A note on relativity before Einstein

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Keswani and Kilmister [1983] did not mention Larmor as one of the precursors of special relativity. The Lorentz transformations, written here in a form similar to that used by Poincaré [1905],

$$\begin{aligned} x_1 &= \ell\beta^{-1}(x - vt) \\ y_1 &= \ell y \\ z_1 &= \ell z \\ t_1 &= \ell\beta^{-1}\left(t - vx/c^2\right) \end{aligned} \tag{1}$$

where

$$\beta = (1 - v^2/c^2)^{1/2} \text{ and } \ell = 1,$$

were first posited by Larmor [1897] although Voigt [1887] had studied these equations with $\ell = \beta$. Since $\ell \neq 1$, Voigt's equations do not form a group (Poincaré [1905]), and do not satisfy the principle of relativity. Larmor's contribution is not well known perhaps because he derives the transformations in two steps and never sets them down in their modern form. His book (Larmor [1900]) is slightly easier to follow than the earlier paper and we consider that work here.

Larmor ([1900], p. 167) first considers the transformation from the rest system (x, y, z, t) to a moving system (x', y', z', t''),

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t'' &= t - \epsilon v x'/c^2 \end{aligned} \tag{2}$$

where

$$\epsilon = (1 - v^2/c^2)^{-1},$$

and demonstrates that electrical and optical phenomena observed in the moving system are independent of velocity to the first order in v/c. He remarks that the time variable is reckoned from a new origin but does not give any interpretation of this. Poincaré [1900], discussing Lorentz's local time, remarked that it arises

when clocks in a moving reference frame are synchronized by exchanging light signals which are assumed to travel with the same speed against and with the motion of the reference frame; that is by the procedure in special relativity.[1]

Equations (2) incorporate the relativity of simultaneity but not time dilation. This was introduced by Larmor ([1897] and [1900], p. 174) in what he called the 'second-order' transformation,

$$\begin{aligned} x_1 &= \epsilon^{1/2}x' \\ y_1 &= y' \\ z_1 &= z' \\ t_1 &= \epsilon^{-1/2}t'' = \epsilon^{-1/2}t - \epsilon^{1/2}vx'/c^2 \end{aligned} \tag{3}$$

Upon making all the substitutions we find that the co-ordinate systems (x_1, y_1, z_1, t_1) and (x, y, z, t) are related by the Lorentz transformations, Eq. (1). Larmor concludes that the length contraction $\left(x_1 = \epsilon^{1/2}x'\right)$ is predicted by Maxwell's theory. Thus,

> ... if the internal forces of a material system arise wholly from electrodynamic actions between the system of electrons which constitute the atoms, then the effect of imparting to a steady material system a uniform velocity of translation is to produce a uniform contraction of the system in the direction of the motion, of amount $\epsilon^{-1/2}$ (Larmor [1900], p. 176).

He regards the time dilation ($t_1 = \epsilon^{-1/2}t''$), which he described by saying 'the scale of time is enlarged', in the same way;

> ... individual electrons describe corresponding parts of their orbits in times shorter for the (rest) system in the ratio $\epsilon^{-1/2}$ (Larmor [1897]).

He also says 'the change in time variable ... involves the Doppler effect on the wavelength' (Larmor [1900], p. 177) but does not enlarge on this obscure but intriguing remark.

If the right hand sides of (3) are multiplied by a factor ℓ, as in (1), the resulting equations are identical to those given by Lorentz [1904]. It appears not to be well known that Lorentz had presented these equations five years before (Lorentz [1899]) arriving at them by two steps as did Larmor, with the difference that Lorentz starts from a first order transformation which includes the length contraction. Lorentz's ([1895], [1899]) interpretation of the length contraction

[1]The calculation (which Poincaré does not give) is simple. The times it takes light to travel along the x-axis between a clock at the origin and a clock at x' in the out and back directions are $t_o = x'/(c - v)$ and $t_b = x'/(c + v)$ when the reference system is moving with speed v in the x direction. Taking the zero point for t and t'' when the origins of the rest and moving systems coincide, the time co-ordinates of the arrival of the signal at x' are $t = t_o$ and $t'' = \frac{1}{2}(t_o + t_b)$, from which the local time (Eq. 2) follows. Poincaré however, gives the result $t' = t - vx'/c^2$ which is the local time often used by Lorentz before 1904. Lorentz and Poincaré may have omitted the term ϵ when discussing first order theories.

is similar to Larmor's and he shows also that Michelson and Morley's famous experiment would always give a null result if any transparent media were placed in the path of either light ray provided that in addition to the length contraction, the time of vibration of the 'ions' of the media was greater for a moving system than for a system at rest.

Lorentz [1904] seems to underrate his own work of 1899, which besides the second order transformations includes a discussion of the variation of mass with velocity. He also seems to underrate Larmor's work (Lorentz [1902]). Perhaps this was because Larmor does not include the term ℓ, and never shows that it must be unity for all velocities. Nevertheless, the credit for the first presentation of the Lorentz transformations, including the crucial time dilation, belongs to Larmor [1897].

References

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