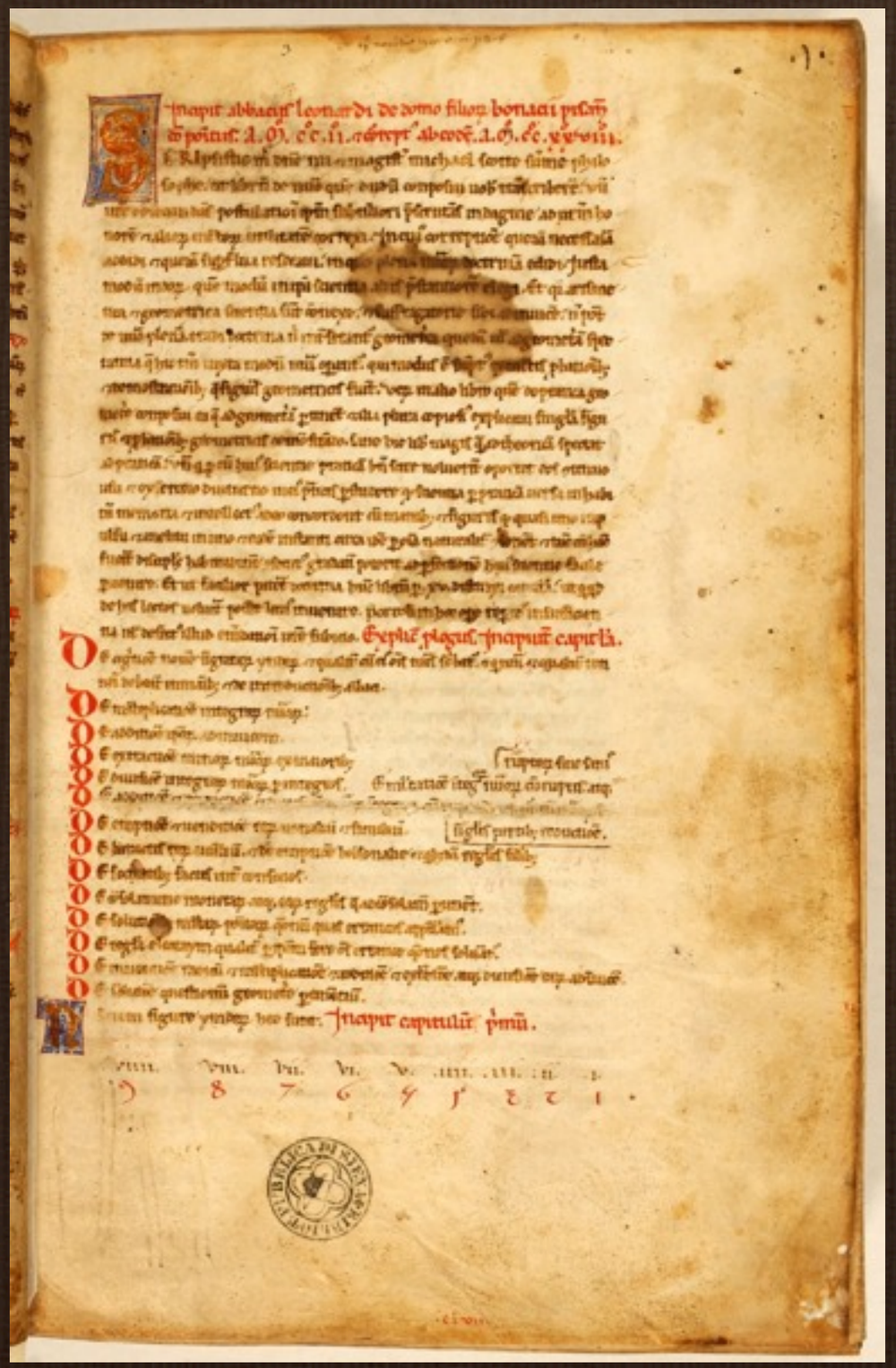
1a



1b

Cont...

The Siena Manuscript

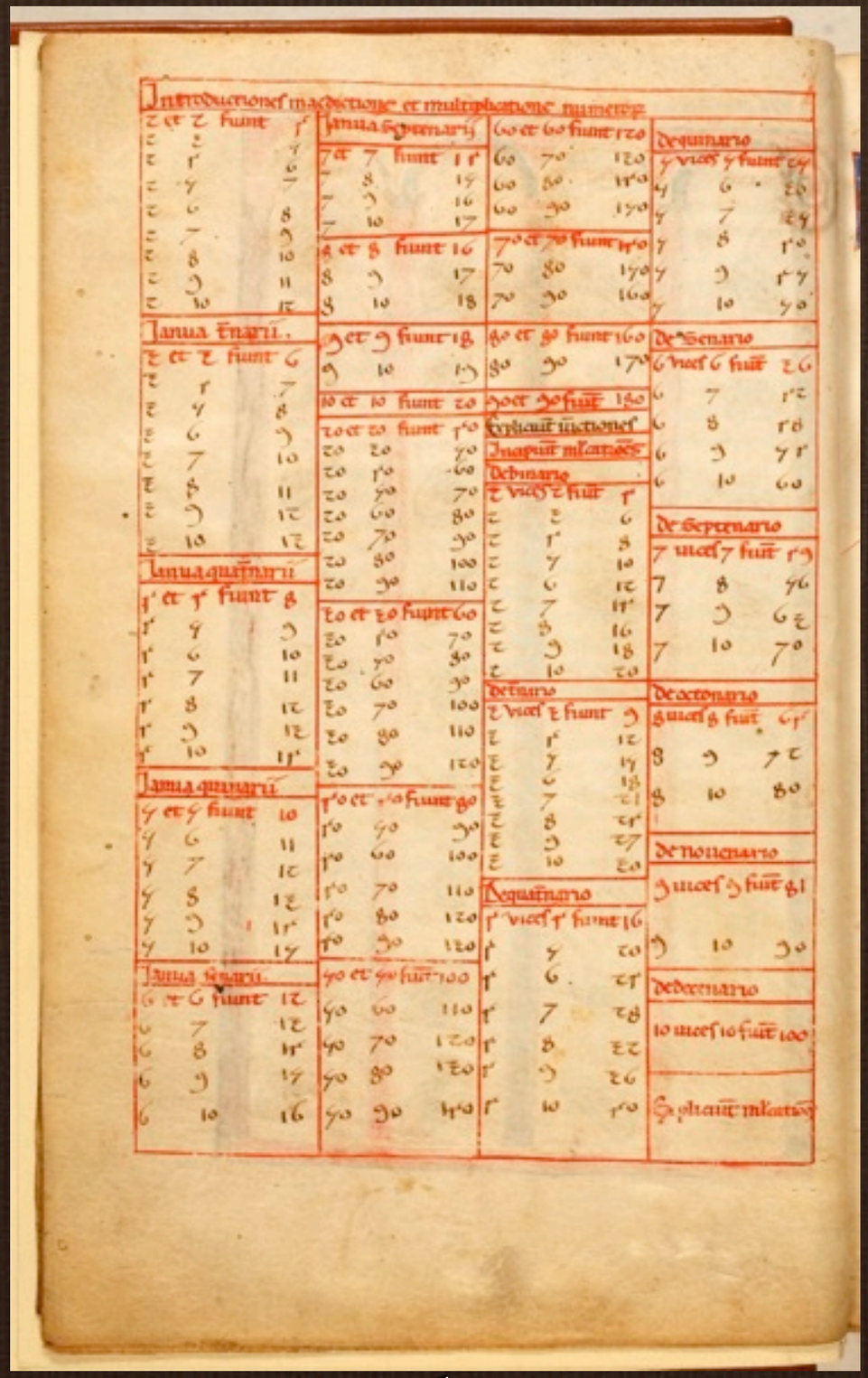


3a

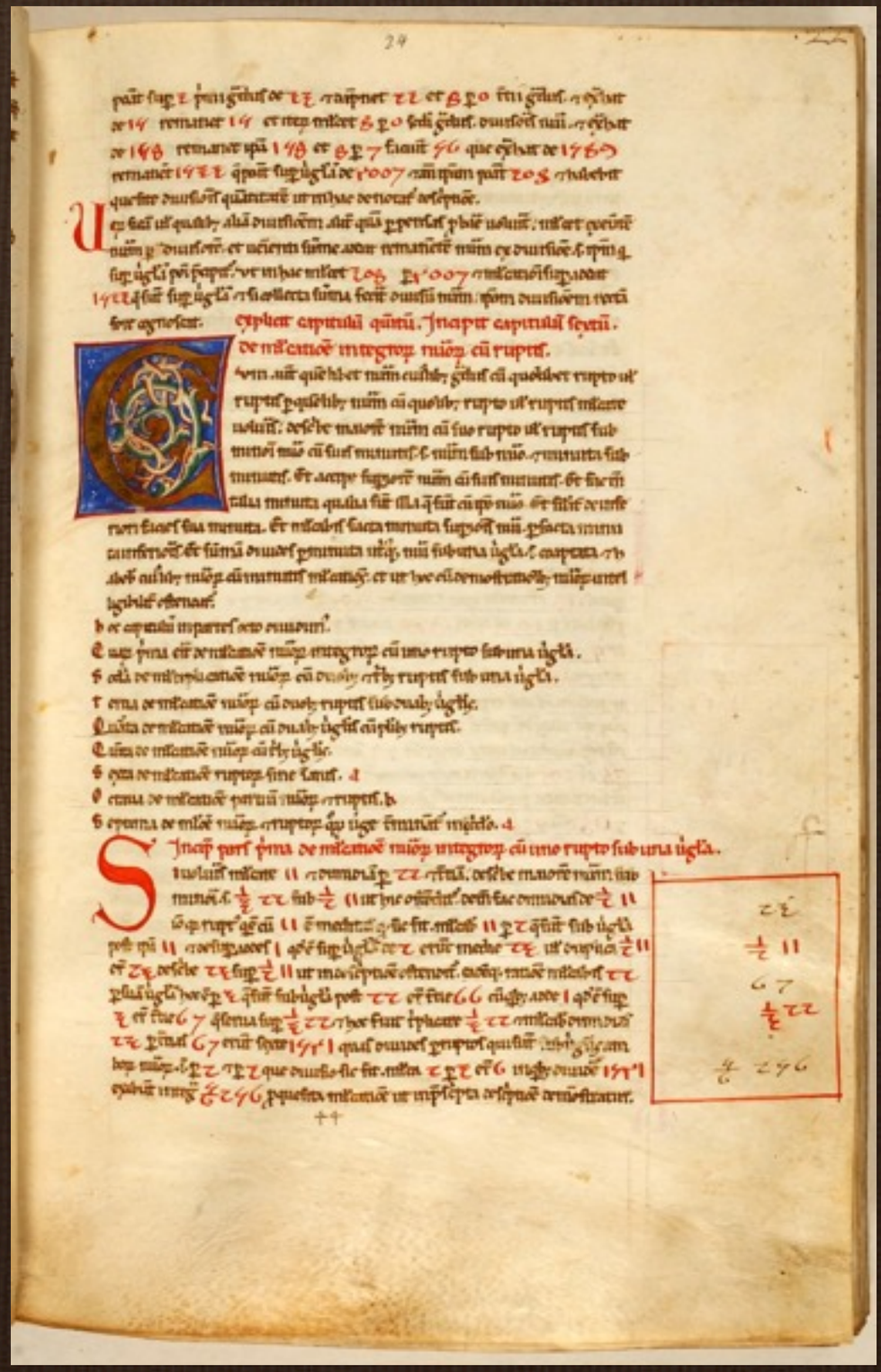
5a

Cont...

The Siena Manuscript



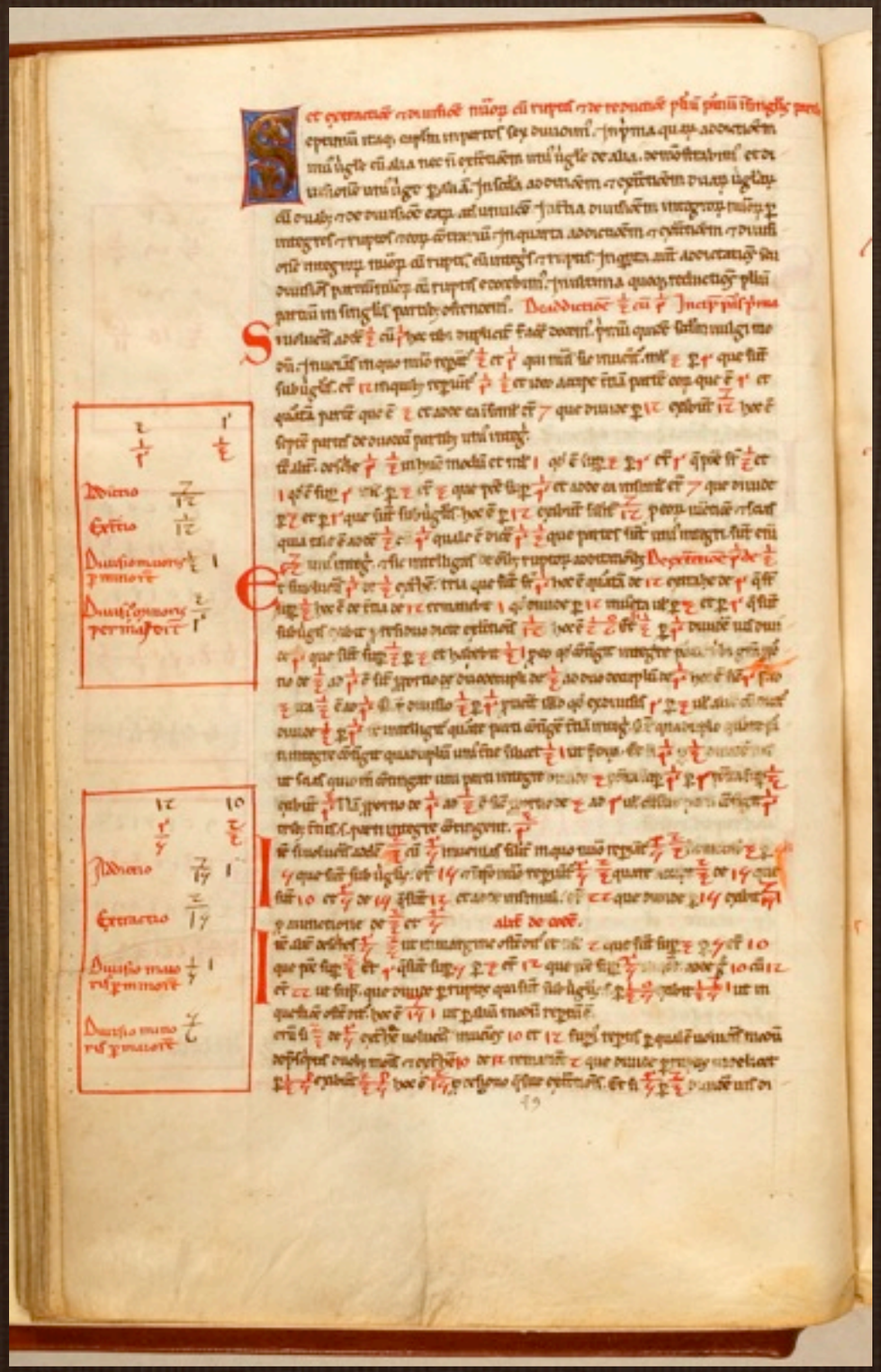
5b



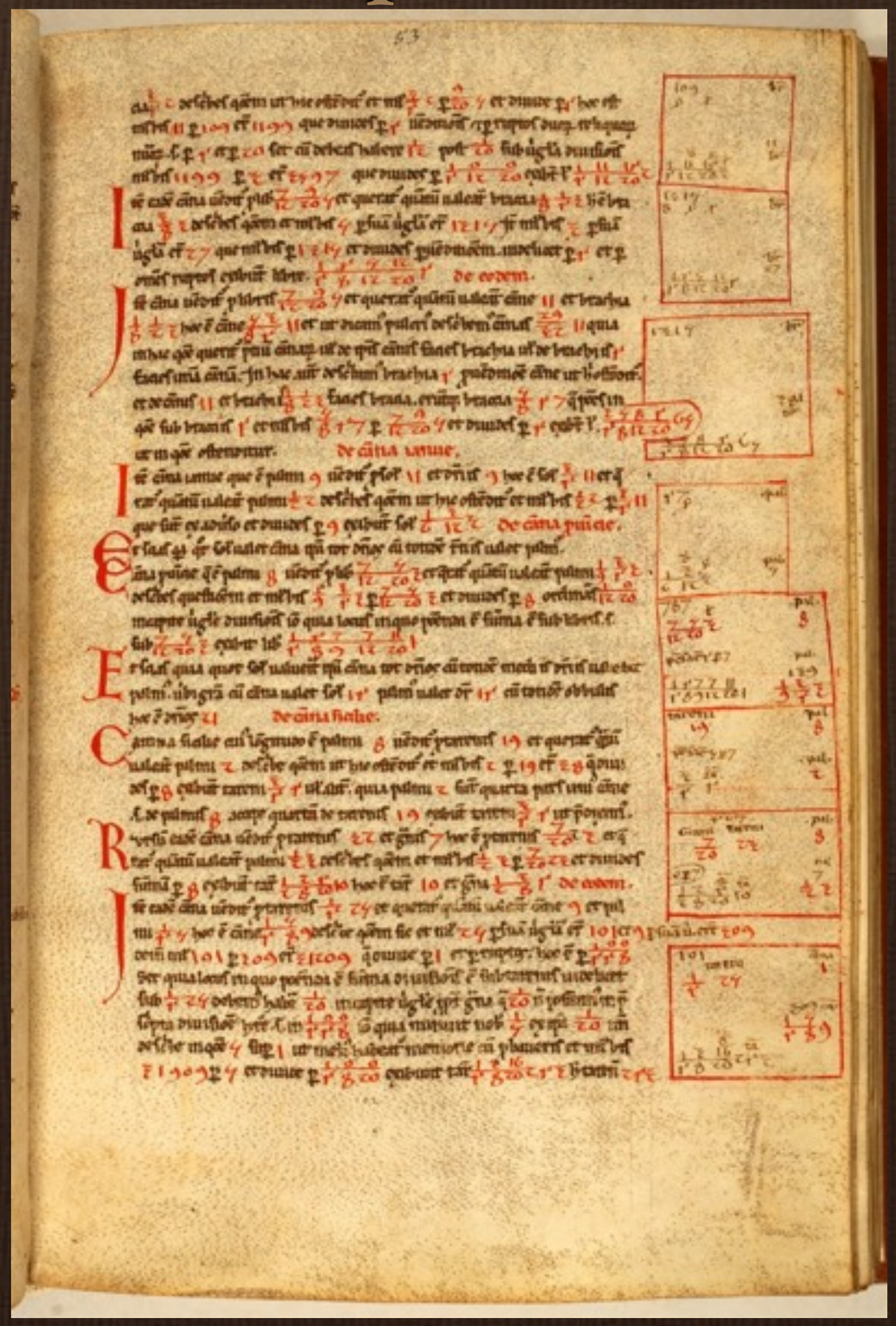
24a

Cont...

The Siena Manuscript



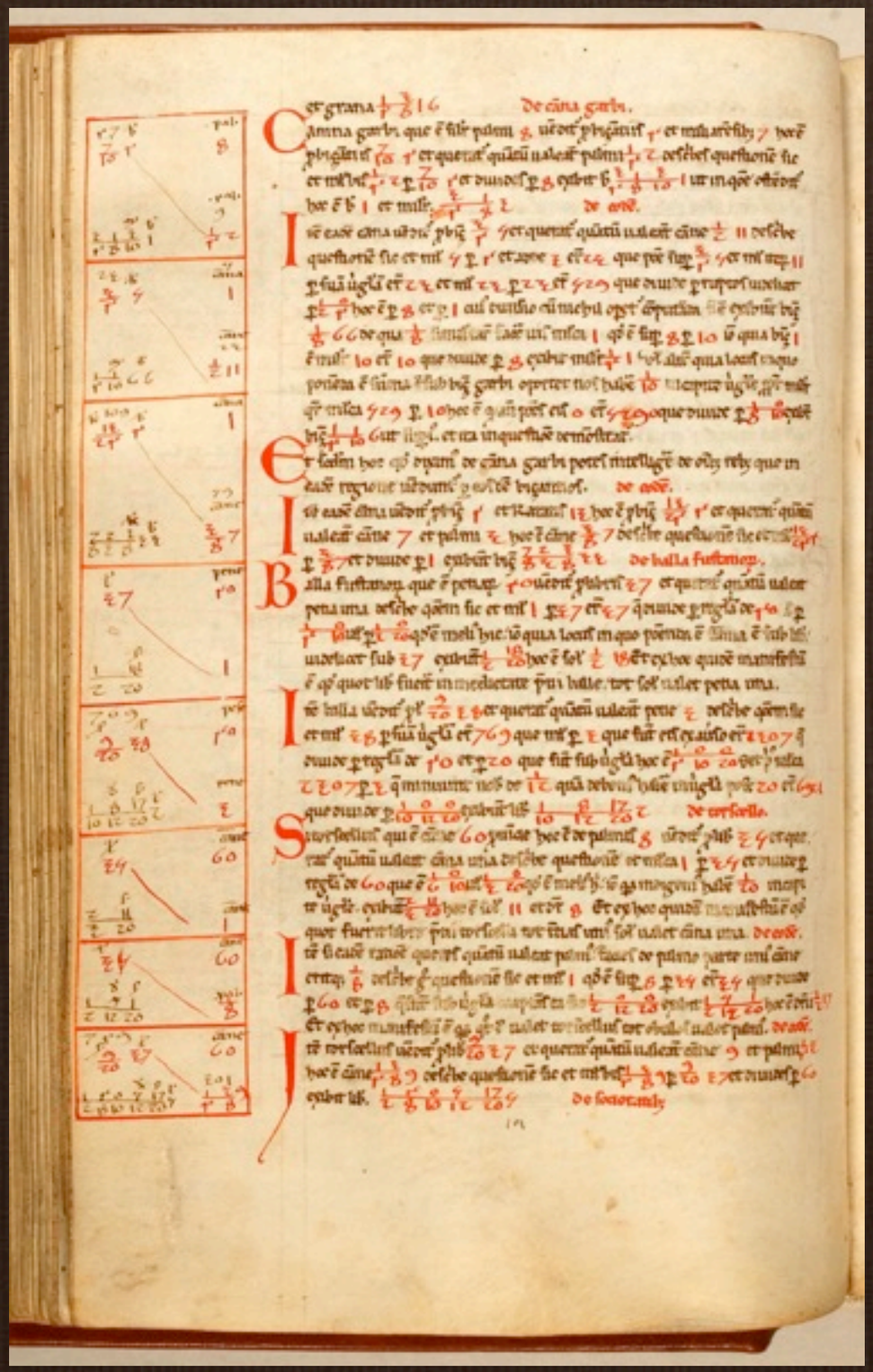
31b



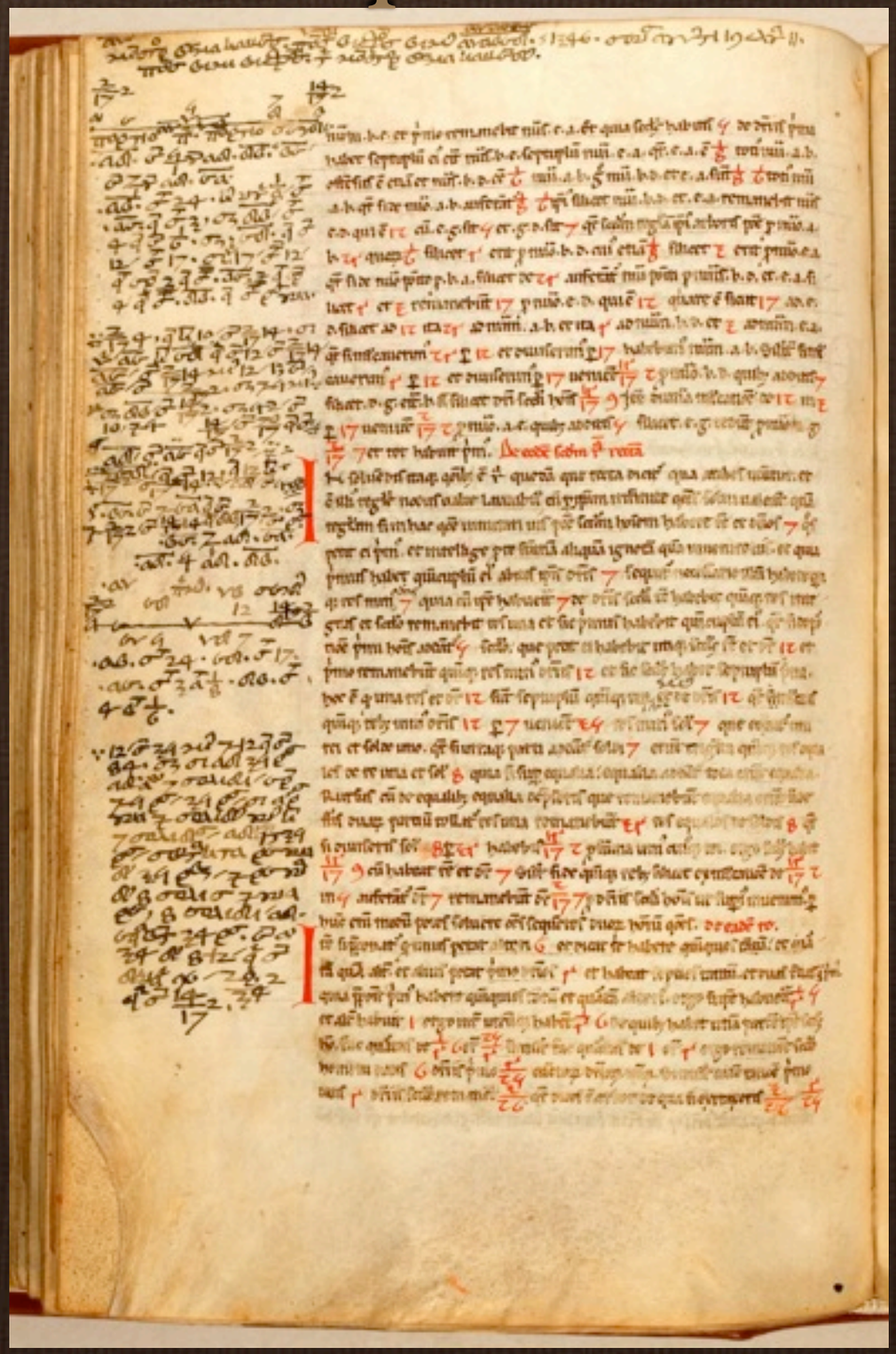
53a

Cont…

The Siena Manuscript



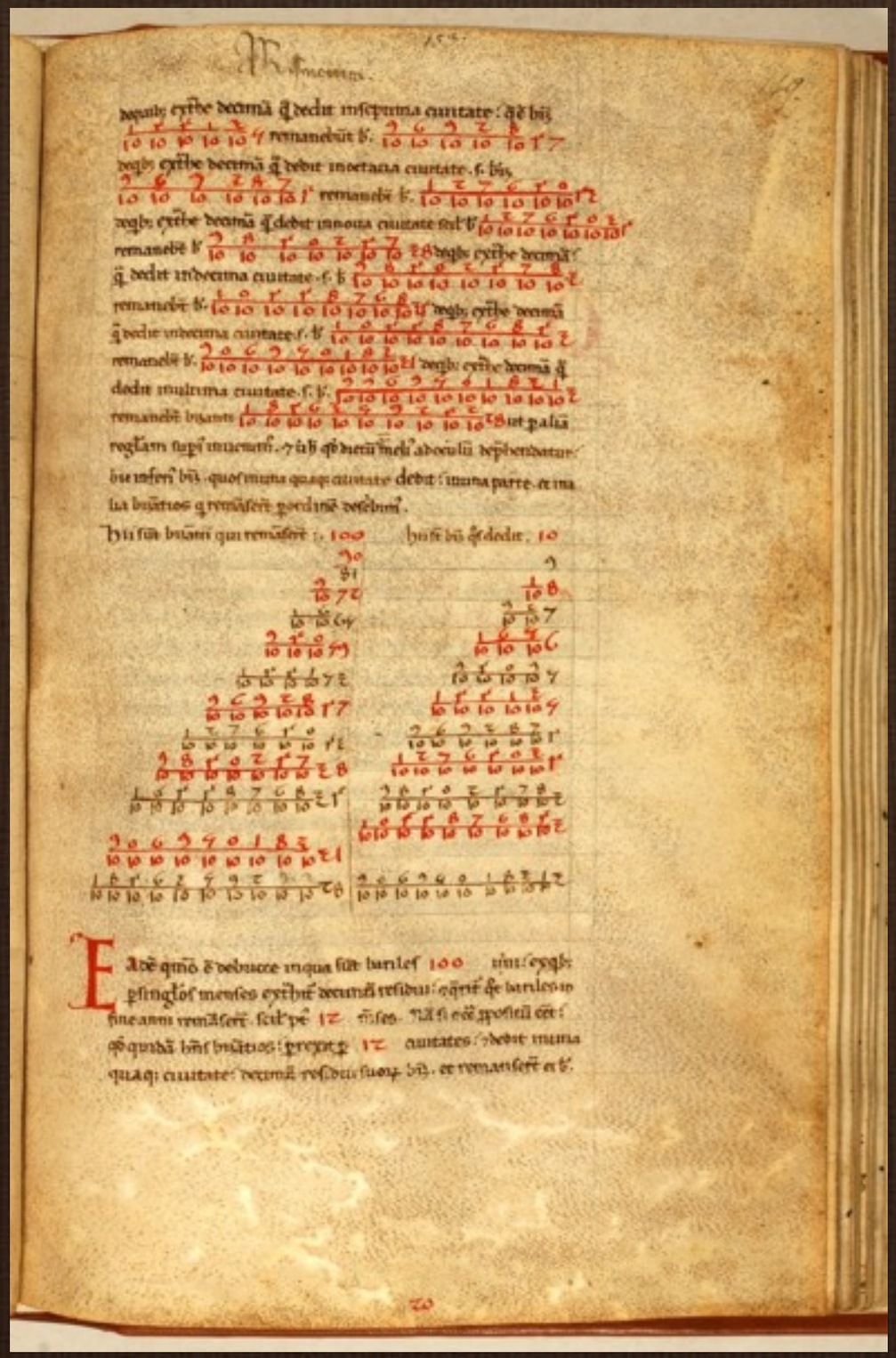
53b



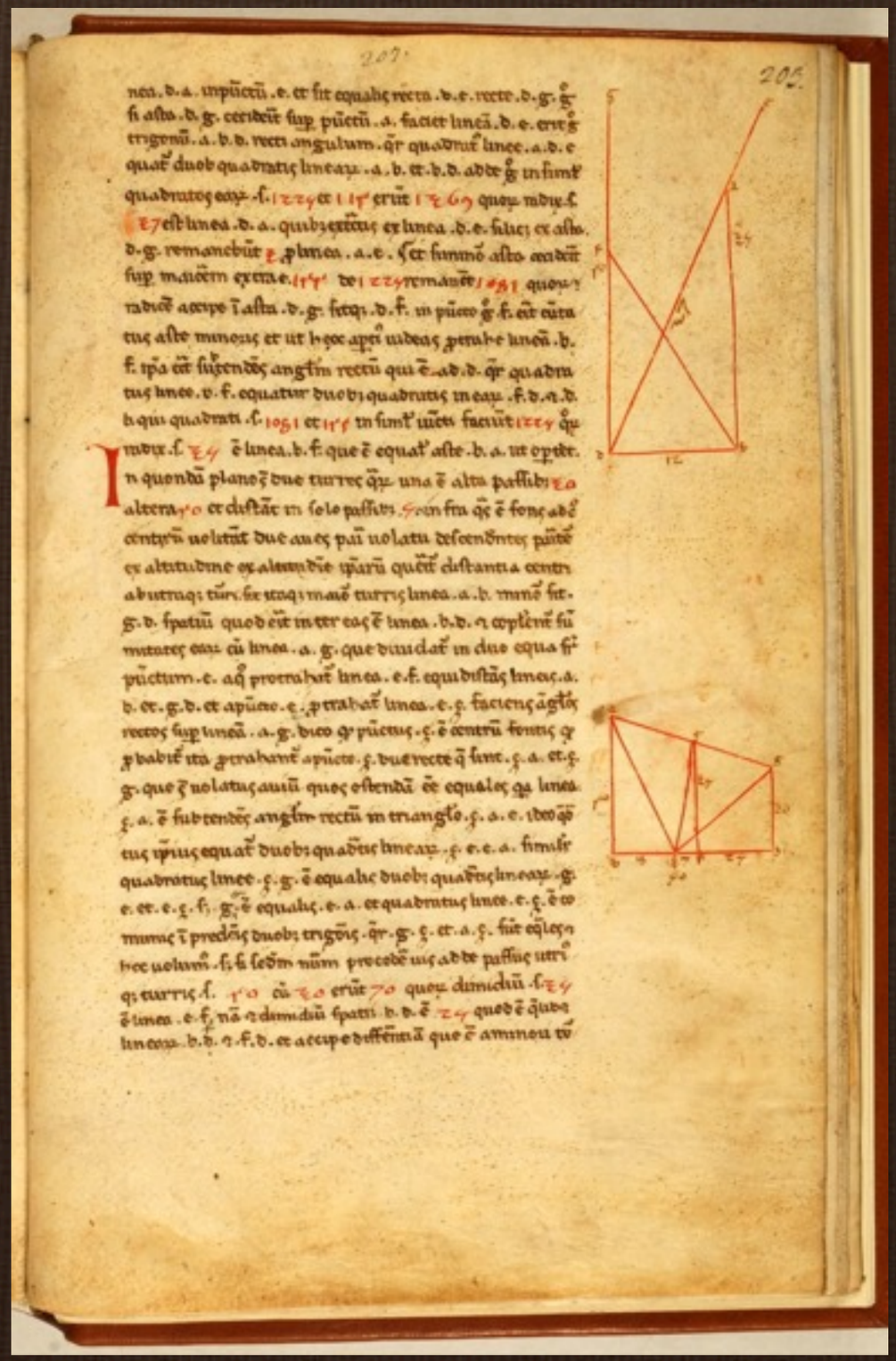
87b

Cont...

The Siena Manuscript



153a



207a

The Florence manuscript

Conv. Soppr.C.1.2616

The Florence manuscript

The volume is bound into book form, thinner than the one in Siena, but about the same height and width. It fastens with two brass clips. There is no writing on the outside cover. Each page is 20 cm wide by 30 cm high, the same size as the one in Siena. Discounting the cover, it measures 4 cm thick, a full centimeter less than L.IV.20. There are 213 pages, each written on both sides, giving 426 sides altogether. Almost the same number as the Siena copy.

On the inside of the front binding are pasted two tiny fractions of manuscript, all that is left of the original cover page. All I could make out when I examined it, and then not with total certainty, was:

Leonardi Pisani Algorism A[ritm]etica

The first page of the manuscript itself bears the following legend at the top of the page:

A C Leonardus pisanus Algorisma & Geometrie est Abbacie florenty

At least, that's what I transcribed. The writing is badly faded.

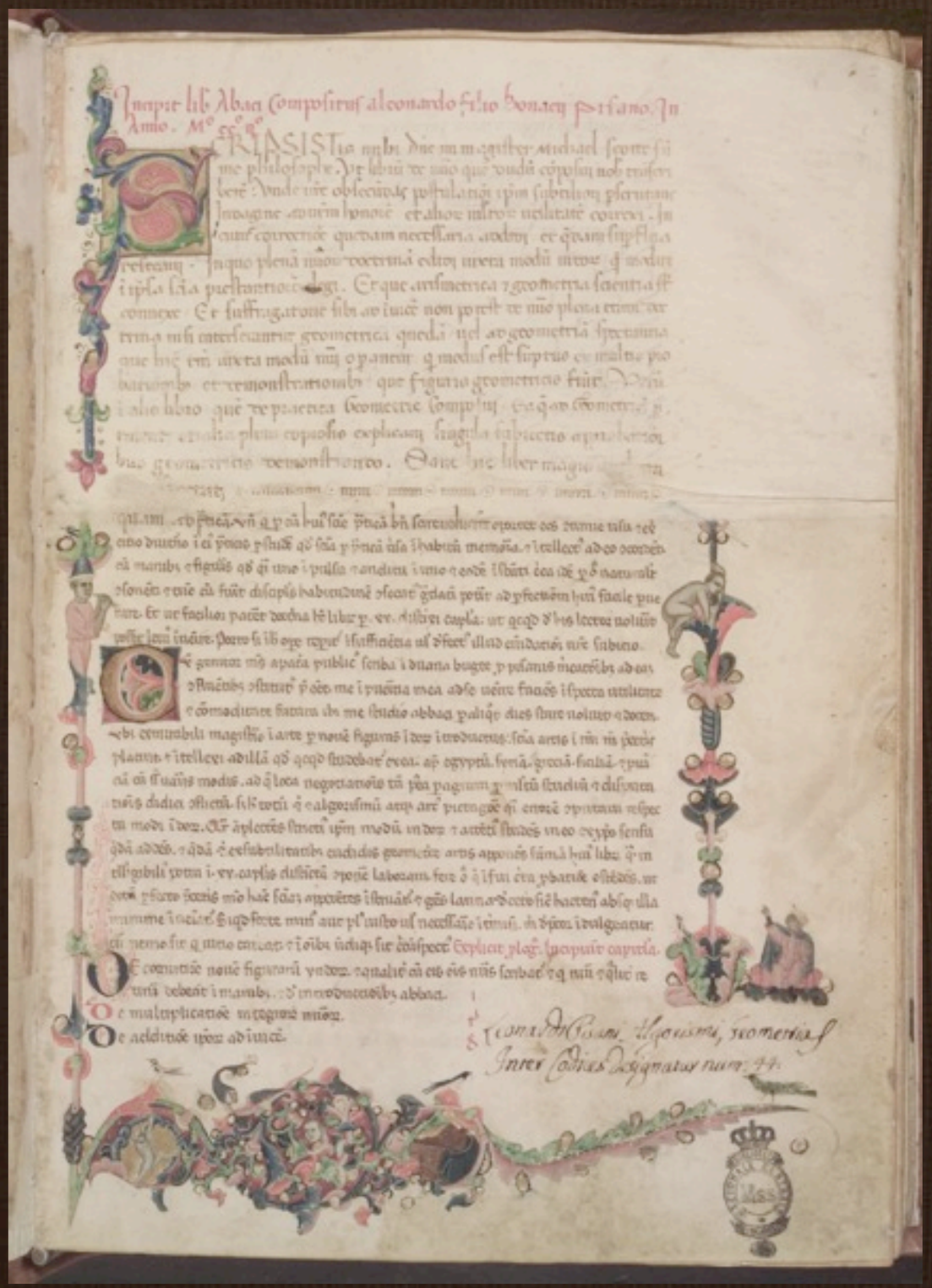
Despite some faded pages (some so badly it is hard to discern the text), overall the manuscript seems to be in much better condition than the one in Siena. The paper is dry but not brittle, and feels slightly thinner than that of the Siena copy — which explains why the bound volume is much thinner, even though the page count is almost the same. None of the pages have holes or worn edges, as is the case with L.IV.20. The basic color scheme is the same, with the Latin text in a brown-black ink and all numerals in red. The scribe who had written it certainly had the greater artistic flair of the two, decorating the margins of many pages with fancy swirls, and making much more extensive use of fancy red letters to begin paragraphs and large red, blue, and gold fancy letters to start new sections. The paper of many pages has turned brown or grey and become spotty. On some pages you can see the guidelines the scribe had used to line up the text. But all in all, it is in great shape, given its age.

The Florence National Central Library

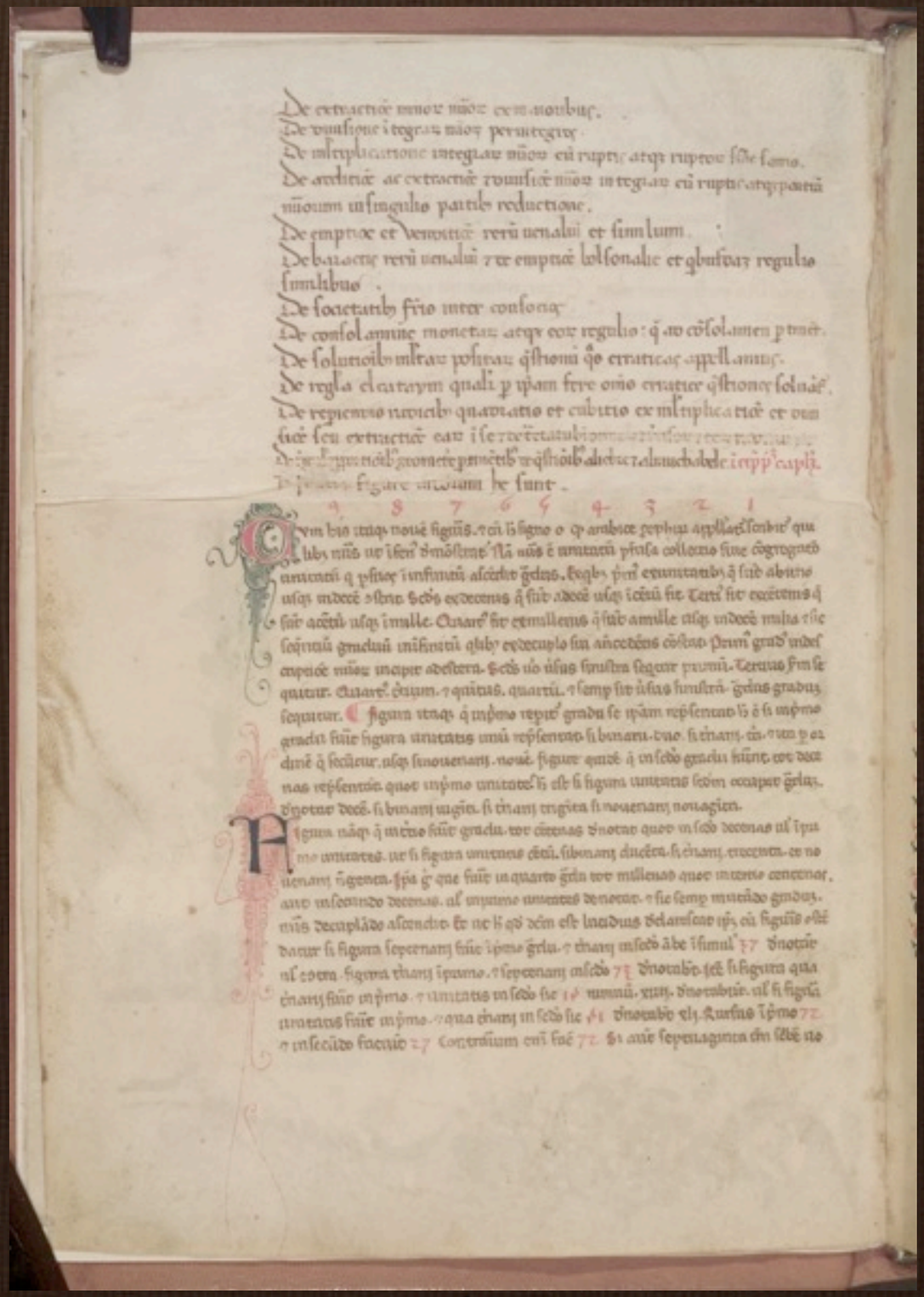
The Florence manuscript is believed by some scholars to be about the same age as the one in Siena, though others say it is more recent, though no more so than the early part of the 14th Century. It is kept in the Biblioteca Nazionale Centrale di Firenze (BNCF), the "Florence National Central Library").

The BNCF occupies an imposing, stone building on the Piazza dei Cavalleggeri, a small square on the north bank of the River Arno. The manuscript bears the reference number Conventi Soppressi ("Supressed Convent" texts) C.1.2616. You have to register with the library to view it in the reading room.

The Florence Manuscript



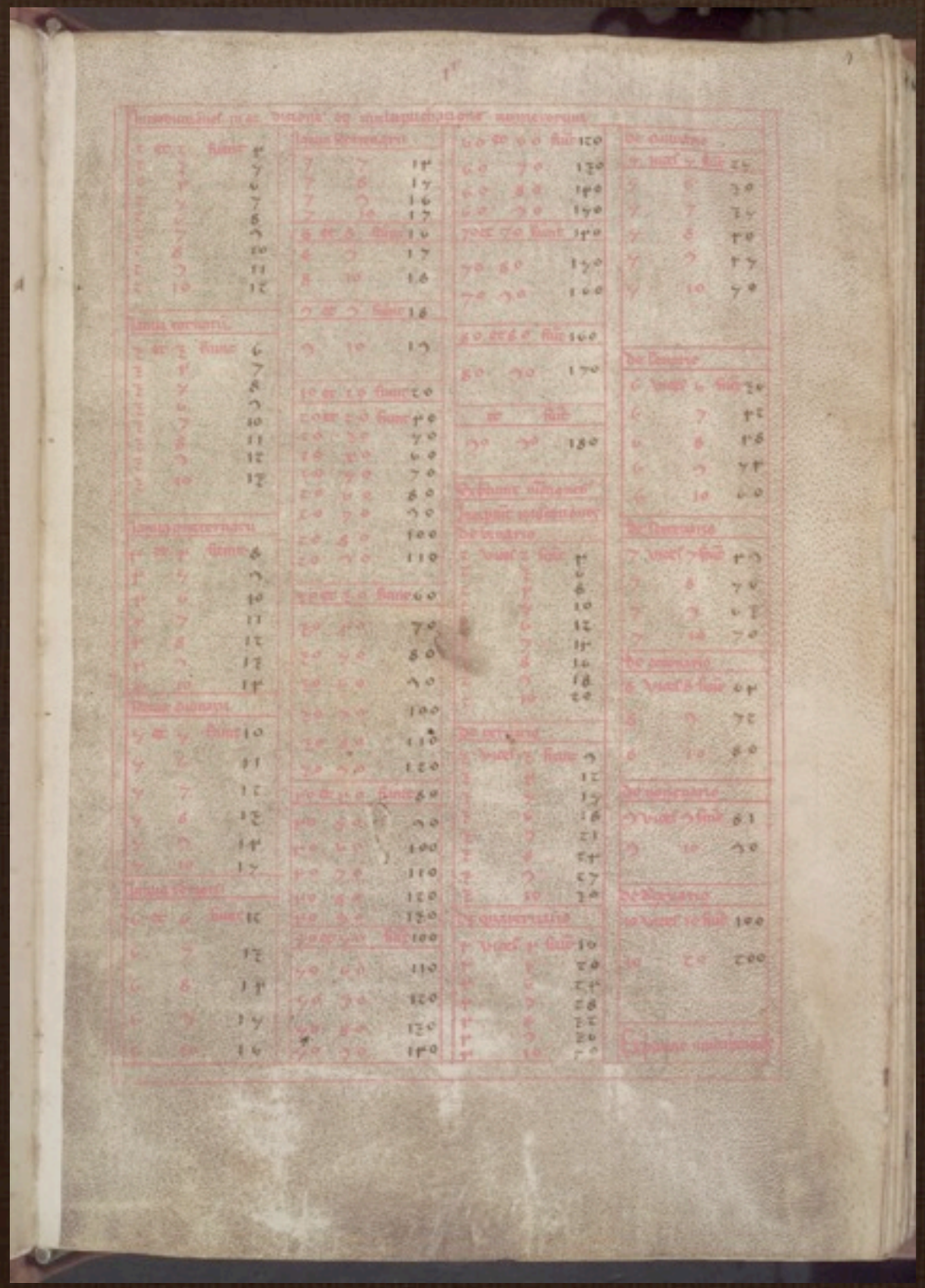
1a



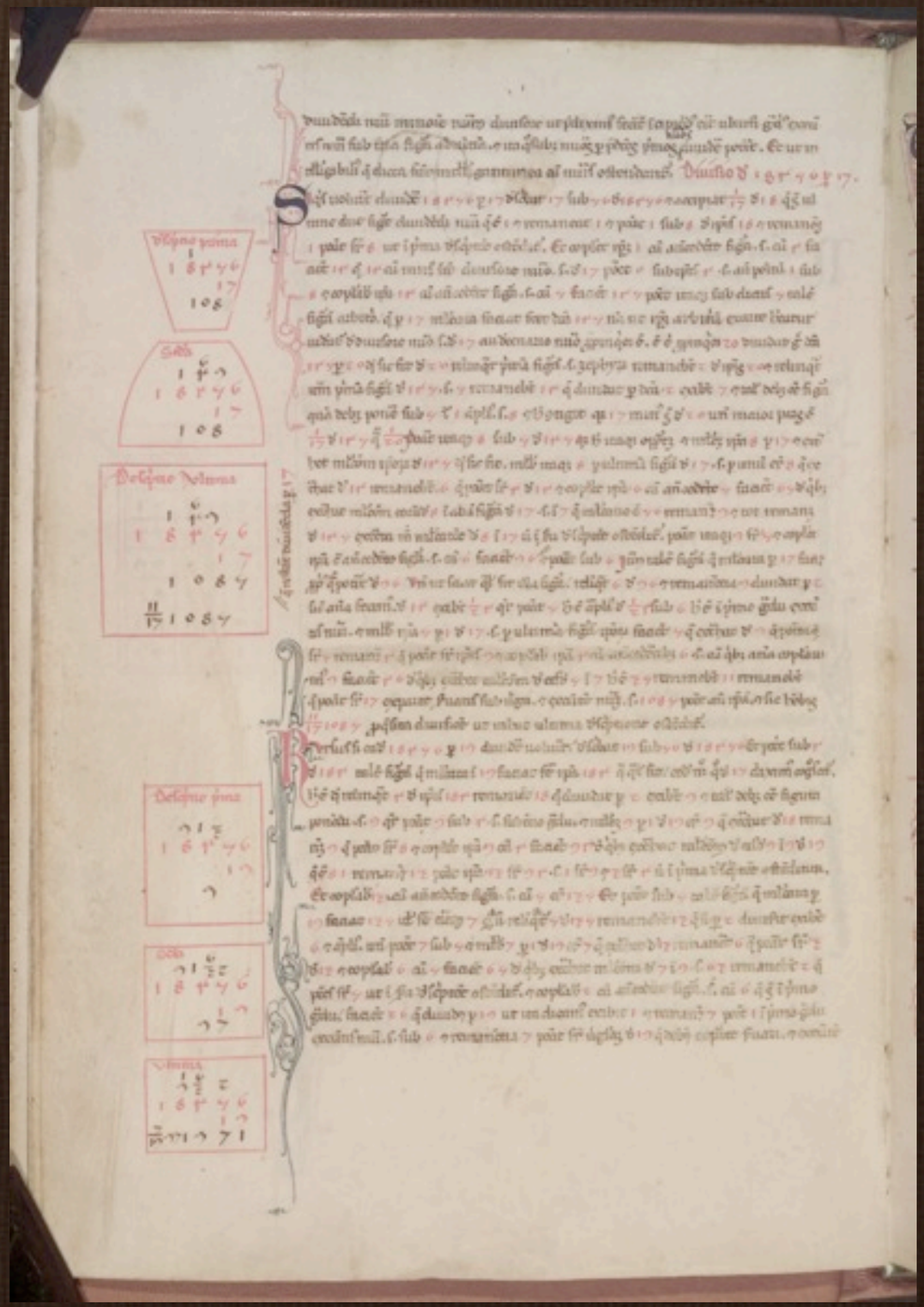
1b

Cont...

The Florence Manuscript



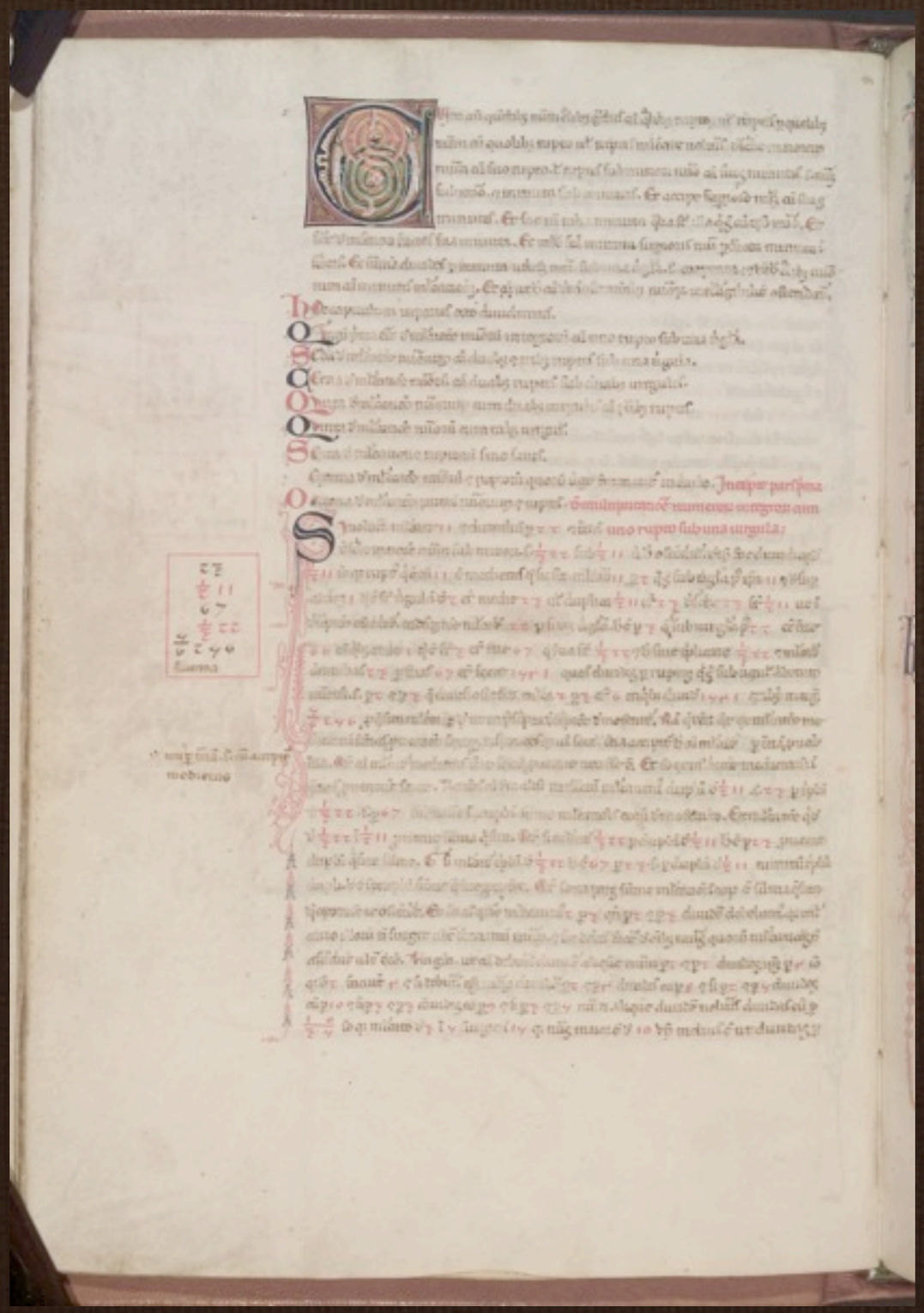
4a



14b

Cont...

The Florence Manuscript



20a

27b

Other *Liber abbaci* manuscripts

In all, fourteen copies of Leonardo's 1228 edition of *Liber abbaci* still exist in one form or another. Seven of them are substantial and seven mere fragments, consisting of between one-and-a-half and three of the book's fifteen chapters.

Of the seven reasonably good copies, three are complete or almost complete and are generally regarded as the most significant. All three are in Italy: the Siena manuscript L.V.20, the Florence manuscript Conv. Soppr. C.1.2626, and a manuscript in the Vatican Library in Rome that bears the reference mark Vatican Palatino #1343. The Vatican manuscript, from which Chapter 10 is missing, is believed to date back to the late thirteenth century.

Of the remaining eleven manuscripts, four are housed in the BNCF (along with Conv. Soppr. C.1.2626), one is in the Biblioteca Laurentiana Gadd in Florence (Gadd. Reliqui 36, dated to the 14th century), one in the Biblioteca Riccardiana in Florence, one in the Biblioteca Ambrosiana in Milan, one in the Biblioteca Nazionale Centrale in Naples, and three in Paris (one in the Bibliothèque Mazarine, two in the Bibliothèque National de France).

"Liber abbaci" in print

Modern editions

The Boncompagni edition

IL

LIBER ABBACI

DI

LEONARDO PISANO

PUBBLICATO

SECONDO LA LEZIONE DEL CODICE MAGLIABECHIANO
C. I, 2616, *Badia Fiorentina*, n.° 73.

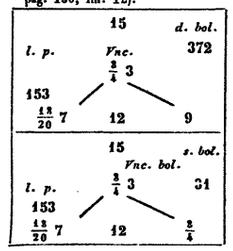
DA

BALDASSARRE BONCOMPAGNI

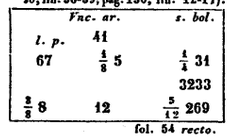
SOCIO ORDINARIO DELL'ACCADEMIA PONTIFICIA DE'NUOVI LINCEI, E SOCIO
CORRISPONDENTE DELL'ACCADEMIA REALE DELLE SCIENZE DI TORINO,
DELLA REALE ACCADEMIA DELLE SCIENZE DI NAPOLI,
E DELLA PONTIFICIA ACCADEMIA DELLE SCIENZE
DELL'ISTITUTO DI BOLOGNA

ROMA
TIPOGRAFIA DELLE SCIENZE MATEMATICHE E FISICHE
VIA LATA NUM.° 211
MDCCCLVII.

« 372 et .... pone » (fol. 53 *verso*, lin. 25-35 ; pag. 129, lin. 30 — pag. 130, lin. 12).

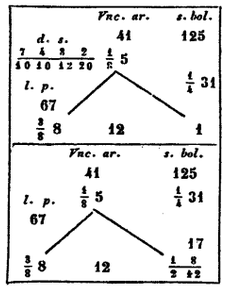


« pensam .... euitabis » (fol. 53 *verso*, lin. 36-39; pag. 130, lin. 12-17).



fol. 54 *recto*.

« Et probabis .... pones ipsas » (fol. 54 *recto*, lin. 1-12; pag. 130, lin. 17-34).



que est $\frac{4\ 0\ 0}{8\ 4\ 84}$, et per 12, et per ruptos, hoc est per $\frac{4\ 0\ 0\ 0\ 0\ 0}{8\ 4\ 4\ 84\ 12\ 20}$; euitabis inde $\frac{4}{8}$ de 15, hoc est quod multiplicabis 9 per tertiam partem de 15, scilicet per 5; que per 153, erunt 6885; que diuides per $\frac{4\ 0\ 0\ 0\ 0}{4\ 4\ 84\ 12\ 20}$, exibit $\frac{4\ 3\ 2\ 7\ 14}{2\ 8\ 84\ 12\ 20}$. In alia uero descriptione multiplicabis 3, que sunt super 4 per 15; que per 153, erunt similiter 6885; que diuides per 31, et per 12, et per omnes ruptos, hoc est per $\frac{4\ 0\ 0\ 0\ 0}{4\ 4\ 84\ 12\ 20}$, exibunt $\frac{4\ 3\ 27\ 4\ 4}{2\ 8\ 84\ 12\ 20}$, hoc est denarii $\frac{4\ 3\ 27}{2\ 8\ 84}$ 13.

De eodem.

Item sint libre $\frac{5\ 9}{12\ 20}$13 cuiusdam bolsonalie, que sit ad uncias $\frac{4}{8}$ 5 argenti; et in libra ipsius sint soldi 31, et denarii 3, hoc est soldi $\frac{4}{4}$ 31; et libra argenti ualeat libras 8, et soldos 7, et denarios 6, hoc est libras $\frac{3}{8}$ 8: fac soldos de libris $\frac{5\ 9}{12\ 20}$13, erunt soldi $\frac{5}{12}$ 269; quos pones sub soldis $\frac{4}{4}$ 31, ut hic ostenditur; et multiplicabis 269 per suam uirgulam, erunt 3233; que pone super $\frac{5}{12}$ 269, et super ipsa pone pensam ipsorum, que est 10, per 11 : similiter facies de $\frac{4}{8}$ 5, et habebis 41 super ipsa, quorum pensa est 8: hoc idem facies de $\frac{3}{8}$ 8, et habebis 67; et pro pensa 1, et super $\frac{4}{4}$ 31 habebis 125: deinde multiplicabis 3233 per 41, et per 67, et per 4, que sunt super uirgula de 31; et diuides summam per regulam de 125, que est $\frac{4\ 0\ 0}{5\ 5\ 5}$, et per 12, et per ruptos trium reliquorum numerorum, scilicet per 12, et per 8, et per 8, et aptabis ruptos, et euitabis, | et probabis, semper exibunt libre $\frac{4\ 3\ 7\ 8\ 8\ 46}{2\ 6\ 10\ 10\ 12\ 20}$ 30 pro pretio dictarum librarum 13, et soldi 9, et denarii 5.

De eodem.

Et si pretium unius soldi eiusdem bolsonalie reperire uolueris, describes 1 sub $\frac{4}{4}$ 31, et multiplicabis ipsum 1 per 41, erunt 41; que multiplicabis per 67, erunt 2747; que relinques multiplicare per 4, que sunt sub uirgula post 31, et non diuides per 4, que sunt in regula de 8, que sunt sub 41: ergo diuides 2747 per 125, et per 12, et per 2, que remanent de 8, que sunt sub uirgula sub 41, et per 8, que sunt sub uirgula sub 67; et aptabis ruptos, exibunt $\frac{7\ 4\ 8\ 2}{10\ 10\ 12\ 20}$, ut in hac descriptione ostenditur; hoc est parum minus de denariis $\frac{4}{2}$ 27, uidelicet $\frac{8}{9}$ unius denarii, minus per unamquamque libram; et hoc cognoscitur ita: quod pretium soldi est denariorum $\frac{7\ 4}{10\ 10}$ 27, hoc est denarii $\frac{47}{100}$ 27; a quibus usque in denariis $\frac{4}{2}$ 27 desunt $\frac{3}{100}$ unius denarii : ergo si unicuique soldi defuerint $\frac{3}{100}$ unius denarii et libre, scilicet soldi 20, deerunt $\frac{60}{100}$, hoc est $\frac{8}{5}$ unius denarii, ut prediximus.

De eodem.

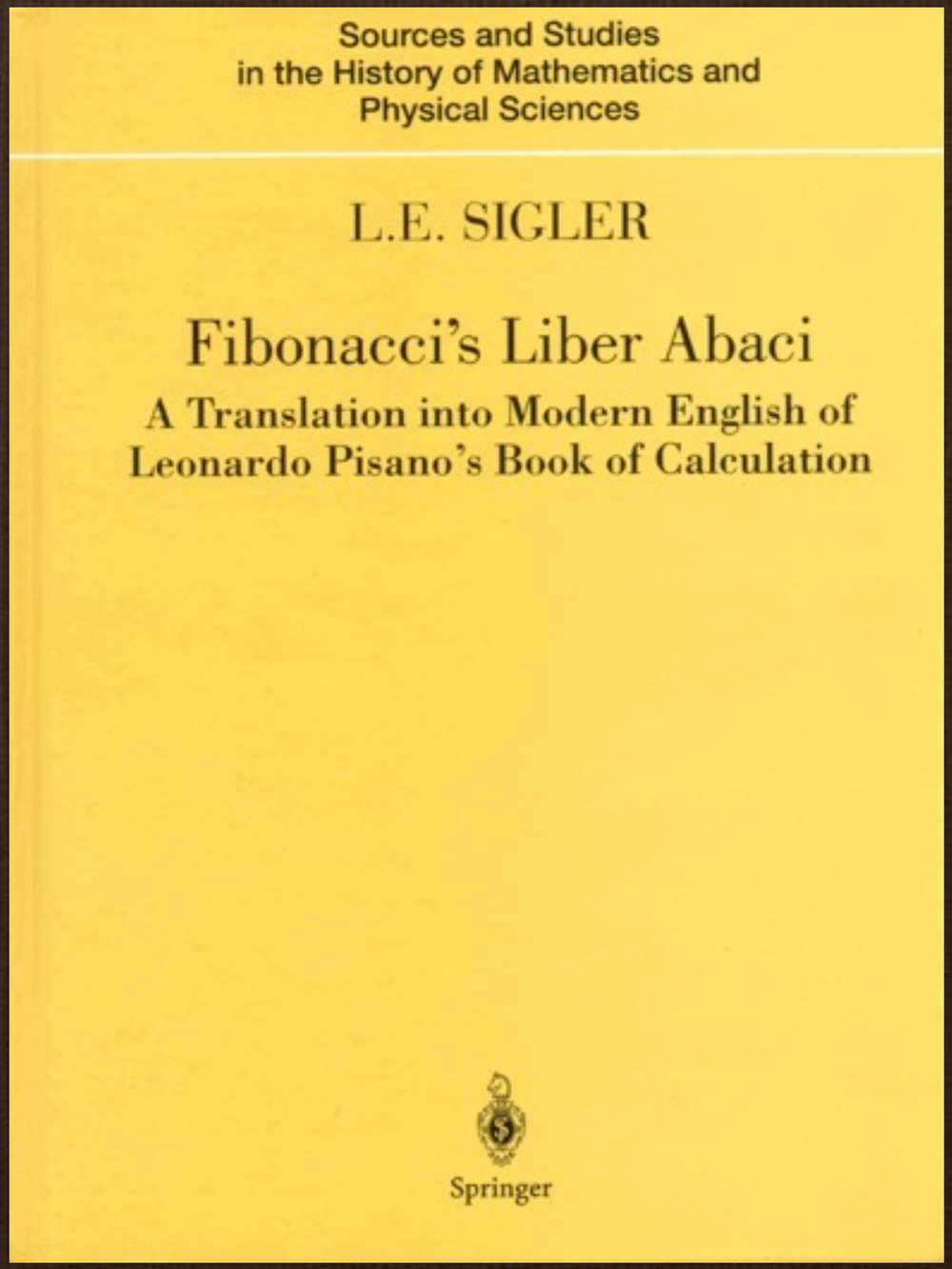
Item si queratur pretium de denariis $\frac{4}{2}$ 8 eiusdem bolsonalie; aut de soldis $\frac{4}{4}$ 31 facies denarios, qui sunt 375; et pones sub eis dictos denarios $\frac{4}{2}$ 8; uel de ipsis denariis $\frac{4}{2}$8 facies partes unius soldi, scilicet $\frac{4\ 8}{2\ 12}$; et pones ipsas sub soldis $\frac{4}{4}$ 31, ut sint soldi sub soldis, ut in hac descriptione ostenditur ; et multiplicabis denarios 8 per suam uirgulam, erunt 17; que multiplicabis per 41; que per 67; que per 4, que sunt sub uirgula post 31; et diuides per 125, et per 12, et per ruptos, scilicet per $\frac{4\ 0}{2\ 12}$, et per 8, et per 8; et euitabis, et coaptabis, et exibunt $\frac{4\ 6\ 5\ 4\ 7\ 4}{2\ 8\ 10\ 10\ 12\ 20}$, hoc est denarii $\frac{4\ 6\ 5\ 4}{8\ 8\ 10\ 10}$19.

Item quidam habet soldos 11, et denarios 7, hoc est soldos $\frac{7}{12}$ 11 cuiusdam bolsonalie, que est ad uncias $\frac{4}{4}$ 3; et intrant in libra ipsius bolsonalie soldi 28, et denarii $\frac{4}{2}$ 5, hoc est soldi $\frac{4\ 5}{2\ 12}$ 28; et libra argenti ualet libras $\frac{7}{20}$ 8; describes questionem, ut hic ostenditur, et multiplica 28 per suam uirgulam, erunt 683 : similiter multiplica omnes numeros per suas uirgulas; et habebis 13 super $\frac{4}{4}$ 3, et 167 super $\frac{7}{20}$ 8, et 139 super $\frac{7}{12}$ 11.

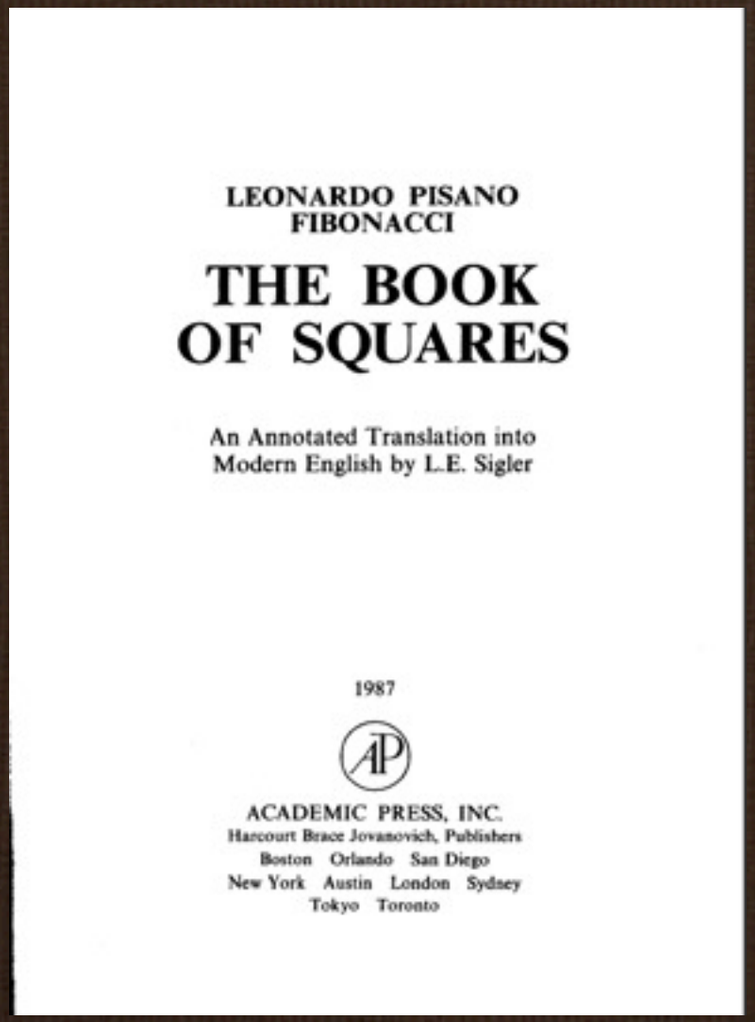
The first (and only) printed edition of the original Latin text, published in Rome by Baron Baldassarre Boncompagni in 1857, based on the Florence manuscript.

The English translation

The only translation of *Liber abbaci* from the original Latin is the English edition by Laurence Sigler (shown here in his office at Bucknell University in Pennsylvania), which was published in 2002, exactly 800 years after Leonardo completed the first edition. Sadly, Sigler died in 1997, before the book appeared. His wife Judith completed the final manuscript preparation. Sigler based his translation on the Boncompagni edition.



Modern translations of other Leonardo works

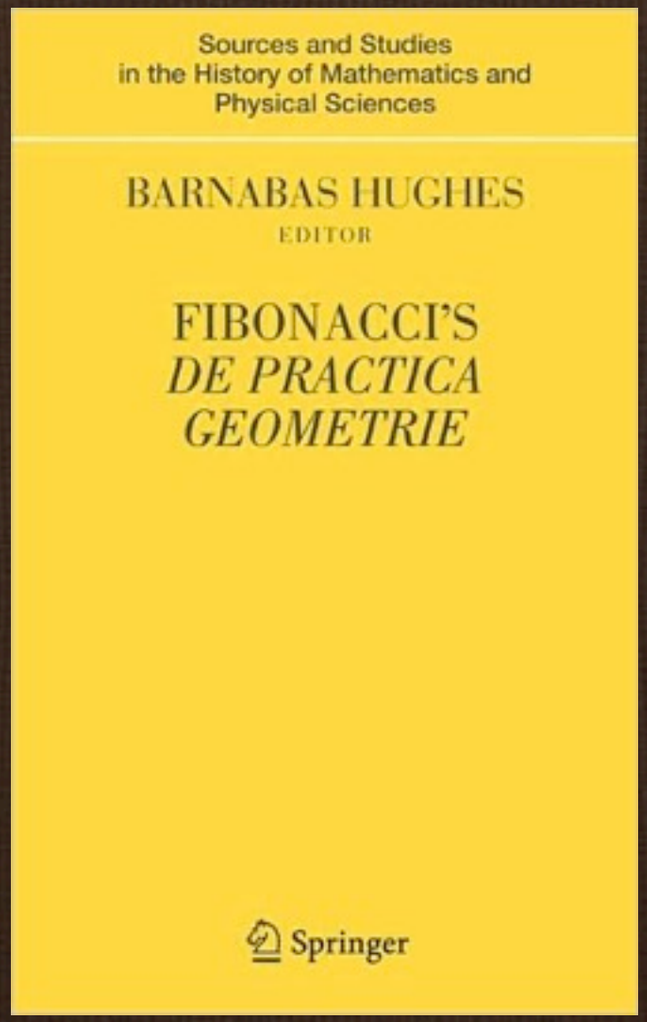

LEONARDO PISANO
FIBONACCI

THE BOOK
OF SQUARES

An Annotated Translation into
Modern English by L.E. Sigler

1987

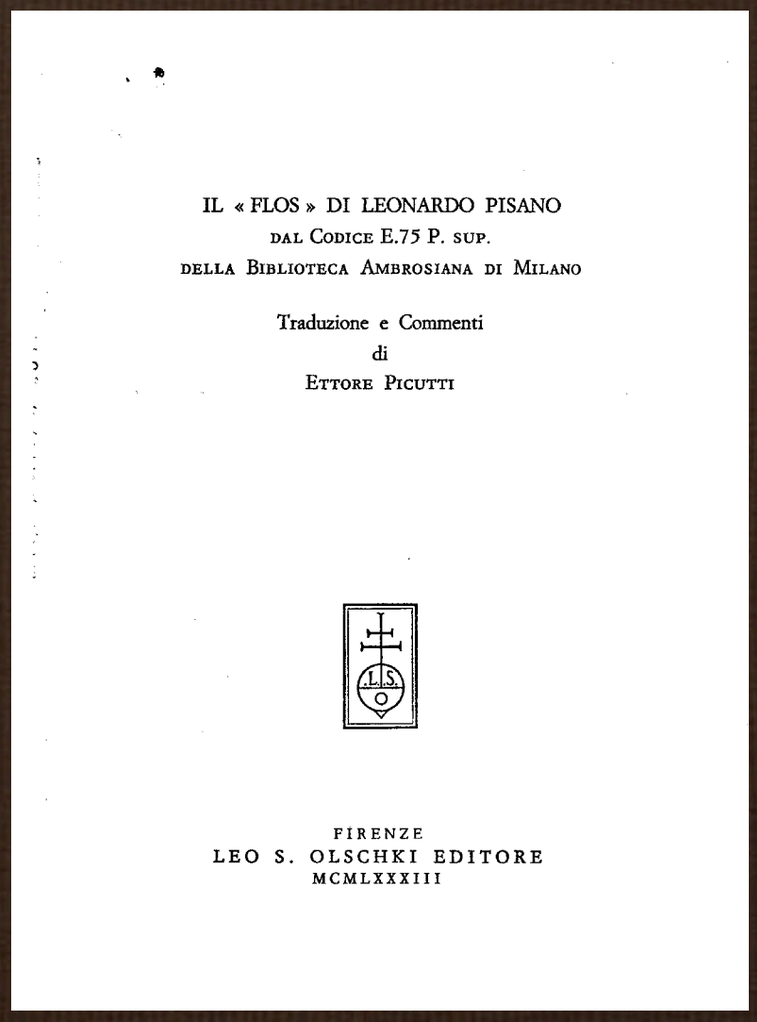
ACADEMIC PRESS, INC.
Harcourt Brace Jovanovich, Publishers
Boston Orlando San Diego
New York Austin London Sydney
Tokyo Toronto


Sources and Studies
in the History of Mathematics and
Physical Sciences

BARNABAS HUGHES
EDITOR

FIBONACCI'S
DE PRACTICA
GEOMETRIE

Springer


IL « FLOS » DI LEONARDO PISANO
DAL CODICE E.75 P. SUP.
DELLA BIBLIOTECA AMBROSIANA DI MILANO

Traduzione e Commenti
di
ETTORE PICUTTI

L.S.

FIRENZE
LEO S. OLSCHKI EDITORE
MCMLXXXIII

Laurence Sigler also translated Leonardo's 1225 book *Liber quadratorum* into English. Barnabas Hughes published an English translation of *De Practica Geometrie* (1220).And Ettore Picutti translated *Flos* (1225) into modern Italian.

Artistic inspirations

Artistic inspirations

Chimney of Turku Energia, Turku, Finland, featuring the Fibonacci sequence in 2m high neon lights. By Italian artist Mario Merz for an environmental art project (1994).

Fibonacci gold pendant.

Designer Mia Schmallenbach based the sizes of the kitchen knives in her *Meeting* set on the Fibonacci sequence.

Fibonacci waistcoat by Jackie Wills.

Stamps


NICHTSBURG &
ZILCHSTADT
POSTAGE
2004
30
MIDENIKA
Fibonacci


C. 1202 - Leonardo Fibonacci
The Commonwealth of
DOMINICA
55¢

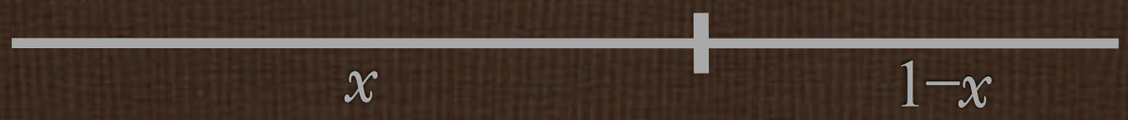
Fibonacci and the Golden Ratio

The Golden Ratio

In his classic geometry book *Elements*, the great Greek mathematician Euclid described the following procedure to determine a ratio of two lengths:

> Divide a line into two so that the ratio of the whole to the larger part equals the ratio of the larger part to the smaller.

If you choose units so the length of the given line is 1, that means you have to find a point a distance x along the line, as shown



so that the following equation is satisfied:

$$\frac{1}{x} = \frac{x}{1-x}$$

The number Euclid asks for is then $1/x$.

To determine x, rewrite the equation as:

$$x^2 + x - 1 = 0$$

Solve this (say by using the familiar quadratic formula) and you get two roots, one positive the other negative. Since x is a length, the positive root is the one you want. It is

$$x = (\sqrt{5} - 1)/2$$
$$= 0.61803 \text{ (approx.)}$$

Contemporary mathematicians call this number the Golden Ratio.

Cont...

The Golden Ratio

Over the past 150 years or so, authors of magazine articles, popular science books, art books, and even mathematics textbooks have made some amazing claims about the Golden Ratio, a phenomenon that grew even more frenetic with the growth of the Web. Most of those claims are without foundation, and the ones that can be tested empirically (such as the claim that the Golden Ratio is the aspect-ratio of the rectangle that the human eye finds most pleasing) have been shown repeatedly and with absolute certainty to be false. Among the (other) false claims are: that the ancient Egyptians used the Golden Ratio to design the pyramids; that the Greeks used it in the design of the Parthenon, as did the architects who designed the United Nations building in New York City; that Da Vinci, Boticelli, Seurat and a host of other artists used it in their paintings (though Paul Sérusier, Juan Gris, Giro Severini, and Salvador Dali did flirt with the number for a while); that musicians used it in their music (with the possible exception of Debussy, though the evidence that he did is decidedly weak and inconclusive); that aspects of the human body exhibit the Golden Ratio; and on the list goes. In my book *The Man of Numbers: Fibonacci's Arithmetic Revolution*~ I give references to scholarly works where you will find each of these claims soundly debunked. So much for the popular Golden Ratio Myth.

Fibonacci gets into the picture because one of the few claims about the Golden Ratio that is valid is that it is related to the Fibonacci sequence.

Cont...

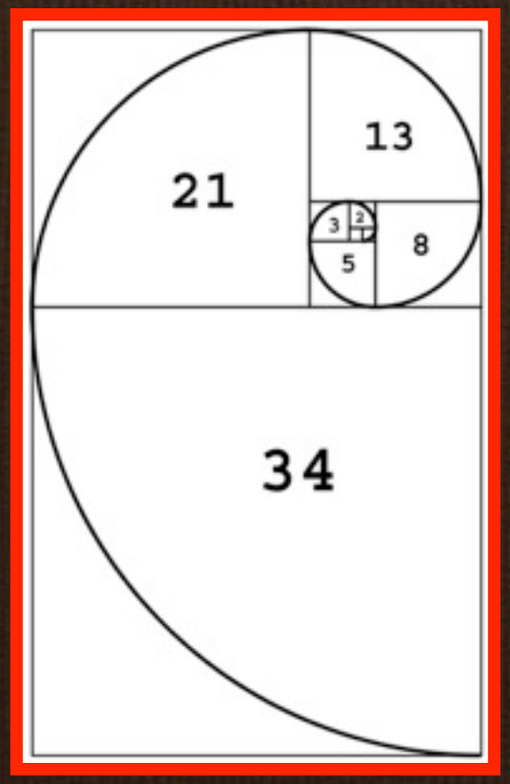
The Golden Ratio

If you calculate the successive ratios of each Fibonacci number to its successor in the sequence, the answers get closer and closer to the Golden Ratio (0.61803...):

1/2 = 0.5, 2/3 = 0.666, 3/5 = 0.6, 5/8 = 0.625, 8/13 = 0.615384, 13/21 = 0.619047, 21/34 = 0.617647, etc.

The accuracy improves to as many decimal places as you want. (The process never stops, since the Golden Ratio is an irrational number, meaning its decimal expansion continues for ever without ever settling into a repeating pattern.)

This is an interesting mathematical result, and provides part of the explanation for the frequent occurrence of the Fibonacci numbers in nature. But it also means the good name of Leonardo Fibonacci is tainted by association with the many false claims about the Golden Ratio. In particular, you often see references to Fibonacci illustrated with a photograph of the Nautilus shell, as in the example to the right. To be sure, you can use the Fibonacci sequence to draw a nice spiral (see left), called the Fibonacci spiral, and that spiral very closely approximates one that turns by a constant angle equal (in radian measure) to the Golden Ratio (and hence called the Golden Spiral). Unfortunately, the belief that the Nautilus shell has the form of the Golden Spiral is another of those false beliefs about Euclid's number. To be sure, the Nautilus shell is a spiral, and it is moderately close to spiraling by a constant angle, but that angle is not the Golden Ratio. Not even close. So there is no connection.



And that is why this topic is tucked away at the end of this book!