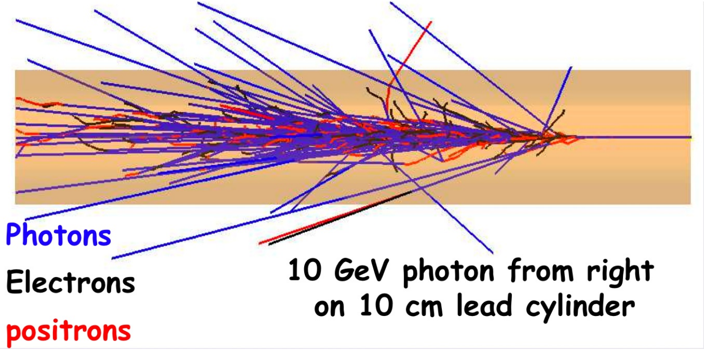
PHYS 5012
Radiation Physics and Dosimetry
Lecture 3

Tuesday 12 March 2013

Interactions of Photons with Matter



What are the dominant photon interactions?

Physical Processes (cont.)

Compton scattering, photoelectric absorption and pair production are the three main energy transfer mechanisms in photon interactions with matter. Each of these processes can dominate under specific conditions determined chiefly by the incident photon energy $h\nu$ and atomic number Z of the absorber.

Compton Scattering – kinematics (cont.)

When $h\nu$ becomes comparable to $m_e c^2$, the simple classical description of electromagnetic waves scattering off an electron in terms of harmonic oscillators breaks down. Both quantum and relativistic effects need to be considered in the *kinematics* and *dynamics* of the interaction. The quantization of the electromagnetic field is equivalent to breaking it up into particles (photons). Photon energies are comparable to the electron rest-mass energy, so the centre-of-mass of the scattering is moving in the observer frame, which results in a Doppler shift in the photon energy. In the observer frame, the decrease in photon energy appears as electron recoil energy. This results in the following kinematic relation derived in the last lecture (c.f. eqn. 23):

$$h\nu' = \frac{h\nu}{1 + \varepsilon(1 - \cos\theta)}$$

Compton Scattering – dynamics

Relativistic and quantum corrections to the dynamics of the scattering process take into consideration that the interaction is electromagnetic, and the magnetic moment associated with the electron spin cannot be ignored. Dirac's relativistic theory of the electron posits that for each value of momentum p_{e}, there are *two* corresponding energy states:

$$E = \pm\sqrt{(p_{\mathrm{e}}c)^2 + (m_{\mathrm{e}}c^2)^2} \tag{1}$$

The normally unobserved negative energy states are capable of playing a role in the electromagnetic interaction between a photon and an electron. In quantum electrodynamics, these interactions involve an initial state, an intermediate state and a final state. Energy does not need to be conserved in the intermediate state.

- Klein-Nishina differential cross section for Compton scattering measures *probabilty* of photon re-emission into solid angle $d\Omega = d\cos\theta\, d\phi$ as a result of a Compton interaction between an incident photon and a free electron
- exact expression is derived from quantum electrodynamics:

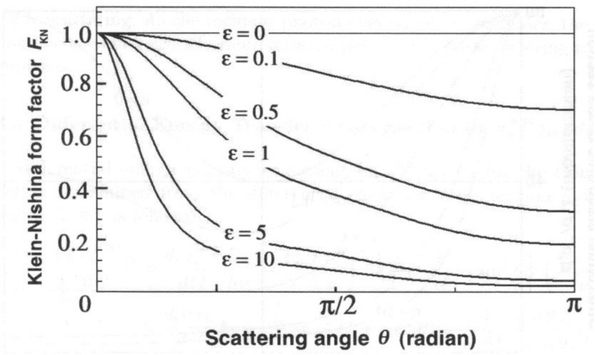
$$\boxed{\frac{d_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}}{d\Omega} = \frac{1}{2} r_{\mathrm{e}}^2 \left(\frac{\nu'}{\nu}\right)^2 \left(\frac{\nu'}{\nu} + \frac{\nu}{\nu'} - \sin^2\theta\right)} \tag{2}$$

- can be written in terms of Thomson differential cross section, $d_{\mathrm{e}}\sigma_{\mathrm{T}}/d\Omega = \frac{1}{2} r_{\mathrm{e}}^2 (1 + \cos^2\theta)$, and a *form factor*:

$$\frac{d_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}}{d\Omega} = \frac{d_{\mathrm{e}}\sigma_{\mathrm{T}}}{d\Omega} F_{\mathrm{KN}} \tag{3}$$

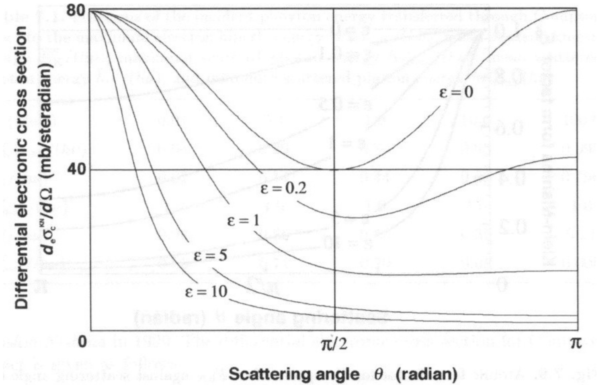
- Klein-Nishina form factor:

$$F_{\mathrm{KN}} = [1+\varepsilon(1-\cos\theta)]^{-2} \times \left\{1+\frac{\varepsilon^2(1-\cos\theta)^2}{[1+\varepsilon(1-\cos\theta)](1+\cos^2\theta)}\right\} \quad (4)$$



The Klein-Nishina form factor plotted against scattering angle for different incident photon energies $\varepsilon = h\nu/m_e c^2$. (Fig. 7.12 in Podgoršak).

Klein-Nishina differential cross section



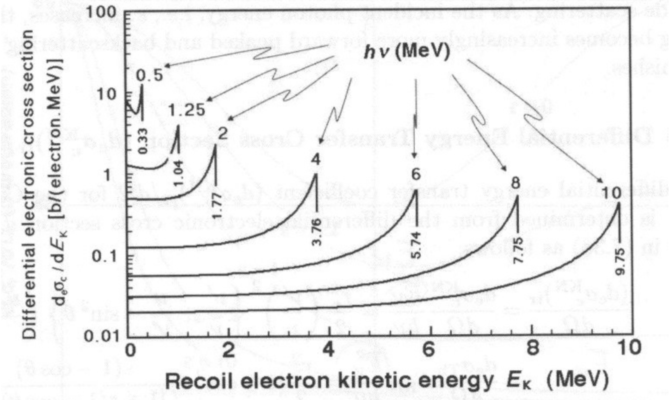
- forward scattering ($\theta \to 0$) and backward scattering ($\theta \to \pi$) have equal probability in Thomson limit ($\varepsilon \to 0$)
- probability of backscattering decreases with increasing $\varepsilon \Rightarrow$ forward beaming of photon re-emission

Energy distribution of recoil electrons

The differential electronic Klein-Nishina cross section can also be expressed as a function of the recoil electron kinetic energy, E_K, rather than scattering angle θ since $E_K = E_K(\theta)$ (c.f. eqn. 24 in previous lecture):

$$\begin{aligned} \frac{d_e\sigma_c^{KN}}{dE_K} &= \frac{d_e\sigma_c^{KN}}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dE_K} \\ &= \frac{\pi r_e^2}{\varepsilon h\nu}\left[2 - \frac{2\xi_K}{\varepsilon(1-\xi_K)} + \frac{\xi_K^2}{\varepsilon^2(1-\xi_K)^2} + \frac{\xi_K^2}{(1-\xi_K)}\right] \end{aligned} \tag{5}$$

where $\xi_K = E_K/h\nu$.



The Klein-Nishina differential electronic cross section for Compton scattering plotted as a function of recoil electron kinetic energy E_K. For a given photon energy, the maximum recoil energy is indicated. (Fig. 7.16 in Podgoršak).

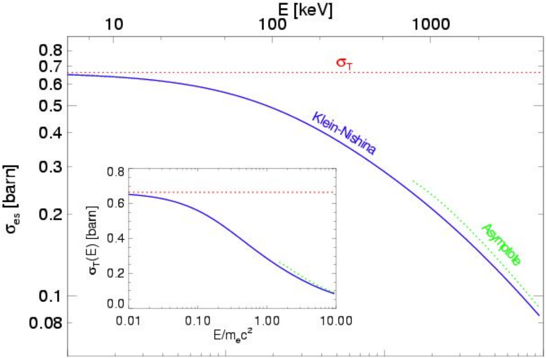
- energy distribution peaks sharply near

$$(E_K)_{max} = \frac{2\varepsilon}{1+2\varepsilon} h\nu$$

c.f. eqn. (27) in last lecture

Total electronic Klein-Nishina cross section

$$
\begin{aligned}
{}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}} &= \int \frac{d_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}}{d\Omega}\, d\Omega = 2\pi \int_{-1}^{+1} \frac{d_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}}{d\Omega}\, d\cos\theta \\
&= 2\pi r_{\mathrm{e}}^2 \left\{ \frac{1+\varepsilon}{\varepsilon^2} \left[\frac{2(1+\varepsilon)}{1+2\varepsilon} - \frac{\ln(1+2\varepsilon)}{\varepsilon} \right] \right. \\
&+ \left. \frac{\ln(1+2\varepsilon)}{2\varepsilon} - \frac{1+3\varepsilon}{(1+2\varepsilon)^2} \right\}
\end{aligned} \tag{6}
$$



Limiting solutions:

- $\varepsilon \ll 1$:

$$ {}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}} \approx \frac{8\pi}{3} r_{\mathrm{e}}^2 \left(1 - 2\varepsilon + \frac{26}{5}\varepsilon^2 - \frac{133}{10}\varepsilon^3 + \frac{1144}{35}\varepsilon^4 - \ldots \right) $$

- $\varepsilon \longrightarrow 0$: ${}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}} \approx \frac{8\pi}{3} r_{\mathrm{e}}^2 = {}_{\mathrm{e}}\sigma_{\mathrm{T}} = 6.65 \times 10^{-29}\,\mathrm{m}^2 = 0.665\,\mathrm{b}$ =Thomson limit
- $\varepsilon \gg 1$: ${}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}} \approx \pi r_{\mathrm{e}}^2 (1 + 2\ln\varepsilon)/(2\varepsilon) \propto (h\nu)^{-1}$
- at high photon energies, Klein-Nishina cross section declines rapidly with respect to the Thomson cross section

Energy Transfer

Mean fraction of incident photon energy $h\nu$ transferred to kinetic energy $E_{\rm K}$ of the recoil electron is an average of the fractional kinetic energy $E_{\rm K}/h\nu$ weighted over the probability distribution $P(\theta)$ for Compton scattering in direction θ, integrated over all scattering angles:

$$\frac{\overline{E_{\rm K}^{\rm C}}}{h\nu} = \frac{\int \frac{E_{\rm K}}{h\nu} P(\theta)\, d\cos\theta}{\int P(\theta)\, d\cos\theta} \tag{7}$$

where

$$P(\theta) = \frac{1}{{}_{\rm e}\sigma_{\rm c}^{\rm KN}} \int \frac{d_{\rm e}\sigma_{\rm c}^{\rm KN}}{d\Omega}\, d\phi = \frac{2\pi}{{}_{\rm e}\sigma_{\rm c}^{\rm KN}} \frac{d_{\rm e}\sigma_{\rm c}^{\rm KN}}{d\Omega} \tag{8}$$

and $E_{\rm K} = E_{\rm K}(\theta)$ is given by eqn. (24) in the last lecture. Note that $\int P(\theta) d\cos\theta = 1$.

We can now write

$$\begin{aligned}\frac{\overline{E_K^C}}{h\nu} &= \frac{2\pi}{{}_e\sigma_c^{KN}} \int_{-1}^{+1} \frac{E_K}{h\nu} \frac{d_e\sigma_c^{KN}}{d\Omega}\, d\cos\theta \\ &= \frac{2\pi}{{}_e\sigma_c^{KN}} \int_{-1}^{+1} \frac{(d_e\sigma_c^{KN})_{tr}}{d\Omega}\, d\cos\theta \\ &= \frac{({}_e\sigma_c^{KN})_{tr}}{{}_e\sigma_c^{KN}} \end{aligned} \quad (9)$$

where

- $({}_e\sigma_c^{KN})_{tr} =$ electronic energy transfer cross section
- $(d_e\sigma_c^{KN})_{tr}/d\Omega =$ differential energy transfer cross section

Differential energy transfer cross section

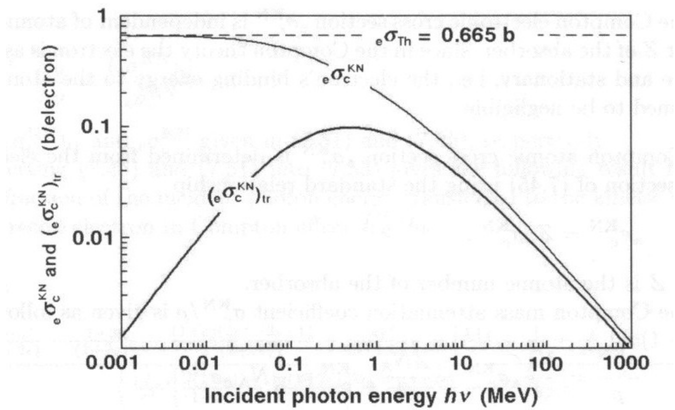
$$\frac{(d_e\sigma_c^{KN})_{tr}}{d\Omega} = \frac{d_e\sigma_c^{KN}}{d\Omega}\frac{E_K}{h\nu} = \frac{1}{2}r_e^2\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu'}{\nu}+\frac{\nu}{\nu'}-\sin^2\theta\right)\left(\frac{\nu-\nu'}{\nu}\right) \quad (10)$$

where $E_K = h\nu - h\nu'$ is the kinetic energy imparted to the recoil electron. From eqn. (24) in the last lecture, the θ dependence is: $E_K/(h\nu) = \varepsilon(1-\cos\theta)[1+\varepsilon(1-\cos\theta)]^{-1}$

- Total energy transfer cross section:

$$({}_e\sigma_c^{KN})_{tr} = \int \frac{d({}_e\sigma_c^{KN})_{tr}}{d\Omega}\, d\Omega = 2\pi\int_{-1}^{+1}\frac{d({}_e\sigma_c^{KN})_{tr}}{d\Omega}\, d\cos\theta$$

see eqn. (7.108) in Podgoršak for full solution.



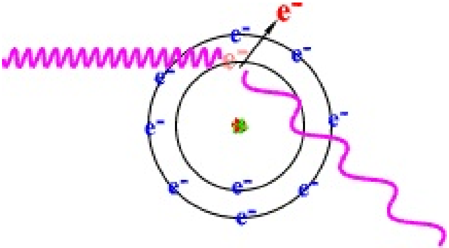
Recall from eqn. (9) that

$$\frac{\left({}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}\right)_{\mathrm{tr}}}{{}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}} = \frac{\overline{E_{\mathrm{K}}^{\mathrm{C}}}}{h\nu}$$

Atomic cross section

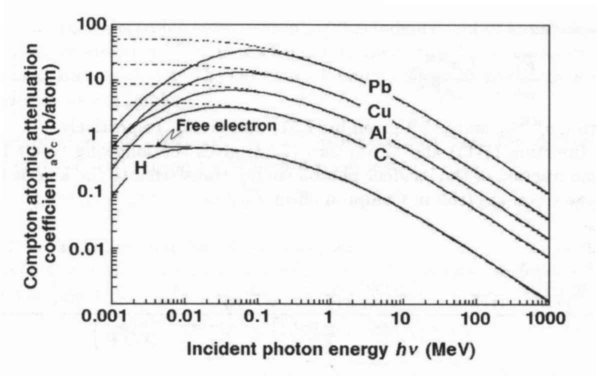
- ${}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}$ is for free electrons $\Rightarrow$ independent of Z
- at high photon energies ($h\nu \gg E_{\mathrm{B}}$, where $E_{\mathrm{B}} =$ electron binding energy), total Compton cross section for entire atom is

$$ {}_{\mathrm{a}}\sigma_{\mathrm{c}}^{\mathrm{KN}} = Z \left({}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}} \right) \qquad (11) $$



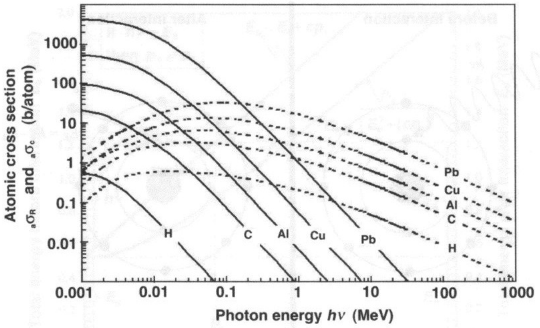
Binding energy effects

- assumption of free electrons breaks down for photon energies $h\nu \sim E_B =$ electron binding energy
- ${}_a\sigma_c^{KN}$ overestimates effective Compton atomic cross section ${}_a\sigma_C$ at low $h\nu$, especially for high Z material



The Compton atomic cross section ${}_a\sigma_C$ (solid curves) compared to the Klein-Nishina atomic cross section ${}_a\sigma_c^{KN} = Z_e\sigma_c^{KN}$ (dashed curves), demonstrating electron binding effects at low $h\nu$. (Fig. 7.19 in Podgoršak).

- binding energy correction to Compton atomic cross section usually has negligible effect on overall attenuation because other processes (e.g. Rayleigh scattering, photoelectric effect) are usually more important than Compton scattering at low $h\nu$ and high Z



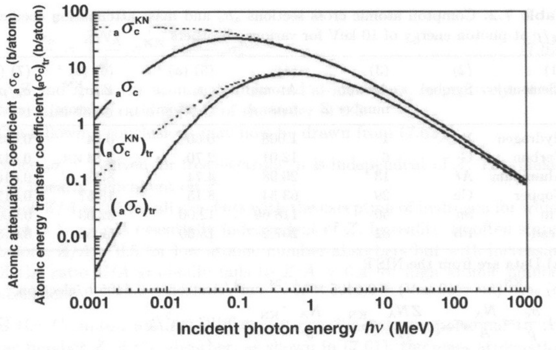
The atomic cross section for Rayleigh scattering (solid curves) compared to that for Compton scattering (dotted curves) plotted against incident photon energy for varying Z atoms. (Fig. 7.25 in Podgoršak)

- Compton mass attenuation coefficient

$$\frac{\sigma_{\rm c}}{\rho} = \frac{N_{\rm A}}{A}\,{}_{\rm a}\sigma_{\rm C} \tag{12}$$

- Compton mass energy transfer coefficient

$$\left(\frac{\sigma_{\rm c}}{\rho}\right)_{\rm tr} = \frac{\sigma_{\rm c}}{\rho}\frac{\overline{E_{\rm K}^{\rm C}}}{h\nu} \tag{13}$$



Atomic attenuation coefficient and atomic energy transfer coefficient for lead. (Fig. 7.22 in Podgoršak)

Example: For $h\nu = 1\,\mathrm{MeV}$ photons incident on lead ($Z = 82$, $A = 0.2072\,\mathrm{kg}$), the Compton atomic cross section is ${}_{\mathrm{a}}\sigma_{\mathrm{C}} = 1.72 \times 10^{-27}\,\mathrm{m}^2$. This can be calculated directly from the expression for ${}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}$ given by (6) and then using ${}_{\mathrm{a}}\sigma_{\mathrm{C}} = Z\,{}_{\mathrm{e}}\sigma_{\mathrm{c}}^{\mathrm{KN}}$ (since binding energy corrections are negligible at this $h\nu$). The Compton mass attenuation coefficient is

$$\frac{\sigma_{\mathrm{c}}}{\rho} = \frac{N_{\mathrm{A}}}{A}\,{}_{\mathrm{a}}\sigma_{\mathrm{C}} = \frac{6.022 \times 10^{23}}{0.2072\,\mathrm{kg}} 1.72 \times 10^{-27}\,\mathrm{m}^2 = 5.00 \times 10^{-3}\,\mathrm{m}^2\,\mathrm{kg}^{-1}$$

The values can be checked by going to the NIST/Xcom database:

`www.nist.gov/pml/data/xcom`

The average fractional recoil energy is obtained by inserting $\varepsilon = 1.96$ into the full solution given by eqn. (7.112) in Podgoršak:

$$\frac{\overline{E_{\mathrm{K}}^{\mathrm{C}}}}{h\nu} \approx 0.440$$

Photoelectric Effect

- interaction between photon and tightly bound orbital electron
- photon completely absorbed, electron ejected
- momentum transfer to atom, but recoil negligible due to relatively large nuclear mass, so energy conservation is:

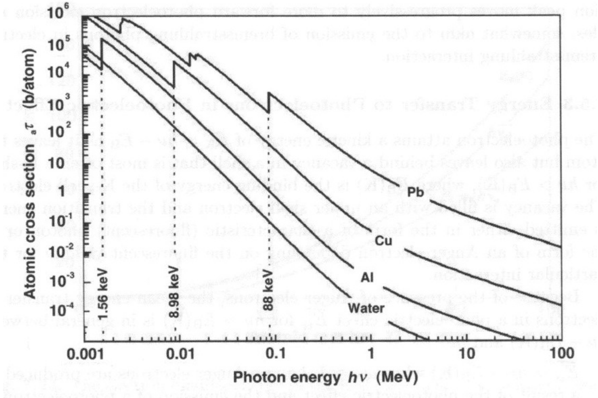
$$E_{\mathrm{K}} = h\nu - E_{\mathrm{B}} \qquad \text{photoelectron kinetic energy} \qquad (14)$$

$E_{\mathrm{B}} =$ binding energy of electron orbital

- approx. 80% occur with K-shell electrons
- resulting shell vacancy quickly filled by a higher shell electron; resulting transition energy released either as:
  – characteristic (fluorescent) photon
  – Auger electron
  probability determined by fluorescent yield

Atomic cross section

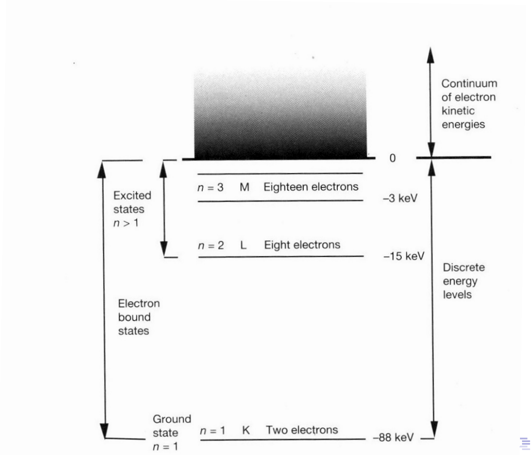
- ${}_{\mathrm{a}}\tau =$ atomic cross section for photoelectric effect
- function of $h\nu$, exhibits characteristic "sawtooth" behaviour: sharp discontinuities coinciding with E_{B} of a particular shell – absorption edges



Atomic cross section for photoabsorption. Energies of K-shell ionisation are indicated. (Fig. 7.28 in Podgoršak.)

e.g. Lead has prominent edges at the following ionisation energies for respective shells:

K edge	88.0 keV
L_1 edge	15.9 keV
L_2 edge	15.2 keV
L_3 edge	13.0 keV
M edge	3.9 keV



3 distinct regions characterise ${}_{\mathrm{a}}\tau$:

1. in immediate vicinty of absorption edge: ${}_{\mathrm{a}}\tau$ poorly known; for K-shell electrons, ${}_{\mathrm{a}}\tau_{\mathrm{K}} \propto \varepsilon^{-3}$ assumed
2. away from absorption edge: cross section for K-shell electrons is

$$ {}_{\mathrm{a}}\tau_{\mathrm{K}} \approx \sqrt{32}\,\alpha^4\,{}_{\mathrm{e}}\sigma_{\mathrm{T}} Z^n\,\varepsilon^{-7/2} \tag{15} $$

   where $\alpha = 1/137 =$ fine structure constant, $n \approx 4 - 4.6$ is a power index for Z dependence
3. $\varepsilon \gg 1$:

$$ {}_{\mathrm{a}}\tau_{\mathrm{K}} \approx 1.5\,\alpha^4\,{}_{\mathrm{e}}\sigma_{\mathrm{T}} Z^n \varepsilon^{-1} \tag{16} $$

   where $n \approx 4.6 - 5$

where $\varepsilon = h\nu/m_{\mathrm{e}}c^2$.

Energy transfer

- Auger electrons sometimes produced $\Rightarrow$ mean energy transfer to electrons as a result of photoelectric effect can be in range

  (no Auger electron) $h\nu - E_{\mathrm{B}} \lesssim \overline{E_{\mathrm{tr}}^{\mathrm{PE}}} \lesssim h\nu$ (Auger electron)

- fluorescent yield for K-shell, ω_K:

  $\omega_K = 1 \Rightarrow$ characteristic emission only, no Auger electrons

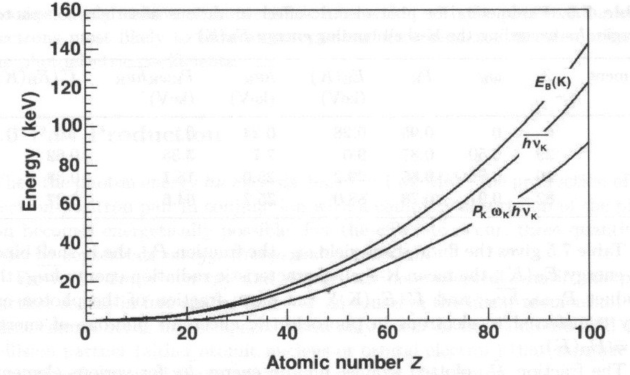
  $\omega_K = 0 \Rightarrow$ no characteristic emission, Auger electrons only

- in general,

$$\overline{E_{\mathrm{tr}}^{\mathrm{PE}}} \approx h\nu - P_K \omega_K \overline{h\nu_K} \quad (17)$$

  where $P_K =$ probability of photoelectric interactions that occur in K-shell for photons with $h\nu > E_{\mathrm{B}}(K)$

- $\overline{h\nu_K}$ = K-shell weighted mean of all possible fluorescent transitions ($L \to K$, $M \to K$); K_α usually most probable, giving $\overline{h\nu_K} \approx 0.86 E_\mathrm{B}(K)$



K-shell binding energy, $E_\mathrm{B}(K)$, weighted mean characteristic X-ray energy for all transitions to K-shell, $\overline{h\nu_K}$, and mean energy of K-shell characteristic emission, $P_K \omega_K \overline{h\nu_K}$. (Fig. 7.29 in Podgoršak.)

Example: Consider $h\nu = 0.5\,\text{MeV}$ photons incident on lead, which has $E_\text{B}(K) = 88\,\text{keV}$. Suppose the photoelectric effect occurs and is immediately followed by a forbidden $K_{\alpha_3}(L_1 \to K)$ transition, with ejection of an Auger electron from the L_2 shell. The total energy transferred to electrons = photoelectron energy + Auger electron energy:

$$E_\text{tr} = [\,h\nu - E_\text{B}(K)\,] + [\,E_{K\alpha_3} - E_\text{B}(L_2)\,]$$

But $E_{K\alpha_3} = E_\text{B}(K) - E_\text{B}(L_1)$, so this simplifies to

$$E_\text{tr} = h\nu - E_\text{B}(L_1) - E_\text{B}(L_2) \approx 0.469\,\text{MeV} \approx 0.94h\nu$$

For all K-shell transitions, the *average* energy transfer is

$$\begin{aligned}\overline{E_\text{tr}^\text{PE}} = h\nu - P_K\omega_K\overline{h\nu_K} &\approx 500\,\text{keV} - 60\,\text{keV} \\ &= 0.440\,\text{MeV} \approx 0.88h\nu\end{aligned}$$

(where $P_K\omega_K\overline{h\nu_K} \approx 60\,\text{keV}$ was obtained from the previous figure.)

Mass coefficients for the photoelectric effect

- mass attenuation coefficient:

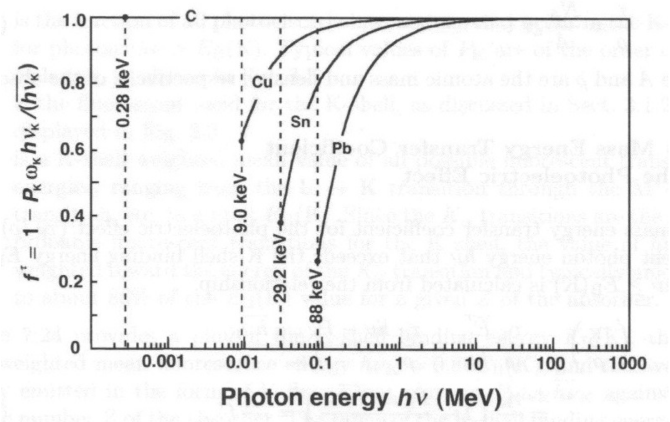
$$\frac{\tau}{\rho} = \frac{N_A}{A}\,{}_a\tau \tag{18}$$

- mass energy transfer coefficient:

$$\left(\frac{\tau_K}{\rho}\right)_{tr} = \frac{\tau}{\rho}\frac{\overline{E_{tr}^{PE}}}{h\nu} = \frac{\tau}{\rho}\left(1 - \frac{P_K\omega_K\overline{h\nu_K}}{h\nu}\right) = \frac{\tau}{\rho}\bar{f}_{PE} \tag{19}$$

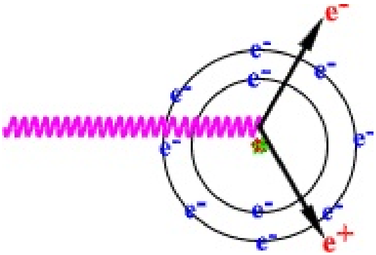
where $\bar{f}_{PE}$ = mean fraction of energy $h\nu$ transferred to electrons, for $h\nu > E_B(K)$

- $\bar{f}_{\text{PE}} \rightarrow 1$ for low-Z absorbers because Auger effect is more prevalent (i.e. fluorescent yield $\omega_K \approx 0$)



Mean fraction of photon energy $h\nu$ transferred to electrons in a K-shell photoelectric interaction.

Pair Production



- production of electron-positron $(e^- - e^+)$ pair resulting from photon interaction with atomic nucleus
- incident photon energy must exceed threshold $2m_ec^2 = 1.02\,\mathrm{MeV}$
- triplet production $(e^- - e^- - e^+)$ results when incident photon interacts with orbital electron; higher threshold energy required: $4m_ec^2 = 2.044\,\mathrm{MeV}$ (see next lecture for derivation)

Relativistic kinematics

Particle 4-momenta:

$$\text{photon: } p_\gamma = \frac{h\nu}{c}(1, \hat{\mathbf{k}})$$

where $\hat{\mathbf{k}}$ = unit vector in direction of photon 3-momentum (i.e. propagation direction).

$$\text{electron and positron: } p_{e^-} = \left(\frac{E}{c}, \mathbf{p_{e^-}}\right) \ , \ p_{e^+} = \left(\frac{E}{c}, \mathbf{p_{e^+}}\right)$$

where $\mathbf{p_{e^-}}$ and $\mathbf{p_{e^+}}$ are the electron and positron 3-momenta, with $|\mathbf{p_{e^-}}| = \gamma\beta m_e c = |\mathbf{p_{e^+}}|$ (must have same magnitude, but can have different direction). Must also consider 4-momentum of atom, $p_a = (E_a/c, \mathbf{p_a})$, which can gain recoil energy.

Conservation of 4-momentum:

$$\text{before:}\ p_\gamma \ = \ p_{e^-} + p_{e^+} + p_a \ \ \text{after}$$

Note that the modulus of a 4-vector $A = (A^0, \mathbf{A})$ is:

$$A^2 = A^\mu A_\mu = -(A^0)^2 + \mathbf{A} \cdot \mathbf{A}$$

which implies that $(p_\gamma)^2 = 0$ always and $(p_{e^-})^2 = -m_\mathrm{e}^2 c^2$. If we square the conservation equation above, then

$$(p_\gamma)^2 = (p_{e^-} + p_{e^+} + p_a)^2 = 0$$

Now consider the case where $p_a = 0$. We have

$$(p_{e^-})^2 + 2p_{e^-}p_{e^+} + (p_{e^+})^2 = 0$$

$$\implies -2m_\mathrm{e}^2 c^2 + 2\left(\frac{E^2}{c^2} + \mathbf{p_{e^-}} \cdot \mathbf{p_{e^+}}\right) = 0$$

$$\Longrightarrow 2(\gamma^2 - 1)m_{\mathrm{e}}^2c^2(1 + \cos\theta_e) = 0$$

which can only be satisfied if the electron and positron are emitted in exactly opposite directions, with separation angle $\theta_e = \pi$.

In general, the atom must gain some recoil energy from the collision with its nucleus. Because of its relatively large mass, however, the recoil gained by the atom will be small.

Energy transfer to electrons and positrons in pair production interactions:

$$(E_{\mathrm{K}}{}^{\mathrm{pp}})_{\mathrm{tr}} = h\nu - 2m_{\mathrm{e}}c^2 \quad (20)$$

is the total kinetic energy gained by the particles (ignoring atom recoil energy). Generally, the electron and positron can be emitted with different kinetic energies, but the average energy transferred to each is

$$\overline{E_{\mathrm{K}}}^{\mathrm{pp}} = \frac{1}{2}(h\nu - 2m_{\mathrm{e}}c^2) \quad (21)$$

For triplet production, the amount of energy converted to mass is also $2m_{\mathrm{e}}c^2$, but the kinetic energy is shared amongst 3 charged particles, so the mean energy transfer is

$$\overline{E_{\mathrm{K}}}^{\mathrm{tp}} = \frac{1}{3}(h\nu - 2m_{\mathrm{e}}c^2) \quad (22)$$

Example: For a $2\,\mathrm{MeV}$ photon, the average energy of charged particles resulting from pair production in the nuclear field is

$$\overline{E_{\mathrm{K}}}^{\mathrm{pp}} = \frac{1}{2}(h\nu - 2m_{\mathrm{e}}c^2) = \frac{1}{2}(2\,\mathrm{MeV} - 1.022\,\mathrm{MeV}) = 0.489\,\mathrm{MeV} = 0.245\,h\nu$$

and in the electron field, the average energy is $\overline{E_{\mathrm{K}}}^{\mathrm{tp}} = 0$ because $2\,\mathrm{MeV}$ is less than the threshold energy $4m_{\mathrm{e}}c^2 = 2.04\,\mathrm{MeV}$ needed for triplet production.

Nuclear screening occurs for $h\nu > 20\,\mathrm{MeV}$ photons that interact with the nuclear Coloumb field outside the K-shell; effective nuclear field is screened by two K-shell electrons and the interaction cross section is reduced.

Atomic cross section

- General form for pair production atomic cross section in field of nucleus or orbital electron is

$$\boxed{{}_{\mathrm{a}}\kappa = \alpha r_{\mathrm{e}}^2 Z^2 \, P(\varepsilon, Z)} \qquad (23)$$

$P(\varepsilon, Z) =$ complicated function

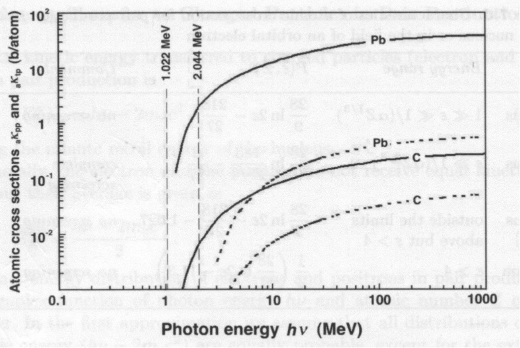
field	energy	$P(\varepsilon, Z)$
1. nucleus	$1 \ll \varepsilon \ll (\alpha Z^{1/3})^{-1}$	$\frac{28}{9}\ln(2\varepsilon) - \frac{218}{27}$
2. nucleus	$\varepsilon \gg (\alpha Z^{1/3})^{-1}$	$\frac{28}{9}\ln(183 Z^{-1/3}) - \frac{2}{27}$
3. nucleus	$\varepsilon > 4$	$\frac{28}{9}\ln(2\varepsilon) - \frac{218}{27} - 1.027$
4. electron	$\varepsilon > 4$	$Z^{-1}\left[\frac{28}{9}\ln(2\varepsilon) - 11.3\right]$

- nuclear screening only important in case 2
- $\varepsilon > 4$ in case 3 must lie outside the limits of cases 1 and 2

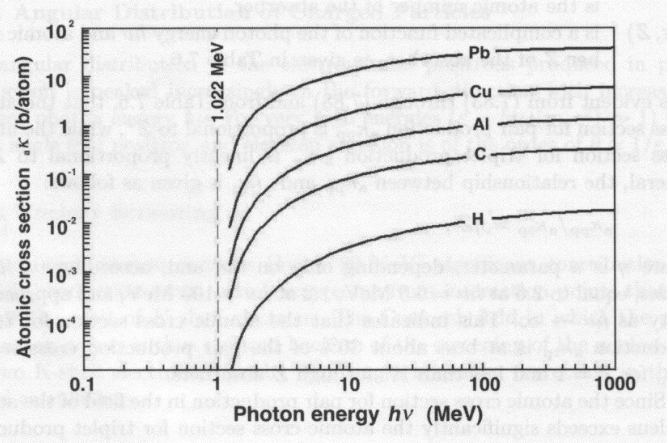
- $P(\varepsilon, Z) \propto Z^{-1}$ for electron field $\Rightarrow$ triplet production usually makes negligible contribution to total cross section:

$$ {}_{\mathrm{a}}\kappa = {}_{\mathrm{a}}\kappa_{\mathrm{pp}} + {}_{\mathrm{a}}\kappa_{\mathrm{tp}} = {}_{\mathrm{a}}\kappa_{\mathrm{pp}} \left[1 + (\eta Z)^{-1}\right) \qquad (24) $$

where $\eta \to 1$ as $h\nu \to \infty$



Atomic cross sections for pair production (solid curves) and triplet production (dashed curves) for a high-Z and low-Z absorber. (Fig. 7.35 in Podgoršak.)



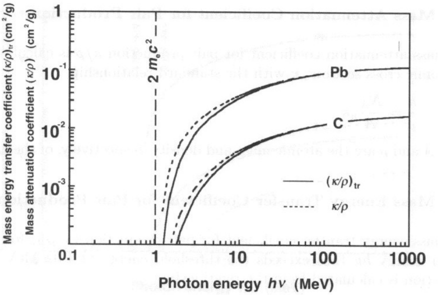
Total atomic cross sections for pair production (including triplet production) for different Z. (Fig. 7.36 in Podgoršak.)

- mass attenuation coefficient for pair production:

$$\frac{\kappa}{\rho} = \frac{N_A}{A}\,{}_a\kappa \qquad (25)$$

- mass energy transfer coefficient:

$$\left(\frac{\kappa}{\rho}\right)_{tr} = \frac{\kappa}{\rho}\frac{\overline{E_{tr}^{pp}}}{h\nu} = \frac{\kappa}{\rho}\left(1 - \frac{2m_e c^2}{h\nu}\right) \qquad (26)$$



Mass energy transfer coefficient (solid curves) and mass attenuation coefficient (dashed curves) for pair production. (Fig. 7.38 in Podgoršak.)

Summary of Photon Interactions

Tabulation of interactions and symbols used:

	electronic cross section $[\mathrm{m}^2]$	atomic cross section $[\mathrm{m}^2]$	linear attenuation coefficient $[\mathrm{m}^{-1}]$
Thomson scattering	${}_{\mathrm{e}}\sigma_{\mathrm{T}}$	${}_{\mathrm{a}}\sigma_{\mathrm{T}}$	σ_{T}
Rayleigh scattering	–	${}_{\mathrm{a}}\sigma_{\mathrm{R}}$	σ_{R}
Compton scattering	${}_{\mathrm{e}}\sigma_{\mathrm{C}}$	${}_{\mathrm{a}}\sigma_{\mathrm{C}}$	σ_{C}
Photoelectric effect	–	${}_{\mathrm{a}}\tau$	τ
Pair production	–	${}_{\mathrm{a}}\kappa_{\mathrm{pp}}$	κ_{pp}
Triplet production	${}_{\mathrm{e}}\kappa_{\mathrm{tp}}$	${}_{\mathrm{a}}\kappa_{\mathrm{tp}}$	κ_{tp}

Tabulation of attenuation coefficients:

Total linear and mass attenuation coefficients are a sum of the *partial* linear and mass attenuation coefficients for individual photon interactions:

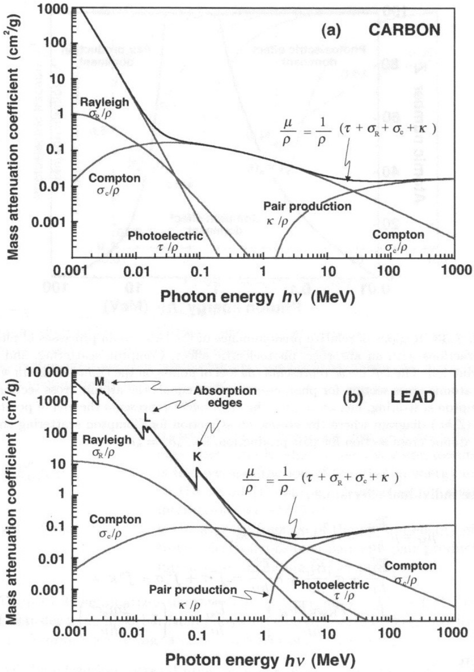
$$\mu = \tau + \sigma_{\mathrm{R}} + \sigma_{\mathrm{C}} + \kappa \tag{27}$$

$$\mu_{\mathrm{m}} = \frac{\tau}{\rho} + \frac{\sigma_{\mathrm{R}}}{\rho} + \frac{\sigma_{\mathrm{C}}}{\rho} + \frac{\kappa}{\rho} \tag{28}$$

Similarly, the total atomic cross section is

$${}_{\mathrm{a}}\mu = \mu_{\mathrm{m}} \frac{A}{N_{\mathrm{A}}} = {}_{\mathrm{a}}\tau + {}_{\mathrm{a}}\sigma_{\mathrm{R}} + {}_{\mathrm{a}}\sigma_{\mathrm{C}} + {}_{\mathrm{a}}\kappa = Z\,{}_{\mathrm{e}}\mu \tag{29}$$

where ${}_{\mathrm{e}}\mu =$ total electronic cross section.


(a) CARBON
Mass attenuation coefficient (cm²/g)
1000
100
10
1
0.1
0.01
0.001
Rayleigh
σR/ρ
Compton
σc/ρ
Photoelectric
τ/ρ
Pair production
κ/ρ
Compton
σc/ρ
μ/ρ = 1/ρ (τ + σR + σc + κ)
0.001
0.01
0.1
1
10
100
1000
Photon energy hν (MeV)
(b) LEAD
Mass attenuation coefficient (cm²/g)
10 000
1000
100
10
1
0.1
0.01
0.001
M
L
K
Absorption edges
Rayleigh
σR/ρ
Compton
σc/ρ
Pair production
κ/ρ
Photoelectric
τ/ρ
Compton
σc/ρ
μ/ρ = 1/ρ (τ + σR + σc + κ)
0.001
0.01
0.1
1
10
100
1000
Photon energy hν (MeV)

For compounds or mixtures, the mass attenuation coefficient is a summation of a weighted average of its constituents:

$$\mu_{\mathrm{m}} = \Sigma_i w_i \frac{\mu_i}{\rho} \tag{30}$$

where $w_i =$ proportion by weight of i-th constituent.

Example: Water, H_2O, has

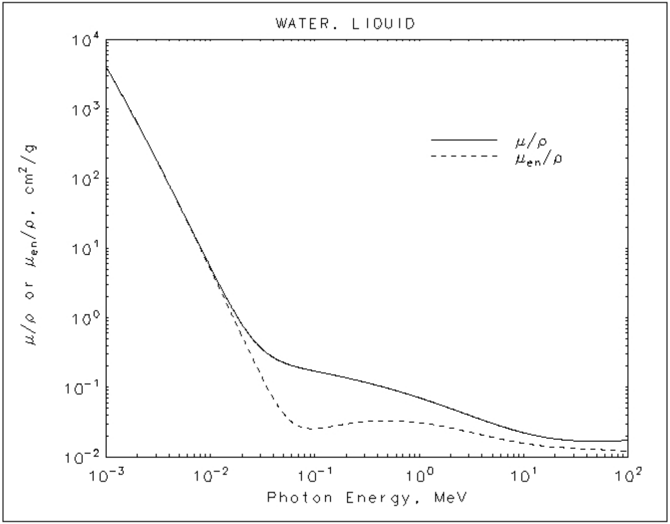
$$w_H = \frac{2 \times 1.0079}{2 \times 1.0079 + 15.999} = 0.1119$$

$$w_O = \frac{15.999}{2 \times 1.0079 + 15.999} = 0.8881$$

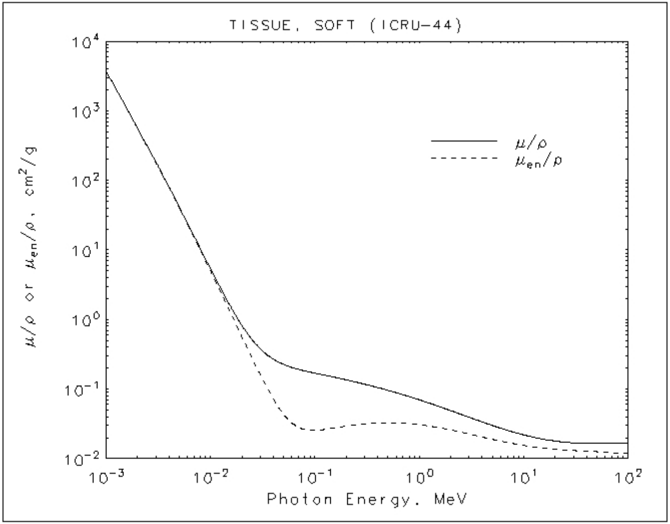
For $1\,\mathrm{MeV}$ photons, $\mu_H/\rho = 1.26 \times 10^{-2} \mathrm{m}^2\mathrm{kg}^{-1}$ and $\mu_O/\rho = 6.37 \times 10^{-3} \mathrm{m}^2\mathrm{kg}^{-1}$ (data from the NIST/Xcom database), so

$$\frac{\mu}{\rho} \approx 7.07 \times 10^{-3}\ \mathrm{m}^2\mathrm{kg}^{-1}$$

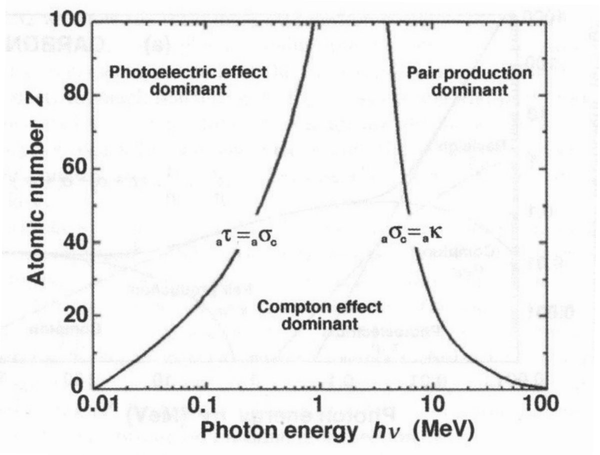
Mass attenuation coefficient for water (NIST):



Mass attenuation coefficient for soft tissue (NIST):



Comparison of the three main photon energy transfer processes:



- photoelectric effect dominates at low $h\nu$ and high Z
- pair production dominates at high $h\nu$ and high Z
- Compton scattering dominates over a broad range in $h\nu$ for low/moderate Z (including water and tissue)

Energy transfer coefficient is the sum of the partial energy transfer coefficients for the photoelectric effect, Compton scattering and pair production:

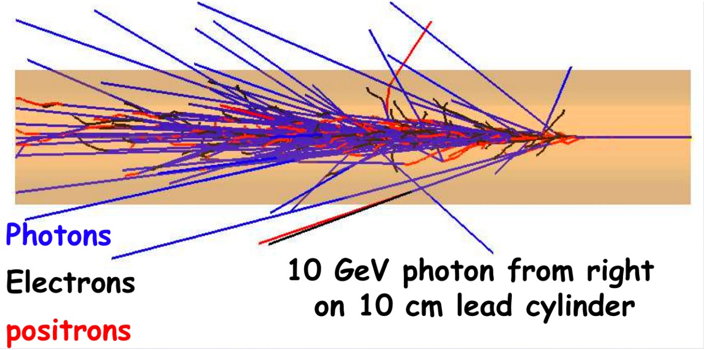
$$\mu_{\mathrm{tr}} = \mu \frac{\overline{E}_{\mathrm{tr}}}{h\nu} = \tau \frac{\overline{E}_{\mathrm{tr}}^{\mathrm{PE}}}{h\nu} + \sigma_{\mathrm{C}} \frac{\overline{E}_{\mathrm{tr}}^{\mathrm{C}}}{h\nu} + \kappa \frac{\overline{E}_{\mathrm{tr}}^{\mathrm{pp}}}{h\nu} = \tau \bar{f}^{\mathrm{PE}} + \sigma_{\mathrm{C}} \bar{f}^{\mathrm{C}} + \kappa \bar{f}^{\mathrm{pp}} \quad (31)$$

where each $\bar{f}$ is the average fraction of incident photon energy $h\nu$ transferred to electrons (and positrons) by the corresponding physical process, with

$$\bar{f}^{\mathrm{PE}} = 1 - \frac{P_K \omega_K \overline{h\nu}_K}{h\nu} \quad (32)$$

$$\bar{f}^{\mathrm{C}} = \frac{\overline{E}_{\mathrm{tr}}^{\mathrm{C}}}{h\nu} \quad (33)$$

$$\bar{f}^{\mathrm{pp}} = 1 - \frac{2 m_{\mathrm{e}} c^2}{h\nu} \quad (34)$$



What is the dominant photon interaction initially?