On the Establishment of Fundamental and Derived Units, with Special Reference to Electric Units. Part I

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IN a recent article entitled *On Electric and Magnetic Units and Dimensions*,[1] I endeavored to summarize the fundamental ideas and equations involved in the so-called *absolute* systems of electric and magnetic units. Subsequent correspondence has indicated the need of a supplementary discussion of some of the questions involved in that paper. These questions relate, in general, to the methods employed in the establishment of electric and magnetic units. There is, however, no difference in principle between the methods employed for electric units and for other types of units such as mechanical units. In fact the difficulties commonly believed to be peculiar to systems of electric units occur to an equal degree in other divisions of physical science. Hence Part I of the present paper is devoted to a general discussion of the methods that have been used in the establishment of physical units, with particular emphasis on the dimensions of the resulting units. Electric units are discussed in this portion of the paper, but only in order to illustrate general principles, and to show the close parallelism between electric and other types of units.

In Part II electric and magnetic units will be discussed in more detail, with specific reference to certain criticisms of my previous paper. In that paper I had no intention nor desire to contribute anything of an original nature, and I adopted merely the general method of treatment and point of view of the great majority of authorities in the field. This fact was made clear by an explicit reference, in the case of each important statement, to one authoritative source, at least, where an identical or similar statement occurs. However, there are at least two distinctly different methods of treatment of electric and magnetic units to be found in recent literature on the subject, and both of these methods will be presented in the present paper.

There exist in the literature many detailed discussions of physical units, but such accounts usually stress the numerical relations involved, and subordinate the matter of dimensions. It is, however, the assigned dimensions that indicate most concisely the principles involved in the establishment of units. Thus two different units for the same quantity may be almost identical in magnitude, but may differ in dimensions due to the distinctly different assumptions involved in their definitions. In this paper we shall be concerned primarily with this question of definitions and resulting dimensions of units, and only incidentally with their magnitudes.

Fundamental Units

Physical units are divided into two classes, *fundamental* or *primary* units, and *derived* or *secondary* units. Most of the difficulties experienced in this subject are connected with the various possible ways that have been used, or may be used, to obtain derived units. I shall start, however, with a brief discussion of fundamental units.

A fundamental unit is distinguished by the fact that it is entirely independent of all other units, in respect both to its *magnitude* and to its *dimension* or *dimensions*. It is customary to assign to each fundamental unit a specific dimension. Thus the *unit* of length is said to have the *dimension* of length. Because, however, of the arbitrary character of dimensions, as presented so ably by Bridgman,[2] the choice and number of fundamental units are arbitrary. This statement will be illustrated presently.

Let us, for the sake of argument, choose as fundamental for a mechanical system, units of length, mass and time. To these units are assigned the specific dimensions length (L), mass (M) and time (T), respectively. There are now a number of different ways in which each of these units may be chosen, but the resulting units fall in general into two distinct classes:[3] (1) The first

[1] R. T. Birge, Am. Phys. Teacher **2**, 41 (1934).

[2] P. W. Bridgman, *Dimensional Analysis*, rev. ed. (Yale Univ. Press, 1931). See also J. C. Oxtoby, *What are Physical Dimensions?* Am. Phys. Teacher **2**, 85 (1934).

[3] In the preliminary draft of this paper I adopted the analysis presented by G. Mie, *Elektrodynamik* [Vol. 11 of Wien-Harms, *Handbuch der Experimentalphysik* (1932),

class comprises units that are local in character, not exactly reproducible if destroyed, and subject to possible change with time; (2) The second class comprises units that are believed to be universal in character, indestructible, and not subject to change with time.

As examples of the first class, consider the following units of length. The adopted unit[4] may be the length of any arbitrarily chosen body. Thus the present c.g.s. standard of length is the length, at 0°C, of a particular bar of platinum-iridium deposited at the International Bureau of Weights and Measures. This so-called prototype meter is obviously local in character, and subject to possible change with time. If destroyed it could not be reproduced exactly. A better standard, from this point of view, would be the circumference of the earth, because of its more permanent character. But the earth is believed to be cooling and shrinking in size, and therefore even such a standard does not have the desired constancy with time. This is quite aside from the practical difficulty of producing a *laboratory* standard that will accurately represent some designated fraction of the earth's circumference. From 1799 to 1872 the prototype meter was, as a matter of fact, merely an auxiliary laboratory standard, designed to represent the 10^{-7} part of the earth's quadrant, so that during that period the circumference of the earth was the primary standard of length. In 1872, however, this definition was abandoned and the actual meter bar that had been constructed in 1799 was chosen as the primary standard of length.

The present c.g.s. standard of mass is, like the prototype meter, an arbitrarily chosen body known as the prototype kilogram, and is thus an example of the first class of fundamental units. On the other hand, the c.g.s. unit of mass, from 1799 to 1872, was defined as the mass of 1 cm^3 of water, at its point of maximum density. This definition makes density a primary unit, and mass a *derived* unit; hence this unit of mass belongs in the section on derived units, and is considered in more detail there. In place of the mass of the prototype kilogram, which may change with time, and is not exactly reproducible, one might choose the mass of the earth as the standard of mass. This would obviously be a poor standard, because of the difficulty in measuring it with any accuracy, in terms of laboratory standards.

Although the earth represents an unsatisfactory standard as regards length or mass, its motion has always been used in science to define the unit of time. The direct standard, in this case, is the length of the sidereal day, since this is a quantity that has no annual variation. For practical purposes, however, the *mean* solar day has been chosen, and the unit of time (the second) is defined as the 1/86,400 part of this latter standard. Such a unit of time is vastly superior to the period of some officially designated laboratory pendulum, but even the period of rotation of the earth is believed by astronomers to be slowly changing with time, as a result of tidal action. Thus all three of the present adopted primary c.g.s. units fall in class one.

The desirability of choosing standards that fall in class two is now well recognized, but unfortunately it is in general not possible to reproduce them in the laboratory with the desired accuracy. One exception is the wavelength of light, which can be measured with very great accuracy. Thus it has been found experimentally that the standard meter equals 1,553,164.24 wavelengths of red cadmium light, in "normal" air at 15°C, with a probable error of only one part in several million. If now we *define* the meter as this number of wavelengths (which corresponds to defining the wavelength as $6438.4691 \times 10^{-10}$ m), we have, in principle, adopted a certain wavelength of light as our unit of length. This unit is believed to be universal and constant in time. It is evidently reproducible, or more precisely, the laboratory standard chosen to represent some multiple of it is reproducible, if destroyed. Such a redefinition of the meter was suggested in 1923

p. 429]. His analysis, however, makes the proper classification of many units ambiguous, and the classification presented here has evolved as a result of private correspondence particularly with Professors Duane Roller and D. L. Webster.

[4] Each *unit* is represented by a *standard*, or some combination of standards, which possesses the property whose unit is being established. Thus any material object possessing length may be chosen as the *standard* of length, and the invariability of the *unit* of length will depend upon the invariability of the chosen *standard*. The invariability of the latter is necessarily a postulate, the reasonableness of which must be based on general considerations. The unit may be identical in magnitude with the standard, or may be some multiple or sub-multiple of it, but in this paper, purely for simplicity, we shall in general assume that the unit and the standard are identical.

by the International Committee on Weights and Measures, and various National laboratories are now carrying out new determinations of the length of the meter in terms of wavelengths of light, preparatory to making this change.[5]

As further examples of fundamental units falling in class two one may cite the mass of an electron, or of a proton, as the unit of mass, and the period of an atomic vibration as the unit of time. Except for purely experimental difficulties of measurement, these quantities, or some multiple of them, would constitute ideal units. If in place of length and time one should adopt length and velocity as the fundamental units of kinematics, with specific dimensions L and V respectively, the velocity of light in empty space, c, immediately suggests itself as a satisfactory unit of velocity falling in class two, Now, as will be discussed more fully in Part II, c plays a unique role among the universal constants. As a measure of the speed of a photon it is to be classified with constants like e (the charge of the electron), M_P (the mass of the proton), and h (the Planck constant). Such constants refer to properties of *elementary particles* and are suitable for use as primary units, each with a specific dimension. But as discussed in the next section, c may also play the role of a factor of proportionality in the equation of a derived unit, and as such may be assigned *zero* dimension.

Certain units are intermediate in type between class one and class two. These units usually apply to *intensive* quantities, rather than *extensive*. An extensive quantity has the property of addition, whereas an intensive quantity does not. Thus since two bodies, each of mass 1 g, have together a mass of 2 g, mass is an extensive quantity. But the density of two bodies, each of 10 $g \cdot cm^{-3}$ density, is still 10 $g \cdot cm^{-3}$, which shows that density is an intensive property. As an example of such an intermediate type of unit, consider the maximum density of water. This is a unit that does not change in time, and in one sense is not local in character. Yet it cannot be considered a universal constant in the sense that e, M_P and h are universal constants. The resistivity of mercury, at 0°C, is a further example of this intermediate type of unit. As already noted, the original definition of the metric unit of mass made mass a derived unit; the fundamental units, as chosen in 1799, were those of length, density and time, the unit of density being the maximum density of water. One further example is the international ohm, whose present definition, as shown in the next section, is equivalent to the assignment of an arbitrary numerical value to the product of the density and the resistivity of mercury, at 0°C.

[5] For an account of this work see W. E. Williams, Nature 135, 459, 496, 917 (1935).

Derived Units

The magnitude and dimensions of a derived unit depend upon the magnitude and dimension of one or more arbitrarily chosen fundamental units. The dependence is shown by an *equation*, which is used to define the derived unit. This equation also contains a *factor of proportionality* to which, in general, one may assign an arbitrary numerical value and *also* arbitrary dimensions (*including zero*). It is this arbitrary character of the so-called factors of porportionality that is primarily responsible for the almost unlimited possibilities present in the assignment of the magnitude and dimensions of a derived unit. Most of the uncertainty and lack of agreement regarding units and dimensions seems to center at just this point, and because of that fact a rather extended treatment follows of various types of derived units and their defining equations.

(a) As a first example consider *volume*. For the purpose of defining unit volume we choose a cube, and then its volume is given directly by (length)^3. If length is taken as a primary unit, the dimensions of the derived unit of volume are L^3, and this unit is most simply the volume of a cube of unit side. Thus the c.g.s. unit of volume is 1 cm^3. More generally one may write for volume,

$$V = kl^3. \tag{1}$$

Thus volume, in U. S. gallons, is given by

$$V = \frac{1}{231} l^3, \tag{2}$$

where l is expressed in inches. In order to *define* the gallon the constant k has been given the *arbitrary* numerical value 1/231 and *zero* dimension. There would be no possible advantage in assigning any specific dimension to the constant, and this has never been proposed.

On the other hand, if one were so illogical as to choose an arbitrary unit of volume, that is, one not specifically related to the volume of a unit cube, then k can be interpreted as the measured ratio between these two possible units of volume, and its numerical value must be determined *experimentally*. Thus if one had adopted the gallon as the unit of volume, and the centimeter as the unit of length, during the period when the inch and yard were still primary units in this country, unrelated to the metric units,[6] then Eq. (1) would have had the form

$$V=\frac{1}{231a^3}l^3, \tag{3}$$

where V is the volume of a cube in gallons, l the length of a side in centimeters, and a the number of centimeters in one inch, as experimentally determined.

(b) The volume of a rectangular parallelopiped is given by the product of three different lengths, and it seems illogical not to use the same unit in the case of each measured length. However, the common use of "board foot" in the lumber industry, and of "acre foot" in irrigation engineering indicates the practical usefulness of such *mixed units* of volume. A very conspicuous example of the use of a mixed unit in science is furnished by the measurement of angle. Since angle is the ratio of arc to radius, the only logical scientific unit is the radian. All other units, such as revolution, degree, etc., correspond to the adoption of one unit of length for the radius, and another unit of length for the arc. Since angle is the ratio of two lengths its unit is necessarily dimensionless. Thus it is possible to have more than one dimensionless unit for certain quantities by the simple artifice of using simultaneously units of different magnitudes for the same sort of primary quantity (in this case *length*) in the defining equation.

[6] The U. S. yard is now, legally, merely 3600/3937 m.

[7] On the other hand, since angle has a physical character and is not a pure number, those who believe that quantities possess *absolute* dimensions, indicative of their physical nature, are forced to extend the theory of dimensions to include the idea of direction. The most ambitious attempt of this kind is possibly that by W. W. Williams, Phil. Mag. (5) **34**, 234 (1892), who writes the dimensions of angle as XY^{-1}. I have not seen his general scheme of dimensions used by any subsequent writer.

(c) Let us consider the equation *density = mass/volume*, or

$$d=m/V. \tag{4}$$

This equation may be interpreted in three different ways. In the first place mass and length may be taken as primary quantities,[8] with volume defined by Eq. (1). Then density is a derived quantity, of dimensions ML^{-3}, and its unit, as defined by Eq. (4), is unit mass per unit volume. This corresponds to the present adopted c.g.s. system in which unit density is 1 $g\cdot cm^{-3}$, and the maximum density of water must be determined *experimentally*.[9]

In the second place, the units of length and density may be taken as primary, of dimensions L and D, respectively. Mass is then a derived unit, of dimensions L^3D, and defined by Eq. (4). This corresponds to the c.g.s. system, as it existed from 1799 to 1872. The unit of length was then, as now, the centimeter. The unit of density was the maximum density of water, and the resulting unit of mass, from Eq. (4), was the mass of unit volume of water, at its point of maximum density. The kilogram was merely a laboratory standard, constructed to represent as nearly as possible the mass of 1000 cm^3 of water but, as we now know, failing to do so by about 28 parts in a million.[9]

[8] Every equation used in science, such as Eq. (4), gives a relation between *numerical measures* of certain quantities, not between the physical quantities themselves [see, for instance, V. F. Lenzen, *The Nature of Physical Theory* (John Wiley and Sons, 1931), pp. 29–30]. Thus in Eq. (4) d is the numerical measure of density, *relative* to the adopted unit, and hence may be represented symbolically by $n[d_0]$, where n is a pure number and $[d_0]$ signifies the unit. The dimensions of d are necessarily those of the unit, and it is to the *unit*, not to the physical quantity, that one assigns a dimension or dimensions. As the title of this paper indicates, one chooses primary and derived units, but not primary and derived quantities.

This point has recently been emphasized by H. Abraham in an excellent critical paper entitled *A Propos des Unités Magnétiques* (Bull. Nat. Res. Council **93**, 8, 1933, especially pp. 10–11). However, it is still common practice to speak, as I have just done, of an equation as expressing a relation between primary and derived quantities. Such forms of expression are used repeatedly by Bridgman,[2] who nevertheless was one of the first to insist on the distinction between the numerical measure and the thing itself. In my previous paper[1] I followed Bridgman in the use of such uncritical forms of expression, which Abraham considers are very unfortunate and likely to lead to misunderstanding. With very few exceptions the mode of expression used in the present paper conforms to that preferred by Abraham.

[9] Its latest value is 0.999972 ± 0.000001 $g\cdot cm^{-3}$; see V. Stott, Nature **124**, 622 (1929).

In the third place the units of mass and density may be taken as primary, with dimensions M and D. The unit of volume then has the dimensions MD^{-1}. If the unit of density is again the maximum density of water, the derived unit of volume is, by Eq. (4), the volume of unit mass of water when at maximum density. This is the present definition of the *milliliter*,[10] provided the gram is taken as the unit of mass, or of the *liter*, if the kilogram is chosen. Hence the milliliter and the liter are derived units of volume (or so-called "capacity") that correspond to mass and density taken as primary units. They do *not* belong to the present accepted c.g.s. system, in which length, mass and time are the primary units. The liter and the milliliter are, however, used almost exclusively by chemists as units of volume.

Eq. (4) does not contain a factor of proportionality. In order once again to illustrate certain general principles, let us insert such a factor and write

$$d = k\frac{m}{V}. \tag{5}$$

It is now possible to choose arbitrarily a unit of density, regardless of the fact that units of mass and volume have already been adopted. Thus we may *define* the maximum density of water as unity, and at the same time retain the gram as the unit of mass, and the cm^3 as the unit of volume. Then k may be interpreted as a pure number which gives the ratio between unity and the maximum density of water in grams per cm^3. As experimentally determined[11] $k = 1.000028 \pm 0.000001$. On the other hand, the value of k may be *defined* as 1.000028 (with no error), and the resulting maximum density of water, using Eq. (5), is then unity (± 0.000001), an *experimental* value that may at any time be changed as a result of more accurate measurements.

The insertion of the k in Eq. (5) has thus given a certain freedom not permitted by Eq. (4). It has allowed *either* the adoption of a unit of density *independent* of the adopted units of mass and volume, *or* the retention of a unit of density depending on the units of mass and volume, but with an *arbitrarily* altered magnitude (the alteration being given by the adopted value of k). Both of these changes concern only the *magnitude* of the unit of density. The insertion of k gives, however, a similar increased freedom with regard to dimensions. Thus if it is desired to retain for the unit of density the dimensions ML^{-3}, one merely assigns to k zero dimension and interprets k as the ratio of two units of density, *each* of dimensions ML^{-3}. But one may also assign to the unit of density the specific dimension D. Then the resulting dimensions of k are DL^3M^{-1}. In other words density has now been chosen as an additional *primary* unit, and this is not an unnatural procedure in the case just mentioned where an *arbitrary* magnitude has been adopted for this unit. It is important to note, however, that the freedom to assign an arbitrary magnitude to a unit and the freedom to assign dimensions are entirely independent. It is customary to assign to the unit of density, even in Eq. (5), the dimensions ML^{-3}, and this custom seems to have resulted from the notion that these dimensions in some way give us more information about the "physical nature" of density than the specific dimension D would do. But as pointed out in footnote 8, dimensions are assigned to *units*, not to the physical quantities themselves, and Bridgman's theory of dimensions stresses the fact that the dimensions of a unit give us no information about the intrinsic nature of the physical quantity of which it is the unit.

(d) A rather detailed treatment has been given of Eqs. (4) and (5), since they typify the principles under discussion. In the remaining illustrations the general principles are quite similar and therefore need not be repeated in detail. The next equation to be considered is *velocity = length/time*, or

$$v = l/t. \tag{6}$$

Here, as in Eq. (4), any two of the three units involved can be taken as fundamental. The c.g.s. system takes the units of length and time, with velocity as a derived unit (1 $cm \cdot sec.^{-1}$). Let us, however, use Eq. (6) to *define a unit of time*, with any unit of length, such as the centimeter, as a primary unit, of dimension L, and any

[10] Sometimes denoted by *cc*. See reference 9.

[11] It is important to note that although $1/k = 0.999972$, this is now considered a *pure number* giving the ratio of two units (just like the 3 in *yards = feet/3*), whereas in Eq. (4) this number expresses the measure of a density with dimensions ML^{-3}.

convenient velocity (speed), such as the mean speed of the earth in its orbit, as a *primary* unit of velocity, of dimension V. The derived unit of time thus defined by Eq. (6) has the dimensions LV^{-1}.

Now an entirely unique velocity is that of light in empty space, c. This is called a universal constant and as succeeding illustrations will show, any universal constant *connected with the properties of empty space*, such as c, G, ϵ_0 and μ_0, may be employed, for the sake of defining a unit, like a mere factor of proportionality to which arbitrary magnitude and dimension (including *zero* dimension) may be assigned. In the theory of relativity it is desirable to call c unity and dimensionless. This defines the unit of time as the time required for light to travel the unit of length, and gives time and length the same dimension. In other words, in the equation $t=(1/c)l$, the $1/c$ is to be considered a mere *factor of proportionality*, just like the G in Eq. (9) ahead, and the a in Eq. (10), and like them it may be assigned *zero* dimension as the basis of a certain definite system of units. But in problems of science not directly concerned with relativity, such a *reduction* in the number of primary units is likely to prove more a hindrance than a help, and hence there is at the moment little advocacy of it.

(e) As a fifth illustration let us consider possible units of *force*. If force is considered as defined by Newton's second law of motion, the unit of force is necessarily a derived unit and in its simplest form is the force required to give unit acceleration to unit mass.[12] In the c.g.s. system this unit is called the dyne and has dimensions MLT^{-2}. More generally the resulting derived unit of force is to be obtained from the equation

$$F=kma. \tag{7}$$

The dyne then corresponds to $k=1$ and dimensionless. To obtain a new derived unit let k represent the dimensionless number 1/980.665. Then unit force is that force which will give to unit mass (the gram) an acceleration of 980.665 $\text{cm}\cdot\text{sec.}^{-2}$. This is the present accepted definition of the *gram-weight*.[13] The dimensions of this unit are MLT^{-2}, just as in the case of the dyne.

One can, however, define *gram-weight* in an entirely different manner, with entirely different dimensions as the result. Let us for the moment ignore Newton's second law, and define the unit of force as the force (of gravity) acting on unit mass, when placed at a specified point on the earth's surface. The magnitude of this unit can be recorded in terms of the extension of a spring balance, on which unit mass is hung. Since the magnitude of this new derived unit of force depends solely on the adopted unit of mass, it has merely the dimension M, in place of MLT^{-2}. One can now discover, experimentally, that force applied to mass produces acceleration. When the defined unit of force is applied to unit mass, it is found that an acceleration of g_0 $\text{cm}\cdot\text{sec.}^{-2}$ is produced, where g_0 is the *experimentally* measured acceleration of gravity at the point on the earth's surface specified in the definition. Newton's second law, in terms of the new unit of force, accordingly takes the form

$$F=\frac{1}{g_0}ma=m(a/g_0). \tag{8}$$

In this equation g_0 is, as just noted, an experimentally determined acceleration, with a unit of 1 $\text{cm}\cdot\text{sec.}^{-2}$, of dimensions LT^{-2}. The unit of a has the same dimensions. Hence, in Eq. (8) a/g_0 is a pure number and the dimensions of the unit of F are merely those of the adopted unit of mass, in agreement with our original conclusion. If g_0 turns out by chance to be $980.665\pm r$, where r is the probable error of measurement, the magnitude of this new "gram-weight" is the same as the first, except for possible errors of measurement. But the meanings of the two are entirely different. The pure number 1/980.665 used in Eq. (7) does not *necessarily* have anything to do with the acceleration of gravity, although the name "gram-weight" obviously implies such a relation. In general, Eq. (7), with k as *any* pure number, necessarily defines the unit of force as one giving to unit mass an acceleration of $1/k$ times the adopted unit of acceleration. Eq. (8), on the other hand, defines the unit of force as one giving to unit mass the acceleration of gravity at a

[12] We here neglect the relativity difference between $d(mv)/dt$ and ma.

[13] Int. Crit. Tables 1, 42 (1926).

specified point on the earth. If $1/k$ is chosen to equal g_0, the two units are equal in *magnitude*, but are *not* equal in dimensions.

A third, and still different unit of force may be obtained from the equation

$$F=G\frac{mm'}{r^2}. \tag{9}$$

Here G is the factor of proportionality, to which an arbitrary numerical value and arbitrary dimensions can be assigned. It is also called a universal constant—the constant of gravitation. If now one wishes to preserve the derived c.g.s. unit of force, *as well as* the primary c.g.s. units of mass and length, G becomes an experimental magnitude, equal to 6.670×10^{-8} dyne·cm^2·g^{-2}, with dimensions $M^{-1}L^3T^{-2}$. But one can equally well give to G the arbitrary value unity and *zero* dimension, and thus *define* a new unit of force, with dimensions M^2L^{-2}. This might be called the "gravitational unit of force." The dimensions and value of k in Eq. (7) are now just the reciprocal of those given previously for G.

It should be noticed that in all three Eqs. (7), (8) and (9) the c.g.s. primary units of length, mass and time have been adopted, and in each case the resulting unit of force is a derived unit. Yet each of the three units of force has different dimensions. This illustrates the arbitrary character of the dimensions of a derived unit, even after assuming a given set of primary units, It is, of course, also possible to make force one of the primary units, and this has often been advocated. If length and time are also assumed as primary units, each of the equations (7), (8) and (9) serves to define a unit of mass, which now becomes a derived unit, and in the case of each equation the derived dimensions of mass are different. For example, if the adopted unit of force happens to have the magnitude of the present gram-weight, and if k in Eq. (7) is unity, the resulting unit of mass equals in magnitude 980.665 g of the c.g.s. system. If the adopted unit of force equals the present pound-weight, the resulting unit of mass is about 32.2 lbs., known as the *slug*.

(f) An equation precisely similar in form to Eq. (9) occurs in electrostatics. This is the equation

$$F=a\frac{ee'}{r^2} \tag{10}$$

for the force between two charges in vacuum. Webster[14] has called a the *constant of electrostatics*. Like G, it can be considered a universal constant, and like G it can *also* be considered a mere factor of proportionality to which can be assigned unit value and *zero* dimension. If this is done one obtains the *absolute e.s. system of units*. There is, however, an historical distinction between Eqs. (9) and (10) that apparently accounts for the fact that a and G are not ordinarily accorded the same treatment. When Eq. (9) was formulated, units of force, mass and length were already in common use. It was accordingly natural to use Eq. (9) to *define* the value and dimensions of G, and this is still done in the modern c.g.s. system. But when Eq. (10) was formulated, no unit of charge had been adopted, and the equation therefore furnished a convenient means of defining such a unit, by giving a an arbitrary value and dimensions. If, however, one adopts an arbitrary (primary) unit of charge, *or* if a unit such as the *absolute e.m. unit* is *derived* from other relations, then Eq. (10) serves to *define* the numerical value and dimensions of a. The result is c^2, with dimensions L^2T^{-2}, in case the absolute e.m. unit of charge is used. The reciprocal of a is often called the "dielectric constant of vacuum," and the origin and desirability of such a designation will be considered in Part II of this paper.

(g) As a final illustration of derived units consider the international ohm. This is defined as the resistance of a column of mercury at 0°C, of uniform cross section, 106.3 cm long, and 14.4521 grams mass. It is desirable to replace this definition by a defining equation, and for this purpose we write the known experimental relation

$$R=\rho l/A, \tag{11}$$

where R is the resistance of a uniform column l cm in length and A cm^2 in cross section; ρ is merely the factor of proportionality and is called the resistivity. Since $l\cdot A=V$, the volume of the column, and $d=m/V$, as in Eq. (4), one can rewrite Eq. (11) as

$$R=\rho l^2d/m, \tag{12}$$

[14] D. L. Webster, Am. Phys. Teacher **2**, 149 (1934).

where now R is the resistance of a uniform column l cm long, of mass m, and density d. In order that R may be in international ohms it is necessary that for mercury at 0°C, $R=1$ when $l=106.3$ cm and $m=14.4521$ g. Hence

$$\rho d = Rm/l^2 = 1\times 14.4521/(106.3)^2 = (1.27898\cdots)\times 10^{-3} \quad (13)$$

and

$$R = (1.27898\cdots)\times 10^{-3} l^2/m. \quad (14)$$

Eq. (14) thus gives the resistance R, in international ohms, of a uniform column of mercury, at 0°C, of measured length l and mass m. One is now able to measure an unknown resistance, in international ohms, by adjusting a column of mercury until its resistance equals that of the unknown, and then measuring its length and mass. Thus the international ohm can be said to be *defined* by Eq. (12), when ρd is arbitrarily put equal to $(1.27898\cdots)\times 10^{-3}$. To get the actual value of ρ we must know the value of d. The best experimental value is [15] 13.5951 $\text{g}\cdot\text{cm}^{-3}$. With this value ρ becomes[16] 0.9407668×10^{-4} int. ohm·cm. But the value of ρ has nothing to do with the definition of the present accepted international ohm, contrary to the statement of Mie.[3]

Thus far nothing has been said about the dimensions of the units of R and ρ. Even if one assumes the c.g.s. units of l, d, and m, Eq. (12) still leaves the dimensions of R and ρ indeterminate. In the Giorgi system of electric units the international ohm is a primary unit, of specific dimension Ω. Hence the unit of ρ has the dimensions $\Omega\cdot L$, and is called the ohm-centimeter. If the international ohm is considered merely a laboratory standard representing the *absolute* ohm, which in turn is merely a name for 10^9 absolute e.m. units of resistance, then the dimensions of the international ohm are those of the absolute ohm, namely LT^{-1}, and the dimensions of the unit of ρ are L^2T^{-1}. Thus Eq. (14) defines the magnitude of the international ohm but not its dimensions, so long as the dimensions of the factor of proportionality $(1.278\cdots)\times 10^{-3}$ are left unspecified.

[15] R. T. Birge, Rev. Mod. Phys. **1**, 1 (1929).
[16] The original Siemens unit of resistance was designed to make $\rho = 1\times 10^{-4}$.

The last two illustrations show that the arbitrary magnitude and dimensions of derived electric units are due to the arbitrary character of the factor of proportionality that occurs in the defining equation. But the preceding illustrations have shown that a similar factor of proportionality occurs, or may be inserted, in the defining equations of all types of derived units, and leads to an equal arbitrariness in the magnitude and dimensions of the unit. Furthermore every electric and magnetic quantity can be put into some one or more equations of the type of (10) and (11), which contain quantities whose units have already been defined in the c.g.s. mechanical system. Hence by assigning to the factor of proportionality no *new* primary dimension—in particular by assigning *zero* dimension—any electric or magnetic unit can be made a *derived* unit whose dimensions include *only* those of the c.g.s. mechanical system. The resulting system of units so derived is called an *absolute* electric or magnetic system. The factors of proportionality are sometimes called *universal constants*, but they deserve that name no more and no less than the factor in any equation that defines a mechanical unit.

The important fact is that, after one has chosen a limited number of *primary* units, each with an assigned dimension, the magnitude and dimensions of the units of all other quantities can be made to depend upon the magnitude and dimensions of the primary units. The chosen primary units may or may not include an electric or magnetic unit. The c.g.s. mechanical system of units is usually called an absolute system, and hence an electric or magnetic system that includes no new primary units is called an absolute electric or magnetic system. Controversies in this field arise primarily from the innumerable possibilities in the defining of derived electric and magnetic units. The illustrations given here have been chosen to show that these difficulties are by no means confined to such units.

I am indebted to many persons for suggestions in regard to the material of Part I of this paper. I am especially indebted to Professor V. F. Lenzen for many helpful discussions.