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Of pots and holes: Einstein's bumpy road to general relativity

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General relativity in the *Annalen* and elsewhere

Readers of this volume will notice that it contains only a few papers on general relativity. This is because most papers documenting the genesis and early development of general relativity were not published in *Annalen der Physik*. After Einstein took up his new prestigious position at the Prussian Academy of Sciences in the spring of 1914, the *Sitzungsberichte* of the Berlin academy almost by default became the main outlet for his scientific production. Two of the more important papers on general relativity, however, did find their way into the pages of the *Annalen* [35,41]. Although I shall discuss both papers in this essay, the main focus will be on [35], the first systematic exposition of general relativity, submitted in March 1916 and published in May of that year.

Einstein's first paper on a metric theory of gravity, co-authored with his mathematician friend Marcel Grossmann, was published as a separatum in early 1913 and was reprinted the following year in *Zeitschrift für Mathematik und Physik* [50,51]. Their second (and last) joint paper on the theory also appeared in this journal [52]. Most of the formalism of general relativity as we know it today was already in place in this Einstein-Grossmann theory. Still missing were the generally-covariant Einstein field equations.

As is clear from research notes on gravitation from the winter of 1912–1913 preserved in the so-called "Zurich Notebook,"[1] Einstein had considered candidate field equations of broad if not general covariance, but had found all such candidates wanting on physical grounds. In the end he had settled on equations constructed specifically to be compatible with energy-momentum conservation and with Newtonian theory in the limit of weak static fields, even though it remained unclear whether these equations would be invariant under any non-linear transformations. In view of this uncertainty, Einstein and Grossmann chose a fairly modest title for their paper: "Outline ("Entwurf") of a Generalized Theory of Relativity and of a Theory of Gravitation." The Einstein-Grossmann theory and its fields equations are therefore also known as the "Entwurf" theory and the "Entwurf" field equations.

Much of Einstein's subsequent work on the "Entwurf" theory went into clarifying the covariance properties of its field equations. By the following year he had convinced himself of three things. First, generally-covariant field equations are physically inadmissible since they cannot determine the metric field uniquely. This was the upshot of the so-called "hole argument" ("Lochbetrachtung") first published in an appendix to [51].[2] Second, the class of transformations leaving the "Entwurf" field equations invariant was as broad

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[1] An annotated transcription of the gravitational portion of the "Zurich Notebook" is published as Doc. 10 in [11]. For facsimile reproductions of these pages, a new transcription, and a running commentary, see [89].

[2] See Sect. 2 for further discussion of the hole argument.

as it could possibly be without running afoul of the kind of indeterminism lurking in the hole argument and, more importantly, without violating energy-momentum conservation. Third, this class contains transformations, albeit it of a peculiar kind, to arbitrarily moving frames of reference. This, at least for the time being, removed Einstein's doubts about the "Entwurf" theory and he set out to write a lengthy self-contained exposition of it, including elementary derivations of various standard results he needed from differential geometry. The title of this article reflects Einstein's increased confidence in his theory: "The Formal Foundation of the General Theory of Relativity" [30]. As a newly minted member of the Prussian Academy of Sciences, he dutifully submitted his work to its *Sitzungsberichte*, where the article appeared in November 1914. This was the first of many papers on general relativity in the *Sitzungsberichte*, including such gems as [37] on the relation between invariance of the action integral and energy-momentum conservation, [36,40] on gravitational waves, [39], which launched relativistic cosmology and introduced the cosmological constant, and [43] on the thorny issue of gravitational energy-momentum.

In the fall of 1915, Einstein came to the painful realization that the "Entwurf" field equations are untenable.[3] Casting about for new field equations, he fortuitously found his way back to equations of broad covariance that he had reluctantly abandoned three years earlier. He had learned enough in the meantime to see that they were physically viable after all. He silently dropped the hole argument, which had supposedly shown that such equations were not to be had, and on November 4, 1915, presented the rediscovered old equations to the Berlin Academy [31]. He returned a week later with an important modification, and two weeks after that with a further modification [32,34]. In between these two appearances before his learned colleagues, he presented yet another paper showing that his new theory explains the anomalous advance of the perihelion of Mercury [33].[4] Fortunately, this result was not affected by the final modification of the field equations presented the following week.

When it was all over, Einstein commented with typical self-deprecation: "unfortunately I have immortalized my final errors in the academy-papers";[5] and, referring to [30]: "it's convenient with that fellow Einstein, every year he retracts what he wrote the year before."[6] What excused Einstein's rushing into print was that he knew that the formidable Göttingen mathematician David Hilbert was hot on his trail.[7] Nevertheless, these hastily written communications to the Berlin Academy proved hard to follow even for Einstein's staunchest supporters, such as the Leyden theorists H. A. Lorentz and Paul Ehrenfest.[8]

Ehrenfest took Einstein to task for his confusing treatment of energy-momentum conservation and his sudden silence about the hole argument. Ehrenfest's queries undoubtedly helped Einstein organize the material of November 1915 for an authoritative exposition of the new theory. A new treatment was badly needed, since the developments of November 1915 had rendered much of the premature review article of November 1914 obsolete.

In March 1916, Einstein sent his new review article, with a title almost identical to that of the one it replaced, to Wilhelm Wien, editor of the *Annalen*.[9] This is why [35], unlike the papers mentioned so far, can be found in the volume before you.[10] Many elements of Einstein's responses to Ehrenfest's queries ended up in this article. Even though there is no mention of the hole argument, for instance, Einstein does present

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3 Einstein stated his reasons for abandoning the "Entwurf" field equations and recounted the subsequent developments in Einstein to Arnold Sommerfeld, 28 November 1915 [15, Doc. 153].

4 See [21] for an analysis of this paper. That Einstein could pull this off so fast was because he had already done the calculation of the perihelion advance of Mercury on the basis of the "Entwurf" theory two years earlier (see the headnote, "The Einstein-Besso Manuscript on the Motion of the Perihelion of Mercury," in [11, pp. 344–359]).

5 "Die letzten Irrtümer in diesem Kampfe habe ich leider in den Akademie-Arbeiten [. . .] verevigt." This comment comes from the letter to Sommerfeld cited in note 3.

6 "Es ist bequem mit dem Einstein. Jedes Jahr widerruft er, was er das vorige Jahr geschrieben hat." Einstein to Paul Ehrenfest, 26 December 1915 [15, Doc. 173].

7 See [8,91,94] for comparisons of the work of Einstein and Hilbert toward the field equations of general relativity.

8 See [68] for discussion of the correspondence between Einstein, Ehrenfest, and Lorentz of late 1915 and early 1916.

9 Einstein to Wilhelm Wien, 18 March 1916 [15, Doc. 196].

10 The article is still readily available in English translation in the anthology *The Principle of Relativity* [73]. Unfortunately, this reprint omits the one-page introduction to the paper in which Einstein makes a number of interesting points. He emphasizes the importance of Minkowski's geometric formulation of special relativity, which he had originally dismissed as "superfluous

the so-called "point-coincidence argument", which he had premiered in letters to Ehrenfest and Michele Besso explaining where the hole argument went wrong.[11] The introduction of the field equations and the discussion of energy-momentum conservation in the crucial Part C of the paper – which is very different from the corresponding Part D of [30] – closely follows another letter to Ehrenfest, in which Einstein gave a self-contained statement of the energy-momentum considerations leading to the final version of the field equations.[12] Initially, his readers had been forced to piece this argument together from his papers of November 1914 and 1915. As Einstein announced at the beginning of his letter to Ehrenfest: "I shall not rely on the papers at all but show you all the calculations."[13] He closed the letter asking his friend: "Could you do me a favor and send these sheets back to me as I do not have this material so neatly in one place anywhere else."[14] Einstein may very well have had this letter in front of him as he was writing the relevant sections of [35].

This paper presents a happy interlude in Einstein's ultimately only partially successful quest to banish absolute motion and absolute space and time from physics and establish a truly general theory of relativity.[15] When he wrote his review article, Einstein still thought that general covariance automatically meant relativity of arbitrary motion. The astronomer Willem de Sitter, a colleague of Lorentz and Ehrenfest in Leyden, disabused him of that illusion during a visit to Leyden in the fall of 1916. A lengthy debate ensued between Einstein and De Sitter in the course of which Einstein introduced the cosmological constant in the hope of establishing general relativity in a new way, involving what he dubbed "Mach's principle" in [41].[16] In this paper he proposed a new foundation for general relativity, replacing parts of the foundation laid in [35]. This may well be why he published [41], like [35], in the *Annalen*. Despite its brevity, this then is the other major paper on general relativity contained in this volume.

Einstein had another stab at an authoritative exposition of general relativity in the early twenties, when he agreed to publish a series of lectures he gave in Princeton in May 1921. They appeared two years later in heavily revised form [46].[17] The Princeton lectures superseded the 1916 review article as Einstein's authoritative exposition of the theory, but the review article remains worth reading and is of great historical interest.

In [35] the field equations and energy-momentum conservation are not developed in generally-covariant form but only in special coordinates. Einstein had found the Einstein field equation in terms of these coordinates in November 1915. As explained above, this part of [35] is basically a sanitized version of the argument that had led Einstein to these equations in the first place. The manuscript for an unpublished appendix [13, Doc. 31] to [35] makes it clear that as he was writing his review article, he was already considering redoing the discussion of the field equations and energy-momentum conservation in arbitrary coordinates. In November 1916, he published such a generally-covariant account in the Berlin *Sitzungsberichte* [37].This paper is undoubtedly much more satisfactory mathematically than the corresponding part of [35] but it does not offer any insight into how Einstein actually found his theory. Reading [37], without

erudition" ("überflüssige Gelehrsamkeit"; [88, p. 151]), and the differential geometry of Riemann and others for the development of general relativity. He also acknowledges the help of Grossmann in the mathematical formulation of the theory.

11 See Sect. 2 for further discussion of the point-coincidence argument.

12 Einstein to Paul Ehrenfest, 24 January 1916 or later [15, Doc. 185].

13 "Ich stütze mich gar nicht auf die Arbeiten, sondern rechne Dir alles vor."

14 "Es wäre mir lieb, wenn Du mir diese Blätter [...] wieder zurückgäbest, weil ich die Sachen sonst nirgends so hübsch beisammen habe."

15 There are (at least) two separate issues here [20, p. 12–15]. The first issue is whether all motion is relative or whether some motion is absolute. Put differently, is space-time structure something over and above the contents of space-time or is it just a way of talking about spatio-temporal relations? The second issue concerns the ontological status of space-time. Is space-time structure supported by a space-time substance, some sort of container, or is it a set of relational properties? The two views thus loosely characterized go by the names of 'substantivalism' and 'relationism', respectively. Newton is associated with substantivalism as well as with absolutism about motion, Leibniz with relationism as well as with relativism about motion (see, e. g., [2, introduction]; [61, Chap. 8]). It is possible, however, to be an absolutist about motion and a relationist about the ontology of space-time. Although the jury is still out on the ontological question, I shall argue that, while non-uniform motion remains absolute in general relativity, the ontology of space-time in Einstein's theory is best understood in relational rather than substantival terms.

16 See Sect. 2 for further discussion of Mach's principle.

17 The Princeton lectures are still readily available in English translation as *The Meaning of Relativity* [47].

having read the November 1915 papers and the 1916 review article, one easily comes away with the impression that Einstein hit upon the Einstein field equations simply by picking the mathematically most obvious candidate for the gravitational part of the Lagrangian for the metric field, namely the Riemann curvature scalar. This is essentially how Einstein himself came to remember his discovery of general relativity. He routinely trotted out this version of events to justify the purely mathematical speculation he resorted to in his work on unified field theory.[18] The 1916 review article preserves the physical considerations, especially concerning energy-momentum conservation, that originally led him to the Einstein field equations, arguably the crowning achievement of his scientific career.

The balance of this essay is organized as follows. Einstein's review article is divided into five parts. The two most important and interesting parts are part A, "Fundamental Considerations on the Postulate of Relativity" (Sects. 1–4) and part C, "Theory of the Gravitational Field" (Sects. 13–18). These two parts are covered in Sects. 2 and 3, respectively. These two sections can be read independently of one another.

The disk, the bucket, the hole, the pots, and the globes[19]

Part A of [35] brings together some of the main considerations that motivated and sustained its author in his attempt to generalize the principle of relativity for uniform motion to arbitrary motion. On the face of it, the arguments look straightforward and compelling, but looking just below the surface one recognizes that they are more complex and, in several cases, quite problematic.

Einstein [35, p. 770] begins with a formulation of the principle of relativity for uniform motion that nicely prepares the ground for the generalization he is after. Both in Newtonian mechanics and in special relativity there is a class of reference frames in which the laws of nature take on a particularly simple form. These inertial frames all move at constant velocity with respect to one another. In the presence of a gravitational field the laws of nature will in general not be particularly simple in any one frame or in any one class of frames. The simplest formulation is a generally-covariant one, a formulation that is the same in all frames, including frames in arbitrary motion with respect to one another. In this sense of relativity, general covariance guarantees general relativity (ibid., 776). This does not mean that observers in arbitrary motion with respect to one another are physically equivalent the way observers in uniform relative motion are. In that more natural sense of relativity, general relativity does not extend special relativity at all.

Einstein's equating of general relativity with general covariance comes in part from a conflation of two different approaches to geometry, a "subtractive" or "top-down" approach associated with the Erlangen program of Felix Klein, and an "additive" or "bottom-up" approach associated with modern differential geometry, which goes back to Bernhard Riemann [86]. In Klein's "subtractive approach" one starts with a description of the space-time geometry with all bells and whistles and then strips away all elements deemed to be descriptive fluff only. Only those elements are retained that are invariant under some group of transformations. Such groups thus characterize the essential part of the geometry. The geometrization of special relativity by Hermann Minkowski (1909) is a picture-perfect example of Klein's "subtractive" approach. Consider Minkowski space-time described in terms of some Lorentz frame, i. e., coordinatized with the help of four orthogonal axes (orthogonal with respect to the standard non-positive-definite Minkowski inner product). Which Lorentz frame is chosen does not matter. The decomposition of space-time into space and time that comes with this choice is not an essential part of the space-time geometry and neither is the state of rest it picks out. These elements are not invariant under transformations of the Lorentz group characterizing the geometry of Minkowski space-time. For instance, a Lorentz boost will map a worldline of a particle at rest onto a worldline of a particle in uniform motion. Lorentz invariance in special relativity is thus directly related to the relativity of uniform motion. The privileged nature of the whole class of uniform motions

[18] For further discussion of Einstein's distorted memory of how he found his field equations and the role it played in his propaganda for his unified field theory program, see [66, Sect. 10] and [102], respectively. John Norton [87], however, accepts that Einstein actually did find the Einstein field equations the way he later claimed he did.

[19] I am indebted to Christoph Lehner for his incisive criticism of earlier versions of many of the arguments presented in this section (cf. [64]). For his own take on some of the issues discussed here, see [72].

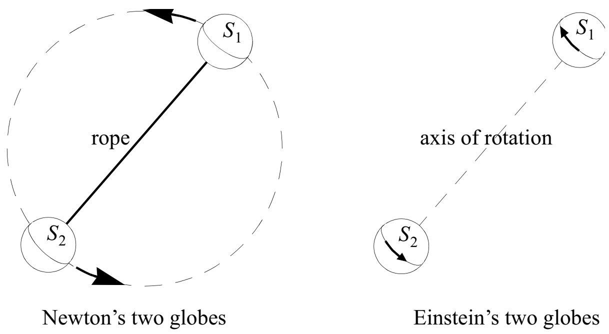

Fig. 1 Absolute rotation.

is an essential part of the geometry. Lorentz transformations will map the set of all possible worldlines of particles at rest in some Lorentz frame onto itself.

In the Riemannian "additive approach" one starts from a bare manifold, a set of points with only a topological and a differential structure defined on it, and adds further structure to turn it into a space-time. Such further structure will typically include an affine connection and a metric so that it becomes possible to tell straight lines from crooked ones and talk about distances. To make sure that no superfluous elements enter into the description of the space-time geometry everything is done in a coordinate-independent manner, if not coordinate-free (i. e., without ever introducing coordinates at all) then at least in a generally-covariant way (i. e., in a way that is exactly the same no matter what coordinates are chosen). Such generally-covariant descriptions can be of space-times with or without preferred states of motion. This already makes it clear that general covariance *per se* has nothing to do with relativity of motion.[20]

Einstein used general covariance in two different ways in his 1916 review article. In Sect. 3, he used it in the spirit of Klein's "subtractive approach" to isolate the essential elements of general relativistic space-times [84,86]. In Sect. 2, he used it for the implementation of a peculiar principle of relativity distinctly his own.

In Sect. 2 Einstein explained his objection against preferred frames of reference and argued for the need of general covariance using a variant of a thought experiment Newton had used to illustrate that rotation is absolute. These two thought experiments are illustrated in fig. 1, Newton's on the left, Einstein's on the right.

Consider Newton's experiment first [6, p. 12], [7, p. 414]. Two globes, S_1 and S_2, connected by a rope are rotating around their center of gravity far away from any other gravitating matter. Can this situation be distinguished from a situation in which the two globes are not rotating but moving at constant speed in a straight line at a fixed distance from each other? The answer is yes: the tension in the rope will be greater when the globes are rotating.

Einstein asks us to consider the two globes in relative rotation around the line connecting their centers. He has no use for the rope. Newtonian theory tells us that it makes a difference whether S_1 or S_2 is truly rotating. A rotating globe bulges out at its equator. This, Einstein argues, violates Leibniz's principle of sufficient reason. The situation looks perfectly symmetric: S_2 rotates with respect to S_1 and S_1 rotates with respect to S_2. Yet, unless the two globes both happen to rotate with half their relative angular velocity, they behave differently. There is no observable cause to explain this difference in behavior. The Newtonian explanation – that the globe's rotation with respect to (the set of inertial frames of) Newton's absolute space rather than its rotation relative to the other globe is what causes it to bulge out – is unsatisfactory, because the purported cause is not an "observable empirical fact" ("beobachtbare Tatsache"; [35, p. 771]). Special relativity, Einstein claims, inherits this "epistemological defect" ("erkenntnistheoretische[n] Mangel"; ibid.), to which he had been sensitized by Ernst Mach. Situations with two objects in relative motion, such as the globes S_1 and S_2 always look symmetric, regardless of whether the motion is uniform or not, but when the motion is non-uniform the two objects will in general behave differently.

[20] See [85] for a review of the (philosophy of) physics literature on the status of general covariance in general relativity.

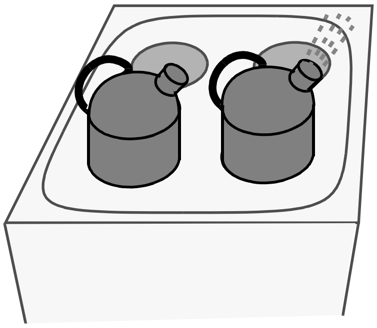

Fig. 2 Absolute space seems to violate the principle of sufficient reason.

The British philosopher of science Jon Dorling [19] was the first to put his finger on the fallacy in Einstein's reasoning. Imagine attaching ideal clocks to both globes somewhere on their equators. Use these clocks to measure how long one revolution of the other globe takes. According to Newtonian kinematics, the two clocks will record the same time for one revolution. According to special-relativistic kinematics, they will in general record different times because of the phenomenon of time dilation. The difference will be greatest when one of the two globes is at rest with respect to some inertial Lorentz frame in Minkowski space-time. Focus on this special case. The clock on the inertially moving globe measures a longer period of revolution than the clock on the non-inertially moving globe. This is just a variant of the famous twin-paradox scenario. The point of introducing these clocks is to show that the situation of the two globes in relative rotation to one another is *not* symmetric, not even at the purely kinematical level. It therefore need not surprise us that it is not symmetric at the dynamical level either. In the special case in which one globe is moving inertially only the other globe, the one with the lower clock reading, bulges out at its equator.

In Chap. 21 of his popular book on relativity, Einstein [38, p. 49], [48, p. 72] used a charming analogy to get his point across. It can also be used to illustrate Dorling's rejoinder. Consider two identical pots sitting on a stove, only one of which is giving off steam (see Fig. 2). One naturally assumes that this is because only the burner under that one is on. It would be strange indeed to discover that the burners under both pots are turned on (or, for that matter, that both are turned off). That would be a blatant violation of the principle of sufficient reason. Einstein's example of the two globes is meant to convince us that both Newtonian theory and special relativity lead to similar violations of this principle. As with the two pots on the stove, there is no observable difference between the two globes, yet they behave differently. The analogy works for Newtonian theory but not for special relativity. With the kinematics of special relativity, the analogy breaks down immediately. The situation with the two globes does not *look* the same to observers on the two globes, so there is no reason to expect the two globes to *behave* the same.

If we take Einstein at his word in 1916 – that preferred frames of reference are objectionable because they lead to violations of the principle of sufficient reason – we must conclude that Einstein was worried about a problem he had already solved with special relativity by making temporal distances between events, like spatial distances between points, dependent on the path connecting them. Einstein's underestimation of what he had achieved with special relativity compensates for his overestimation of what he had achieved with general relativity. Contrary to what Einstein believed when he wrote his review article in 1916, general covariance does not eliminate absolute motion.

For the further development of physics it was a good thing that Einstein did not fully appreciate what he had accomplished with special relativity. In trying to solve a problem that, unbeknownst to himself, he had already solved, Einstein produced a spectacular new theory of gravity.

The fundamental insight that Einstein would base his new theory on came to him while he was working on a review article on special relativity [24]. Sitting at his desk in the Swiss patent office in Berne one day it suddenly hit him that someone falling from the roof would not feel his own weight.[21] He later called it "the

[21] Einstein related this story in a lecture in Kyoto on December 14, 1922 [1, p. 15].

best idea of my life."[22] It told Einstein that there was an intimate connection between acceleration – the kind of motion he wanted to relativize – and gravity. In [26, pp. 360, 366], he introduced the term "equivalence principle" for this connection. Einstein wanted to use this principle to extend the relativity principle from uniform to non-uniform motion.

The equivalence principle explains a striking coincidence in Newton's theory. To account for Galileo's principle that all bodies fall with the same acceleration in a given gravitational field, Newton had to assign the same value to two conceptually clearly distinct quantities, namely inertial mass, the measure of a body's resistance to acceleration, and gravitational mass, the measure of a body's susceptibility to gravity. The equivalence principle removes the mystery of the equality of inertial and gravitational mass by making inertia and gravity two sides of the same coin.

Einstein only formulated the equivalence principle along these lines in his second paper on the foundations of general relativity [41]. In its mature form, the equivalence principle says that inertial effects (i. e., effects of acceleration) and gravitational effects are manifestations of one and the same structure, nowadays called the inertio-gravitational field. How some inertio-gravitational effect breaks down into an inertial component and a gravitational component is not unique but depends on the state of motion of the observer making the call, just as it depends on the state of motion of the observer how an electromagnetic field breaks down into an electric field and a magnetic field [63, pp. 507–509], [64]. In other words, what is relative according to the mature equivalence principle is not motion but the split of the inertio-gravitational field into an inertial and a gravitational component.

Einstein initially did not distinguish these two notions carefully and instead of unifying acceleration and gravity, thereby implementing what I shall call the relativity of the gravitational field, he tried to reduce acceleration to gravity, thereby hoping to extend the relativity principle to accelerated motion. Invoking the equivalence principle, one can reduce a state of acceleration in a gravitational field (i. e., free fall) to a state of rest with no gravitational field present. The man falling from the roof of the Berne patent office and a modern astronaut orbiting the earth in a space shuttle provide examples of this type of situation. One can similarly reduce a state of acceleration in the absence of a gravitational field to a state of rest in the presence of one. An astronaut firing up the engines of her rocket ship somewhere in outer space far from the nearest gravitating matter provides an example of this type of situation. This then is the general principle of relativity that Einstein was able to establish on the basis of the equivalence principle: two observers in non-uniform relative motion can both claim to be at rest if they agree to disagree on whether or not there is a gravitational field present.

This principle is very different from the principle of relativity for uniform motion. Two observers in uniform relative motion are physically equivalent. Two observers in non-uniform relative motion obviously are not. Sitting at one's desk in the patent office does not feel the same as falling from the roof of the building, even though the man falling from the roof can, if he were so inclined, claim that he is at rest and that the disheveled patent clerk whose eyes he meets on the way down is accelerating upward in a space with no gravitational field at all. Likewise, the astronaut accelerating in her rocket in outer space can claim that she is at rest in a gravitational field that suddenly came into being when she fired up her engines and that her hapless colleague, who was hovering in space next to the rocket at that point, is now in free fall in that gravitational field. Despite this nominal relativity of acceleration, the two astronauts will experience this situation very differently.[23]

The physical equivalence in the paradigmatic examples examined above is not between the observers in relative motion with respect to one another, it is between the man at his desk in the first example and the

[22] "der glücklichste Gedanke meines Lebens" [14, Doc. 31, p. 21]. For discussion of this oft-quoted passage, see, e. g., [88, p. 178], [63, pp. 507–509], [64].

[23] As late as November 1918 – more than half a year after clarifying the foundations of general relativity [41] – Einstein saw fit to publish an account of the twin paradox along these lines [44]. This 1918 paper not only offered a solution for a problem that had already been solved, it also raised suspicion about the earlier solution by suggesting that the problem called for general relativity. Einstein thus bears some responsibility for the endless confusion over the twin paradox, which is nothing but a vivid example of the path dependence of temporal distances in special as well as in general relativity.

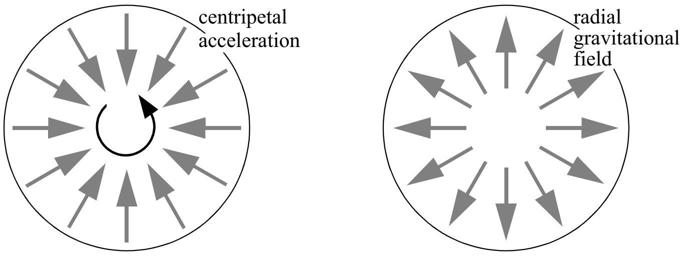

Fig. 3 Rotating disk.

astronaut inside the rocket in the second, and between the man falling from the roof in the first example and the astronaut outside the rocket in the second. Resisting the pull of gravity and accelerating in the absence of gravity feel the same. Likewise, free fall in a gravitational field and being at rest or in uniform motion in the absence of a gravitational field feel the same. These are examples of inertial and gravitational effects that are physically indistinguishable and that get lumped together in the new taxonomy for such effects suggested by the mature equivalence principle. This is arguably one of Einstein's greatest contributions to modern physics. The peculiar general relativity principle for which Einstein originally tried to use the equivalence principle did not make it into the canons of modern physics. It was nonetheless extremely important as a heuristic principle guiding Einstein on his path to general relativity.

The equivalence principle, understood as a heuristic principle, allowed Einstein to infer effects of gravity from effects of acceleration in Minkowski space-time. The most fruitful example of this kind was that of the rotating disk, which is discussed in Sect. 3 of the review article [35, pp. 774–775] and which played a pivotal role in the development of general relativity [96].

Consider a circular disk serving as a merry-go-round in Minkowski space-time (see Fig. 3). Let one observer stand on the merry-go-round and let another stand next to it. The person next to the disk will say that he is at rest and that the person on the disk is subject to centrifugal forces due to the disk's rotation (see the drawing on the left in Fig. 3). Invoking the equivalence principle, the person on the disk will say that she is at rest in a radial gravitational field and that the person next to the disk is in free fall in this field (see the drawing on the right in Fig. 3).[24]

Now have both observers measure the ratio of the circumference and the diameter of the disk. The person next to the disk will find the Euclidean value π. The person on the disk will find a ratio greater than π. After all, according to special relativity, the rods she uses to measure the circumference are subject to the Lorentz contraction, whereas the rods she uses to measure the diameter are not.[25] The spatial geometry for the rotating observer is therefore non-Euclidean. Invoking the equivalence principle, Einstein concluded that this will be true for an observer in a gravitational field as well. This then is what first suggested to Einstein that gravity should be represented by curved space-time.

To describe curved space-time Einstein turned to Gauss's theory of curved surfaces, a subject he vaguely remembered from his student days at the *Eidgenössische Technische Hochschule* (*ETH*) in Zurich. He had learned it from the notes of his classmate Marcel Grossmann. Upon his return to their *alma mater* as a full professor of physics in 1912, Einstein learned from Grossmann, now a colleague in the mathematics department of the *ETH,* about the extension of Gauss's theory to spaces of higher dimension by Riemann and others.[26] Riemann's theory provided Einstein with the mathematical object with which he could unify the effects of gravity and acceleration: the metric field. The metric makes it possible to identify lines of

[24] For the person on the disk, the person standing next to it is rotating and subject to both centrifugal and Coriolis forces. If this person has mass M and moves with angular frequency ω on a circle of radius R, the centrifugal force is $M\omega^2 R$. The Coriolis force provides a centripetal force twice that size, both compensating the centrifugal force and keeping the person in orbit.

[25] This simple argument has been the source of endless confusion. Einstein's clearest exposition can be found in two letters written in response to a particularly muddled discussion of the situation (see Einstein to Joseph Petzoldt, 19 and 23 August 1919 [16, Docs. 93 and 94]).

[26] For accounts of how Einstein's collaboration with Grossmann began, see the Kyoto lecture [1, p. 16] as well as [88, p. 213] and [54, pp. 355–356].

extremal length in curved space-time, so-called metric geodesics. In Riemannian geometry these are also the straightest possible lines, so-called affine geodesics.[27] Free fall in a gravitational field and being at rest or in uniform motion in the absence of a gravitational field are represented by motion along geodesics. Resisting the pull of gravity and accelerating in the absence of gravity are represented by motion that is not along geodesics. In the scenario envisioned in the twin paradox, the stay-at-home moves on a geodesic, whereas the traveller does not. In the example of the rotating disk, the person next to the disk is moving on a geodesic, whereas the person on the disk is not.

No coordinate transformation can turn a geodesic into a non-geodesic or vice versa. Observers moving on geodesics and observers moving on non-geodesics are physically not equivalent to one another. This is just another way of saying that general relativity does not extend the principle of relativity for uniform motion to arbitrary motion. Both in special and in general relativity there are preferred states of motion, namely motion along geodesics.[28]

The relativity principle that Einstein had established on the basis of the equivalence principle, however, will be satisfied if all laws of this new metric theory of gravity, including the field equations for the metric field, are generally covariant. In that case one can choose any wordline, a geodesic or a non-geodesic, as the time axis of one's coordinate system. An observer travelling on that worldline will be at rest in that coordinate system. If her worldline is not a geodesic, she will attribute the inertial forces she experiences to a gravitational field, which will satisfy the generally-covariant field equations. A geodesic and a non-geodesic observer in an arbitrarily curved space-time can thus both claim to be at rest if they agree to disagree about the presence of a gravitational field. As I mentioned above, however, it seems more natural to call this relativity of the gravitational field than relativity of motion.

Either way it was a serious setback for Einstein when his search for field equations in the winter of 1912–1913, undertaken with the help of Grossmann and recorded in the "Zurich Notebook", did not turn up any physically acceptable generally-covariant candidates. The problem continued to bother him in the months following the publication of the "Entwurf" field equations. Eventually (but not ultimately), Einstein made his peace with the limited covariance of these equations. In August 1913, in a vintage Einstein maneuver, he convinced himself that he had not been able to find generally-covariant field equations simply because there were none to be found. Einstein produced two arguments to show that generally-covariant field equations are physically unacceptable. Both arguments are fallacious, but both significantly deepened Einstein's understanding of his own theory.

The first argument, which can be dated with unusual precision to August 15, 1913, was that energy-momentum conservation restricts the covariance of acceptable field equations to linear transformations.[29] Einstein soon realized that the argument turns on the unwarranted assumption that gravitational energy-momentum can be represented by a generally-covariant tensor. The argument was retracted in [52, p. 218, note]. The general insight, however, that there is an intimate connection between energy-momentum conservation and the covariance of the field equations survived the demise of this specific argument and was a key element in Einstein's return to generally-covariant field equations in the fall of 1915 (see Sect. 3 below).

[27] The affine connection, which was not introduced until after the formulation of general relativity, is better suited to Einstein's purposes than the metric [100].

[28] A German *Gymnasium* teacher, Erich Kretschmann [70], clearly formulated what it takes for a space-time theory to satisfy a genuine relativity principle. Kretschmann first pointed out that a theory does not satisfy a relativity principle simply by virtue of being cast in a form that is covariant under the group of transformations associated with that principle. With a little ingenuity one can cast just about any theory in such a form. Einstein [41] granted this point, but did not address Kretschmann's proposal in the spirit of Klein's Erlangen program to characterize relativity principles in terms of symmetry groups of the set of geodesics of all space-times allowed by the theory. In special relativity, this would be the group of Lorentz transformations that map the set of all geodesics of Minkowski space-time, the only space-time allowed by the theory, back onto itself. The set of geodesics of all space-times allowed by general relativity has no non-trivial symmetries, so the theory fails to satisfy any relativity principle in Kretschmann's sense. Einstein still did not comment on Kretschmann's proposal after he was reminded of it in correspondence (see Gustav Mie to Einstein, 17–19 February 1918 [15, Doc. 465]). For further discussion of Kretschmann's proposal, see [84, Sect. 8], [85, Sect. 5] and [93].

[29] The date can be inferred from Einstein to H. A. Lorentz, 16 August 1913 [12, Doc. 470], in which Einstein mentions that he had found the argument the day before. For discussion of the argument and its flaws, see [82, Sect. 5].

By the time Einstein retracted this first argument against generally-covariant field equations, it had already been eclipsed by a second one, the infamous hole argument mentioned in the introduction.[30] A memo dated August 28, 1913, in the hand of Einstein's lifelong friend Michele Besso and found among the latter's papers in 1998, sheds some light on the origin of this argument.[31] Shortly after the publication of [50], Einstein and Besso had done extensive calculations to see whether the "Entwurf" theory can account for the anomalous advance of the perihelion of Mercury (see note 4 above). In this context Besso had raised the question whether the field equations uniquely determine the field of the sun [11, Doc. 14, p. 16]. This query may well have been the seed for the following argument recorded in the Besso memo:

> The requirement of covariance of the gravitational equations under arbitrary transformations cannot be imposed: if all matter were contained in one part of space and for this part of space a coordinate system [is given], then outside of it the coordinate system could, except for boundary conditions, still be chosen arbitrarily, so that a unique determinability of the g's cannot occur.[32]

This argument, presumably communicated to Besso by Einstein, turns into the hole argument when space is replaced by space-time and the regions with and without matter are interchanged. In the published version of the argument the point is that the metric field in some small matter-free region of space-time – the "hole" from which the argument derives it name – is not uniquely determined by the matter distribution and the metric field outside the hole.

The hole argument works as follows. Suppose we have generally-covariant field equations that at every point set the result of some differential operator acting on the metric field $g_{\mu\nu}$ equal to the energy-momentum tensor $T_{\mu\nu}$ of matter at that point. Consider a matter distribution such that $T_{\mu\nu}(x) = 0$ for all points inside the hole. Suppose $g_{\mu\nu}(x)$ is a solution for this particular matter distribution. Now consider a coordinate transformation $x \to x'$ that only differs from the identity inside the hole and express the energy-momentum tensor and the metric field in terms of the new primed coordinates. Because of the general covariance we assumed, the field equations in primed coordinates will have the exact same form as the field equations in unprimed coordinates and $\{g'_{\mu\nu}(x'), T'_{\mu\nu}(x')\}$ will be a solution of them. This will still be true – and this is the key observation – if we read x for x' everywhere. Since the energy-momentum tensor vanishes inside the hole and since the coordinate transformation $x \to x'$ is the identity outside the hole, $T'_{\mu\nu}(x) = T_{\mu\nu}(x)$ everywhere. That means that both $g_{\mu\nu}(x)$ and $g'_{\mu\nu}(x)$ are solutions of the field equations in unprimed coordinates for one and the same matter distribution $T_{\mu\nu}(x)$. These two solutions are identical outside the hole but differ inside. The matter distribution (along with boundary conditions for $g_{\mu\nu}$) thus fails to determine the metric field inside the hole uniquely. The only way to avoid this kind of indeterminism, the argument concludes, is to rule out field equations that retain their form under transformations $x \to x'$ such as the one that was used in the construction of the alternative solution $g'_{\mu\nu}(x)$ from the original solution $g_{\mu\nu}(x)$.[33]

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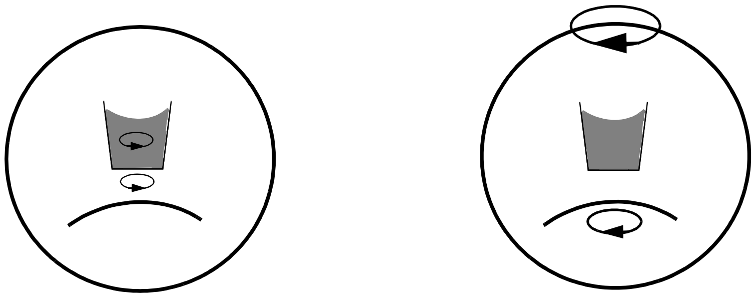
[30] See [82] and [97] for the classic historical discussions of the hole argument. The argument has also spawned a huge philosophical literature following the publication in [23] of an argument inspired by and named after Einstein's. See, e. g., [5, 83], [20, Chap. 9], [76], and [98, 99].

[31] For detailed analysis of this memo, see [65].

[32] "Die Anforderung der Covarianz der Gravitationsgleichungen für beliebige Transformationen kann nicht aufgestellt werden: wenn in einem Teile des Raumes alle Materie enthalten wäre und für diesen Teil ein Coordinatensystem, so könnte doch ausserhalb desselben das Coordinatensystem noch, abgesehen von den Grenzbedingungen, beliebig gewählt werden, so dass eine eindeutige Bestimmbarkeit der gs nicht eintreten könne." See Fig. 2 of [65] for a facsimile of the page of the Besso memo with this passage.

[33] Another passage in the Besso memo quoted above makes it clear that, even in the embryonic version of the hole argument, Einstein saw the inequality $g_{\mu\nu}(x) \neq g'_{\mu\nu}(x)$ as expressing indeterminism, *not*, as older commentators have suggested (see, e. g., [88, p. 222]), the inequality $g_{\mu\nu}(x) \neq g'_{\mu\nu}(x')$, which merely expresses the non-uniqueness of the coordinate representation of the metric field. Besso wrote: "If in coordinate system 1 [with coordinates x], there is a solution K_1 [i. e., $g_{\mu\nu}(x)$], then this same construct [modulo a coordinate transformation] is also a solution in [coordinate system] 2 [with coordinates x'], K_2 [i. e., $g'_{\mu\nu}(x')$]; K_2, however, [is] also a solution in 1 [i. e., $g'_{\mu\nu}(x)$]" ("Ist im Coordinatensystem 1 eine Lösung K_1, so ist dieses selbe Gebilde auch eine Lösung in 2, K_2; K_2 aber auch eine Lösung in 1"). See [65, Sect. 4] for further discussion.



bucket & water rotating
shell at rest

bucket & water at rest
shell rotating

Fig. 4 Mach's response to Newton's bucket experiment.

Einstein never explicitly retracted the hole argument in print and it was only after he had returned to generally-covariant field equations in November 1915 that he at least addressed the issue in correspondence. Before we turn to this denouement of the hole story, however, we need to examine another strand in Einstein's quest for a general relativity of motion that made it into the 1916 review article.

During the period that he accepted that there could not be generally-covariant field equations, Einstein explored another strategy for eliminating absolute motion. This strategy was directly inspired by his reading of Ernst Mach's response to Newton's famous bucket experiment [6, pp. 10–11]; [7, pp. 412–413].[34] When a bucket filled with water in the gravitational field of the earth is set spinning, the water will climb up the wall of the bucket as it catches up with the bucket's rotation. Newton famously argued that it cannot be the relative rotation of the water with respect to the bucket that is causing this effect.[35] After all, the effect increases as the relative rotation between water and bucket decreases and is maximal when both are rotating with the same angular velocity. The effect, according to Newton, was due to the rotation of the water with respect to absolute space. Mach argued that Newton had overlooked a third possibility: the effect could be due to the relative rotation of the water with respect to other matter in the universe. "Try to fix Newton's bucket," he challenged those taken in by Newton's argument, "and rotate the heaven of fixed stars and then prove the absence of centrifugal forces" [74, p. 279].[36] Mach implied that it should make no difference whether the bucket or the heavens are rotating: in both cases the water surface should become concave. Mach's idea is illustrated in Fig. 4, depicting the bucket, the water, and the earth sitting at the center of a giant spherical shell representing all other matter in the universe.[37] According to Mach, the water surface should be concave no matter whether the bucket and the water or the earth and the shell are rotating. According to Newtonian theory, however, the rotation of the shell will have no effect whatsoever on the water in the bucket.

For most of the reign of the "Entwurf" theory and beyond, Einstein was convinced that this was a problem not for Mach's analysis but for Newton's theory and that his own theory vindicated Mach's account of the bucket experiment. In the spirit of the equivalence principle, Einstein [30, p. 1031] argued that the centrifugal forces responsible for the concave surface of the water in the rotating bucket might just as well be looked upon as gravitational forces due to distant rotating masses acting on the water in a bucket at rest.

[34] Looking back on this period in late 1916, Einstein wrote about these Machian ideas: "Psychologically, this conception played an important role for me, since it gave me the courage to continue to work on the problem when I absolutely could not find covariant field equations" ("Psychologisch hat diese Auffassung bei mir eine bedeutende Rolle gespielt; denn sie gab mir den Mut, an dem Problem weiterzuarbeiten, als es mir absolut nicht gelingen wollte, kovariante Feldgleichungen zu erlangen." Einstein to Willem de Sitter, 4 November 1916 [15, Doc. 273]).

[35] For Newton this argument was not so much an argument for absolute space or absolute acceleration as an argument against the Cartesian concept of motion [71], [61, Chap. 7].

[36] "Man versuche das Newtonsche Wassergefäß festzuhalten, den Fixsternhimmel dagegen zu rotieren und das Fehlen der Fliehkräfte nun nachzuweisen" [75, p. 222]. For extensive discussion of Mach's response to Newton's bucket experiment and Einstein's reading and use of it, see [3].

[37] The sad faces that one can discern in these drawings may serve as warning signs that the arguments of Mach and Einstein do not hold up under scrutiny.

To guarantee that Einstein's theory predicts that we get the same concave water surface in both cases in Fig. 4, the field equations need to satisfy two requirements. First, the Minkowski metric expressed in terms of the coordinates of a rotating frame of reference has to be a solution of the vacuum field equations. This was the kind of requirement that Einstein retreated to when he accepted that general covariance could not be had. He hoped that the "Entwurf" field equations would at least allow the Minkowski metric expressed in the coordinates of arbitrarily moving frames as vacuum solutions. Second, the metric field produced by the shell near its center has to be the Minkowski metric in rotating coordinates.

The "Entwurf" field equations satisfy neither of these two requirements. Einstein went back and forth for two years on whether or not the Minkowski metric in rotating coordinates is a vacuum solution. A sloppy calculation preserved in the Einstein-Besso manuscript reassured him in 1913 that it is [11, Doc. 14, pp. 41–42]. Later in 1913 Besso told him it is not.[38] Einstein appears to have accepted that verdict for a few months, but in early 1914 convinced himself on general grounds that it had to be.[39] That the "Entwurf" theory thus seems to account for the bucket experiment along Machian lines is hailed as a great triumph in the systematic exposition of the theory of late 1914 [30, p. 1031]. In September 1915, possibly at the instigation of Besso, Einstein redid the calculation of the Einstein-Besso manuscript and discovered to his dismay that his friend had been right two years earlier.[40] A month later Einstein replaced the "Entwurf" field equations by equations of much broader and ultimately general covariance. The Minkowski metric in its standard diagonal form is a solution of these equations. Their covariance guarantees that it is a solution in rotating coordinates as well.

The second requirement is satisfied neither by the "Entwurf" theory nor by general relativity, although it took a long time for Einstein to recognize this and even longer to accept it. When he calculated the metric field of a rotating shell in 1913 using the "Entwurf" field equations, he chose Minkowskian boundary conditions at infinity and determined how the rotating shell would perturb the metric field of Minkowski space-time.[41] This perturbation does indeed have the form of the Minkowski metric in rotating coordinates near the center of the shell but is much too small to make a dent in the water surface. More importantly, treating the effect of the rotating shell as a perturbation of the metric field of Minkowski space-time defeats the purpose of vindicating Mach's account of the bucket experiment. In this way, after all, the leading term in the perturbative expansion of the field acting on the bucket will not come from distant matter at all but from absolute space, albeit of the Minkowskian rather than the Newtonian variety. This problem will arise for any non-degenerate physically plausible boundary conditions [101, p. 38]. Einstein seems to have had a blind spot for the role of boundary conditions in this problem.

It is important to note that even if both requirements were satisfied, so that the water surfaces have the same shape in the two situations shown in Fig. 4, we would still not have reduced these two situations to one and the same situation looked at from two different perspectives. Consider the shell in the two cases. Its particles are assumed to move on geodesics in both cases, but while the case with the rotating shell requires cohesive forces preventing them from flying apart,[42] the case with the shell at rest does not. This in and of itself shows that the rotation of the water and bucket with respect to the shell and the earth is not relative in the "Entwurf" theory or in general relativity. Einstein added the metric field to the shell, the earth, the bucket, and the water – the material components that for Mach exhausted the system – and the relation between field and matter is very different in the two situations shown in Fig. 4.

Provided that the first of the two requirements distinguished above is satisfied, however, Einstein's own peculiar principle of relativity is satisfied in this case. We can start from the situation on the left with the bucket rotating in Minkowski space-time and transform to a rotating frame in which the bucket is at rest.

[38] This can be inferred from the Besso memo discussed above [65, Sect. 3].

[39] See Einstein to H. A. Lorentz, 23 March 1915 [15, Doc. 47].

[40] See Einstein to Erwin Freundlich, 30 September 1930 [15, Doc. 123].

[41] This calculation can be found in the Einstein-Besso manuscript [11, Doc. 14, 36–37]. For further analysis of this calculation, see [65, Sect. 3].

[42] Without such cohesive forces, to put it differently, the shell, as the source of the inertio-gravitational field, will not satisfy the law of energy-momentum conservation as it must both in the "Entwurf" theory and in general relativity (see Sect. 3).

We would have to accept the resulting unphysical degenerate values of the metric at infinity,[43] but we can if we want. Invoking the equivalence principle, an observer at rest in this frame can claim to be at rest in a gravitational field. This observer will claim that the centrifugal forces on the water come from this gravitational field and that the particles that make up the shell are in free fall in this field, which exerts centrifugal as well as Coriolis forces on them.[44] The shell, however, is not the source of this field.

Einstein conflated the situation on the left in Fig. 3, redescribed in a coordinate system in which the bucket is at rest, with the very different situation on the right. He thus believed that meeting the first of the two requirements distinguished above sufficed for the implementation of a Machian account of the bucket experiment. This is clear from a letter he wrote in July 1916. Explaining to Besso how to calculate the field of a rotating ring, a case very similar to that of the rotating shell which Einstein himself had considered in the Einstein-Besso manuscript (see note 41), he wrote

> In *first* approximation, the field is obtained easily by direct integration of the field equations.[45] The second approximation is obtained from the vacuum field equations as the next approximation. The first approximation gives the Coriolis forces, the second the centrifugal forces. That the latter come out correctly is obvious given the general covariance of the equations, so that it is of no further interest whatsoever to actually do the calculation. This is of interest only if one does not know whether rotation-transformations are among the "allowed" ones, i. e., if one is not clear about the transformation properties of the equations, a stage which, thank God, has definitively been overcome.[46]

The general covariance of the Einstein field equations does guarantee that the Minkowski metric in rotating coordinates is a vacuum solution. But it does not follow that this metric field is the same as the metric field near the center of a rotating shell. This would follow if the two situations in Fig. 4 were related to one another simply by a transformation to rotating coordinates. But, notwithstanding Einstein's suggestion to the contrary, they are not.

Einstein's correspondence with Hans Thirring in 1917 shows that this misunderstanding persisted for at least another year and a half.[47] When Thirring first calculated the metric field inside a rotating shell, he was puzzled that he did not simply find the Minkowski metric in rotating coordinates as he expected on the basis of remarks in the introduction of [30]. He asked Einstein about this and Einstein's responses indicate that he shared Thirring's puzzlement and expected there to be an error in Thirring's calculations. When he published his final results, Thirring [101, pp. 33, 38] explained that the metric field inside a rotating shell is not identical to the Minkowski metric in rotating coordinates because of the role of boundary conditions. He cited Einstein and De Sitter [39, 17] for the discussion of the role of boundary conditions. But although they were at the focus of his discussions with De Sitter, Einstein did not breathe a word about boundary conditions in his letters to Thirring.

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43 The components $g_{14} = g_{41} = \omega y$, $g_{24} = g_{42} = -\omega x$, and $g_{44} = 1 - \omega^2 r^2$ of the Minkowski metric in a coordinate system rotating with angular velocity ω around the z-axis go to infinity as $r \equiv \sqrt{x^2 + y^2}$ goes to infinity.

44 For a particle of mass m rotating with angular frequency ω at a distance r from the axis of rotation the centrifugal force and the Coriolis force add up to a centripetal force of size $m\omega^2 r$ needed to keep the particle in its circular orbit (cf. note 24). This is explained in Einstein to Hans Thirring, 7 December 1918 [15, Doc. 405].

45 Note that no mention is made of the Minkowskian boundary conditions that Einstein had used in his calculation for the case of a rotating shell.

46 "Das Feld in *erster* Näherung ergibt sich leicht durch unmittelbare Integration der Feldgleichungen. Die zweite Näherung ergibt sich aus den Vakuumfeldgleichungen als nächste Näherung. Die erste Näherung liefert die Korioliskräfte, die zweite die Zentrifugalkräfte. Dass letztere richtig heraus kommen, ist bei der allgemeinen Kovarianz der Gleichungen selbstverständlich, sodass ein wirkliches Durchrechnen keinerlei Interesse mehr hat. Dies Interesse ist nur dann vorhanden, wenn man sich nicht weiss ob Rotations-transformationen zu den 'erlaubten' gehören, d. h. wenn man sich über die Transformationseigenschaften der Gleichungen nicht im Klaren ist, welches Stadium gottlob endgültig überwunden ist." Einstein to Michele Besso, 31 July 1916 [15, Doc. 245]. For further discussion, see [62, Sect. 11], [65, Sect. 3].

47 See Hans Thirring to Einstein, 11–17 July 1917 [15, Doc. 361], Einstein to Thirring, 2 August 1917 [15, Doc. 369], Thirring to Einstein, 3 December 1917 [15, Doc. 401], and the letter cited in note 44.

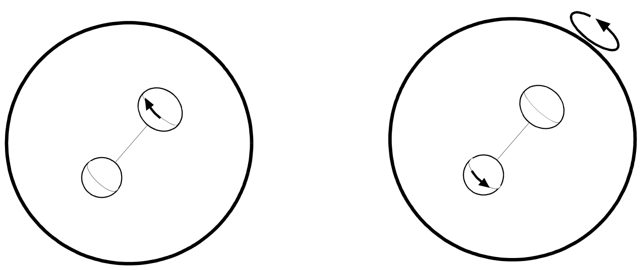

Fig. 5 Einstein's Machian solution to the problem of the two globes.

Not surprisingly, given the above, Einstein's account of the two globes rotating with respect to one another in Sect. 2 of the 1916 review article is modelled on his Machian account of Newton's bucket experiment. This is illustrated in Fig. 5, the analogue of Fig. 4. Recall the puzzle that Einstein drew attention to: why do the two globes take on different shapes, one becoming an ellipsoid, the other retaining its spherical shape?[48] Einstein [35, p. 772] identifies distant masses as the cause of this difference. He does not elaborate but after the discussion of Einstein's account of the bucket experiment, it is easy to fill in the details.

For Einstein, the distant masses (once again represented by a large shell in Fig. 5) function as the source of the metric field in the vicinity of the two globes. The globe that is rotating with respect to this metric field is the one that bulges out at the equator. As with the motion of the bucket with respect to the shell, the motion of the bulging globe with respect to the shell is not relative: the situation on the left in Fig. 5 with the bulging globe rotating and the shell (and the other globe) at rest is not equivalent to the situation on the right with the bulging globe at rest and the shell (and the other globe) rotating in the opposite direction. The relation between matter and metric field is different in these two cases. For one thing, the boundary conditions at infinity are different.

Still, Einstein's idiosyncratic relativity principle based on the equivalence principle – or, what amounts to the same thing, the relativity of the gravitational field – is satisfied in this case. Depending on which perspective we adopt in the situation depicted on the left in Fig. 5, that of an observer on the bulging globe or that of an observer on the other globe, we will interpret the forces on the bulging globe either as gravitational or as inertial forces. It is important that all perspectives are equally justified. Otherwise, as Einstein points out, we would still have a violation of the principle of sufficient reason. Einstein's analysis of the example of the two globes thus becomes an argument for general covariance. Note that general covariance in this context serves the purpose not of making rotation relative but of making the presence or absence of the gravitational field relative.[49]

As in the case of the bucket experiment, Einstein overlooked the role of boundary conditions. He proceeded as if the distant matter fully determines the metric field. References to motion with respect to the metric field could then be interpreted as shorthand for motion with respect to the sources of the field. But the metric field is determined by material sources plus boundary conditions. General relativity thus retains vestiges of absolute motion. This point was driven home by Willem de Sitter in discussions with Einstein in Leyden in the fall of 1916,[50] although Einstein's letters to Thirring a full year later give no indication that their author was aware of the problem. This is all the more puzzling since by that time Einstein had come up with an ingenious response to De Sitter.

In his paper on cosmology published in February 1917, Einstein [39] circumvented the need for boundary conditions by eliminating boundaries! He proposed a cosmological model that is spatially closed. The metric

[48] In terms of the somewhat more technical language that has meanwhile been introduced the simple answer of [19] to this puzzle is that the spherical globe moves on a geodesic, while the ellipsoidal one does not. Hence, the symmetry between the two globes in Einstein's example is illusory, like the symmetry between the two twins in the twin paradox, and there is nothing puzzling about them behaving differently.

[49] A similar way of interpreting the covariance properties of the "Entwurf" theory and general relativity in terms of the relativity of the gravitational potential or the gravitational field rather than in terms of the relativity of motion was proposed in [77, 78]. See Gustav Mie to Einstein, 30 May 1917 [15, Doc. 346].

[50] See the editorial note, "The Einstein–De Sitter–Weyl–Klein Debate", in [15, pp. 351–357].

field of such a model could thus be attributed in full to matter. He picked a model that was not only closed but static as well. To prevent this model from collapsing he had to modify the Einstein field equations and add a term with what came to be known as the cosmological constant. This term produces a gravitational repulsion, which exactly balances the gravitational attraction in the model.

De Sitter [18] promptly produced an alternative cosmological model that is also allowed by Einstein's modified field equations. This model is completely empty. Absolute motion thus returned with a vengeance. Einstein's modified field equations still allow space-times with no matter to explain why test particles prefer to move on geodesics. Before publishing his new solution, De Sitter reported it to Einstein.[51] In his response Einstein finally articulated the principle that he had tacitly been using in his Machian accounts of Newton's bucket experiment and of his own variant on Newton's thought experiment with the two globes. He wrote:

> It would be unsatisfactory, in my opinion, if a world without matter were possible. Rather, the $g^{\mu\nu}$-field should be *fully determined by matter and not be able to exist without it*.[52]

This passage is quoted in the postscript of [18]. Einstein rephrased and published it as "Mach's principle" in [41]: "The [metric] field is *completely* determined by the masses of bodies."[53] In a footnote he conceded that he had not been careful in the past to distinguish this principle from general covariance. A day after submitting [41], he submitted [42] in which he argued that there was matter tugged away on a singular surface in De Sitter's cosmological model. In that case the De Sitter solution would not be a counter-example to Mach's principle. The following June, Einstein had to admit that this singular surface is nothing but an artifact of the coordinates used.[54] The De Sitter solution is a perfectly regular vacuum solution and thus a genuine counter-example to Mach's principle after all. Einstein never retracted his earlier claim to the contrary, but he gradually lost his enthusiasm for Mach's principle over the next few years. The principle is still prominently discussed in the Princeton lectures, but its limitations are also emphasized ([46, pp. 64–70], [47, pp. 99–108]).

Much of the appeal of Mach's ideas disappears when one switches from a particle to a field ontology. Among the sources of the metric field in general relativity is the electromagnetic field. Mach's principle then amounts to the requirement that the metric field be reduced to the electromagnetic field. But why privilege one field over another? Einstein, to my knowledge, never explicitly raised this question, but by the early 1920s he was trying to unify the electromagnetic field and the metric field rather than trying to reduce one to the other.

Einstein thus accepted that the shape of the water surface in Newton's bucket and the bulging out of one of the globes in his own thought experiment is caused by the rotation of the water and the globe with respect to the metric field and that the metric field cannot be reduced to matter. Even in general relativity these effects are the result of acceleration with respect to space(-time) just as in Newtonian theory and in special relativity. That does not mean, however, that Einstein's objection in the 1916 review article, that Newtonian theory and special relativity violate the principle of sufficient reason, now also applies to general relativity. Space-time in general relativity, Einstein argued in his Princeton lectures, is a *bona fide* physical entity to which causal efficacy can be ascribed.[55] Unlike Newtonian absolute space or Minkowski space-time, he pointed out, space-time in general relativity both acts and is acted upon ([46, p. 36], [47, pp. 55–56]). As Misner et al. [80, p. 5] put it: "*Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve*." Newtonian absolute space and Minkowski space-time only do the former. This is how Einstein was able to accept that general relativity did not eradicate absolute motion (in the sense of motion with respect to space-time rather than with respect to other matter).

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51 Willem de Sitter to Einstein, 20 March 1917 [15, Doc. 313]

52 "Es wäre nach meiner Meinung unbefriedigend, wenn es eine denkbare Welt ohne Materie gäbe. Das $g^{\mu\nu}$-Feld soll vielmehr *durch die Materie bedingt sein, ohne dieselbe nicht bestehen können*." Einstein to Willem de Sitter, 24 March 1917 [15, Doc. 317].

53 "Das G-Feld ist restlos durch die Massen der Körper bestimmt" [41, p. 241, note].

54 Einstein to Felix Klein, 20 June 1918 [15, Doc. 567].

55 In 1920, Ehrenfest and Lorentz arranged for a special professorship for Einstein in Leyden. In his inaugural lecture, Einstein [45] talked about the metric field as a new kind of ether.

As the simple solution of [19] to Einstein's problem of the rotating globes shows, it is not necessary to turn space-time into a causally efficacious substance to avoid violations of the principle of sufficient reason. In the course of developing general relativity, Einstein in fact provided ammunition for a strong argument *against* a substantival and in support of a relational ontology of space-time. This argument is based on the resolution of the hole argument against generally-covariant field equations.

Einstein first explained what was wrong with the hole argument, which can be found in four of his papers of 1914,[56] in a letter to Ehrenfest written about a month after reaffirming general covariance in November 1915.[57] He told Ehrenfest that the hole argument should be *replaced* by a new argument that has come to be known as the "point-coincidence argument".[58] A week later he told Besso the same thing.[59] The field equations, Einstein argued, need not determine the metric field uniquely, only such things as the intersections of worldlines, i. e., the "point coincidences" from which the new argument derives its name. Generally-covariant field equations will certainly do that. Two years earlier Besso had suggested that the escape from what was to become the hole argument might be that only worldlines need to be determined uniquely, but that suggestion had immediately been rejected.[60] In August 1913, Einstein had no use for an escape from the hole argument. The argument was mainly a fig leaf at that point for Einstein's inability to find generally-covariant field equations. Now that he had found and published such equations, however, he did need a escape from the hole argument. It is probably no coincidence (no pun intended) that five days before the letter to Ehrenfest in which the point-coincidence argument makes its first appearance a paper by Kretschmann [69] was published in which the notion of point coincidences, if not the term, is introduced [59, p. 54]. Kretschmann thus provided Einstein with just the right tools at just the right time.

In his letters to Ehrenfest and Besso, Einstein did more than substitute the point-coincidence argument for the hole argument. He also explained in these letters, albeit rather cryptically, where the hole argument went wrong.[61] The notion of point coincidences almost certainly helped Einstein put his finger on the problem with the hole argument. Once again consider the transformation $x \to x'$ used in the hole construction. Suppose two geodesics of the metric field $g_{\mu\nu}(x)$ intersect one another at a point inside the hole with coordinates $x = a$. Let the primed coordinates of that point be $x' = b$. In the metric field $g'_{\mu\nu}(x)$, obtained from $g'_{\mu\nu}(x')$ by reading x for x', the two corresponding geodesics will intersect at the point $x = b$. If the two points labeled $x = a$ and $x = b$ can somehow be identified before we assign values of the metric field to them, $g_{\mu\nu}(x)$ and $g'_{\mu\nu}(x)$ describe different situations. This suggests that the escape from the hole argument is simply to deny that bare manifold points can be individuated independently of the metric field. Solutions such as $g_{\mu\nu}(x)$ and $g'_{\mu\nu}(x)$ related to one another through Einstein's hole construction dress up the bare manifold differently to become a space-time. The original solution $g_{\mu\nu}(x)$ may dress up the bare manifold point p to become the space-time point P where two geodesics intersect, whereas the alternative solution $g'_{\mu\nu}(x)$ dresses up the bare manifold point q to become that same space-time point P. If the bare manifold points p and q have their identities only by virtue of having the properties of the space-time point P, there is no difference between $g_{\mu\nu}(x)$ and $g'_{\mu\nu}(x)$. Point coincidences can be used to individuate such space-time points. Most modern commentators read Einstein's comments on the hole argument in his letters to Ehrenfest and Besso in this way.[62]

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56 [51, p. 260], [52, pp. 217–218], [29, p. 178], [30, p. 1067].

57 Einstein to Paul Ehrenfest, 26 December 1915 [15, Doc. 173].

58 For discussions of the point-coincidence argument, see [59, 83, 97–99], [65, Sect. 4], and, especially, [58].

59 Einstein to Michele Besso, 3 January 1916 [15, Doc. 178].

60 Immediately following the passage quoted in note 32, Besso's memo of 28 August 1913 says: "It is, however, not necessary that the g themselves are determined uniquely, only the observable phenomena in the gravitation space, e. g., the motion of a material point, must be" ("Es ist nun allerdings nicht nötig, dass die g selbst eindeutig bestimmt sind, sondern nur die im Gravitationsraum beobachtbaren Erscheinungen, z.B. die Bewegung des materiellen Punktes, müssen es sein"). Appended to this passage is the following comment: "Of no use, since with a solution a motion is also fully given" ("Nützt nichts, denn durch eine Lösung ist auch eine Bewegung voll gegeben"). For further discussion, see [65, Sect. 3].

61 A clearer version can be found in a follow-up letter: Einstein to Paul Ehrenfest, 5 January 1916 [15, Doc. 180].

62 See the papers cited in note 58.

This resolution of the hole argument amounts to an argument against space-time substantivalism. What it shows is that there are many indistinguishable ways of assigning spatio-temporal properties to bare manifold points. According to Leibniz's "Principle of the Identity of Indiscernibles" all such assignments must be physically identical. But then the points themselves cannot be physically real for that would make the indistinguishable ways of ascribing properties to them physically distinct. This argument can be seen as a stronger version of a famous argument due to Leibniz himself against Newton's substantival ontology of space [23]. In his correspondence with Clarke, Leibniz objected that on Newton's view of space as a container for matter God, in creating the universe, had to violate the principle of sufficient reason [2, p. 26].[63] Without any discernible difference, He could have switched East and West, to use Leibniz's own example, or shifted the whole world to a different place in Newton's container. The Principle of the Identity of Indiscernibles tells us that these indistinguishable creations should all be identical. That in turn leaves no room for the container. In the hole argument, a violation of determinism takes over the role of the violation of the principle of sufficient reason in Leibniz's argument. This is what makes the hole argument the stronger of the two. In this secular age it is hardly a source of great distress that God for no apparent reason had to actualize one member of a class of empirically equivalent worlds rather than another. Ruling out determinism, however, as the substantivalist seems forced to do is clearly less palatable. To give determinism as much as "a fighting chance" [20, p. 180] in general relativity, we had thus better adopt a relational rather than a substantival ontology of space-time.

After his return to general covariance, Einstein never mentioned the hole argument again in any of his publications, but he did use the point coincidence argument in the 1916 review paper. He did not use it as part of an argument against space-time substantivalism, however, but as another argument against preferred frames of reference [35, pp. 776–777]. Since all our measurements eventually consist of observations of point coincidences, he argued,[64] and since such point coincidences are preserved under arbitrary coordinate transformations, physical laws should be generally covariant. Reformulated in the spirit of Klein's Erlangen program:[65] the group of general point transformations preserves the set of point coincidences, which supposedly exhaust the essential content of the geometries allowed by general relativity. Of course, this group of transformations also preserves the difference between geodesics and non-geodesics. Einstein nonetheless continued to tie general covariance to the relativity of motion. Revisiting the foundations of general relativity two years later, Einstein no longer used the point-coincidence argument to *argue* for a relativity principle but to *define* it:

> *Relativity principle*: The laws of nature are merely statements about point coincidences; the only natural way to express them is therefore in terms of generally-covariant equations.[66]

As Einstein realized at this point, this principle will only give full relativity of motion in conjunction with the other two principles on which he based his theory in 1918, the equivalence principle and Mach's principle. Only a few months later, as we saw earlier, Einstein had to concede that the latter does not hold in general relativity.

[63] For detailed analysis of this argument, see [20, Chap. 6]. For an abridged annotated version of the Leibniz-Clarke correspondence, see [61, Chap. 8].

[64] Earman [20, p. 186] thus charges Einstein with "a crude verificationism and an impoverished conception of physical reality." For a detailed critique of this reading of the point-coincidence argument, see [58].

[65] In one of his physics textbooks, Sommerfeld [95, pp. 316–317] citing [46], explicitly endorses such a reading of Einstein's presentation of general relativity.

[66] "*Relativitätsprinzip*: Die Naturgesetze sind nur Aussagen über zeiträumliche Koinzidenzen; sie finden deshalb ihren einzig natürlichen Ausdruck in allgemein kovarianten Gleichungen" [41, p. 241].

The "fateful prejudice" and the "key to the solution"[67]

In his search for satisfactory field equations for $g_{\mu\nu}$ in 1912–1913, Einstein had consciously pursued the analogy with Maxwell's theory of electrodynamics.[68] He continued to pursue this analogy in developing a variational formalism for the "Entwurf" theory in 1914 [52,30].[69] In his lecture on gravity at the 85th *Naturforscherversammlung* in Vienna in the fall of 1913 [28], he had already shown that the "Entwurf" field equations, like Maxwell's equations, can be cast in the form "divergence of field = source." In Maxwell's equations, $\partial_\nu F^{\mu\nu} = j^\mu$, the tensor $F^{\mu\nu}$ represents the electromagnetic field and the charge-current density j^μ its source. In Einstein's gravitational theory, the source is represented by the sum of $T^{\mu\nu}$, the energy-momentum tensor of 'matter' (which can be anything from particles to an electromagnetic field) and $t^{\mu\nu}$, the energy-momentum pseudo-tensor of the gravitational field itself.

Einstein used the energy-momentum balance law,

$$T^{\mu\nu}{}_{;\nu} = 0 \tag{1}$$

(with the semi-colon indicating a covariant derivative), to identify both the pseudo-tensor $t^{\mu\nu}$ and the expression for the gravitational field. He had derived this equation early on in his work on the metric theory of gravity as the natural generalization of the special-relativistic law of energy-momentum conservation, $T^{\mu\nu}{}_{,\nu} = 0$ (with the comma indicating an ordinary coordinate derivative).[70] For a charge distribution described by the four-current density j^μ in an electromagnetic field $F^{\mu\nu}$, we have

$$T^{\mu\nu}{}_{,\nu} = j_\nu F^{\mu\nu} \ , \tag{2}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the electromagnetic field.[71] The right-hand side has the form "source × field." It gives the density of the four-force that the electromagnetic field exerts on the charges, or, equivalently, the energy-momentum transfer from field to charges. The equation $T^{\mu\nu}{}_{;\nu} = 0$ (where $T^{\mu\nu}$ is the energy-momentum tensor of arbitrary 'matter' again) can be interpreted in the same way. Eq. (1) can be rewritten as

$$\mathfrak{T}^{\alpha}_{\mu,\alpha} - \left\{ \begin{matrix} \beta \\ \mu\alpha \end{matrix} \right\} \mathfrak{T}^{\alpha}_{\beta} = 0 \ , \tag{3}$$

where $\mathfrak{T}^{\mu}_{\nu} \equiv \sqrt{-g} T^{\mu}_{\nu}$ is a mixed tensor density, and where

$$\left\{ \begin{matrix} \beta \\ \mu\alpha \end{matrix} \right\} \equiv \frac{1}{2} g^{\beta\rho} (g_{\rho\mu,\alpha} + g_{\rho\alpha,\mu} - g_{\mu\alpha,\rho}) \tag{4}$$

are the Christoffel symbols of the second kind. Because $T^{\mu\nu}$ is symmetric, eq. (3) can be further reduced to:

$$\mathfrak{T}^{\alpha}_{\mu,\alpha} - \frac{1}{2} g^{\beta\rho} g_{\rho\alpha,\mu} \mathfrak{T}^{\alpha}_{\beta} = 0 \ . \tag{5}$$

Compare the second term on the left-hand sides of eqs. (3) and (5) to the term on the right-hand side of eq. (2). All three terms are of the form "source × field." In eqs. (3) and (5), these terms represent the density of the four-force that the gravitational field exerts on matter (the source $\mathfrak{T}^{\alpha}_{\beta}$), or, equivalently, the energy-momentum transfer from field to matter. One can thus read off an expression for the gravitational field from

[67] This section is based on [66].

[68] See [90] for a detailed reconstruction of Einstein's reliance on this analogy.

[69] See Sect. 3 of [66] for a detailed analysis of this variational formalism the use of which runs like a red thread through Einstein's work on general relativity from 1914 through 1918.

[70] See the "Zurich Notebook", [p. 5R] [11, Doc. 10, p. 10]; a facsimile of this page graces the dust cover of this volume. For analysis of this page, see [87, Appendix C] and [67, Sect. 3].

[71] Cf., e. g., [35, Sect. 20, 814–815], eqs. (65), (65a), (66), and (66a) for $g_{\mu\nu} = \eta_{\mu\nu}$, the flat Minkowski metric.

these terms. Using the field equations to eliminate $\mathfrak{T}^{\alpha}_{\beta}$ from these terms and writing the resulting expression as a divergence, $\partial_\alpha \mathfrak{t}^{\alpha}_{\mu}$, one can identify the gravitational energy-momentum pseudo-tensor ($\mathfrak{t}^{\mu}_{\nu} \equiv \sqrt{-g}t^{\mu}_{\nu}$).

Einstein used the notation $\Gamma^{\mu}_{\alpha\beta}$ for the components of the gravitational field. Unfortunately, different forms of eq. (1) lead to different choices for $\Gamma^{\mu}_{\alpha\beta}$. Eq. (5) gives

$$\Gamma^{\mu}_{\alpha\beta} \equiv -\frac{1}{2} g^{\mu\rho} g_{\rho\alpha,\beta} \ , \tag{6}$$

which is essentially the gradient of the metric tensor. This makes perfect sense since the metric is the gravitational potential in Einstein's theory. But the relation between field and potential could also be the one suggested by eq. (3):

$$\Gamma^{\mu}_{\alpha\beta} \equiv -\left\{ \begin{matrix} \mu \\ \alpha\ \beta \end{matrix} \right\} . \tag{7}$$

With this definition of the field there are three terms with a gradient of the potential. In fact, $\Gamma^{\mu}_{\alpha\beta}$ in eq. (6) is nothing but a truncated version of $\Gamma^{\mu}_{\alpha\beta}$ in eq. (7).

For the "Entwurf" theory Einstein chose definition (6) (omitting the minus sign).[72] It was only in the fall of 1915 that he realized that he should have gone with definition (7) instead. In Einstein's own estimation this was a crucial mistake. In the first of his four papers of November 1915, he wrote:

> This conservation law [in the form of eq. (5)] has led me in the past to look upon the quantities [in eq. (6)] as the natural expressions of the components of the gravitational field, even though the formulas of the absolute differential calculus suggest the Christoffel symbols [. . .] instead. *This was a fateful prejudice.*[73]

Later that month, after he had completed the theory, he wrote in a letter:

> *The key to this solution* was my realization that not [the quantities in eq. (6)] but the related Christoffel symbols [. . .] are to be regarded as the natural expression for the "components" of the gravitational field.[74]

To understand why eq. (6) was a "fateful prejudice" and eq. (7) was "the key to [the] solution," we need to look at Einstein's 1914 derivation of the "Entwurf" field equations from the action principle $\delta J = 0$. The action functional,

$$J = \int H \sqrt{-g} d^4x \ , \tag{8}$$

is determined by the Lagrangian H (Hamilton's function in Einstein's terminology). Drawing on the electrodynamical analogy, Einstein modelled H on $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$, the Lagrangian for the free Maxwell field:

$$H = -g^{\mu\nu}\Gamma^{\alpha}_{\beta\mu}\Gamma^{\beta}_{\alpha\nu} \ . \tag{9}$$

Inserting (minus) eq. (6) for the gravitational field and evaluating the Euler-Lagrange equations, he recovered the vacuum "Entwurf" field equations

$$\partial_\alpha \left(\sqrt{-g} g^{\alpha\beta} \Gamma^{\lambda}_{\mu\beta} \right) = -\kappa \mathfrak{t}^{\lambda}_{\mu} \ , \tag{10}$$

[72] See [30, p. 1058, eq. (46)]. In a footnote on p. 1060, Einstein explains why he used eq. (5) rather than eq. (3) to identify $\Gamma^{\mu}_{\alpha\beta}$

[73] "Diese Erhaltungsgleichung hat mich früher dazu verleitet, die Größen [. . .] als den natürlichen Ausdruck für die Komponenten des Gravitationsfeldes anzusehen, obwohl es im Hinblick auf die Formeln des absoluten Differentialkalküls näher liegt, die Christoffelschen Symbole statt jener Größen einzuführen. *Dies war ein verhängnisvolles Vorurteil*" [31, p. 782]; my emphasis.

[74] "*Den Schlüssel zu dieser Lösung* lieferte mir die Erkenntnis, dass nicht [. . .] sondern die damit verwandten Christoffel'schen Symbole [. . .] als natürlichen Ausdruck für die "Komponente" des Gravitationsfeldes anzusehen ist." This comment comes from the letter to Sommerfeld cited in note 3. The emphasis is mine.

where the left-hand side is essentially the divergence of the gravitational field and where

$$\kappa \mathfrak{t}^{\lambda}_{\mu} = \sqrt{-g}\left(g^{\lambda\rho}\Gamma^{\alpha}_{\tau\mu}\Gamma^{\tau}_{\alpha\rho} - \frac{1}{2}\delta^{\lambda}_{\mu}g^{\rho\tau}\Gamma^{\alpha}_{\beta\rho}\Gamma^{\beta}_{\alpha\tau}\right) \tag{11}$$

is the energy-momentum pseudo-tensor for the gravitational field ([30, p. 1077, eq. (81b)]). This quantity was chosen is such a way that the energy-momentum balance law (5) can be written as a proper conservation law:

$$\partial_{\lambda}(\mathfrak{T}^{\lambda}_{\mu} + \mathfrak{t}^{\lambda}_{\mu}) = 0\ . \tag{12}$$

The field equations in the presence of matter are found by adding $-\kappa T^{\lambda}_{\mu}$ to the right-hand side of eq. (10) on the argument that all energy-momentum – of matter and of the gravitational field itself – should enter the field equations the same way. One thus arrives at:

$$\partial_{\alpha}\left(\sqrt{-g}g^{\alpha\beta}\Gamma^{\lambda}_{\mu\beta}\right) = -\kappa\left(\mathfrak{T}^{\lambda}_{\mu} + \mathfrak{t}^{\lambda}_{\mu}\right). \tag{13}$$

Having the four-divergence operator ∂_{λ} act on both sides of eq. (13), one sees that energy-momentum is conserved if and only if[75]

$$B_{\mu} \equiv \partial_{\lambda}\partial_{\alpha}\left(\sqrt{-g}g^{\alpha\beta}\Gamma^{\lambda}_{\mu\beta}\right) = 0 \tag{14}$$

Einstein showed that these same conditions also determine the covariance properties of the action (8).[76] He argued that the corresponding Euler-Lagrange equations – i. e., the vacuum “Entwurf” field equations (10) – will inherit these covariance properties from the action. Since $T^{\mu\nu}$ is a generally-covariant tensor, the four conditions $B_{\mu} = 0$ would then determine the covariance properties of the full “Entwurf” field equations (13) as well.

How do these conditions select transformations that leave the “Entwurf” field equations (13) invariant? Start with a metric field $g_{\mu\nu}$ given in coordinates x^{α} that satisfies both the field equations (13) and conditions (14). Now consider a transformation from x^{α} to x'^{α} under which $g_{\mu\nu}$ goes to $g'_{\mu\nu}$. Einstein believed that $g'_{\mu\nu}$ would also be solution of the field equations (13) *if and only if* $g'_{\mu\nu}$ satisfies conditions (14). The transformations picked out by the conditions $B_{\mu} = 0$ are thus of a somewhat peculiar nature. The condition selects transformations from x^{α} to x'^{α} leaving the “Entwurf” field equations invariant *given* a metric field that is a solution of the field equations in the original -coordinates. Because of their dependence on the metric, Einstein called such transformations “non-autonomous” (“unselbständig”) at one point.[77] The Italian mathematician Tullio Levi-Civita wrote the Minkowski metric in two different coordinate systems and showed that both forms satisfy the condition $B_{\mu} = 0$, while only one form is a (vacuum) solution of the “Entwurf” field equations.[78] Despite this clear-cut counter-example, Einstein stubbornly continued to believe that the condition guaranteeing energy-momentum conservation was also the necessary and sufficient condition for a solution of the “Entwurf” field equations in one coordinate system to be a solution in some other coordinate system. In Einstein’s defense, it must be said that he was on to an important result even if his math did not quite add up. The connection between the invariance of the action for the “Entwurf” field equations (as opposed to the field equations themselves) and energy-momentum conservation is a special case of one of Noether’s celebrated theorems connecting symmetries and conservation laws.[79] Einstein had intuitively recognized this special case almost five years before Emmy Noether [81] published the general

[75] Einstein had learned the hard way that he had better make sure that the field equations be compatible with energy-momentum conservation. In 1912 he had been forced to modify the field equation of his theory for static gravitational fields because the original one violated energy-momentum conservation [27, Sect. 4].

[76] See [66, Sect. 3.3].

[77] Einstein to H. A. Lorentz, 14 August 1913 [12, Doc. 467].

[78] Tullio Levi-Civita to Einstein, 28 March 1915 [15, Doc. 67].

[79] See [4] for an insightful analysis of Noether’s theorems. For historical discussion, see [91,92], and [94].

theorem. "What can be more beautiful," he had written in 1913 when he believed that energy-momentum conservation restricted the covariance of acceptable field equations to linear transformations (see Sect. 2), "than that the necessary specialization [of admissible coordinate systems] follows from the conservation laws?"[80]

Given this clarification of the structure of the "Entwurf" theory, one can understand why Einstein felt that the time was ripe for an authoritative self-contained exposition of the theory. The result was [30], which appeared in November 1914.

In the fall of 1915, a number of worrisome cracks were beginning to show in the "Entwurf" edifice. Most importantly, Einstein was finally forced to accept that the "Entwurf" field equations are not invariant under the (non-autonomous) transformation to rotating coordinates in the special case of the standard diagonal Minkowski metric (see the discussion in Sect. 2 and the letter cited in note 40). His Machian solution to the problem of Newton's rotating bucket experiment required the Minkowski metric in rotating coordinates to be a vacuum solution of the field equations. The "Entwurf" field equations were no longer acceptable now that it had become clear that they do not meet this requirement. Einstein needed new field equations.

If we take him at his word when he identified definition (6) of the gravitational field as a "fateful prejudice" and definition (7) as "the key to [the] solution," a plausible scenario of how Einstein found the successor(s) to the "Entwurf" field equations suggests itself.[81] The scenario runs as follows. Einstein decided to keep the Maxwell-inspired Lagrangian (9), with the exception of the immaterial minus sign,

$$H = g^{\mu\nu}\Gamma^{\alpha}_{\beta\mu}\Gamma^{\beta}_{\alpha\nu} \ , \tag{15}$$

and change only the definition of the gravitational field entering into it. Inserting eq. (7) into eq. (15), setting $\sqrt{-g} = 1$ in eq. (8) for the action J, and evaluating the Euler-Lagrange equations for the resulting variational problem $\delta J = 0$, he arrived at:[82]

$$-\partial_\alpha \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} + \begin{Bmatrix} \alpha \\ \beta\mu \end{Bmatrix} \begin{Bmatrix} \beta \\ \nu\alpha \end{Bmatrix} = 0 \ . \tag{16}$$

Einstein had encountered these two terms before. They are two of the four terms in the Ricci tensor, a direct descendant of the Riemann curvature tensor. The other two terms are[83]

$$\partial_\mu \begin{Bmatrix} \alpha \\ \alpha\ \nu \end{Bmatrix} - \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} \begin{Bmatrix} \beta \\ \alpha\ \beta \end{Bmatrix} . \tag{17}$$

Introducing the quantity

$$T_\nu \equiv \begin{Bmatrix} \alpha \\ \alpha\ \nu \end{Bmatrix} = \partial_\nu \left(\lg\sqrt{-g}\right) , \tag{18}$$

which transforms as a vector under unimodular transformations (i. e., transformations with a Jacobian equal to one), one recognizes that expression (17) is the covariant derivative of T_ν,

$$\partial_\mu T_\nu - \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} T_\alpha = T_{\nu;\mu} \ , \tag{19}$$

80 "Was kann es schöneres geben, als dies, dass jene nötige Spezialisierung aus den Erhaltungssätzen fliesst?" Einstein to Paul Ehrenfest, before 7 November 1913 [12, Doc. 481].

81 The bulk of [66] is concerned with making the case for this scenario on the basis of all extant primary source material.

82 This calculation can be found in Sect. 15 of [35].

83 See [35, p. 801, eq. (44)] for this decomposition of the Ricci tensor.

which transforms as a tensor under unimodular transformations. Since the full Ricci tensor is a generally-covariant tensor and one half transforms as a tensor under unimodular transformations, the other half (i. e., the left-hand side of eq. (16)) must transform as a tensor under unimodular transformations as well.

In the "Zurich Notebook" Einstein had actually looked carefully into the possibility of using this unimodular tensor as the basis for gravitational field equations.[84] One of the problems that had defeated him back then was that he could not show that such equations would be compatible with energy-momentum conservation. In the course of developing his variational formalism for the "Entwurf" theory in 1914, Einstein had learned how to deal with that problem (cf. eqs. (12)–(13) above). This made field equations based on the unimodular tensor in eq. (16) extremely attractive.

By this time, October 1915, Einstein had been struggling with the intractable covariance properties of the "Entwurf" field equations for almost three years. Changing the definition of the gravitational field in the Lagrangian for the "Entwurf" theory had now led him back to field equations covariant under a broad class of transformations. The only fly in the ointment was the hole argument, according to which there could be no such field equations. Sooner or later he would have to deal with this objection of his own making. But that could wait. As we saw Sect. 2, he only addressed this issue in correspondence of late December and early January.

So Einstein went ahead and decided on the vacuum field equations

$$\partial_\alpha \Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu} = 0\ , \tag{20}$$

which are obtained by replacing the Christoffel symbols in eq. (16) by minus the components $\Gamma^\alpha_{\mu\nu}$ of the gravitational field (see eq. (7)). He generalized these equations to situations with matter present by putting (minus) the energy-momentum tensor for matter on the right-hand side

$$\partial_\alpha \Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu} = -\kappa T_{\mu\nu}\ . \tag{21}$$

With Hilbert in hot pursuit, he rushed these equations into print [31]. He realized soon afterwards that they were still not quite right. Within a three-week span, he published two modifications of the equations [32, 34]. The second time he got it right. He had found the generally-covariant field equations still bearing his name.

In Sects. 14–18 on the field equations and energy-momentum conservation in [35], the reader is spared the detour through the erroneous field equations of November 1915. Using the derivation rehearsed in one of his letters to Ehrenfest (see note 12), Einstein introduced the correct equations right away, albeit not in their generally-covariant form, but, as in the November 1915 papers, in unimodular coordinates (picked out by the condition $\sqrt{-g} = 1$). It turns out that Einstein's generalization of eq. (20) to eq. (21) violates the requirement that all energy-momentum enter the field equations the same way. This becomes clear when the vacuum field equations are rewritten in terms of the energy-momentum pseudo-tensor for the gravitational field in the new theory. This pseudo-tensor is found in the same way as the one for the "Entwurf" theory in eq. (11) and has the exact same structure:

$$\kappa t^\lambda_\sigma = \frac{1}{2}\delta^\lambda_\sigma g^{\mu\nu}\Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu} - g^{\mu\nu}\Gamma^\alpha_{\mu\sigma}\Gamma^\lambda_{\alpha\nu} \tag{22}$$

[35, p. 806, eq. (50)]. Eq. (22) is obtained from eq. (11) by setting $\sqrt{-g} = 1$ (reflecting the restriction to unimodular coordinates), introducing an overall minus sign (since the Lagrangians (9) and (15) have opposite signs), and – most importantly – replacing definition (6) of the components of the gravitational field $\Gamma^\alpha_{\mu\nu}$ by definition (7). With the help of eq. (22) and its trace, $\kappa t \equiv \kappa t^\lambda_\lambda = g^{\mu\nu}\Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu}$, the vacuum field equations (20) can be rewritten as (Ibid., 806, eq. (51)):

$$\partial_\alpha (g^{\nu\sigma}\Gamma^\alpha_{\mu\nu}) = -\kappa\left(t^\sigma_\mu - \frac{1}{2}\delta^\sigma_\mu t\right). \tag{23}$$

[84] See [67], Sect. 5.5, for a detailed analysis of the relevant pages of the notebook, [pp. 22L–24L] and [pp. 42L–43L] [11, Doc. 10, pp. 43–47 and pp. 7–9].

Notice how closely this equation (along with eq. (22) for t^σ_μ) resembles the vacuum "Entwurf" field equations (10) (along with eq. (11) for $\mathfrak{t}^\lambda_\mu = \sqrt{-g} t^\lambda_\mu$). The crucial difference – besides immaterial minus signs, factors of $\sqrt{-g}$, and a slightly different ordering of indices – is the presence of the trace term $(1/2)\delta^\sigma_\nu \kappa t$ on the right-hand side of eq. (23). On the by now familiar argument that all energy-momentum enter the field equations the same way, this means that the field equations in the presence of matter should likewise have a term with the trace of the energy-momentum tensor of matter (Ibid., 807, eq. (52)):

$$\partial_\alpha (g^{\nu\sigma}\Gamma^\alpha_{\mu\nu}) = -\kappa \left([t^\sigma_\mu + T^\sigma_\mu] - \frac{1}{2}\delta^\sigma_\mu [t+T] \right) \tag{24}$$

Now recall that eq. (23) is just an alternative way of writing eq. (20). Eq. (24) can thus also be written as:

$$\partial_\alpha \Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu} = -\kappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \tag{25}$$

(Ibid., 808, eq. (53)). In November 1915, Einstein had found his way from eq. (21) to eq. (25) following a more circuitous route.

In the unimodular coordinates used in this calculation $\sqrt{-g} = 1$ and the quantity T_μ in eq. (18) vanishes. This means that expression (17) vanishes as well and that the Ricci tensor reduces to the left-hand side of eq. (25) (cf. eq. (16)). The field equations (25) can thus be looked upon as generally covariant equations expressed in unimodular coordinates. The corresponding generally-covariant equations are:

$$R_{\mu\nu} = -\kappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right), \tag{26}$$

where $R_{\mu\nu}$ is the Ricci tensor (denoted by $B_{\mu\nu}$ in ibid., 801, eq. (44)). The reader, I trust, will immediately recognize eq. (26) as the Einstein field equations.

Returning now to eq. (24), the original form of the field equations in unimodular coordinates, one sees that energy-momentum is conserved, i. e., $T^{\mu\nu}{}_{;\nu} = 0$, or, equivalently, $\partial_\sigma (t^\sigma_\mu + T^\sigma_\mu) = 0$, if and only if

$$\partial_\sigma \left[\partial_\alpha (g^{\nu\sigma}\Gamma^\alpha_{\mu\nu}) - \frac{1}{2}\kappa\delta^\sigma_\mu (t+T) \right] = 0 \,. \tag{27}$$

These conditions are the analogues of the conditions $B_\mu = 0$ in the "Entwurf" theory (cf. eqs. (12)–(14)). Using the trace of eq. (24), $\partial_\alpha (g^{\nu\sigma}\Gamma^\alpha_{\sigma\nu}) = \kappa(t+T)$, to replace $\kappa(t+T)$ by an expression in terms of the metric and its derivatives, one can rewrite eq. (27) as

$$\partial_\sigma \left[\partial_\alpha \left(g^{\nu\sigma}\Gamma^\alpha_{\mu\nu} \right) - \frac{1}{2}\delta^\sigma_\mu \partial_\alpha \left(g^{\nu\beta}\Gamma^\alpha_{\beta\nu} \right) \right] = 0 \,. \tag{28}$$

Einstein showed that these four relations, unlike the conditions $B_\mu = 0$, hold identically. In other words, the field equations (24) guarantee energy-momentum conservation without the need for restrictions on admissible coordinates over and above the condition $\sqrt{-g} = 1$ for unimodular coordinates (ibid., 808–810, Sect. 17–18).

This came as no surprise to Einstein. He expected there to be a close connection between covariance of the field equations and energy-momentum conservation. In [30] he had shown that the four conditions $B_\mu = 0$ (eq. (14)) that together with the "Entwurf" field equations guarantee energy-momentum conservation double as the conditions restricting the range of coordinate systems in which these field equations hold. The dual role of such conditions played a central role in the breakthrough of November 1915.[85] What made the field equations (21) replacing the "Entwurf" field equations in [31] so attractive was that the range of

[85] This is one of the central claims argued for in [66, see, in particular, Sects. 6–7].

coordinate systems in which they hold was restricted only by one condition, viz. that the determinant of the metric transform as a scalar, as opposed to the four conditions restricting the range of coordinate systems in which the "Entwurf" equations hold. Given the dual role of these conditions in the "Entwurf" theory, this suggested to Einstein that it would suffice to add this one condition on g to the new field equations to guarantee energy-momentum conservation in the new theory. In [31] he showed that the analogues of the four conditions $B_\mu = 0$ in the new theory can indeed be replaced by one condition on g. He presumably expected this condition to be that g transform as a scalar, since this expresses the restriction to unimodular transformations. Instead he found that g cannot be a constant, which though compatible with unimodular transformations is a stronger condition and rules out unimodular *coordinates* (with $g = -1$).

The transition from the field equations of the first November paper [31] to those of the second and the fourth [32, 34] was driven by the desire to change the field equations in such a way that the condition that g cannot be a constant can be replaced by the more congenial condition $\sqrt{-g} = 1$ for unimodular coordinates. These amended field equations could then be looked upon as generally-covariant field equations expressed in special coordinates. In [32] this goal is reached at the expense of the assumption, largely discredited at that point, that all matter (represented by $T_{\mu\nu}$) somehow consists of electromagnetic fields (in which case the trace T vanishes). In [34] generally-covariant field equations in unimodular coordinates are obtained without specifying $T_{\mu\nu}$ but by adding a term with the trace T to the right-hand side of eq. (20).

The "Zurich Notebook" shows that Einstein had already considered adding such a term three years earlier to render field equations based on the Ricci tensor compatible with energy-momentum conservation in the weak-field limit.[86] What had stopped him from doing so was that the resulting weak-field equations rule out a spatially flat metric of the form $g_{\mu\nu} = (-1, -1, -1, c^2(x, y, z))$. For such a metric the ten components of the gravitational potential reduce to one component, the variable speed of light $c(x, y, z)$ of the theory for static fields of [26, 27]. Einstein firmly believed that this was how weak static fields had to be represented in his theory.[87]

This changed only when he calculated the perihelion advance of Mercury on the basis of the field equations of [32]. He realized that if $\sqrt{-g} = 1$ and g_{44} is variable, the components g_{ij} cannot all be constants [82, p. 147]; [21, pp. 144–145]. This removed his old objection to adding a term with the trace T to the field equations. At that point Einstein realized that such a trace term was needed anyway to make sure that all energy-momentum enter the field equations in exactly the same way. This told him that he had finally got it right. He had found the Einstein field equations in unimodular coordinates (see eq. (25)).

In his 1916 review article Einstein still derived the field equations in unimodular coordinates only. The manuscript for an unpublished appendix to the article [13, Doc. 31] shows that he at least started an alternative discussion of the field equations and energy-momentum conservation in arbitrary coordinates. The numbering of the sections in this document suggests that at one point he considered substituting this discussion for the one in unimodular coordinates. He then considered adding it as an appendix. In the end he did neither. Instead he published the generally-covariant treatment separately a few months later [37].

In this paper he derived the generally-covariant field equations from an action principle with the Riemann curvature scalar as the Lagrangian. Terms with second-order derivatives of the metric in this quantity do not contribute to the action integral, so the effective Lagrangian becomes:

$$\sqrt{-g}g^{\mu\nu}\left[\left\{\begin{matrix}\beta\\ \mu\,\alpha\end{matrix}\right\}\left\{\begin{matrix}\alpha\\ \nu\beta\end{matrix}\right\}-\left\{\begin{matrix}\alpha\\ \mu\nu\end{matrix}\right\}\left\{\begin{matrix}\alpha\\ \alpha\,\beta\end{matrix}\right\}\right]. \tag{29}$$

For $\sqrt{-g} = 1$ the second term vanishes (cf. eq. (18)) and the expression reduces to the Lagrangian (15) used in the November 1915 papers and in the 1916 review article. [37] fills two important gaps in [35]. First, Einstein derived the generally-covariant version of the identities (28), which in conjunction with the

[86] See the "Zurich Notebook", [p. 20L] [11, Doc. 10, p. 39]. For analysis of this page, see [67, Sect. 5.4.3].

[87] Einstein checked and confirmed this assumption on at least two occasions, in the "Zurich Notebook", [p. 21R] [11, Doc. 10, p. 42] (for analysis see [67, Sect. 5.4.4 and 5.4.6]), and in Einstein to Erwin Freundlich, 19 March 1915 [15, Doc. 63]

field equations imply energy-momentum conservation. These generally-covariant identities are the now famous contracted Bianchi identities. Second, Einstein showed – proceeding exactly the way he did in the premature review article [30] – that the identities guaranteeing energy-momentum conservation are a direct consequence of the covariance of the action functional. Einstein had thus, in a mathematically impeccable way, found a special case of one of Noether's theorems published two years later.

From a purely mathematical point of view, the discussion of the field equations and energy-momentum conservation in [37] is far more elegant than in [35]. This more elegant treatment, however, obscures the way in which Einstein found the Einstein field equations. It makes it look as if it was a matter of picking the most obvious candidate for the Lagrangian, the Riemann curvature scalar, at which point everything else fell into place. Ironically, this is exactly what Einstein in his later years came to believe himself, in part no doubt because it made his successful search for the field equations of general relativity look so similar to his fruitless search for a unified field theory. The clumsier discussion in unimodular coordinates in [35], however, may serve as a reminder that – whatever he believed, said, or wrote about it later on – Einstein only discovered the mathematical high road to the Einstein field equations after he had already found these equations at the end of a bumpy road through physics. Serving as road signs were Newton's gravitational theory, Maxwell's electrodynamics, and such key results of special relativity as the law of energy-momentum conservation. Considerations of mathematical elegance played only a subsidiary role.

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