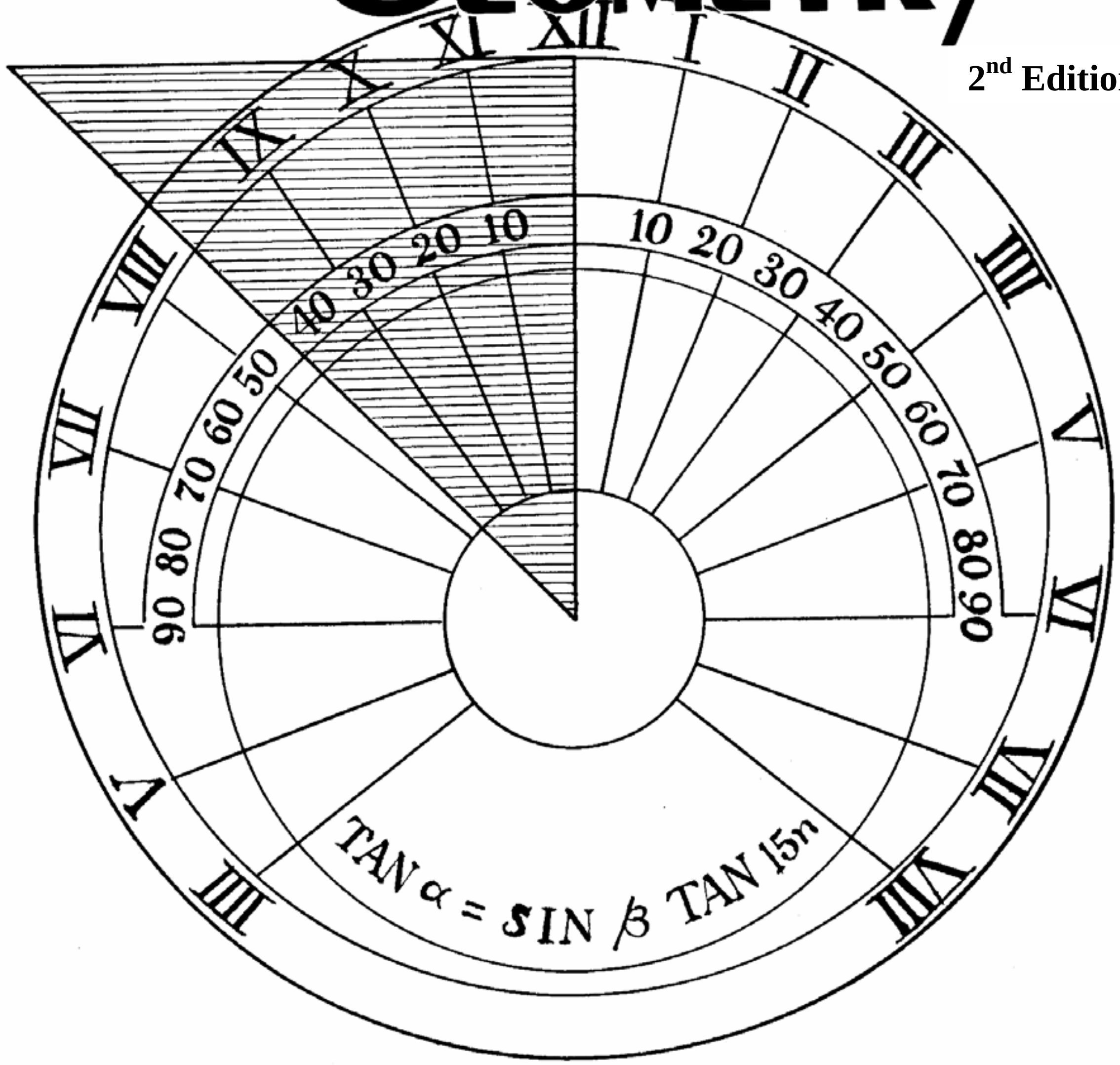
The Sundial and Geometry

2nd Edition



AN INTRODUCTION FOR THE CLASSROOM

BY

LAWRENCE E. JONES

THE SUNDIAL AND GEOMETRY

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INTRODUCTION 2005

The expanding membership of NASS, the North American Sundial Society, has created several requests from new members and from teachers for introductory level materials about sundials. When Fred Sawyer asked me if my book *The Sundial and Geometry* could be updated and reprinted, I was delighted to say yes.

However, the book had been out of print for some years and needed some dusting off and recharging. I asked Mr. Sawyer to do this, and he has done a magnificent job getting the history into proper order and updating where newer information has become available. He has improved the nomogram in section 8 with a new computer generated version which greatly improves the precision.

INTRODUCTION 1979

The genesis of this booklet goes back quite a few years to my early teaching days when I wrote out steps to make a sundial for a class that was learning to use a protractor. During succeeding years, as youngsters responded, my interest in sundials grew, and applications to all levels of classes began to present themselves.

The earliest attempts to use the sun for indicating time are lost in history. The use of the sundial for regulating time reached its zenith during the seventeenth and eighteenth centuries. Hundreds of texts were written on the subject, and every aspect of sundial theory was explored. Some of the books available today are listed in the bibliography. These books are essentially modern renditions of the old texts, and this booklet is drawn from them. No originality is claimed by this writer other than an occasional expression of a point of view or a variation in the sequence of presentation.

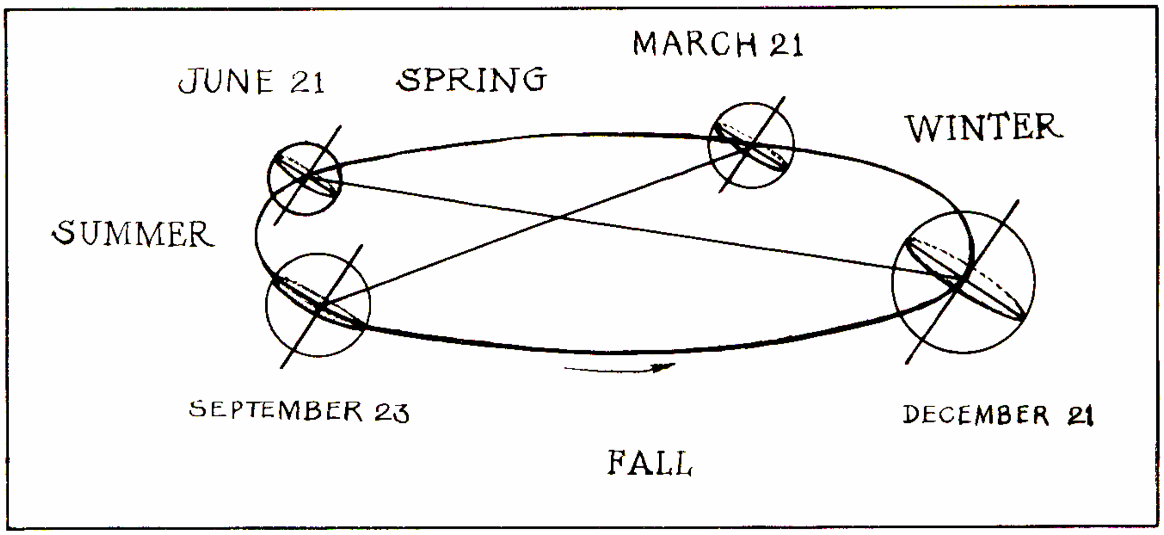
The heart of this booklet is Chapter II, "Delineation Of The Horizontal Sundial." Applications from this chapter require only minimal mathematical understanding. The material collectively is well within the reach of high-school mathematics. No effort has been made to list specific applications to schoolroom use; in this I follow the principle that self-discovery and personalized presentation are fundamental to good teaching.

Particular thanks are due my colleagues Mr. Jack Allen for the drawings of the historic sundials and Mrs. Elizabeth Meigs for her help with the details of the manuscript.

I sincerely hope this material will find its way, through devoted teachers, to students in classrooms and that from it, their understanding of mathematics and life itself may be enlightened even a little.

QUI DOCET DISCIT
(He who teaches, learns)

I



The earth obeys two rules of motion. It rotates on its axis to give us day and night. It orbits the sun to give us the seasons.

HISTORY OF THE SUNDIAL

Section 1 *The Classsical Period*

A sundial, in its broadest sense, is any device that uses the motion of the apparent* sun to cause a shadow or a spot of light to fall on a reference scale indicating the passage of time. The invention of the sundial is lost in the obscurity of ancient times. But we can imagine some of the factors that led up to its invention.

Very early in our history, humans must have observed the shadows cast by trees and noticed that the shadows grew shorter as the morning wore on and then grew longer again after midday. Thus, if a shadow a little after sunup was twice as long as the height of a stick, then, in the afternoon, when the shadow was again twice as long as the height of the stick, there would be that same amount of time left before sundown (*see Figure 1*).

* It seems to us, as it did to the ancients, that the earth is stationary and the sun moves across the sky. Actually, the sun is stationary (as shown by Nicolaus Copernicus, A.D. 1543) and the earth is in motion, spinning on its axis as it orbits around the sun. However, the shadows would be the same no matter which heavenly body was in motion. The "apparent" sun is understood to be the actual sun we see – that does indeed 'appear' to us - as opposed to the fictional mean sun that we will discuss later.

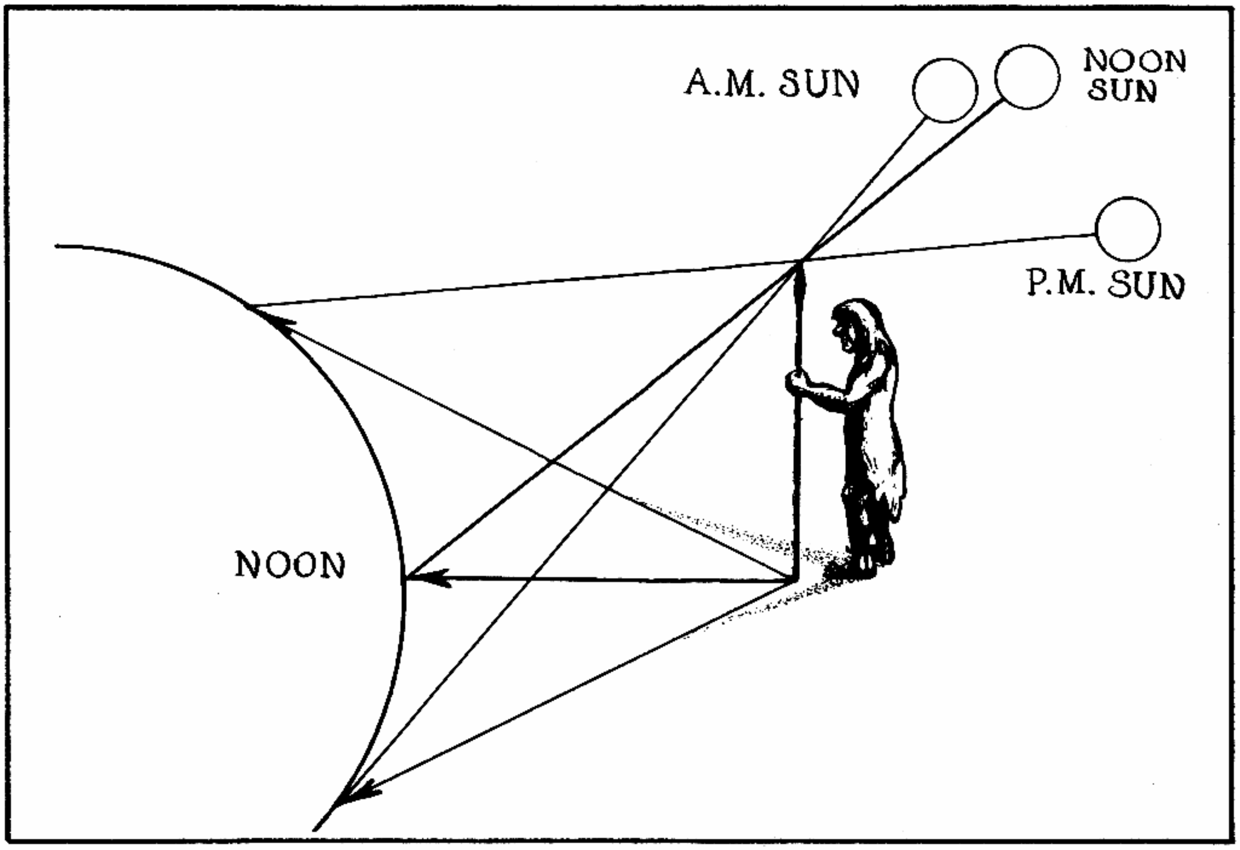


Fig.l The length of the shadow is a crude indication of the passage of time. A morning shadow of given length (direction not considered) and an afternoon shadow of the same length would represent equal periods of time before or after noon.

The shadow of a vertical object like the spear, or in later times a tall obelisk, can be used in two ways to indicate the passing of time. The first method, which uses the changing length of the shadow, is illustrated above. The second method uses the changing direction of the shadow. In the morning as the sun rises in the east, the shadow points west. Then, as the day advances, the shadow first swings to the north and then to the east, where it points when the sun sets in the west.

The direction method has been preferred historically over the length of shadow method. The problem with the latter is that shadows are shorter in the summer than in the winter because the earth is tilted toward the sun in summer and away from the sun in the winter.

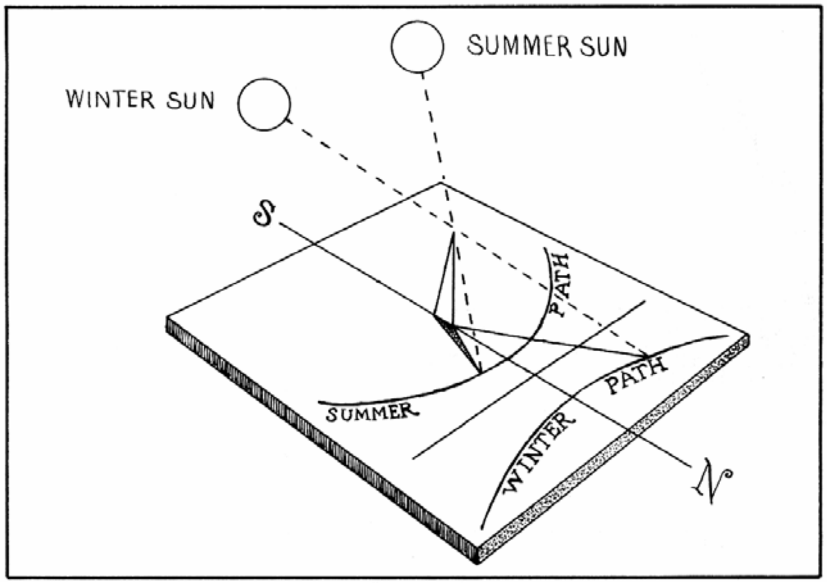


Fig.2 The length of the noon shadow in summer is GS, and in winter it stretches to GW. The same is true for any other hour of the day.

The summer and winter paths of the sun can easily be traced out by experiment. With a gnomon* secured firmly in place, simply set a stone at the various positions of the tip of the shadow five or six times each day, then connect these positions. The result is a graphic illustration of the relationship of the sun to the seasons. The shortest shadow is traced about June 21, and the longest about December 22. The straight-line shadow occurs about March 21 and again about September 23. Such a basic knowledge of the seasons was essential to the earliest agricultural societies, so we might assume that their need gave rise to the first primitive sundials. The Greek historian Herodotus (484-425 B.C.) stated in his writings that the sundial originated in Babylonia in the fertile valleys of the Tigris and Euphrates rivers.

As time went on, people's needs grew more complex and so did the design of the sundial. A major change was that markings were introduced to show the passage of the hours of the day. The oldest sundial in existence falls into this latter category. It is of Egyptian origin and dates to the time of Thutmose III (fifteenth century B.C.). This portable stone device, housed today in the Berlin Museum, is L-shaped and may have been topped by another straight piece as shown in the figure (*see Figure 3*). Different versions of how this dial was used have been offered. According to one view, the cross piece was placed facing east before noon and facing west after noon, and the shadow of the top bar indicated the time among the markings on the lower bar. Another view is that the dial (perhaps without the top bar but with some other sort of capstone) was always placed directly towards the sun, so that the shadow of the elevated portion indicated the time by where it fell among the hour markings.

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* The object that casts the shadow, such as a pole or obelisk, is generally called the "gnomon," a word of Greek origin meaning "one who knows."

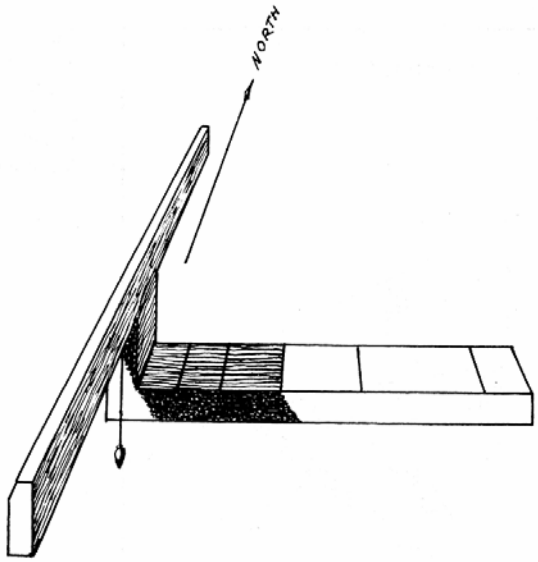


Fig. 3. The oldest sundial extant. c1500 B.C.

A number of simple sundial designs were developed in the Greek classical period. Aristarchus of Samos (3rd century B.C.) is said to have designed the sundial called a "hemispherium" (*see Figure 4*). A stone hemisphere was hollowed out; then the gnomon, a vertical pin, was set in the center point. The tip of the pin traced, in reverse, the path of the sun as it moved across the sky. Vertical markings on the surface divided the daylight period into twelve temporary hours[*], and horizontal lines delineated the seasons or months. A similar type of sundial was found at the base of Cleopatra's Needle in Alexandria when that site was excavated in 1852 (*see Figure 5*). It is now in the British Museum.

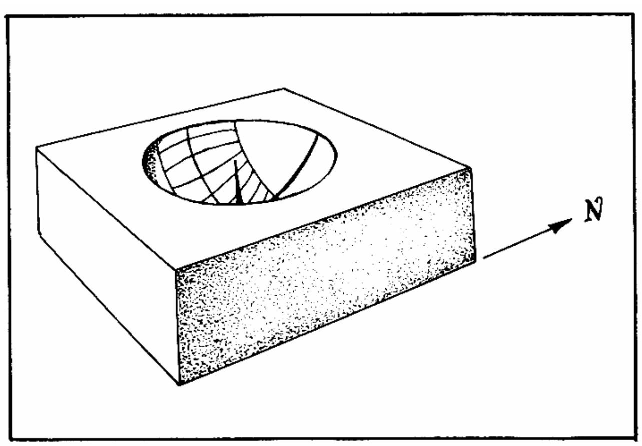


Fig. 4: The hemispherium of Aristarchus.

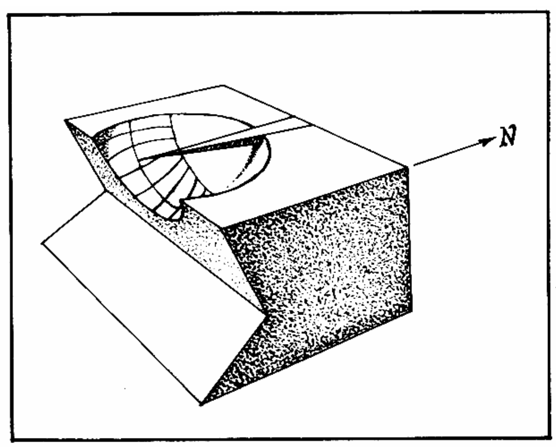


Fig. 5: The hemicyclium of Berossos

The pelekinon is another sundial of the Greek period. The word ‘pelekinon’ comes from the Greek word for a double-headed axe, which perfectly describes the appearance of the temporary hour lines on this sundial. A perpendicular pole served as the gnomon, and the dial face showed not only the seasonal lines but also the hour lines. This sundial could be drawn on vertical as well as horizontal surfaces (see Figure 7).

[*] A temporary hour is 1/12th the time that the sun is above the horizon on any given day. Temporary hours are longer in the summer and shorter in the winter. They were the common measure of time until about the end of the 14th century.

Although mathematical knowledge, such as astronomy, the geometry of Euclid (300 B.C.), and the foundations of trigonometry of Hipparchus (160-125 B.C.) was growing, there is simply no evidence to suppose that any methods other than empirical ones were used to construct hour lines. However, the seasonal lines, hyperbolic in shape, could by this period be accurately calculated mathematically for any latitude.

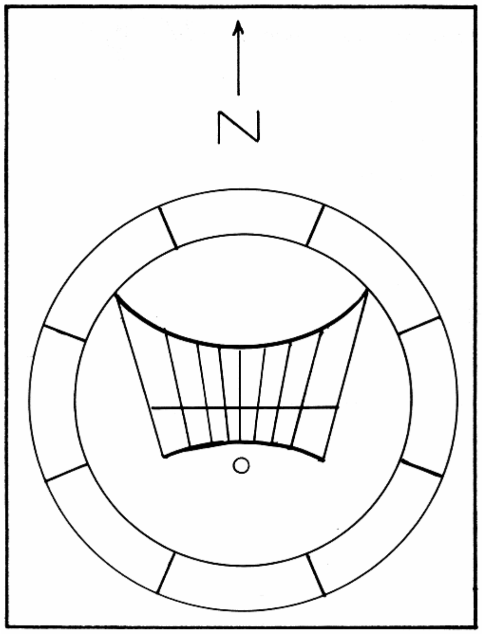


Fig. 6: Pelekinon dial.

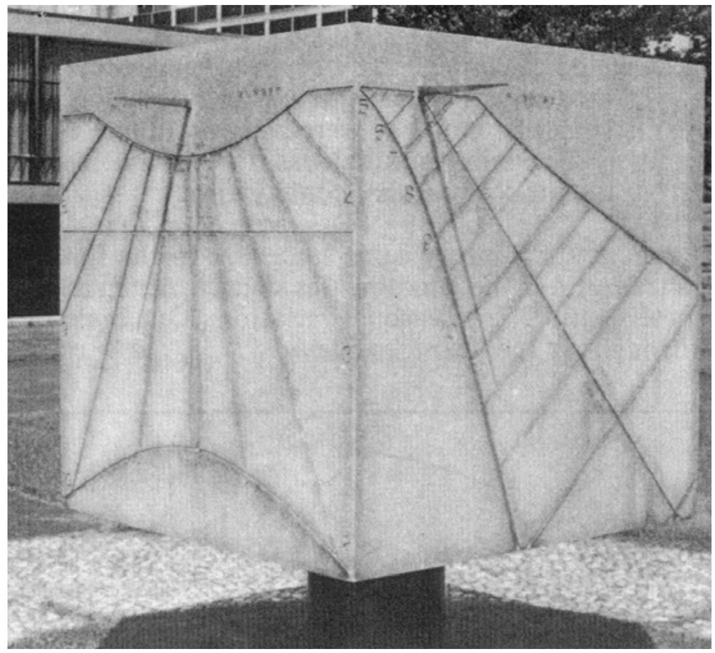
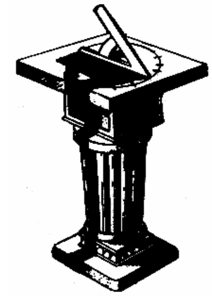


Fig. 7: Modern Pelekinon Cube Dial. Bloomfield CT

Though each of the new designs had brought improvements, all of the early sundials indicated the time as measured in temporary hours; that is, they were based on a division of daylight into twelve equal periods. Since the sun's position in the sky changes every day, the length of the day, sunup to sundown, changes too. Therefore, each of the twelve periods in a summer day was much longer than each in a winter day. In addition to this seasonal change, there was a change between day and night. For example, each period in a summer "day" was much longer than each period in a summer "night."

This arrangement may seem strange to us today, but it was the way time was reckoned for over 2500 years. This method of timekeeping based on temporary hours remained in use until about the fourteenth century, when mechanical clocks became popular. They were less ‘versatile’ than sundials and could not indicate hours of varying length; their unit of time had to be one of equal length throughout the year. As the popular notion of timekeeping changed, so did the design of sundials, and sundials were developed that can also measure time in equal hours.

Section 2 *The Beginnings of Modern Dialing*



Over the centuries, many types of clepsydra (water clock, literally water thief) and clepsammia (sandglass or hourglass) were popular as timekeeping instruments, but they were all subordinate to the sundial. However, very little is known about exactly how and when sundials made the transition from indicating temporary hours to indicating equal hours, i.e., hours that are each one twenty-fourth of an average day.

By the end of the 10th century, Arab astronomers certainly were aware of 'the great discovery' (*see Figure 8*) of modern gnomonics – that using a gnomon that is parallel to the earth's axis will produce sundials whose hour lines indicate equal hours on any day of the year. A dial of this type was made by the astronomer Ibn al-Shatir for the Umayyad Mosque in Damascus in 1371. It is the oldest polar-axis sundial still in existence.

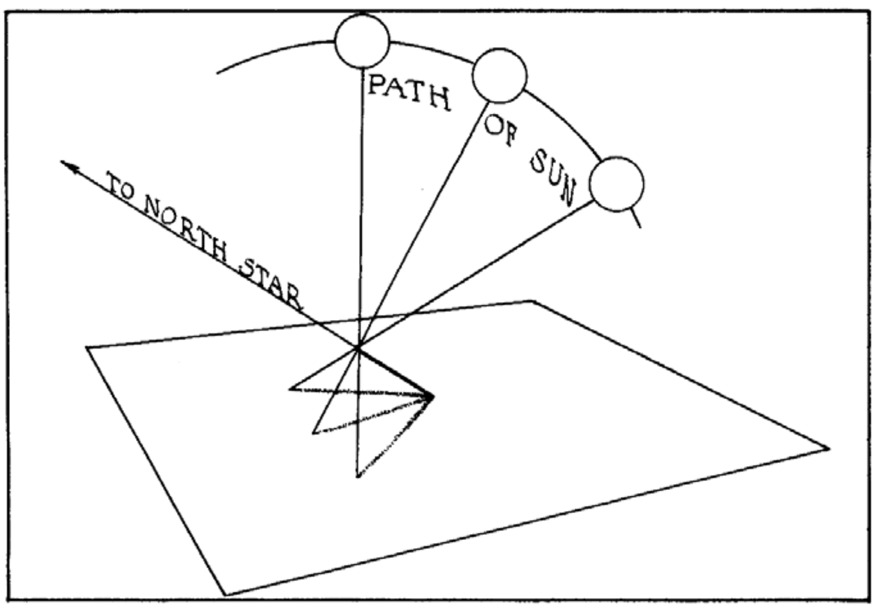


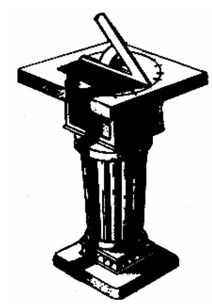
Fig. 8. The great discovery

It took much longer for the idea of a polar-axis sundial to make its appearance in Europe. The oldest extant examples are probably a 1446 dial in Germany and several Austrian dials from the period 1447 - 1457. However, none of the many 14th and 15th century European texts on sundial construction mentions the idea of a polar-axis gnomon to indicate equal hours.

The first books to deal with modern dialing, using equal hours, appeared in the 16th century. Sebastian Münster published *Compositio Horologiorum* in Basel in 1531, and Oronce Finé published *Protomathesis* in Paris in the following year. Münster's book was the first treatise on the new form of sundials; for this reason, he is sometimes referred to as the 'Father of Modern Gnomonics'. However, almost nothing in his book is claimed as original, and he clearly wrote it more as a comprehensive summary of contemporary techniques than as the introduction of a revolutionary new idea.

After the 16th century, everyone in Europe, it would seem, was conversant with the art of dialing. Visitors to the birthplace of Sir Isaac Newton are dutifully shown the ceiling dial made by young Isaac. He used a mirror on the windowsill to reflect the sun's rays onto the ceiling where hour lines had been drawn. The art of dialing, or gnomonics, held an honored place in the school curriculum. Every mathematics book had chapters on dialing, and many books were devoted entirely to the subject.

The earliest mechanical clocks were woefully inaccurate. Even the pendulum clocks of the 1700's needed a standard for setting and checking. The sundial served that purpose. By the time of the American Revolution, however, mechanical clocks had greatly improved and had begun to replace the sundial as the timepiece of choice.



Section 3 *The Colonial Period*

In early America a sundial was a luxury and, therefore, rare. By the 1820s, Eli Terry, with waterpower and mass-production methods, had placed an inexpensive wooden-works clock in nearly everyone's home. The sundial was more or less driven into retirement as a garden ornament. Nevertheless, dialing had its enthusiasts well after its decline in popularity.

The earliest sundial to reach the continent was probably the one erected by Champlain in 1608. It was placed on the building of the Habitation, which marked the founding of Quebec City.[1] The earliest sundial in the U.S. arrived in 1638 (*see Figure 9*). It accompanied a clock commissioned by Governor John Endicott of Massachusetts. The clock and the sundial were made in London, but the sundial was calibrated for Boston and was used as the standard by which to set and regulate the clock.[2]

[1] Malcolm M. Thomson, "Sundials," *The Physics Teacher Journal*, Volume 10, Number 3.

[2] Leonard V. Short, Jr. "The Endicott Sundial," *Sundials and More* (booklet), New Ipswich, New Hampshire.

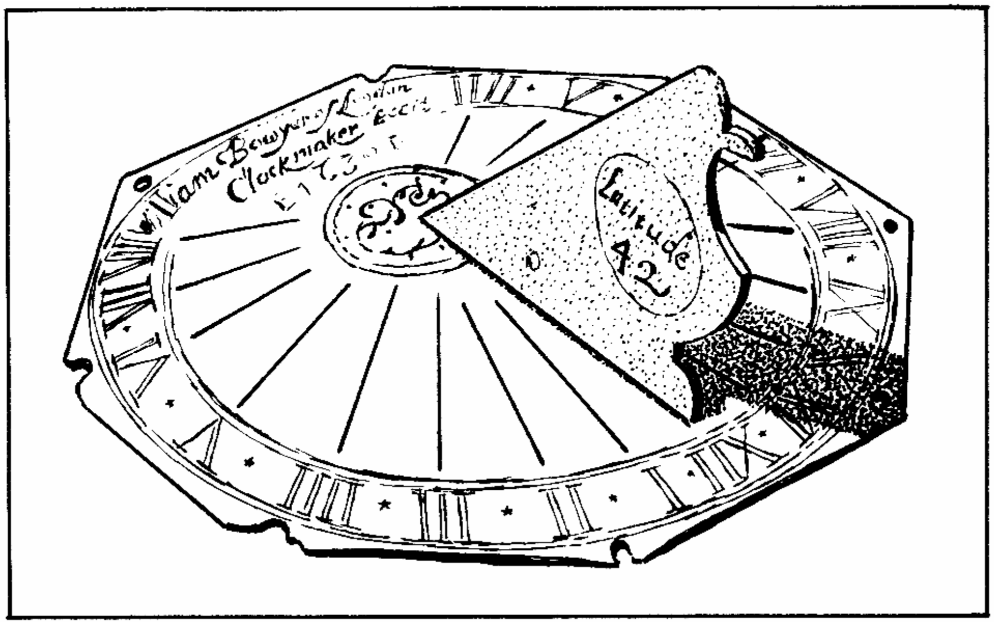


Fig. 9. The Endicott sundial. This sundial is on exhibit at the Peabody Essex Museum in Salem, Massachusetts. (Drawing by Jack Allen, from a photograph by Leonard V. Short.)

Another historic sundial was placed on the State House in Boston when it was built in 1713. That wood dial was lost long ago, but when the building was refurbished in 1957, a new one was made from an old engraving showing the original dial (*see Figure 10*). This replica dial has since been removed.

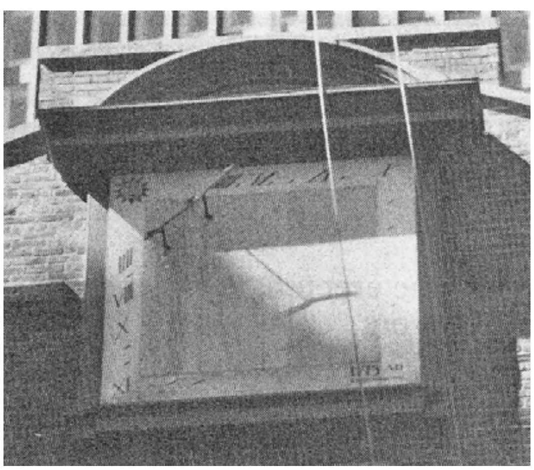


Fig. 10. Replica of 1713 Dial on the Old State House, Boston, MA.

Fig. 11. A slate dial, 1722. Found in Nova Scotia.

A slate sundial, dated 1722, was recovered during excavations and restoration of the Fortress of Louisbourg, in Nova Scotia (*see Figure 11*). The motto, incomplete because of a missing piece, probably read Sum Sine Sole Nihil, which means "Without the Sun I am Nothing." [*]

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[*] Thomson, "Sundials".

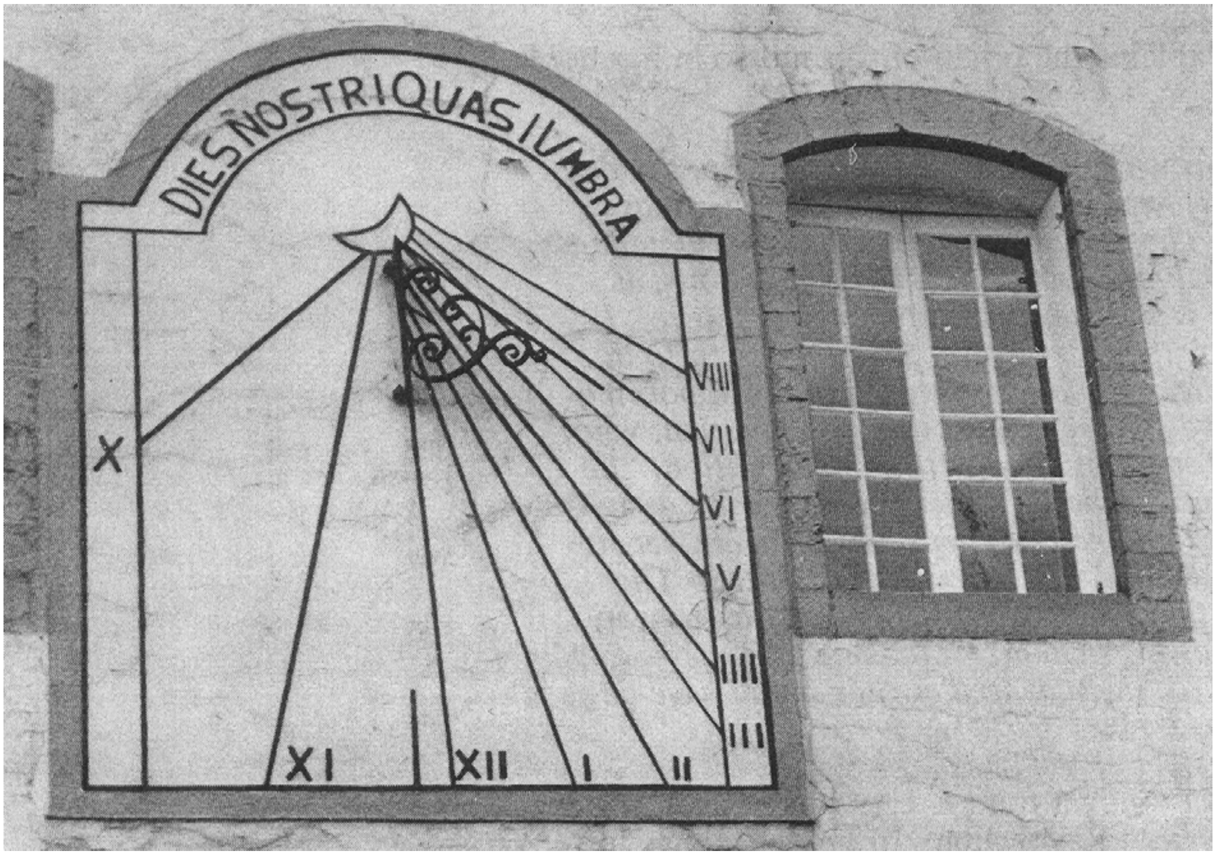
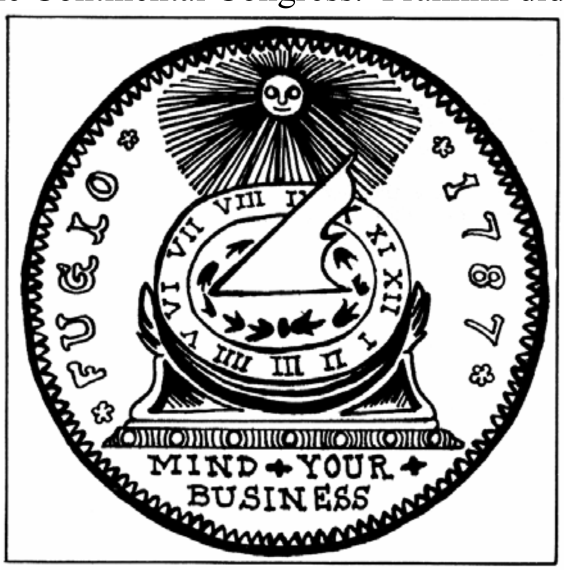


Fig. 12. Dial at Quebec Seminary, 1773. Motto is I Chronicles 29:15. Our Days on Earth Are Like a Shadow.

Benjamin Franklin was an enthusiastic student of the sundial. It was through his influence that the first U.S. coin, the *Fugio Cent*, was cast with a sundial design and the motto "Mind Your Business." It was minted in 1787 under the authority of the Continental Congress. Franklin did not originate the motto; he brought it



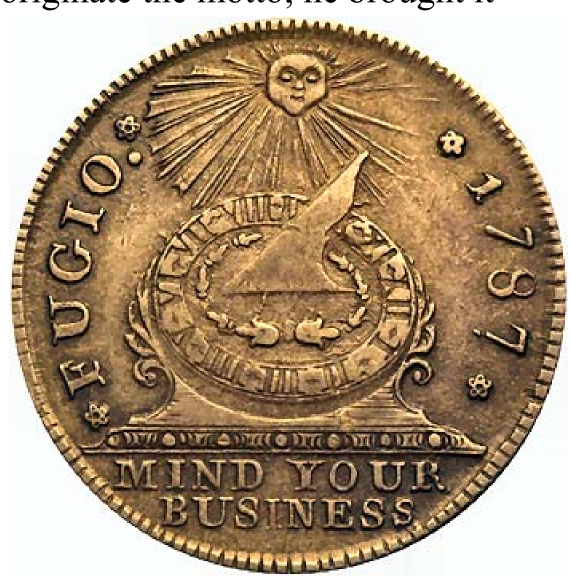


Fig. 13. The Fugio Cent, 1787.

back from London where he had spent considerable time studying the English postal service. On the Post Office building there was a sundial with a similar motto.

Alice Morse Earle explains the origin of this motto in her book, *Sun-Dials and Roses of Yesterday*:

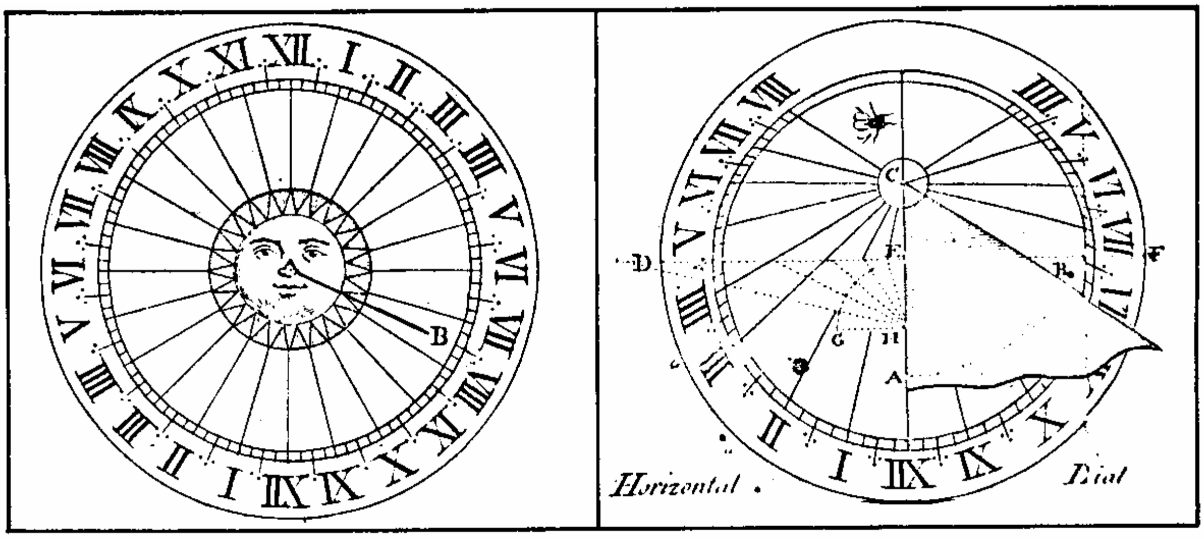
Legend has it that when the dial was first put up, the dial maker enquired if he should paint a motto under it as was customary. The Benchers assented and instructed him to call at the library on a certain day and hour, at . which time they would have agreed upon the motto. It appears, however, that they had totally forgotten to do this, and when the dial maker appeared at the appointed time and enquired for the motto, the lone librarian, who knew nothing about it, replied with some irritation, 'Be Gone About Your Business." The dial maker, by design, or mistake, chose to take this curt reply as an answer to his enquiry and the dial was accordingly painted. The Benchers, when they saw it, decided that it was very appropriate and that they would let it stand, chance having done their work for them as well as they could have done it for themselves.

George Washington is known to have carried a pocket dial in preference to a watch in the American Revolution. In 1811 Thomas Jefferson, in a letter to Charles Clay, tells how he amused himself during an illness by calculating a sundial scaled for each five minutes of time. He sent his directions along so that Mr. Clay's young son could make the sundial.

In France, as late as the beginning of the twentieth century, one of the leading railroads used the sundial to regulate the watches of its trainmen. This dial was a precisely machined sun clock called a heliochronometer.

There is a common but erroneous belief that a sundial is not very accurate. The sundial indicates local time (also called sun time), while our watches indicate Civil time, known in the United States as Standard Time. The inaccuracy is not in the sundial, but in the translation from local time to Civil time. Most persons are not aware that these two times can differ by fifteen minutes or more, and so, unfortunately, the casual observer declares the sundial inaccurate. How this easy translation is done will be presented in later chapters.

II



Charles Leadbetter, Mechanick Dialling, London, 1737

DELINEATION OF THE HORIZONTAL SUNDIAL

Section 4 ***The Classroom Sundial***

A sundial consists of two parts. The *Dialface* is a flat surface on which are drawn the lines that label the hours of time. The Gnomon is a triangular shape that rises perpendicular above the dial face with its slanted edge equal in degrees to the local latitude. When the sun shines, the gnomon casts a shadow on the dialface. The time is read from the scale on the hour lines at the point indicated by the shadow of the gnomon.

The hour lines drawn on the dialface are calculated from the sundial formula

$$\alpha = \arctan(\sin \beta \tan 15n)$$

where n is the number of hours before or past noon, β is the local latitude, and α is the angle value that determines where to draw the hour lines. Before making the classroom sundial, a table giving the α values must be consulted, or a formula should be used to determine the α values for your exact latitude.*

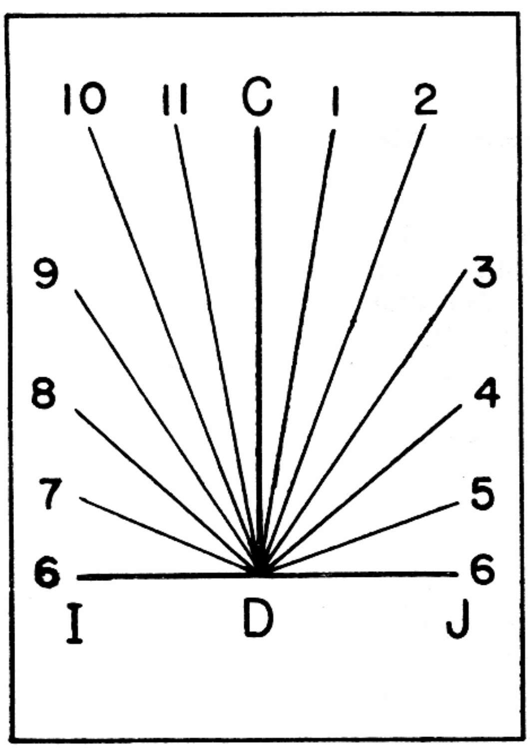
* Latitude and longitude can be found from a map of your local area in the Rand McNally Road Atlas. On the Internet, such sites as www.multimap.com or www.multimap.com also provide this information.

TABLE I

Hour Angles (α) For Various Latitudes

Hour AM	Hour PM	n	Latitude (β) 30°	36°	42°	48°	54°	60°
12:00		0	0.0	0.0	0.0	0.0	0.0	0.0
11:00	1:00	1	7.6	9.0	10.2	11.3	12.2	13.1
10:00	2:00	2	16.1	18.7	21.1	23.2	25.0	26.6
9:00	3:00	3	26.6	30.4	33.8	36.6	39.0	40.9
8:00	4:00	4	40.9	45.5	49.2	52.2	54.5	56.3
7:00	5:00	5	61.8	65.5	68.2	70.2	71.7	72.8
6:00	6:00	6	90.0	90.0	90.0	90.0	90.0	90.0
5:00	7:00	7	118.2	114.5	111.8	109.8	108.3	107.2

On the dialface, the morning angles are measured to the left of the noon line, and the afternoon angles are measured to the right of the noon line.



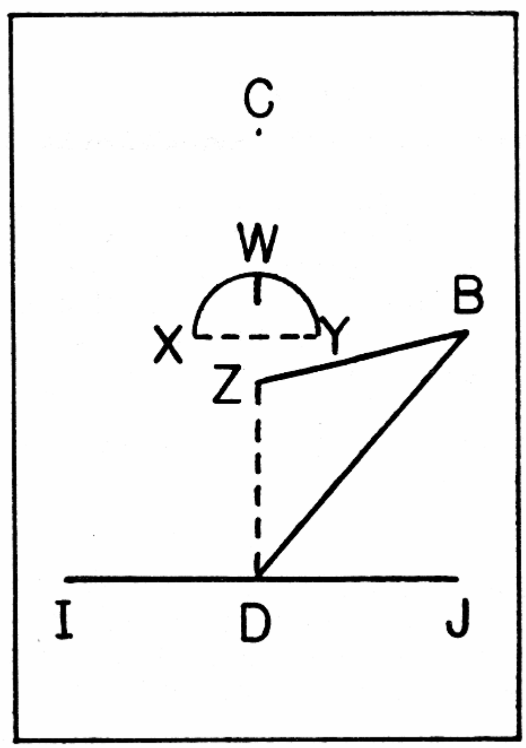


Fig. 14. Dialface layout and cutout for gnomon.

Directions for Making a Classroom Sundial (*see Figure 14*)

1. On a 3 x 5 card, lay out line DC perpendicular to IJ.

2. Place a protractor on D along DC and measure each hour angle for your latitude. Indicate each measure around the edge of the protractor with a dot. The morning hour angles are measured to the left of the noon line DC, and the afternoon hour angles, which are symmetric to those of the morning, are measured to the nght of the noon line.

3. Connect D with each dot. These are the hour lines, which are labeled from 6 A.M. at point I, through 12 noon at point C, and on to 6 P.M. at point J (*see Figure 14a*).

4. Measure angle CDB equal to your latitude (*see Figure 14b*, which is the same as *Figure 14a* but with the hour marks left out for clarity). Draw XY perpendicular to DC and in line with point B. Connect B with some point Z on line DC. Draw a semicircle using XY as its diameter.

5. With a sharp blade, cut along lines BD and BZ, and around the semicircle. Also cut a one-quarter inch slit at W into the semicircle flap.

6. Crease the semicircle and the triangle so they stand up along the lines XY and DZ respectively. The slit in the semicircle will fit on the triangle to hold the gnomon erect.

7. Place the sundial in the sun. The dialface must be perfectly level and the noon line DC point to true north. D is the south end and C is the north end. (Procedure for finding true north is in Section 5.)

8. As a final step, the sundial should be adorned with an appropriate motto (*see Figure 15*).

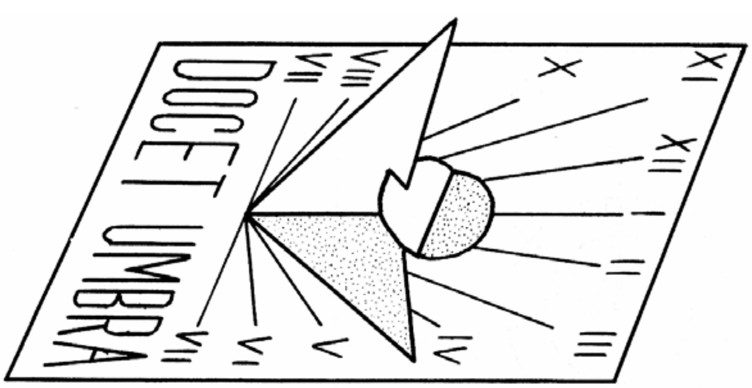
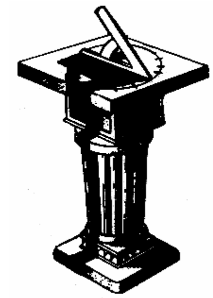


Fig. 15. Motto – The Shadow Teaches. The hours are labeled for Daylight Saving Time.

In summer, Daylight Saving Time is in use, so it would be desirable to have the sundial scaled accordingly. To do so, each hour label is increased by one. Thus the DI line becomes 7 A.M., the DC line becomes 1 P.M., and the DJ line becomes 7 P.M. (*see Figure 15*).

Section 5 ***On Finding True North***



Use of the magnetic compass is not a reliable way of establishing a meridian line, *i.e.*, finding the north-south line. The compass points to the magnetic pole, which (in 2004) is just to the west of Ellesmere Island, Nunavut, Canada, considerably distant from the actual north pole. In the eastern United States this causes the compass to decline towards the west by as much as 20 or more degrees, while in the western United States the compass can decline towards the east by as much. A line of near zero declination runs west of the Great Lakes, through the western portions of Wisconsin and Illinois, splitting Mississippi in half and entering the Gulf of Mexico around New Orleans. The pole and this line are constantly shifting. So, unless the exact current declination for your area is known and compensated for, the compass does not indicate true north.

The sun rises in the east and sets in the west and must, therefore, cross the north-south line. This crossing occurs at midday. The following method for finding true north is based on this principle.

Draw two concentric circles on a flat board and place a nail at the center of the circles. The nail must be absolutely erect. Place the board perfectly level and locate it so that the sun will fall on it from mid-morning to mid-afternoon.

As the morning shadow shortens, mark the spot where the tip of the shadow touches the outer circle. When the afternoon shadow lengthens, mark where the

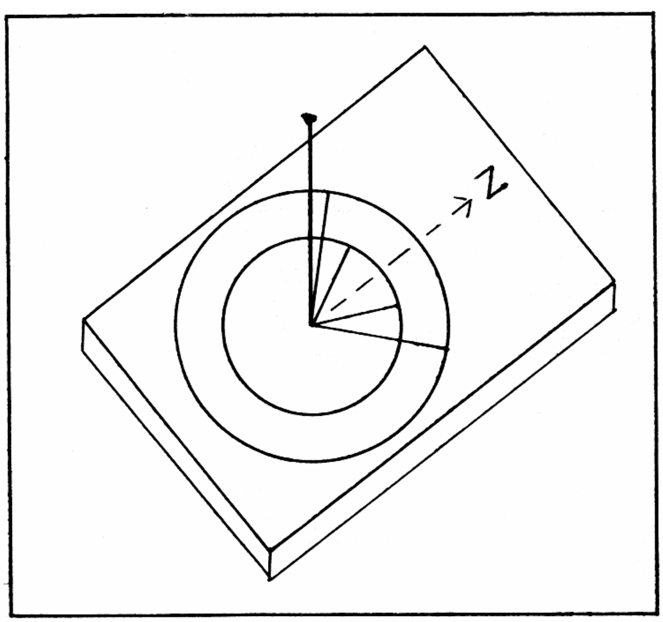
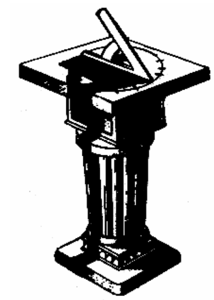


Fig. 16. Find true north.

tip of the shadow touches the outer circle again. Make two similar marks on the inner circle as a check or in case the sun goes behind a cloud at the critical time.

Connect the two points on each circle with the center point, and bisect each angle. The composite of these two bisectors will be a true north-south line (*see Figure 16*).



Section 6 ***The Geometric Method***

Historically, geometric methods preceded trigonometric methods for finding the hour lines. Indeed, the geometric methods remained the more popular of the two. Each writer apparently felt obliged to devise his or her own method. Originality seemed more important than simplicity, for many of the old methods are very difficult to follow. The mathematical foundations for these methods were frequently omitted.

The method presented here is reasonable and easily followed. The necessary tools are protractor, straightedge, and compass.

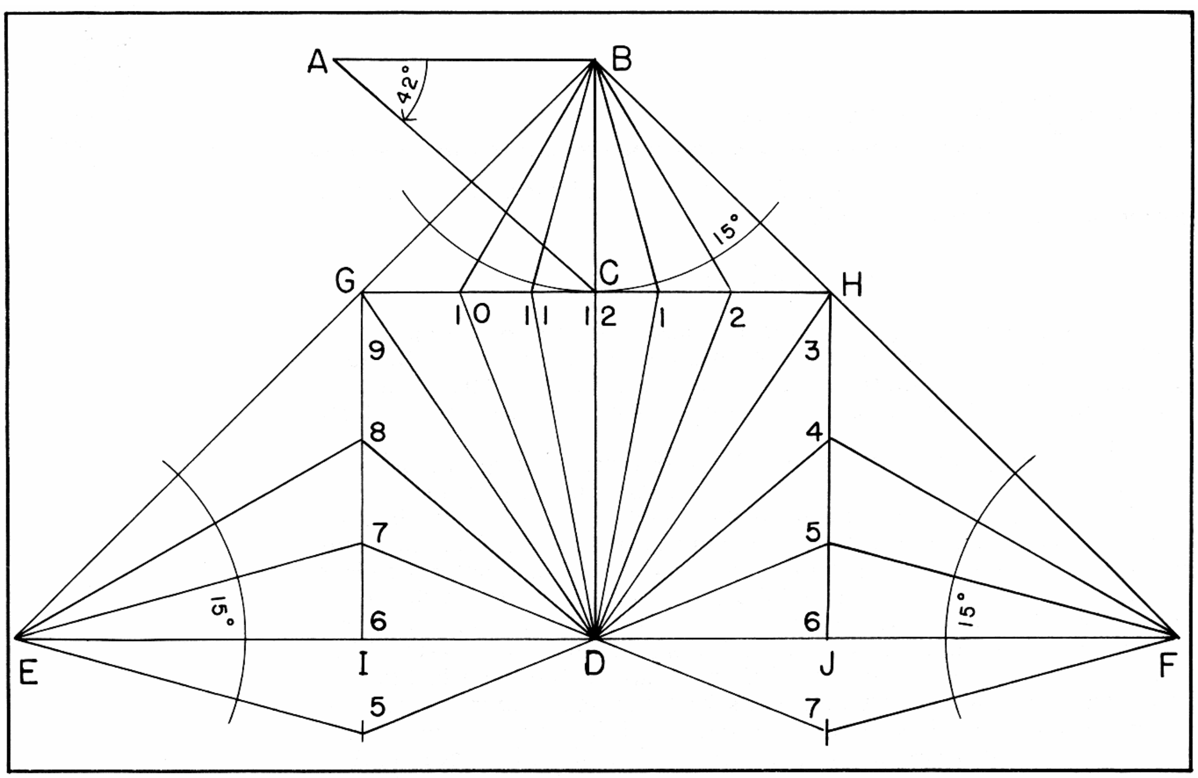
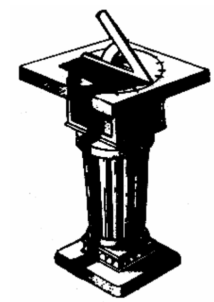


Fig. 17. Geometric layout for latitude 42°.

Geometric Layout for the Sundial Hour lines (*see Figure 17*)

1. Draw line AB and erect a perpendicular from B.
2. At A, draw AC, making angle BAC equal to your latitude.
3. Measure CD equal to CA.
4. Draw EF perpendicular at D with DE and DF equal to DB.
5. Connect B with E, and B with F.
6. Make GH perpendicular at C, and drop GI and HJ perpendicular to EF.
7. At E, B, and F, draw arcs of radius BC.
8. Divide these arcs into 15-degree sectors, and extend each radius until it meets a side of the rectangle GHIJ.
9. The lines drawn from D to these points of intersection are the hour lines for a sundial at your latitude.
10. The dialface may be circular, elliptic, pentagonal, hexagonal, or as you please. The desired shape is placed symmetrically on the DC line. If you are making a sundial of material heavier than cardboard, you must allow for the thickness of the gnomon (as explained on page 29).



Section 7 *The Sundial Formula*

The above construction, and also the table of hour angles in an earlier paragraph, are based on the sundial formula:

$$\tan\alpha = \sin\beta \tan 15n\,.$$

This formula will generate the hour angles for the garden-type horizontal sundial. The value α is the angle measure for each hour line, the value β is the local latitude, and the value n is the number of hours from noon. The formula is necessary because the angles for the hour lines are all different.

To understand the derivation of this formula, it is helpful first to consider a simpler type of sundial on which all the hour shadow lines are uniformly spaced at 15 degrees. Such a dial is called an equatorial dial, taking its name from the equator of the earth. Suppose the earth to be hollow and the outside globe to be transparent. Suppose also that the equator is a plane through the center of the earth, and the axis that connects the north and south poles is a long straight rod. Suppose further that there is a fictitious sun that rotates around the earth at a uniform speed and that it stays within the extended plane of the equator. This fictitious sun is often called the mean sun. It requires twenty-four hours for the mean sun to travel one revolution, through the 360 degrees of the equator. If lines were drawn radiating from the center of the earth and spaced every 15 ($360 \div 24$) degrees, each line would represent one hour of time. Any particular hour of the day would be indicated by the shadow cast by the axis rod. The illustration of this, in Figure 18, is an equatorial sundial.

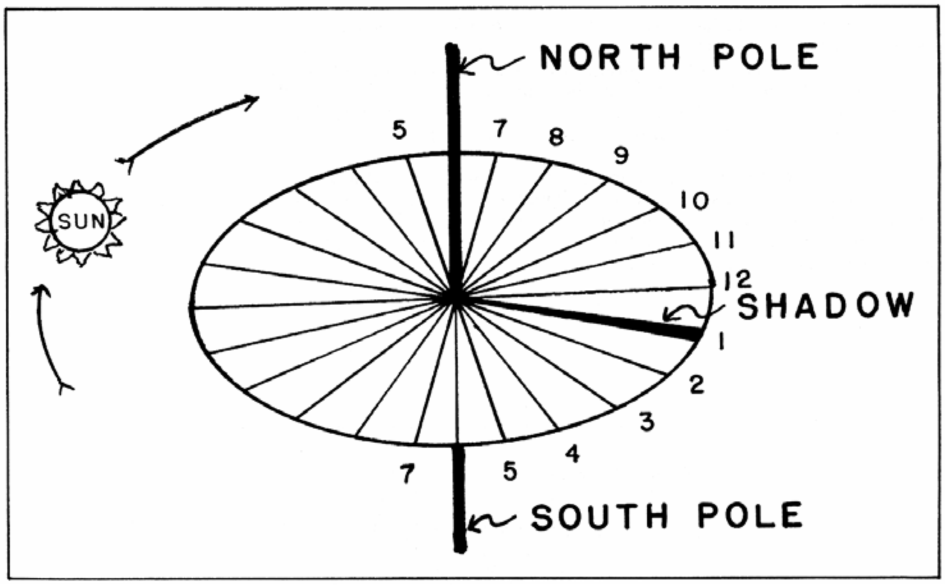


Fig. 18. The sun revolving around the earth's axis casts a shadow on the equatorial plane.

Actually the earth is not hollow, nor is it transparent, but if an equatorial dial were made and placed on the surface of the earth at the equator, with its circular dialface in line with the equatorial plane and its rod parallel to the earth's axis, it would indicate each hour in exactly the same way. In fact this equatorial dial could be placed anywhere on earth, as long as the circular dial face remained parallel to the earth's equatorial plane, and its rod (gnomon) was parallel to the earth's north-south axis (*see Figure 19*).

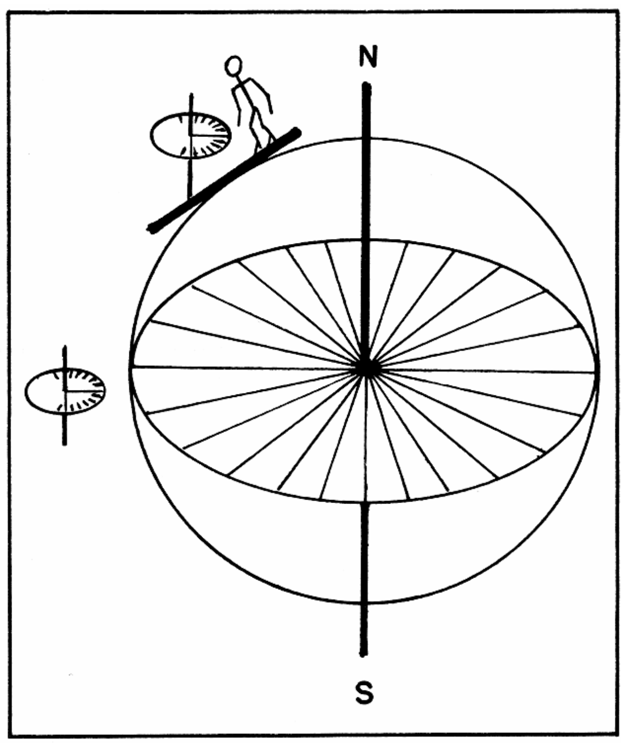


Fig. 19: The equatorial dial, when moved away from the equator, will be slanted to a person viewing it.

The earth is inclined at 23.4 degrees with respect to the real sun, and because of this, the shadow appears on the top side of the equatorial dialface in summer, and on the underside of the dial face in winter. Thus, for year round use, the equatorial sundial requires the hour lines to be drawn on both surfaces.

An equatorial sundial at some northern latitude will appear tilted because we view it from the perspective of our local horizon. It is desirable, however, that a sundial face be parallel to the local horizon. The solution, then, is to translate the hour lines from the equatorial dial to a horizontal dialface. Notice in Figure 20 that the angle between the gnomon and the horizontal dialface must be equal to the local latitude angle.

The plane of the local horizon, shown oblique (inclined) in Figure 20, is shown level, as we experience it, in Figure 21. A vertical plane has been added. The equatorial dial gnomon, although still parallel to the earth's axis, becomes oblique in perspective to a person standing on the earth's surface at some distance from the equator.

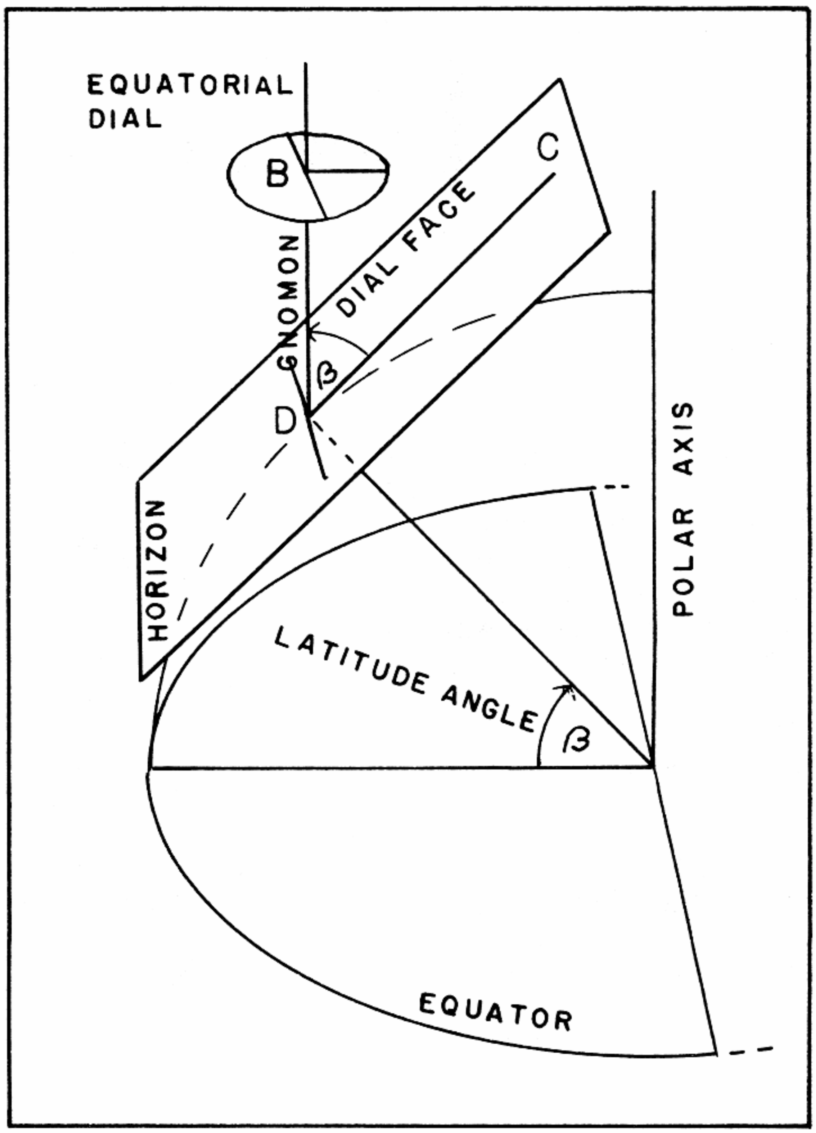


Fig. 20: The equatorial dial is oblique to the horizon at an angle equal to the latitude.

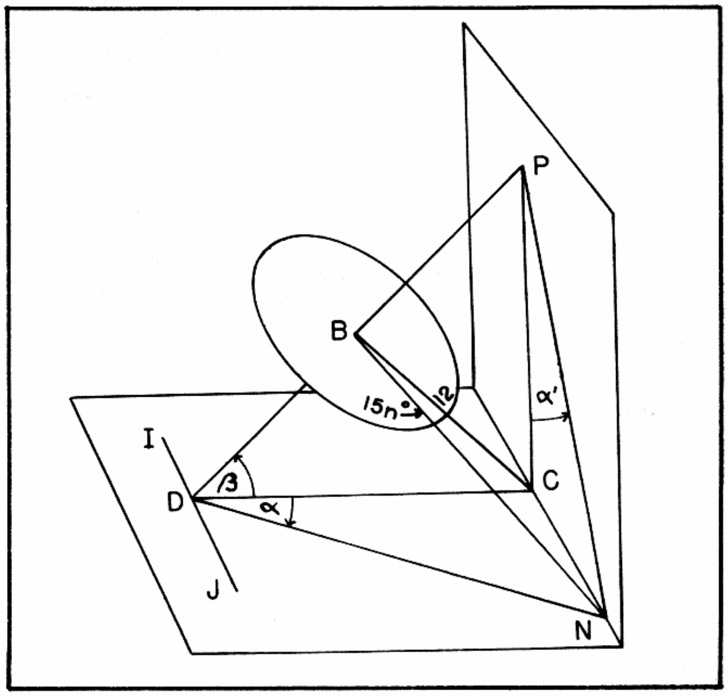


Fig. 21 The equatorial dial in relation to the horizontal and vertical dials.

Derivation of the Sundial Formula

Observe in Figure 21 that:

∠DCP = ∠BCN = ∠DCN = ∠DBC = right angle
Let ∠CDN = α = hour angle
Let ∠CDB = β = latitude
Let BN = extension of hour line from the equatorial dial
For 1 o'clock, ∠CBN = 15°
For *n* o'clock, ∠ = 15*n*°

In ΔDBC: $\sin\beta = BC/DC$, so $BC = DC\sin\beta$

In ΔCBN: Let ∠CBN = 15*n*°
$\tan 15n = CN/BC$
$CN = BC\tan 15n = DC\sin\beta\tan 15n$

In ΔDCN: $\tan\alpha = CN/DC = \sin\beta\tan 15n$

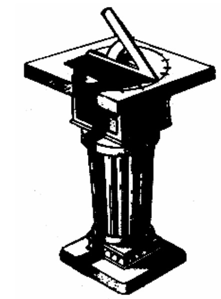
Horizontal sundial formula:

$$\tan\alpha = \sin\beta\tan 15n$$

The formula for the vertical sundial can be found in similar manner to be:

$$\tan\alpha' = \cos\beta\tan 15n$$

Section 8 ***The Alignment Chart Method***



Geometric constructions and trigonometric formulae have been presented. There is a third technique by which a sundial may be delineated called dialing scales. The alignment chart or nomogram presented here (*Figure 23*) is original to this writer, but there is no question that others have also created the same or similar scales.* Mine is based on the basic sundial formula $\tan\alpha = \sin\beta \tan 15n$ and is, therefore, a convenient way to determine the hour angles for any latitude.

The alignment chart is used by holding a string taut across the chart (*see Figure 22*). The string (or ruler!) must cross the appropriate latitude on the left scale labeled "Latitude", and the other end of the string must cross the desired hour on the right scale marked "Hour". The reading where the string crosses the inner scale marked "Degree" will be the correct angle to use on the sundial face. The afternoon hours are measured to the right of the 12 o'clock noon line, and the morning hours, which are symmetric to those of the afternoon, are measured to the left. The 6 o'clock line is perpendicular to the noon line.

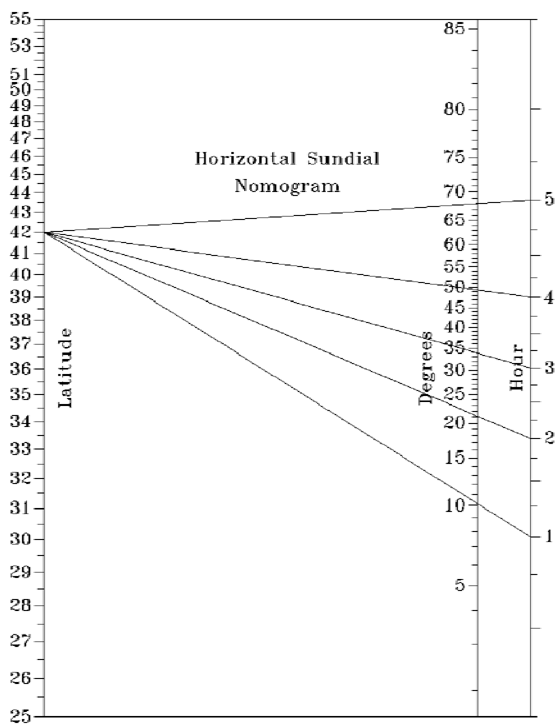


Fig. 22. Nomogram for sundial hour angles (latitude 42°).

* Frank Cousins (*see Bibliography*) presents a dialing scale that is triangular in pattern. This scale, invented in 1638 by Samuel Foster, has the further advantage that, instead of simply calculating the required angle, it allows you to lay the angle out on the dial face.

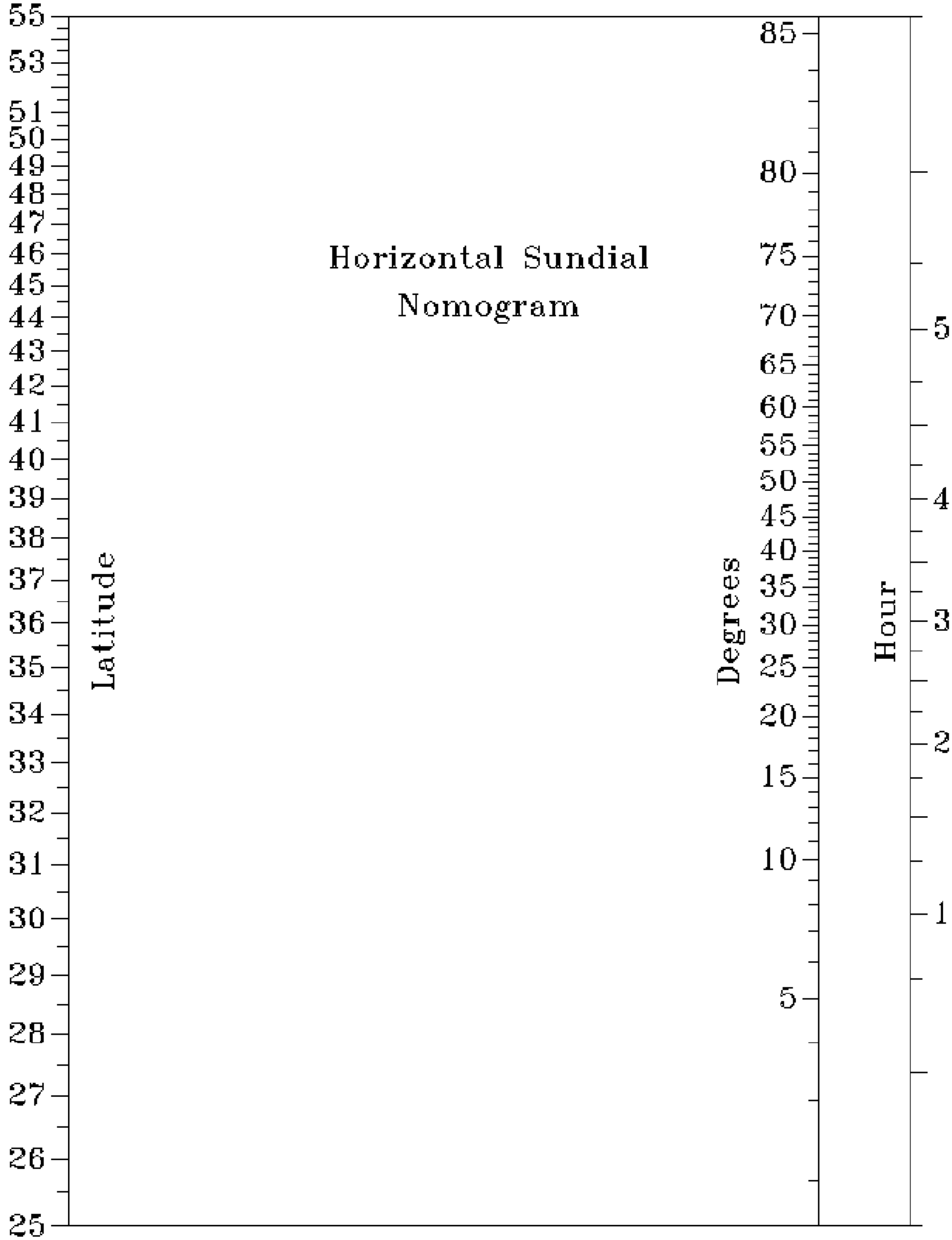
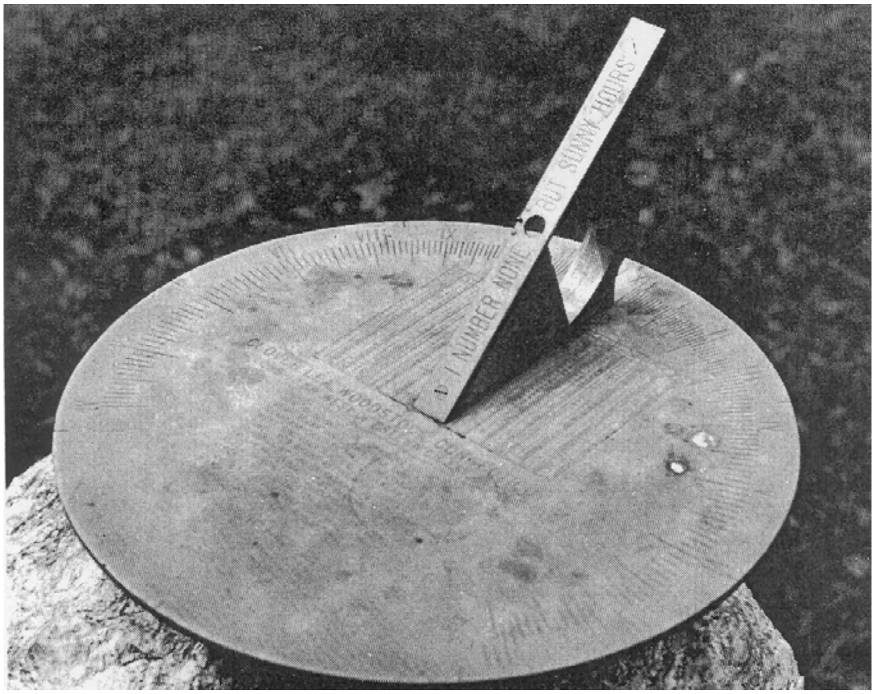


Fig. 23. Nomogram for sundial hour angles (as redrawn by F.W. Sawyer)

III



Etched dialface c1920. The month is read from the inside circular scale of the declination of the sun. With the date known, the equation of time chart is consulted, and the sundial reading is converted to clock time.

CORRECTIONS TO THE SUNDIAL READING

Section 9 *Conversion from Sun Time to Clock Time*

The natural phenomenon of the earth's varying speed in its elliptic orbit and the manner in which the earth's axis tilts away from or towards the sun as the seasons change make the hours from the real sun irregular and uneven, whereas the mean sun assumes a uniform speed and equal hours. Clocks also operate at a uniform speed, with equal hours, giving mean time. Although the difference between sun time and clock time may vary by as much as seventeen minutes, neither time should be thought of as less accurate than the other. It is simply a matter of translation. This difference is known as the equation of time,* which is shown in Figure 24 as a graph. During the months when the sundial is ‘slow’, the correction is added to the sundial reading; during the months when the sundial is ‘fast’, the correction is subtracted from the sundial reading. The derivation of the equation of time will be investigated in the following section.

* In astronomy, the word ‘equation’ refers to an amount which must be added to or subtracted from the result of an observation or calculation in order to compensate for a known cause of error or irregularity. Do not confuse this use of the word with the more familiar mathematical equation.

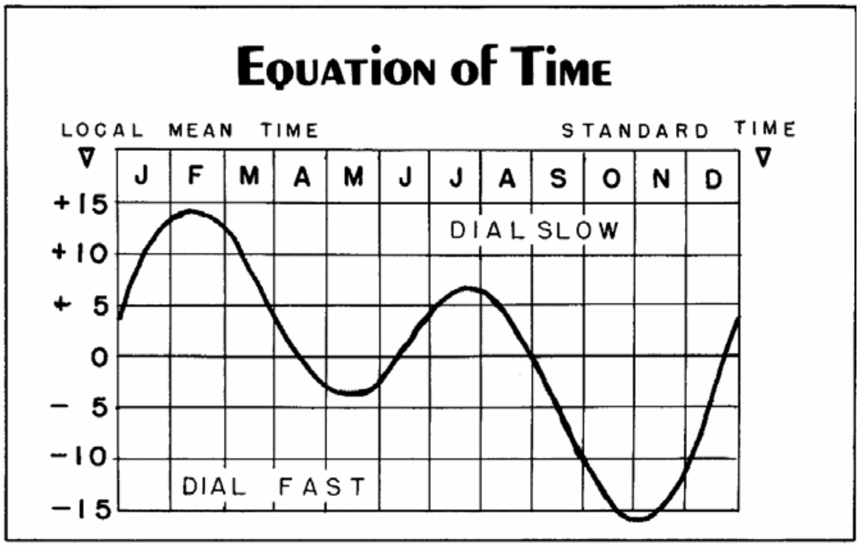


Fig. 24

A second correction of sundial time first became necessary in the 1880s as the rai1roads, figuratively speaking, brought cities closer together. To understand this, it is necessary to define noon as that time in the day when the sun is at its highest point and crosses the local meridian. This time is assigned 12 o'clock. When people lived pretty much in their own areas, it didn't matter that when it was noon in Boston it was only 11:14 in Cleveland, and when the sun traveled westward to cross the meridian at Cleveland it was 12:44 in Boston. However, this was a great inconvenience to the railroad in making timetables for its trains.

The solution came with the invention of zone time, also called standard time. Starting at Greenwich, England, time zones were laid around the earth roughly every 15 degrees of longitude. This placed four time zones across the United States *(see Figure* 25). Meridians 75, 90, 115, and 120 are at the center of the Eastern time zone, the Central time zone, the Mountain time zone, and the Pacific time zone respectively. The lines separating time zones are very irregular because they follow geographic and population clusters for convenience. The Eastern Standard Time zone, for instance, extends eastward from the 75th meridian to include Boston and westward to include Cleveland, and all clocks in the zone will be set at the same time, based on 12 noon when the sun crosses the 75th meridian.

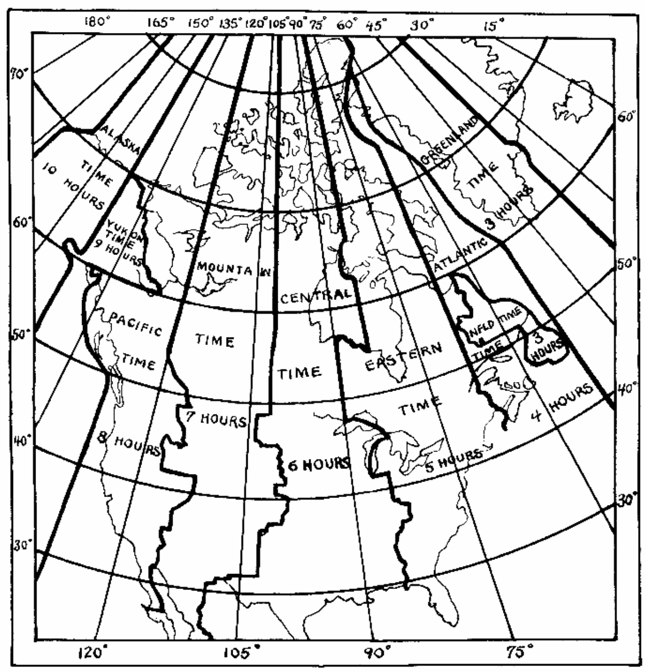


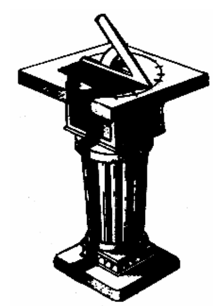
Fig. 25. The time zones of North America

The sundial will still indicate the local time, even though all clocks will indicate the same standard time. In Boston (71st meridian) the sundial will indicate noon when the sun is on the meridian, but Eastern Standard Time (EST) won't indicate noon until the sun reaches the 75th meridian. Since the sun requires 4 minutes to travel one degree westward, it will require 16 minutes to travel 4 degrees of longitude from Boston to the central meridian of the time zone. Therefore, in Boston the sundial will always be 16 minutes fast. At 12 noon EST the sundial at Cleveland (83rd meridian) is 8 degrees of longitude away, so the sun will require 32 (8 x 4) minutes to reach there, and, therefore, a sundial in Cleveland will always be 32 minutes slow. This is called longitude correction, and once it is determined for any area, it remains the same through the year.

RULE: *To find the longitude correction, find the difference between the local longitude and the appropriate central meridian. Multiply this difference by four to get the minutes of correction. For areas east of the central meridian, the sundial is fast, so subtract this correction from the sundial reading to get standard time. For areas west of the central meridian the sundial is slow, so add this correction to the sundial reading to get standard time.*

These two corrections can be combined into a single correction by adding the longitude correction to the equation of time correction and writing the combined values as the scale on the right side of the chart in Figure 24. For example, Hartford (72nd meridian) is 3 degrees from the central meridian, so it is 12 minutes fast. The correction is made by subtracting 12 from the scale on the left, giving the values: +3, -2, -7, -12, -22, -27. These numbers are written on the right side of Figure 24. Using this new scale, the graph will translate sundial time in Hartford to standard time within one to two minutes of accuracy.

Section 10 ***The Equation of Time***



Just as one can drive a car without understanding how the engine works, one can also use the equation of time graph *(Figure* 24) without understanding the details of its derivation. In a similar manner, casual readers may omit this section without jeopardizing their understanding of the following sections.

There are two factors that enter into the equation of time. The real sun moves with irregular speed relative to the earth, and its orbit is oblique to the equatorial plane of the earth. These two factors should first be considered separately and then together.

The irregular speed of the real sun westward through the heavens is caused by the eccentricity of the earth's orbit. Kepler long ago discovered that the earth's orbit is elliptical rather than circular. He also showed that the earth will sweep equal area in equal time, as shown in Figure 26. The sun is at one focal point of the ellipse; thus, the radius in winter (in the northern hemisphere) is less than in summer; hence, the earth's speed along the orbit must be faster in winter and slower in summer. This change in speed effects a change in the length of the average day. A day is caused by the earth's rotation on its own axis 360 degrees from noon to noon. But as it rotates on its axis, the earth is also in orbital motion, and must, therefore, rotate more than 360 degrees to complete one day (*see Figure* 27). In winter, when the earth orbits faster than average, it travels farther, so it requires additional rotation to complete the day. In summer, the slower speed of orbit requires less than average rotation to complete the day.

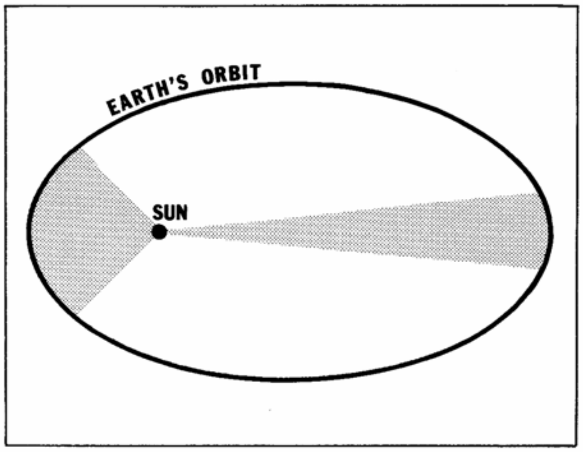


Fig. 26. Equal area in equal time. (Ellipticity greatly exaggerated.)

This factor alone would cause the equation of time (the difference between sun time and clock time) to be zero on January 1 and July 1, and the values between would reach approximately eight minutes.

Perhaps this phenomenon can be more readily visualized if we let the average excess beyond the 360 degrees rotation be identified with the fictitious mean sun in orbit at a uniform speed and let the change to more or less than average be identified with the real sun in apparent orbit at speeds sometimes faster and sometimes slower than the mean sun. Both suns will complete one full orbit in one year, but their relative positions will vary back and forth.

The equation of time is identified by the distance the two suns are apart. When the real sun is at its slower-than-average speed, it trails the mean sun. It will slowly accelerate and begin to catch up, which it does when at its fastest speed. At the moment it passes the mean sun, it starts to decelerate; however, it is still moving faster than

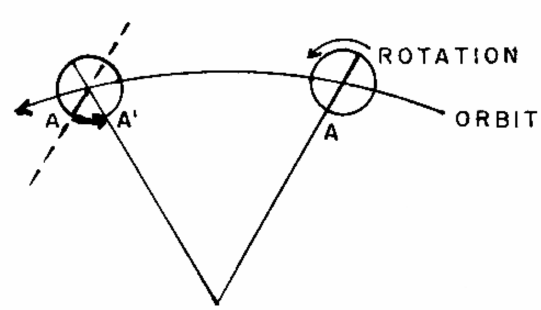


Fig. 27. Angle AA’ is the additional rotation beyond 360° required to complete a day.

the mean sun, so it pulls ahead. As it continues to slow down, it will reach speeds less than that of the mean sun. The mean sun, at its constant speed, begins to catch up, doing so when the real sun is at its minimum speed; one year is completed, and the process begins again.

The fastest and the slowest speeds occur on January 1 and July 1, so the two suns are coincident on those dates, making the equation of time zero on those dates.

The plane in which the real sun orbits around the earth is oblique by 23.44 degrees to the equatorial plane of the earth. This oblique orbit is called the ecliptic and is caused by the earth's axis being tilted 23.44 degrees to the plane of its own orbit around the sun. This is what causes the sun to reach 23.44 degrees above the earth's equatorial plane on June 21 (summer solstice), and to dip 23.44 degrees below the earth's equatorial plane on December 21 (winter solstice). This angle of declination of the sun is zero on March 21 and September 22 (the equinoxes) when the sun crosses the equatorial plane.

To study the effect of the inclination of the earth, the two suns may again be used. The real sun will follow the ecliptic around the earth, traveling at a uniform rate. The mean sun will orbit in the equatorial plane at the same uniform rate as the real sun. Let the two suns be together at the vernal equinox, as shown in Figure 28.

In a given length of time, the mean sun will travel along the equator as vector M. In the same given time, the real sun follows the ecliptic along vector R for the same distance, but its horizontal component R', projected onto the equator, is less than M. Here the mean sun is ahead of the real sun, and will continue to gain until the real sun has climbed to its maximum latitude, 23.44 degrees, where its direction becomes parallel to the equator and its honzontal component equals R and equals M. However, as the real sun approaches its maximum height, its projection has grown larger than the vector M, and therefore the real sun, by the time it reaches its solstice, has caught up to the mean sun so that they are both at the same meridian.

By this factor alone, the equation of time (the difference) will be zero four times a year: the two equinoxes and the two solstices, or about the 21st of March, June, September, and December. Its greatest difference, about ten minutes, will fall in between these dates.

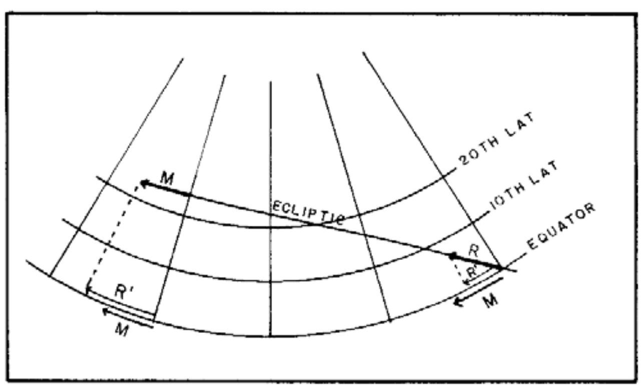


Fig. 28. This chart shows the effect of the 23½° inclination of the earth to the equatorial plane.

To find the complete equation of time, these two factors must be combined *(see Figure* 29). The value of the eccentricity factor is plotted as a dotted line. The value of the inclination factor is plotted as a dashed line. The combination is the solid line, which is determined by adding the ordinates of the two factors for each abscissa value. The actual equation of time will be zero four times a year, about April 25, June 14, September 1, and December 25. Notice that the equation never differs by more than seventeen minutes.

One further point should be mentioned. If the equation of time is plotted on a horizontal scale of minutes, and the months are plotted on a vertical scale

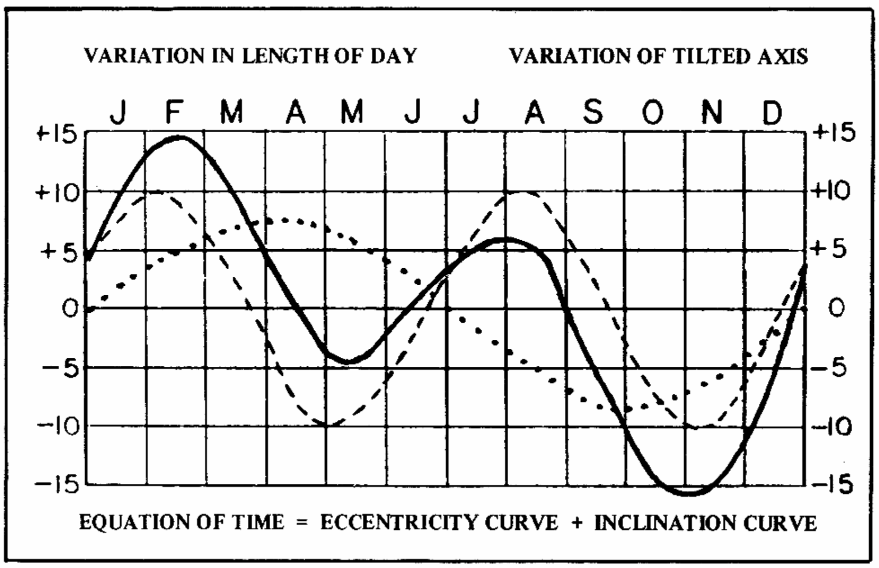
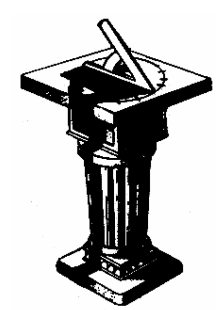


Fig. 29. The equation of time is a composite of the additions of the ordinates of the two sine-like graphs.

according to their declination values of the sun, the resulting graph is the analemma, the large figure-eight shaped diagram found on globes of the world.

Section 11 ***Adjusting the Dial for Exact Longitude***

The longitude correction, once determined for a given location, remains the same throughout the year. This correction can, therefore, be incorporated into the delineation of the hour lines.



To adjust the geometric construction given in Figure 17 to include this correction, subtract the local longitude from the central meridian longitude of the appropriate time zone (as detailed in the previous section). The longitude in Figure 30, for example, is 72 degrees, and the central meridian is 75, so the correction is 3 degrees. Since the location is east of the central meridian, the correction is positive, so it is added to the right of the BC line, creating a BC' line that is then used as the base line for measuring the 15-degree arcs. A negative correction is called for if the location is west of the central meridian, and the correction would be measured to the left of the BC line. The correction in Figure 30 is positive 3 and is, therefore, measured counterclockwise.

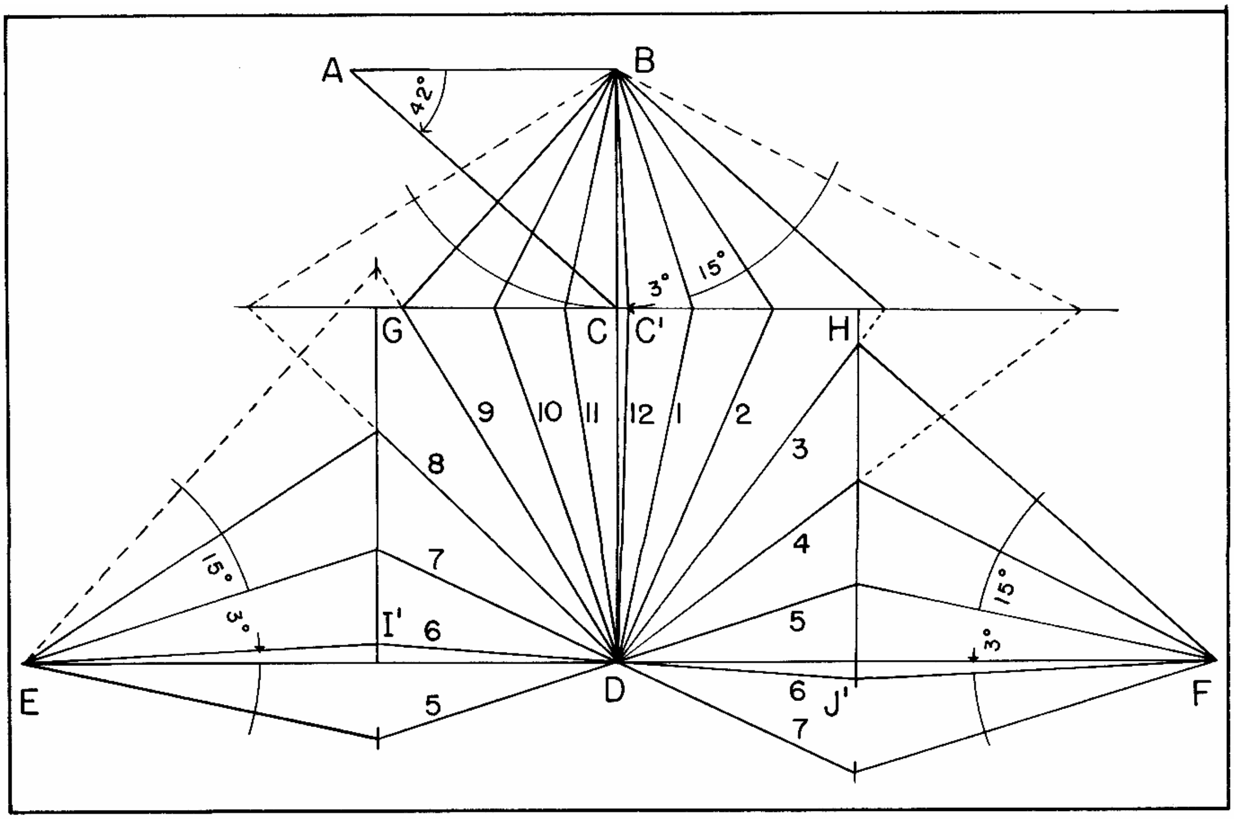


Fig. 30. Layout for latitude 42° longitude 72°.

To alter the sundial formula to include this correction, first determine the correction in degrees and then convert to minutes of time. Each degree of longitude represents 4 minutes of time. For Figure 30 the correction is 12 (3 x 4) minutes. The number of minutes is next converted to a decimal fraction of the hour, which for the example is 0.2 hours (12 min./60 min. per hour). The correction of 0.2 is added to the *n* in the formula, which becomes:

$$\tan\alpha = \sin 42 \tan 15(n + 0.2)$$

To express the corrected formula in general, let *f* be the decimal factor (a positive or negative value), which is added algebraically to *n:* $\tan\alpha = \sin\beta \tan 15(n + f)$

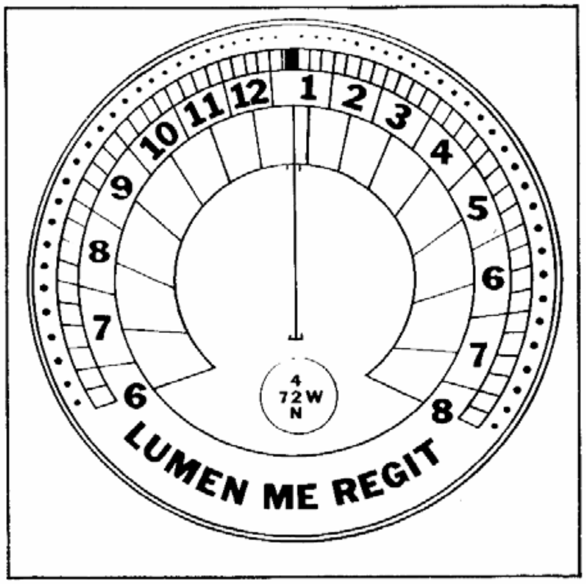


Fig. 31. Completed dialface for 12 minutes longitude correction. Light rules me.

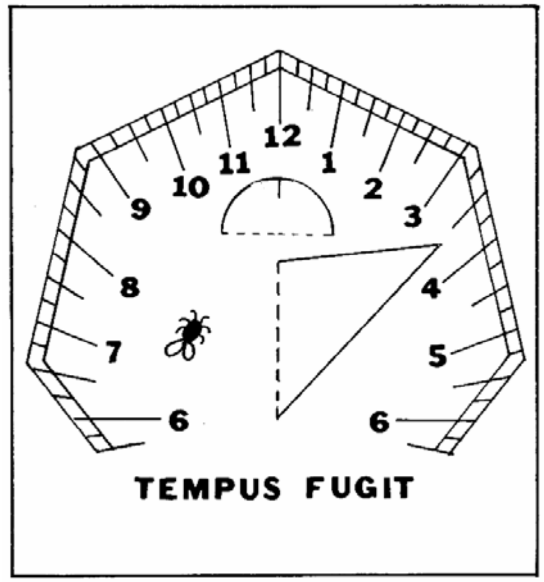


Fig. 32. An elaborate classroom cardboard sundial. Motto: Time flies. The picture of the fly is a pun on the motto.

In making the cardboard classroom sundial, no allowance is necessary for the thickness of the gnomon.

However, if a dial of heavier material is desired, say one-quarter or one-half inch plywood, pine wood, or metal, an allowance must be made for the thickness of the gnomon. Instead of one line DC (as in Figures 14a and 17), there must be two parallel lines DC the same distance apart as the thickness of the gnomon *(see Figure* 33). The A.M. hours would be measured from the left parallel DC line, and the P.M. hours would be measured from the right parallel DC line. Note, however, the hours before 6 A.M. and after 6 P.M. are measured from the opposite DC line; *e.g.*, the 7 P.M. line is an extension of the 7 A.M. line.

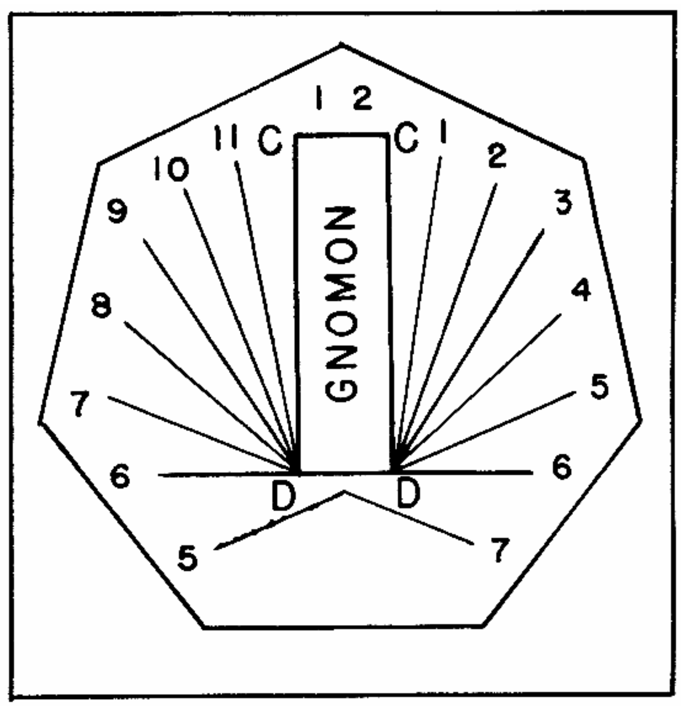
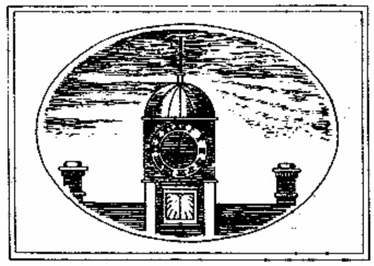


Fig. 33: Allowance for gnomon thickness.

IV



THE CLOCK AND THE DIAL.

A CLOCK happened to fall into conversation with a Dial. One cloudy forenoon, when the sun did not shine, the Clock said to the Dial, "what a mean slavery do you undergo; you cannot tell the hour unless the sun pleases to inform you. I can tell the hour at any time, and would not be in such a dependent state as you are in for the world: night and day are both alike to me: it is just now twelve o'clock." Upon this the sun shone forth from under the cloud, and shewed the exact time of the day: it was half an hour past twelve. The Dial then replied to the Clock, "you may now perceive that boasting is not good: for you see you are wrong: it is better to be under direction and follow truth, than to be eye to one's self and go wrong."

BEWICK'S FABLES, 1820

SOME CLASSICAL CONSIDERATIONS

Section 12 *Mottoes and Meditations*

Like the weather, time is a popular subject for discourse, ranging from casual conversation about its fleeting nature to profound interpretation of time as a fourth dimension. Perhaps it would be well to pause a moment to meditate on time.

Time is: Too slow for those who wait,
Too swift for those who fear,
Too long for those who grieve,
Too short for those who rejoice.
But for those who love-time is eternity. – Henry Van Dyke

Lost, yesterday, somewhere between sunrise and sunset, two golden hours, each set with sixty diamond minutes. No reward is offered, for they are gone forever!
- Horace Mann

Time is the most undefinable yet paradoxical of things; the past is gone, the future is not come, and the present becomes the past, even while we attempt to define it, and, like the flash of the lightning, at once exists and expires. - John Cotton

To every thing there is a season, and a time to every purpose under the heaven; A time to be born, and a time to die….A time to plant, and a time to pluck up that which is planted….-Ecc. 3:1-8

...he drew a dial from his poke, and looking on it with lack-luster eye, Says, very wisely, 'It is ten o'clock'. - William Shakespeare

O God! methinks it were a happy life
To be no better than a homely swain;
To sit upon a hill, as I do now,
To carve out dials quaintly, point by point,
Thereby to see the minutes how they run,
How many make the hour full complete;
How many hours bring about the day;
How many days will finish up the year;
How many years a mortal man may live. - William Shakespeare

Hide not your talents. They for use were made.
What's a sundial in the shade ? -Benjamin Franklin

Much of the charm of sundials is in the mottoes they display. Some are crude witticisms, some reflect on life, some admonish or warn not to linger, some are hopeful, some are philosophical, some are religious, all are fascinating.

Inanimate objects often seem to develop a personality of their own. We look at them as almost human. For instance, craftspersons often personify their tools, and musicians treat their instruments like friends. Computer operators frequently think their machines are human - they receive information, make decisions, and communicate back in words.

The sundial, too, becomes to many a friend to admire. For a sundial is a sentinel, always ready to give its knowledge to any viewer. Not only does it give the time on a sunny day, but it tirelessly waits out the cloudy sky, ready to give its truth even if only for the instant or two when the sun is able to break a slight path through the clouds.

Many are the lessons the silent sundial teaches. And we learn so much more when a sundial speaks to us in mottoes. Its messages are succinct, yet often cryptic; they always speak some good advice.

The Clock and the Dial

A Clock happened to fall mto conversatIon with a Dial. One cloudy forenoon, when the sun did not shine, the Clock said to the Dial, "what a mean slavery do you undergo; you cannot tell the hour unless the sun pleases to inform you. I can tell the hour at any time, and would not be in such a dependent state as you are in for the world: night and day are both alike to me: it is just now twelve O clock. Upon this the sun shone forth under the cloud, and shewed the exact time of the day: it was half an hour past twelve. The Dial then replied to the Clock, "You may now perceive that boasting is not good: for you see you are wrong: it is better to be under direction and follow truth, than to be eye to one's self and go wrong.

-Bewick's Fables, 1820

All in good time.

As time goes by.

A little at a time.

Time marches on.

Today is the tomorrow
You worried about yesterday.

As a thief in the night

As time and hours do pass away,
So doth the life of man decay.

Time on my hands.

A time to live.

A stitch in time saves nine.

Remember the time. ..

A clock the time may wrongly tell;
I, never, if the sun shine well.

Taken Idly Motivates Evil.

So teach us to number our days that we may apply our hearts unto wisdom.

CARPE DIEM Use well the day.
COELI ENARRANT GLORIAM DEI The heavens declare the glory of God.
DEFICIT SOL, NEMO RESPICIT When the sun sinks, of the dial no one thinks.
DISCE DIES NUMERARE TUOS Learn to number thy days.
DOCEO HORAS I teach the time.
DUM LICET UTERE While time is given, use it.
EX IIS UNAM CAVE Beware of one of them.

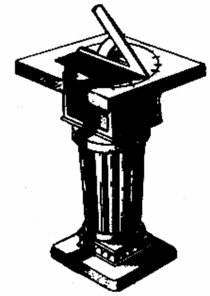
FELICIBUS BREVIS, MISERIS HORA LONGA The hour is short to the happy, Long to the miserable.
FESTINA LENTE Make haste slowly.
FIAT LUX Let there be light.
GOA BOU TYO URB US IN ESS Go about your business.
HORAM SOLE NOLENTE NEGO I tell not the hour when the sun will not.
HORAS NON NUMERO NISI SERENAS I count the bright hours only.
IN COELO QUIES In heaven is rest.
LAUS DEO Praise be to God.
LEX DEI LUX DIEI God’s law may be read in the light of the sun.
OMNES VULNERANT, ULTIMA NECAT Each one wounds, the last one kills.
OMNIA FALCE METIT TEMPUS Time reaps all things with a scythe.
PEREUNT ET IMPUTANTUR They (the hours) pass and are placed to our account.
QUI DOCET DISCIT One who teaches, learns.
SERIUS EST QUAM COGITAS It is later than you think.
SIC TRANSIT HORA Thus passes the hour.
SOL EST REGULA The sun is the rule.
SOL OMNIBUS LUCET The sun shines for all.
STA PROMISSIS Stand by your promises.
TEMPUS VINCIT OMNIA Time conquers all.
TRANSIT HORA, MANENT OPERA Time passes, deeds remain.
ULTIMA LATET UT OBSERVENTUR OMNES The last is hidden so we have to watch them all !
UTERE NON REDITURA Use it, it won't come back.
VERA LOQUI AUT SILERE Speak the truth or be silent.
VITA UMBRA Life is a shadow.

Let not the sun go down upon your wrath.
Time waits for no man (*gnomon*).
Now is yesterday's tomorrow.
I also am under authority.

Life’s but a shadow, man’s but dust
This diall says: dy all we must.

As true as the dial to the sun
Although it be not shone upon.

The light of the sun is too bright for our eyes;
By its shadow then the time we derive.



The three dials considered in previous sections are the equatorial dial, the horizontal dial, and the vertical dial facing south. These sundials are part of a family of sundials for which the gnomon is parallel to the polar axis of the earth. Sundials based on this property are collectively referred to as the classical sundials. Several more of this group are of interest.

The North-Facing Vertical Dial

The dialface of a vertical dial facing north is the same as the south-facing dial, except that the hour lines are viewed from the north side. The gnomon on the north side is a continuation of the gnomon on the south side. The diagrams in Figure 34 show this relationship more clearly. Suppose that the dial faces are painted onto the two sides of a transparent dial.

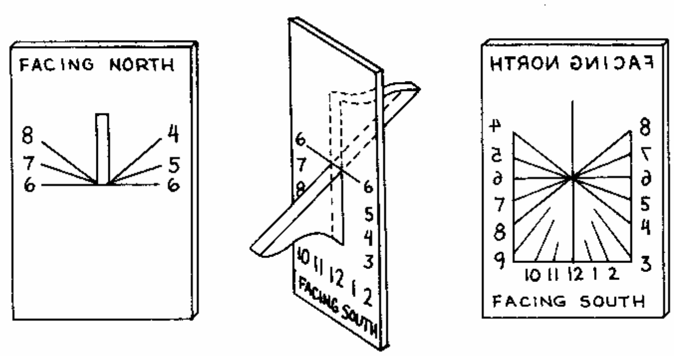


Fig. 34. Vertical dials facing south and north.

Fig. 35. Classical Cube Dial at Woods Hole Oceanographic Institution, Woods Hole, Cape Cod, MA.

When the sun is shining on the south face, it cannot also be shining on the north face. In general, the south vertical dial receives sunlight from 6 A.M. to 6 P.M., and the north vertical dial receives sunlight from sunrise to 6 A.M. and again from 6 P.M. to sunset.

The north vertical dial is seldom used by itself; it usually appears on cube dials and pillar dials in combination with sundials facing the other three cardinal directions *(see Figure* 35).[*]

[*] The North face of this cube dial is not visible in the photograph. For over 30 years the gnomon on the north face has been upside-down – so the dial does not work in the early morning or late afternoon.

The East and West Vertical Dials

As the south and north dials on the previous page have the same hour lines, so also do the east and west dials. The gnomon on both the east and west dials will be parallel to the dialface but rotated to equal the latitude angle *(see Figure* 35). The mean sun rotates around the gnomon in a circular plane that is perpendicular to the gnomon, so every 15 degrees of rotation represents one hour of time. The construction layout is shown in Figure 36. The radius of the generating circle must equal the height that the gnomon is above the dial face.

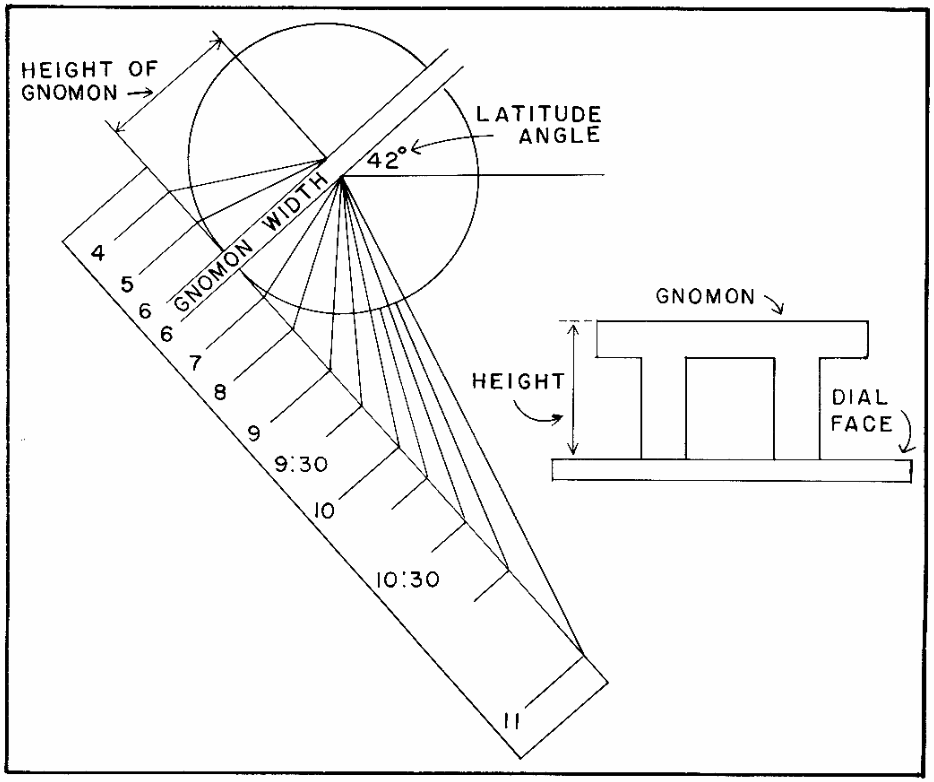


Fig. 36. Geometric layout for vertical dial facing east.

The distance that each hour line is from the base of the gnomon can easily be found. The formula is: $d = h \tan 15n$ where n, as before, is the number of hours away from noon; h is the height the gnomon is above the dial face. The sun will shine on the east dial during the morning hours, and on the west dial during the afternoon hours.

The Polar Dial

The polar dial is neither horizontal nor vertical but is raised at the north end by rotation around the west-east axis to an angle equal to the local latitude. The geometric construction is shown in Figure 37. The construction and the formula are the same as used for the east and west dials. The only difference is the period of sunlight. The polar dial should show the hours after 6am and before 6pm.

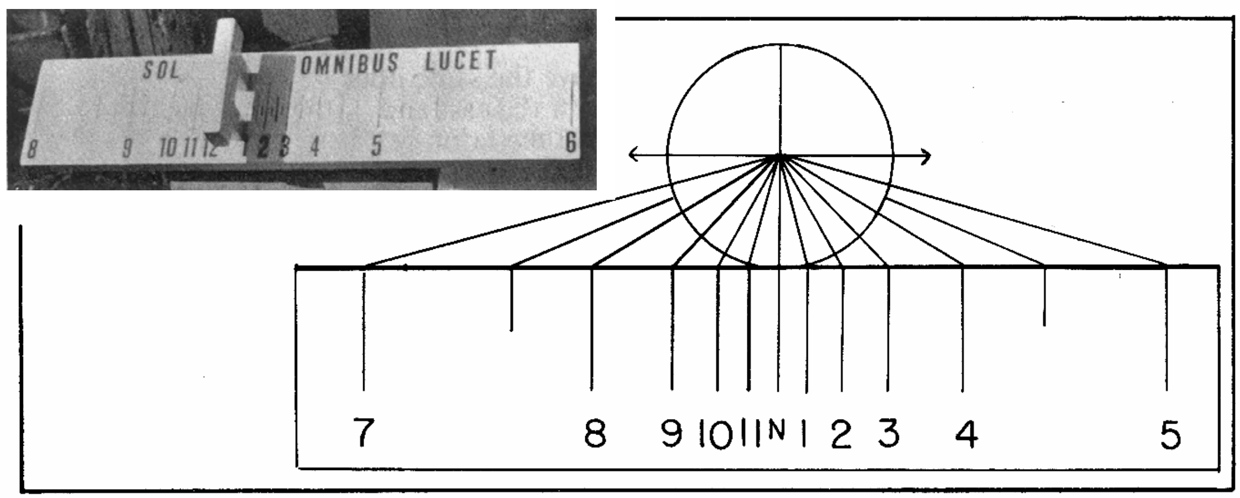
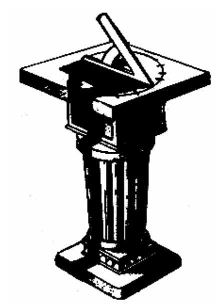


Fig. 37. Geometric layout for a polar dial. Motto: The sun shines for everyone .



Section 14 ***A Pseudo-Universal Sundial***

When the derivation of a sundial formula does not use a specific latitude angle, that dial is known as a universal dial and may be used at any latitude. Such a dial must, however, be oriented so that the gnomon is parallel to the polar axis of the earth. Of the dials discussed so far, the equatorial dial, the armillary dial, and the polar dial are universal dials.

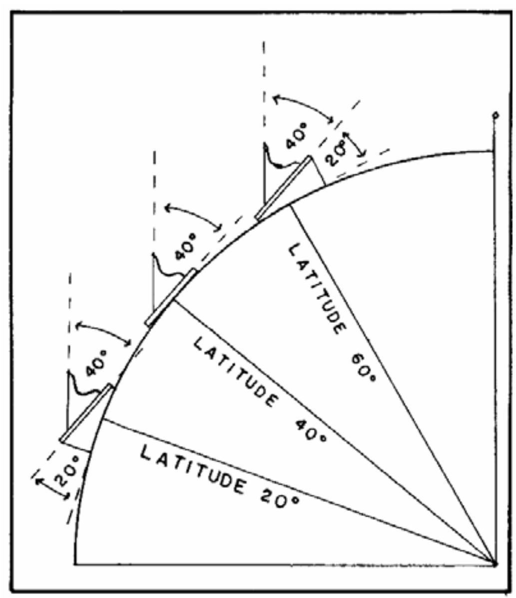


Fig. 38. Wedging can be used to adjust a sundial for a different latitude.

The horizontal sundial, on the other hand, is made for a specific latitude. It will not indicate time correctly unless it is used at the latitude for which it was calculated. But there might be a time when a horizontal dial must be used at some other latitude. Fortunately, there is a way to devise a pseudo-universal dial. Simply wedge the dial face on its west-east axis until the shadow-casting edge of the gnomon becomes parallel to the earth's axis. For instance, a sundial made for a latitude of 40 degrees and used at latitude 20 degrees will have to be wedged 20 degrees from the south end; or, the same dial used at 60 degrees latitude would be wedged 20 degrees from the north end. The wedge angle is found as follows:

wedge angle = new latitude - dial latitude

When the wedge angle is negative, the wedge is applied from the south end. When the wedge angle is positive, the wedge is applied from the north end *(see Figure* 38).

A similar technique will allow a south vertical dial to be used on a wall that does not face true south; *i.e.*, on a declining wall. In this instance, however, the wedge is applied from the west side or the east side. In Figure 40 the wall declines 9 degrees to the east of south, so the wedges are applied from the east side.

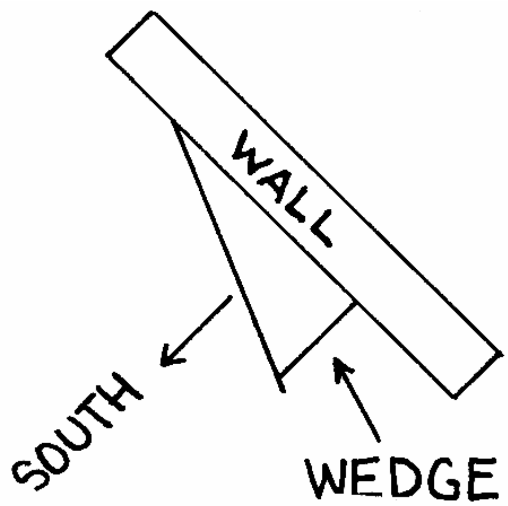
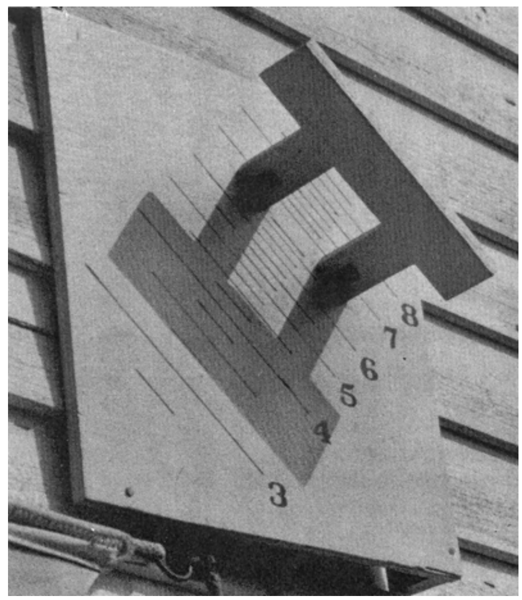


Fig. 39 .



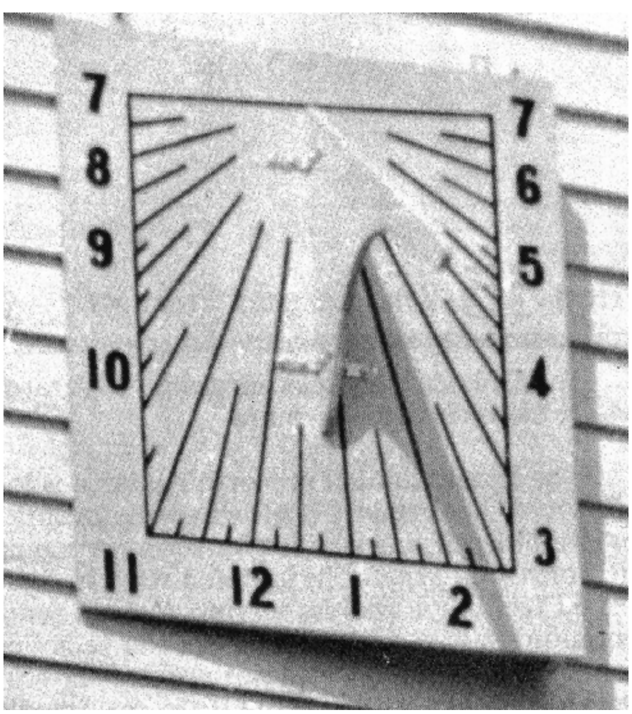
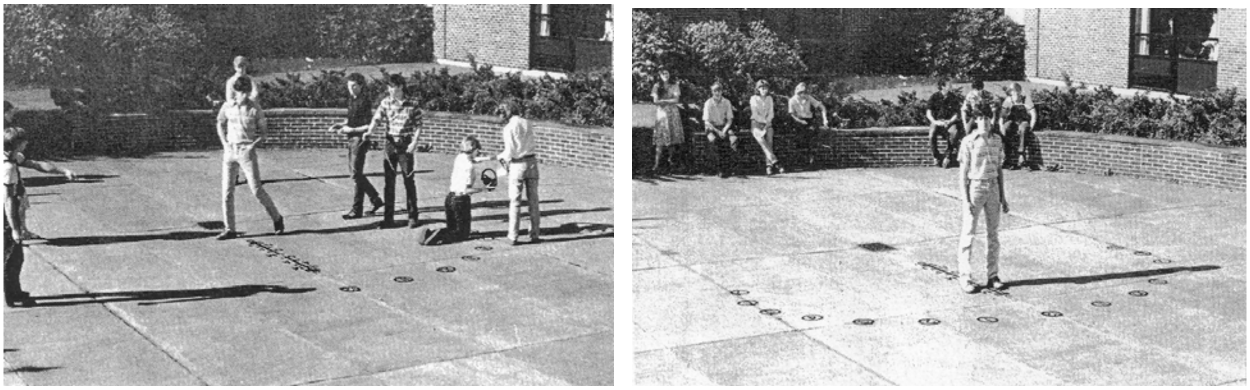


Fig. 40: A vertical line south dial wedged 9 degrees to accommodate a west declining wall. The vertical line west dial is also wedged 9 degrees.

V



Students using the analemmatic sundial (be your own gnomon).

FOUR SPECIAL SUNDIALS

Section 15 ***The Largest Sundial***

People invariably seek information regarding the largest sundial. Two large sundials come to mind.

Possibly the largest sundial in North America is located at Carefree, Arizona *(see Figure 41*). Certainly it offers aesthetic enrichment, and perhaps it also serves as an identifying symbol for sunny Arizona. This sundial is of the classic horizontal type. The gnomon rises 35 feet, and the edge of the gnomon is 62 feet long. Besides serving its main function of casting a shadow, the gnomon's 4-foot width contains solar cells that utilize the sun's heat to supply a nearby building with hot water. The scale of hours on the dialface requires a circle that is 90 feet in diameter; the scale is graduated at ten-minute intervals.

Fig. 41 Possibly the largest sundial in North America. Carefree, Arizona.

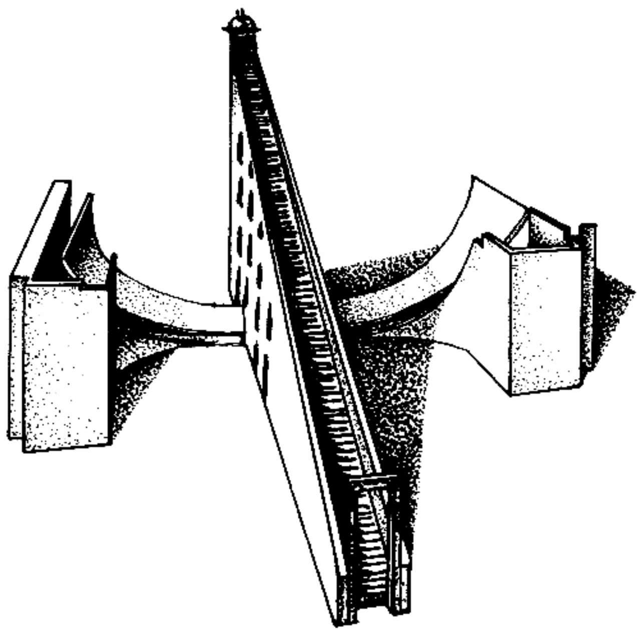


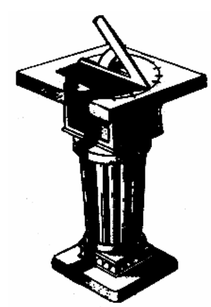
Fig. 42. The largest sundial in the world. Jaipur, India.

The largest sundial in the world was built in 1724 at Jaipur, India, by the Maharaja Sawai Jai Singh II *(See Figure* 42). This sundial is of the armillary type but with a traditional triangular gnomon that rises 90 feet above the earth. The slanted edge is 118 feet long and 12 feet wide with stairs leading to a small gazebo at the top. The scale of hours is marked on two quadrants of 90-foot radius that serve as the periphery of an equatorial-type dial. The shadow moves across the curved surface about a foot in five minutes with graduations marked for each minute of time.

This massive structure covers almost an acre, and neither words nor picture can adequately convey the awesome grandeur of its reality .

But on the surrealistic side, the giant gnomon with its staircase does suggest a loading ramp to a fueled and ready moon rocket. The shadow moves across the great arc indicating the countdown. One can easily imagine hearing the flight engineer saying, "All systems are go. Four inches and counting."

Section 16 ***The Pelekinon Sundial***



The pelekinon dial was discussed in a preceding section because it represented an advance in the history of the sundial. The caveman realized that a vertical gnomon casts a shadow of varying length during the day; therefore, he could use the length of the shadow as a gauge of the passing of the daylight hours.

By the time of the Egyptians, it was known that the length of the noon shadow in the summer was shorter than the length of the noon shadow in the winter, thus the passing of the seasons could also be recorded.

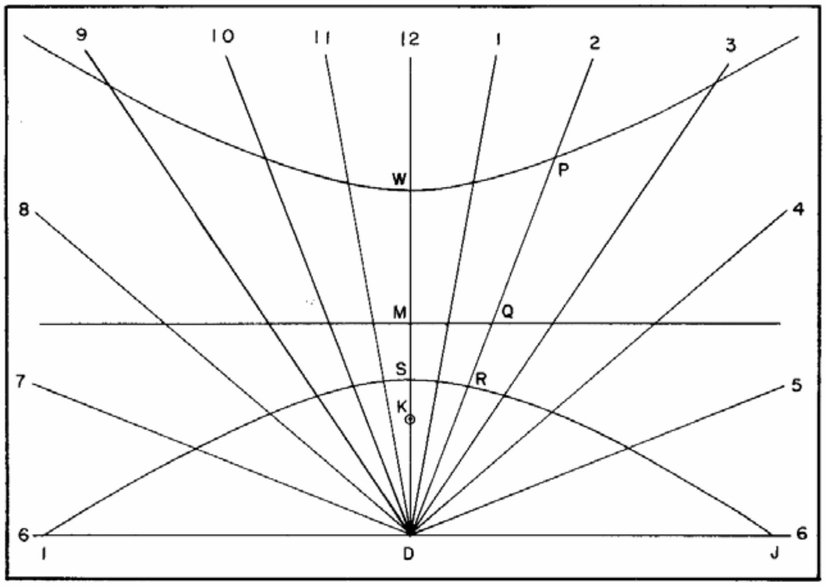


Fig. 43. The zodiac lines are drawn over the hour lines to create the Pelekinon sundial.

The Greeks, in addition, realized that the path traced by the tip of the vertical gnomon traced the path of a hyperbola except at the equinoxes when the path was a straight line.

By Roman times more lines were added that could show the hours of the day using the direction as well as the length of the shadow (*see Figure* 43). But these were all traced out by empirical methods. During the Golden Age, the astronomers and mathematicians caught up with this early sundial and provided geometric constructions and formulae which allowed the pelekinon to be determined exactly.

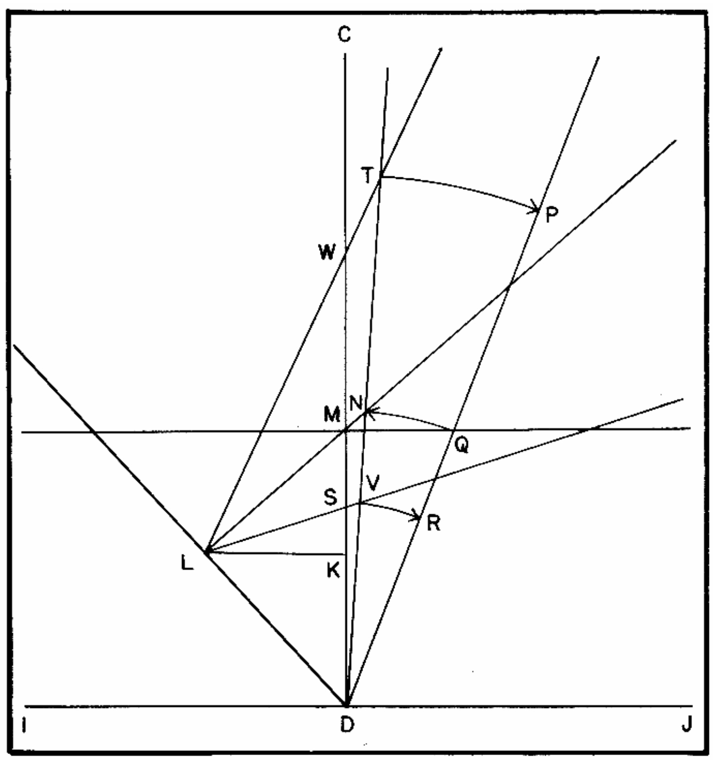


Fig. 44. Geometric construction of the zodiac lines.

But the construction and calculation methods for finding the hour lines of the horizontal sundial have been fully detailed in earlier sections. These same hour lines are used for the pelekinon. The calculation of the hyperbolic arcs showing the seasons, however, requires advanced understanding of both astronomy and spherical trigonometry. Therefore, with its proof omitted, a construction method will be outlined to draw the seasonal lines: the greatest hyperbolic arc occurs about December 21, and is called the winter solstice; the shortest hyperbolic arc occurs about June 21, and is called the summer solstice; about March 21 and September 21, the equinox occurs, which is the time of equal day and night, and it plots as a straight line. These lines are often called the zodiac lines.

To Construct the Zodiac Lines

Refer to Figure 44:

1. Draw a line DC, running North-South.

2. At D, measure the latitude angle, ∠CDL, in degrees.

3. From L, drop a perpendicular line to CD at K. The length LK is the height of the gnomon, but the pin is shown lying flat. In use, it is rotated to straight-up position.

4. At L, draw a line perpendicular to LD, cutting DC at M.

5. From LM, measure ∠MLW = 23.5 degrees, cutting DC at W.
   From LM, measure ∠MLS = 23.5 degrees, cutting DC at S.
   (lines LW, LM, LS are known in the old dialing books as a *Trigon)*

6. Draw a line through M that is perpendicular to DC. This is the required equinoctial line of the pelekinon. (The branches of the hyperbola will pass through points Wand S.)

7. From D, draw an hour line. This illustration uses the regular 2 o'clock hour line of the horizontal dial. This line cuts the equinoctial line at Q.

8. Using a compass, swing an arc of radius DQ to cut line LM at N. Extend the line DN, which cuts line LW at T and lines LS at V.

9. Using the compass again, swing an arc of radius DV cutting line DQ at R. Swing another arc of radius DT cutting line DQ at P.

10. Point P is a point of the desired upper hyperbola branch. Point R is a point of the desired lower hyperbola branch.

11. Repeat steps 7 through 10 with the other hour lines.

12. Connect these points with a smooth line to complete each branch of the hyperbola.

The pelekinon is now completed as shown in Figure 43. The original triangular gnomon of the horizontal dial is now replaced by a pin at point K. The height of the pin must equal the length LK. The time is indicated by the shadow of the tip of the pin. Because the shadow always stays between the branches of the hyperbola, the hour lines above and below the branches may be removed *(see Figure* 45).

The hyperbola for the other months can be drawn in a similar manner. The appropriate angles of declination of the sun are given in table form in the appendix. It is effective to place a mark on the noon line for each month where the shadow will fall.

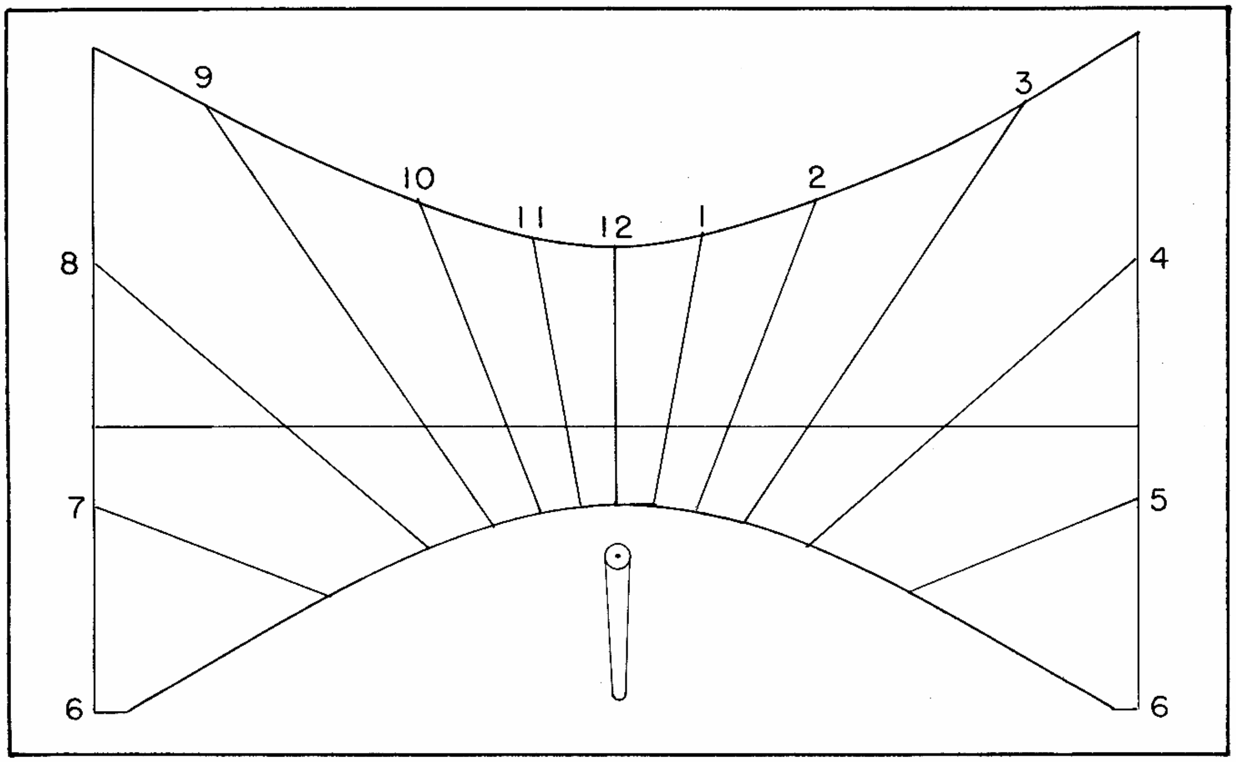
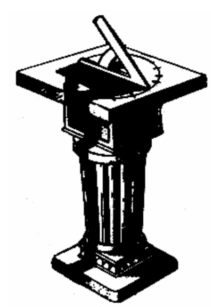


Fig. 45. Completed Pelekinon sundial. The pin, drawn flat, must be erect to the dialface for use.

Section 17 *The South Vertical Declining Sundial*



To place a south vertical dial directly onto the side of a building would require that the building wall face true south. This usually is not the case. Therefore, it is necessary to adjust the hour lines of the sundial to the declination of the wall. Declination is the number of degrees the wall differs from exact south *(see Figure* 46). A wall that faces easterly, in a counter-clockwise rotation from exact south, is said to have an east declination. A wall that faces westerly, in a clockwise rotation from exact south, is said to have a west declination.

The first step is to determine the angle of declination of the wall. Two practical ways are at hand: (1) Attach a small shelf to the wall, and make it exactly level.

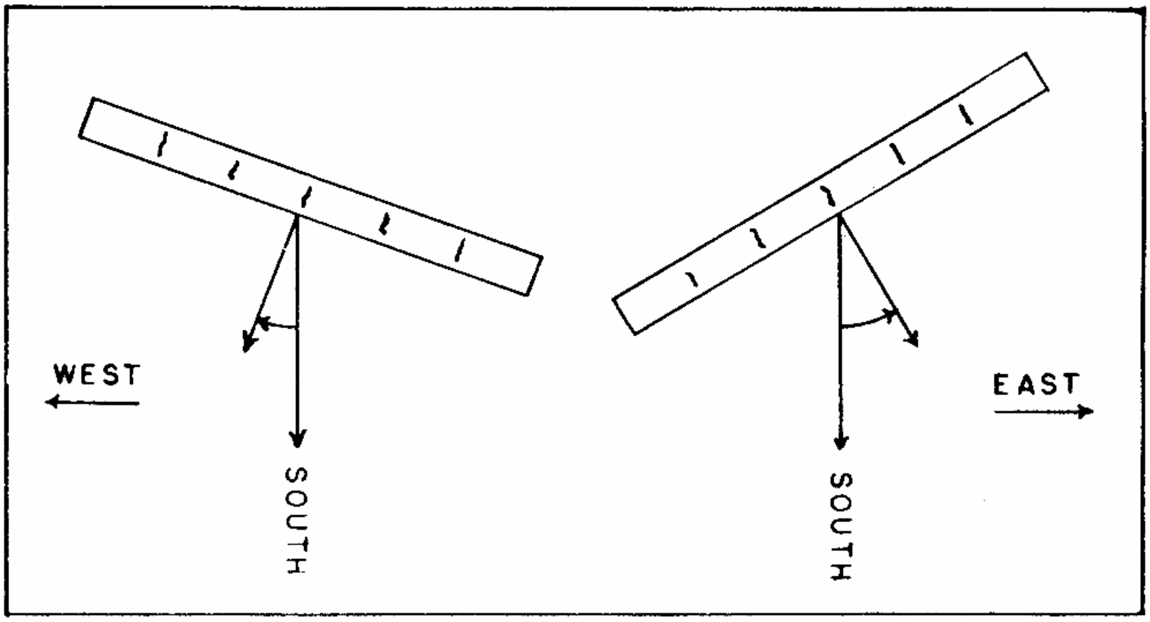


Fig. 46. 20° declination westward. 30° declination eastward.

Determine true south by the pin and concentric circle method described earlier (in Section 5). (2) Attach a small shelf as in the first case, and then arrange a plumb line above it *(see Figure* 47). Combine the equation of time correction with the longitude adjustment for the exact time of solar noon.

For instance, if the sun is eight minutes fast, record where the shadow of the plumb line falls exactly eight minutes before noon. Use a good watch that has been set by radio time. On the other hand, if the sun is five minutes slow, record the shadow of the plumb line exactly five minutes after noon. The recorded shadow

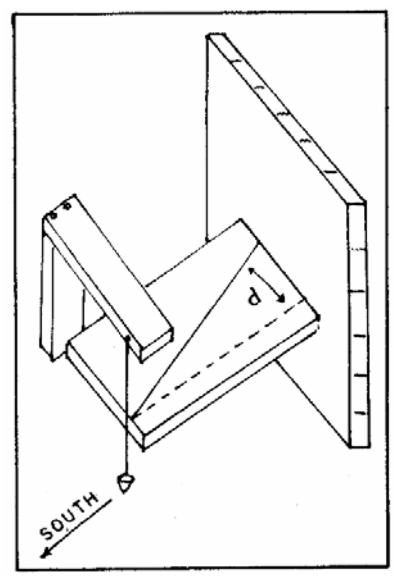


Fig. 47. To determine the declination of a wall, the shadow is cast at exact noon.

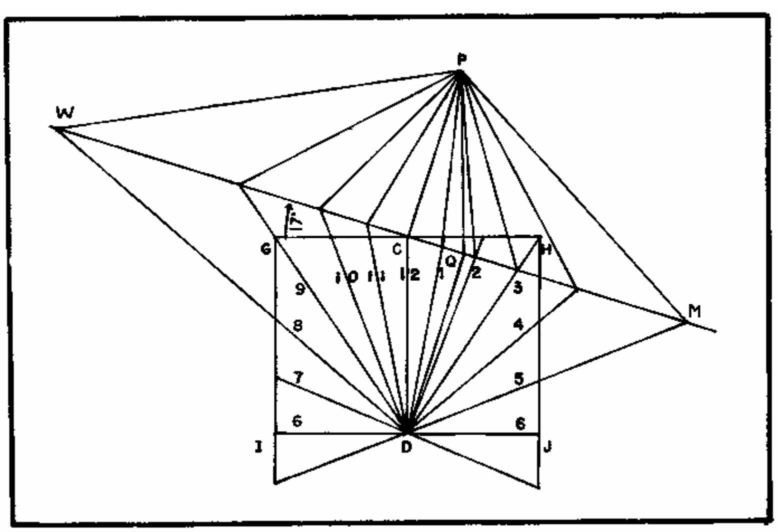


Fig.48. Geometric layout for south vertical declining sundial.

line is the true south-north line. The angle this line makes with a horizontal line perpendicular to the wall is the angle of declination.

A perspective view is seen in Figure 48, in which a regular horizontal sundial is resting on a horizontal plane and is properly aligned on the north-south line. The wall is declined from the proper direction. The construction to be described will translate the hour lines from the horizontal dial to the vertical declined surface. The right triangle DCP is the gnomon triangle, with the edge DP (not drawn in Figure 48) parallel to the earth's axis.

Geometric Construction for South Vertical Declining Sundial (Figure 48)

1. HGIJ is a regular horizontal sundial.
2. The declination (westerly in this illustration) is measured from the line GCH at C.
3. From C, draw CP perpendicular to WM. The length CP is determined from the gnomon triangle DCP *(see Figure 49a)*.
4. Extend the hour lines of the horizontal dial until they meet the declination line WM.
5. Connect each of the points of step 4 with point P. These lines are the hour lines for the vertical declining dial. To see this in perspective, fold the drawing along the declination line bringing the flap up to a vertical position. In this folded position, the gnomon triangle DCP is oblique to the face of the vertical dial. A gnomon perpendicular to the vertical dial is preferred.
6. From D, construct a line perpendicular to the declination line WM at the point Q. Connect PQ. In the folded position, the triangle DQP becomes the gnomon triangle that is perpendicular to the vertical dialface. (Notice that the shadow-casting hypotenuse DP is the same line in both gnomon triangles.)

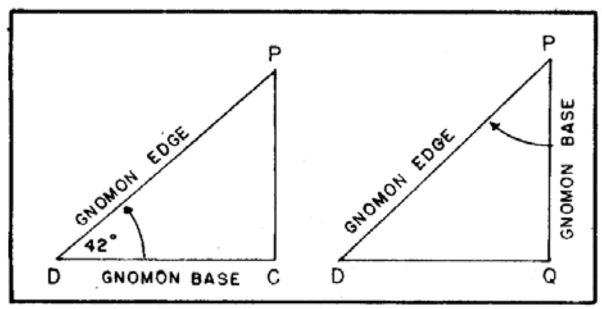


Fig. 49.

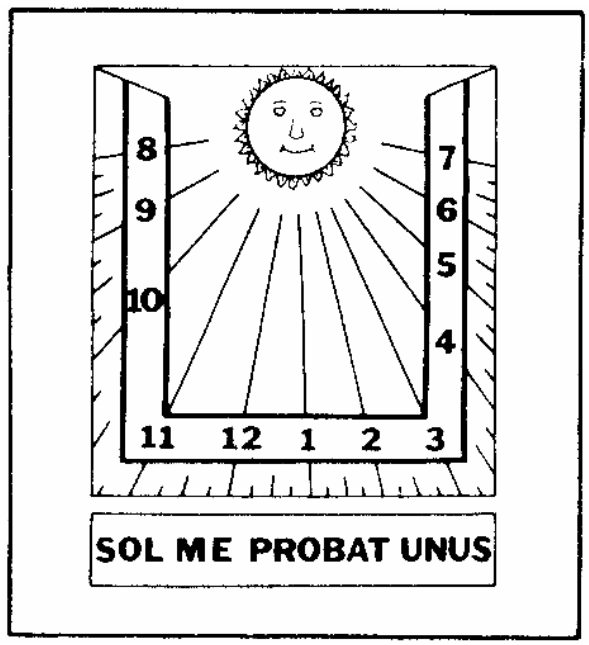


Fig. 50. Completed vertical sundial with 17° west declination. Motto: Only the sun can prove me useful.

7. To construct the gnomon triangle DQP it is necessary to construct a right triangle with sides DQ and PQ *(see Figure 49b)*.
8. A completed drawing is shown in Figure 50.

For the reader who enjoys a formula approach, the following formula, based on the steps on the prior page, is offered.

$$\tan\alpha' = \frac{\cos\beta}{\cot 15n \cos d + \sin\beta \sin d}$$

Derivation of the South Vertical Declining Dial Formula

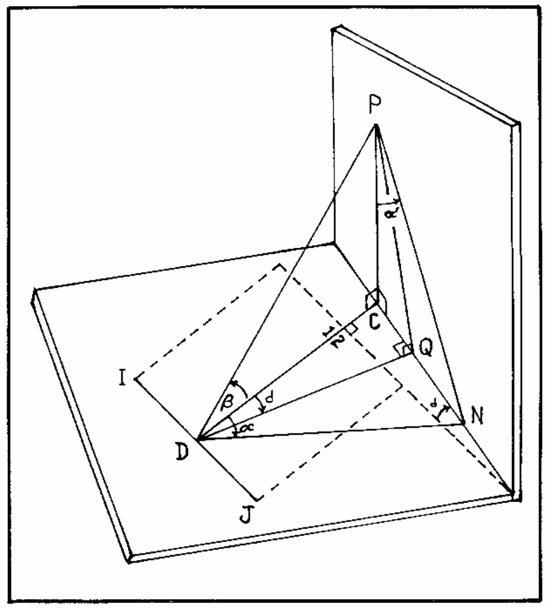


Fig. 51. A horizontal dial in relation with a vertical declined dial.

In Figure 51, $\angle CDN = \angle\alpha$ is the hour angle of the horizontal dial. $\angle CPN = \angle\alpha'$ is the desired hour angle of the south vertical declining dial. Let DC = unit length = 1.

1) In oblique ΔDCN,

$$\frac{CN}{\sin\alpha} = \frac{DC}{\sin\angle CND} = \frac{DC}{\sin(90+d-\alpha)}, \text{ so}$$

$$CN = DC\sin\alpha / \sin(90+d-\alpha).$$

$$[\angle CND = 180 - \alpha - (90-d) = 90 + d - \alpha]$$

2) In right ΔPCD, $\tan\beta = PC/DC$, so $PC = DC\tan\beta$

3) In right ΔPCN, $\angle CPN = \angle\alpha'$.

$$\tan\alpha' = CN/PC = \frac{DC\sin\alpha}{\sin(90+d-\alpha)} \cdot \frac{1}{DC\tan\beta} = \frac{\sin\alpha}{\tan\beta\sin(90+d-\alpha)}$$

Recall that $\sin\alpha = \tan\alpha\cos\alpha$

$$\tan\alpha' = \frac{\tan\alpha\cos\alpha}{\tan\beta\left[\sin(90+d)\cos\alpha - \tan\alpha\cos\alpha\cos(90+d)\right]}$$

$$= \frac{\tan\alpha\cos\alpha}{\frac{\sin\beta}{\cos\beta}\cos\alpha\left[\sin(90+d) - \tan\alpha\cos(90+d)\right]}$$

Recalling the standard horizontal sundial formula, $\tan\alpha = \sin\beta\tan 15n$,

$$\tan\alpha' = \frac{\tan 15n}{\frac{1}{\cos\beta}\left[\sin(90+d) - \sin\beta \tan 15n \cos(90+d)\right]}$$

$$= \frac{\tan 15n \cos\beta}{\sin(90+d) - \sin\beta \tan 15n \cos(90+d)}$$

$$= \frac{\cos\beta}{\cot 15n \sin(90+d) - \sin\beta \cos(90+d)}$$

$$= \frac{\cos\beta}{\cot 15n \cos d + \sin\beta \sin d}$$

which is the desired formula giving the required hour angle *α'* in terms of the latitude *β*, and the declination *d* of the wall. The number of hours from noon, *n*, is negative for the morning hours and positive for the afternoon hours. Now, two angles are needed. One for the gnomon, and one for placing the gnomon.

∠QPD is the angle at which the new gnomon will rise from the vertical dialface.

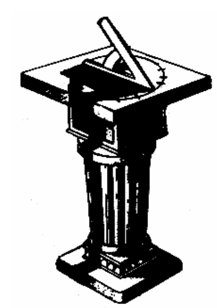
In right ΔDCP: $\cos\beta = DC/DP = 1/DP$. In right ΔDQC: $\cos d = DQ/DC = DQ$.

In right ΔDQP: $\sin\angle DPQ = DQ/DP = \cos d \cos\beta$

∠CPQ is the angle the gnomon is rotated from the vertical line PC.

In right ΔDCP: $\tan\beta = PC/DC = PC/1 = PC$.
In right ΔDQC: $\sin d = CQ/DC = CQ/1 = CQ$.

In right ΔPCQ: $\tan\angle CPQ = CQ/PC = \sin d/\tan\beta = \sin d \cot\beta$.



Section 18 ***"Be Your Own Gnomon"***

The classical sundials presented above utilize the motion of the sun relative to its path around the earth's polar axis. Two other perspectives can also be utilized for indicating time. The altitude of the sun is a measure of how high the sun is above the horizontal plane, such as the caveman's pole dial where the length of the shadow changed according to the height of the sun during the day. There are many modern (Golden Age and later) portable dials based on the sun's altitude. These dials do not require orienting to true north; they are used by pointing the sundial towards the sun.

The azimuth of the sun is a measure of rotation of the sun around the pole of the horizontal plane; *i.e.*, around an obelisk or vertical gnomon of our horizon. The analemmatic dial is an azimuth-type dial (*see Figure* 52). It consists of an elllpse on which the hour markings are placed and a vertical gnomon that can be positioned on a scale of months. The time is indicated on the ellipse at the point crossed by the shadow of the gnomon.

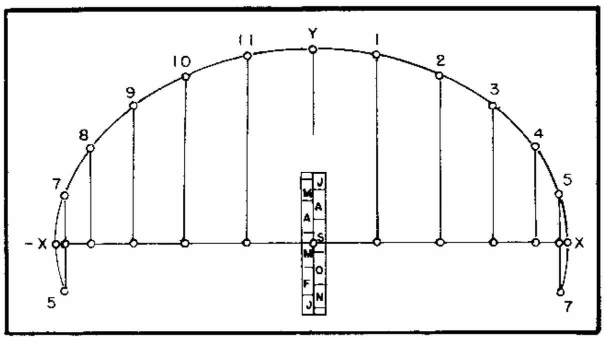


Fig. 52. Analemmatic sundial by plotting abscissas and ordinates.

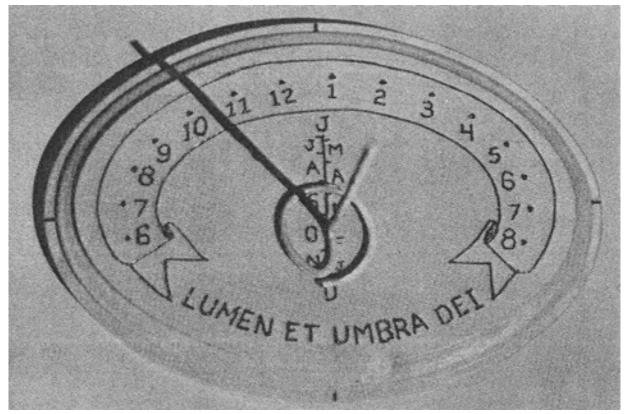


Fig. 53. Analemmatic Dial. Motto: The Light and Shadow of God

The derivation of the formulae necessary for constructing an analemmatic dial is somewhat involved. Although the formulae require a knowledge of astronomy and solid geometry, they are convenient to use in their completed form. Let a north-south line, and an east-west line be taken as axes. The hour points on the ellipse are given by the equations:

$$x = \sin 15n \quad y = \sin\beta \cos 15n,$$

Table A

Hour		X	Y
12		0.000	0.766
11	1	0.259	0.740
10	2	0.500	0.663
9	3	0.707	0.542
8	4	0.866	0.383
7	5	0.966	0.198
6	6	1.000	0.000
5	7	0.966	- 0.198

where n is the number of hours away from noon (thus n = 1 for both 11 A.M. and 1 P.M.), and β is the local latitude. Suppose an analemmatic dial is desired for Winnipeg, which has a latitude of β = 50°. Repeated applications of these formulae are summarized in Table A.

The scale of months must next be calculated, and the formula for this is:

$$M = \tan d \cos\beta$$

Table B

Month	Declination	M
1 Jan	- 23.13	- 0.275
1 Feb	- 17.30	- 0.200
1 Mar	- 8.00	- 0.090
1 Apr	+ 4.25	+ 0.048
1 May	+ 15.00	+ 0.172
1 Jun	+ 22.00	+ 0.260
21 Jun	+ 23.45	+ 0.279
1 Jul	+ 23.00	+ 0.273
1 Aug	+ 18.00	+ 0.209
1 Sep	+ 8.50	+ 0.096
1 Oct	- 2.90	- 0.033
1 Nov	- 14.00	- 0.160
1 Dec	- 21.67	- 0.255
21 Dec	- 23.45	- 0.279

where β is the local latitude and d is the declination, or degrees by which the sun is above or below the equatorial plane of the earth. Table B gives the declinations for the first day of each month and the values of M, the measure above or below the x-axis that corresponds to each month. While the declination values are independent of any specific location, the M values are different for each latitude. The scale of months is marked on the y-axis; the width of the scale is optional. The movable gnomon is placed over the current month on the y-axis.

An alternate method of delineating the analemmatic dial is of interest. Draw the ellipse first, then locate the hour marks by rays radiating from the center. To do this, use the coordinate formulae above. By letting n = 6, the point (1,0) is on the x-axis and marks off the length of the major axis of the ellipse. Let this be of unit length. By letting n = 0, the point (0, sin β) is on the y-axis and marks off the minor axis of the ellipse. The equation of the ellipse becomes

$$\frac{x^2}{1^2} + \frac{y^2}{\sin^2 \beta} = 1$$

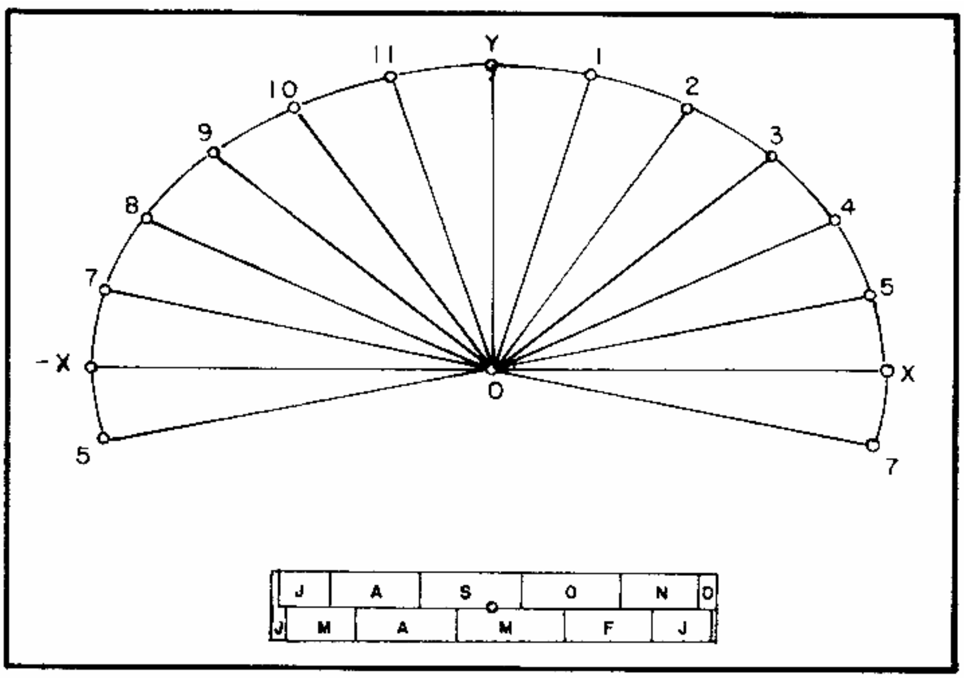


Fig. 54. Analemmatic sundial by central angle and ray. The scale of months is enlarged for clarity. It would be placed on line OY as shown in Figure 52.

After sketching the graph, the hour marks are found by a ray from the center cutting the ellipse. The angle α of the ray is measured to the right or left from OY and is found by the formula

$$\tan\alpha = \frac{\tan 15n}{\sin\beta}$$

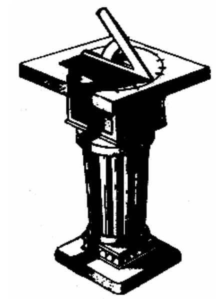
Table C shows the values of α for each hour.

Table C

Hour		α Degrees
12		0
11	1	19.3
10	2	37.0
9	3	52.6
8	4	66.1
7	5	78.4
6	6	90.0
5	7	101.6

Joseph Jérome De Lalande, a French astronomer and dialist of the eighteenth century, suggested that an analemmatic dial be laid out on a garden plot or patio, duly proportioned to about a 20-foot diameter, and the scale of months also properly proportioned but with the gnomon missing. Then the viewer can become the gnomon. By standing appropriately on the month scale, with back to the sun, the viewer will see his or her own shadow stretch across the ellipse and indicate the correct time.

Bibliography – 1979



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11. Daniel, Christopher St. J. H., *Sundials*, Shire Publications, 1986 & 2004.

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The North American Sundial Society has also published on CD several facsimile reproductions of old dialing texts as well as a quarterly journal of (technical) developments in the field of gnomonics.

Appendix I

Mean Value of the Equation of Time
(in Minutes at True Noon)

Day	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	+ 3.4	+13.6	+12.5	+ 4.1	− 2.8	− 2.3	+ 3.6	+ 6.3	+ 0.2	−10.1	−16.3	−11.2
2	3.9	13.7	12.3	3.8	3.0	2.2	3.8	6.2	− 0.1	10.4	16.4	10.8
3	4.3	13.8	12.1	3.5	3.1	2.0	4.0	6.2	0.5	10.8	16.4	10.4
4	4.8	13.9	11.9	3.2	3.2	1.9	4.2	6.1	0.7	11.1	16.4	10.0
5	5.2	14.0	11.7	2.9	3.3	1.7	4.4	6.0	1.1	11.4	16.4	9.6
6	5.7	14.1	11.5	2.6	3.4	1.5	4.6	5.9	1.5	11.7	16.3	9.2
7	+ 6.1	+14.2	+11.2	+ 2.3	− 3.4	− 1.3	+ 4.7	+ 5.8	− 1.8	−12.0	−16.3	− 8.8
8	6.5	14.2	11.0	2.1	3.5	1.2	4.9	5.7	2.1	12.3	16.3	8.3
9	6.9	14.3	10.7	1.8	3.6	1.0	5.0	5.5	2.5	12.6	16.2	7.9
10	7.3	14.3	10.5	1.5	3.6	0.8	5.2	5.4	2.8	12.8	16.1	7.5
11	7.8	14.3	10.2	1.2	3.7	0.6	5.3	5.2	3.2	13.1	16.0	7.0
12	8.2	14.3	10.0	0.9	3.7	0.4	5.4	5.1	3.5	13.4	15.9	6.5
13	+ 8.5	+14.3	+ 9.7	+ 0.7	− 3.7	− 0.2	+ 5.6	+ 4.9	− 3.9	−13.6	−15.8	− 6.1
14	8.9	14.3	9.4	0.4	3.7	0.0	5.7	4.7	4.2	13.8	15.6	5.6
15	9.3	14.2	9.1	+ 0.2	3.7	+ 0.2	5.8	4.5	4.6	14.1	15.5	5.1
16	9.6	14.2	8.9	− 0.1	3.7	0.4	5.9	4.3	5.0	14.3	15.3	4.6
17	9.9	14.1	8.6	0.2	3.7	0.7	6.0	4.1	5.3	14.5	15.1	4.1
18	10.3	14.0	8.3	0.5	3.7	0.9	6.1	3.9	5.5	14.7	14.9	3.6
19	+10.6	+13.9	+ 8.0	− 0.7	− 3.6	+ 1.1	+ 6.2	+ 3.7	− 6.0	−14.9	−14.7	− 3.2
20	10.9	13.8	7.7	0.9	3.6	1.3	6.2	3.5	6.4	15.1	14.5	2.7
21	11.2	13.7	7.4	1.2	3.5	1.5	6.3	3.2	6.7	15.2	14.3	2.2
22	11.5	13.6	7.1	1.4	3.5	1.7	6.3	3.0	7.1	15.4	14.0	1.7
23	11.8	13.5	6.8	1.6	3.4	2.0	6.4	2.8	7.4	15.6	13.7	1.2
24	12.0	13.4	6.5	1.8	3.3	2.2	6.4	2.5	7.8	15.7	13.4	0.7
25	+12.3	+13.2	+6.2	− 1.9	− 3.2	+ 2.4	+ 6.4	+ 2.2	− 8.1	−15.8	−13.1	− 0.2
26	12.5	13.1	5.9	2.1	3.1	2.6	6.4	1.9	8.4	15.9	12.9	+ 0.3
27	12.7	12.9	5.6	2.3	3.0	2.8	6.4	1.7	8.8	16.0	12.5	0.8
28	12.9	12.7	5.3	2.4	2.9	3.0	6.4	1.4	9.1	16.1	12.2	1.3
29	13.1		5.0	2.6	2.8	3.2	6.4	1.1	9.5	16.2	11.9	1.8
30	13.3		4.7	2.7	2.6	3.4	6.4	0.8	9.8	16.3	11.5	2.3
31	+13.4		+ 4.4		− 2.5		+ 6.3	+ 0.5		−16.3		2.8

Appendix II

Mean Value of the Solar Declination

Day	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	−23°01′	−17°09′	−7°40′	+ 4°28′	+15°01′	+22°02′	+23°08′	+18°04′	+8°22′	− 3°06′	−14°22′	−21°46′
2	−22°56′	−16°52′	−7°17′	+ 4°51′	+15°19′	+22°10′	+23°03′	+17°49′	+8°00′	− 3°30′	−14°41′	−21°56′
3	−22°50′	−16°35′	−6°54′	+ 5°14′	+15°37′	+22°17′	+22°59′	+17°34′	+7°38′	− 3°53′	−15°00′	−22°04′
4	−22°44′	−16°17′	−6°31′	+ 5°37′	+15°54′	+22°25′	+22°54′	+17°18′	+7°16′	− 4°16′	−15°19′	−22°13′
5	−22°38′	−15°59′	−6°08′	+ 6°00′	+16°12′	+22°31′	+22°49′	+17°02′	+6°54′	− 4°39′	−15°37′	−22°21′
6	−22°31′	−15°40′	−5°44′	+ 6°22′	+16°29′	+22°38′	+22°43′	+16°46′	+6°31′	− 5°02′	−15°55′	−22°28′
7	−22°24′	−15°22′	−5°21′	+ 6°45′	+16°45′	+22°44′	+22°37′	+16°29′	+6°09′	− 5°25′	−16°13′	−22°35′
8	−22°16′	−15°03′	−4°58′	+ 7°08′	+17°02′	+22°50′	+22°30′	+16°12′	+5°46′	− 5°48′	−16°31′	−22°42′
9	−22°08′	−14°44′	−4°34′	+ 7°30′	+17°18′	+22°55′	+22°23′	+15°55′	+5°24′	− 6°11′	−16°48′	−22°48′
10	−21°59′	−14°25′	−4°11′	+ 7°52′	+17°34′	+23°00′	+22°16′	+15°38′	+5°01′	− 6°34′	−17°05′	−22°54′
11	−21°50′	−14°05′	−3°47′	+ 8°14′	+17°50′	+23°04′	+22°08′	+15°20′	+4°38′	− 6°57′	−17°22′	−22°59′
12	−21°40′	−13°45′	−3°24′	+ 8°36′	+18°05′	+23°08′	+22°00′	+15°02′	+4°16′	− 7°20′	−17°39′	−23°04′
13	−21°31′	−13°25′	−3°00′	+ 8°51′	+18°20′	+23°12′	+21°52′	+14°44′	+3°53′	− 7°42′	−17°55′	−23°08′
14	−21°20′	−13°05′	−2°37′	+ 9°20′	+18°35′	+23°15′	+21°43′	+14°26′	+3°30′	− 8°04′	−18°11′	−23°12′
15	−21°09′	−12°44′	−2°13′	+ 9°42′	+18°49′	+23°18′	+21°34′	+14°07′	+3°07′	− 8°26′	−18°26′	−23°16′
16	−20°58′	−12°24′	−1°49′	+10°03′	+19°03′	+23°21′	+21°24′	+13°49′	+2°43′	− 8°49′	−18°41′	−23°19′
17	−20°47′	−12°03′	−1°25′	+10°24′	+19°17′	+23°23′	+21°14′	+13°29′	+2°20′	− 9°11′	−18°56′	−23°21′
18	−20°35′	−11°42′	−1°02′	+10°45′	+19°30′	+23°24′	+21°04′	+13°10′	+1°57′	− 9°33′	−19°11′	−23°23′
19	−20°22′	−11°21′	−0°38′	+11°06′	+19°43′	+23°25′	+20°53′	+12°51′	+1°34′	− 9°54′	−19°25′	−23°25′
20	−20°09′	−10°59′	−0°14′	+11°27′	+19°56′	+23°26′	+20°42′	+12°31′	+1°11′	−10°16′	−19°39′	−23°26′
21	−19°57′	−10°38′	+0°09′	+11°47′	+20°09′	+23°27′	+20°31′	+12°11′	+0°47′	−10°37′	−19°52′	−23°26′
22	−19°43′	−10°16′	+0°33′	+12°08′	+20°21′	+23°26′	+20°19′	+11°51′	+0°24′	−10°59′	−20°05′	−23°27′
23	−19°29′	− 9°54′	+0°57′	+12°28′	+20°32′	+23°26′	+20°07′	+11°31′	+0°01′	−11°20′	−20°18′	−23°26′
24	−19°15′	− 9°32′	+1°20′	+12°48′	+20°44′	+23°25′	+19°55′	+11°11′	−0°23′	−11°41′	−20°31′	−23°25′
25	−19°00′	− 9°10′	+1°44′	+13°07′	+20°55′	+23°24′	+19°42′	+10°50′	−0°46′	−11°02′	−20°43′	−23°24′
26	−18°46′	− 8°47′	+2°08′	+13°27′	+21°05′	+23°22′	+19°29′	+10°29′	−1°10′	−12°22′	−20°54′	−23°22′
27	−18°30′	− 8°25′	+2°31′	+13°46′	+21°16′	+23°20′	+19°16′	+10°08′	−1°33′	−12°43′	−21°05′	−23°20′
28	−18°15′	− 8°02′	+2°55′	+14°05′	+21°26′	+23°18′	+19°02′	+ 9°47′	−1°56′	−13°03′	−21°16′	−23°18′
29	−17°59′		+3°18′	+14°24′	+21°35′	+23°15′	+18°48′	+ 9°26′	−2°20′	−13°23′	−21°27′	−23°15′
30	−17°43′		+3°41′	+14°42′	+21°44′	+23°11′	+18°34′	+ 9°05′	−2°43′	−13°43′	−21°37′	−23°11′
31	−17°26′		+4°05′		+21°53′		+18°19′	+ 8°43′		−14°03′		−23°07′

Appendix III

Latitude and Longitude for Selected North American Cities

	Lat.	Long.		Lat.	Long.
Akron, OH	41	81	Chicago, IL	42	88
Albany, NY	42	74	Cincinnati, OH	39	85
Albuquerque, NM	35	107	Cleveland, OH	42	82
Allentown, PA	41	75	Concord, NH	43	72
Amarillo, TX	35	102	Columbia, SC	34	81
Annapolis, MD	39	76	Columbus, Ga	32	85
Anchorage, AK	61	150	Columbus, OH	40	83
Ashland, KY	38	83	Corpus Christi, TX	28	97
Atlanta, GA	34	84	Dallas, TX	33	97
Atlantic City, NJ	39	74	Davenport, IA	41	91
Augusta, Ga	33	82	Dayton, OH	40	84
Augusta, ME	44	70	Denver, CO	40	105
Austin, TX	30	98	Des Moines, IA	42	94
Bakersfield, CA	35	119	Detroit, MI	42	83
Baltimore, MD	39	77	Douglass, AZ	31	109
Bangor, ME	45	69	Dover, DE	39	76
Baton Rouge, LA	31	91	Duluth, MN	47	92
Neaumont, TX	30	94	Dutch Harbor, AK	54	167
Billings, MO	45	108	Edmonton, Alta.	54	113
Binghamton, NY	42	76	Elmira, NY	42	77
Birmingham, AL	33	87	El Paso, TX	32	107
Bismark, ND	47	101	Erie, PA	42	80
Boise, ID	44	116	Evansville, IN	38	88
Boston, MA	42	71	Fairbanks, AK	66	148
Brandon, Man.	50	100	Fargo, ND	47	97
Bridgeport, CT	41	73	Flint, MI	43	84
Buffalo, NY	43	79	Ft. Wayne, IN	41	85
Burlington, VT	44	73	Ft. Worth, TX	33	97
Butte, MT	46	113	Fredericton, N.S.	46	67
Calgary, Alta.	51	114	Fresno, CA	37	120
Canton, OH	41	81	Gary, IN	42	87
Caribou, ME	47	67	Grand Forks, ND	48	97
Carson City, NV	39	120	Grand Rapids, MI	43	87
Casper, WY	43	106	Great Falls, MT	48	111
Charleston, SC	34	80	Green Bay, WI	44	88
Charleston, WV	38	82	Greensboro, NC	36	80
Charlotte, NC	35	81	Halifax, N.S.	45	64
Chattanooga, TN	35	85	Harrisburg, PA	40	77
Cheyenne, WY	41	105	Hartford, CT	42	73

	Lat.	Long.		Lat.	Long.
Helena, MT	47	112	Philadelphia, PA	40	75
Honolulu, HI	21	158	Phoenix, AZ	33	112
Houston, TX	30	95	Pierre, SD	44	100
Indianapolis, IN	40	86	Pittsburgh, PA	40	80
Ithaca, NY	42	76	Pocatello, ID	43	112
Jackson, MS	32	90	Portland, ME	44	70
Jacksonville, FL	30	82	Portland, OR	46	123
Jefferson City, MO	39	92	Prince Albert, Sask.	53	106
Juneau, AK	58	134	Prince George, B.C.	54	123
Kalamazoo, MI	42	86	Prince Rupert, B.C.	54	130
Kansas City, KS	39	94	Providence, RI	42	71
Kiska, Ak	52	182	Pueblo, CO	38	105
Knoxville, Tn	36	84	Quebec, Que.	47	71
Kodiak, Ak	58	153	Raleigh, NC	36	79
Lansing, MI	43	86	Rapid City, SD	44	103
Lincoln, NB	41	97	Regina, Sask.	50	105
Little Rock, AR	35	92	Richmond, VA	38	77
London, Ont.	43	81	Roanoke, VA	37	80
Los Angeles, CA	34	118	Rochester, MN	44	92
Louisville, KY	38	86	Rochester, NY	43	78
Lowell, MA	43	71	Rockford, IL	42	89
Lubbock, TX	34	102	Sacramento, CA	39	121
Madison, WI	43	89	Saint John, N.B.	45	66
Manchester, NH	43	71	Saint John's, Newf.	47	53
Marquette, MI	45	87	St. Louis, MO	39	90
Memphis, TN	35	90	St. Michael, Ak	63	162
Miami, FL	26	80	St. Paul, MN	45	93
Milwaukee, WI	43	88	St. Peterburgh, FL	28	82
Minneapolis, MN	45	93	Salem, OR	45	123
Mobile, AL	31	88	Salt Lake City, UT	41	112
Montgomery, AL	33	86	San Antonio, TX	29	98
Montpelier, VT	44	73	San Diego, CA	33	117
Montreal, Que.	46	74	San Francisco, CA	38	122
Nashville, TN	36	87	San Jose, CA	37	122
New Bedford, MA	42	71	Santa Fa, NM	36	106
New Haven, CT	41	73	Saskatoon, Sask.	52	107
New Orleans, LA	30	90	Sault St. Marie, Ont.	47	84
New York, NY	41	74	Savannah, GA	32	81
Niagara Falls, NY–Ont.	43	79	Scranton, PA	41	76
Norfolk, VA	37	76	Seattle, WA	48	122
Oakland, CA	38	122	Shreveport, LA	32	94
Oklahoma City, OK	36	98	Sioux Falls, SK	44	97
Omaha, NB	41	96	Sitka, AK	57	135
Orlando, FL	28	81	South Bend, IN	42	86
Ottawa, Ont.	45	76	Spokane, WA	48	117
Peoria, IL	41	90	Springfield, IL	40	90

	Lat.	Long.
Springfield, MA	42	73
Springfield, MO	37	93
Sudbury, Ont.	47	81
Sydney, N.S.	46	60
Syracuse, NY	43	76
Tacoma, WA	47	122
Tampa, FL	28	83
Thunder Bay, Ont.	48	89
Timmins, Ont.	48	81
Toledo, OH	42	84
Topeka, KS	39	96
Toronto, Ont.	44	80
Trenton, NJ	40	75
Tucson, AZ	32	111
Tulsa, OK	36	96
Utica, NY	43	75
Vancouver, B.C.	49	123
Washington, DC	39	77
Waterloo, IO	42	92
Wheeling, WV	40	81
Wichita, KS	38	97
Winnipeg, Man.	50	97
Winston-Salem, NC	36	80
Worcester, MA	42	72
Youngstown, OH	41	81