Post-Crisis Pricing of Swaps using xVAs

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Abstract

OTC derivative pricing has been relying on the risk neutral pricing framework presented by (Black and Scholes, 1973) under the assumptions of funding at the risk free rate and the ability to perfectly replicate derivatives to fully hedge all potential risks. The financial crisis of 2008 proved that these assumptions were indeed too simplistic. This has led to a new era of derivative pricing, where issues as counterparty credit risk, funding costs and costs of capital must be considered, as perfect hedging of these risks is not fully possible.

This thesis provides an application to price interest rate swaps (IRS) and fixed-fixed cross currency swaps (CCS). It expands the risk neutral pricing method to use a multi-curve approach to account for tenor risk of the floating leg of IRS contracts. Furthermore, counterparty credit risk, funding costs and costs of capital are quantified as an extra Value Adjustment (xVA) to the risk neutral price. This adjustment consists of different xVA terms each accounting for different adjustments. Credit Value Adjustment (CVA) accounts for the costs of counterparty risk. Funding Value Adjustment (FVA) covers costs of funding cash flows of the swap contract or cash flow mismatches with the hedge, which constitutes the Hedging Value Adjustment (HVA) and furthermore covers funding costs of collateral posted to mitigate potential credit risk, resulting in the Collateral Value Adjustment (CollVA). Margin Value Adjustment (MVA) covers the costs of funding initial margin collateral posted for further credit risk reduction. Capital Value Adjustment (KVA) provides the cost of required regulatory capital, which must be held against losses. Moreover, this paper includes theoretical discussions of xVA risk management, optimization of the xVA charge, and xVA hedge strategies.

Monte Carlo simulation of the underlying stochastic risk factors is used to estimate the different xVA charges. This thesis assumes that interest rate dynamics follow one-factor CIR models and that default intensity of the counterparty follows a JCIR model. We provide the framework to calibrate these models to market data, respectively swap rates for the CIR model and the iTRAXX Crossover Index credit spreads for the JCIR model using non-linear GRG optimization of the model parameters. Dependency between default intensity and interest rates is based on Cholesky factorization of the Gaussian variables and evaluation of discretization schemes of the stochastic models contribute to enhance simulation precision. FX rates are deterministically calculated from the stochastic OIS rates in EUR and USD in accordance with the interest rate parity.

The pricing model results reveal that the total xVA charge increases for longer contract maturities, increasing floating tenor, and is much higher for fixed-fixed CCS than for IRS contracts,

as CCS contracts depend on both interest rates and FX rates. Collateral agreements prove to significantly reduce the CVA, HVA, and KVA charge. However, this comes at the costs of higher CollVA and potentially also higher MVA if adding an initial margin. Entering a reverse swap contract with a central clearing party (CCP) to hedge market risk covers most or all of the HVA and generally lowers the xVA charge. Combining collateralization and a market risk hedge lowers the costs of collateral as collateral held from either the counterparty or the CCP can be rehypothecated and hence posted as collateral at the other party. This thesis furthermore finds, that the optimal initial margin must depend on factors determining the expected exposure and credit risk. Sensitivity analyses demonstrates model robustness. Results of changes in crucial input factors are in accordance with theory and MC noise has an inconsiderable effect on results.

The xVA pricing model presented in this paper illustrates the quantitative complexity of pricing and risk management of even plain vanilla derivatives when including xVAs. Appropriate pricing models must be able to price infinitely many different derivative contracts with the estimated xVA charge on each single trade as a marginal charge dependent on the entire existing trading book and all current counterparties. Developing optimal model solutions to accomplish this complexity at a reasonable computational effort is the most difficult task financial engineers currently face. Besides, inclusion of different xVA terms is still under intense debate by both theorists and financial professionals.

Acknowledgment

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Nomenclature

BS model	Black and Scholes model
CCP	Central Clearing Party
CCR	Counterparty Credit Risk
CCS	Cross-Currency Swap
CDS	Credit Default Swap
CIR model	Cox-Ingersoll-Ross model
CollVA	Collateral Value Adjustment
CSA	Credit Support Annex
CVA	Credit Value Adjustment
DVA	Debit Value Adjustment
EE	Expected Positive Exposure
ES	Expected Shortfall
FVA	Funding Value Adjustment
FX	Foreign Exchange
GRG	Generalized Reduced Gradient optimization method
HVA	Hedging Value Adjustment
IMA	Internal Model Approach of market risk
IMM	Internal Model Method of counterparty credit risk
IRB	Internal Rating Based Approach of market risk
IRS	Interest Rate Swap
ISDA	International Swaps and Derivatives Association

JCIR model	Jump-Diffusion CIR model
KVA	Capital Value Adjustment
LGD	Loss Given Default
LVA	Liquidity Value Adjustment
MC	Monte Carlo
MR charge	Market Risk charge
MTA	Minimum Transfer Amount
MVA	Margin Value Adjustment
NEE	Expected Negative Exposure
OIS	Overnight Indexed Swap Rate
OR charge	Operational Risk charge
OTC	Over-The-Counter
PDE	Partial Differential Equation
PD	Probability of Default
PFE	Potential Future Exposure
RARoC	Risk Adjusted Return on Capital
RoC	Return on Capital
RWA	Risk Weighted Assets
SA-CCR	Standardized Approach for Counterparty Credit Risk
SMM	Standardized Measurement Method of market risk
VaR	Value at Risk
xVA	Extra Value Adjustment

Part I

Formalia

1.1 Introduction

> *"The gap that had traditionally existed between counterparty risk people, derivatives pricing, and hedging people needs to be closed. On one side, it does not make sense to consider derivatives as platonic instruments living in a world of their own without being affected by credit, default, liquidity, funding and collateral ... when even a vanilla instrument portfolio is embedded in such risks, it becomes the most formidable derivative to be priced and managed -" (Brigo et al., 2013)*

The financial crisis in 2008 turned the view on derivative markets completely upside down. Issues, which had earlier been deemed insignificant and thus been ignored in the financial market turned out to have a huge impact on derivative costs, which completely undermined the simplistic assumptions of pricing models for many simple derivative instruments. This started a new era in the field of derivatives pricing. Financial engineers, who had been busy focusing on exotic derivatives, had to return to the plain vanilla instruments, acknowledging that these in fact included complexities requiring the same attention as exotic derivatives (Brigo et al., 2013). The simple assumptions for derivative pricing did no longer provide even a decent approximation of the fair derivative value.

This thesis focuses on vanilla interest rate swap contracts[1] nominated in a single or dual currency, which used to be priced and managed in a simple way. The pre-crisis pricing of plain vanilla swaps simply used a single interest rate curve. Discounting of the swap cash flows was with the same interest rates, as those used to forward the floating rate payments (Eriksson, 2012). Hence, such method assumed no significant risk in the tenor of the floating reference rate (Bianchetti, 2008).

Financial institutions were assumed to be default free due to top credit ratings. The effect of the credit risk related to the counterparty was not an issue when pricing the swap at the trading desk, but instead left to be handled by the credit risk department (Brigo et al., 2013). The contract collateral agreements were assumed to provide perfect collateralization meaning that neither the financial institution nor the counterparty would have leftover credit risk exposure. The effect of margining costs of such collateral agreements were disregarded due to the assumption of funding at the risk free rate. Liquid capital markets and high bank ratings made it possible to achieve close to risk free funding of the swap (Kenyon and Stamm, 2012). Required regulatory capital holding as buffer against unexpected losses were not very strict before the the Capital Requirements Directive IV in the EU and Dodd-Frank in the US (Kenyon and Stamm, 2012) and target return on capitals were set as guidelines for the derivative desk rather than charged individually to each single trade (Gregory, 2015a). Hence, the costs of such capital was not a pricing issue (Kenyon and Green, 2014).

The financial crisis demonstrated how these simple methods of derivative pricing did not result in a fair value of the contract as many risks were ignored. One of these being the basis swap spread[2], which rose to a significant size due to higher credit risk on longer tenors. Hence, forwarding the floating payments of a swap and discounting with a risk free proxy had to be with

[1] See Appendix A for the fundamentals of swap contracts.
[2] See Appendix A.2 and Appendix A.3.

two different rates (Linderstrøm and Rasmussen, 2011). For this reason, this thesis presents the multi-curve approach to account for tenor specific risk premiums on the floating payments of swaps (Bianchetti, 2008).

The acceptance of many new risk factors following in the post-crisis years, have led to a whole new pricing field for OTC derivatives, including different risk factors under the overall term xVA[3]. This adjustment provides a the risk premium estimate to be included in the fair price of an OTC derivative so that potential funding and capital costs and the default risk of the counterparty are accounted for (Gregory, 2015a). There are many different value adjustments under the field of xVA, including:

- CVA measuring the costs of counterparty default on a derivative contract with positive value outstanding.
- DVA being equivalent to the CVA for the counterparty, which account for the benefit of defaulting while the value of the derivative contract is negative.
- FVA accounting for the costs of funding cash flow mismatches of the derivative contract (HVA) and funding of collateral posted against a positive market value of the contract to mitigate CVA (CollVA).
- MVA covering costs of funding initial margins to be posted besides the ordinary collateral.
- KVA accounting for the costs of capital required by regulators to be held against unexpected losses.

Dealing with these value adjustments will be the main topic of this thesis paper. We present the different value adjustments and estimate them to price plain vanilla IRS contracts or fixed-fixed CCS contracts while including the possibility to hedge market risk with a reverse trade at a Central Clearing Party (CCP) and allow for different choices of cash collateralization agreements.

We consider a numerical model approach based on Monte Carlo simulations of the values, collateralization and value adjustments calculated from relevant input factors and underlying risk factors to estimate the xVA charges. These xVA estimates are determined from stochastic processes for underlying interest rates in two currencies and the default risk of the counterparty. The stochastic models are calibrated to market data. As these factors depend on one another, a simulation framework provides a better solution than approximations or closed formulas not able to include such cross-gamma effects or account for complexities of the different value adjustments like details of collateral agreements.

We perform a sensitivity analysis for the most crucial model inputs including the model choice for underlying risk factors to capture potential model risk and avoid biased results. We discuss backtest of the model and potential Monte Carlo noise in results in order to ensure confident estimates of the xVA charge.

[3] Extra Value Adjustment.

Bearing in mind the significance of the different xVAs, we suggest approaches to optimize the overall xVA premium and discuss potential hedging strategies and risk management measures to lower xVA risk.

This thesis aims to give an application to pricing with xVAs relevant for medium-sized financial institutions with respect to their minor corporate customers. In practice, the financial institution can often consider themselves default free relative to the corporate counterparty, which reduces the CVA charge to a unilateral case (Brigo et al., 2013). Swap contracts can include collateral agreements, but are not limited to such. The models presented in this thesis have the purpose of being sufficiently complex to describe issues rising from the field of xVAs and provide an appropriate amount of robust details about the different xVA terms allowing for discussion of xVA risk management and hedging. However, the model is not complete in the sense of being directly implementable and usable for xVA desks of financial institutions, where the xVA charge should be seen as the marginal effect with respect to the entire derivatives portfolio, which includes complex interdependency modelling. To keep the main focus on xVAs and acknowledge the real life challenge of balancing between model complexity and the computational challenge of measuring xVAs (Ruiz, 2015), plain one-factor models are used to describe the interest rate dynamics while the default intensity measure of credit risk includes a jump diffusion.

1.2 Motivation

Modern pricing of interest rate derivatives has been an important and highly debated topic since the financial crisis. Academic literature on the multi-curve approach for pricing interest rate derivatives existed before the financial crisis. Despite the fact that this theory of market segmentation of different tenors was already accepted as the theoretical correct pricing method, the low basis spread between tenors made the difference negligible in practice (Bianchetti, 2008). Thus, the same interest curve was used to forward floating rates and discount all cash flows (Tuckman and Serrat, 2012). The basis spreads between different tenors increased significantly during the financial crisis and have not been negligible since then meaning that the multi-curve approach is now used in practice. Hence, this addition to interest rate derivative pricing makes it a relevant starting point for this thesis.

The years following the financial crisis triggered reflection towards the field of derivatives for academics, practitioners and regulators. The academic world acknowledged that issues of funding, capital requirement, and counterparty credit risk would no longer be possible to separate from derivative pricing, but instead had to be an integrated part of pricing models. For these issues, simplistic assumptions did no longer hold. Even pricing of simple interest rate derivatives became a rather complex affair. In practice, the ignorance of complexity in derivatives pricing resulted huge losses to the financial sector, which motivated the financial institutions to increase sophistication of their models to correct pricing errors, which could cause similar losses in the future. Regulators recognized that the regulatory framework of Basel II did not efficiently capture all relevant risks. In Basel II only the counterparty default and credit migration risk was addressed within the framework to calculate required regulatory capital. But during the financial crisis these risks only accounted for one-third of the losses, whereas the latter two-thirds came from marking-to-market CVA changes from higher credit risk volatility. This led to inclusion of a CVA capital charge within Basel III expected to double required regulatory capital for counterparty credit risk compared to Basel II (BCBS, 2011). The new value adjustment in derivative pricing for CVA, later followed by funding value adjustment (FVA) and capital value adjustment (KVA), have along with other valuation adjustments created the field of xVAs. This new field in financial engineering is still under development and intensely debated among academics as well as practitioners.

This thesis introduces xVAs on a multi-curve pricing model applied to price swap contracts. The types of swap contracts covered is vanilla IRS contracts and fixed-fixed CCS contracts. These instruments are a relevant choice as they used to be priced in a very simple manner[4]. Especially the IRS contracts are relevant to apply the new pricing methods to, as they account for a majority of the total derivatives markets in both market value and notional volume[5]. Adjusting the price of such contracts hence affect a major part of the fixed income derivatives markets.

To be able to develop a model capable of estimating the xVAs, we create a stochastic model to simulate the dynamics of underlying risk factors: interest rates and default intensity. The entire field of stochastic interest rate models and intensity models for credit risk is in itself an extension of the curriculum provided at the MSc. Finance program at Aarhus BSS. Hence, with this thesis we seize the opportunity to extend our skills in financial engineering to interest rate derivatives and be able to add credit risk modelling allowing for inclusion of advanced topics such as counterparty risk, funding costs, capital requirements, and collateralization. Creating a discrete simulation model furthermore requires additional theory on discretization schemes of stochastic processes.

From our job experience within capital markets of the financial sector, we have both experienced the importance of interest rate derivatives, especially interest rate swaps in the total derivatives markets. The field of xVA receives a lot of attention, whether the purpose is to strengthen pricing competitiveness with better models or to comply with regulatory requirements on risk management. This practical experience has played an important role in the motivation for the choice of thesis topic. Our background also inspired the choice to create a pricing model in the perspective of a medium-sized bank with a funding spread to interbank rates, but with a negligible relative default risk compared to its corporate counterparties.

1.3 Research Methods

This section outlines the thesis problem statement and describes the conceptual research framework used, including general assumptions and limitations of the thesis. This includes a walkthrough of the overall structure of the different parts included in this paper.

The research methods used for this thesis associates with the methodology widely accepted in finance research following an empirical and critical rational ideology based on a positivistic

[4]See Appendix A.4
[5]See Table A.1 and A.2 in Appendix A

view[6] (Holm, 2011),(Ryan et al., 2002). Within this methodology of research, the development of theories origins from some fundamental axioms[7] resulting in a number of core assumptions and models as described by (Lakatos and Musgrave, 1970). Even though these models have some simplistic assumptions inconsistent with reality, they provide a comprehensive framework necessary for development of general models to formulate theories (Ryan et al., 2002).

Some of the assumptions can be relaxed to account for actual abnormalities relevant for the research results. E.g., the discussions of xVAs would be a relaxation of the strict general assumption of perfect capital markets implying no transaction and funding at a risk free rate. This thesis focus on fundamental research on the effect and nature of the xVAs. The model aims to describe the nature and optimization of xVA charges for swap contracts issued by financial intermediaries with high internal validity in such a manner decribed by (Ryan et al., 2002). For this reason, the stochastic models describing dynamics of the underlying risk factors are limited to simplified choices, as these simply serve as input for the xVA model.

In order to secure the validity of the results as suggested by (Kane, 2004), this thesis provides an in depth sensitivity analysis, which takes into account potential misspecifications of the model emerging from the choice of assumptions. Data used for the thesis are all actual price quotes available on (Bloomberg, 2015) or in the dataset of this thesis in order to allow for replication of presented results. We stretch that as the model results are made for certain instruments, maturities and a proxy for counterparty risk, these results might not yield a strong external validity in such a manner decribed by (Ryan et al., 2002). Hence, the model should not be used for such (Kane, 2004), but serve as input for a general discussion on xVA and as a framework to develop an internal xVA model.

1.3.1 Problem Statement

The purpose of this thesis is to design a sophisticated model for pricing plain vanilla IRS contracts and CCS contracts issued by a medium-sized financial institution to a minor corporate customer. Post-crisis pricing of swaps has become far more complex, considering multiple-curve pricing approach and a xVA charge including xVA terms like[8] CVA, FVA, MVA, and KVA, while also allowing for different choices on collateral agreements. Besides pricing, the purpose of thesis is to describe the nature of the different xVA terms and suggest sophisticated ways of managing and hedging xVA risk.

The overall research question is stated as follows:

- *How can financial institutions set up a sophisticated pricing framework for plain vanilla IRS and CCS contracts with corporate customers when taking into account xVAs?*

In order to design the swap pricing model and evaluate the implications of xVAs, the following sub-questions supplement the overall research question:

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[6]The acceptance and discussion of the actual existence of a methodology in finance could be debated as in (Frankfurter, 2007).

[7]The most well-know axioms being the von Neumann-Morgenstern axioms of the rational investor.

[8]See Section 1.1.

- *How can a simulation-pricing framework of swap contracts based on the multi-curve approach be designed?*
- *What is the nature of xVAs and how can such adjustment be included?*
- *What significance do the xVAs have on the price and what affect the relative impact of the different xVAs?*
- *How can financial institutions manage the xVA risks and is it possible to create approximate hedge strategies for the xVAs?*

1.3.2 Thesis Structure

Following this introducing part, the thesis has the theoretic framework separated into three parts. Part V to VII covers analysis and discussion of the model results, risk management and conclusions.

- **Part II** covers the theory and models needed for pricing swap contracts. Section 2.1 evaluates the increase in the basis swap spread during the financial crisis. Section 2.2 describes the framework of the multi-curve approach. Section 2.3 introduces stochastic interest rate and default intensity models and includes a framework for model calibration.
- **Part III** presents different approaches and issues of Monte Carlo simulation. Section 3.1 describes the purpose of using MC when pricing with xVAs. Section 3.2 shortly introduces the basic principles of MC simulation and measurements of simulation efficiency. Section 3.3 explains issues and challenges in MC simulation when estimating xVAs. Section 3.4 presents the appropriate discretization schemes for the dynamics of the risk factor models required for simulation purpose.
- **Part IV** deals with the theory and nature of xVA. Section 4.1 introduces the field of xVA terms and give a short overview of the different kinds of xVA components and discuss how these have changed the know pricing framework. Section 4.2 focus on the nature of CVA, exposure calculation with and without collateral, and estimation of CVA. Section 4.3 evaluates the theory of FVA and the different FVA elements including estimation of these. Section 4.4 discuss determination of initial margin and estimation of MVA. Section 4.5 provides a discussion on capital requirements and present regulatory capital charges using the standardized approach for counterparty risk and the advanced approach for market risk in the Basel regulatory framework. These serve as base for KVA estimation.
- **Part V** presents the xVA pricing model for IRS and fixed-fixed CCS contracts and evaluates model results and sensitivity. Section 5.1 provides a primer on content of the model results, while Section 5.2 discusses consideration and assumptions on model input and market data. Section 5.3 provides interpretation of the model

results while section 5.4 performs a sensitivity of input parameters and discuss model robustness.

- **Part VI** discuss the challenge of managing xVA. While section 6.1 provides an introduction to xVA managment, section 6.2 suggests optimization of contract terms to reduce the total xVA charge. Section 6.3 presents ways to construct approximate hedges on some of the xVA risk, while section 6.4 discuss risk management supplements to hedges.

- **Part VII** is the conclusion of the thesis summarizing previous parts and main lessons learned from the analysis.

1.3.3 Software

The software used to make the necessary modelling and simulations has been carried out in Microsoft Excel, including the Excel Visual Basic Application (VBA) and Excel the add-ins *Solver* and *Goal Seek*. In addition, the collected data have been obtained from (Bloomberg, 2015).

1.3.4 General Assumptions

Fundamental assumptions within the framework of financial research apply to this thesis unless stated otherwise. To keep a well-structured and tractable model, assumptions are only relaxed when aiming to answers relevant questions of the thesis problem statement[9], acknowledging that xVAs are indeed extensions and reinterpretation of the traditional assumptions within derivative pricing. Hence, the aim is to keep the model comprehensive and avoid drawing wrong conclusions as a result of assumption choices (Ryan et al., 2002). This subsection discusses general assumptions and the consequences of relaxing assumptions.

The basic von Neumann-Morgenstern axioms (Elton et al., 2011) assume a rational market participants or a rational representative agent in the financial market (Munk, 2011) even though debated and falsified by many researchers like (Barberis and Thaler, 2003), (Kahneman and Tversky, 1979). We do not pursue complexities within the field of the individual investor and their preferences and maintain the rationality assumption.

Assuming strong market efficiency will consequently eliminate long-term arbitrage opportunities. This requires all market participants to have full market information and be able to exploit arbitrage opportunities, which drags the market into equilibrium (Wilmott, 2007). For this to be possible, we must also assume complete markets in the sense that all risks can be perfectly hedged with a replication trading strategy (Munk, 2011). Thus, derivatives are assumed to be purely redundant market instruments of underlying assets and an assumed riskless asset. As no arbitrage can exist the perfect replication trading strategy of the derivative must have the same value as the derivative itself. Otherwise, arbitrage opportunities would mean that it is possible to earn a

[9]See Section 1.3.1.

- certain non-negative return at a negative initial costs or
- possible non-negative profit with no initial costs

by making an opposite position in the perfect replication strategy and exploit possible over- or undervaluation of the derivative (Glasserman, 2003). The no-arbitrage assumption does not restrict the derivative instruments to conform to the law of one price. When accounting for funding costs, which differs among financial intermediaries, this leads to different prices of identical derivative instruments depending on the creditworthiness of the issuing intermediary (Kenyon and Stamm, 2012). Furthermore, different choices of regulatory framework for required capital results in unique capital costs for derivative contracts depending on the issuing financial institution (Ruiz, 2015). However, this still obeys the rule of no arbitrage, as intermediaries in principle cannot charge much more for the derivative than their costs of the replication strategy (Glasserman, 2003). Event studies like (MacKinlay, 1997) is an example of research that falsifying even the medium efficiency of the market. Although, this is not taken into further consideration in this thesis. However, Section 4.1.1 describes how xVA challenge the no-arbitrage opportunity.

The perfect capital market assumption allows investors to borrow at the risk free rate and trade without incurring any transaction costs or be subject to any taxes (Hull, 2012). Tax issues are ignored in this thesis, though warehousing credit risk does result in profits and losses, which are subject to taxes and hence create the Tax Value Adjustment (TVA) (Kenyon and Green, 2015b). Acknowledging FVA, we cannot allow borrowing at the risk free rate. However, this assumption does not affect the risk-neutral pricing principle of the swap contracts when FVA is estimated separately. We also relax the assumptions of no transaction costs, as significant transaction costs are indeed related to collateralization and potentially close-out issues of swap contracts (Brigo et al., 2013).

The Martingale approach assumes that a risk neutral martingale measure can be used as well as the actual drift of an asset to discount the expected payoff of a derivative under the risk neutral probability measure[10], resulting in the fair derivative value. This is the case, as the PDE[11] is constructed to be independent of risk preferences and expected asset return premium (Hull, 2012). The choice of martingale measure or numeraire allows for a time varying discount rate (Glasserman, 2003),

$$r(\tau) = e^{\int_t^T r(u)du}$$

Hence, assuming that the OIS curve is a decent proxy for the risk free rate[12], it can be used to discount swap cash flows[13]. This thesis uses the Martingale approach, which with the numeraire above provides the fair value of the swap contract

$$V_t = E_t^Q \left[r(\tau) \cdot V_T \right]$$

where V_T is the expected payoff at maturity under the risk neutral probability measure, E_t^Q,

[10]See Appendix C.1 for a presentation of the risk-neutral probability measure.
[11]See Appendix C.6 for construction of the BS partial differential equation.
[12]Although, the OIS rate is not completely risk free, as it is still an unsecured lending rate.
[13]See Appendix A.3 for Bootstrapping of the discount curve.

also known as the $\mathbb{Q}$-Martingale (Munk, 2011).

When pricing single interest rate payoffs at time T, it is convenient to use the discount factor from t to T as the martingale measure. A forward rate under this measure has the Martingale property[14]. Hence, this is the T-Forward Martingale or the $\mathbb{Q}^T$-Martingale. This differs from the regular $\mathbb{Q}$-Martingale with the following adjustment

$$dz_t^{\mathbb{Q}^T} = dz_t^{\mathbb{Q}} - \beta_t^T dt$$

Where β_t^T is the price volatility from time t to T (Munk, 2011). Hence, the uncertainty element is not included in this measure as it takes the forward expectations.

Day count conventions might differ for different instruments, which slightly influences the interests accrued. For simplicity this thesis disregards day count issues and estimate interests in fractions of years. Date data are converted assuming 360 days a year ignoring leap years. The amount of trading days varies in different years and on different exchanges. This thesis assumes 260 trading days a year as (Ruiz, 2015).

1.3.5 Limitations

The main goal of this thesis is to introduce additions to traditional derivative pricing. The multi-curve pricing approach and inclusion of xVA in pricing makes it possible to extend the model in unlimited possible ways for different products and different assumptions. This thesis applies the most relevant methods to a case, which takes the viewpoint of a financial institution pricing a single contract[15] with corporate customers and focus solely on plain vanilla swaps. This aims to give a well-structured insight into dealing with xVAs in pricing and keep a tractable model. The results should be seen as indicative for the different risks, but not as generalizable for more complex products or all counterparties.

The simulation model exemplifies pricing of IRS and CCS contracts with maturity up to 10 years with two different floating tenor rates[16]. The swap contracts can include either unilateral or bilateral collateralization. The model aims to present the methods and challenges of including these additional risks, which in reality extends to deal with myriad of derivatives with different features, underlying instruments and maturities jointly rather than isolated. Hence, it is important to notice, that the xVA framework provided in this thesis focus only on a per trade basis. In reality, this should be seen as the marginal effect dependent on the existing trading book. The marginal effect of xVAs from each trade will depend on the existing exposure of the overall derivative portfolio, which might reverse the isolated xVA effect of a single derivative. This creates a quantitative challenge far beyond the scope of this thesis. Therefore, the focus is on describing factors affecting the different xVA terms and their relative contribution to the total xVA charge.

The xVA charge is the sum of different components. New components have been introduced gradually, and both practitioners and academics still debate on the appropriateness and market

[14]See Appendix C.
[15]Ignoring netting effects present when extending to a portfolio view.
[16]3M EURIBOR and 6M EURIBOR.

standard for many of these. This thesis includes CVA, MVA, and KVA, while CollVA and HVA contributes to FVA also included. Components like TVA[17], LVA[18] and DVA[19] are excluded. DVA is the most crucial of those, but ignored by assuming the financial institution issuing the swap contract to be relative default free compared to their counterparty[20]. Besides DVA has some very undesirable features (Brigo et al., 2013), which makes it quite controversial[21].

This thesis assumes use of the Standardized Approach described by Basel II (Choudhry et al., 2012) for counterparty credit risk while having approval by regulatory authorities to use the Advanced Approach for market risk.

Calibration of short-term interest rate models is based on the current term structure and provides interest rate dynamics for simulation purpose. Similarly default intensity is calibrated on index CDS series. Although, taking the current term structure into account, and deriving the forward rates from this, there are more sophisticated models available including time-inhomogeneity, additional factors or recalibration (Brigo and Mercurio, 2007), (Hull, 2012), (Wilmott, 2007). However, using such models would increase complexity and computational time moving away from the main focus of this thesis being the challenge of xVA pricing.

[17]Tax Valuation Adjustment.
[18]See section 4.3.1.
[19]See Apendix E.
[20]Even though the presence of FVA based on funding costs above the risk free rate illustrates the fact that the financial institution is not actually default free.
[21]See Appendix E for a short presentation of DVA and a discussion of the controversy about DVA.

Part II

Risk Factor Modelling

2.1 Post-crisis Lessons for Interest Derivatives

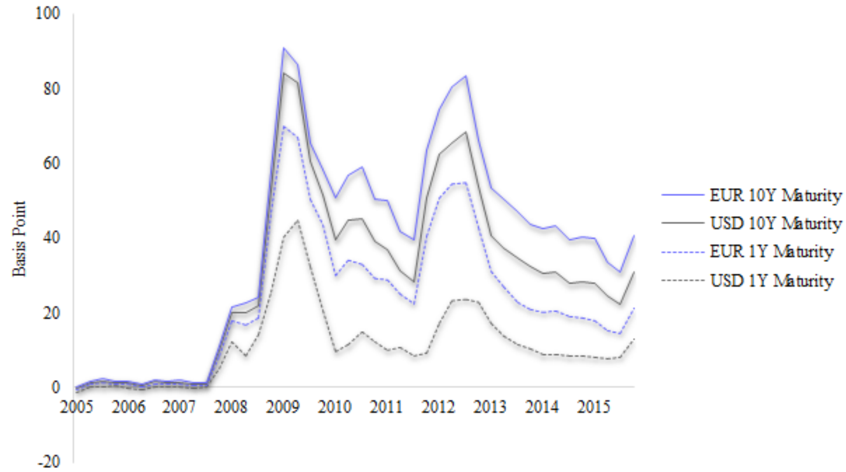
Since the market of interest rate derivatives started back in the 1970s, the market has evolved rapidly, but the pricing framework remained rather simplistic[22] for e.g. IRS contracts. The idea was to construct an interest rate curve or calibrate a model to determine the curve from market prices. This could give the appropriate interest rates needed to forward all floating cash flows of a swap contract. The same interest rates where used to discount all cash flows to find the present value of the swap (Linderstrøm and Rasmussen, 2011). The underlying assumption was that tenor specific reference rate like the 3-month LIBOR were all risk free. Thus, making the funding of future cash flows equal to the forward rates (Tuckman and Serrat, 2012). Hence, meaning no basis spread between different tenors. This made it possible to price single-currency swaps with just one interest rate curve (Eriksson, 2012). The main concern debated within this pricing framework was limited to the bootstrapping and fitting of the curve[23] (Eriksson, 2012).

In advance of the financial crisis, OTC-derivative prices relied solely on the present value of future cash flows, ignoring which parties entered the contract. Counterparty risk was a hedging task for the risk management desk, which assumed a perfect and simple hedging strategy to be available, so that counterparty risk was de facto eliminated (Brigo et al., 2013). Assuming a perfect hedge, such would provide all funding of collateral required to be posted against a trade with a negative market value, so that collateralization would be costless. The absence of any counterparty risk and costs of collateralization meant that funding costs equaled the risk free rate for all financial institutions (Kenyon and Stamm, 2012). This corresponded to the theory of using the same curve for funding and discounting mentioned above. Hence, neither of these factors were considered to effect on the derivative price.

The financial crisis of 2008 led to market participants being much more concerned about credit and liquidity risks (Eriksson, 2012). The default and bailout of banks made the market participants realize that the assumption of interbank rates being risk free did not hold and that the liquidity in the money market could in fact dry out. This meant that investing in e.g. the 3-Month EURIBOR rather than the overnight rate increased the risk that the counterparty could default before the next payment, which would yield a loss for uncollateralized lending (Kenyon and Stamm, 2012), (Hull and White, 2012). Hence, the basis swap spread[24] between the OIS and the different tenors increased drastically by the end of 2007 and 2008. Even though the basis spread stabilized at lower level in the following years, it has not ever since 2007 been negligible for even the most liquid tenor reference rates, as can be seen in figure 2.1.

[22] See Appendix A.4.
[23] See Appendix A.3.
[24] See Appendix A.2.

Figure 2.1: 3MV6M Basis Spread



Source: Author's own creation

With a risk premium occurring on longer tenors due to increased default risk of counterparties, the forwarding of the floating payments of interest rate derivatives (like the floating leg of an IRS) would be different depending on the tenor. E.g. the forward rate of a 3-month EURIBOR should be bootstrapped from the spot rates of the 3-Month EURIBOR, as 6-Month EURIBOR forward rates would include a premium from fixing the payment for 6 months rather than 3 months at a time (Kenyon and Stamm, 2012). So in order to forward the floating cash flows of a swap, it is necessary to derive a curve specific to that tenor risk premium and discount with the risk free curve to find the present value. This resulted in a multi-curve approach for pricing single currency swaps and other interest rate derivatives.

The assumption of risk free funding being possible for financial institutions collapsed along with financial markets in light of the credit events of banks like Lehman Brothers and Bear Sterns (Linderstrøm and Rasmussen, 2011). This led to breakdown of the risk free assumption on the interbank rate. Financial institutions did in fact have risk of defaulting, which affected their funding costs in the market. This led to higher expenses related to entering swap contracts - collateralized or not, as collateral itself also requires funding (Brigo et al., 2013). These issues fostered the FVA relating to the unsecured funding of the cash flows and funding of collateral and MVA accounting for funding of initial margin required in many collateral agreements for additional security (Gregory, 2015a). A risk free hedge is not possible as used to be assumed, and these costs must be included in the derivative price.

Counterparty credit risk of OTC derivatives became apparent in the financial crisis of 2008. The risk of counterparties defaulting on derivative contracts with a positive market value would mean a loss for the other party. Besides losses from actual defaults, the value loss from increasing credit risk alone created paper losses as the value of contracts had to be written down. The losses of increasing credit risk alone accounted for twice the actual losses from defaults (BCBS, 2011). To adjust the value of a derivative for this risk, CVA was introduced. The CVA can be mitigated by the counterparty posting collateral, but not eliminated as perfect collateralization in continuous time is necessary to remove CVA completely, but then again not possible in reality

(Brigo et al., 2013).

Basel III added an additional capital charge to account for the CVA risk of credit derivatives along with already existing capital charges. The amount of capital required became significant and KVA was introduced in order to ensure that their target return on capital could be honored by the trading desk (Gregory, 2015a).

Considering the lessons learned from the financial crisis of 2008, the traditional pricing approach for OTC interest rate derivatives requires an extension to be priced using more than a single yield curve and adjusted for the different risk factors related to costs counterparty risk, funding costs, and capital costs.

2.2 Multi-curve Pricing Approach

Section 2.1 evaluates how the post crisis lack of liquidity and increase in credit risk led to a tenor spread premium. This was apparent from the increase in the basis swap spread for longer tenors. As a result of this, the forward curves had different rates depending on the tenor. The multi-curve approach use this tenor specific curve for forwarding, while using a proxy for the risk free rate as discount curve.

First subsection adds some theoretical points to construction of the discount curve. Next follows the derivation of single forward curves under the no-arbitrage condition, which later extends to a multi-curve framework. This includes evidence that the basis swap spread accounts for the difference between the discount curve and the forward curve. Last subsection evaluates multi-curve pricing in a regime with two currencies necessary for pricing of a fixed-fixed CCS contract.

2.2.1 Discount Curve

The discount curve is the only component in the single-curve approach and still has huge importance in the multi-curve pricing approach, as the same curve applies for discounting cash flows for all different tenors. Different from the derivation of forward curves, there is no market consensus on the choice of discount curve. As discussed in Appendix A.3 discount curves can be bootstrapped from different liquid instruments. This can give inconsistency between different market participants depending on their choice of instruments for bootstrapping (Bianchetti, 2008). Alternatively, the OIS curve is widely used as a proxy for the risk free rate. This is justified with the OIS having only a 1-day tenor and with the calculation of the rate being based upon contributions only by solid market participants (Tuckman and Serrat, 2012). Clearinghouses like LCH Clearnet Group Ltd use the OIS curve as the risk free proxy for discounting (Kenyon and Stamm, 2012). The OIS base itself on different reference rates. The USD OIS is the Fed Funds rate while the EUR OIS is the EONIA rate (Tuckman and Serrat, 2012).

Trying to smooth the different maturities of instruments into a curve, different interpolation methods can be used to fit the term structure. Instead of interpolating complete curves, this thesis uses interest rate models calibrated based on tenor relevant instruments[25]. Construction

[25]Though Nelson-Siegel Parameterization is used to provide holes in the term structure needed to calibrate

of curves with either interpolation or simulation makes the pricing of interest rate derivatives an approach relative to the quoted market prices market prices (Ametrano and Bianchetti, 2009). This means the no-arbitrage assumption cannot be consistent at all points on the curve, though the market microstructure and transaction costs make these small arbitrage opportunities impossible to exploid in practice (Bianchetti, 2008).

2.2.2 Single Forward Curve

This subsection uses the no-arbitrage assumption from Section 1.3.4 to derive the discount and forward curve, C^d_{OIS} and C^f_{OIS}, used in the single-curve pricing framework presented by (Bianchetti, 2008) and (Ametrano and Bianchetti, 2009). The notation ignores subscripts of the forward and discount rates, as there is only a single curve[26] within this framework. The no-arbitrage condition implies that

$$D(t, T_2) = D(t, T_1) \cdot D(t; T_1, T_2)$$

with $t \leq T_1 \leq T_2$. Thus meaning, that the discount rate from t to T_2 must equal the product of the discount rate from t to T_1 and the time t forward discount rate from time T_1 to T_2. Hence, the forward discount rate is given by

$$D(t; T_1, T_2) = \frac{D(t, T_2)}{D(t, T_1)} = \frac{1}{1 + F(t; T_1, T_2) \cdot \tau} \tag{2.1}$$

where $F(t; T_1, T_2) = \frac{1}{\tau} \cdot \left((\frac{1}{D(t;T_1,T_2)} - 1 \right)$ and $\tau = T_2 - T_1$. Rearranging for $F(t; T_1, T_2)$ gives

$$F(t; T_1, T_2) = \frac{D(t, T_1) - D(t, T_2)}{\tau - D(t, T_2)}$$

Moving on to continuous time the discount rates can be written at time t as an integral of instantaneous forward rates, f, with the reference rate tenor starting at t_0:

$$D(t_0, T) = e^{-\int_{t_0}^{T} f(t_0, u) du}$$

Here f is given as

$$f(t_0, T) = -\frac{\partial}{\partial t} log D(t_0, t) \ |_{t=T}$$

thus the forward rate curve C^f is given with the function of f for different maturities up to $T \geq t_0$.

$$C^f = \{T \rightarrow f(t_0, T)\}$$

the interest rate model.

[26]The OIS curve.

A continuous spot discount rate d is instead

$$d(t_0, T) = -\frac{1}{\tau} e^{D(t_0,t)}$$

with $\tau = T - t_0$. Hence, in the same way the spot discount rate for different maturities up to $T \geq t_0$ is

$$C^d = \{T \to d(t_0, T)\}$$

Note that even though pricing using a single curve, the forwarding of cash flows uses the forward rate curve, while the present value discount with the spot discount rate curve or accumulated sum of forward discount rate curves to the time of the cash flow. Yet, taking the basis spread into account, the single curve approach would allow for arbitrage opportunities (Tuckman and Serrat, 2012).

2.2.3 Multiple Forward Curves

The multi-curve approach aims to incorporate the effect of the basis swap spread, which since the crisis has been significant. The approach accepted a segmented market for the different tenors, meaning that each forward curve would follow different dynamics for the interest rate process (Ametrano and Bianchetti, 2009). This second extension is covered in Section 2.3.1.

To account for the different tenors spreads, it is necessary to derive forward rate curves for each different tenor. The derivation of these curves works just as in Section 2.2.2. Hence, subscripts cannot be ignored, because in

$$C_x^f = \{T \to f_x(t_0, T)\}$$

with

$$f_x(t_0, T) = -\frac{\partial}{\partial t} log D_x(t_0, t) \ |_{t=T}$$

where $x = \{d, f_1, f_2, \ldots, f_n\}$, f_n are different floating tenors like 3-month, 6-month, 12-months etc. Thus, when determining the future cash flows of the floating rate with a certain tenor under the no-arbitrage condition, implying that forward rates can be extracted from the forward curve with the same tenor

$$F_x(t; T_1, T_2) = \frac{D_x(t, T_1) - D_x(t, T_2)}{\tau - D_x(t, T_2)}$$

with $t \leq T_1 \leq T_2$ and $\tau = T_2 - T_1$ (Bianchetti, 2008). Hence, the expected cash flow under the $\mathbb{Q}_d^{T_i}$-Martingale, where i is the ith floating cash flow

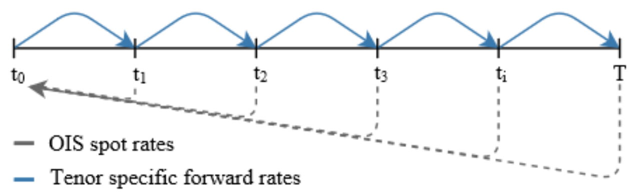
$$c_i^{fl} = c^{fl}(t, T_i) = E_t^{\mathbb{Q}_d^{T_i}}[H(F_x)]$$

with H being the notional value and d is denoting that the numeraire for this $\mathbb{Q}_d^{T_i}$-Martingale measure is the discount curve $D_d(t, T_i)$. The discount curve must still be unique at all tenors, as the no-arbitrage condition is violated under the risk neutral pricing approach if this is not the case (Bianchetti, 2008). Thus, the single discount curve derived in Section 2.2.2 is used to assign the appropriate discount rates to the different cash flows (both legs of the swap). From Appendix A.4 we know that the swap value is the sum of all discounted cash flows

$$V_j = \sum_{i=j}^{m} \tau \cdot D_d\left(T_j, T_i\right) \cdot H \cdot \left(r^{fix} - E_t^{\mathbb{Q}_d^{T_i}-}\left[(F_x)\right]\right) \tag{2.2}$$

with m total cash flows and r^{fix} interest rate on the fixed leg of the swap. Figure 2.2 gives an intuitive graphical illustration of the multi-curve pricing approach.

Figure 2.2: Multiple Approach



Source: Author's own creation

2.2.4 Relation Between Different Tenors

The relationship between the different tenor curves under condition of no-arbitrage assumes independence of interest rate dynamics. The relationship in Section 2.2.2 does not hold when the forward curve and discount curve have different tenor due to the forward basis spread making the two curves different. This is the case, when acknowledging that different tenor curves are not identical. This is adjustable by including the spread between the discount curve, d, and forward curve, f, with a longer tenor than the OIS (Bianchetti, 2008). Deriving the forward discount rates from equation 2.1 with the basis swap spread taken into account gives

$$D_f\left(t; T_1, T_2\right) = \frac{1}{1 + \left[F_d\left(t; T_1, T_2\right) + BA_{f\,d}\left(t; T_1, T_2\right)\right] \cdot \tau} \tag{2.3}$$

with $BA_{f\,d}\left(t; T_1, T_2\right)$ being the basis forward spread between the discount and the forward tenor at that certain point in maturity. This basis forward equals[27]

$$BA_{f\,d}\left(t; T_1, T_2\right) = F_f\left(t; T_1, T_2\right) + F_d\left(t; T_1, T_2\right) \tag{2.4}$$

(Tuckman and Serrat, 2012) provides evidence that using the forward basis swap spreads between two tenors, it is possible to use the spread and one tenor curve to derive the other. Hence, for different maturities the forward curve can be recursively calculated from the discount curve and the forward curve basis spread (Bianchetti, 2008)

$$D_f\left(t, T_i\right) = \frac{D_d\left(t, T_i\right)}{D_d\left(t, T_{i-1}\right) - D_d\left(t, T_i\right) \cdot BA_{f\,d}\left(t; T_{i-1}, T_i\right) \cdot \tau} \cdot D_f\left(t, T_{i-1}\right)$$

[27]This is a reduced version of (Bianchetti, 2008) as a result of disregarding day count conventions.

and vice versa for the discount curve

$$D_d(t,T_i) = \frac{D_f(t,T_i)}{D_f(t,T_{i-1}) - D_f(t,T_i)\cdot BA_{f\,d}(t;T_{i-1},T_i)\cdot\tau}\cdot D_d(t,T_{i-1})$$

As mentioned the derivation of the relationship between basis swap spreads for different tenors and the different forward curves is just a statistical measure of the difference between the discount curves at some certain maturities, but does not account for dependence of the dynamics of the underlying interest rates (Ametrano and Bianchetti, 2009). Using the current forward rate is just an approximation. Theoretical correct pricing would require the use of expectations under $\mathbb{Q}_d^{T_i}$-Martingale measure and involves dynamic properties of the interest rate markets for the different curves, meaning stochastic models for the interest rates (Bianchetti, 2008). The dynamics can be different so that calibrating interest rate models to market data on different tenors yields slightly different parameters and stresses the segmented market of tenors.

2.2.5 Cross-Currency Basis Spread

Previous subsection, only considered the multi-curve framework in a single currency. However, the relationship between similar tenor curves in two different currencies is similar to the relation between different tenor curves in a single currency. (Brigo and Mercurio, 2007) shows that in the risk neutral pricing framework, changing the measurement from foreign currency $\mathbb{Q}^f$ to domestic currency $\mathbb{Q}^d$ is equivalent to change the numeraire from $r^f(t)$ to $\frac{r(t)}{\mathbb{Q}^d}$, so that the value is equivalent at time t. (Brigo and Mercurio, 2007) also proves, that this relation holds, when moving to the $\mathbb{Q}^T$-forward measure. This means that cash flows in foreign currency are possible to discount with the suitable spot discount curve of that currency and convert at the spot exchange rate $S(t)$

$$D^{f\to d}(t,T_i) = S(t)\cdot D_d^f(t,T_i)$$

Alternatively, one can derive the forward exchange rates at all cash flows to convert all foreign cash flows to domestic currency and then discount using the suitable domestic spot curve

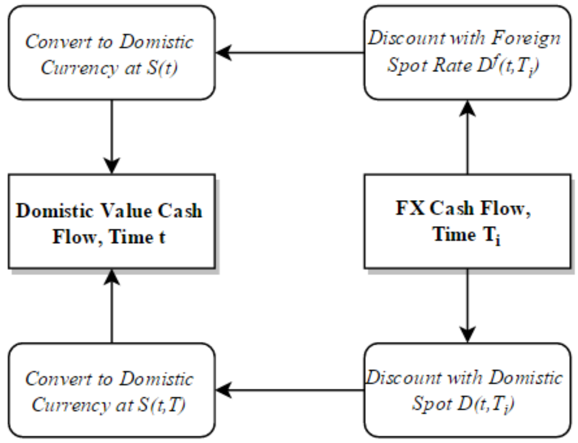
$$D^{f\to d}(t,T_i) = S(t,T_i)\cdot D_d(t,T_i)$$

Figure 2.3 illustrates this procedure as in (Bianchetti, 2008), indicating that these two methods must be equivalent

$$S(t)\cdot D_d^f(t,T_i) = S(t,T_i)\cdot D_d(t,T_i)$$

to fulfill the no-arbitrage condition.

Figure 2.3: Measurement with Two Currencies



Source: Author's own creation inspired by (Bianchetti, 2008)

The spread between two currency floating rates with identical tenors, has the same relations as expressed with basis swap spreads between two different tenors with a single currency. This is characterized as the cross-currency basis spread (Brigo et al., 2013).

For explicit pricing formulas, the change in numeraire of currency requires a quanto-adjustment (Bianchetti, 2008). However, this will not be covered in this thesis, as short-rate models are used for numerical pricing, which as mentioned by (Hull, 2012) eliminates the need for quanto-adjustment.

(Brigo et al., 2013) rearranges equation 2.1 to determine the forward exchange rates

$$S\left(t, T_i\right) = \frac{S\left(t\right) \cdot D_d^f\left(t, T_i\right)}{D_d\left(t, T_i\right)} \tag{2.5}$$

Modelling stochastic interest rates in both currencies, the forward exchange rate can be determined as a function in equation 2.5 to ensure no-arbitrage. This relationship is known as the interest rate parity (Hull, 2012). However, this ignores the cross-currency basis spread above. Hence, such a currency basis depends on issues related to collateralization agreement on the CCS market quotes, which mean this effect will be embedded in our choice of collateral agreement specified in Section 4.2.1 (Brigo et al., 2013).

2.3 Stochastic Models

This section of the thesis presents the dynamic models used for the underlying risk factors in the simulation price model. This is respectively a model to describe the interest rate dynamics and a model to simulate credit risk of counterparties. This thesis uses

- one-factor CIR model[28] with constant parameters calibrated to swap rates for the dynamics of interest rates.

[28] Cox-Ingersoll-Ross model presented in 1985 (Hull, 2012).

- a JCIR model[29] with constant parameters calibrated to index CDS credit rates as a stochastic intensity model for the default probability of the counterparty.

Though these models are quite simple, they provide suitable dynamics for interest rates and survival probability and remain tractable when computing the xVA terms in the simulation of the IRS and CCS valuation. The following subsection suggests a simple way to include correlation between the models.

For the purpose of CCS pricing, this thesis assumes independence of forward exchange rates and credit risk. This simplification might over- or underestimate CVA, but is a fair approximation for the EUR/USD currency cross, but not as good for emerging market currencies, where the exchange rate might be critical to a counterparty operating in countries with that currency (Brigo et al., 2013). Including the correlation between exchange rate and credit risk is also a very complex matter, and thus left out. It makes less sense to include this, when assuming a deterministic model for exchange rates as described in Section 2.2.5.

The final part of this section provides insights into the art of model calibration and presents formulas and discuss issues necessary for calibration of CIR models for interest rate dynamics and JCIR model for default intensities. Model calibration to market data provides the dynamic properties of different tenor segments of interest rates, which as explained in Section 2.2.4 results in a theoretical correct price[30].

2.3.1 Interest Rate Model

This subsection evaluates the theoretical framework of the calibrated one-factor CIR short-rate model, which describes the dynamics of the interest rates for different tenors. Appendix D evaluates general properties of interest rate models, while Appendix D.2 focus on the general properties of a square-root process like the CIR model.

The one-factor CIR model follows a mean reverting process for dr_t

$$dr_t = \kappa \left[\theta - r_{t-1}\right] dt + \beta \sqrt{r_{t-1}} dz_t \tag{2.6}$$

where the parameters are held constant. The properties of the different parameters are as described in D.2. With fixed parameters the future short rate is non-central χ^2-distributed under the risk-neutral measure (Glasserman, 2003).

The risk-neutral interest rate curve shape derived from a CIR process depends on the parameters. Assuming $\kappa > 0$, the curve is decreasing for $r \geq \theta$, increasing for $0 \leq r \leq \frac{\theta}{\sqrt{\kappa^2+2\beta^2}}$ and is humped with an increase followed by a decrease for $\theta < r < \frac{\theta}{\sqrt{\kappa^2+2\beta^2}}$. If $\kappa \leq 0$, the curve is also increasing for $0 \leq r \leq \frac{\theta}{\sqrt{\kappa^2+2\beta^2}}$ and instead humped for $r > \frac{\theta}{\sqrt{\kappa^2+2\beta^2}}$ (Munk, 2011). However, in this case the yield curve cannot be decreasing. One of the weaknesses of one-factor models like the CIR model is that they cannot generate curves, which first decrease for short maturities, but increases for larger maturities, as is occasionally observable in reality (Hull, 2012). Furthermore, one-factor models allow only one hump changing the direction or steepness

[29] Jump-Diffusion CIR Model (Brigo and Mercurio, 2007).

[30] Assuming the chosen short-rate model is the true model.

of the curve. In reality, there might be different increasing and decreasing relationships along the different maturities of the curve, which one-factor models cannot capture. Another problem with one-factor models is that shifts in the curves from interest rates changes are parallel, which implies the unrealistic assumption of perfect correlation between different maturities. Empirical studies show, that a perfect correlation is far from realistic. E.g. parallel shifts only account for around 10% of change in bond yields, while non-parallel shifts have twice that effect (Phoa and Shearer, 1997). Short-term and long-term rates also might not even move in the same direction as a change in the short interest rate occurs (Munk, 2011).

It is possible to mitigate these errors by extending the model with further factors like the Longstaff-Schwarz model extension of the CIR model (Munk, 2011) or allow for time-varying parameters when calibrating the model like the CIR++ model (Brigo et al., 2013). Extending to additional factors would provide further possible shapes of the interest rate curve. An extension to a time-inhomogeneous calibrated model will improve the precision, but still lack the ability to capture curve movement of short-term and long-term maturity in different directions as response to a change in the short-term interest rate. Using one-factor models to value financial instruments, the calibration must be on the same type of instruments in order to give a valid price estimate. Otherwise, one-factor models are inferior to two-factor models in terms of pricing. For hedging purpose, calibrated one-factor models are not even good approximations of the two-factor models, which become superior (Munk, 2011), (Hull, 2012). Extension to a two-factor CIR model will complicate the calibration and interpretation of the model. Hence, to keep tractability when including xVAs, this thesis sticks to the rather simple calibrated one-factor CIR model.

The CIR models have a shortcoming to other short-rate models when it comes to calibrating a time-inhomogeneous model for forward rate curves. To avoid $\theta(t)$ ever being negative, the following restriction is required for the relationship between forward rates $f_x(T)$, initial interest rate r_0 and the derivative of non-centrality parameter in the χ^2-distribution $b'(T)$[31]

$$f_x(T) \geq r_0 \cdot b'(T)$$

Thus, even the CIR++ model cannot be calibrated to all shapes of term structures (Munk, 2011).

For this reason and for simplicity, this thesis settles for a time-homogeneous calibrated CIR model. This means, that it does not ensure a perfect no-arbitrage model with the term structure as input, but rather produces the term structure as output (Hull, 2012). It is important to note as mentioned in Appendix D.2 that the CIR model as a square-root model has an advantage over Ohrnstein-Uhlenbeck processes[32]. This is due to the fact, that interest rates cannot become negative, which is generally a good assumption when using nominal interest rates[33]. Even though the CIR model was originally for real interest rate dynamics, it is in practice also used for nominal rates (Munk, 2011).

[31]See formula for $b(\tau)$ in Appendix D.2.
[32]See Appendix D.1.
[33]Though as proved by current EUR market rates not realistic.

2.3.2 Counterparty Credit Model

This subsection presents a simple way to model credit risk as a stochastic default intensity model. This provides an extension of the CIR model with a jump diffusion, which makes it more suitable to measure default probabilities of counterparties.

As mentioned in Appendix D.2 a square-root process like the CIR model must obey the restriction $\beta^2 \leq 2\varphi$ in order to exclusively produce positive values. This restriction is less problematic for modelling interest rates. However, for modelling stochastic default intensity, the volatility might be much higher than allowed by the restriction. This will possibly violate the restriction. Adding a jump diffusion to the stochastic process keeps the possible higher variation in default intensity while avoiding violation of the restriction (Brigo et al., 2013). This transforms equation 2.6 into a jump diffusion model

$$d\lambda_t = \kappa \left[\theta - \lambda_{t-1}\right] dt + \beta \sqrt{\lambda_{t-1}} dz_t + dJ_t^{\alpha,\gamma} \tag{2.7}$$

The model is now describing the default intensity λ_t. The additional feature added is the jump $dJ_t^{\alpha,\gamma}$. (Brigo and Mercurio, 2007), (Brigo and El-Bachir, 2010), and (Brigo et al., 2013) provides the following notation for the jump at time t in discrete form

$$J_t^{\alpha,\gamma} = \sum_{i=1}^{M_t} Y_i$$

Here Y_i follows an exponential distribution $Y_i \sim f(x_i, \gamma)$ with x_i an i.i.d. random generated positive number (Brigo and Mercurio, 2007) with $\gamma > 0$ being a rate parameter determining the size of each jump. Larger γ increases the size of all jumps (Brigo et al., 2013), (Brigo and El-Bachir, 2010). The probability of each jump follows a Poisson distribution (Hull, 2012)

$$P\left(Y_i \mid \alpha\right) = \frac{e^{-\alpha}\alpha^i}{i!}$$

which is the probability for a certain amount of events for a given period of time. The amount of events at each time interval, is independent of other periods, but has the same probability for equally large time intervals, thus making it depend on the size of the time interval (Keller, 2009). $J_t^{\alpha,\gamma}$ equals the cumulative Poisson distribution

$$J_t^{\alpha,\gamma} = \sum_{i=1}^{M_t} Y_i = \sum_{i=1}^{n} Y_i \frac{e^{-\alpha}\alpha^i}{i!}$$

where $\alpha > 0$ is the arrival rate for the frequency of jumps and n is the number of jumps (Brigo and El-Bachir, 2010). Thus, a larger α increases the probability of the Y_i^{th} jump. M_t denotes a Poisson distribution variable with arrival rate α.

Due to the nature of the Poisson distribution described above, the jump process is uncorrelated with the current default density. Thus adding an unpredictable element to the probability of default, which seems more aligned with reality. This feature is not available for structured models, which is an alternative way to model default rather than intensity models (Brigo et al., 2013). Jump diffusions make the intensity model consistent with empirical data (Brigo and El-Bachir, 2010).

Note that as the jumps are only positive, the long-term mean changes to $\theta = \widetilde{\theta} + \frac{\alpha\gamma}{\kappa}$. Hence, including the drift effect of the jump to the drift term of the process changes equation 2.7 to

$$d\lambda_t = \kappa \left[\widetilde{\theta} - \lambda_{t-1}\right] dt + \beta \sqrt{\lambda_{t-1}} dz_t + dJ_t^{\alpha,\gamma} - \alpha\gamma\, dt$$

with $dJ_t^{\alpha,\gamma}$ now having a zero mean (Brigo and El-Bachir, 2010).

The JCIR model does as other intensity models not provide the precise time of default, but instead the evolvement of default probability over time. This is acceptable to calculate CVA on the exposure to the counterparty (Gregory, 2015a).

2.4 Correlation of Credit Risk and Interest Rate

The CIR model for interest rate dynamics in Section 2.3.1 and the JCIR model for default intensity Section 2.3.2 have so far not included any dependency with one another. However, (Brigo et al., 2013) shows that interest rate and default intensity correlate and that this has a significant impact on the counterparty risk. This subsection provides a simple framework to include correlation between the two stochastic models and discusses the limitations of this method.

The nature of stochastic uncertainty for the two processes both arise from Gaussian variables[34]. Moreover, the jump diffusion is independent of time. Thus, linear dependency modelling through Pearson correlation measure of the Brownian motions creates the decent dependency between the interest rate model and default intensity model (Munk, 2011).

However, the jump being exponential would not allow for this simple linear correlation, if two jump processes should depend on one another (e.g. when taking into account correlation between default intensity of more counterparties). So when valuating counterparty risk in a joint framework rather than an isolated case, this would require more efficient methods to model dependency structures like copula functions (Brigo et al., 2013), (Glasserman, 2003). In such cases, linear dependence structures through the Gaussian variables become too weak in case of one counterparty defaulting (Glasserman, 2003). Shared jumps for dependent default intensity processes could improve the linear correlation, but would still not be efficient (Brigo et al., 2013). Hence, heavy dependence in a crisis cannot be appropriate captured by the normal distribution, which would require a rather heavy tailed distribution (Munk, 2011).

As mentioned above, dependency for the JCIR and the CIR model is applicable through the Brownian motion independent of the rest of the models (Glasserman, 2003). Each Brownian motion is a Gaussian variable i.e. that it follows a standard normal distribution

$$z_i \sim N(0,1) \tag{2.8}$$

[34]The Brownian motions in diffusion process.

Equation 2.8 in matrix form for multiple independent Brownian motions becomes

$$Z \sim N(0, I) \tag{2.9}$$

where Z is the vector of i independent Brownian motions. Thus, the covariance matrix is an identity matrix. Dependent Brownian motions will change equation 2.9 to

$$Z \sim N\left(0, \sum\right) \tag{2.10}$$

where $\sum$ is the covariance matrix. In order to move from the independent case in equation 2.9 to the dependent case in equation 2.10 it is possible to use the linear transformation property[35] for equation 2.10

$$AZ \sim N\left(0, A \sum A^T\right) \tag{2.11}$$

where A is a matrix, satisfying the condition $\sum = AA^T$ (Glasserman, 2003). Equation 2.11 is still following a normal distribution. By knowing AZ and be able to generate Z by equation 2.9 dependence is created through the construction of matrix A alone and thus achieve the result of equation 2.10. In other words, by determining the elements of A from the correlation or covariance matrix, AZ will be a vector with dependent standard normal distributed numbers following the specified correlation matrix (Hull, 2012).

The Cholesky factorization is a way to derive A as a lower triangular matrix, which is convenient as it halves the calculations compared to the covariance matrix. This solution satisfies $\sum = AA^T$, where A^T then must be equal to the upper triangular matrix. We can rewrite the condition in terms of elements in A_{ij}

$$\sum_{ij} = \sum_{k=1}^{j} A_{ik} A_{jk}$$

to be able to solve for each element sequentially (Glasserman, 2003)

$$A_{ij} = \frac{\left(\sum_{ij} - \sum_{k=1}^{j-1} A_{ik} A_{jk}\right)}{A_{jj}} \tag{2.12}$$

As already mentioned, two processes must be correlated in this model framework. With $\sum = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$[36], then using equation 2.12 (Glasserman, 2003)

$$A = \begin{bmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{1-\rho^2\sigma_2} \end{bmatrix}$$

[35]This covariance matrix must be a symmetric $d\ x\ d$ matrix and positive definite meaning that all eigenvalues of elements in the matrix must be positive in order to allow for linear transformation, as using negative eigenvalues to scale the matrix would reverse the direction of elements when multiplying with the eigenvalue matrix. A Covariance matrix is so by construction (Wilmott, 2007).

[36]where $\sigma_{21} = \sigma_{12} = \sigma_1\sigma_2\rho_{12}$. And ρ_{12} is the Pearson correlation measure of the two processes.

And (Hull, 2012)

$$AZ = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{1-\rho^2\sigma_2} \end{bmatrix}$$

(Brigo and Pallavicini, 2008) uses the above approach for correlating stochastic interest rate and default models for pricing IRS contracts. They show that higher correlation between the interest rate model and default intensity makes CVA for the receiver leg of IRS contracts decrease. This supports the idea of wrong way risk or cross-gamma effects, as a counterparty paying fixed rate will lose market value with interest rates dropping and vice versa for a receiver leg, which affects the credit risk of the counterparty due to such losses (Gregory, 2015a). A drop in interest rates often relates to economic recession times, while the interest rate increases in times with recovery. (Duffee, 1998) supports this by showing how corporate bond yield spreads[37] tighten with increasing Treasury bill yield[38]. These issues support the necessity of dependence between interest rate and default probability as the cross-gamma sensitivity of these two underlying risk factors increase the total sensitivity of the xVA terms.

Introducing correlation between the two diffusion models will theoretically introduce dependency between interest rates and default intensity model parameters. This means that calibration of the models must be jointly to account for the dependency. This will increase complexity tremendously. (Brigo and Mercurio, 2007), however, shows that even when calibrating parameters for two stochastic models jointly including high correlation for CDS contracts[39], calibrating the parameters for two stochastic models jointly using a high correlation, parameters does not differ noteworthy from a separated calibration case[40]. Hence, it is then possible to calibrate both models individually based on their most relevant instruments[41] and afterwards set the correlation parameter, ρ, to the desired level based on historical data or market-implied correlation.

2.5 Calibration of Diffusion Models

This subsection presents the process for calibrating the fixed parameters for the time-homogeneous one-factor models described in Section 2.3.1 and Section 2.3.2. First is a discussion on the advantage of calibrating model parameters. Next, a short theoretical discussion on different calibration methods follows, which includes a brief evaluation of the non-linear Generalized Reduced Gradients (GRG) optimization method used in this thesis. The interest model calibration fits swap rates calculated by the model to quoted market rates. Hence, there is a presentation on the derivation of swap rates taken the CIR model as given. The default-intensity model calibration fits index CDS prices to market prices of the iTRAXX Europe Crossover Index. To achieve this,

[37] Indicating the risk premium of default risk compared to the risk free interest rate.

[38] Used as proxy for the risk free rate.

[39] Which are heavily dependent on the parameters of the default intensity.

[40] Differences in value between using joint and separated calibration fall within a small fraction of the bid-ask spread.

[41] Calibration of an interest rate from the same instruments as those optimal for modelling default intensity is not suitable. From Section 2.3.1 we described that calibration of the one-factor interest rate model needed to be from the same instruments as those that the model aims to price to be a suitable model.

the subsection also includes a derivation of a CDS pricing formula taking the JCIR model as given.

2.5.1 The Purpose of Calibration

Calibration is an iterative process to determine the parameters in a stochastic model. These iterations ensure that the fixed parameter are set so that the model implied values fit current quoted market values of such instruments as precise as possible. Thus, meaning minimizing the sum of squared errors between market quoted rate and model implied rates (Keller, 2009)

$$SSE = \sum_{j=1}^{k} \left(x_j - x_j^*\right)^2$$

with x_j the model rate and the mid of the quoted market rate x_j^* for the j^{th} instrument. This makes the term structure of the interest rate model as close as possible to the current term structure of the chosen instruments and thus more suitable as input for pricing these models (Brigo and Mercurio, 2007).

Calibration of stochastic models has the purpose to ensure that the pricing model is closer to reality. This calibration can be to historical prices or to current market prices, the latter assuming the risk-neutral measure, while when also combined with historical prices allows for estimation with time-inhomogeneous parameters. However, even though the risk-neutral measure produces the best indications of current market conditions, it does not acknowledge that the presence of risk premiums causes a systematic bias. Another downside is the potential procyclicality in the stochastic models due to solely relying on current prices for calibration, which might over- or underestimate the underlying volatility in the model. Even though these downsides occur, calibration to market prices is consistent with the underlying risk neutral approach, provides best indications of current market conditions, and is an accepted general market practice (Gregory, 2015a)[42].

The calibrated models only fit well, if the model chosen represents a good fit for the actual fundamental dynamics. As discussed in Section 2.3.1 the CIR model already limit the possible term structure, which in reality can take more forms. Furthermore, negative interest rates are not possible, but present in current market data evaluated in Section 5.2.1. As the actual dynamics have an infinite number of maturities, the calibration based on only a finite number of market quotes will be an approximation. It is necessary to choose a decent amount of market quotes to match the infinite number of different maturities properly (Brigo and Mercurio, 2007). As mentioned in Section 2.3.1, one-factor models only provide a good base for pricing the same instrument as those used to calibrate the model. For this reason, calibration of the CIR models[43] is to market quotes of swap rates of generic IRS contracts. Calibration of the default intensity model uses market quotes of the iTRAXX Europe Crossover Index credit spread to fit the default intensities from the JCIR model. To avoid violating constraints of the models all parameters

[42] Market prices are sometimes used only for calibrating the drift, while market volatility is mixed with historical time-series (Gregory, 2015a).

[43] Different tenors and currencies.

for all dynamic models are restricted to positive values (Munk, 2011), (Brigo et al., 2013).

2.5.2 Non-Linear GRG Optimization

Calibration of optimal model parameters is a non-linear mathematical optimization issue. There are essentially six types of procedures to solve constrained nonlinear optimization programming (NLP) problems.

- Successive Linear Programming
- Successive Quadratic Programming
- Generalized Reduced Gradients
- Penalty or Barrier Functions Methods
- The Augmented Lagrangian functions
- The Methods of Feasible Directions (or Projections)[44]

The three first methods above are have proven to be more efficient than the last three (Pike, 1986).

For this thesis, the GRG method is chosen to solve the NLP problem. This method is included in the Premium Solver Pro in Microsoft Excel, which can handle up to 500 decision variables and 250 constraints (Systems, 2011).

The GRG algorithm is a non-linear extension of the Simplex Method for Linear Programming[45]. It is shown that the implementation is both robust and efficient (Lasdon et al., 1978), (Systems, 2011). As the process iterates from point to point, GRG make adjustments for the variables, so that the constraint remains satisfied, finding the optimal solution.

The basic idea behind GRG is to first make use of Taylor expansions to linearize the NLP problem and then constraint equations to find a feasible optimal local solution for the NLP problem. From this point the reduced gradient algorithm incrementally adjusts parameters until reaching an optimum (Bazaraa et al., 2013), which satisfied the specified conditions (Pike, 1986). As the algorithm at each increment chose the best marginal improvement, it will move towards and optimal solution. However, this might not be the global optimum as choosing the best marginal improvements at each increment might just lead in the direction of a local optimum depending on the starting point of the algorithm. Hence, the only way to ensure, that the optimal solution found is indeed the global optimum and not just a local optimum, the method has to be repeated with different starting values to uncover different optimums and pick the best of those (Systems, 2011). An in depth discussion of the mathematical procedure is beyond the scope of this thesis.

[44]Also called the methods of restricted movement.
[45]See e.g. (Balakrishnan et al., 2013) for evaluation of linear programming.

2.5.3 Calibration of CIR Model on Swap Rates

Calibration of the CIR model sets the parameters that minimize the error of model estimated swap rates relative to market quoted swap rates. This requires a model to estimate swap rates based on the CIR model. Swap rates as derived in Appendix A.6 equals[46]

$$r^*_{t=0} = \frac{1 - D\left(0, T_n\right)}{\tau \cdot \sum_{i=1}^{n} D\left(0, T_i\right)}$$

The discount factors depend on the interest rate and can be seen as a zero-coupon bond with a notional of $D\left(T, T, r\right) = 1$

$$r^*_{t=0} = \frac{1 - D\left(0, T_n, r\right)}{\tau \cdot \sum_{i=1}^{n} D\left(0, T_i, r\right)}$$

Thus, discount factors can be calculated given that the CIR model describes underlying interest rate dynamics. (Munk, 2011) shows that the pricing formula

$$D\left(t, T, r\right) = e^{-a(\tau) - b(\tau) \cdot r} \tag{2.13}$$

where $\tau = T - t$ with restriction

$$\frac{1}{2}\beta^2 \cdot b\left(\tau\right)^2 + \kappa \cdot b\left(\tau\right) + b'\left(\tau\right) - 1 = 0$$
$$a'\left(\tau\right) - \kappa \cdot \theta \cdot b\left(\tau\right) = 0$$

is the solution for the PDE when r follows a CIR model. The terminal condition $D\left(T, T, r\right) = 1$ is as well satisfied at T with $\tau = 0$. Solving the restrictions for $a\left(\tau\right)$ and $b\left(\tau\right)$ equals

$$a\left(\tau\right) = -\frac{2\kappa \cdot \theta}{\beta^2}\left(ln\left(2\omega\right) + \frac{1}{2}\left(\kappa + \omega\right) \cdot \tau - ln\left[\left(\omega + \kappa\right)\left(e^{\omega \cdot \tau} - 1\right) + 2\omega\right]\right)$$

$$b\left(\tau\right) = \frac{2\left(e^{\omega\tau} - 1\right)}{\left(\omega + \kappa\right)\left(e^{\omega \cdot \tau} - 1\right) + 2\omega} \tag{2.14}$$

with $\omega = \sqrt{\kappa^2 + 2\beta^2}$. With these CIR dynamics, optimal estimate of the parameters r_0, θ, κ and β can be calibrated from quoted swap rates.

It is important to notice, that the model chosen for swap rates assume that there is not counterparty risk, which contradicts the very foundation of this thesis. So deriving these under the assumption of no counterparty risk is only an approximation. However, for this thesis it has been reasonable to assume unilateral case with the financial institution assumed relatively default free and able to hedge market risk with another default free institution[47]. As the swap rate quotes are market prices for such financial institution counterparties, ignoring counterparty risk, have little effect on a simple model like the one-factor CIR model.

[46] Assuming valuation at $t = 0$.
[47] See Section 1.3.5.

2.5.4 Calibration of JCIR Model for Default Intensity

The JCIR model calibration has the purpose of achieving a decent proxy for the dynamics of the default intensity based on credit spreads. This requires a model for CDS pricing assuming that dynamics of default intensity are given by the JCIR model. In Appendix B the fair CDS spread is given as

$$k_t^* = \frac{LGD \cdot \sum_{i=1}^{n} \lambda\left(T_{i-1}, T_i\right) \cdot D\left(t, T_i\right)}{\tau \cdot \sum_{i=1}^{n} \lambda\left(t, T_i\right) \cdot D\left(t, T_i\right)} \tag{2.15}$$

with $\lambda\left(T_{i-1}, T_i\right)$ the current default intensity and LGD the loss given default. (Brigo et al., 2013), (Brigo and Mercurio, 2007) and (Brigo and El-Bachir, 2010) provide a closed form solution for survival probabilities[48], which satisfies the PDE

$$1 - \lambda\left(t, T_i\right) = a\left(\tau\right) \cdot \tau e^{b(\tau) \cdot \lambda_t} \tag{2.16}$$

where $\tau = T - t$. Solving for $a\left(\tau\right)$ and $b\left(\tau\right)$ equals

$$a\left(\tau\right) = \left(\frac{2\omega e\left(\frac{\omega+\kappa}{2} \cdot \tau\right)}{\left(\omega+\kappa\right)\left(e^{\omega \cdot \tau} - 1\right) + 2\omega}\right)^{\frac{2\kappa \cdot \theta}{\beta^2}} \left(\frac{2\omega e^{\left(\frac{\omega+\kappa+2\gamma}{2}\tau\right)}}{\left(\omega+\kappa+2\gamma\right)\left(e^{\omega \cdot \tau} - 1\right) + 2\omega}\right)^{\frac{2\alpha \cdot \gamma}{\beta^2 - 2\kappa \cdot \gamma - 2\gamma^2}}$$

$$b\left(\tau\right) = \frac{2\left(e^{\omega \cdot \tau} - 1\right)}{2\omega + \left(\kappa + \omega\right)\left(e^{\omega \cdot \tau} - 1\right)}$$

with $\omega = \sqrt{\kappa^2 + 2\beta^2}$. This solution does, however, collapse, if $\gamma = \frac{\omega-\kappa}{2}$, as $\frac{2\alpha \cdot \gamma}{\beta^2 - 2\kappa \cdot \gamma - 2\gamma^2} \rightarrow \frac{2\alpha \cdot \gamma}{0}$. Hence to avoid this $\left(\frac{2\omega e\left(\frac{\omega+\kappa+2\gamma}{2} \cdot \tau\right)}{\left(\omega+\kappa+2\gamma\right)\left(e^{\omega \cdot \tau} - 1\right) + 2\omega}\right)^{\frac{2\alpha \cdot \gamma}{\beta^2 - 2\kappa \cdot \gamma - 2\gamma^2}} = 1$ when $\gamma = \frac{\omega-\kappa}{2}$, which reduces the model to the CIR model.

To be able to find the CDS spread the above formula can estimate default probabilities[49]

$$\lambda\left(T_{i-1}, T_i\right) = \mathbb{Q}\left(T_{i-1} < \tau^* \leq T_i \mid \tau > t\right) = 1 - \mathbb{Q}\left(T_{i-1} < \tau^* \geq T_i \mid \tau > t\right)$$

The discount rates in Section 2.15 following the CIR-model correlate with the default intensity. Hence, parameters for the CIR model should principally be jointly calibrated on the CDS spread. However, as mentioned in Section 2.4 and by (Brigo and El-Bachir, 2010), though this dependency exists, it has a negligible effect on the CDS spread. Thus, it is reasonable to assume a deterministic term structure based on the OIS rates derived from the CIR model with parameters calibrated to swap rates as in Section 2.5.3 without causing a significant effect on the model parameters.

This thesis takes the perspective of a medium-sized financial institution in an international scale with a corporate counterparty as mentioned in Section 1.3.5. Consequently, it is unlikely, that a liquid single-name CDS if any would with just a little liquidity would exist on such a counterparty. To account for this issue, parameters are calibrated to fit market quotes of a

[48] $Survival\ Probability = 1 - \lambda$.

[49] Where τ^* is the default time.

CDS index, namely the iTRAXX Europe Crossover Index. The iTRAXX Europe Crossover Index CDS consists of 75 actively traded European companies, which are characterized as non-investment grade and have no rating[50] (Markit, 2014b). Thus, making it a suitable proxy of default probability for minor corporates. As mentioned in Appendix B.3, such an index spread could roughly be the average spread of single-name CDS contracts of all companies included in the index. However, it is not a precise measure as:

1. Not all companies potentially using this as proxy are in fact included in the index.
2. As Appendix B.3 shows, the payout features of the iTRAXX indices differ from a normal single-name CDS therefore making the CDS pricing formula unsuitable.

Nevertheless, no better public data are available and the simple assumption of a representative index at least provides a decent range for the default intensity. If having at hand historical credit risk data on the specific counterparty available, such can serve as a base for the individual credit risk, but would also not provide a full perspective of future credit risk. This shows that credit modelling is a very delicate matter. Determining λ_0, θ, κ, β, γ and α of the JCIR model is sub-optimally achieved by calibrating them to fit market quotes of iTRAXX Europe Crossover Index CDS spreads paid quarterly, which makes the present value of the contract equal to 0 (Markit, 2012).

An important issue to mention is that the calibration of default intensity assumes a constant LGD, which means a constant recovery rate. With LGD having a huge effect on the CVA, so this might seem like a very strong assumption. However, there are two arguments in favor of leaving LGD constant. First, it is almost impossible to calibrate recovery rates to any instruments other than the single-name CDS contract or CDS indices, which are also used to calibrate default intensity[51]. Hence, calibrating two variables to one instrument is an under-defined problem and thus has infinite solutions. The second argument relates a lot to the first, because the credit spread is what is actually traded in the market, not the default intensity. As can be seen from equation 2.15, both LGD and default intensity determines the credit spread jointly. In other words, the calibrated default intensity only shows the actual probability of default, if the LGD accounts for the actual recovery rate. Otherwise, the probability of default shows a pseudo probability of default given a fixed LGD. Nevertheless, the credit spread is what effects the CVA. Hence, we should not care whether the LGD differs, as this adjusts default intensity to account for the actual LGD expected by market through the quoted credit spreads (Ruiz, 2015)[52].

[50] By Fitch, S&P or Moody's rating agency.

[51] Some theoreticians propose to use Digital CDS contracts (pay full notional at default) or Recovery Default Swaps (Swap a fixed recovery rate for the actual recovery rate on a notional amount). However, in reality these instruments are rare and extremely illiquid, hence making them less attractive to use (Ruiz, 2015).

[52] We do, however, acknowledge, that the choice of LGD will have some effect as default intensity is used isolated for estimation of MVA in Section 4.4.1 and partially for estimation of KVA in Section 4.5.2.

Part III

Monte Carlo Simulation

3.1 Purpose of Monte Carlo Simulation

For purpose of pricing plain vanilla derivatives, analytical approaches exists as a direct way of pricing without any great computational effort. However, most of these analytical techniques depend on some simple assumptions, which will be violated when introducing the effect of xVAs. Even efficient numerical procedures like finite difference methods is not applicable (Wilmott, 2007), as different risk factors and variables required for pricing with inclusion of xVA demand a high dimensional pricing framework. Hence, in order to be able to calculate the right risk exposure with respect to underlying dynamics, Monte Carlo simulation is considered the contemporary valuation method within the financial service industry (Gregory, 2015a).

MC simulation provides the necessary flexibility to model collateral, funding and path dependent features, offering more holistic pricing (Gregory, 2015a). Albeit a powerful approach, simulations for xVA purposes has become the greatest quantitative challenge for the financial service industry (Ruiz, 2015). Estimates only converge as the number of simulated scenarios increase. Getting rid of this MC noise within the estimates by increasing the number of simulations increases the computational time exponentially. Simulating derivative prices becomes time consuming when including different features and dependencies. However, as dependency between different counterparties and trades is present, the entire OTC portfolio of a financial institution is interdependent, affecting each single derivative price. Thus, the computational requirements for joint dependency simulations explodes with the number of trades (Ruiz, 2015). Besides price estimates, calculation of sensitivities, exposure allocations to different counterparties, and other metrics are required intra-day at the xVA desk (Ruiz, 2015). To solve this challenge, there have been different ways to approximate some elements to reduce the computational effort at the expense of flexibility in the pricing framework. It is also important to know, if any estimates can be approximated rather than simulated, as avoiding this might ease the simulation burden.

This thesis only focus on pricing swaps individually. Including xVAs on a per trade basis is a numerical challenge in itself. Considering a whole portfolio with interdependencies, pricing marginal exposure for each individual derivative with respect to one another will complicate this matter further, which illustrates the magnitude of the xVA challenge.

Section 3.2 provides a general discussion on the art of MC simulation and efficiency. Section 3.3 further provides the steps necessary for estimating xVAs in general, and Section 3.4 presents methods of discretization for the stochastic dynamics of the different underlying risk factors.

3.2 The Basics of Simulation

The concept of simulation is to generate an amount of different scenarios each reflecting a potential future scenario. Random numbers serve as input for the stochastic models providing randomness for the dynamics of factors determining each future scenario. If the probabilistic distribution of the random numbers are set to fit the real dynamic behavior of different risk factors, then with a larger number of scenarios the average will converge to the expected value (Glasserman, 2003). Each scenario must be calculated based on the current information and be independent of other scenarios (Ruiz, 2015). Following the Martingale approach described in Section 1.3.4, it is possible to estimate the xVA charges in different scenarios. The mean value of all scenarios represent the expected value

$$E\left[V_t\right] = \hat{V}_{t,n} = \frac{\sum_{i=1}^{n} V_{t,i}}{n}$$

This expected value converges to the fair value if the estimator becomes unbiased, as the amount of simulations increase

$$\lim_{n\to\infty} E\left[\hat{V}_{t,n}\right] = V_t$$

Hence, the central limit theorem Glasserman (2003). The estimator converges to the standard normal distribution in the limit

$$\frac{\hat{V}_{t,n} - V_t}{\sigma_{V_t}/\sqrt{n}} = N\left(0,1\right)$$

Thus, when rearranging the simulation error or MC noise, this is approximately

$$\hat{V}_{t,n} - V_t \approx N\left(0, \frac{\sigma_{V_t}^2}{n}\right) \tag{3.17}$$

This confirms that in order to achieve a lower variance and converge to the true value, the amount of simulations should be increased (Glasserman, 2003). The reduction of the MC noise is only $\frac{1}{\sqrt{n}}$, meaning that the amount of simulations must be quadrupled in order to half the MC noise (Hull, 2012).

As computational time constraints the maximum possible amount of simulations, the efficiency of the simulation method can be compared if setting a time limit, s, and calculate the duration of each simulation, τ. Thus, using the time-constrained integer amount of simulations $n = \left[\frac{s}{t}\right]$ the highest simulation efficiency can be achieved (Glasserman, 2003).

The convergence to an unbiased estimator can remove potential small sample biases. However, some sources of biasness will persist for infinite samples and thus make the estimator inefficient (Glasserman, 2003). The most obvious persistent bias is the discretization error, which will be discussed in Section 3.4. It is possible to reduce biases and errors with different techniques. However, doing so will increase the computational time of each simulation. For this reason, the above efficiency measure will be relevant when measuring efficiency of including different variance reduction techniques, selective sampling or as described in Section 3.4 a more precise discretization.

3.3 Simulation Process for Pricing with xVAs

Pricing derivatives with xVAs potentially depends on numerous underlying risk factors determining cash flows and exposures. To take these appropriately into account MC simulation is the only possible method, which computationally can handle the high dimensionality occurring from different risk factors and decision variables (Gregory, 2015a).

(Ruiz, 2015) divides the simulation process into three overall steps:

1. Risk factors
2. Pricing
3. Calculation of risk metrics to estimate xVA charges

As many risk factors potentially affect the prices and exposures of a derivative contract, it is important to choose enough relevant risk factors to achieve a realistic model, but at the same time keep the model tractable and limit the computational effort (Gregory, 2015a).

Using stochastic models to describe discrete paths for the risk factors provides an approximation of the underlying dynamics up to maturity. The discrete dynamics of the underlying risk factors allow calculation of the market price and exposure of the derivative contract at all discrete time point for each simulated path (Ruiz, 2015). Such simulations should follow a pathwise simulation scheme, which moves from each discrete point in time to the next. Using pathwise simulation rather than direct simulation of each discrete point in time set from time 0 allows for collateral modelling (Gregory, 2015a). A discrete number of calculated prices and exposure only approximates a continuous process. The swap exposure is highly discontinuous over time due to payments at discrete points in time or potential options features at certain points in time (Gregory, 2015a). Thus, the discretization grid should cover payment intervals and other possible discontinuous features or in addition include these points in time in the discretization grid (Ruiz, 2015). There is, however, still a tradeoff between accuracy and computational effort, if a derivative contracts has many payments or option features.

Knowing the discrete exposure of the swap contract over the contract lifetime, one can determine mark-to-market collateral directly on the model including terms of the collateral agreement. Thus, it is possible to estimate the effect and costs mitigating credit exposure through dynamic collateral posting. Including collateral and hedging in the simulation we can examine the effect and dependency between different xVA terms and be able to discuss optimization of the total xVA charge. Approximations for the effect of collateral do exists. However, such approximations do not capture the effect of having certain limits for the mark-to-market amounts or the gap risk between each possible margin call very well (Gregory, 2015a).

When calibrating the underlying risk factor models, it is important to acknowledge the dependency of the different risk factors, so the pricing based upon these does not become naïve (Gregory, 2015a). Dependency modelling becomes more time consuming and must be rather sophisticated when considering more counterparties (Ruiz, 2015). As discussed in Section 2.4, this thesis only looks at individual pricing of swaps without considering netting effects of different counterparties or contracts. Hence, a linear correlation between interest rates and default probability can be used to model dependency. In reality, simulation of underlying risk factors and pricing are sometimes separated. Risk factors might be calibrated to historical data rather than current market data[53]. Doing so, reduces the simulations required for convergence of the price towards the true value (Ruiz, 2015). It is important to notice, that financial

[53]Especially if used for risk management rather than pricing.

institutions might already calculate some of the underlying risk factors with credit or interest rate models for other purposes than xVA estimation. Hence, such results can be reused to ease the computational burden of the xVA terms.

Including details of e.g. the collateral agreement to fine tune the estimate should be held up against the computational burden of including such details. If the computational burden increase, less simulations are possible to make. This increases the MC noise, which might conceal detailed features anyway. Using the same seed value for the random number generation used for the stochastic element fixes the MC noise. Accepting this fixed MC noise, it is possible to separate it from the analysis of minor changes. This will require a very careful understanding of the magnitude of the MC noise (Ruiz, 2015).

3.4 Discretization of Stochastic Models

Section 2.5.3 and Section 2.5.4 provide solutions to the PDE given the dynamics assumed for the underlying risk factors. This allows for an analytical solution or an exact price estimate with just a single step for each path simulation rather than a discrete path (Glasserman, 2003). Nonetheless, in order to model collateral and estimate different xVA terms, this requires the discrete exposure profile of each simulation (Gregory, 2015a)[54]. Hence, a discretized version of the stochastic diffusion models will be necessary. The downside of this is longer computational time, as each simulation will include simulation of many discrete steps rather than a single step. In discrete time, the total time interval of the simulation path, $\tau = T - t$, is separated into smaller intervals $\Delta t = \frac{\tau}{n}$ summing up to the total time (Glasserman, 2003). As both the CIR model and JCIR model have a PDE[55], their discretization will converge to the continuous process in the mean square limit (Wilmott, 2007). A simple Euler Discretization the CIR model is given as

$$r_{t+\Delta t} = r_t + \kappa \left[\theta - r_t\right] \Delta t + \beta \sqrt{r_t} \Delta Z \tag{3.18}$$

where $\Delta Z = \varepsilon \sqrt{\Delta t}$[56] (Munk, 2011). This discretization causes a risk of negative rates not allowed in the models. A simple way to avoid this is to adjust the model to

$$r_{t+\Delta t} = r_t + \kappa \left[\theta - r_t\right] \Delta t + \beta \sqrt{max\left(0, r_t\right)} \varepsilon \sqrt{\Delta t} \tag{3.19}$$

However, both the discretization schemes of the CIR model above do not exactly follow the non-central χ^2-distribution, which is the case for the continuous CIR model[57]. Hence, they cause a discretization error. For this reason, this thesis use the more accurate discretization scheme presented below.

Using equation D.2 from Appendix D.2 adjusted to constant parameters and inserting $\theta = \frac{\varphi}{\kappa}$, the degrees of freedom are given as

$$a = \frac{4\theta \cdot \kappa}{\beta^2}$$

[54] Furthermore, if path-dependent features are included this also requires discretization.

[55] See Appendix C.6 for an evaluation of PDE.

[56] See Appendix C.2.

[57] See Section 2.3.1

and the non-centrality parameter (inserting equation for c into b)

$$b = \frac{4\kappa e^{-\kappa\Delta t}}{\beta^2 \left(1 - e^{-\kappa\Delta t}\right)} \cdot r_i$$

(Glasserman, 2003) shows, that random non-central $\chi^2(a,b)$-distributed variable will equal a squared Gaussian variable with mean b plus a χ^2-distributed number with $a-1$ degrees of freedom.

Using this, equation 3.18 in an accurate discrete form is[58]

$$r_{t+\Delta t} = \frac{\beta^2 \left(1 - e^{-\kappa\Delta t}\right)}{4\kappa} \chi^2(a,b) = \frac{\beta^2 \left(1 - e^{-\kappa\Delta t}\right)}{4\kappa} \left(Z + \sqrt{b}\right)^2 + \chi^2_{a-1}$$

Extending the discrete CIR model to the JCIR model is straightforward as the jump and the diffusion term are independent of one another[59]. The process from above can be written as

$$\lambda_{t+\Delta t} = \frac{\beta \left(1 - e^{-\kappa\Delta t}\right)}{4\kappa} \left(Z + \sqrt{b}\right)^2 + \chi^2_{a-1} + \lambda_t \sum_{i=1}^{M_{t\Delta}} Y_i$$

Thus, the jumps are simply added at the time step in which they occur (Brigo et al., 2013). The increase ratio in default intensity at each jump comes from the intensity parameter γ in the exponential distribution (Glasserman, 2003). (Atkinson, 1979) describes methods to generate random Poisson distributed variables with the sequential search method as a simple way to determine the number of occurrences. With this method, the random probability deciding the number of jumps for a given period comes from a uniform distributed number, U. The Poisson random variable is then generated from a sequential loop through the cumulative Poisson distribution with arrival rate α until $U < P_i$, where P_i is the cumulative Poisson probability for i occurrences. This method gives an easy way to determine the amount of jumps, as long as α is constant. This method, however, becomes computational heavier as α increases. Random uniform variables, U_i, are used to assign the point in time, τ_i, from $[t,T]$, where each jumps occur, which (Mikulevicius and Platen, 1988) proves to be a valid method estimation in a discrete simulation model. In the discrete model given above, the jumps will then occur in the interval including the jump time given by $U_i \cdot n$, where n is the total number of discrete steps in a simulation path.

As mentioned in Section 2.5.4 the size of each jump is determined by a random exponential variable with the rate parameter γ. Which is generated as

$$X = \gamma e^{-\gamma \cdot U}$$

with U a random uniform number (Gentle, 2006). The discretization scheme of the CIR model and the JCIR model ensure that they follow a non-central χ^2-distribution consistently, as Δt increases. The distribution of the brute discretization in equation 3.18 produces heavier

[58]This method to generate random $\chi^2(a,b)$ only holds, if $a > 1$.
[59]See Section 2.5.4.

left tail and a higher mode[60] compared to the exact distribution in equation 3.19 (Glasserman, 2003).

To generate random χ^2-distributed numbers (Glasserman, 2003) shows, that this is a special case of generating random γ distributed numbers, which has density function

$$f(x) = f_{\alpha,\beta}(x) = \frac{1}{\Gamma(\alpha) \cdot \beta^{\alpha}} \cdot x^{\alpha-1} e^{\frac{x}{\beta}}$$

with $x \geq 0$, $\mu = \alpha \cdot \beta$ and $\sigma = \alpha \cdot \beta^2$. Setting $\beta = 2$ and $a = \frac{a}{2}$, where a are the degrees of freedom in the χ^2-distribution, the γ distribution equals a χ^2-distribution[61]. This thesis uses the GKM1 algorithm[62] in (Fishman, 2013) to generate the random $\chi^2(a)$ when $a > 2$ which is required for the algorithm to be valid (Glasserman, 2003). When $0 < a \leq 2$ the GS* algorithm[63] in (Fishman, 2013) is instead used. The GKM1 and GS* algorithm only generate $\gamma(\alpha, \beta)$, where $\beta = 1$. Hence multiplying the random γ distributed number with $\beta = 2$, will produce $\chi^2(a)$ as desired (Kroese et al., 2013).

The Gaussian variables, follow the standard normal distribution with the density function

$$Z = \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

with $-\infty < x < \infty$. To generate random normal distributed variables, this thesis uses the algorithm by (Marsaglia and Bray, 1964) as described by (Glasserman, 2003).

[60] The mode is the most frequent number in the sample. See (Keller, 2009).
[61] Assuming that α is an integer (Kroese et al., 2013). (Glasserman, 2003), however, shows, the γ distribution is valid for all $\alpha > 0$, also non-integers.
[62] Based on (Cheng and Feast, 1979).
[63] Based on (Ahrens and Dieter, 1974).

Part IV

xVAs

4.1 Introducing xVAs

This section presents the theory and application of xVA charges within a simulation model including stochastic models for the underlying risk factors. As mentioned in Setion 2.1, the financial crisis of 2007 made it obvious that the Black-Scholes equation based on risk neutral pricing and strict assumptions did not fit reality (Ruiz, 2015). Counterparty risk, which was already known but assumed insignificant, had to be included (Tuohy and Dooseman, 2014).

This section evaluates the weaknesses of the BS model compared to reality and introduces the necessary extension to xVA charges, providing a more sophisticated era of derivative pricing, though still being under development. First subsection provides an overall discussion of the need for xVA pricing. The following sections deal with different price adjustment terms of xVA: CVA, FVA, MVA, and KVA. It is important to notice that it is widely debated whether to include the credit risk of the derivative issuer, DVA, as well. This thesis does not include DVA. A short evaluation and discussion the controversy of DVA is included in Appendix E. This part of the thesis provides theory and estimation of each xVA term, while Part VI covers management and hedging issues for all xVA terms.

As mentioned in Section 3.3, MC simulation is the most appropriate way to estimate different xVA terms. The precision depends on the quality of the models for the underlying risk factors used as input for the calculation of the different xVA terms. MC simulation is more sophisticated than different approximations, if using the suitable stochastic models for the underlying risk factor dynamics, as it is possible to include further complexities than when using analytical or semi-analytical approaches (Gregory, 2015a). Thus, this thesis will focus solely on implementation of xVA charge estimation in a simulation framework and disregard other approaches. This part of the thesis takes the underlying risk factor model for granted because Part II and Part V discuss these issues. However, we use the setup provided by Basel III under assumptions described in 4.5.1 for the calculation of regulatory capital resulting in the KVA charge.

4.1.1 Pricing Beyond the Black-Scholes Equation

The classic BS risk neutral framework has been the market standard providing a straight forward pricing framework based on the following assumptions (Hull, 2012), (Ruiz, 2015):

- The existence of a risk free rate, which has original been assumed constant, but can be stochastic using numerical procedures. Unlimited lending and borrowing will be available at this risk free rate.
- Returns follow an underlying diffusion model, which has been extended to more advanced dynamic models over time.
- No transaction costs, taxes, and the option to trade fractions of securities in continuous time to create a perfect hedge strategy.
- The assumption of no-arbitrage in the market.

Some of these assumptions are also mentioned in Section 1.3.4. We still accept some of these assumptions, but extend them to be far more realistic. The rule of no-arbitrage does still apply to some extent. Thus, we can accept using stochastic model dependent on the no-arbitrage assumption for the underlying risk factors and use risk neutral valuation of the price.

However, the OIS rate is now used as a proxy for the risk free rate though it is not completely risk free and hence challenge the no-arbitrage assumptions required in Section 2.2.3. Even though using no-arbitrage to allow for a risk neutral valuation, arbitrage opportunities can occur from the pricing on the OTC market. The reason being, that it is difficult to exploit small arbitrage opportunities due to transaction costs and limited lending possibilities. It is also not possible to create a perfect hedge portfolio as trading cannot be in fractions of securities and in continuous time, which creates discrepancies between the derivative and the hedge strategy (Ruiz, 2015).

Different from exchange traded derivatives, OTC derivatives carry counterparty credit risk, which is not included in the original BS model, which assumes that both parties would fulfill their contractual obligations (Hull and White, 2014b). (Burgard and Kjaer, 2011a) try to adjust the BS PDE for counterparty credit risk. They also try to take into account the spread between the risk free rate and the actual funding rate, which are the actual costs of lending cash rather than risk free rate. Hence, relaxing some of the BS assumptions. However, Burgard and Kjaer (2011a) still operate in a continuous time framework causing discrepancies between the derivative contract and the hedge strategy, which in reality cannot be adjusted in continuous time. They also disregard transactions costs. Furthermore, (Burgard and Kjaer, 2011a) shows that the Feynman-Kac risk neutral valuation is possible when simply including the adjustment of CVA to the traditional risk neutral value and (Hull and White, 2014b) illustrates that this can be extended to include collateral agreements. Thus, this thesis still relies on the PDE with extensions to its shortcomings to provide the theoretical fair price when the xVA charge is simply added.

FVA and KVA do not follow the risk neutral adjustment. These values depend on choices of xVA management, which in reality goes beyond each single trade, as it depends on the overall portfolio and xVA hedging of a financial institution individually. This creates potential arbitrage opportunities, which violate the risk neutral pricing. However, in practice it is difficult to exploit such opportunities for third parties as the OTC trades are private transactions between two parties (Ruiz, 2015). Though FVA and KVA are controversial to the risk neutral pricing, (Ruiz, 2015), (Gregory, 2015a) and (Brigo et al., 2013)[64] still include them to improve simulation model complexity to e.g. also take the effect of collateral agreements into account. The simulation allows for direct estimation of CVA and FVA independent of the hedge of these, as hedging in reality is a huge challenge[65]. Thus, xVA is still an art as much as it is a science, as there is yet no market consensus on inclusion and method for estimating KVA and FVA (Ruiz, 2015), (Brigo et al., 2013).

This thesis includes most of these xVA terms, as they account for actual costs, which the financial institutions face when dealing with derivatives. These costs depend on the funding curve and regulatory demands on the financial institution, which are not identical among differ-

[64] KVA is newer to academia than (Brigo et al., 2013), which does hence not cover KVA.
[65] See Section 6.3.

ent financial institutions. Thus, a violation of the law of one price, which has been a mandatory assumption for derivative pricing for years (Kenyon and Stamm, 2012).

With this in mind, this thesis defines the theoretical fair price of a derivative within the xVA framework as the risk neutral present value of the derivative plus the present value of the xVA charge (Ruiz, 2015) including CVA, FVA, MVA, and KVA (but excluding DVA[66]). Hence, to break even, the financial institution should only enter swaps where they can achieve a positive present value after including the xVA costs.

4.2 Credit Value Adjustment

Credit risk measures the risk that the counterparty of the contract does not fulfill the binding contract obligations due to default (Gregory, 2015a). In terms of pricing of loans, risk measures quantify a premium to account for the particular risk of a loan. To estimate such, risk statistical measures as the Credit VaR have been popular. Credit VaR assumes a given fixed distribution for the return from the loan with a volatility of this value caused by potential change in credit rating and possible default. The Credit VaR given the assumed distribution is the maximum value that can be lost with a certain probability and time horizon (Choudhry et al., 2012). The time horizon can e.g. be set equal to the maturity of an OTC derivative.

However, Credit VaR only captures the current default intensity of the counterparty, but this might change through the lifetime of the derivative contract. It also does not take into account the directional way risk coming from the correlation between risk factors (e.g. interest rates and default intensity). Due to this correlation, the risk of counterparty default increase when the value of a swap increases from falling interest rates. This drives up the likelihood that that potential positive value will be lost[67] (Blundell-Wignall and Atkinson, 2010). As evaluated in Section 1.2 losses due to actual default of the counterparty only accounted for one-third of the OTC derivative losses during the financial crisis, while the latter came from higher credit volatility due to increasing risk of default. Furthermore, OTC derivatives like swap contracts have opposing cash flows, which will mean that the cash flows paid partially offsets some of the potential losses on cash flows received (Gregory, 2015a). For a fixed-floating IRS contract, these cash flows depend on the change of the floating interest rate. Again, the directional way correlation between interest rates and default intensity must be included (Brigo et al., 2013).

The CVA measure addresses the issues mentioned above. CVA is a price adjustment to the risk neutral price of the derivative. Hence, it is based upon expectations under the $\mathbb{Q}$-Martingale estimated from the real world underlying risk factors (Brigo et al., 2013). In other words, CVA is the present value difference between assuming a risk free counterparty and introducing counterparty risk (Brigo et al., 2013). This section presents CVA. First subsection presents the necessary exposure measures; first in an uncollateralized case and then extended to include collateral agreements. Next subsection shows how to estimate CVA in the simulation model based on exposure and default metrics. This thesis only considers a unilateral case of CVA

[66]See Appendix E.
[67]See Section 2.2.4 for discussion of correlation and directional way risk.

adjusting for counterparty risk, but ignores subtracting own credit risk (DVA), assuming the issuer default free relative to their counterparty[68].

4.2.1 Credit Exposure

Credit risk will depend on three components changing over the time horizon (Ruiz, 2015):

- Probability of default
- Loss given default[69]
- Exposure of the derivative contract

Current exposure equals the current market value of the derivative contract at any point in time. Assuming a unilateral case, this equals 0, if the market value is negative (Gregory, 2015a)[70]. The expected exposure is then the estimation of the current exposure at a future point in time (Brigo et al., 2013):

$$EE_t = [V_t, 0]^+ \tag{4.20}$$

This market value is only realizable as long as the counterparty has not yet defaulted. Thus, at each point in time the counterparty risk will consist of the current exposure times the probability of default and LGD.

Uncollateralized Exposure

Using equation 2.2 from Section 2.2.3 yields the price of an IRS contract in a multi-curve framework. (Munk, 2011) shows that forward interest rates F_x, of the floating rate given the CIR model dynamics is

$$F_x(t, \tau, r) = r_t + \kappa[\theta - r] \cdot b(\tau) - \frac{1}{2}\beta^2 \cdot r_t \cdot b(\tau)$$

Where r_t is the spot rate at time t, $b(\tau)$ is similar to equation 2.14 in Section 2.5.3 and $\tau = T_{k+1} - T_k$ with T_k being the time the floating rate was last fixed. The parameters are equal to those calibrated for the CIR model. Extraction of the forward and discount curves follows under the $\mathbb{Q}^T$-Martingale measure evaluated in Section 1.3.4.

Applying the multi-curve approach, by using as input:

- calibrated OIS rate parameters for the discount curve
- relevant calibrated tenor specific rate parameters for the forward curve

to price the swap contract at any point in time, it is possible to calculate the expected exposure at all points in time. Hence, we can derive the exposure structure from all positive values during the lifetime of the contract discretely. This will in practice only be a discrete approximation, as

[68] See Appendix E for a short description of DVA and a discussion of the controversy of including DVA.

[69] LGD is assumed constant in this thesis but as mentioned in Section 2.5.4 indirectly incorporated in the PD dynamics.

[70] Negative values would be used in the bivariate case with inclusion of own default risk.

the exposure changes in continuous time. However, as in many other cases this value converges towards the true value, as the number of steps increase (Gregory, 2015a).

For the purpose of pricing CCS contracts (Brigo et al., 2013) provides a pricing function for respectively a fixed-fixed CCS contracts paying the foreign currency leg ignoring the currency basis for the reason provided in Section 2.2.5

$$V = \left(H \cdot \sum_{i=j}^{m} \tau \cdot D_d\left(T_j, T_i\right) \cdot r - H^f \cdot \sum_{i=j}^{m} \tau \cdot S_i \cdot D_d\left(T_j, T_i\right) \cdot r^f \right) + \left(H - S\left(T_j, T_m\right) \cdot H^f\right) \cdot D_d\left(T_j, T_m\right)$$

where r and r^f are the fixed rates and H and H^f are the notional value of respectively the domestic and foreign currency leg of the swap. S_i is the spot exchange rate, which for simplifications this in the thesis follows the deterministic forward rates at initiation of the contract. Using these deterministic spot FX rate and the stochastic interest to calculate the forward FX rates $S\left(T_j, T_m\right)$ following equation 2.5 in Section 2.2.5, the interest rate stochasticity in both currencies provide some variation in the FX forward rates at different points in time.

The exposure structure of an IRS contracts is not continuous increasing over time, but peaks around 1/3 of the total maturity of the contract[71]. After this point, exposure start to decrease. This exposure profile arises from the exchange of cash flows during the life of the contract amortizing the potential value over time, which at a certain point prevails against the higher uncertainty of exposures further in the future. The total exposure will be higher with longer time to maturity and can be asymmetric, if the exchange of cash flows between the two parties do not occur with the same frequency. The shape of the term structure and the leg received will decide the direction of this asymmetry (Gregory, 2015a).

As the exposure at any point in time is $EE_t = \left[V_t, 0\right]^+$, this has an option-like payoff. (Sorensen and Bollier, 1994) indeed shows that the unilateral exposure equals the value of a swaption with strike 0 and maturity at time t. This simplification is precise in the uncollateralized case, but would need an approximation for collateralization rather than the more precise modelling with MC simulation (Gregory, 2015a). The semi-analytical approach also lacks other flexibilities, as the need to incorporate changes future market practices and regulations (Gregory, 2015a).

Collateralized Exposure

The exposure profile described so far does not consider collateralization. Collateral is an asset passed to the other party without changing ownership, when the contract between the two parties has a positive value for the party receiving the collateral. Holding collateral to cover exposure will reduce the loss[72] of the counterparty defaulting and hence mitigate the credit risk, as subtracting the value of the collateral from the value of the derivative contract minimizes the exposure or reset it to 0 (Gregory, 2015a). Posting collateral based on the exposure is to mark-

[71] Under the simple assumption of $V_t \sim N\left(0, \beta\sqrt{t}\tau\right)$ and a strictly positive upward sloping yield curve (Gregory, 2015d).

[72] There are few extreme cases for CDS contracts, where collateralization might increase exposure due to strong dependency (Brigo et al., 2013).

to-market, meaning that the collateral is supposed to equal the exposure. Collateral posted in this way is the variation margin (Gregory, 2015a). Some gap risk might exist from remaining exposure existing with a collateral agreement. This risk might occur due to depreciation of the value of the collateral assets, new exposure occurring since last collateral post, or from the gap between exposure and thresholds on collateral payments (Ruiz, 2015). For simplification, this thesis only allows collateralization to be with cash payments in domestic currency. This reduce the gap risk to concern choice of margin period for collateral posting, thresholds/initial margins, and transfer sizes.

The Credit Support Annex OTC derivative trades are not enforced to include a collateral agreement, but since the financial crisis, doing so is quite typical (Gregory, 2015a). Such agreements usually follow a Credit Support Annex (CSA), which is a part of the ISDA Master Agreement for best practices for OTC transactions (Brigo et al., 2013). A CSA agreement is a legal contract, which contains terms for all relevant issues in connection with collateralization structure and determination of key parameters in the collateral agreement (Ruiz, 2015). Most of these features allow for different optionalities (Brigo et al., 2013). The weakness of the CSA agreement is, that agreement terms are made upon upfront and can only be changed if both parties agree to do so. Thus, making the CSA vulnerable to changing market conditions (Gregory, 2015a). (ISDA, 2014) shows, that in 2013, 90% of all non-cleared OTC transactions were subject to a collateral agreement. 87% of these collateral agreements were CSA agreements.

The structure of the CSA agreement resolves issues as eligible collateral, collateral legal ownership, interests on collateral, rehypothecation, and whether the collateral agreement is unilateral or bilateral (Ruiz, 2015):

- As mentioned above this thesis only consider cash payments as eligible collateral for simplicity, which eliminates issues regarding the choice of collateral to deliver, change in collateral value and the potential need of haircuts on risky collateral. These issues would make collateral modelling even more complex.
- Issues of legal ownership influence the right to coupon and dividend payments on collateral, which has an impact on the collateral value. These issues are not relevant as this thesis assumes only domestic cash payments.
- Posting collateral will require funding or inclusion of the opportunity costs of the collateral cash. A compensation interest rate is typically paid; often the OIS rate (Ruiz, 2015). However, as Section 2.1 explains, the funding rate of financial institutions is individual and significant above the OIS rate in most cases. The mismatch between those two lead to funding costs, which are evaluated in Section 4.3.1.
- ISDA defines rehypothecation as the right to reuse collateral held (Ruiz, 2015). This allows the collateral receiver to sell or repo out assets or to invest cash received to earn interest, which can finance the interest payment to the collateral payer. Furthermore, rehypothecation allows collateral received to be posted as collateral elsewhere. However, this also means that collateral posted at the counterparty might be at risk

if the counterparty defaults. Hence, collateral that can be rehypothecated will less effectively reduce counterparty risk from the view of the counterparty (Gregory, 2015a). The fine balance between funding costs and counterparty risk determines whether rehypothecation is attractive. This thesis allows for rehypothecation of the variation margin.

- Lastly, it must be agreed whether the CSA agreement is unilateral or bilateral. In a bilateral agreement both parties post collateral to cover counterparty exposure, while in a unilateral case one party is willing to bear the entire default risk of the counterparty. It must be said, that if a party does not post collateral, this will increase the CVA charge on that party. However, as mentioned in Section 1.3.5 in contracts with minor corporate counterparties, these might accept the financial institution as being risk free relative to themselves, justifying a unilateral CSA agreement. However, (Ruiz, 2015) argues that as even minor corporates have increased their management of credit risk, they will not accept the above assumption. This thesis allows for both unilateral and bilateral collateral agreements.

Besides the structural decisions in the CSA agreement, it also defines numerical values for margining frequency, thresholds and initial margins (Ruiz, 2015):

- As mentioned earlier, collateral posting frequency cannot be in continuous time. Furthermore, in reality posting a high margin frequency will result in increasing transaction costs as each posting has operational costs. (RMA and Deloitte, 2015) reports that collateral management for most financial institutions is still a manual task handled by the back office. Hence, it is necessary to find an optimum between the effect of posting balanced against the operational costs of collateral management (Ruiz, 2015). It is possible to set a fixed margin period, which is the time frequency between each mark-to-market of exposure. The larger margin period, the larger margin period of risk, which allows for change in collateral value and exposure (Gregory, 2015a). Another parameter to alter costs is the threshold exposure acceptable before making the first margin call for post of collateral. Also setting a minimum transfer amount (MTA) and agree on rounding of the collateral calculations minimizes the costs of transactions due to less frequent margining and fewer disputes[73] about market value at the expense of higher gap risk.

- In case of a counterparty default the variation margin might not cover all costs of terminating the contract. Besides the effect of margin period and threshold, close-out costs might occur. These represent the costs replacing the contract with another counterparty, e.g. to retain a hedge of market risk (Brigo et al., 2013). Replacement costs are likely above the current market value of the contract due to potential change in own credit risk or to transaction costs[74]. Besides there will be a period of risk

[73]Rounding agreements are used to minimize disputes. Longer margin period increases the risk of disputes, as this increase the chance of different value estimates (Gregory, 2015a).

[74]Transaction costs might e.g. come from potentially having to accept a less favorable CSA agreement with the new counterparty.

up to and after default before close-out is completed. During this margin period of risk, no additional collateral is posted, market prices of a replacement contract might increase, and the value of collateral might depreciate (Gregory, 2015a). Additional collateral might be required upfront to account for this additional risk not covered by the variation margin (Gregory, 2015a). This is the initial margin, which will be relevant for the MVA in Section 4.4.

The size of the different parameters might be linked to the credit quality of the counterparty, so that lower rated counterparties will be required more frequent posts and have lower minimum thresholds and MTAs. This might also influence the requirement of an initial margin (Gregory, 2015a). The flexibility of choosing different structure and setting different key parameters for the CSA agreement clearly has great impact on CVA. However, this must be put up against the funding costs (FVA and MVA), which follow. (RMA and Deloitte, 2015) shows that this flexibility is indeed used in practice to differ between counterparties, as 80% of the financial institutions surveyed only used the same standard CSA agreement with less than 10% of their collateralized counterparties.

Modelling Collateral Suggestions of simple analytical approximations to include the effect of collateral on the exposure as in (Gregory, 2015b) exist. However, these only provide approximate measures and overlook the effect of different terms within a CSA agreement. Thus, simulating the collateralization process provides the flexibility to include more features. However, this might give some computational challenges if setting a very short margin period, as this will affect the amount of discrete steps in the simulation. The simple collateral calculation algorithm has the following steps at each discrete point in time (Ruiz, 2015)[75]:

1. First, collateral shall only be posted, if $V_i > Thr$, which means that the value is above the threshold margin.
2. Collateral held since last margining, C_i^{before}, is subtracted from the current value of the swap contract $V_i - C_i^{before} = \Delta C_i$
3. If $\Delta C_i > MTA$, which is the minimum transfer amount then $C_i^{after} = C_i^{before} + round\,(\Delta C_i)$ else $C_i^{after} = C_i^{before}$.

Threshold and initial margin are both two sides of the same coin as an initial margin can be seen as a negative threshold (Gregory, 2015a). Merging these two effects into the initial margin, IM[76], the collateralized exposure can be calculated subtracting the collateral and initial margin hold before marking-to-market at each point in time (Gregory, 2015a)

$$EE_i = \left[V_i - C_i^{before} - IM\right]^+ \tag{4.21}$$

Not being able to post collateral in continuous time, the exposure in equation 4.21 measures the gap risk. Hence, in this discrete model ΔV from $i-1$ to i yields the gap risk, which equals the

[75] Assuming collateral can only be cash in domestic currency, the algorithm must not correct for changed value of collateral or changes in the exchange rates.

[76] Negative if a threshold.

market exposure above if positive. This is under the assumption that $\Delta t = Marging\ Period$ (Ruiz, 2015). This thesis assumes quarterly margining for all maturities.

4.2.2 Estimating CVA

CVA expectations taken are under the $\mathbb{Q}$-Martingale, as the CVA is a pricing adjustment to a derivative. (Brigo et al., 2013) uses the indicator function $1_{\{t<u<T\}}$, to indicate that default occurs between time t and T. If default happens at time $u < T$ then the value V_u (if positive) will be the value lost at default, when multiplying with LGD (Tuohy and Dooseman, 2014), Discounting this loss to present value will yield the simple formula for the uncollateralized unilateral CVA (Brigo et al., 2013)

$$CVA_t = E_t\left[LGD \cdot 1_{\{t<u<T\}} \cdot D(t,u) \cdot (V_u)^+\right]$$

Replacing the value with the expected exposure EE_t, we can use either equation 4.20 or 4.21 depending whether the derivative contract is uncollateralized or collateralized

$$CVA_t = E_t\left[LGD \cdot 1_{\{t<u<T\}} \cdot D(t,u) \cdot EE_t\right] \tag{4.22}$$

The CVA_t can be seen as a call option, as it only takes other values than 0 in case of default. The premium of such an option is the subtracted value of the default free IRS contract compared to the contract with default risk. The effect of adding default risk will require an interest rate model to price the value at default. Hence, pricing an IRS contract with CVA is not model independent, as is the case for the IRS price is in itself (Brigo et al., 2013). A discrete version of equation 4.22 is a good approximation with a decent number of steps, as postponing the time of default to the next T_i after default will then have a fading effect on the price (Brigo et al., 2013)

$$CVA_t = LGD \cdot \sum_{i=1}^{m} \tau \cdot E_t\left[1_{\{T_{i-1}<u<T_i\}} \cdot D(t,T_i) \cdot EE_i\right] \tag{4.23}$$

with $\tau = T_i - T_{i-1}$. It is possible to factor the indicator function $1_{\{T_{i-1}<u<T_i\}}$, inside the summation into the probability of default, λ_i, at each point in time under the assumption, that exposure and default intensity is uncorrelated (Brigo et al., 2013). This would mean that default intensity would be model independent. However, as already shown in Section 2.4, there is a significant correlation between default intensity and the interest rates, which determine the value of default free the swap contract. This is the argument in favor of including a model for default intensity and model correlation with the interest rate model (Gregory, 2015a). Then equation 4.23 can be rewritten as (Gregory, 2015a), but including discounting

$$CVA_t = LGD \cdot \sum_{i=1}^{m} \tau \cdot D(t,T_i) \cdot EE_i \cdot \lambda_i \tag{4.24}$$

assuming a fixed LGD as discussed in Section 2.5.4.

(Gregory, 2015a) further suggest that instead of using the EE_i to account for the exposure from step $i-1$ to i, one should instead use the average of EE_i and EE_{i-1}, which gives a more precise discrete approximation of CVA. Hence, this thesis uses the average exposure at each step, EE_i^* instead in equation 4.24. The discount factors used are the forward rates for T_i set at t (Gregory, 2015a). This means that these are determined under the $\mathbb{Q}^T$-Martingale evaluated in Section 1.3.4.

(Gregory, 2015c) provides a simple analytical approximation of CVA, which equals the CDS spread times the average expected exposure. The average exposure is calculated from a chain of swaption prices of different maturities approximating the exposure profile as described in Section 4.2.1. However, on top the shortcoming mentioned in Section 4.2.1, CDS prices are very illiquid and only available for very few counterparties. Hence, the approximation would not be very useful in practice.

4.3 Funding Value Adjustment

Funding of a derivative contract has always been necessary. Cash flows of a contract requires funding, collateral posted requires funding, and hedge of short position calls for funding (Brigo et al., 2013). For derivative contracts like swaps, cash payments flow from both counterparties, collateral agreements can be bilateral, and long positions requires a short hedge. These effects make it likely that there will also be both cash inflows and outflows. Cash outflows require funding while cash inflows reduce funding requirements or can be lend out (Ruiz, 2015).

Before the financial crisis of 2008, the market considered financial institutions risk free. Hence, there was no significant difference between the borrowing and lending rate in the market, both close to the interbank rate[77] (Gregory, 2015a). In that case, unlimited funding and lending at the risk free rate is possible (Hull, 2012). Hence, the mismatch of cash flows had no influence on the value, as this could be hedged away perfectly at no additional costs.

As described in Section 2.1, it is now very clear that financial institutions are not risk free. Funding rates for financial institutions have become individual with respect to their actual credit quality, while lending out excess liquidity at lowest risk[78] would earn far less. Hence, funding has become an issue increasing the costs of derivative contracts. Management of this issue cannot just be passed on to the treasury department of the financial institution. It must be included in the price of the derivatives, if a contract should be profitable (Brigo et al., 2013). As funding rates are different for each financial institution, the price of an identical contract will be different. Thus, violating the law of one price ruling the BS framework (Kenyon and Stamm, 2012). Besides, there is a difference between unsecured funding and repo funding secured with collateral (Piterbarg, 2010). When the counterparty post collateral, they will usually be entitled to a return on such collateral often equal to the OIS rate plus/minus a spread. However, when the accrued interests are lower than the discount rate[79], posting collateral is costly for the counterparty (Gregory, 2015a). Funding issues are different for collateralized and uncollateralized derivative contracts, which apart from funding of collateral also accounts for funding of any gap risk. Adjusting for these issues have significant effect on the price (Piterbarg,

[77] Interbank rates like the LIBOR rate was for the same reason seen as the risk free rate.
[78] The OIS rate.
[79] The OIS rate is used.

2010).

There is still much debate about, whether to charge FVA on OTC derivatives. Critics as (Hull and White, 2012) oppose the inclusion of FVA. They argue that the reason CVA and DVA should be included is that they include the economic value of default issues of both parties in the contract. They add that a part of the FVA captures the risk that the financial institution might default on its other liabilities in case of default, which should increase their funding costs. This change should account for the same as the FVA, which means including FVA would be double counting, when valuing liabilities under the $\mathbb{Q}$-Martingale (Hull and White, 2012). (Ruiz, 2015) instead argues that for the DVA and FVA to be the same, the CDS spread when a counterparty hedge against our default risk should be equal to our bond yield spread over the risk free rate, which should cover the same default risk. However, due to the lack of liquidity in bond market, these yields are higher (Gregory, 2015a), which requires at least a liquidity adjustment when including DVA (Ruiz, 2015). This part of DVA (Hull and White, 2014a) finds justifiable. For the remaining part of FVA, (Hull and White, 2012) sees the funding costs and derivative desk as two different things. Allocating the overall funding costs to each single derivative would be the same as foregoing investments with positive net present value, because inclusion of the funding issues, which should be separated (Berk and DeMarzo, 2013). (Ruiz, 2015) instead argues that the funding can be seen as the costs of selling a derivative, equivalent to material costs of manufacturing a physical product. Hence, if funding costs are not included, derivatives might be sold with a negative value incurred for the financial institution.

(Burgard and Kjaer, 2011b) argue, that it is possible to eliminate funding costs in two different ways, which allows funding costs to be ignored:

1. If the value of the derivative is positive and there is a gap between the exposure and collateral this difference requires funding. Assuming that this derivative is usable as collateral somewhere else or the value can be repoed due to its positive value, this will provide the necessary funding, just as assets as stocks are self-financing.

2. The difference between the funding liability and the derivative value at default can be balanced away by actively trading own bonds of different seniority, so that there should not be charged higher funding costs.

(Gregory, 2015a) strongly rejects these two methods. Assuming that positive derivative values is usable as collateral for funding is far from reality. (Burgard and Kjaer, 2011b) admits this including the fact, that even if allowed, the collateral value of the derivative would be subject to substantial haircuts creating a gap in collateralization. (Burgard and Kjaer, 2011b) argues, that taking a portfolio perspective will partially solve this problem. The issue of free trade of own bonds seems very naïve due to lack of liquidity in the bond market in general.

DVA is not included in this thesis. Hence, it seems even more relevant to include FVA to account for the costs from the risk of own default. This also seems to be the industry approach towards FVA, although there is not yet a clear market standard on the method of doing so (Gregory, 2015a). 50% of questioned banks by (Solum, 2014) already include FVA while another 40% have planned to implement FVA.

The FVA is covering different types of funding adjustments, which also might depend on collateralization of the derivative contract. (Ruiz, 2015) groups this into 3 sub categories: CollVA, HVA and LVA. First subsection evaluates these. Second subsection provides suggestions to estimate CollVA and HVA with MC simulation building and expanding on the exposure calculations evaluated in Section 4.2.1. Note that the mismatch between cash and collateral comes from both additional cash required or received. Hence, the FVA has both a negative funding adjustment called the Funding Cost Adjustment (FCA) and Funding Benefit Adjustment (FBA). The sum of these makes the FVA and can thus be both a negative or positive price adjustment to the derivative contract. Benefitting from collateral funding wise is only possible assuming rehypothecation (Gregory, 2015a).

4.3.1 Sources of Funding Risk

The different sources of FVA has been divided into Collateral Value Adjustment (CollVA), Hedging Value Adjustment (HVA), and Liquidity Value Adjustment (LVA). The background for each adjustment term follows below.

Collateral Value Adjustment

If an OTC trade is made with a corporate counterparty, the financial institution will likely want to hedge away the market risk of this contract with a standardized contract with another financial intermediary as shown in Appendix A. However, swaps trades nominated in EUR and USD with financial counterparties has clearing obligations with a central clearing counterparty (CCP). This means that there will be a fixed collateral agreement including initial margins[80] required from the CCP (ESMA, 2015). Hence, any discrepancy between the collateral agreements with the counterparty and the agreement with the CCP will incur funding costs. If no hedge with a CCP is made all collateral demands with the counterparty must be funded. If rehypothecation is not allowed this is also the case, but in addition, collateral held does also not provide any funding benefit (Ruiz, 2015).

Allowing collateral in form of other assets than cash and in different currencies might as well result in mismatches creating funding issues (Brigo et al., 2013). However, this thesis assumes only cash settlement in domestic currency.

Even if identical collateral posting happens simultaneously from the corporate counterparty to the CCP, the collateral accrual rates paid by the financial institution might be higher, than what it accrues on their collateral posted at the CCP (Gregory, 2015a).

Modelling the present value of the mismatches in collateral and collateral accrual rates constitutes the CollVA. With the above in mind, market hedging with an offsetting contract at the CCP in this thesis includes bilateral collateral agreements and include initial margin posted to the CCP.

[80] Initial margin issues are evaluated in Section 4.4.

Hedging Value Adjustment

A receiver IRS contract, assuming an upward sloping yield curve will mean that cash outflows are smaller than cash inflows at the beginning of the contract resulting in net cash outflow needing funding. If hedging this receiver IRS with a payer IRS with longer tenor on the floating leg, this will as well create some cash outflow mismatches requiring funding (Ruiz, 2015). With the BS model assuming unlimited borrowing and lending at the risk free rate, hedging a trade or a cash flow with a replicating portfolio is possible at the cost of the risk free rate. Due to the no-arbitrage condition, this will also be the payoff of the hedged position. Hence, no funding costs (Hull, 2012).

In reality, Section 2.1 shows, how the market liquidity died out and funding rates clearly rose above the risk free rate. Hence, the costs of funding and lending are different from one another, as described in Section 4.3. It is clear, that this difference will have an effect on the mismatch in cash flows described above, which gives an additional funding cost or benefit depending on the cash flows (Ruiz, 2015). HVA covers the net present value of funding costs and funding benefits of cash flow mismatches through the lifetime of the derivative contract.

Liquidity Value Adjustment

The introduction to FVA already mentioned that the credit risk premium of a firm differs when looking at the bond yield premium[81] or the credit spread paid on CDS protection on the company. As the risk premium of default must be the same in both cases, the difference observed comes from the lack of liquidity in the bond market increasing this spread and therefore also funding costs (Gregory, 2015a). This is an extension to the CVA and the DVA (Hull and White, 2014a). Including risk of own default, the spread between bond yield and CDS spread provides a discount to DVA (Ruiz, 2015), as the financial institution fund itself with the funding spread, which is higher than the CDS spread on the financial institution, hence leading to higher funding costs. Funding does not solely come from issuance of bonds, but can include many different types of short-term borrowings, which might potentially lower funding costs. Hence, the internal funding curve of the financial institution might be lower than the bond yield curve. Still, the liquidity spread seems apparent. From the CVA point, it is far more difficult to adjust for the counterparty funding curve, which is unknown, even if knowing or estimating the counterparty funding curve by a market proxy (Gregory, 2015a). (Morini and Prampolini, 2011) illustrates different liquidity effects with pure positive or negative exposure profiles and concludes that an extension for derivative contracts like swaps where exposure can change between positive and negative is yet to be developed.

Disregarding DVA, the LVA on this side of the adjustment is not relevant to make in this thesis. Estimating the CVA side LVA is extremely complex. First, choosing a funding curve proxy for the counterparty might not be very precise. Second, adding the liquidity effect of swap contracts is yet unsolved (Morini and Prampolini, 2011). Hence, this thesis also disregards the liquidity adjustment for the counterparty side. In most cases, LVA has a negligible effect on the derivative value in practice (Ruiz, 2015), and ignoring LVA for both parties might even

[81] Premium on top of the risk free rate accounting for the default risk of the company.

out much of the value adjustment. (Ruiz, 2015) furthermore argues that including CollVA and HVA, the effect of DVA and LVA will only reflect future liabilities, and charging such up front would be the same as borrowing the present value of expected future liabilities and hold the cash. In practice, liability funding happens ad-hoc as they occur rather than upfront, when entering the deal, as doing otherwise would result in huge cash holdings.

4.3.2 Estimating FVA

The three sources of funding risk adjustment described in Section 4.3.1 can simply be estimated independently in the simulation and added together to get the total FVA (Ruiz, 2015)

$$FVA = CollVA + HVA + LVA$$

where this thesis ignores LVA[82]. The funding adjustments depend on the expected future value, which is simply the sum of the negative expected exposure and the positive expected exposure, which are used to estimate respectively CVA in Section 4.2.2 and DVA in Appendix E.1 (Gregory, 2015a). EE_t is an approximate base to estimate the FCA, while NEE_t is the same to estimate the FBA. This, however, is the uncollateralized case, where $FVA = HVA$. Making a general and more precise calculation of FVA account for the fact that collateralized trades, require estimation of HVA and CollVA based on the hedging cash and collateral surplus/deficit depending on the exposure (Ruiz, 2015). These estimations follow below.

Estimating CollVA

The CollVA depends on the expected collateral exposure after margining in each period

$$EE_t^{Collateral} = C_i^{recieved} - C_i^{paid}$$

Assuming bilateral collateral agreements, if the market value of the trade with the counterparty is positive (negative), then $C_i^{recieved}$ is collateral posted by the corporate counterparty (CCP hedge), while C_i^{paid} is the collateral to be posted to the CCP hedge (corporate counterparty)[83]. In a unilateral collateral agreement with the counterparty collateral is only received. We can define

$$PEE_t^{Collateral} = \left[C_i^{recieved} - C_i^{paid}\right]^+$$

as the maximum of 0 and the collateral gap to fund, while

$$NEE_t^{Collateral} = \left[C_i^{recieved} - C_i^{paid}\right]^-$$

[82]See Section 4.3.1.

[83]Depending on the precision of the hedge there might be cases, where both swap contracts are either positive or negative resulting in collateral exclusively paid or received.

is the minimum of 0 and the excess collateral held. (Ruiz, 2015) separates the CollVA into Collateral Cost Adjustment (CollCA) and Collateral Benefit Adjustment (CollBA),

$$CollVA = CollCA + CollBA$$

The CollCA (CollBA) will depend on the positive (negative) expected collateral exposure

$$CollCA = \sum_{i=1}^{m} \tau \cdot PEE_i^{Collateral} \cdot D\left(t, T_{i+1}\right) \cdot s_i^{borrow}$$

$$CollBA = \sum_{i=1}^{m} \tau \cdot NEE_i^{Collateral} \cdot D\left(t, T_{i+1}\right) \cdot s_i^{lend}$$

with s_i^{borrow} and s_i^{lend} being the borrowing and lending rate (Ruiz, 2015) and $\tau = T_i - T_{i-1}$. These effects, account for a collateral deficit to be funded at T_{i+1} at s_i^{borrow}, while a collateral surplus accrues s_i^{lend} received at T_{i+1}.

As mentioned in Section 4.3.1, collateral agreements often obligate the holder of collateral to accrue interest to counterparty on their posted collateral (Gregory, 2015a). If this rate provided by the hedging position is different from the agreement with the counterparty, then this creates an additional cost to CollVA, which we refer to as Accrued Interests,

$$AI_i = \tau \cdot -1_{\{V_i > 0\}} \left(-C_{i-1}^{CCP} \cdot r_{i-1}^{AI,\,CCP} + C_{i-1}^{C} \cdot r_{i-1}^{AI,\,c}\right) + \tau \cdot 1_{\{V_{i-1} < 0\}} \left(C_{i-1}^{CCP} \cdot r_{i-1}^{AI,\,CCP} - C_{i-1}^{C} \cdot r_{i-1}^{AI,\,c}\right)$$

At time T_{i-1}, if the value and thus exposure is positive, then the financial institution holds collateral from the corporate counterparty C^C, on which it pays an interest rate of $r^{AI,\,c}$, while posting C^{CCP} at the CCP receiving a rate of $r^{AI,\,CCP}$ and vice versa for negative exposure. Furthermore, all positive AI_i gap requires funding

$$AI_i^* = AI_i + 1_{\{AI_i > 0\}} \cdot AI_i \cdot S_i^{borrow}$$

while negative AI_i gap instead earn the lending rate. Hence,

$$CollVA = CollCA + CollBA + AI$$

with

$$AI = \sum_{i=1}^{m} \tau \cdot AI_i^* \cdot D\left(t, T_i\right)$$

This only allows for different accruing interest rates agreements with respectively counterparty and CCP, but not for different accruing rates between the counterparty and the financial institution.

Estimating HVA

HVA depends on the expected cash flow exposure in each period before margining. If netting exposure of the swap contract with the counterparty and the hedge with a CCP is positive, the cash flow exposure is also positive and requires funding[84]. The total cash flow exposure of the trade and hedge at each point in time shows the mismatch requiring funding or providing excess cash. (Ruiz, 2015) estimates HVA as

$$HVA = HCA + HBA$$

with HCA the funding costs of mismatch and HBA the funding benefits. (Gregory, 2015a) provides the following discrete approximations

$$HCA = \sum_{i=1}^{m} \tau \cdot PEE_t^{Hedge} \cdot D(t, T_{i+1}) \cdot s_i^{borrow}$$

$$HBA = \sum_{i=1}^{m} \tau \cdot NEE_i^{Hedge} \cdot D(t, T_{i+1}) \cdot s_i^{lend}$$

$$PEE_i^{Hedge} = \left[EE_i^c + EE_i^{CCP}\right]^+$$

$$NEE_i^{Hedge} = \left[EE_i^c + EE_i^{CCP}\right]^-$$

meaning, that if at time i the uncollateralized net exposure of the trade and hedge as described above is positive, this contributes to the PEE_i^{Hedge} while contributing to the NEE_i^{Hedge} if negative.

4.4 Margin Value Adjustment

As argued in Section 4.2.1, variation margin with collateral still leaves gap risk due to non-perfect collateralization (margin periods, MTA, close-outs, etc.). Hence, additional collateral to cover the gap risk might be required upfront. This collateral is the initial margin. However, initial margin only reduces exposure if segregated, meaning no rehypothecation and that the party posting initial margin has legal priority to reclaim it from the counterparty in case of counterparty default (Gregory, 2015a). The exposure calculation in a bilateral collateral agreement with non-segregated initial margins posted to the counterparty below demonstrate the need to segregate the initial margin (Gregory, 2015a)

$$EE_t = \left[V_i - C_i^{held} + IM_{non-segregated}^{posted}\right]^+$$

If the financial institution has a derivative contract with positive value, but also have initial margin posted at the counterparty without segregation, then this just reduce the net collateral held against the positive market value and increases the exposure (similar to a threshold). If both parties post initial margin, which is not segregated, this would have no effect on exposure, as $IM_{non-segregated}^{posted} - IM_{non-segregated}^{recieved} = 0$. Trades with a CCP would always require initial margin to be posted (Ruiz, 2015). For trades with corporate counterparties unilateral or bilateral post of initial margin if any can be negotiated, which might depend on the perceived CVA of

[84] The case of course vice versa for a negative net exposure.

the two parties. Initial margins can also be contingent to credit ratings (Gregory, 2015a).

Initial margin might be a fixed amount set in the contract as a percentage of the notional value. However, it might also depend on a dynamic VaR estimation, so in order to reduce the expected exposure to a certain level of risk for a rolling period (Gregory, 2015a). (Kenyon and Green, 2014), provides methods to calculate VaR within a simulation without double simulations[85]. For simplifications, this thesis uses Potential Future Exposure, PFE, as the approximation for the VaR. PFE is a confidence level for expected exposure for a certain period into the future given the distribution of future exposure (Brigo et al., 2013). Assuming that the PFE follows a normal distribution as (Ruiz, 2015),

$$PFE_t = \phi^{-1}(1-\alpha)\sqrt{\frac{Margin\,Period}{Trading\,Days}} \cdot \sigma$$

where $\phi^{-1}(1-\alpha)$ is the inverse standard normal distribution with α confidence level, the $Margin\ Period$ is the number of days between each adjustment of the initial margin, $Trading\ Days$[86] is the number of days a year with active trading, and σ is the volatility of the swap exposure. This thesis defines the initial margin as the PFE of the current exposure, so that it varies over time, even when assuming a fixed volatility of the swap exposure. If using time-inhomogeneous risk factor models, volatility could vary based on earlier periods in the simulation and PFE could be set with respect to the notional value of the contract, improving the efficiency of initial margin. (BCBS, 2015) recommends that 10-day margin period of risk be used in bilateral agreements, with a distribution and volatility calibrated to historical data including a financial stress period. To account for the lack of a varying volatility, this thesis set the margin period of risk equal to the margin period a quarter and set $\alpha = 1\%$. CCPs normally use dynamic initial margins with 5-day time horizon (Gregory, 2015a). Hence, the dynamic initial margin provided in this section is to be posted at the CCP in the estimation model. This should also account for the marginal contribution to the CCP default fund, which is not accounted for separately (Gregory, 2015a). The CCP on the contrary does not need to post any initial margin themselves, as they have very low counterparty risk.

Posting initial margin when segregated requires funding of the entire margin. Thus, it should be included in the pricing of a derivative contract (Gregory, 2015a). The present value of these costs form the Margin Value Adjustment (MVA).

4.4.1 Estimating MVA

(Gregory, 2015a) provides the discrete approximation for MVA

$$MVA = \sum_{i=1}^{m} \tau \cdot EIM_i \cdot D(t, T_i) \cdot \left(s_{i-1}^{borrow} - r_{i-1}^{IA}\right) \cdot (1 - \lambda_{i-1}) \tag{4.25}$$

where s_i^{borrow} is the funding rate, r^{IA} is the interest earned or paid on initial margin, λ_{i-1} the

[85] Simulations inside the simulations.

[86] Trading days per year is fixed at 260 as described in Section 1.3.4.

probability of counterparty default[87] and

$$EIM_i = PFE_i\left(T_i, T_{i+Margin\,Period}\right)$$

if the initial margin is set dynamical and

$$EIM_i = IM_0$$

if the initial margin is fixed at initiation of the derivative contract.

The formulas provided above only cover the costs of initial margin posted at the corporate counterparty and the CCP. However, if the counterparty post initial margin to the financial institution as well, then accrued interests on these require funding as well, because rehypothecation is not possible. Hence, financing interests to be paid on the initial margin posted by the counterparty cannot be generated from the collateral itself. If the accrued rate of the CCP and the corporate counterparty is different, then their respective contribution with their respective interest rates must be separately calculated using equation 4.25.

4.5 Capital Value Adjustment

Basel provides an overall regulatory framework of reporting and determining capital requirements for financial institutions. The goal is to ensure, that financial institutions have reasonable estimates of their risks and a sufficient level of capital to cover unexpected losses. Regulatory capital requirements force financial institutions to finance loans or derivative contracts partially with equity capital. The main risks of financial institutions relate to credit risk, market risk, liquidity risk, operational risk and geographical risk (Choudhry et al., 2012). For OTC derivatives, required capital mainly focus on the market risk and credit risk. To maximize the shareholder value, the return on the required regulatory capital tied to a derivative contract must at least equal the target return demanded by shareholders of the financial institution.

Measures as RARoC[88] used to be the guidelines account for the costs of capital used for the trading desk (Ruiz, 2015). Managing this return on capital used to be ad-hoc with capital limits or incremental capital requirements for different types of derivatives and counterparties. However, the capital requirements of Basel III implemented over the last years has significantly increased the capital requirements and complicated measurement of the required regulatory capital. Along with higher capital requirements, costs of raising new capital have increased in light of the financial crisis (Gregory, 2015a). Hence, fostering the need for a Capital Value Adjustment (KVA)[89], ensuring the correct capital charge on derivative trades has been crucial to manage the overall RARoC goals of financial institutions correctly (Ruiz, 2015). KVA does

[87] (Gregory, 2015a) use the joint default probability of own default and counterparty default. As this thesis does not estimate risk of own default, this is left out of the calculation.

[88] Risk Adjusted Return on Capital: $RARoC = \frac{Expected\,Profit}{Required\,Capital}$.

[89] Using "K" for Capital in KVA is to distinguish the name from CVA (Kenyon et al., 2014). Even if this make little sense in English, we use this abbreviation, as we consider ourselves more loyal to finance jargon than the English language.

not just estimate current capital requirements, but the total expected capital requirements of a derivative contract until maturity (Sherif, 2015).

Estimating KVA is more subjective than other xVA terms, as predicting future regulatory capital requirements is extremely difficult. Regulatory changes might revise whether available hedge instruments for counterparty risk can achieve relief on regulatory capital, change regulations on required capital, or change capital charge exceptions on certain instruments (Gregory, 2015a). (Sherif, 2015) discusses, how counterparties current exempted of the CVA capital charge in EU, will probably not be able to keep this advantage in the future. Hence, trying to include predictions on how regulations might change and affect required capital is very difficult to price and will give very subjective KVA estimates.

Regulatory capital result in individual issue costs of derivatives for different financial institutions, which means that there cannot be a single risk-neutral derivative price assumed with the BS model (Kenyon and Green, 2014). Besides, KVA gives concerns about overlaps with other xVAs terms. One issue being double counting of own credit risk, if including DVA. However, KVA is beyond the BS model, knowing that not all risk is perfectly possible to hedge. Setting capital aside covers unexpected losses, which might occur along the way. Hence, setting stricter capital constraints would decrease the DVA not making them count the same thing (Ruiz, 2015). As this thesis focus on unilateral counterparty credit risk, the overlap to DVA is not really an issue.

CVA capital charge is a part of the KVA estimating the capital to cover the risk of CVA changes over time, which some argue is already included in the CVA, which equals the costs of hedging the counterparty risk (Gregory, 2015a). However, setting aside capital for losses does not mean that the will not occur. Hence, the costs of hedging of expected losses also still exist (Sherif, 2015). Besides, only hedging CVA with a single-name CDS on the counterparty provides a full capital relief. But such instruments are only available on a few major counterparties (Gregory, 2015a).

The capital required to be held against a derivative contract is usually the maximum of the economic capital, which is the internal target of the financial institution for capital held and the required capital by the regulatory authority. In reality, the regulatory requirements attract all attention by financial institutions (Ruiz, 2015). With many financial institutions criticizing e.g. the CVA capital charge of being too high[90], they would likely perceive regulatory capital as the largest compared to their internal economic capital measure. (Brigo et al., 2013) criticizes the way regulators try to standardize models to calculate required capital, as this might cause misleading capital requirements depending on the regulatory framework used by a financial institution. Inadequate capital requirements can lead to regulatory arbitrage exploitation increasing procyclicality and bubbles in the financial industry. Nevertheless, calculation of regulatory capital exists. This thesis assumes required regulatory capital to be the base for estimation of KVA rather than having to discuss economic capital targets.

This section first presents methods to calculate the regulatory capital under relevant assumption on the regulatory framework for each source of capital charge (standardized or advanced).

[90]See Section 4.5.1.

Second subsection provides a method to estimate KVA based on regulatory capital and discusses measures for costs of capital.

4.5.1 Regulatory Capital

There are three main sources determining regulatory capital (Gregory, 2015a):

- Counterparty Credit Risk capital charge (CCR)
- CVA capital charge (CVA Capital)
- Market Risk capital charge (MR).

The methods for calculating these capital charges depend on the regulatory status of the financial institution. In general, financial institutions can either use the Standardized Approach or apply for the Advanced Approach (Choudhry et al., 2012). To make things even more complex, the use of a standard approach and advanced approach might differ for the different sources of regulatory capital. For counterparty credit risk an Internal Rating Based Approach (IRB) allow the use internal default probability measures for credit risk[91], while the Standardized Approach does not. If furthermore allowed to use the Internal Model Method (IMM) for credit risk, this allows use of internal measures of exposure at default (Ruiz, 2015). Similar for market risk capital charges, a financial institution can use either the Standardized Measurement Method (SMM) or seek approval to use the Internal Model Approach (IMA) by national regulation authorities (Ruiz, 2015), which allows the use of an internal model to estimate the VaR for market risk (Kenyon and Green, 2014). Using advanced approaches for respectively counterparty credit risk (IMM) and market risk (IMA) requires separate approval (Kenyon and Stamm, 2012).

This thesis focus on a medium-sized financial institution with IMA approval for estimation of market risk but without IMM approval for counterparty credit risk. Thus, the standardized methods are used to determine CCR charge and CVA capital based on advanced internal ratings of credit risk, while the internal model method is used for the MR charge.

Ideally the MR risk is completely hedged away with a reverse swap contract with a CCP. (Kenyon et al., 2014), however, proves that this is only true if the hedge is perfect. As discussed in 4.3.2 a perfect hedge is not always available. Thus, some market risk still exists due to the mismatch of trade and hedge. (Kenyon et al., 2014) provides evidence that this market risk has a significant effect on the total value of an IRS contract when including xVAs. To account for this issue, we includes a simplified model estimation of the MR charge.

The regulatory framework of Basel includes two additional components of regulatory capital: The Incremental Risk Charge (IRC) and the Operational Risk (OR). The marginal effect of a trade will have no significant effect on the operational costs. Hence, OR charge is disregarded as in (Ruiz, 2015). IRC is a market risk measure VaR measure with a longer time horizon than the other VaR measures defining regulatory capital. The purpose of the IRC charge is to capture effects of larger price moves and potential loss of liquidity, which are unlikely to happen within a short time horizon. The IRC charge also includes a component called the Comprehensive

[91] Advanced IRB if allowing internal measures of both default intensity and LGD.

Risk Measure (CRM), which has the purpose of capturing correlation effects on basket credit products. (Ruiz, 2015) argues that this measure in many cases is negligible for most instruments. Only some instruments would have a high IRC charge due to low liquidity (covered by the IRC) or because the product has strong inner correlations as CDO contracts (covered by the CRM). To avoid unnecessary complexity from introducing another IMA model for the IRC charge, this thesis disregards the IRC capital requirement, as it would not be worth considering the effect on an IRS or a CSS contract.

Basel defines required capital as 8% of the Risk Weighted Assets, RWA. All capital requirements are determined in terms of RWA (Choudhry et al., 2012).

CCR Charge

The CCR charge has the goal of quantifying the default risk of counterparties. The measure is normally on a portfolio level, but for this thesis, we look at the isolated effect of a single trade. The CCR charge is given by

$$RWA_{CCR} = RW \cdot EAD$$

where RW is the risk weight of the counterparty based on their credit quality. This measurement is identical for all internal rating approaches and given by the formula (Kenyon et al., 2014)

$$RW = 12.5 \cdot LGD \cdot \left(\phi \left(\frac{\phi^{-1}(\lambda)}{\sqrt{1-\rho}} + \phi^{-1}(0.999) \sqrt{\frac{\rho}{1-\rho}} \right) - \lambda \right) \cdot \frac{1 + (M - 2.5)\, b}{1 - 1.5b} \tag{4.26}$$

with

$$\rho = 0.12 \cdot \frac{1 - e^{-50 \cdot \lambda}}{1 - e^{-50}} + 0.24 \frac{1 - \left(1 - e^{-50 \cdot \lambda}\right)}{1 - e^{-50}}$$

$$b = (0.11852 - 0.0578 \cdot ln\, \lambda)^2$$

and λ the expected default intensity over one year, M is the maturity of the contract, and ϕ is the cumulative standard normal distribution. We use the closed form solution for survival probability in equation 2.16 assuming the JCIR model to derive the expected one-year survival probability. This is subtracted from 1 to provide λ.

Basel requires a floor on the default intensity (Kenyon and Green, 2014).

$$\lambda = [0.03, E(\lambda)]^+$$

EAD is the exposure at default. With the IMM approach, an internal model can be used to calculate EAD (Ruiz, 2015). If a financial institution does not have IMM approval, as is the case used in this thesis, there are currently three different standardized methods (Gregory, 2015a): Standardized Method (SM), Current Exposure Method (CEM) and the Standardized Approach for Counterparty Credit Risk (SA-CCR)[92]. SA-CCR will replace CEM and SM from 2017 (Ruiz, 2015). Hence, the SA-CCR method will be most adequate to use in this thesis.

SA-CCR calculates the EAD from the replacement costs and from future potential exposure.

[92] CEM method being more advanced than SM method and SA-CCR being the most sophisticated.

However, it is important to notice, that these two measures differ from EE_t and PFE_t. The replacement costs is calculated as (BCBS-279, 2014)

$$RC = [V - IM]^+$$

for uncollateralized contracts[93] and

$$RC = [V - IM, Thr + MTA - NICA]^+ \tag{4.27}$$

for collateralized contracts[94]. Thr is the threshold, MTA is the minimum transfer amount and

$$NICA_t = C_t^{recived} - C_{t_{non-segregrated}}^{posted} \tag{4.28}$$

Hence,

$$EAD = \alpha \cdot (RC_t + PFE_t^*) \tag{4.29}$$

α is multiplier also used in the IMM approach, where it varies due to portfolio size, default intensity, correlation, and confidence level of the model (Gregory, 2015a). For SA-CCR, $\alpha = 1.4$ (Ruiz, 2015). PFE_t is not an internal model estimate as in Section 4.4.1, but is an add-on value defined by (BCBS-279, 2014) as

$$PFE_t^* = mult \cdot AddOn$$

where

$$mult = min\left[1.5\% + (1 - 5\%)\, e^{\frac{V-C}{2\cdot(1-5\%)\cdot AddOn}}\right]$$

$mult$ is a multiplier ensuring that contracts with positive collateralized exposure are charged 1, meaning the full $AddOn$ while over-collateralized contracts with NEE_t are charged less, as they are less risky (BCBS-279, 2014)[95]. The $AddOn$ depends on the primary underlying risk factors divided into asset classes

$$AddOn = SF \cdot EffetiveNotional$$

Both factors depend on the underlying asset class of the derivative. SF has the purpose of approximating the positive expected exposure, EE_t, of the notional value by accounting for the volatility of the underlying asset class. For interest rate products $SF = 0.5\%$ and for FX products $F = 4\%$. As a CCS contracts depends on both risk factors, the $AddOn$ must be calculated for both risk factors (BCBS-279, 2014)[96]. The effective notional is calculated by the formula[97] (BCBS-279, 2014)

$$EffetiveNotional = \delta \cdot d \cdot MF$$

[93]However, allowing for an initial margin.
[94]If collateral in other forms than cash is possible, this requires the appropriate haircuts on IM and C.
[95]If $AddOn$ is very close to 0, calculating $mult$ in the model will not be possible. However, this reflects $mult$ being very close to 1, which can then be assumed when $AddOn$ is too small.
[96]In this thesis $AddOn$ for CCS is calculated for respectively the effect of domestic interest rate (EONIA) and FX rate.
[97]The $EffectiveNotional$ is in absolute values if the product includes FX risk.

where $\delta = \pm 1$ is a supervisory delta with sign depending on a positive or negative relation between the derivative value and the underlying risk factor. d is a trade-level adjusted discount factor

$$d = \frac{1 - e^{-0.05M}}{0.05 \cdot M}$$

if the swap is a spot trade (Kenyon and Green, 2014)[98]. M is the maturity of the contract. MF represents the time risk horizon and differs between trades with and depends on trade and collateral agreement[99].

$$MF^{Uncollateralized} = \sqrt{\frac{min\,[M; Trading\,Days]}{Trading\,Days}}$$

$$MF^{Collateralized} = \frac{2}{3}\sqrt{\frac{MPR}{Trading\,Days}}$$

if the swap contract is a spot trade. MPR depends on the margin frequency. Margining is less frequent than daily, $MPR = 20$ for non-CCP trades and $MPR = 5$ for CCP trades[100] (BCBS-279, 2014).

Trades cleared at a CCP used as hedging instruments in this thesis, will also be subject to a capital charge. However, as central clearing houses have a very low risk, (BCBS-282, 2014) only charge 2% risk weight to EAD calculated with the SA-CCR method. M must minimum be 10 days (BCBS-282, 2014).

CVA Charge

As mentioned in Section 2.1, two thirds of the increase in counterparty risk came from higher credit risk volatility, which was not covered by the capital charges already in place. Instead of adjusting the existing VaR calculation, Basel III added the CVA capital charge as an additional charge to take CVA risk into account (Gregory, 2015a). Not having IMM approval, this subsection focus on the Standardized CVA charge formula, which should approximate the 10-day VaR[101] of CVA with 99% confidence (Ruiz, 2015). EU regulation, exempt CCP transactions between clearing member and clients from the CVA charge (Gregory, 2015a). Hence, this thesis ignores such charge for the hedge with a CCP. However, to account for potential changes in future regulation, the current exemption for non-financial counterparties below the clearing obligation in the EU is ignored, as these indeed carry the highest counterparty risk and because the EBA[102] suggests this exemption be removed (EBA, 2015).

[98] We did discover a discrepancy between (Kenyon and Green, 2014) including M in the divisor of d and (BCBS-279, 2014) not doing so. Testing this issue in our model, there is little doubt, that (BCBS-279, 2014) has made a typo in their guidelines by excluding M in the divisor resulting in discount factors well above 1.

[99] This thesis assumes 260 trading days as described in Section 1.3.5.

[100] If there are outstanding margining and valuation disputes, the MPR should be the double number of days.

[101] See Section 4.5.1 for a specification of the VaR-measure.

[102] European Banking Authority.

(Kenyon et al., 2014) provides the standardized formula

$$RWA_{CVA} = \sqrt{MF} \cdot 2.33 \cdot \left\{ \left(0.5RW \cdot \left(M \cdot EAD - M^H \cdot H^H\right) - RW_\ell^H \cdot M_\ell^H \cdot H_\ell^H\right)^2 + 0.75RW^2 \left(M \cdot EAD - M^H H^H\right)^2 \right\}^{\frac{1}{2}}$$

with RW the counterparty risk weight, M the swap contract maturity and MF the time risk horizon equal to those described in Section 4.5.1. EAD also equals equation 4.29 in Section 4.5.1 but is discounted with d from Section 4.5.1.

The CVA capital charge allows for a direct hedge of the counterparty risk with a single-name CDS and for partial hedge with an index CDS. $\left(0.5RW \cdot \left(M \cdot EAD - M^H \cdot H^H\right) - RW_\ell^H \cdot M_\ell^H \cdot H_\ell^H\right)^2$ can be seen as the systematic risk of the CVA, which can be fully hedged with a single-name CDS or an Index CDS. $0.75RW^2 \cdot \left(M \cdot EAD - M^H \cdot H^H\right)^2$ represents the unsystematic CVA risk, which can only be fully hedged with a single-name CDS, as it is counterparty specific (Gregory, 2015a).

H^H is the CDS hedge notional, while M^H is the hedge maturity. This is similar with M_ℓ^H and H_ℓ^H for the Index CDS hedges, but includes the risk weight of the CDS Index RW_ℓ^H. To achieve a full hedge effect, the choice of notional and maturity for the hedge must match respectively EAD and maturity for the swap contract (Gregory, 2015a). In absence of a hedge of counterparty risk with the above mentioned instruments the formula reduces to (Kenyon et al., 2014)

$$RWA_{CVA} = 2.33\sqrt{MF} \left\{ (0.5RW \cdot M \cdot EAD)^2 + 0.75RW^2 \cdot (M \cdot EAD)^2 \right\}^{\frac{1}{2}}$$

When using this standardized formula to calculate the CVA capital charge for the entire trading book of a financial institution, assuming counterparties with the same level of credit risk

$$\frac{RWA_{CVA}}{N} = 2.33\sqrt{MF} \left(\sum_{i=1}^{N} 0.5RW \cdot M \cdot EAD \right)^2 \sqrt{0.25 + \frac{0.75}{N}}$$

Hence, an increasing number of counterparties adds a diversification effect with an implicit assumed correlation of 25% and decreases the incremental CVA capital charge (Gregory, 2015a). The CVA charge has been quite controversial. First, it gives a very conservative measure of risk resulting in high capital requirements. Second, only allowing for capital relief to single-name CDS and Index CDS contracts remove incentive of financial institutions to hedge CVA at all if none of the allowed hedge instruments above are available. The motive being that other hedge possibilities might actually increase total costs due to capital charges on the hedge outweighing the potential CVA reduction. For this reason, the capital charge on other hedges is exempted from capital charge in the US and Canada. A last issue is that the capital relief for CDS contract might increase demand for such, which will drive up the credit spread in the market[103]. Up to 50% of the CDS spread alone represents the capital relief provided by a single-name CDS rather than the default probability and LGD (Gregory, 2015a).

[103]This effect being even stronger as the single-name CDS market is relative illiquid.

MR Charge

Basel III updated the traditional way to estimate the MR charge originally based on a 10-day VaR estimation with a 99% confidence level (Ruiz, 2015). Instead of VaR Basel III suggests to use the Expected Shortfall, ES. VaR shows the 99^{th} percentile of random CVA losses given a distribution, while ES shows the expected CVA loss beyond the 97.5^{th} percentile of a given distribution (Brigo et al., 2013). Hence, ES includes the effect of heavy tail observations (BCBS-265, 2013). If tails are small and losses are close to normal distributed, then the difference between the two measures becomes insignificant (Ruiz, 2015). The data for the distribution must represent a stressed scenario, as it is most critical that capital is able to absorb losses in stressed periods (BCBS-265, 2013). Besides, the risk horizon fixed at 10 days will in the future instead depend on the underlying risk factor to adjust for their individual riskiness (Ruiz, 2015). (BCBS-265, 2013) fix the risk horizon for interest rate and FX products at 20 days. With these changes added to the formula provided by (Ruiz, 2015)

$$RWA_{MR} = 12.5 \cdot (3 + x + y) \cdot s \cdot ES_{97.5\%,20d}$$

x is determined by the regulatory authorities based on backtest on the model performance. Backtest performance is measured by exceptions quantified as the number of historical observations on a time horizon of the last year with actual losses that went above the losses predicted by ES. x varies from 0 to 1 depending on the number of exceptions observed. For 10 or more exceptions[104] $x = 1$. y is an add-on, which can be set by national regulators (Ruiz, 2015).

This thesis assumes a normal distribution of the losses for simplification, as for the dynamic measure of initial margin in Section 4.4 If the tail risk is very small, there would be no significant difference between using ES and VaR[105]. As VaR is a simpler measure, we will use VaR instead of ES assuming also a fixed expected volatility of the swap exposure and $\alpha = 1\%$

$$RWA_{MR} = 12.5 \cdot (3 + x + y) \cdot \phi^{-1}(0.99) \sqrt{\frac{20}{Trading\ Days}} \sigma_s$$

To adjust for a stress scenario, the expected volatility will be adjusted the maximum volatility with a $\alpha = 5\%$ significance level

$$\sigma_s = \sigma \cdot \phi^{-1}(0.95)$$

This should set a high volatility improving the confidence of VaR estimate. To account for potential model inadequacy, x is set to 1, while $y = 0$. σ is the expected volatility of the hedged exposure and fixed based on 10.000 simulation of the model.

[104] Even though not in the Basel framework, some financial institutions were assigned a higher add on than 1 during the financial crisis of 2008 (Ruiz, 2015).

[105] As discussed in Section 5.4 heavy tails of the total xVA charge do in fact seem to be an issue, which suggests using ES.

4.5.2 Estimating KVA

As described in Section 4.5.1 the Regulatory Capital charge equals $RWA \cdot 8\%$. Hence, the expected capital equals

$$EK_t = (RWA_{CCR,t} + RWA_{CVA,t} + RWA_{MR,t}) \cdot 8\%$$

(Gregory, 2015a) provides following discretized approximation of KVA

$$KVA = \sum_{i=1}^{m} \tau \cdot EK_i \cdot r_c \cdot (1 - \lambda_i)$$

where r_c is the costs of capital and λ_i is the default intensity[106].

The costs of capital measure is often defined as the weighted average cost of capital, r_{WACC}, as it weighs the cost of capital financed by debt and by equity with the capital structure of the company (Berk and DeMarzo, 2013). However, as almost all Tier1 capital[107] must be equity, using return on capital, RoC, as the measure of capital costs is a decent approximation (Gregory, 2015a). (Kenyon and Green, 2015b) shows that not all CVA can be perfectly hedged, resulting in left over credit risk providing profit or losses, which will be subject to tax. Accounting for this effect would require a Tax Valuation Adjustment (TVA), which will be left out in this thesis. However, it is important to state, that tax effects will lower the realized RoC, so that the gross RoC required should be set higher. (Finansrådet, 2015) shows that the pre-tax RoC equals 8.3% for the last 10 years for financial institutions in Denmark, while (Gregory, 2015a) argues that gross RoC for OTC derivatives should be in the region of 15-20%. Hence, the costs of capital should be set to reflect the individual goal of financial institution, which might vary quite a lot depending on the risk appetite of the shareholders of the financial institution.

[106] (Gregory, 2015a) uses the joint default probability of own default and counterparty default. As this thesis does not estimate risk of own default, this is left out of the calculation.

[107] Tier1 Capital must consist of at least 85% common equity or retained earnings, while the latter 15% might include deferred taxes, investments in other financial institutions and mortgage service rights (Choudhry et al., 2012).

Part V

Interpretation of Model Result

5.1 A Primer on Model Results

This part of the thesis presents the complete pricing model of a single IRS or CCS contract based on simulation on the underlying risk factors described in Section 2.3 and Section 3.4. The multi-curve approach is used for the risk-neutral price as described in Section 2.2 and estimation of the different xVA terms are included with the setup described in Part IV, which extends the model beyond the BS framework.

The first section discusses data input for the model and describes supplementary model assumptions related to practical issues of setting up the model to market data and to limit simulation time and complexity. Second section presents model results for respectively payer and receiver IRS contracts or fixed-fixed CCS contracts. This includes the effect from market risk hedges and collateral agreements on different xVA terms and total xVA charge. Third section analyze the model sensitivity towards crucial inputs and the calibration process and discuss suitability of the model assumptions to ensure that these do not significantly alter the model results. Subsequently, the sensitivity analysis includes evaluation of the simulation precision.

5.2 Market Data and Model Input

The parameters of the underlying risk factor models are calibrated to market data achieved from closing market prices on 2^{nd} November 2015 from (Bloomberg, 2015). This section provides a closer description of these data and discusses treatment of missing data issues. Successively, computational assumptions are evaluated.

5.2.1 Interest Rates

The midpoint of quoted market prices of swap rates with semi-annual payments serve as input for the interest rate model calibration. Hence, the CIR model parameters rely on the current term structure of swap rates up to a maturity of 10 years. This thesis includes the OIS curve as discount curve and uses either a 3-month or 6-month tenor nominated in EUR to forward the floating leg interest rates for IRS contracts. Valuation of fixed-fixed CCS contracts uses the USD OIS rate, which along with the EONIA discount factors to determine the deterministic EUR/USD FX rate. Hence, we need swap rates with the following reference rate:

- EONIA as the EUR OIS rate.
- Fed Funds as the USD OIS rate.
- EURIBOR 3-month and 6-months.

(Bloomberg, 2015) provides generic instruments for EONIA and Fed Funds, which eliminates accrued interest issues. However, only few maturities of the EURIBOR swap rates have quotes for generic market prices available. To resolve this issue, this thesis uses generic market quotes for the basis swap spreads:

- EONIA against 3-month EURIBOR

- 3-month EURIBOR against 6-month EURIBOR

added to the EONIA rates to provide the discrete term structure of swap rates for the two respective EUROBOR reference rates. As described in Section 2.2.4 these spreads are only statistical approximations at the discrete maturities. However, using the rates calculated from the spreads as input for a stochastic model provides dynamics of sensitivity to different maturities.

Swap rate maturities with semi-annual interval up to 10 years provide the term structure input for model calibration. However, for some of these maturities[108], no quoted prices are available. This thesis uses Nelson-Siegel parameterization described in Appendix A.3 to approximate[109] missing quotes in the term structure. The CIR model[110] is used to calculate the OIS rate for discounting and the tenor curve for forwarding[111], which allows calculation of swap rates. Calibration optimize the input parameters for the CIR models for different tenors to provide the lowest squared sum of errors compared to the actual swap rates. The calibration restricts $\alpha = \frac{4\theta \cdot \kappa}{\beta^2} > 1$ to avoid violation of the algorithms generating random χ^2 variables for the non-central χ^2 discretization of the CIR model. Including the restriction $\beta^2 > 2\varphi$ ensures that interest rates can only be positive[112].

The use of the CIR model results in some practical challenges, as the current EUR tenor rates at the low end of the term structure is currently negative, which violates the CIR model. However, using other models like Vasicek would take out the relationship between current rate and the volatility (Munk, 2011), which results in a less realistic model as described in Section 2.5.3. Hence, this thesis accepts the calibration effect of negative rates. To be able to model negative rates, the simulation uses an interest rate of 0 to approximate the incremental change while interest rates are negative. This incremental change is added to the negative rate, thus eliminating the diffusion element of the CIR model until $r_t > 0$.

Calibration of the CIR model to the different EUR tenors and for the OIS in USD show different tenors and currencies accounts for the difference in dynamics between the different rate curves. Macroeconomic shocks affecting either the Gaussian variable or the drift[113] should be the same for all interest rates in the same currency, which only differ by their sensitivity determined by the model parameters. Hence, this thesis uses the same Gaussian variable and same series of random number for the random χ^2 variable in all models rather than creating dependency through correlation. The difference in calibrated parameters still provides different dynamics of the process. Nevertheless, it is unlikely, that the US OIS will experience the same macroeconomic shocks as the EUR interest rates. Hence, the Gaussian variable for the Fed Funds rate is instead linearly correlated with the EONIA rate through Cholesky factorization of the two independent Gaussian variables, which is evaluated further in Section 2.4.

[108] Maturities of 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5 and 9.5 years.
[109] Nelson-Siegel parameterization with 4-parameters fitted by using the non-linear GRG method in Excel Solver Tool as described in Section 2.5.2.
[110] See Section 2.5.3.
[111] See Section 2.2.
[112] This restriction is not required if rates can also take the value of 0 (Munk, 2011). However, the effect of this restriction does not change the calibration outcome for any of the CIR models calibrated in this thesis.
[113] The drift is non-central χ^2-distributed.

5.2.2 Default Intensity

The credit risk of the counterparty is calculated under the assumption of a fixed $LGD = 40\%$, which should only have a minor effect on the overall result, as Section 2.5.4 shows, that default intensity is adjusted to account for the assumption of LGD.

As mentioned in Section 2.5.4 default risk is calibrated to iTRAXX Europe Crossover Index, as this provides a decent approximation for companies with credit rating levels below investment grade. iTRAXX series exists for time horizon of 3, 5, 7, and 10 years (Markit, 2014b). But as the JCIR model has six parameters the calibration problem would be underdefined when using generic instruments. Hence, this thesis uses the prices of actual iTRAXX series with 5 and 10 year maturity, which are issued semi-annually[114]. Therefore, such provide the entire credit risk term structure. However, as the market quotes are not from the date of issue, quotes include some accrued effects on the credit spreads, which are unaccounted for by the JCIR model using semi-annual valuations of the quarterly payments[115] To achieve cleaner input data, we assume for simplification that the accrued effect is a linear function of time. Hence, fitting credit spreads to the payment date is a proportional adjustment of each single spread with the day difference to the nearest payment date

$$\frac{k_t^*}{t} \cdot T_i = k_{T_i}^*$$

Furthermore, some iTRAXX series are not currently quoted in the market and inconsistencies between the 5 to 10 year instruments result in a not completely smooth term structure. We accept this incomplete and wavy term structure as simple parameterization like the Nelson-Siegel used for the missing data in Section 5.2.1 uses only four parameters to describe the term structure[116]. As the JCIR model includes six parameters to be estimated, Nelson Siegel parameterization would turn calibration into an underdefined problem, which would result in an insignificant jump diffusion.

5.2.3 Risk Factor Correlation

As mentioned in Section 2.4, interest rates and default intensity have some negative correlation creating wrong way risk. This correlation is captured with a linear correlation in the Gaussian variable in the JCIR model using Cholesky factorization. Nevertheless, the correlation of credit spread and interest rates is difficult to estimate on market data (Gregory, 2015a). This thesis estimates a simple correlation parameter using Pearson's ρ (Keller, 2009) based on quarterly[117] data using a 10-year historical time horizon of the generic 5 year iTRAXX Europe Crossover Index credit spread and the EONIA swap rate (Bloomberg, 2015). As expected, this results in a minor negative correlation between the EONIA swap rates and European credit spreads.

In the same way Pearson's ρ is used as the linear correlation parameter on quarterly data between the EONIA and Fed Funds rate using a historical time horizon of 10 years (Bloomberg, 2015). This reveals quite a strong positive correlation between the two OIS rates.

[114] March and September 20.

[115] iTRAXX indices have quarterly payments (Markit, 2014b).

[116] See Appendix A.3.

[117] Quarterly changes are used as this equals the time grid in the simulation model defined in Section 5.2.7.

As this thesis assumes deterministic FX rates, no multivariate variables are required as only two risk factors are stochastic. The assumption on the FX is that future FX rates can be calculated under the interest rate parity[118] based on forward OIS rates of the different currencies calculated from the CIR model with spot rates depending on one another.

5.2.4 Calibration Method

Section 2.5.2 explains that the GRG method of non-linear approximation used by Excel's *Solver* tool has the weakness, that the reduced gradient algorithm only continues changing the model parameters until an optimum is found without checking if this is just a local optimum or also the global optimum. To ensure to achieve the optimal parameters, the calibration takes off from fixed starting values, found to achieve the best optimal of solution by trial and error.

5.2.5 Fixed Rates and Notional Value in Foreign Currency

The fixed leg interest rate of the swaps is set as input before taking xVA terms into consideration. For an IRS contract the fixed leg must be set so that the present value of the swap at initiation equals 0. This is done with the Microsoft Excel *Goal Seek* function, which iterates the fixed rate input for the IRS contract to get a value as close as possible to 0. If the fixed rate set for the hedge with the CCP is higher than the fixed rate set for the contract with the counterparty due to different floating tenors, the counterparty is instead charged the fixed rate of the CCP to cover additional costs of the market risk hedge. The *Goal Seek* function looks for the nearest local optimum of a single variable without restrictions as the *Solver* function evaluated in Section 2.5.2 does for multiple variables. Hence, as also mentioned in Section 5.2.4 different starting values must be tried to ensure, that the global optimum is used as input for the fixed rate (Brandewinder, 2009).

For CCS contracts, the fixed leg in domestic currency is set to 1% while the fixed leg in the foreign currency is achieved iterative in the same manner as the fixed leg for an IRS contract. The notional value of the foreign leg is set to be exact equal to the notional of the domestic leg at the current spot FX rate, which is in practice often subject to some rounding (Hull, 2012).

5.2.6 Random Numbers

The simulation model generates different random variables from algorithms described Section 3.4. These all use uniform random numbers, which are generated with the Microsoft Excel VBA function *rnd*(). This function uses a linear congruential generator[119] as described in (Glasserman, 2003).

5.2.7 Time Grid and Cash Flows for the Simulation

At each point in time exposure for the xVA terms is estimated from the risk neutral value of the swap using the multi-curve approach. The time grid for the simulation is fixed to $\tau = T_i - T_{i-1} =$

[118] See Section 2.2.5.
[119] With constants $a = 1140671485$ and $c = 12820163$ (Microsoft, 2015).

0.25. This ensures that swaps with a rolling 3-month tenor rate can be priced at each fixing of the floating tenor rate. To account for the fact, that payments are only semi-annually, equation 2.2 is modified to measure only payments at steps, where these occur

$$V_t = \sum_{i=1}^{m} 1_{\{T_{i-1}<CF_i<T_i\}} \cdot D_d\left(t, T_i\right) \cdot H \cdot \left(E_t^{\mathbb{Q}_d^{T_i}} \left[\left(F_x\right)\right] - r^{fix} \right)$$

where CF_i is time of the ith cash flow. Hence, the model only values cash flows in periods where payments actually occur. For a 3-month tenor floating rate swap with semi-annual payments, the floating rate payment is the simple average of the two 3-month rates set at each quarter. Hence,

$$V_t = \sum_{i=1}^{m} 1_{\{T_{i-1}<CF_i<T_i\}} \cdot D_d\left(t, T_i\right) \cdot H \cdot \left(\frac{E_t^{\mathbb{Q}_d^{T_{i-1}}} \left[\left(F_x\right)\right] + E_t^{\mathbb{Q}_d^{T_i}} \left[\left(F_x\right)\right]}{2} - r^{fix} \right)$$

The model can price for all swap maturities, but should only be used for swaps with a maturity up to 10 years, as this reflects the time horizon used to calibrate the underlying risk factor models used as input for the pricing formula.

5.3 Model Results

This section provides model results for both IRS contracts and fixed-fixed CCS contracts using different maturities. Subsequently follows examination of the effect of a market risk hedge with a CCP and different collateral agreements. The model provides the present value of the different xVA terms for swap contracts with the fixed leg set so that risk neutral value is set to 0[120]. Apart for the present value, the simulation model also approximates the xVA charges in terms of basis points to be added to the fixed interest rate. This makes a better comparable for the magnitude of different xVA terms of contracts with different maturities and notional values. It is possible to charge counterparties the costs of xVA along with coupons payments rather than up front. Though the xVA charge added to the coupons must be higher than the net present value used here to account for the management and risk of those payments (Ruiz, 2015). Hence, the xVA charge provided in terms of basis points only serve as a comparable proxy between different types of swaps and maturities. All swaps have notional value of the domestic leg of 1,000,000 EUR. The hedge exposure volatility used as input for the MR charge has been estimated with the model for each relevant case[121] based on 10,000 simulations.

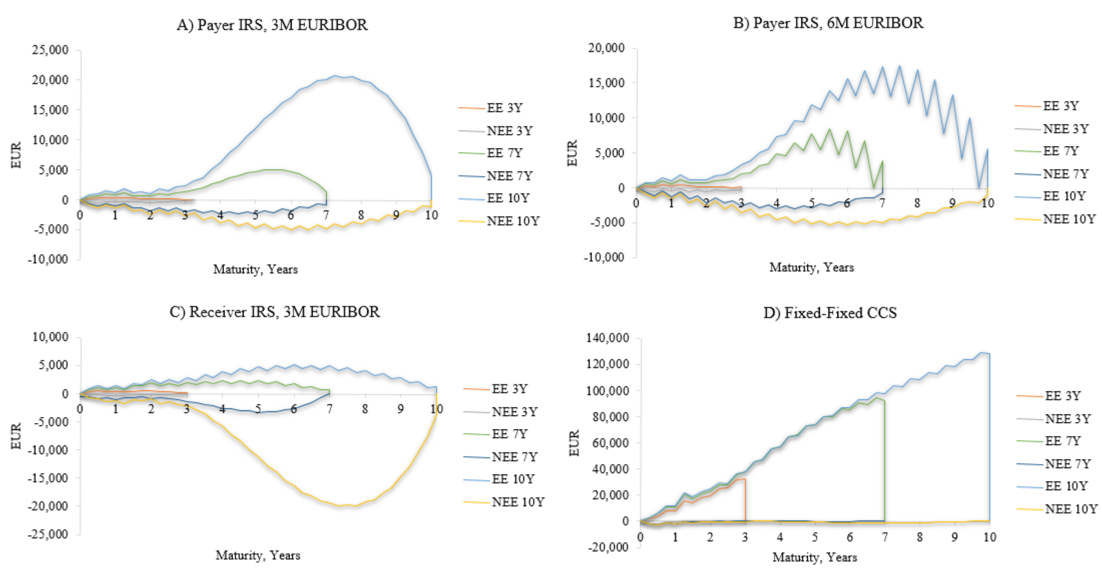
[120] Section 5.2.5 provides an exception to this.

[121] As the hedge exposure differs depending on swap maturity, floating leg tenor, choice of market risk hedge, and choice of collateral agreement.

5.3.1 Swap Property Results

This subsection evaluates how xVA terms and exposure differ for respectively payer IRS contracts, receiver IRS contracts and fixed-fixed CCS contracts with different maturities. All cases are unhedged, uncollateralized contracts, and have semi-annual payments. Thus, no MVA or CollVA. As mentioned in Section 4.2.1, Figure 5.1 illustrates that expected exposure consistently increases for contracts with longer maturity. The expected exposure is also not constant throughout the lifetime of the swap as cash flows further from today yield higher uncertainty. The amortizing effect of the swaps as time passes by provides a maximum exposure for IRS contracts around 2/3 of the total maturity, which differs from the conclusions made by (Gregory, 2015d). However, the results by (Gregory, 2015d) follow a simplified assumption that swap volatility follows a normal distribution and that the yield curve is upwards sloping with strictly positive interest rates. This thesis shows the effect of the current negative EUR interest rates and the model assumptions to deal with this evaluated in Section 5.2.1. When rates are initially negative, only the drift determined by the χ^2-variable controls interest rates in order for the CIR model to work. With the current term structure, rates are not expected above 0 before after 3 years. Hence, up till that point no interest rate volatility through the Gaussian variable exists for the exposure.

Figure 5.1: Expected Exposure Profiles



Source: Author's own creation

The payer and the receiver swap contracts are mirrored cases of one another with the NEE of the payer swap equal to the EE of the receiver swap with same maturity. Due to the upward sloping term structure of interest rates, the NEE of the payer swap contract is lower than EE and vice versa for the receiver swap contract. As mentioned in Section 2.2.3 the longer tenor on the floating leg of a payer IRS contract increases risk and thus exposure as can be seen when comparing A) and B) in Figure 5.1. For the CCS swap contracts the exposure profile is quite different. First of all, NEE is almost identical for different maturities, while this is very different for the EE. Second, EE does not drop before maturity. Both of these issues relate to the FX effect. As explained in Appendix A.2, the notional value of a CCS contract is exchanged at maturity at the FX rate equal to the spot rate at initiation of the contract. Hence, increasing EUR/USD FX rates increase the positive cash flow from exchange of notional at maturity. The calibrated parameters for respectively the EONIA rate and the Fed Funds rate result in a lower long-term rate for EONIA, $\varphi = \theta\kappa = 0.22\%$, than for Fed Funds, $\varphi = \theta\kappa = 0.67\%$. Hence, the deterministic model using the interest rate parity expects an increasing EUR/USD FX rate. This explains the strongly increasing EE. NEE would possible be larger were it not for the choice of a deterministic FX model, as this would allow for simulation scenarios with decreasing EUR/USD exchange rates.

Table 5.1 provides the xVA estimates for the different contracts mentioned above. For the payer swaps, CVA, FVA, and KVA all increase with longer maturity for both floating tenor rates, which provides a higher xVA charge for longer maturities. The relative effect of KVA decreases for both case 1-3 and 4-6 as maturity increase from 3 to 7 years, but becomes relative larger for the 10-year maturity as KVA does not capture the actual exposure, but only approximates it with standardized calculations. CVA, on the other hand increases relatively as time increase for the 6-month floating tenor, while however decreasing for the case 3 with a 6-month floating tenor. FVA increases relatively more than CVA with longer maturities. The reason for this is that funding of the positive exposure for short maturities will more likely be at negative or low interest rates due to the current negative short rates, which provide low average costs, while these costs for longer term maturities will likely converge towards the long-term rate of EONIA, $\varphi = \theta \cdot \kappa = 0.22\%$, added the funding spread of 100 bp increasing funding costs for longer tenors[122].

Longer floating tenor increase the xVA charge comparing case 1-3 with case 4-6 as a result of the higher exposure, except when comparing case 1 to case 4, where the xVA charge is significantly higher for the shorter tenor. Though FVA is significantly higher for case 1 than for case 4 and CVA is borderline to be significantly higher, the KVA charge of case 4 is higher for the shorter term, as the average exposure of the shorter tenor is higher at the beginning of the contract life time. KVA exposure is discounted with the d factor in Section 4.5.1 not following actual term structure of discount rates. This likely overweighs the exposure in the short end of the contract though actual discount rates do not. For longer floating tenor rate, CVA and KVA increase faster for longer maturities. HVA on the other hand increase slightly less as maturity increases from case 4-6, as lower payment frequency is less sensitive to increasing maturity.

Examining case 7-9 with the receiver swap contracts with similar maturity, the current term structure gives a much lower exposure, comparing C) with A) in Figure 5.1. As a logical result, the total xVA charge is also lower for receiver swap contracts. Though, for case 7 with only 3-year maturity the absolute xVA charge is above that of a similar payer swap. This is due to negative interest rates in the short term, which result in cash inflows for the receiver contract, however

[122]Section 5.4 examines model sensitivity towards choice of funding rate and calibrated input for the risk factor models.

Table 5.1: Simulation Results for Uncollateralized Swap Contracts

Case:	***1**: 3&6*, Payer, 3Y.*				***2**: 3&6*, Payer, 7Y.*				***3**: 3&6*, Payer, 10Y.*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*
xVA Total	42.54	74.10	[41.68 ; 43.40]	0.28	2826.86	10523.10	[2704.47 ; 2949.25]	8.09	15982.35	39490.74	[15523.04 ; 16441.66]	32.48
CVA	3.10	16.24	[2.92 ; 3.29]	0.02	674.72	3763.77	[630.95 ; 718.50]	1.91	2578.92	9604.77	[2467.21 ; 2690.63]	5.21
CollVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
HVA	5.32	12.03	[5.18 ; 5.46]	0.03	769.71	4423.57	[718.26 ; 821.16]	2.19	4005.68	14972.11	[3831.55 ; 4179.82]	8.11
FVA	5.32	12.03	[5.18 ; 5.46]	0.03	769.71	4423.57	[718.26 ; 821.16]	2.19	4005.68	14972.11	[3831.55 ; 4179.82]	8.11
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
KVA	34.11	54.98	[33.47 ; 34.75]	0.22	1382.42	3105.16	[1346.31 ; 1418.54]	3.94	9397.75	18346.08	[9184.36 ; 9611.13]	19.08
Case:	***4**: 6&6**, Payer, 3Y.*				***5**: 6&6**, Payer, 7Y.*				***6**: 6&6**, Payer, 10Y.*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*
xVA Total	39.82	84.39	[38.84 ; 40.80]	0.26	3761.06	15779.70	[3577.53 ; 3944.60]	10.79	18539.25	46124.89	[18002.78 ; 19075.73]	45.32
CVA	3.45	26.34	[3.15 ; 3.76]	0.02	1054.70	6777.48	[975.87 ; 1133.53]	3.03	3001.62	11636.84	[2866.27 ; 3136.96]	13.70
CollVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
HVA	4.82	17.09	[4.62 ; 5.02]	0.03	1023.44	6439.73	[948.54 ; 1098.34]	2.94	4332.94	17359.12	[4131.04 ; 4534.84]	16.41
FVA	4.82	17.09	[4.62 ; 5.02]	0.03	1023.44	6439.73	[948.54 ; 1098.34]	2.94	4332.94	17359.12	[4131.04 ; 4534.84]	16.41
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
KVA	31.55	53.16	[30.93 ; 32.16]	0.21	1682.92	3769.09	[1639.08 ; 1726.76]	4.83	11204.70	21700.25	[10952.31 ; 11457.09]	30.39
Case:	***7**: 3&6*, Receiver, 3Y.*				***8**: 3&6*, Receiver, 7Y.*				***9**: 3&6*, Receiver, 10Y.*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*
xVA Total	56.45	51.61	[55.85 ; 57.05]	-0.38	1464.56	1035.67	[1452.52 ; 1476.61]	-4.23	6486.13	4271.94	[6436.45 ; 6535.82]	-13.24
CVA	2.55	4.54	[2.50 ; 2.61]	-0.02	25.97	42.39	[25.48 ; 26.47]	-0.10	108.89	285.98	[105.56 ; 112.21]	-0.26
CollVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
HVA	6.46	5.56	[6.39 ; 6.52]	-0.05	85.24	47.72	[84.69 ; 85.80]	-0.27	264.11	157.83	[262.27 ; 265.94]	-0.58
FVA	6.46	5.56	[6.39 ; 6.52]	-0.05	85.24	47.72	[84.69 ; 85.80]	-0.27	264.11	157.83	[262.27 ; 265.94]	-0.58
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
KVA	47.44	43.96	[46.93 ; 47.96]	-0.32	1353.34	976.37	[1341.99 ; 1364.70]	-3.91	6113.14	4053.43	[6065.99 ; 6160.28]	-12.48
Case:	***10**: 3&6*, CCS, 3Y.*				***11**: 3&6*, CCS, 7Y.*				***12**: 3&6*, CCS, 10Y.*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*
xVA Total	1977.48	1576.76	[1959.15 ; 1995.82]	11.44	41803.59	53810.12	[41177.73 ; 42429.45]	116.70	128932.66	116952.88	[127572.39 ; 130292.92]	265.34
CVA	217.08	541.14	[210.79 ; 223.38]	1.26	7502.66	25738.16	[7203.31 ; 7802.02]	20.94	16092.41	39334.19	[15634.92 ; 16549.90]	33.12
CollVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
HVA	343.18	252.64	[340.24 ; 346.11]	1.99	5612.00	13357.59	[5456.64 ; 5767.36]	15.67	14670.98	26158.83	[14366.73 ; 14975.23]	30.19
FVA	343.18	252.64	[340.24 ; 346.11]	1.99	5612.00	13357.59	[5456.64 ; 5767.36]	15.67	14670.98	26158.83	[14366.73 ; 14975.23]	30.19
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
KVA	1417.22	911.70	[1406.62 ; 1427.83]	8.20	28688.93	21790.88	[28435.48 ; 28942.37]	80.09	98169.27	69676.55	[97358.87 ; 98979.67]	202.03

* *3-month floating tenor paid semi-annualy*

** *6-month floating tenor paid semi-annualy*

*** *Confidence Interval*

Source: Author's own creation

accruing a negative interest rate so that FVA increases, while CVA is lower. The simple KVA discount factor as mentioned above also overweighs positive the short term exposure compared to case 1, which result in a higher KVA. However, case 8-9 with longer maturities have a much lower xVA charge than their corresponding payer swap contracts. Case 8-9 have very small CVA and FVA charge, so that KVA contributes most of the total xVA charge for those cases.

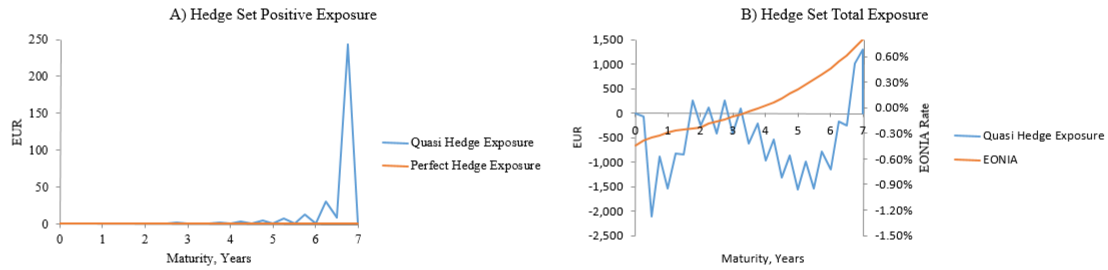
As in Section 4.5 it must be mentioned, that in reality KVA is calculated on a portfolio base, hence the gain from netting trades provides a much lower exposure on a portfolio level for the KVA, which is beyond the scope of this thesis to analyze. Looking at a single trade, KVA clearly accounts for the largest part of the xVA charge in an uncollateralized case. KVA is not completely linked to the current exposure, as EAD is predefined by (BCBS-279, 2014). This is evident when comparing payer and receiver swap contracts as maturity increases. For both types, the KVA charge increases as maturity increase, but represents relatively less of the total xVA charge at higher maturities for payer swaps, which has an actual higher exposure increasing CVA and FVA.

Case 10-12 reveal that the xVA charge for fixed-fixed CCS contracts increase more in proportion with maturity than IRS contracts. This is a result of expecting an upward sloping EUR/USD FX rate. The xVA charge is also multiple times bigger than for the IRS contracts with a similar maturity due to the higher exposure from the FX risk with KVA accounting for the largest part of the xVA charge. This high KVA charge is a result of the exposure to FX added in the SA-CCR method and the CVA charge of calculating EAD rapidly increasing exposure as the FX risk, SF, in Section 4.5.1 is weighted eight times higher than SF for interest rates.

5.3.2 Market Hedge Results

This subsection evaluates how xVA charges for a 7-year payer IRS with semi-annual payments change with respect to decisions on hedging market risk by entering a receiver swap contract with a CCP. In the perfect hedge case, a 6-month tenor floating leg IRS contract is hedged with a similar receiver contract. However, such perfect hedge instruments are not always available or liquid, especially when requiring a hedge of a customized swap contract with the counterparty. For this reason, this subsection also presents a case where a payer IRS contract with a 3-month rolling tenor floating leg with semi-annual payments is hedged with a 3-month tenor floating leg swap with quarterly payments. Contracts with a CCP will always require a bilateral collateralization for security including a dynamic initial margin to cover potential gap risk between collateral margining. Calculation of the dynamic initial margin assumes a fixed 25% swap volatility input. Collateral agreements with the CCP include rounding of collateral posts to lot sizes of 100 and the MTA of 100.

Figure 5.2: Hedge Profiles



Source: Author's own creation

A) in Figure 5.2 shows the net exposure of the IRS contract with the counterparty and the hedge with a CCP. The perfect hedge leaves no open exposure of market risk left, while the quasi hedge still has some open exposure due to cash flow mismatches between the two contracts, which must be funded and therefore result in some HVA charge.

Table 5.2: Simulation Results for Market Hedging

Case:	***5**: 6&6**, Payer, 7Y, No Hedge.*				***13**: 6&6**, Payer, 7Y, Hedge.*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*
xVA Total	3761.06	15779.70	[3577.53 ; 3944.60]	10.79	2585.76	7130.14	[2502.83 ; 2668.69]	7.42
CVA	1054.70	6777.48	[975.87 ; 1133.53]	3.03	781.89	3951.87	[735.92 ; 827.85]	2.24
CollVA	0.00	0.00	[0.00 ; 0.00]	0.00	120.21	52.93	[119.59 ; 120.83]	0.34
HVA	1023.44	6439.73	[948.54 ; 1098.34]	2.94	0.00	0.00	[0.00 ; 0.00]	0.00
FVA	1023.44	6439.73	[948.54 ; 1098.34]	2.94	120.21	52.93	[119.59 ; 120.83]	0.34
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	59.59	143.07	[57.92 ; 61.25]	0.17
KVA	1682.92	3769.09	[1639.08 ; 1726.76]	4.83	1624.07	3623.20	[1581.93 ; 1666.22]	4.66
Case:	***2**: 3&6*, Payer, 7Y.*				***14**: 3&6*, Payer, 7Y, Hedge.*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*
xVA Total	2826.86	10523.10	[2704.47 ; 2949.25]	8.09	1585.97	2842.33	[1552.91 ; 1619.03]	4.53
CVA	674.72	3763.77	[630.95 ; 718.50]	1.91	317.75	1459.30	[300.78 ; 334.72]	0.89
CollVA	0.00	0.00	[0.00 ; 0.00]	0.00	146.43	84.16	[145.45 ; 147.41]	0.40
HVA	769.71	4423.57	[718.26 ; 821.16]	2.19	-173.46	1117.90	[−186.47 ; −160.46]	-0.25
FVA	769.71	4423.57	[718.26 ; 821.16]	2.19	-27.04	1140.03	[−40.30 ; −13.78]	-0.04
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	74.47	162.34	[72.58 ; 76.35]	0.19
KVA	1382.42	3105.16	[1346.31 ; 1418.54]	3.94	1220.79	2373.72	[1193.18 ; 1248.40]	3.48

** 3-month floating tenor paid semi-annualy*

*** 6-month floating tenor paid semi-annualy*

**** Confidence Interval*

Source: Author's own creation

Table 5.2 shows the estimated xVA charge for the different two hedging cases. The overall xVA charge is significantly lower for the quasi hedges than the perfect hedge, which is unrelated to the hedge exposure, but is a result of two issues: First, the quasi hedge cash flows mismatches create funding issues. However, negative interest rates and a strong negative funding hedge exposure from 3.5 years and 3 years forth[123] provides a cash inflow to be lend out at a positive

[123]The cash inflow from the negative exposure occurring from initiation till 1.5 later will, however, be expected

OIS rate from year 4 and forth. The lending gain outweighs funding costs of funding at a spread to the OIS rate charged on cash outflows from positive exposure mainly present at the last year of the contract. B) in Figure 5.2 shows expected interest rates and hedge exposure of the quasi hedge indicating the situation described above. Second, the KVA charge is higher for the perfect hedge, though market risk is eliminated. But as shown in Section 5.3.1 exposure for 6-month tenor is higher than for 3-month tenor, which is also strongly significant when comparing case 2 and 5 without hedges. Table 5.3 shows the average capital charges on cases 13-14. The SA-CCR charge on the CCP hedge is lower for the perfect hedge than for the quasi hedge and the MR charge is 0. Nevertheless, the MR charge on the quasi hedge is still close to 0. The much lower SA-CCR charge and regulatory CVA charge on the counterparty for case 14 makes the total capital requirements of case 13 higher due to the higher tenor risk of the floating leg. Hence, the higher KVA charge does not relate to the choice between a perfect and a quasi hedge, but rather the floating tenor exposure. Comparing case 13 and 14, the perfect hedge would be preferred if isolating the hedging effect from issue of a higher tenor and the current negative interest rates providing a negative funding rate for shorter maturities.

Table 5.3: Average Regulatory Capital, Risk Weighted Assets

	***14**: 3&6*, Payer, 7Y, Hedge.*	***13**: 6&6**, Payer, 7Y, Hedge.*
	Quasi Hedge	*Perfect Hedge*
Avg. $RWA_{CCR_{Counter\ Party}}$	2912.78	4329.21
Avg. $RWA_{CCR_{Central\ Clearing\ Party}}$	61.26	50.26
Avg. CVA_{Charge}	23401.96	31685.85
Avg. MR_{Charge}	0.001	0.000

Source: Author's own creation

The CVA charge from Table 5.2 also shows, that the CVA is significantly higher for case 13, while the costs of the collateral agreement with the CCP (CollVA and MVA) in case 14 are slightly higher than case 13. This reduce the advantage for the quasi hedge in terms of the total xVA charge. Collateral issues will be covered in the next subsection. Compared to case 2 and 5, the total xVA charge for both case 13 and 14 is much lower. This provides a strong incentive to hedge market risk to reduce the CVA and HVA significantly[124], KVA moderately (though only borderline significant for the perfect hedge), while accepting a small MVA and CollVA from the collateral requirements of the CCP. The quasi hedge is cheaper under the current term structure affecting the exposure profile and a funding rate fixed at OIS plus 100 bp, which provide a HVA benefit besides the lower CVA and KVA of the shorter tenor rate. This HVA benefit would not necessarily be present in case for changes in the interest rate dynamics or funding spread[125]. Besides significantly lower xVA charges, the volatility of the xVA terms is more than halved for both floating tenors compared to the unhedged cases, except for CollVA and MVA of course not present for unhedged contracts.

to lend at negative interest rates.

[124]The perfect hedge eliminates HVA.

[125]Section 5.4.2 evaluates the effect of risk factor input parameters and funding spread.

5.3.3 Collateralization Results

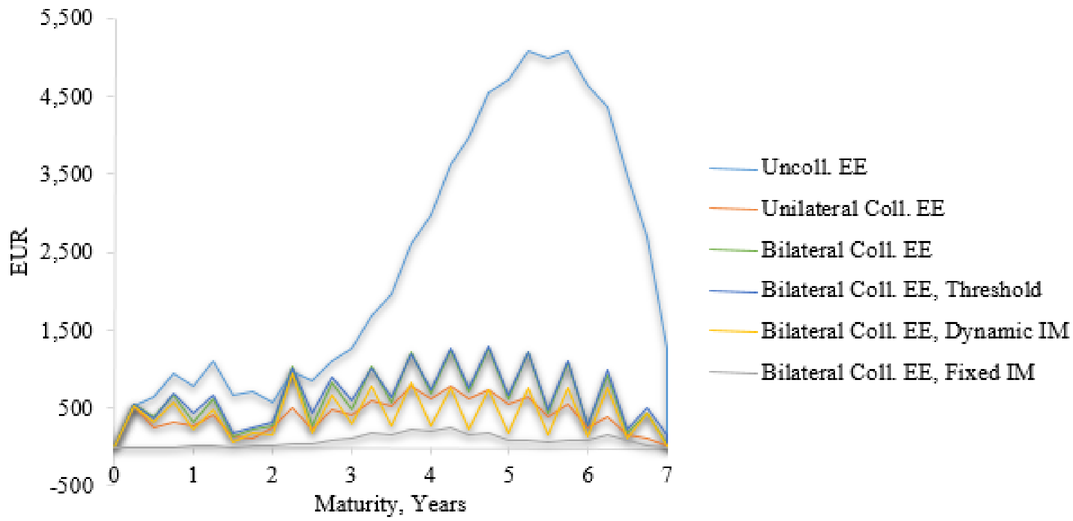
This subsection evaluates how collateral can substantially reduce the exposure of a contract and how initial margin can cover some of the gap risk between margining. Subsequently, this subsection evaluates effect on funding costs, if a market risk hedge with a CCP is added. This subsection evaluates a 7-year payer swap contract with a 3-month floating tenor and semi-annual payments in 5 different cases of CSA agreements for collateralization:

- Case 15: Unhedged contract with a unilateral CSA agreement
- Case 16: Unhedged contract with a bilateral CSA agreement
- Case 17: Unhedged contract with a bilateral CSA agreement allowing a threshold of 1,000 for both parties.
- Case 18: Unhedged contract with a bilateral CSA agreement and a fixed segregated initial margin of 0.5% of the notional IRS value posted by both both parties.
- Case 19: Unhedged contract with a bilateral CSA agreement and a segregated dynamic initial margin assuming a fixed 25% swap exposure volatility posted both by the counterparty and to the counterparty.
- Case 20: Contract with a quasi hedge (3-month floating tenor and quarterly payments) with bilateral CSA agreement with both counterparty and with the CCP including a dynamic initial margin with the same assumptions as in case 4 for the counterparty, the financial intermediary, and to the CCP.

CSA agreements for all cases above include rounding of collateral posts to a lot size of 100. The CSA agreement with the counterparty include a 200 MTA for both the financial intermediary and the counterparty, while the MTA with the CCP hedge is 100 for both parties. The margin period of collateral and initial margin is quarterly and there are no thresholds[126]. Funding of payments or collateral is at the OIS rate plus a spread of 100 bp, while lending of excess cash and rehypothecation of collateral earns the OIS rate. Collateral held accrues the OIS rate between the financial intermediary and the corporate counterparty, while only OIS minus 25 bp between the CCP and the financial intermediary.

[126] Except in case 17.

Figure 5.3: Expected Positive Exposure



Source: Author's own creation

Figure 5.3 demonstrates how the exposure of a contract is strongly reduced as collateral is posted against exposure. Some gap exposure remains open due to the discrete margin period, minimum transfer amounts, and rounding conventions. If collateral is to be posted much more frequently and for smaller amounts, this would require the model to take into account operational costs of margining, which would then have significant effect on the optimal margining period, MTA, and rounding conventions. Including a threshold give slightly less exposure reduction, but should also result in lower collateral costs. Unilateral collateralization is slightly more efficient than bilateral, as collateral posted at the counterparty at last collateral margining might also be at risk, if the contract has turned to be in-the-money in the bilateral case. It might, however, not be possible to convince counterparties to carry additional costs of posting collateral if the same is not done by the financial institution issuing the contract, as this in fact increases their exposure by their posted collateral, if such is not segregated (Gregory, 2015a). However, as seen from case 15 and 16 in Table 5.4 bilateral agreements on the other hand further increases the xVA charge. (Gregory, 2015a) compares the increase in xVA charge of a bilateral CSA agreement against no CSA agreement from the counterparty point of view. (Gregory, 2015a) concludes that the counterparty benefits significantly more from receiving collateral than not doing so. This indicates that rational counterparties should prefer bilateral CSA agreements rather than accept a unilateral CSA agreement, though the former increase the xVA charge to the counterparty, as this lower the total costs of the counterparty.

Adding either a fixed or dynamic initial margin to the contract terms further covers some or all of the gap risk depending on the amount of initial margin. As seen in Figure 5.3 an initial margin of 0.5% of the notional IRS value almost eliminates exposure. A dynamic initial margin tries to avoid over- or underestimating the required initial margin, as 99% of additional exposure should be covered[127].

[127] Based on the assumption in Section 4.4.

Table 5.4: Simulation Results for Collateralization

Case:	***15**: 3&6*, Payer, 7Y, No Hedge, Unilateral CSA*				***16**: 3&6*, Payer, 7Y, No Hedge, Bilateral CSA*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	bp	NPV	σ	$CI^{***}_{\alpha=5\%}$	bp
xVA Total	2559.79	12423.90	[2415.29 ; 2704.29]	7.35	3207.44	13218.60	[3053.69 ; 3361.18]	9.20
CVA	53.43	225.01	[50.81 ; 56.05]	0.15	62.31	253.22	[59.36 ; 65.25]	0.18
CollVA	1243.75	7619.60	[1155.13 ; 1332.37]	3.57	1386.60	8168.26	[1291.60 ; 1481.61]	3.98
HVA	682.51	3731.31	[639.11 ; 725.91]	1.96	717.91	3987.17	[671.54 ; 764.29]	2.06
FVA	1926.26	11345.70	[1794.30 ; 2058.22]	5.53	2104.52	12148.36	[1963.22 ; 2245.82]	6.04
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	0.00	0.00	[0.00 ; 0.00]	0.00
KVA	580.10	1171.97	[566.47 ; 593.73]	1.66	1040.61	1127.49	[1027.49 ; 1053.72]	2.99
Case:	***17**: 3&6*, Payer, 7Y, No Hedge, Bilateral CSA, Thr=1000*				***18**: 3&6*, Payer, 7Y, No Hedge, Bilateral CSA, IM=5000*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	bp	NPV	σ	$CI^{***}_{\alpha=5\%}$	bp
xVA Total	3473.18	12093.35	[3332.52 ; 3613.83]	9.94	3339.51	13990.36	[3176.79 ; 3502.23]	9.56
CVA	62.60	239.08	[59.82 ; 65.38]	0.16	20.25	138.25	[18.65 ; 21.86]	0.04
CollVA	1373.09	7372.82	[1287.34 ; 1458.84]	3.92	1432.31	8752.22	[1330.52 ; 1534.11]	4.09
HVA	709.75	3649.41	[667.31 ; 752.20]	2.01	735.92	4210.21	[686.95 ; 784.89]	2.09
FVA	2082.84	11017.15	[1954.70 ; 2210.98]	5.93	2168.23	12954.96	[2017.56 ; 2318.91]	6.20
MVA	0.00	0.00	[0.00 ; 0.00]	0.00	366.34	178.54	[364.26 ; 368.42]	1.03
KVA	1327.74	1116.30	[1314.75 ; 1340.72]	3.79	784.68	990.63	[773.16 ; 796.20]	2.23
Case:	***19**: 3&6*, Payer, 7Y, No Hedge, Bilateral CSA, IM=Dynamic*				***20**: 3&6*, Payer, 7Y, Hedge, Bilateral CSA, IM=Dynamic*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	bp	NPV	σ	$CI^{***}_{\alpha=5\%}$	bp
xVA Total	3155.82	12778.04	[3007.20 ; 3304.44]	9.03	1733.21	3372.83	[1693.98 ; 1772.44]	4.95
CVA	22.40	98.99	[21.25 ; 23.55]	0.04	17.75	46.23	[17.21 ; 18.29]	0.03
CollVA	1380.53	7585.86	[1292.30 ; 1468.76]	3.94	905.56	3319.68	[866.95 ; 944.17]	2.58
HVA	716.32	3762.30	[672.56 ; 760.08]	2.03	-167.70	976.85	[−179.06 ; −156.34]	-0.24
FVA	2096.85	11342.04	[1964.93 ; 2228.77]	5.99	737.86	2351.11	[710.52 ; 765.21]	2.09
MVA	185.62	841.04	[175.84 ; 195.41]	0.51	186.60	570.44	[179.97 ; 193.24]	0.51
KVA	850.95	725.49	[842.51 ; 859.38]	2.42	790.99	556.88	[784.52 ; 797.47]	2.25

** 3-month floating tenor paid semi-annualy*

*** 6-month floating tenor paid semi-annualy*

**** Confidence Interval*

Source: Author's own creation

Table 5.4 shows the xVA charge and relationship between the different xVA terms when introducing collateralization. First examining the effect on CVA, it is clear that any form of collateral agreement presented above strongly decrease the credit risk of the counterparty contrary to case 2 with no CSA agreement. Case 15 has a slightly lower CVA than case 16 due to the additional exposure reduction of unilateral CSA agreements as mentioned above. CVA mitigation effect would expectedly be lower for case 17 allowing for thresholds compared to case 16, but is insignificant for the results in Table 5.4. Adding either dynamic or fixed initial margin as in respectively case 19 and 18 further reduces CVA significantly.

The difference in choosing between unilateral and bilateral CSA agreements favor the unilateral in terms of KVA reduction as a result of risk on collateral posted at the counterparty, which increases exposure of case 16 as mentioned above. Allowing for threshold increases KVA of case 17 compared to case 16 as the threshold is included in RC formula in the EAD calculation in the SA-CCR and the CVA regulatory capital charge. Inclusion of an initial margin lowers KVA significantly when comparing case 18-19 to case 16-17. Throughout case 15-19 KVA is lower than for case 2.

FVA is not significantly different from one another comparing case 15 and 16. This is the same when allowing for threshold in case 17. Still, it must be considered that the uncertainty on CollVA and HVA account for most of the uncertainty in the xVA charge of collateralized contracts as can be seen from the standard deviation for case 15-20 in Table 5.4. Hence, one should be careful to conclude on a small sample of simulations[128].

Including initial margin comes at the costs of an MVA charge, which is significantly higher using a fixed initial margin as in case 18 than a dynamic initial margin adjusted to exposure as in case 19. However, case 18-19 have significantly lower KVA and CVA than case 16, with case 19 reducing CVA and KVA the most.

The overall xVA charge becomes lower for case 15 than case 16. This is mainly due to the difference in KVA charge already evaluated. Allowing for a threshold in case 17 seems to make the xVA charge even higher, as the KVA charge increase further without any significant decrease in CollVA from the less strict collateral agreement. The potential reduction of CVA is limited as CVA is moderate due to the chosen proxy index. This makes the total xVA charge higher for case 16-19 than for case 15. However, the xVA charge of case 15 is borderline to be significantly lower than case 2. For the bilateral CSA agreements, collateral posted at the counterparty potentially increase $NICA_t$ in equation 4.28 and hence the estimated exposure for KVA. Higher counterparty risk favors collateralization. Adding a dynamic initial margin does not provide a significant decrease in xVA, as the effect of CVA and KVA reduction, though very significant, are lower than the MVA charge for the initial margin. A fixed initial margin at 0.5% of the notional IRS value in case 19 seems to be excessive given the expected exposure. Yet, this depends on the maturity of the contract as seen in Section 5.3.1 and the default intensity[129]. Section 6.2 evaluates this relationship further.

Case 20 combines the quasi hedge in case 14 from Section 5.3.2 with the bilateral CSA agreement including a dynamic initial margin from case 19. Doing so further reduces CVA and

[128]See Section 5.4.3 for further discussion of this issue.
[129]See Section 5.4.2 for results on changing default intensity.

even KVA[130] (though the hedge with the CCP will require capital), as the total hedge exposure becomes very small as described in Section 5.3.2.

Allowing for rehypothecation of the variation margin, collateral posted by the counterparty can be passed on to the CCP, if the swap contract with the counterparty is in-the-money. If the contract with the counterparty is out-of-the-money, the hedge contract is almost always in-the-money[131]. Hence, collateral posted by the CCP can be passed to the counterparty. This reduce collateral funding to depend only on the mismatch between the counterparty and the CCP contract value and collateral agreement details, significantly reducing CollVA. Yet, the difference in the accrued interest rates on the different collateral agreements still exist.

The hedge also partially covers cash flow mismatches and make the HVA charge negative for the same reasons as in Section 5.3.2. With a market quasi risk hedge, the xVA charge of case 20 is also significantly lower than case 2 even with the credit risk only moderately high. The xVA charge of case 20 is also only significantly higher than case 14 in Section 5.3.2 by a landslide. Case 20 indicates that adding a hedge provides a powerful advantage not only to the reduction of HVA, but also to the cost of collateralization for the reasons provided above. Besides the reduction in the xVA charge, adding the hedge reduces the volatility of all xVA terms as can be seen from the standard deviation of case 20 in Table 5.4, which reduces the total volatility of the xVA charge by 74% compared to case 16 and 68% compared to case 2.

5.4 Sensitivity Analysis

This section discusses backtest of the model and relevant test of sensitivities towards changes in underlying risk factors or input assumptions for the model. It furthermore discusses the precision and convergence of the simulations accounting for potential MC noise of the model.

5.4.1 Model Backtest

The model includes the flexibility to estimate xVAs for different kinds of swap contracts with different floating tenor and maturity. These can include a market risk hedge and allows for both unilateral and bilateral CSA agreements with both the counterparty and the CCP. The CSA agreements can have optionality on numerical parameters[132] and the possibility to include either a fixed or dynamic initial margin. Such flexibilities complicate the overall model, and it must be tested thoroughly to ensure that implementation of such options work as intended. E.g. that the effect of hedge and collateral produce the intended exposure reductions while increasing funding costs captured by CollVA and MVA. The amount of strictness of the collateral agreement through rounding issues, MTAs, and thresholds should be apparent on the positive exposure and the cost of collateral. Section 5.3.3 illustrates that the effect of collateral requirements and initial margin have a clear effect on the exposure reduction while on the other hand increasing respectively CollVA and MVA. The choice of hedge also seems to work as expected, as positive

[130] Although the hedge with the CCP is subject to the SA-CCR capital charge.

[131] If e.g. tenor rate or other contract features are different, the hedge is not an exact mirror of the contract with the counterparty.

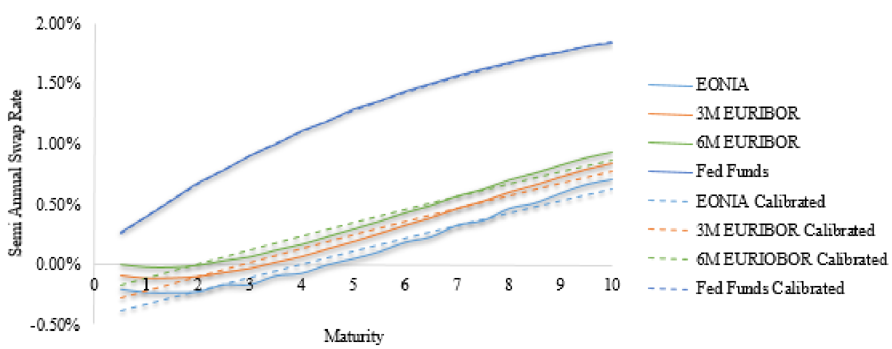
[132] See Section 4.2.1.

exposure is greatly reduced with a quasi hedge, while eliminated completely with a perfect hedge. These relationships should also hold, as the input for risk factors change. Testing this will be one of the goals of the scenario analysis provided in Section 5.4.2.

As mentioned in Section 1.3, the main goal of the model in this thesis is to provide an internal valid model for IRS contracts and fixed-fixed CCS contracts with maturity up to 10 years given the choices mentioned above. Model validity is highest for the IRS contracts, as the one-factor model dynamics for interest rates are calibrated to IRS rates. Pricing with respect to these underlying interest rates provides low external validity for other products as mentioned in Section 2.5.3. Still, the fixed-fixed CCS contract, is somehow similar in structure, making pricing with the same risk factor model for interest rates reasonable. One should, however, acknowledge that the simplistic assumption of deterministic FX rates likely provides a less valid result than that of the IRS contracts, as the CSS exposure clearly is much more sensitive to the change in FX rates than interest rates, which can be seen in Section 5.3.1. The current interest rate term structure should be evaluated as well.

The CIR model cannot describe a term structure with decreasing interest rates in the short maturity reversing to increasing rates in the longer term. Hence, the term structure must be strictly upward or downward sloping allowing for a hump changing direction (except of course the hump from downward to upward sloping) as evaluated in Section 2.5.3.

Figure 5.4: Swap Rate Term Structure



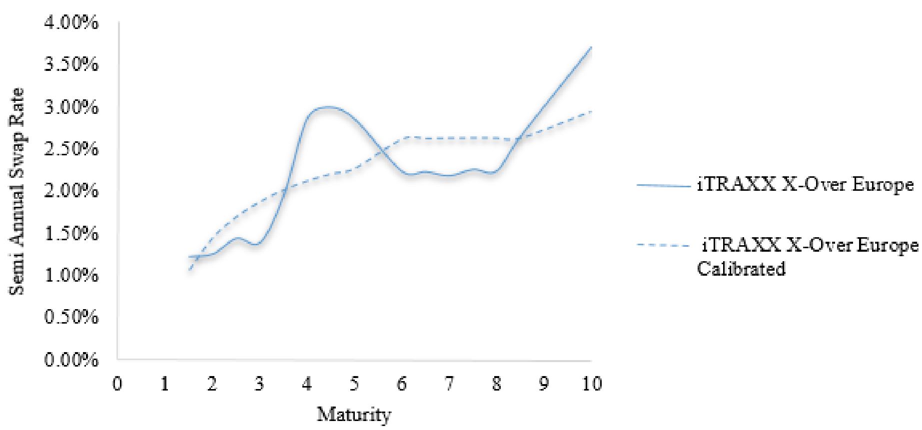
Source: Author's own creation

Monitoring the current term structure in Figure 5.4 this seems to be slightly violated for the EONIA and EURIBOR rates, which have a slight decrease up to a maturity of two years. Besides the actual term structure includes two humps: One from decreasing to increasing, but also one flattening the term structure at the longer maturity, which cannot be described by a one-factor model as stated in Section 2.5.3. Hence, extending the CIR model to more factors or time-varying parameters would circumvent this issue. However, extending the CIR model changes the focus of this thesis, which rather focus on estimating the xVA charge, taking the underlying risk factors as given. In reality, the xVA model of a financial institution should take as input risk factor output from the models already developed in the organization to increase consistence and sophistication of the xVA model and lower computational effort rather than do the same simulations a second time for the xVA model (Ruiz, 2015). As mentioned in

Section 2.3.2 adding a jump diffusion makes the model dynamics of credit spreads consistent with empirical data. The prevalent issues for the default intensity input is respectively:

- The simplified assumption of using the pricing function for a single-name CDS contract to price an index CDS contract, which has a slightly different payout structure evaluated in Appendix B.3.
- The choice of a proxy, which at best captures the systematic credit risk of the counterparty but not takes into account idiosyncratic risk as mentioned in Section 2.5.4 and Section 4.5.1.
- The rather wavy term structure described in Section 5.2.2, which makes it difficult to fit parameters to the model and results in a strong reversion speed of κ and a strong jump diffusion effect as can be seen in Figure 5.5.

Figure 5.5: Credit Rate Term Structure



Source: Author's own creation

If internal models used to rate the specific counterparty is available, this can be used or combined with the JCIR model to make a more sophisticated model for default intensity of the counterparty. This would increase internal validity of the model if used by financial institution estimating their xVA charge.

5.4.2 Scenario Analysis

Estimation of first-order and second order sensitivities increase the computational time of the model manifold (Ruiz, 2015). (Kenyon and Green, 2015a) shows how such sensitivities can be calculated using analytic measures or an algorithmic differentiation to optimize computational time when requiring a large number of different sensitivity measures. However, presenting such methods is beyond the scope of this thesis. This section analyses the model sensitivity with respect to some of the most critical input factors for two reasons: Firstly, it checks the sensitivity of the results presented above towards realistic market changes. Secondly, it ensures that the model behaves as expected towards changes in such input factors. Table 5.5 shows the 5 different cases tested against the base case equal to case 20 in Section 5.3.3.

Case 21 checks the effect of doubling the chosen MTA on the bilateral CSA agreement with the counterparty keeping everything else equal. This case is not significantly different from case 20 for any of the xVA terms including the overall xVA charge, except for a significant yet small increase in the CVA charge. Hence, the increase in MTA would have to be higher to provide a significant difference on the costs of collateralization (CollVA) and potential gap risk increasing the CVA charge further. The change would also effect EAD in the SA-CCR charge negatively as can be seen from equation 4.27. Nevertheless, the minor MTA increase does not seem to have significant effect on KVA. It is worth noticing, that volatility increases slightly for all xVA terms, which indicates possible higher variability of expected xVA charges.

Case 22 investigates the spread in accrual rate between the two collateral agreements. Hence, it doubles the negative spread to the OIS rate to be accrued on collateral posted at or by the CCP, while holding the accrual rate for the agreement with the counterparty constant at the OIS rate. As for the case above this change does not have any significant change on either the total xVA charge or any of the xVA terms. However, it does seem to increase the volatility of all different xVA charges.

Case 23 looks at the effect of increased funding costs with a fixed 200 bp spread between funding and lending rate[133] instead of 100 bp as assumed in case 20. In reality, such a change would follow as a consequence of a decline in the credit rating of the financial institution, though a perfect parallel shift would be unlikely (Phoa and Shearer, 1997). Different from case 21-22, the change in case 23 affects the overall result with a significant increase in the total xVA charge caused by a significant change in MVA and CollVA, as the costs of funding mismatch in collateral and funding initial margin increase. It does, however not significantly affect HVA, which remains a benefit for the reasons discussed in Section 5.3.2.

Case 24 evaluates the choice on return of capital. If the financial institution requires a higher target return on the capital held due to regulatory demands, this would increase the costs of such capital and thus the KVA charge. This is indeed the case even with the return rate only increasing from 8.3% in case 20 to 10% in case 24. KVA increases significantly leading to a significant increase the total xVA charge, while the other xVA terms do not change significantly as expected.

Cases 25-26 take a closer look at the underlying risk factors, which strongly depend on model choice and the current market outlook for interest rates and credit risk. Hence, changing the underlying assumption for calibration checks the robustness of the model assumptions, while also indicating, how changes in market conditions would affect the xVA charge and the different xVA terms.

As mentioned in Section 5.4.1 the one-factor CIR model cannot perfectly describe the current term structure with short term falling rates followed by an upward slope term structure and does not accept negative interest rates. For this reason, case 25 provides results of a recalibration of the stochastic interest rate models restricting the current short rate to be strictly positive.

[133] With the lending rate equal to the OIS rate.

Table 5.5: Simulation Results on Scenario Analysis

Case:	***20****: 3&6*, Payer, 7Y, Hedge, Bilateral CSA, IM=Dynamic*				***21****: Case 20,* ***change:*** *MTA=400*				***22****: Case 20,* ***change:*** $Coll_{AI,CCP}$*=-50*			
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*
xVA Total	1733.21	3372.83	[1693.98 ; 1772.44]	4.95	1733.45	3719.96	[1690.18 ; 1776.72]	4.95	1883.05	3840.39	[1838.39 ; 1927.72]	5.38
CVA	17.75	46.23	[17.21 ; 18.29]	0.03	20.12	57.53	[19.45 ; 20.79]	0.04	18.01	52.19	[17.40 ; 18.61]	0.03
CollVA	905.56	3319.68	[866.95 ; 944.17]	2.58	914.29	3730.00	[870.90 ; 957.67]	2.60	896.22	3848.78	[851.46 ; 940.98]	2.55
HVA	-167.70	976.85	[−179.06 ; −156.34]	-0.24	-177.81	1115.96	[−190.79 ; −164.83]	-0.25	-165.33	1177.13	[−179.02 ; −151.64]	-0.24
FVA	737.86	2351.11	[710.52 ; 765.21]	2.09	736.48	2622.74	[705.98 ; 766.98]	2.09	730.89	2680.59	[699.71 ; 762.06]	2.07
MVA	186.60	570.44	[179.97 ; 193.24]	0.51	191.11	626.97	[183.82 ; 198.40]	0.53	184.03	611.11	[176.93 ; 191.14]	0.51
KVA	790.99	556.88	[784.52 ; 797.47]	2.25	785.74	563.40	[779.19 ; 792.30]	2.23	950.12	681.38	[942.20 ; 958.05]	2.70

Case:	***23****: Case 20,* ***change:*** *Funding spread=200*				***24****: Case 20,* ***change:*** *RoC=10%*				*Input: Calibrated Model*				
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*		3M EURIBOR	6M EURIBOR		PD
xVA Total	2212.44	3684.30	[2169.59 ; 2255.29]	6.33	1883.05	3840.39	[1838.39 ; 1927.72]	5.38	r_0	-0.4407%	-0.3366%	λ_0	1.8595%
CVA	17.40	41.34	[16.92 ; 17.88]	0.03	18.01	52.19	[17.40 ; 18.61]	0.03	κ	0.0057	0.0143	κ	0.6792
CollVA	1271.74	3500.21	[1231.03 ; 1312.45]	3.63	896.22	3848.78	[851.46 ; 940.98]	2.55	θ	0.3851	0.1634	$\hat{\theta}$	0.0200
HVA	-161.13	954.71	[−172.23 ; −150.02]	-0.23	-165.33	1177.13	[−179.02 ; −151.64]	-0.24	β	0.0661	0.0684	β	0.0212
FVA	1110.61	2554.47	[1080.90 ; 1140.32]	3.16	730.89	2680.59	[699.71 ; 762.06]	2.07				γ	0.2314
MVA	295.97	697.82	[287.86 ; 304.09]	0.83	184.03	611.11	[176.93 ; 191.14]	0.51				α	0.0786
KVA	788.46	538.25	[782.20 ; 794.72]	2.24	950.12	681.38	[942.20 ; 958.05]	2.70					

Case:	***25****: Case 20,* ***change:*** *CIR recalibration;*$r_0 > 0$				***26****: Case 20,* ***change:*** *JCIR recalibration;5Y*				*Input: Recalibrated Model Parameters*				
	NPV	σ	$CI^{***}_{\alpha=5\%}$	*bp*	*NPV*	σ	$CI^{***}_{\alpha=5\%}$	*bp*		3M EURIBOR	6M EURIBOR		PD
xVA Total	2085.01	2656.01	[2054.12 ; 2115.90]	6.00	1950.82	6933.70	[1870.18 ; 2031.47]	5.58	r_0	0.0000%	0.0000%	λ_0	0.8298%
CVA	18.72	40.17	[18.26 ; 19.19]	0.05	24.01	67.37	[23.23 ; 24.80]	0.05	κ	0.0007	0.0015	κ	0.0460
CollVA	996.70	2411.52	[968.65 ; 1024.75]	2.87	1028.32	7732.11	[938.39 ; 1118.25]	2.93	θ	1.2195	0.8950	$\hat{\theta}$	0.3330
HVA	-172.67	768.19	[−181.61 ; −163.74]	-0.25	-215.83	2454.58	[−244.38 ; −187.28]	-0.31	β	0.0420	0.0510	β	0.0006
FVA	824.03	1649.64	[804.84 ; 843.22]	2.37	812.49	5303.67	[750.81 ; 874.18]	2.31				γ	0.0124
MVA	212.79	456.68	[207.48 ; 218.11]	0.61	195.59	975.31	[184.24 ; 206.93]	0.54				α	0.1133
KVA	1029.47	651.06	[1021.89 ; 1037.04]	2.96	918.73	964.42	[907.51 ; 929.95]	2.61					

* *3-month floating tenor paid semi-annualy*

** *6-month floating tenor paid semi-annualy*

*** *Confidence Interval*

Source: Author's own creation

Hence, avoiding the issues of dealing with negative rates described in Section 5.2.1.

Table 5.5 shows how such a calibration result in a decreasing κ and β, while θ increases considerably. This decreases the long run rate, $\kappa \cdot \theta = \varphi$, from 0.22% to 0.09%. Comparing case 25 to case 20, the higher interest rate result in a significant higher xVA charge driven by significant increase in KVA, CollVA, and MVA. Strictly positive rates clearly increase the costs of both funding collateral and accruing interest on initial margin, while the higher KVA is an indirect result of increased standardized EAD because of a higher swap exposure. Hence, the simplicity of the regulatory framework results in a more critical change in exposure as value increase compared to a smoother adjustment in the more sophisticated internal model taking into account the actual underlying interest rates when discounting. Besides, a small borderline significant increase in CVA follows from the increased part of exposure not being hedged. The volatility of all xVA charges except KVA increase as a result of this change in the parameters of the underlying interest rate models.

Lastly, case 25 evaluates the sensitivity towards calibration of model parameters for the JCIR model providing the default intensity. Section 5.4.1 evaluates that using iTRAXX Europe Crossover Index as proxy for a counterparty might not be a perfect fit. Nevertheless, not having specified a genuine counterparty, it makes more sense to focus on the issue of a wavy term structure of the default rates resulting a jump in the term structure from 5-year to 10-year iTRAXX series, which might be a result of illiquidity of 10-year series or segmented markets[134]. To try see, how this might affect results, Table 5.5 provides a recalibration of the JCIR input parameters only based on the 5-year iTRAXX series, acknowledging that longer maturities are not as well described by the new parameters. κ, β, and γ decrease considerable, while α increases slightly and θ increases significantly. Though the variation both β and the jump diffusion decrease significantly, the long-run default intensity, $\tilde{\theta} = \theta + \frac{\alpha \cdot \gamma}{\kappa}$, increases greatly from 4.68% to 36.78% as reversion speed, κ, of the calibration for case 25 is only a fraction of that in the calibration for case 20. However, jump effects lose most of their magnitude as a result of γ dropping.

Table 5.5 indicates that the higher long run default intensity effect outweighs the lower volatility through β and the jump diffusion. As expected such a difference creates a significantly higher CVA and KVA charge. Higher default intensity drives up KVA being the logic consequence of a higher expected default intensity entering the calculation of the RW in equation 4.26, while CVA is directly linked to default probability of exposure in equation 4.24. Volatility of CVA increases, while the more stable default intensity with less jump effect reduce KVA volatility. The result furthermore indicates higher volatility on funding issues (FVA and MVA), while the CollVA and HVA charge respectively are borderline significantly higher and lower for case 26 than case 20. Yet, these should not depend on change in default intensity, as this has no effect on exposure and collateral. It is quite likely, that these changes prove insignificant when increasing the amount of simulations. These changes become insignificant when increasing the amount of simulations.

Based on the different scenario analyses of sensitivity, the results indicate that the model

[134](Choudhry et al., 2012) discuss the segmented market hypothesis and liquidity preference theory.

responds as expected to changes made, though minor changes in the collateral agreements might not be significant. Potentially, a much larger amount of simulations or more extreme changes in those scenarios are needed to provide a significant result. Nevertheless, the volatility remains high for a great number of simulations, which might indicate heavy tails rather than just MC noise evaluated below.

5.4.3 Simulation Precision

All results presented in Section 5.3 and Section 5.4 are achieved using 20,000 simulations. The confidence intervals in the model output tables are calculated as (Hull, 2012)

$$\hat{\mu} \pm \frac{\rho^{-1}(0.95) \cdot \hat{\sigma}}{\sqrt{M}}$$

where $\hat{\mu}$ is the estimated value with $\hat{\sigma}$ standard deviation using M simulations. Reducing the spread of the confidence intervals further would require a great increase in the number of simulations, as the effect of more simulations decrease with the square root. Extending the model of this thesis to achieve more precise estimates at potentially less computational effort can be done with several variance reduction methods as observation sampling, antithetic variables or control variables as described by (Glasserman, 2003). Furthermore, using quasi MC methods moves away from generating completely random numbers and instead use low-discrepancies sequences to ensures a more even distribution, making convergence nearly proportional with M (Glasserman, 2003). Such an extension is possible to use if using another algorithm than (Marsaglia and Bray, 1964) to determine the random Gaussian variables (Glasserman, 2003).

Equation 3.17 in Section 3.2 provides an approximate measure of MC noise in terms of a standard normal distribution with mean 0. Isolating the standard deviation makes

$$\hat{\sigma}_{MC} = \frac{\hat{\sigma}}{\sqrt{M}}$$

Table 5.6 provides measurement of the MC noise for the case 20 in Section 5.3.3.

Table 5.6: Monte Carlo Noise

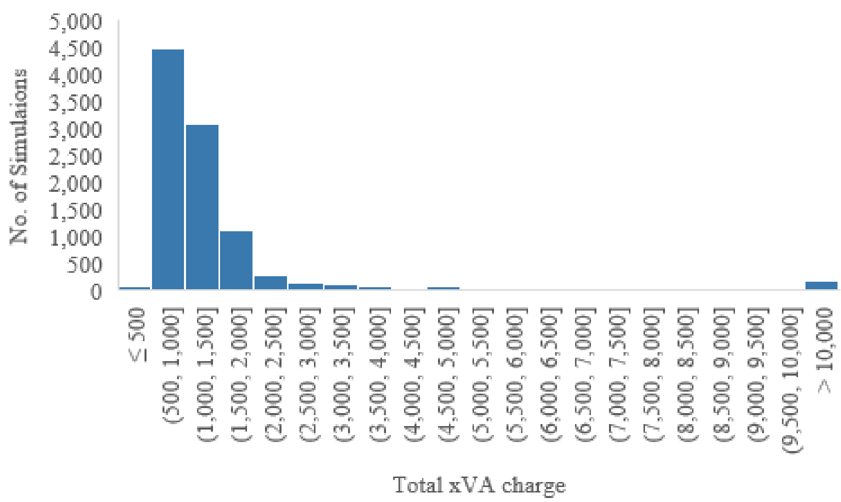
Case:	**2**: *3&6*, Payer, 7Y.*		
	NPV	σ	$\hat{\sigma}_{MC}$
xVA Total	1733.21	3372.83	23.85
CVA	17.75	46.23	0.33
CollVA	905.56	3319.68	23.47
HVA	-167.70	976.85	6.91
FVA	737.86	2351.11	16.62
MVA	186.60	570.44	4.03
KVA	790.99	556.88	3.94

Source: Author's own creation

Interestingly, though the volatility of xVA remains quite high at 20,000 simulations, only $\frac{\hat{\sigma}_{MC}}{\hat{\sigma}} = 0.71\%$ of this variation is approximately a result of the MC noise. It seems reasonable that conclusions drawn using a 95% confidence intervals on 20,000 simulations provides a result only marginally blurred by the MC noise. To half the current MC noise, M must equal 80,000. With each simulation of a contract with 7-year maturity equal to 15-20 milliseconds, using 20,000 simulation takes approximately one hour. If using some of the methods described above to increase convergence of the simulation, this is to be compared against a case with $n = \left[\frac{s}{t}\right]$ simulations[135] in order to measure the MC efficiency.

As mentioned in Section 3.3 one should be careful interpreting insignificant changes, as these might as well be a result of the MC noise and does not ensure a 95% significance. Furthermore, the assumption by (Glasserman, 2003) that the MC noise follows a normal distribution with a 0 mean might be very misguiding. The distribution of the the xVA charge simulations forming the results in Table 5.6 presented in Figure 5.6 reveals that the distribution has very heavy right tale. One potential cause can be that the model has a relative flat default intensity with a strong downwards reversion from the CIR part of the JCIR as discussed in Section 5.4.2. However, the jump effect does in rare cases provide extremely high jumps in default intensity, hence, CVA and KVA will explode in extreme cases compared to their average value. In extreme cases of the interest rate model, this can increase FVA and MVA radically.

Figure 5.6: Simulation Distribution of Total xVA Charge



Source: Authors own creation

If adding to the model the option to select a seed for the simulation process, this gives a fixed MC noise. Such an adjustment to the model would require seed input for each path, which would complicate the simple structure of adding simulations. Hence, such an extension is left to further research.

[135]See Section 3.2.

Part VI

Managing xVAs

6.1 Introducing xVA Management

This part uses the conclusions from the model results presented in Section 5.3 to evaluate optimization the overall xVA charge and methods to hedge and manage xVA risk. There are strong overlaps and interdependences between the different xVA terms. As a results, financial institutions have moved towards centralization of xVA management at a xVA desk (Carver, 2013). Section 5.3 demonstrated how the relative importance of different xVA terms changed with choices of hedging and collateral agreements and potentially lowered the total xVA charge. Hence, depending on the type of derivative contract it is possible to optimize the contract terms to reduce the total xVA charge. First section discuss optimization of the total xVA charge as the price competitiveness of the financial institution, does not care about the size of each xVA, but the overall xVA charge (Ruiz, 2015). This includes model results on setting an optimal initial marging. Second section discusses hedging possibilities besides the market risk hedge included in the model. Last section accounts for more general risk management approaches by the xVA desk, combining hedging and optimal xVA charge with further risk management strategies.

6.2 xVA Optimzation

For the purpose of approximating an optimal choice of contract terms minimizing the total xVA charge, this thesis focus on a payer IRS contract with 3-month tenor on the floating leg paid semi-annually having a quasi hedge.

As mentioned in Section 5.3.2, the perfect hedge would be a more optimal choice of hedge in eliminating market risk, though this cannot be seen from the results, as the longer tenor effect of the floating leg is larger than the market risk reduction. However, if a 3-month tenor floating leg paid semi-annually is available or can be replicated with FRA contracts[136] at costs below that of the quasi hedge instrument and the additional xVA reduction from having a perfect hedge, using such as hedge instead, would lower the total xVA charge further.

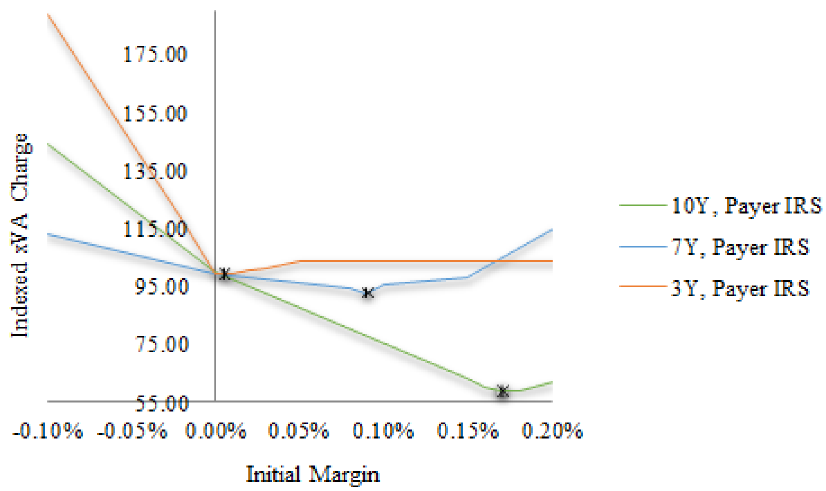
It is quite clear from Section 5.3.3 that unilateral CSA agreements with the counterparty is favorable, but also slightly unrealistic to achieve when assuming a rational counterparty. Hence, we focus on bilateral CSA agreements with the same fixed MTA and rounding as case 16-20 in Section 5.3.3 to avoid conclusions on these two options, as they will definitely be sensitive to introducing operational costs of transferring collateral, which is left out of the model. Furthermore, minor changes in these collateral options do not significantly change the total xVA charge as demonstrated in Section 5.4.2.

Still outstanding is determination of an optimal initial margin or threshold in terms of the overall xVA costs. To analyze this issue and provide approximate indications, Figure 6.1 provides the overall xVA charge as a discrete function of threshold or initial margin requirements. No threshold and a dynamic initial margin is assumed for the CCP while thresholds and initial margins are held equal for both counterparty and the financial institution. Otherwise, it would logically be optimal to allow a huge threshold for the financial institution, which in the limit reduces the bilateral CSA agreement to a unilateral agreement. As mentioned in Section 4.2.1,

[136] Forward rate agreements.

thresholds and initial margin are the exact reverse things of one another, meaning that a threshold can be seen as a negative initial margin and vice versa. For this reason, it makes no sense to include them both simultaneously. Hence, the negative initial margin in Figure 6.1 means that initial margin equals 0, but threshold instead is positive with that amount of the notional swap value. The optimal initial margin is an approximation found by a trial and error process of simulating the total xVA charge for different amounts of initial margin and thresholds for the payer IRS contracts with respectively 3, 7, and 10 years to maturity. The resulting xVA charge is indexed to 100 for $IM = 0.00\%$ to allow for comparison across maturities.

Figure 6.1: xVA Charge



Source: Author's own creation

Figure 6.1 shows that including an initial margin reduces the total xVA charge compared to not doing so or allowing for a threshold. Comparing similar contracts with different maturity, the expected exposure is logically smaller for shorter maturities due to less uncertainty and potential variation, which is a function of time to maturity. This results in a lower optimal level of initial margin for contracts with shorter maturity. For a contract with only 3-year maturity, the optimal initial margin is only slightly above 0 given the current term structure of interest rates and the default intensity providing a low gap risk of exposure. However, in all cases some amount of initial margin is still more optimal than including no initial margin or allowing for a threshold. The optimal xVA charge for the 7-year payer IRS contract is 4.93 bp, which is not significantly different from case 20, which includes a dynamic initial margin. Hence, a dynamic initial margin seems to capture an optimal level of initial margin quite well. (Gregory, 2015a) provides similar results as above for the optimal choice of fixed initial margin with respect to contract maturity, while also providing an increasing relationship with lower credit quality of the counterparty. Knowing this, it might be possible to create an approximate xVA optimization of initial margin based on maturity of the contract and credit quality of the counterparty. However, as shown in Section 5.3 many other factors like hedge choice, floating leg tenor, collateral agreement details, and current term structure of interest rates affect the exposure and xVA charge. Thus, these alter the optimal choice of initial margin. However, using a dynamic initial marging would account for all of the above issues, if the swap exposure volatility used to estimate intitial marging requirments is time-inhomogeneous.

6.3 Hedging xVA

If a perfect xVA hedge existed, this would allow the trading desk to simply use the traditional risk-neutral price under the multi-curve pricing approach while adding the costs to set up the xVA hedge (Ruiz, 2015). Hence, the xVA charge represents the cost of different risks not included in traditional pricing, which the xVA desk can use for hedging purpose. These costs can be seen as the premium of an exotic option with a complex payoff structure determined by the underlying exposures. However, replicating such an option is impossible due to the enormous complexity of the payoff affected by many underlying risk factors also displaying interdependencies. Hence, such a replication is computationally impossible to model exactly. Besides, issues as future changes in regulations includes a great deal of uncertainty, which also cannot be accurately quantified (Gregory, 2015a). Acknowledging these issues, this section suggests different hedge strategies to mitigate some of the xVA risk, accepting that not all xVA risk can be completely covered. Hedging of market risk has already been covered in Section 5.3.2, while Section 4.5.1 explains how such hedge instruments incur a KVA charge, which reduces the hedge efficiency as capital rules are not perfectly risk sensitive (Gregory, 2015a).

The CVA represents the credit risk of the counterparty given the exposure of the contract. If a CDS contract would be available on the counterparty such can be used to hedge the default risk. However, CDS contracts have a fixed notional value, while the CVA risk is contingent on the exposure of the contract, which as shown in Section 5.3.1 changes over time. There have been issued contingent CDS contracts, where the notional value is indexed to the value of a contract or an index, but these are highly illiquid (Gregory, 2015a).

(Ruiz, 2015) instead suggests an approximate static hedging strategy with CDS contracts, which approximately replicates a contingent CDS contract. This is a backward hedging process dividing the expected exposure profile into a discrete number of intervals and calculating their average exposure, EE_i^*, as in Section 4.2.2. Starting with the last time interval, which ends at time T_i, the required notional CDS protection, H_i^H, with maturity, T_i, equals EE_i^*. Moving backwards to the discrete interval ending at $T_{i-1}, H_{i-1}^H = EE_i^*$ notional value of CDS protection with maturity T_{i-1} is either bought or sold short depending on the sign of H_{i-1}^H. Continuing backward until T_0, this approximately hedges the expected exposure at initiation of the contract. However, CVA sensitivity to the underlying market risk factors like the interest and FX rate must also be hedged to try lock in the expected exposure profile. Otherwise, the CVA hedge might be completely off, as changes in underlying risk factors change the actual exposure profile and hence CVA. Creating such a market risk hedge is far from easy. (Ruiz, 2015) suggests first hedging gamma[137] and vega sensitivities[138] of CVA towards the underlying risk factors with

[137] Including cross-gamma effects between interdependent underlying risk factors.
[138] I.e. the second-order sensitivities.

options and use future or swap contracts to hedge the delta sensitivities[139]. However, such hedges must be with a CCP[140] or based on exchange instruments as these will otherwise carry counterparty risk themselves. Still, OTC hedges carry other xVA costs as CollVA, MVA, and HVA.

Taking into account the illiquidity of the CDS protection, this produce transaction costs making the approximate static hedges less attractive. Besides, this thesis focus on minor corporate counterparties, for which single-name CDS protection is not available. Hence, an index CDS like an iTRAXX index can provide a proxy hedge for the systematic risk as mentioned in Section 4.5.1, but cannot protect against actual default and hence results in a less efficient hedge than when using a single name CDS contract. In addition, the iTRAXX index contracts are OTC-trades (Markit, 2014b) leading to an xVA charge itself[141] (Ruiz, 2015).

Hedging of other xVA terms is even more difficult, as hedge instruments seem even less obvious than for CVA. In theory, FVA can be hedged approximately by borrowing forward expected funding needs from the expected positive hedge exposure profile in the same way as the approximate static CVA hedge on the expected exposure profile. However, this also requires hedging of market risks as for CVA, but with respect to FVA sensitivities toward underlying risk factors. Even with this being possible, it requires the possibility to borrow forward from the treasure department of the financial institution, which usually only provides short-term funding of liquidity needs (Ruiz, 2015). Leaving FVA unhedged results in liquidity risk on funding as described in Section 4.3.1 measured through the spread between bond yield and credit rates. This risk might lead to much higher funding costs as Section 5.4 indicates in a simple case of parallel increase in the funding spread. These funding issues would affect the funding of initial margin equivalently. Hence, hedging the MVA charge based on the expected initial margin held, would use the same approach as FVA.

As already mentioned, some of the KVA charge is possible to partially hedge by hedging CVA with CDS or Index CDS hedges as mentioned above to achieve partial relief on the CVA regulatory capital charge (Gregory, 2015a). The rather simplistic Basel framework to calculate exposure provided in Section 4.5.1 does not capture exposure as precisely as CVA. Hence, KVA exposure to the underlying risk factors is slightly different than CVA exposure. On top of counterparty risk hedges, achieving capital relief in the CVA capital charge by setting up a market risk hedge, minimizes the exposure and lowers the SA-CCR charge[142] and MR charge as evaluated in Section 5.3.2. Yet, the most difficult issue of KVA still remains. Namely, hedging the risk of changes in the regulatory framework, which is next to impossible to predict at a long time horizon, when thinking of the numerous changes in regulations presented just in the past decade.

[139]I.e. the first-order sensitivities.
[140]Hedge instruments at a CCP are still subject to SA-CCR capital charge resulting in KVA costs.
[141]Mainly costs of collateral if traded with a CCP, as low gap exposure and default risk makes CVA negligible, while KVA only occurs from the SA-CCR charge.
[142]Even though the CCP hedge is subject to SA-CCR charge as evaluated in Section 5.3.2.

6.4 xVA Risk Management

The hedging strategies provided in Section 6.3 are only approximations. Trying to make a precise hedge might result in higher transaction costs than the value at risk. It is, hence a strategic decision to what extend the xVA desk should actively try hedging the xVA risk or instead set aside the xVA charge as a capital buffer to cover losses and accept the full xVA volatility. Between these two extremes, the xVA desk can act as a profit center like trading desks, potentially profiting from open exposure, but ensuring appropriate risk management (Ruiz, 2015).

Besides hedging the xVA exposure, risk management at the xVA desk includes negotiation of optimal collateral agreements with the counterparties and ensuring market risk hedges with a CCP to optimize the overall xVA charge as covered in Section 6.2 (Gregory, 2015a). However, such must be done with respect to the hedging alternatives of the xVA exposure as described in Section 6.3.

Managing more counterparties, the choice of central clearing of such contracts at a CCP in the same way as the market risk hedges eliminates counterparty risk and allows for compression of all trades. Compression works in the same way as netting trades on a per counterparty basis, though possible across original counterparties when contracts are centrally cleared (Gregory, 2015a). However, clearing fees and costs of compression must be held against the benefit from elimination of counterparty risk.

Besides the analytical mitigation of xVA exposure, risk management can set different limits for the trading desk regarding

- total xVA exposure
- individual limits on the credit risk on different counterparties
- exposure limits towards different underlying risk factors

The xVA desk is responsible to enforce such limits (Ruiz, 2015). Managing the combination of exposure and xVA risk limits, hedging strategies, and optimization of the xVA charge through market risk hedges and collateral agreements becomes an extremely complex matter, which in practice is still under development at the xVA desks of financial institutions.

Part VII

Conclusion

In this thesis, we have presented expansions of the basic risk neutral pricing method of OTC fixed income derivatives, which emerged as a consequence of the financial crisis of 2008. These extensions have been exemplified with a model framework to price single IRS contracts nominated in EUR or fixed-fixed CCS contracts nominated in respectively EUR and USD. The model takes the point of view of a medium-sized financial institution pricing a swap contract with a minor corporate counterparty with credit quality below investment grade. Hence, the counterparty perceives the financial institution to be risk free relative to themselves. While this is the case for the counterparty the financial institution is not risk free from the point of view of financial markets. Therefore, the financial institution can only achieve funding at a risk premium.

This model extends basic pricing of swaps to a multi-curve framework recognizing the existence of a basis spread between different floating tenors due to non-negligible credit risk and liquidity risk on the interbank market. We further present, how basic assumptions for the risk neutral pricing framework of OTC derivatives have been abandoned as a result of acknowledging counterparty credit risk, funding costs, and regulatory capital requirements. These issues became apparent with the financial crisis and resulted in the rise of xVAs to adjust the risk neutral price.

Counterparty credit risk of OTC derivatives must be taken into account. But such exhibits dependency with interest rates. Hence, appropriate inclusion of counterparty credit risk results in model dependency, which is only feasible through numerical pricing, even for plain vanilla interest rate derivatives. The numerical pricing problem includes high dimensionality most efficiently solved with Monte Carlo simulation of the underlying risk factors with dynamics described by stochastic models. A simulation approach furthermore makes it possible to model collateral directly. This thesis examines a CIR model to describe the dynamics of interest rates and a JCIR model to describe the default intensity dynamics of the counterparty as a simple and tractable way to model the underlying risk factors. Dependency modelling of interest rates and default intensity is achieved through Cholesky factorization of the Gaussian variables based on historical correlation. Furthermore, this thesis provides the framework to calibrate the interest rate models to market quotes of swap rates and the default intensity model to quoted credit spreads of the iTRAXX Europe Crossover Index as proxy for counterparty credit risk. Model parameters are fitted using the generalized reduced gradient non-linear optimization method. Suitable discretization schemes are defined for the stochastic risk factor models for simulation purpose to ensure convergence towards the true value.

The stochastic underlying risk factors provide the necessary input to estimate exposure of the swap contract throughout its lifetime, which allows for estimation of the xVA charge divided into different components. CVA measures expected losses on the swap contract due to counterparty default risk. FVA evaluates the costs of funding cash flow mismatches of the contract (HVA) and funding costs of collateral posted (CollVA) to mitigate counterparty exposure and hence CVA. MVA accounts for funding costs of segregated initial margin posted to cover potential gaps between contract exposure and collateral posted. KVA relates to the costs of the capital buffer required by regulatory authorities to be held against unexpected losses. Regulatory capital is determined as the CCR-SA charge and the regulatory CVA charge for counterparty risk following the Basel framework and an internal model to estimate regulatory capital for market risk.

The simulation model based on the above inputs demonstrate that the different estimates of xVA terms depend on swap contract properties. The expected exposure and the total xVA charge increases for swap contracts with longer maturities as these yield higher uncertainty. The current upward sloping EUR interest rate term structures generally result in larger exposure for payer than receiver IRS contracts, which furthermore increases CVA, FVA (through HVA), and KVA for the payer relative to the receiver IRS contracts. Longer tenor on the floating leg does not surprisingly also result in a higher exposure and total xVA charge. Exposure and xVA charge of Fixed-Fixed CCS contracts yield similar relation to the contract maturity, but are much higher than for IRS contracts as it is also sensitive to the FX rate, which for simplicity is assumed to follow the interest rate parity. This makes the FX rate a deterministic function of the stochastic spot interest rates. This upward FX rate term structure overweighs the positive exposure of the fixed-fixed CCS contracts and results in almost negligible negative exposure.

Introducing the option to hedge market risk with an identical or similar reverse swap contract with a CCP greatly reduces exposure and the total xVA charge, though the collateral agreement and initial margin requirements related to the hedge with the CCP result in a CollVA and MVA charge. Mitigating counterparty exposure through a collateral agreement and initial margin requirements further increase respectively CollVA and MVA. Yet, the JCIR model based on the iTRAXX index CDS provides a rather modest level of credit risk. Hence, the funding costs of collateral and initial margin, outweigh the reduction in CVA, HVA, and KVA for the bilateral CSA agreements. This makes the collateralized xVA charge higher than the uncollateralized case, except for unilateral CSA agreements with the counterparty posting collateral. However, combining collateral agreements with a market risk hedge reduces CollVA significantly due to rehypothecation of collateral, which reduce funding requirements to emerge from negative mismatches in collateral between the counterparty and the CCP and hence CollVA. As the hedge also further reduces CVA and KVA, while also providing a HVA benefit as a consequence of the current interest rate term structure, collateral agreements including a market risk hedge result in a much lower xVA charge, even at a low level of credit risk.

The optimal total xVA charge depends on counterparty credit risk, funding costs, capital requirements, and many other issues. Hence, the choice of hedge and collateral might differ much depending on these issues. This thesis demonstrates how an optimal amount of initial margin can be set to minimize the total xVA charge with respect to maturity of the contract, knowing that changes in counterparty credit risk, floating tenor and other features also affect the optimal initial margin post. Such issues can potentially be covered by setting a dynamic initial margin. It is possible to establish different proxy hedges for the risk of the different xVA terms based on the expected exposure profile and xVA sensitivities to different underlying market risk factors. Yet, a perfect hedge is next to impossible to establish as there is a limited access to necessary hedge instruments and as transaction costs of hedge instruments limit the extent to which hedging is efficient. Furthermore, such hedge instruments might carry xVA risk themselves. The xVA desk of a financial institution might enforce overall limits on the xVA risk allowed to account for risks uncovered by collateral and hedging strategies.

Sensitivity analysis of the model includes a back test of the model with a discussion of the input factors and scenario analyses verifying model robustness and internal validity when crucial assumptions change. Subsequently, the scenario analysis evaluates the xVA sensitivities to changes in fixed input assumptions. The model estimates of the different xVA terms in the different scenarios change as anticipated for the most crucial input changes, namely funding costs, target return on capital, and recalibrated parameters for the underlying stochastic risk factor models. On the other hand, minor adjustments of the collateral agreement lack significance in the model estimates. An evaluation of the simulation precision indicates a low MC noise from using 20,000 simulations in the pricing model. Heavy tails in the distribution of simulations have a large impact on the average xVA charge, which might result in a slightly higher MC noise than those estimated.

Though the pricing model presented in this thesis is a simplified version of reality, it proves to be extremely complex and requires a great computational effort. This illustrates the magnitude of the quantitative challenge faced by financial institutions requiring models capable of pricing a tremendous quantity of different OTC derivatives simultaneously to include marginal effects and interdependencies. Sophisticated xVA models must also include a wide arrange of sensitivity measures with respect to underlying risk factors and cross-gamma effects between these to provide a reasonable foundation for risk management. However, an appropriate xVA model must be able to achieve such estimates at a high level of precision without accelerating the computation time. Hence, derivative pricing with inclusion of xVAs still remains a complex matter for financial institutions. Meanwhile the likelihood of additional value adjustments emerging remains, which will increase the complexity of derivative pricing further.

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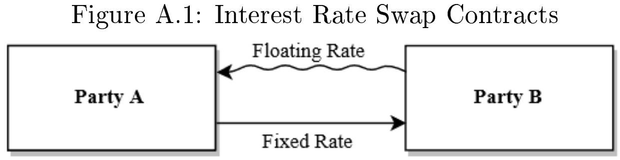
Appendix A

Appendix: Fundamentals of Swap Contracts

This appendix introduces different kinds of simple swap contracts and the fundamentals of swap properties and valuation.

A swap is a financial contract where two parties exchange a stream of cash flows for a predetermined period of time with one another (Munk, 2011). The simplest example is the plain vanilla interest rate swap (IRS) where the exchange of cash flows is defined as interest rates on an agreed notional value. As can be seen in Figure A.1, Party A pays a fixed interest rate throughout the life of the swap contract and receives a floating rate from the counterparty, B, linked to a floating reference rate (e.g. EURIBOR and LIBOR) in exchange[1]. The two different sides of the contract are called the legs. Thus, an IRS has a fixed and a floating leg. In a plain vanilla IRS, the floating rate can be quoted on a day-to-day basis as the overnight indexed swap rate (OIS). Some other very common quotes are the 3-month, 6-month and 1-year reference rate (Wilmott, 2007). The floating payment is based on the prevailing reference rate for the period in the vanilla contracts. If payments are less frequent than the quoted floating rate, it will be rolled over up to the next payment being an average of those rolls. Usually, the fixed rate is set so, that the present value of the swap contract is close to 0.

Figure A.1: Interest Rate Swap Contracts



Source: Author's own creation

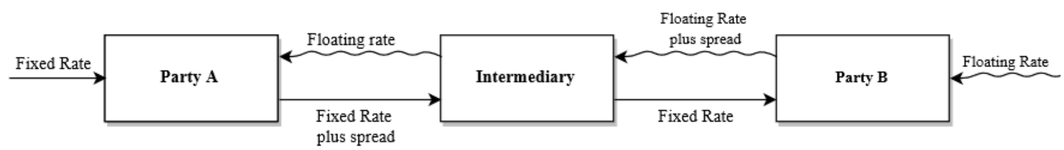
Swaps is an OTC product, which means that it is not traded on an exchange and thus not standardized. This makes it possible to design payments, notional, maturity and other features to fit the parties entering the contract. This has given a variety of more or less exotic swap

[1] This is called a payer swap

types, which besides interest rates also can be based on currencies, equity, credit, commodities ect. (Hull, 2012).

Financial institutions are mainly used as intermediaries between two parties by entering two offsetting swap contracts with two different counterparties to hedge away the interest rate market risk on the cash flows, as can be seen in Figure A.2. This provides a liquid market place for swap contracts as financial institutions act as market maker on swaps quoting a bid-ask spread to account for the risk on their matched swaps contracts, which have to be hedged.

Figure A.2: Interest Rate Swap Contracts through an Intermediary



Source: Author's own creation

Table A.1 and Table A.2 indicate that the swap market have become more liquid over the years. Hence, total notional amount and the total market value have increased, due to the positive development in both interest rate swaps and cross-currency swaps outstanding. In addtion the market share for interest rate swaps and cross-currency swaps has increased quit a lot over the years.

Table A.1: Notional Amounts Outstanding

	07	08	09	10	11	12	13	14
Total derivative contracts	585.932	598.147	603.899	601.046	647.810	635.684	710.632	630.149
Cross-Currency swaps	14.346	14.940	16.509	19.271	22.791	25.420	25.447	24.203
Interest rate swaps	309.588	341.127	349.287	364.377	402.610	372.293	456.725	381.027

Source: (BCBS, 2015)

Table A.2: Gross Market Value

	07	08	09	10	11	12	13	14
Total derivative contracts	15.802	35.280	21.541	21.298	27.296	24.953	18.825	20.880
Cross-Currency swaps	817	1.633	1.042	1.234	1.324	1.258	1.186	1.350
Interest rate swaps	6.182	18.157	12.575	13.138	18.045	17.284	12.918	13.946

Source: (BCBS, 2015)

A.1 Interest Rate Swaps

The plain vanilla IRS is as mentioned above a contract exchanging fixed interest payments for payments determined by a quoted reference rate. The use of IRS contracts is quite popular, as it can be used to hedge or speculate against changing interest rates. An IRS can also transform the payment stream of an asset or a liability, so that a fixed or floating return is reversed, without having to change the asset or liability itself. This can often be done cheaper than replacing an asset or a liability, which explains the popularity of IRS contracts. This is called the competitive advantage argument (Hull, 2012). This theory argues that some companies have a comparative advantage in borrowing money on the floating-rate market, while others have a comparative advantage borrowing in the fixed-rate market. By entering a swap contract, a company is able to borrow at a fixed rate by borrowing in the floating-rate market and convert to a fixed rate using a plain vanilla IRS. Vice versa being the case for the counterpart company, which will in total give a lower interest rate for both (Wilmott, 2007).

This theory has been facing a lot of critique. One can argue, that the floating loan being cheaper for company would only be because the credit risk of a company is smaller for a shorter period of time. Increasing credit risk as time go by would make the lender refuse to roll over the floating rate for the next period without an additional spread to the reference rate (Hull, 2012). If the comparative advantage would still exist taking the credit risk into account for the floating rate, then the gain should disappear due to the rule of no arbitrage.

However, the swap market remains huge, which has in itself given an advantage of being more liquid than many of the bond markets (Wilmott, 2007). From Table A.1 and Table A.2, the IRS market did in December 2014 account for 60.5% of the total derivatives market with a total notional amount of USD 381 trillion and with a market value of almost USD 14 trillion outstanding.

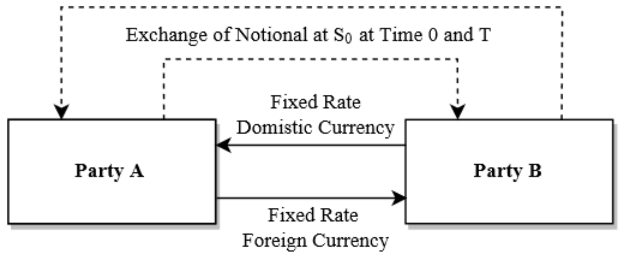
Besides the plain vanilla IRS, the basis swap has also become an important single currency swap.This swap contract exchanges cash flows given by a floating rate for both parties. However, this will be different quoted reference rates, e.g. paying 3 month EURIBOR versus receiving 6 month EURIBOR or paying 3 month EURIBOR versus receiving 3 month LIBOR. Because the different types of reference rates or different quoted maturities carry different credit risk, there will be higher rate on one of the legs. To even this difference, the leg with the least risk will be added a spread. This basis swap spread has become very important to measure the risk of a certain tenor rate.

The OTC-market has developed many types of IRS contracts more complex than the plain vanilla swap IRS. Some like the step-up swaps and the amortizing swaps does not have a constant notional value. Some swaps only exchange cash flows at maturity or pays the observed floating rate for each period rather than the prevailing floating rate. The floating rate might be set to many different reference rates and some cash flows might only be exchanged when the floating rate is within a predefined bound. It is also possible to make swap contracts with embedded options, so that the contract can be cancelled or terminated before the maturity date (Hull, 2012).

A.2 Cross-Currency Swaps

A plain vanilla fixed-fixed cross-currency swap (CCS) is an exchange of fixed interest rate cash flows and the notional at initiation and maturity, where the two legs of the contract are nominated in different currencies, which can be seen from Figure A.3.

Figure A.3: Cross-Currency Swap Contracts



Source: Author's own creation

Using the spot exchange rate the notional values are set so that they approximately are equal. The interest rates are set so that the initial value of the contract is close to 0. Just as for IRS contracts, CCS contracts can be used for speculating and hedging purpose. This will be towards changes in the exchange rate. A CCS contract can also be used to transform liabilities and assets from one currency to another without changing the underlying position. As for IRS, some argue for a comparative advantage from doing so, e.g. if one company pays more for a loan in one currency than in another. Even though some might argue, that such an advantage should disappear in an efficient market the liquidity of the CCS market makes it a cheap way to make transformation of assets and liabilities (Hull, 2012).

Besides the plain vanilla CCS contract, it is possible to enter a basis CCS contract, which exchanges two floating legs. It is also possible to enter Cross-currency IRS contracts combining the feature of a fixed and a floating leg but in two different currencies.

The CCS market did in December 2014 account for 3.8% of the total derivatives markets with a total notional amount of USD 24.2 trillion and a market value of USD 1.35 trillion outstanding, taking from Tables A.1 and Table A.2[2].

A.3 Bootstrapping of the Interest Rate Term Structure

Traditionally when evaluating swap contracts the assumption was that they could be hedged completely and that the discounting curve was the same as the forwarding curve for the floating leg, meaning that there was no basis swap spread on different quoted rate maturities, because the floating tenor rate was risk free. If such was possible, one could price the floating leg simply by calculating the expected future cash flows determined from the forward rates and discount these cash flows back using the same rates (Bianchetti, 2008). Only the floating payment fixed for the next period will be priced, as it might differ from the current risk free rate, which might have moved since last prevailing rate was set. Also the cash flows from the fixed leg can be discounted with the same discount rates. Thus, the present value of the two legs is the value of the swap contract (Hull, 2012).

[2]FX Swaps are not included as CCS, as only the principal is exchanged and the domestic returned at maturity equals the forward price at initiation F_0X and not the spot price S_TX (Amatatsu and Baba, 2008). In that sense (BCBS, 2015) categorize FX swaps in group with FX Forwards.

To construct the interest rate term structure or zero swap curve, liquid instruments for different maturities are used to derive a single discount rate at each maturity. These instruments could e.g. be deposits for the short-term maturities, interest rate futures for the medium-term and IRS on the long-term (Ametrano and Bianchetti, 2009). We need to find generic instruments or instruments with maturity fitting the relevant maturities for which we want to find rates for our curve or alternatively strip instruments with non-fitting maturities from accrued interest (Kenyon and Stamm, 2012). If using bonds for constructing the curve, coupon bonds remain liquid at longer maturities which is often also the case for IRS contracts. However, we need to turn this into synthetic zero-coupon bonds to obtain one single rate (Munk, 2011).

If we need discount rates fitting the maturities quoted in the market, we can use interpolation to fit term structure between the different rates derived. This can e.g. be done with simple linear interpolation, cubic splines or Nelson-Siegel parametrization (Munk, 2011). The latter approach has become rather popular, as it provides a smooth, but still flexible curve with the same constants for all maturities (Munk, 2011). Term structure formulae for zero-coupon rates with Nelson-Siegel parameterization is given by

$$\bar{y}(\tau) = \beta_0 + \beta_1 \cdot \frac{1 - e^{-\tau/\theta}}{\tau/\theta} + \beta_2 \cdot \left(\frac{1 - e^{-\tau/\theta}}{\tau/\theta} - e^{-\tau/\theta} \right)$$

where β_0, β_1, β_2 and θ are constants, which is to be calibrated. β_0 reflects the long-term forward rates, where $\beta_1 e^{-\tau/\theta}$ reflects the effect on the short-term forward rates and $\beta_2 (\tau/\theta) e^{-\tau/\theta}$ reflects the medium-term forward rates. The parameter θ gives the impact of the maturity interval for τ (Munk, 2011).

However, though Nelson-Siegel parameters now have the quasi interpretation of the different parameters, the parameterization is not based on economic arguments, but simply on finding a good fit to the current term structure (Munk, 2011). Hence, interpolated rates are not constrained to follow the rule of no arbitrage, as the only concern is to create the best fit across the entire term structure. (Christensen et al., 2011) however, provide affined modifications of Nelson-Siegel, which abide to the rule of no arbitrage. Another shortcoming is that parameterization does not necessarily show consistency with dynamic short rate models, meaning that interpolation for the sake of dynamics short rate model calibration might be inconsistent. (Bjork and Christensen, 1999) demonstrate that only augmented versions of the Nelson-Siegel parameterization are consistent with the Hull-White Model and Ho-Lee model. (Filipovic, 1999) generalizes this to conclude that no simple interest rate models show consistency with the general Nelson-Siegel.

A.4 Traditional Swap Valuation

As mentioned above the value of a swap contract is determined as the sum of discounted cash flows of the swap over the entire lifetime of the contract (including the cash flows of notional exchange on CCS contract at maturity). (Hull, 2012) discusses two basic approaches to value both IRS and CCS contracts.

Eventhough the principal is not exchanged at maturity for an IRS contract, the cash flows stream of interest payments of each of the legs and a final notional looks similar to a coupon bond. Valuing the fixed and the floating leg separately as respectively a fixed-rate $\left(B_t^{fix}\right)$ and a floating-rate $\left(B_t^{fl}\right)$ coupon bond and subtract them from each other the similar notional will cancel out and the difference in the value of the cash flows will give the value of the IRS $\left(V_t^{IRS}\right)$. Hence,

$$V_t^{IRS} = B_t^{fix} - B_t^{fl} \tag{A.1}$$

Depending on which cash flow is larger, the value of the swap contract will be either negative or positive. Note that as mentioned in Appendix A.3, the floating bond would only consist of next payment and principal as all other payments discounted and forwarded at the same rate gives a present value of 0. For the value of a CCS contract $\left(V_t^{CCS}\right)$ a similar approach applies:

$$V_t^{CCS} = B_t^D - S(t) \cdot B_t^F \tag{A.2}$$

where the spot rate $(S(t))$ is multiplied to the foreign currency bond $\left(B_t^F\right)$ measured in foreign currency and the domestic currency bond $\left(B_t^D\right)$.

The other valuation approach follows that described in A.3. Here each cash flow can be seen as a Forward Rate Agreement (FRA) and thus the whole swap can be seen as a portfolio of FRA contracts. The value of each FRA contracts $\left(V_i^{FRA}\right)$ is given by:

$$V_i^{IRS} = H \cdot \left(r^{fix} - r^{fl}_{(t_{i+1}-t_i)}\right) \cdot D_{(t_{i+1}-t_i)}$$

where D is the discount factor, H is the notional value, r^{fix} is the fixed interest rate, r_i^{fl} is the floating rate from time t_i to t_{i+1}, equal to the time between each payment. Thus the value of an IRS contract would be:

$$V_t^{IRS} = \sum_{i=1}^{n} V_i^{FRA}$$

where n is the number of i cash flows. For a fixed-fixed CCS the same applies, and each cash flow can be seen as an FX forward contract:

$$V_i^{FX\,Forward} = f_{i,\tau} \cdot H \cdot \left(r - r^f\right) \cdot D_{(t_{i+1}-t_i)}$$

where $f_{i,\tau}$ is the forward exchange rate for period i with length $\tau = t_i - t_{i-1}$. The value of the CCS contract would be:

$$V_t^{CCS} = S_0 \cdot H - f_{n,\tau} \cdot H \cdot D_{(t_{i+1}-t_i)} + \sum_{i=1}^{n} V_i^{FX\,Forward}$$

A.5 Swap Valuation in Details

As mentioned in Appendix A.4, the value of a payer swap equals the value of a fixed bond minus the value of a floating bond as in equation A.1. The fixed rate bond value is

$$B_t^{fix} = \tau \cdot H \cdot r^{fix} \cdot \sum\nolimits_{i=i(t)}^{n} D\left(t, T_i\right) \tag{A.3}$$

where τ is the length of each cash flow period[3] and $D\left(t, T_i\right)$ the spot discount rate[4]. The floating rate bond value

$$B_t^{fl} = \tau \cdot H \cdot \sum\nolimits_{i=1}^{n} D\left(t, T_i\right) \cdot F\left(t, T_i, T_{i+1}\right) \tag{A.4}$$

which follows the approach in Section 2.2.3 valuing the present value of the forward interest payments, $F\left(t, T_i, T_{i+1}\right)$ for a fixed cash flow window of τ on the notional H. However, this is the price of the floating bond at time 0. At $t > 0$ the interest payment at the end of the current period is known, and can thus be quoted as the spot rate, which changes equation A.4 to

$$B_t^{fl} = \tau \cdot H \cdot \left[D\left(t, T_{i(t)}\right) r_{T_{i(t)-\tau}}^{T_{i(t)}} + \sum_{i=1+1}^{n} D\left(t, T_i\right) \cdot F\left(t, T_i, T_{i+1}\right) \right]$$

With $r_{T_{i(t)-\tau}}^{T_{i(t)}}$ being the current floating spot rate. Assuming pricing at $t = 0$, this can be ignored. Then inserting the value formula for the floating and the fixed bond gives the value of the swap. Here the value of a payer swap from equation A.1 by inserting A.3 and A.4 is

$$V_0^{Payer} = \tau \cdot H \cdot \sum\nolimits_{i=1}^{n} D\left(t, T_i\right) \cdot F\left(t, T_i, T_{i+1}\right) - \tau \cdot H \cdot r^{fix} \cdot \sum\nolimits_{i=1}^{n} D\left(t, T_i\right) \tag{A.5}$$

Rearranging gives

$$V_0^{Payer} = H \cdot \Big((1 - D(0, T_n) - \tau \cdot r^{fix} \cdot \sum\nolimits_{i=1}^{n} D\left(0, T_i\right) \Big) \tag{A.6}$$

For a CCS contract with two fixed legs as in Appendix A.4 equation A.3 can be substituted into equation A.2

$$V_0^{CCS} = \tau \cdot r \cdot \sum\nolimits_{i=1}^{n} D\left(0, T_i\right) - S\left(0\right) \cdot \tau \cdot r^f \cdot \sum\nolimits_{i=1}^{n} D^f\left(t, T_i\right) \tag{A.7}$$

where r^f is the foreign fixed interest rate and $D^f\left(0, T_i\right)$ the discount factor with a foreign rate. Note that in order to abide the no-arbitrage condition for two currencies in Section 2.2.5, discounting the foreign leg must be with discount rates of that currency in order to convert as the spot rate at $t = 0$ as given above.

[3]From $T_i$to T_{i+1}
[4]From t to T_{i+1}.

A.6 Deriving Swap Rates

The swap rate is the equilibrium fixed rate, which the market is willing to pay for receiving a certain floating tenor rate in exchange over a certain period of time (Hull, 2012). Thus, the swap rate is the fixed interest rate making the value of the swap contract equal to 0, which ensures no arbitrage (Munk, 2011). IRS contracts are often issued at the swap rate, so that the contract has no initial value.

(Munk, 2011), shows that the swap rate can be derived as a weighted average of the present value forward rates determining the cash flows of the floating leg, which comes from setting the value of equation A.5 equal to 0 and rearrange for the fixed rate

$$r_t^* = \frac{\sum_{i=1}^{n} F(t, T_{i-1}, T_i) \cdot D(t, T_i)}{\tau \cdot \sum_{i=1}^{n} D(t, T_i)}$$

Note that τ is the time between each coupon[5]. Pricing at time 0 and similarly rearranging equation A.6, the swap rate can be simplified to

$$r_t^* = \frac{1 - D(0, T_n)}{\tau \cdot \sum_{i=1}^{n} D(0, T_i)}$$

Following the logic for IRS contracts, setting equation A.7 to equal 0 and rearrange for r, the swap rate for CCS contracts must equal

$$r_t^* = \frac{r_0^f \cdot S(0) \cdot \sum_{i=1}^{n} D^f(t, T_i)}{\tau \cdot \sum_{i=1}^{n} D(t, T_i)}$$

[5]E.g. semi-annual coupons would mean that $\tau = 0.5$.

Appendix B

Appendix: Fundamentals of CDS Contracts

Credit default swaps are the most used credit derivatives. It is a contract written on an underlying company, and can be seen as an insurance for the buyer against credit events of the underlying company as e.g. default (Hull, 2012). The buyer of the payer leg pays a predetermined premium, which is a fixed interest rate on the notional value of the CDS contract. If a credit event occurs for the underlying company of the CDS contract, then the holder receives coverage of the LGD times the notional value of the contract. Settlement is either in cash or physical delivery. Physical delivery gives the holder of the CDS contract the right to sell bonds of underlying company of the CDS contract at face value to the counterparty (Munk, 2011). Even though the market for CDS contracts was rapid increasing up until the financial crisis of 2008 (BCBS, 2015), it has been very constant ever since and does to a great extend lack liquidity (Brigo and Mercurio, 2007). This appendix presents a way to value single-name CDS contracts and find the fair CDS spread. It also introduces iTRAXX index as a proxy measure of default risk for companies, on which no single-name CDS contract are available.

B.1 Valuation of Single-name CDS Contracts

A single-name CDS contract is written on one underlying firm. Thus, CDS contracts are mainly available on the largest firms. (Munk, 2011) provides a framework for pricing single-name CDS contracts

$$V_t^{CDS} = V_t^{Prot} - V_t^{Prem} - V_t^{Accr}$$

Here V_t^{Prem} is the value at time t given no default up till time t and without any recovery of value at default. V_t^{Prem} is expressed as

$$V_t^{Prem} = \tau \cdot k \cdot H \cdot \sum_{i=i(t)}^{n} E_t^{\mathbb{Q}} \left[e^{-\int_t^u (r_\tau + \lambda_\tau) d\tau} \right]$$

with $\tau = T_i - T_{i-1}$, k the fixed rate payment for the CDS, H the notional value, r_τ the dynamic interest rate and λ_τ the dynamic default intensity.

V_t^{Prot} takes into account potential recovery of the notional at default. Assuming a fixed recovery rate over time, which gives the a fixed LGD

$$V_t^{Prot} = H \cdot LGD \int_t^{T_n} E_t^{\mathbb{Q}} \left[e^{-\int_t^u (r_\tau + \lambda_\tau) d\tau} \lambda_u \right] du$$

V_t^{Accr} takes into account accrual costs of k to be paid by the buyer of the CDS contract, if default does not happen at τ^* which is $T_i < \tau^* < T_{i+1}$. These accrual costs are $\left[\tau^* - T_{i(\tau)-1}\right] k \cdot H$. Thus,

$$V_t^{Accr} = k \cdot H \int_t^{T_n} E_t^{\mathbb{Q}} \left[e^{-\int_t^u (r_\tau + \lambda_\tau) d\tau} \lambda_u \right] \left[u - T_{i(u)-1} \right] du$$

B.2 Derivation of the CDS Spread

The fair CDS spread is the fixed rate, k_t^*, which makes $V_t^{CDS} = 0$. It is possible to derive the spread as a weighted average of the present value default probabilities with respect to $\frac{LGD}{\tau}$. However, this requires the assumption that LGD is constant for all default times, and that r_τ and λ_τ follow stochastic dynamic models (Munk, 2011). Then

$$k_t^* = \frac{LGD \cdot \sum_{i=1}^{n} \mathbb{Q} \left(T_{i-1} < \tau^* \leq T_i \mid \tau > t \right) \cdot D \left(t, T_i \right)}{\tau \cdot \sum_{i=1}^{n} \mathbb{Q} \left(\tau^* > T_i \mid \tau > t \right) \cdot D \left(t, T_i \right)}$$

Where $\mathbb{Q} \left(T_{i-1} < \tau^* \leq T_i \mid \tau > t \right) = \lambda \left(T_{i-1}, T_i \right)$ is the probability of default between T_{i-1} and T_i while $\mathbb{Q} \left(\tau^* > T_i \mid \tau > t \right) = \lambda \left(t, T_i \right)$ is the accumulated default probability from t to T. Hence,

$$k_t^* = \frac{LGD \cdot \sum_{i=1}^{n} \lambda \left(T_{i-1}, T_i \right) \cdot D \left(t, T_i \right)}{\tau \cdot \sum_{i=1}^{n} \lambda \left(t, T_i \right) \cdot D \left(t, T_i \right)} \tag{B.1}$$

B.3 Index CDS

An index CDS contract gives the buyer an insurance against default on a portfolio of companies. The buyer receive payment as for single-name CDS contracts each time one of the underlying companies experience a credit event[1] (Munk, 2011). The iTRAXX indices provide portfolio protection for an interval of defaults defined by the contract as a percentage of the notional value, H. The intervals of default are called tranches. If an iTRAXX has a tranche of $3\% - 6\%$ this means that losses are covered fully up to 6% of the notional has defaulted. An iTRAXX Index includes 125 companies, and thus each is equally weighted 0.8%[2] of the notional. Each default accounts for $0.8\% \cdot LGD \cdot H$. When $\sum_{i=1}^{n} 0.8\% \cdot LGD \cdot H < 3\% \cdot H$ the default payments

[1] Note that default payment must exceed a threshold to be paid, which is described for the iTRAXX indices below.

[2] $\frac{1}{125} = 0.8\%$.

accumulate, but are not paid out. At $\sum_{i=1}^{n} 0.8\% \cdot LGD \cdot H \geq 3\% \cdot H$, then $3\% \cdot H$ is paid out to the CDS protection buyer by the seller. The following defaults payment are made for each default up to $\sum_{i=1}^{n} 0.8\% \cdot LGD \cdot H \geq 6\% \cdot H$ At this point, default payments only up to the limit of 6% of H are paid. The contract is afterwards unwinded (Brigo and Mercurio, 2007). iTRAXX contracts exists on different sectors, geographic areas and are available with 3, 5, 7, and 10-year maturity (Bloomberg, 2004).

Different CDS indices provide possible proxies for companies without their own single-name CDS available or can be a proxy for a portfolio hedge instrument. To model the actual default intensity accurately is quite complex and would for e.g. iTRAXX indices involve 125 correlated intensity models. The roughly fair index CDS spread should be the average fair CDS spread on the companies included in the underlying index of the index CDS (Munk, 2011). The iTRAXX index spread is fair the coupon to be paid quarterly for the insurance, if the value of this Index CDS should equal 0 (Markit, 2014a).

Appendix C

Appendix: The Nature of Stochastic Calculus

This appendix is a short review on fundamental stochastic calculus used as background to understand this thesis. Stochastic calculus is the modelling of uncertainty, which is used for derivative pricing. A stochastic process can be in either discrete or continuous time. This appendix describes the most important steps to get to the geometric Brownian motion from the Brownian motion, assumptions, stochastic integrals, Itô's Lemma and partial differential equations. Derivation of different properties and proof of the different processes are excluded. The most important assumptions derived from the assumptions of a rational investor, perfect capital markets and full information is the well-know no-arbitrage assumption, which is essential for derivation of the partial differential equation Wilmott (2007).

C.1 Properties of Stochastic Processes

Stochastic processes have three main properties: The Markov property, the Martingale property and the property of quadratic variation.

Having the Markov property, past movements of the process has no effect on the future movements, which means that these are not possible to predict (Hull, 2012). This complies with the efficient market hypothesis. Thus, the conditional distribution of the process does only depend on the current information available and not past information.

Having a Martingale property, the stochastic process does not have any trend relative to the chosen numeraire. This mean that the expected value of the process one incremental step into the future would equal that of today (Wilmott, 2007). This should, however, be seen in the light of the used probability measure i.e. the choice of numeraire. When using a risk-neutral probability measure the expected value of a process only evolves at the risk free rate $(r\,(t_i - t_{i-1})) \cdot P_t = \bar{P}_t e^{r\,dt}$ is a $\mathbb{Q}$-martingale. Thus, $P_t = E_t^{\mathbb{Q}}\left[P_t e^{r(t_i - t_{i-1})}\right]$(Munk, 2011).

The property of quadratic variation ensures that moving from discrete time steps in a process to a continuous time, the sum of the quadratic variations becomes a linear function of time and do not explode or collapse (Wilmott, 2007).

C.2 Brownian Motion

The Brownian motion is a random process, which is used as the element of randomness in other processes like e.g. the geometric Brownian motion. The Brownian motion for a discrete process can be described as:

$$\Delta Z = \varepsilon\sqrt{\Delta t}$$

where Δ is the change in time $(t_{i+1} - t_i)$ and ε is a standard normal distribution, $N(0,1)$ (Munk, 2011). The continuous process for the Brownian motion is expressed as $dz = \varepsilon\sqrt{dt}$.

C.3 Diffusion Processes

A very general diffusion process is what we call an one-dimensional Itô process. This could e.g. be:

$$dx = \mu(t)dt + \sigma(t)dz_t \tag{C.1}$$

Where $\mu(t)dt$ is the drift of the process and $\sigma(t)dz_t$ is the diffusion of the process. $\mu(t)$ and $\sigma(t)$ can be time-inhomogeneous or constant given as μ and σ. dt is the continuous time and dz_t is a Brownian motion in continuous time (Munk, 2011). If we want to describe the price of an asset like a stock, it is more reasonable to make the drift and the diffusion proportional to the price of the asset. Then returns are determined relatively to the stock price. Thus, we scale the process dx with the current value x_t (Hull, 2012):

$$dx = \mu(t)x_t dt + \sigma(t)x_t dz_t$$

Holding both the drift and the diffusion constant we obtain the geometric Brownian motion:

$$dx = \mu x_t dt + \sigma x_t dz_t$$

C.4 Stochastic Integrals

The stochastic integral is the integral of a stochastic process. For a process in discrete time the sum of the process will converge to the stochastic integral as the number of time steps, n, increase and the process converge to continuous time (Wilmott, 2007).

$$\int_t^T f(\tau)\, dx_\tau = \lim_{n\to\infty} \sum_{i=1}^{n} f(t_{i-1})\left(x_{t_i} - x_{t_{i-1}}\right)$$

For the Brownian motion this will be explained as

$$\int_t^T \sigma(u)\, dz_u = \lim_{n\to\infty} \sum_{i=1}^{n} \sigma(t_{i-1})\left(z_{t_i} - z_{t_{i-1}}\right)$$

The main point is that the sum of randomness must converge to one randomness, which is called the mean-square convergence. Thus, the stochastic integral is the limit of the discrete approximation of the continuous process (Munk, 2011). From this we can show, that the expected value is 0 and the variance

$$Var_t\left[\int_t^T \sigma(u)dz_u\right] = \int_t^T E_t\left[\sigma(u)^2\right]dz_u$$

For a process with a drift included the stochastic integral is given by the function

$$x_T = x_t + \int_t^T \mu(\tau)d\tau + \int_t^T \sigma(\tau)dz_\tau$$

If a stochastic process has a stochastic integral the process can be integrated as one step with one randomness. If this is not possible, the process can only be described by an Euler Discretization (Wilmott, 2007).

C.5 Itô's Lemma

Equation C.1 is a one-dimensional Itô process. If we want to describe a process, which depends on time, but also of the dynamics of the process in equation C.1

$$y_t = f\left(x_t, t\right)$$

then we use, what is called Itô's Lemma, which is a Taylor expansion of the function $f\left(x_t, t\right)$:

$$dy_t = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dx + \frac{1}{2}b^2(x_t,t)\frac{\partial^2 f}{\partial x^2}dt$$

Note that different from a normal Taylor expansion this includes the second derivative of x, as this has first order of t, and will then not be negligible, which is the case for normal calculus (Hull, 2012). As Itô's Lemma presents the dynamics of a process of time and the dynamics of another process, which is also stochastic, it can be used to describe the dynamics of derivatives, as these takes values based on underlying assets. Itô's of a geometric Brownian motion is given by:

$$df(x_t,t) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dx + \frac{1}{2}\sigma^2 x_t^2\frac{\partial^2 f}{\partial x^2}dt$$

If we want to express the function as a logarithmic function in order to ensure lognormal returns often, which is often assumed for stochastic processes, this will affect the drift, which must be adjusted with subtraction of half the variance of the process in continuous time (Hull, 2012). For the geometric Brownian motion, taking Itô's of the function $y_t = ln(x)$ gives the dynamics (Wilmott, 2007)

$$dx_t = x_t\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dz$$

C.6 Partial Differential Equations

A partial differential equation (PDE) is the art of constructing a risk free portfolio of a derivative and the underlying asset. By shorting the right fraction of underlying assets per long derivative contract in the portfolio, which equals the first order sensitivity of the derivative, Δ, then the portfolio payoff will be the same no matter how the diffusion part of the stochastic processes changes in continuous time. This does of course only work, if the underlying hedge of the derivative can be traded in continuous time as well and if we assume that fraction of an asset can be sold short without any margin costs. Assuming this, then as the portfolio payoff will not be affected by randomness the payoff is risk free. Thus, the payoff must equal the risk free rate by the assumption of no arbitrage (Hull, 2012). For the geometric Brownian motion the PDE is given as

$$\frac{\partial V}{\partial t} + r(t)x_t\frac{\partial V}{\partial x}dx + \frac{1}{2}\sigma^2 x_t^2\frac{\partial^2 V}{\partial x^2} - r(t)V_t = 0$$

where $r(t)$ is the risk free rate and V_t is the value of the derivative. The PDE proves two important properties:

1. If an analytical pricing formula can be derived, then it must fulfill the condition set by the PDE.

2. Even more important: If all risk is eliminated by the construction of a portfolio, then the value of a derivative is not affected by risk and risk preferences. Hence, one can find the value of the derivative under the simple assumption of a risk neutral world, where the expected return equals the risk free rate for all underlying assets. The value of the derivatives in such a scenario would also hold as the value in the real world. Otherwise, one could make an arbitrage, as all risk can be eliminated with the PDE hedge portfolio.

Appendix D

Appendix: Mean Reverting Diffusion Processes

The geometric Brownian motion described in Appendix C.3 is a widely accepted diffusion process to describe the dynamics of stock prices, as these have an expected rate of return and a volatility relative to the stock price (Hull, 2012). The dynamics of interest rates do, however, seem to behave differently. They tend to exhibit mean-reversion back to a certain level rather than continuously grow with a fixed expected drift. The mean-reversion feature is not well described by the geometric Brownian motion, but is captured in a number of interest rate diffusion models categorized as Ornstein-Uhlenbeck processes and Square-root processes.

D.1 Ornstein-Uhlenbeck Processes

Stochastic processes categorized as Ornstein-Uhlenbeck processes have the time-inhomogeneous dynamics

$$dx_t = \kappa \left[\theta(t) - x_t\right] dt + \beta(t) dz_t$$

where $\theta(t) = \frac{\varphi(t)}{\kappa}$. $\beta(t)$ describes the volatility of the process, but not proportional to the current level of x_t as for the geometric Brownian motion. The drift of the process has now become mean-reverting by introducing the two parameters $\varphi(t)$ and κ. $\varphi(t)$ represents the long term level, which the mean-reversion will drive the diffusion process towards, while κ represents the adjustment speed of the reversion effect. There are also versions of Ornstein-Uhlenbeck processes with κ being time-inhomogeneous. $\theta(t)$ determines the direction of the drift. If $x_t > \theta(t)$ then the drift will be negative, whereas $x_t < \theta(t)$ will make the drift positive. As the process is a random process, the diffusion might drive the x_t further away from $\theta(t)$ in the short run. But as will be shown below through Itô's Lemma and the stochastic integral, the diffusion is (just as for the geometric Brownian motion) normal distributed with mean zero, so that the mean-reversion will converge the process in the long run (Munk, 2011).

(Munk, 2011) illustrates with Itô's lemma for $y_t = f(x_t, t)$ with $f(x,t) = e^{\kappa t}x$

$$dy_t = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dx + \frac{1}{2}b^2(x_t,t)\frac{\partial^2 f}{\partial x^2}dt$$

$$dy_t = (x,t)\frac{\partial f}{\partial t}dt + (x,t)(\varphi(t) - \kappa x_t)\frac{\partial f}{\partial x}dt + \frac{1}{2}\beta(t)^2(x_t,t)\frac{\partial^2 f}{\partial x^2}dt + \frac{\partial f}{\partial x}(x_t,t)\beta(t)dz_t$$

$$dy_t = \varphi(t)e^{\kappa t}x_t dt + \beta(t)e^{\kappa t}dz_t$$

The stochastic integral for the Itô dynamics is given as

$$y_T = y_t + \int_t^T \beta(u)e^{\kappa u}dz_u$$

which by substitution for y and modifications gives

$$x_T = e^{-\kappa T}x_t + \int_t^T \varphi(u)e^{-\kappa(T-u)}du + \int_t^T \beta(u)e^{-\kappa(T-u)}dz_u$$

This prove that the diffusion is normal distributed with mean zero and a variance of

$$Var_t[x_T] = \int_t^T \beta^2(u)e^{-2\kappa(T-u)}du$$

The expected value of the entire process is given by

$$E_t[x_t] = e^{-\kappa\tau}x_t + \int_t^T \varphi(u)e^{-\kappa(T-u)}du \tag{D.1}$$

Examples of Ornstein-Uhlenbeck processes is the Vasicek model, the Hull-White model, and the BDT-model[1] (Hull, 2012).

D.2 Square-root Processes

A category of interest rate term structure models, which differ from the Ornstein-Uhlenbeck processes is the square-root processes. These have the dynamics

$$dx_t = \kappa[\theta(t) - x_t]dt + \beta(t)\sqrt{x_t}dz_t$$

which differ from the Ornstein-Uhlenbeck processes in that the diffusion is scaled by $\sqrt{x_t}$. This ensures that the process cannot take a negative value, as the diffusion will completely disappear if the process hits a value of 0, so that the drift term will force the process back to a positive value. If the variance variable restriction $\beta^2(t) \leq 2\varphi$ holds[2], then the process cannot reach a value of 0 either, as the positive drift will be stronger than the volatility at low values of x_t.

As for the Ornstein-Uhlenbeck processes, we can derive the Itô dynamics and the stochastic

[1] Black-Derman-Toy model.

[2] As in the Ornstein-Uhlenbeck process $\theta(t) = \frac{\varphi(t)}{\kappa}$

integral as done by (Munk, 2011), which shows that the expected value does not change. Thus,

$$E_t\left[x_t\right] = e^{-\kappa\tau}x_t + \int_t^T \varphi\left(u\right)e^{-\kappa(T-u)}du$$

The variance is given as

$$Var_t\left[x_T\right] = \int_t^T \beta^2(u)e^{-2\kappa(T-u)}E_t\left[x_u\right]du$$

Because of the inclusion of the $\sqrt{x_t}$ in the square-root process the variance is a bit more complex. As shown by (Munk, 2011) the process will not be normal distributed with mean and variance as given above. Instead, the process follows a non-central $\chi^2(a, b(\tau))$-distribution with

$$a = \frac{4\varphi(t)}{\beta(t)^2}, \qquad b(\tau) = x_t c(\tau)e^{-\kappa\tau}, \qquad c(\tau) = \frac{4\kappa}{\beta(t)^2(1-e^{-\kappa\tau})} \tag{D.2}$$

with mean and variance as given above. a is the degrees of freedom, while b is a non-centrality parameter. (Munk, 2011) argues that the square-root processes, though more complex, are more suitable to describe the short-term interest rates, as there is empirical evidence that the variance of the interest rate is correlated with the interest rate itself.

Examples of square-root processes is e.g. the Cox-Ingersoll-Ross model (CIR) and the Longstaff-Schwartz model.

Appendix E

Appendix: Debit Valuation Adjustment

CVA is also known as unilateral CVA, as it only takes the credit risk of the counterparty into consideration. In the bilateral case, own credit risk is included as the Debit Value Adjustment (DVA).

This thesis take the assumption that the financial institution issuing the swaps considers itself default free. Hence, financial institutions can charge CVA based on the credit risk of the counterparty, while the counterparty must accept the financial institution as risk free. Before the financial crisis, counterparties seemed to perceive this assumption to be fair. However, the Lehmann collapse in 2008 made it clear, that this was obviously not the case (Kenyon and Stamm, 2012), (Gregory, 2015a). Besides the disputes with the corporate counterparties, DVA is clearly also an issue for interbank trades, as both financial institutions would charge the other party a CVA charge, but ignore their own risk of default. Thus, there would be an asymmetry in the pricing from the point of view of the two different parties (Brigo et al., 2013). Hence, there could never be a price agreement without inclusion of DVA meaning no trades would be made (Gregory, 2015a).

This appendix briefly introduces DVA. First section provides the model framework for estimating DVA. Second section discusses the controversy of DVA and provides arguments for and against inclusion of DVA. This thesis takes the opinion of (BCBS-214, 2011), which base its opinion on a pricing perspective rather than an accounting perspective (Gregory, 2015a). Further arguments follow at the end of Appendix E.2.

E.1 Estimating DVA

To avoid the asymmetry of pricing, DVA takes into consideration the risk of own default. Hence, the exposure for the counterparty will be the market value, which will be lost in case of default by the financial institution. Thus, the negative expected exposure (Gregory, 2015a)

$$NEE_t = [V_t, 0]^-$$

Compared to EE_t, NEE_t focus on cases, where the contract is in-the-money for the counterparty. Hence, the contract has a negative market value for the financial institution. Market value calculations and collateral calculations can be made in the same way as for CVA in Section 4.2.1 and Section 4.2.1. However, in this case, it would seem more normal to use a bilateral collateral agreements, except if the credit risk of the two parties are very different, as we do not assume any of the parties to be relatively risk free compared to the other. Similar to the estimation of CVA in Section 4.2.2, (Brigo et al., 2013) provides the formula for DVA, which are the present value of the expectations to the exposure at the time of default under the risk neutral measure, $\mathbb{Q}$:

$$DVA_t = E_t \left[LGD_b \cdot 1_{\{t<u^1=u_b<T\}} D(t,u) NEE_t \right] \tag{E.1}$$

However, note that the indicator function now includes an additional condition. As default can happen to both parties, the loss of one party will only happen, if the other party has not yet defaulted. Hence, when the first party defaults, the effect of default of the other party is irrelevant, as the contract has already ceased to exist (Gregory, 2015a). The discrete approximation[1] of equation E.1 is (Brigo et al., 2013)

$$DVA_t = LGD_b \sum_{i=1}^{m} E_t \left[1_{\{t<u^1=u_b<T\}} D(t,u) NEE_t \right]$$

As we need to know which party defaults first, it is not possible to use the default probability. The model must find the actual time of default of the first counterparty, which is the stopping time. To model default time structured default models can be used (Munk, 2011). Depending on who defaults first, and whether the exposure is positive for the counterparty, this would be the contribution to DVA or CVA. After this, the contract ceases to exist, and there will not be further exposure. Hence, the possible default of the surviving party has no effect on the CVA and DVA (Brigo et al., 2013). In case neither defaults, there will be no contribution to either CVA or DVA. Achieving these results is clearly model dependent. Besides the correlation between the interest rates and the risk of default for the counterparty needed in the model in Section 4.2.2, the risk model of own default must also have model dependency on both interest rates and the default model for the counterparty (Brigo et al., 2013).

If the counterparty defaults, then this would in principal give them a gain on their own DVA up to the original maturity. The effect of this will, however, be difficult to claim, as it would be a subjective measure, equal to the DVA estimate at the time of counterparty default. In principle, this is addable to the close-out price, if priced into the default case, hence reducing both CVA and DVA. However, there is no easy way to do this correctly for contracts like swaps, which have opposing cash flows (Gregory, 2015a).

[1] This discretization is precise as $\frac{m}{T}$ increases.

E.2 The DVA Controversy

Whether or not to include DVA in pricing of derivatives has raised many arguments. Some of these come from an accounting point of view omitted here, as this thesis concerns the derivative pricing and hedging aspects of xVA charges. The critique of DVA from a pricing and hedging perspective occurred from issues of companies profiting from falling credit quality.

DVA accounts for the risk of own default. If the credit rating drops this would mean, that the risk of own default increases, hence the liability of the cash flows will decrease, as these cash flows are zero in case of default (Brigo et al., 2013). For this reason, and for the complexity of DVA on derivative contracts, the Bank of International Settlements requires DVA be excluded from the price when measuring regulatory capital on a unilateral estimation of credit VaR, meaning disregarding or deducting[2] the DVA:

> *"Therefore, after considering various alternatives, the Basel Committee is of the view that all DVAs for derivatives should be fully deducted in the calculation of CET1. The deduction of DVAs is to occur at each reporting date, and requires deducting the spread premium over the risk free rate for derivative liabilities."* - (BCBS-214, 2011)

Regulatory capital relieves resulting from lower credit rating, would indeed be very counterintuitive with the purpose of regulatory capital trying to reduce the overall risk.

The question then is, whether this DVA gain is realizable as cash profit, or whether it is only senseless paper money. The easiest way to realize the DVA benefit would be by defaulting, but this is outright stupid. As mentioned in Appendix E.1, a positive DVA claim could be made at close-out, but this would be difficult. The DVA claim is a subjective model estimate and counterparties might not recognize the claim, as was the case for many counterparties after the bankruptcy of Lehmann Brothers (Gregory, 2015a). (Ruiz, 2015) does not agree that DVA is next to impossible to realize. (Ruiz, 2015) exemplifies this with a company having a significant drop in their credit rating. Here the counterparty would be willing to pay a premium to this company to unwind the derivative contract between them. The reason being, that this would circumvent a larger loss in case of the very likely default of the company. This premium received by the company is a profit from increasing DVA that the counterparties pay willingly to decrease their CVA. However, (Gregory, 2015a) argues that such cases only occur in case of a great default risk, which would be close to the case of benefitting from own bankruptcy as first stated. Besides (Gregory, 2015a) points out, that when such contracts are unwound, replacement is necessary. For such replacements, the CVA charge by the counterparty would be much larger than this DVA benefit.

A last way to realize the DVA gain, but also for the CVA or xVA desk to manage the DVA risk, would be to create a hedge. For the CVA side such a hedge would likely be with CDS contracts on the counterparty, if such instruments exist or with short positions in bonds of the counterparty. Reversing these methods to hedge of own DVA, it seems far from reality, that any sane counterparty would buy CDS protection on the same company offering such protection agreement. Buying back own bonds would be possible, but they cannot be traded freely (Kenyon

[2]If a bilateral estimation of default risk has been used.

and Stamm, 2012). Besides, at an actual default, the bond hedge would not hedge losses. Neither would selling CDS protection on similar counterparties and indices as proxy for own credit risk (Gregory, 2015a). Hence, the hedging might lower the credit volatility, but not have any effect in case of default or against any firm specific shocks to the credit ratings of the company.

The perspective of this thesis is on derivative pricing and hedging rather than accounting. The pricing takes the viewpoint of a medium-sized financial institution pricing a swap contract with a minor corporate counterparty[3]. Hence, the financial institution might have default risk relative to counterparties on interbank trades or to large corporate clients. However, the credit risk of the financial institution might be as good as risk free relative to a minor corporate counterparty. Considering the issues described in this section, it seems reasonable to disregard DVA from the pricing model. It is also important to note, that FVA covers some of the issues of DVA[4].

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[3]See Section 1.3.5

[4]See Section 4.3.