A Scientific Approach to CAPM and Options Valuation

Emanuel Derman
Columbia University

My Discomfort with CAPM

- Neoclassical finance:
  - Black Scholes derivation depends on replication (elegant)
  - CAPM depends on mean variance optimization (unrealistic)
- Would prefer one approach to both that starts from one logical principle I can accept

General Principle of Valuation

- Finance asks: What return should you expect from taking on risk?
- I claim:
  Equal Risk Should Expect Equal Return μ
- But some risks can be avoided
- Equal Unavoidable Risk Should Expect Equal Return μ
- We know what expected return is, but what is risk?

How Can You Avoid Risk?

- Dilution:
  Combine security with a riskless bond
- Diversification:
  Combine security with many uncorrelated securities
- Hedging:
  Combine security with a correlated security

Specialize:
Define Risk as Std Deviation

- Pretend that risk is the standard deviation σ of returns
- Expected return μ
- This is obviously naive. Of course there are much more general and realistic definitions of risk, but for now we stick with this one....

1
Equal Diluted Risk, Equal Return: All stocks have same Sharpe Ratio

- Combine weight w of a risky stock S (μ, σ) with a weight (1 - w) of riskless bond B $(r, 0)$

$$[w\mu + (1-w)r, w\sigma] = [r + w(\mu - r), w\sigma]$$

- All uncorrelated stocks with risk $w\sigma$ earn excess return $w(\mu - r)$
- One parameter fixes everything
- Same Sharpe ratio for all stocks

$$\frac{(\mu - r)}{\sigma} = \lambda$$

2

Equal Diversified Diluted Risk, Equal Return: Sharpe Ratio is Zero

- Suppose there are countless uncorrelated stocks (μ_i, σ_i)
- Put them all in a portfolio with weights $P = \sum_i w_i S_i$
- Then

$$\mu - r \equiv \sum_i w_i(\mu_i - r) = \sum_i \lambda w_i \sigma_i$$

- But the total risk σ diversifies to zero.

$$\mu - r = \lambda\sigma = \lambda \sum_i \sigma_i = 0$$

- Thus $\lambda = 0$
- Every stock is expected to earn the riskless rate

3
Equal Diversified Hedged Diluted Risk, Equal Return: CAPM

- Suppose there are countless stocks Si (μ_i, σ_i)

  correlated with the market M (μ_M, σ_M)

- Then the market-neutral stock $S_i^M \equiv S_i - \beta_i \left(\frac{|S_i|}{|M|}\right) M$

  has no market risk (is hedged), where $\beta_i = \rho_{iM} \frac{\sigma_i}{\sigma_M}$
- Thus $S^M{}_i$ has zero Sharpe ratio, earns riskless rate r
- Which means

$$(\mu_i - r) = \beta_i (\mu_M - r)$$

4
Stock and Option Have Equal Sharpe Ratio: Black-Scholes

- Stock S, call C

$$\frac{(\mu_S - r)}{\sigma_S} = \frac{(\mu_C - r)}{\sigma_C}$$

- Ito’s Lemma applied to a call C to obtains its expected return and effective volatility leads to Black-Scholes, as originally derived by Black.

- Unified treatment of BS and CAPM