

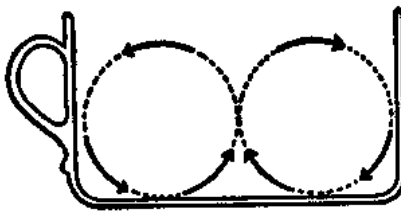
# Einstein's Tea Leaves and Pressure Systems in the Atmosphere

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Tea leaves gather in the center of the cup when the tea is stirred. In 1926 Einstein<sup>1</sup> explained the phenomenon in terms of a secondary, rim-to-center circulation caused by the fluid rubbing against the bottom of the cup. This explanation can be connected to air movement in atmospheric pressure systems to explore, for example, why low pressure systems tend to be stormy and high pressure systems are fair-weather. Here, following Einstein's lead, we revisit the tea leaf phenomenon, make the connection with atmospheric pressure systems and describe an illustrative laboratory experiment.

## Tea Leaves in a Cup

The following quote from Einstein's 1926 article explains the phenomenon, "Imagine a flat-bottomed cup full of tea. At the bottom there are some tea leaves, which stay there because they are rather heavier than the liquid they have displaced. If the liquid is made to rotate by a spoon, the leaves will soon collect in the center of the bottom of the cup. The explanation of the phenomenon is as follows: the rotation of the liquid causes a centrifugal force to act on it. This in itself would give rise to no change in the flow of the liquid if the latter rotated like a solid body. But in the neighborhood of the walls of the cup, the liquid is restrained by friction, so that the angular velocity with which it rotates is less there than in other places nearer the center. In particular, the angular velocity of rotation, and therefore the centrifugal force, will be smaller near the bottom than higher up. The result of this will be a circular movement of the liquid of the type illustrated in Fig. 1 which goes on increasing until, under the influence of ground friction, it becomes stationary. The tea leaves are swept into the center by the circular movement and act as proof of its existence."



**Figure 1.** From Einstein (1926). The arrows show the secondary circulation induced by friction between the stirred tea and the bottom of the cup.

The interface region where friction between the fluid and the container takes place is called the boundary layer. In the cup, the force arising due to radial pressure difference<sup>2</sup> provides the centripetal force required to keep a fluid parcel moving in a circle. i.e.,

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{V^2}{r}, \quad (1)$$

where  $\rho$  is the density of the fluid,  $p$  is the pressure and  $V$  is the fluid velocity and  $r$  is the radial distance from the center. Here the left hand term is called the pressure gradient force per unit mass, arising due to the pressure variation in the radial direction. The partial derivative notation simply means that the pressure difference variations are being evaluated at some constant depth. The right hand term is the centripetal force per unit mass. Outside the boundary layer, where the fluid rotates as a solid body,  $V = \Omega r$ , where  $\Omega$  is the rate of rotation. Within the boundary layer, however, the fluid velocity decreases, say, from  $V$  to  $v$ , and the pressure gradient is not balanced due to this lower velocity. This decrease in velocity leads to a net inward acceleration per unit volume of the fluid within the boundary layer, which has the value  $\frac{V^2 - v^2}{r}$ .

If  $v=V$ , there is no net inward force on the fluid and the fluid would simply move in a circle. If  $v < V$ , then there is a net inward force, and if  $v > V$ , there is a net outward force. The bottom boundary layer of the tea cup corresponds to the former case.

It is insightful for students to view this situation from overhead, not in the usual stationary reference frame, but in a rotating reference frame that has the same angular velocity as the bulk of the fluid in solid body rotation. In this case, it is easier to see the role that the Coriolis force plays in the rotating reference frame. Viewing the cup from the point of view of a reference frame rotating with the bulk of the fluid, the interior fluid is at rest in this frame, and the slower-moving fluid near the bottom appears to move opposite to the rotation of the reference frame velocity  $\Omega$ , with a tangential velocity  $v_t$ . (Note  $v_t=0$  outside the boundary layer and  $v_t = -\Omega r$  right at the bottom, so  $v_t \leq 0$  in the boundary layer). We can write the boundary layer velocity  $v$  in the frame of reference of the tea-cup (i.e. the resting frame) as,

$$v = V + v_t. \quad (2)$$

Substituting (2) in the expression for net inward acceleration,  $\frac{V^2 - v^2}{r}$ , we can re-write it in terms of  $v_t$  and  $\Omega$  as follows:

$$\begin{aligned} \frac{V^2 - v^2}{r} &= -\frac{2Vv_t}{r} - \frac{v_t^2}{r} \\ &= -2\Omega v_t - \frac{v_t^2}{r}. \end{aligned} \quad (3)$$

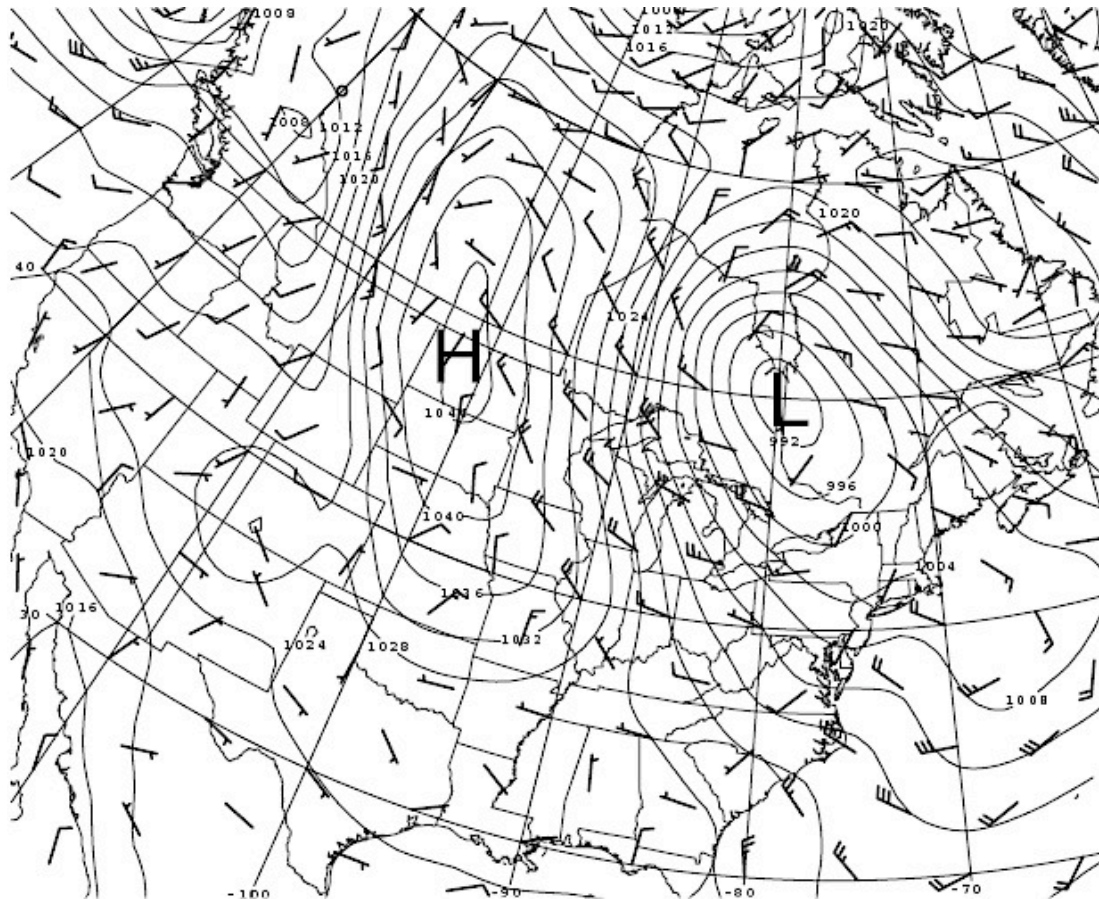
where it has been assumed that  $V = \Omega r$  outside the boundary layer. Here, the first term on the right hand side represents the Coriolis force and the second term represents the centrifugal force (both per unit mass) in the rotating frame<sup>3</sup>. The above expression tells us that the net force is inward, since  $V > v$ . Formally too, we see the Coriolis force ( $-2\Omega v_t$ ) is inward, irrespective of whether the fluid is stirred clockwise or counterclockwise, since  $v_t$  changes sign when  $\Omega$  is

reversed<sup>4</sup>. Thus in the rotating reference frame of the tea leaves phenomenon, the Coriolis force is always directed inward and is greater than the centrifugal term: the fluid within the boundary layer moves towards the center, and takes the tea leaves along with it. This net force leads to a secondary circulation as the fluid moves inward. This secondary circulation must go somewhere, which then demands that the fluid rise near the center and descend near the walls of the cup, just as sketched by Einstein in Fig.1. Take, for example, a tea cup of radius 5 cm, set in motion at  $\Omega=30$  rpm (or  $\pi/2$  rad/s) by a stirring spoon. In the rotating reference frame, at the bottom of the cup, the centrifugal term is  $5\pi^2$  cm/s<sup>2</sup> (0.493 m/s<sup>2</sup>) and directed outward, whereas the Coriolis term is  $10\pi^2$  cm/s<sup>2</sup> (0.986 m/s<sup>2</sup>), and inward. The non-dimensional ratio of centrifugal (or more generally, inertial) term to Coriolis term is called the Rossby number (Ro), after the famous meteorologist, C.G. Rossby. The Rossby number,  $v/(2\Omega L)$  can be easily estimated for flows with characteristic length scale  $L$ , velocity  $v$  and rate of rotation  $\Omega$ . For the tea leaves example, Ro is  $1/2$ , whereas for large scale atmospheric flows, the Rossby number is typically quite small, closer to 0.1. This number is small for flows which are dominated by rotation. Low Rossby number flows have a number of interesting properties.

### Applicability to Pressure Systems

The secondary circulation discussed above has an important analog in atmospheric pressure systems, the circulation of air around them and the associated vertical motion within them. One difference from the tea cup, of course, is that  $\Omega$  must now be interpreted as the rotation rate of the earth, rather than the angular swirl of the tea in the cup: thus  $\Omega$  is constant, does not change sign and is positive. Another is that the radius of curvature for pressure systems can be fairly large reducing centrifugal effects, and leading to a small Rossby number. While centrifugal terms in the rotating frame of the Earth are particularly important for small-scale systems such as tornadoes, they are much less important for large pressure/storm systems. For atmospheric systems, the Coriolis acceleration term is written as  $f \times v_t$  where the Coriolis parameter  $f$  takes into account the component of Earth's rotation in local vertical direction (opposing gravity). For example, at 40N,  $f = 2 \Omega_{\text{earth}} \sin(40) = 6.8 \times 10^{-5} \text{ s}^{-1}$ . For a tornado of radius 0.5km, and wind speed 20 m/s, Rossby number is 571, the centrifugal term dominates and the Coriolis term can be ignored. In contrast, for a pressure system idealized as a circular system of  $r = 1000\text{km}$ , for 5m/s wind, the Rossby number is small, 0.07, and the Coriolis term dominates. While discussing the pressure systems below, we shall assume a small Rossby number, and hence ignore the centrifugal term. In a low pressure system, the air outside the bottom boundary layer is in a balance where, in the rotating frame, the dominant outward Coriolis force (note  $-fv_t > 0$  because  $v_t > 0$  in a cyclone) balances the inward radial pressure gradient. This is called the geostrophic balance. The air rotates cyclonically (in the same sense as the local vertical component of the rotation of the Earth) for this balance to be achieved. For example, in the Northern Hemisphere the air would circulate in a counterclockwise sense outside the bottom boundary layer, parallel to the isobars. As the ground is approached from above, and the influence of friction becomes important, this balance changes. The friction slows down the primary circular motion, and thus the Coriolis force is lower and no longer sufficient for the fluid to flow parallel to the isobars. There is a net inward force, and the air thus moves towards the center of the Low marked L in figure 2, much like the tea leaves example. The converse case is also very interesting. Around a

high pressure system, the air rotation is anticyclonic to achieve balance. In this case for the air outside the bottom boundary layer the inward Coriolis force (note  $-fv_i < 0$  because  $v_i < 0$  in an anticyclone) balances the outward radial pressure gradient.

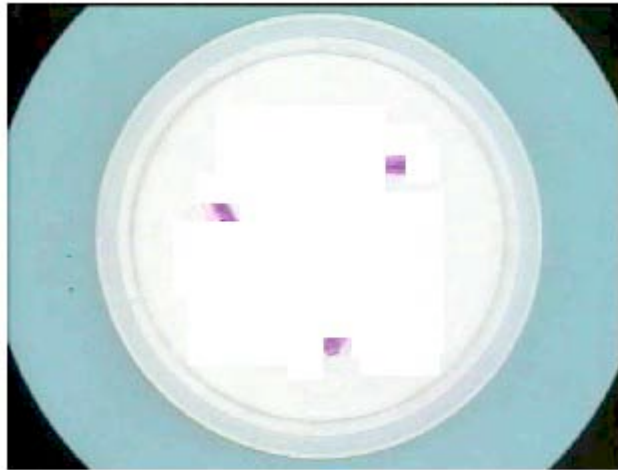


**Figure 2:** Surface pressure field and surface wind at 12GMT on 10th Feb, 2008. The contour interval is 4mbar. High and low pressure systems are marked. The dark segments represent wind arrows, whose arrowhead is not drawn in meteorological plots, by convention. The reader should imagine arrowhead at the end of segment that has no quivers. The quivers are drawn at only one side, at the tail end. The wind blows in the direction of the quiver base to the arrowhead. One full quiver represents a wind of 5m/s. We see air circling in a generally anticlockwise direction around the low but spiraling inwards, and air circling in a generally clockwise direction around the high but spiraling outwards.

As the ground is approached, the friction slows down the primary motion (clockwise in the Northern Hemisphere), which reduces the dominant Coriolis force, while the pressure gradient remains the same and thus there is a net outward force in the bottom boundary layer. The air flow acquires a net radially outward component, as seen in figure 2, for arrows near the High marked H.

## Demonstration using a Rotating Platform

Besides tea leaves stirred in a cup, other classroom demonstrations can show the secondary circulation that arises due to the presence of friction. We discuss a simple laboratory experiment below, which demonstrates these ideas in a more controlled setting and can be readily applied to both atmospheric low and high pressure systems. All one needs is a rotating turntable, a cylindrical container (a large transparent beaker or a cylindrical insert inside a square container works fine), and some potassium permanganate crystals<sup>5</sup>. When rotated at a constant rate, all the water comes into solid body rotation, and so there is no Coriolis or centrifugal accelerations acting. The key experimental requirement is to be able to speed up or slow down (by 10% or so) the rate of rotation of the turntable so as to induce relative motion between the water and the tank, thus creating a frictional boundary layer. The rotating platform can be used in a whole series of experiments to demonstrate atmospheric and oceanic phenomena, as presented in Marshall and Plumb (2007)<sup>6</sup> and the 'Weather in a Tank' website.<sup>5</sup>



**Figure 3:** The experiment is carried out by placing a flat-bottomed, cylindrical container of water on to a turntable and spinning it up to solid body rotation. The rotation is cyclonic (counterclockwise), looking down from the top, to mimic the sense of rotation of the earth. The precise rotation rate is not of great importance, but between 5 and 10rpm works well. After solid body rotation has been achieved (10 minutes or so, depending on the size of the container) three small applications of potassium permanganate crystals are made at the corners of an equilateral triangle, as can be seen in the figure. If solid body rotation has indeed been achieved the crystals should fall vertically (note this is only true at rotation rates that are low enough that the free surface does not become significantly concave, depressed in the middle and rising up to the outside) and settle on the bottom and remain together rather than disperse.

A tank of water is placed on the rotating platform long enough for water to reach solid body rotation, say, about 10 minutes for 10-15 rpm. Then we drop a very few potassium permanganate crystals in an equilateral triangle about the center. This shows up as three small clouds when viewed by a rotating camera (Figure 3). We also drop a few colored paper dots on the surface to see the flow outside the boundary layer. As the table is slowed down by a few rpm (about 10%), the permanganate on the bottom traces the near bottom circulation, which is cyclonic and inward,

just like a low pressure system. The paper dots floating on the surface do not go inward. Why does this happen?



**Figure 4:** To carry out the experiment we first very slightly reduce (by 10% max) the rate of rotation of the turntable. Because of the inertia of the turning fluid, it continues to spin at its original speed and so moves relative to the tank: permanganate streaks are pulled around not in circles as one might initially expect, but rather inward turning, anticlockwise spirals, as can be seen in the top panel. A beautiful symmetric pattern is remarkably easy to achieve. This is analogous to the near-surface flow in a low pressure system, as can be seen by comparing with Fig.2 (see low pressure system). To visualize the flow at the upper surface, we can float a few paper dots on the surface (black dots are the most visible in this application). We observe circular, rather than spiraling, motion. To create an analogy of a high pressure system we now simply increase the speed of the turntable by 10% or so (back up to, roughly, its original speed). We observe the dye streaks on the bottom reversing and, over time, spiraling clockwise and outwards, as can be seen in the lower panel in this figure. This should be compared to the pattern of surface winds that can be seen in the high pressure system marked in Fig.2. From Marshall and Plumb (2007).

The water outside the boundary layer is still rotating with the original fast rotation rate, while the water at the bottom is rotating slower, at the new slower rotation rate. This speed differential, just like the low pressure system leads to an inward flow which is seen in the permanganate streaks at the bottom. (Figure 4). Similarly, the pressure gradient can be reversed by increasing the speed by a few rpm to the original speed, and it leads to permanganate streaks that show an outward anticyclonic flow, analogous to the surface boundary layer of a High pressure system.

This secondary flow in the boundary layer has important implications for movement in the vertical direction. The inward flow associated with a low pressure system leads to rising air near the center of the Low. As this air rises, it expands (pressure always decreases going upward in the atmosphere) and cools<sup>7</sup>. Since the saturation of the air is very strongly dependent on the temperature, as the air cools, it may get saturated, and the water vapor may condense out to form – clouds! This is why the Low pressure systems are the ones associated with stormy weather and precipitation. Conversely, high pressure systems are associated with outward motion in the boundary layer, and hence subsidence. As the air descends, it gets compressed due to the pressure increase, warms, and becomes less and less saturated. Thus the High pressure systems are fair weather systems.

### Acknowledgments

This work is supported by the National Science Foundation via a CCLI-Phase II grant to John Marshall and Lodovica Illari at MIT.

### References:

1. Albert Einstein, “The Cause of the Formation of Meanders in the Courses of Rivers and of the So-Called Baer’s Law.” *Die Naturwissenschaften*, 14, 1926
2. Evidence for this pressure difference can be seen by observing the tilt of the free surface of the fluid which rides up the side of the container, resulting in high pressure in the periphery, low pressure toward the center.
3. In the fixed reference frame, the sum of all external forces provides the centripetal and Coriolis acceleration, while in a rotating reference frame, the sum of external forces on the object are balanced by the centrifugal and the Coriolis force. See the James Bond cartoon, <http://xkcd.com/123/>
4. Since  $v_t = -\Omega r$ , then  $-2\Omega v_t = 2\Omega^2 r$  and so is always positive, corresponding to an inward acceleration.
5. ‘Weather in a tank’ website and project are at <http://paoc.mit.edu/labguide/> and <http://paoc.mit.edu/labguide/ekman.html>. Readers and students can generate their own maps (similar to fig.2) for any day by going to [http://www.paoc.mit.edu/labguide/ekman\\_atmos.html](http://www.paoc.mit.edu/labguide/ekman_atmos.html), clicking on the maps and following subsequent directions.
6. John Marshall and Alan Plumb, *Atmosphere, Ocean and Climate Dynamics, An Introductory Text*, (Academic Press, 2007), p.131.
7. The overwhelming influence of gravity in the vertical direction implies that the pressure variation is much larger in the vertical than it is in the horizontal.