Material Dissipative Conditions and the Impossibility of Complete Recycling

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Abstract

The preservation of the natural environment requires a reduction in material intensity of economic systems. Recycling is a major method for meeting this requirement. One of the most appropriate formulations for economic recycling models is the introduction of recycling sectors and joint production of waste materials. The models are generally checked for the feasibility of net-production. Such models may be able to realize complete recycling material resources, but this is clearly impossible due to the unrecoverable material dissipation in economic production processes. The result is that the models have sometimes reproduced material resources larger than the amount of inputted virgin material. This paper introduces the material dissipative conditions and the material transferability system appropriate for recognizing the material dissipation of economic systems with recycling sectors.

Keywords: Environmental model; Recycling; Material dissipation

1. Introduction

Recycling is believed to be one of the most important activities for the protection of the environment. It reduces not only the volume of waste materials but also resource consumptions. The latter effect makes the construction of mathematical models that represent economic processes of recycling an important issue in economics. Recycling is not merely a production process, but also has a social dimension. Therefore, models are needed that can express the interdependency of economic subjects. General equilibrium or input-output models meet this requirement. Previous research on this subject includes both theoretical and empirical analysis of the recycling process using such a framework (e.g. Washida (1994), Atri and Schellberg (1995), Dobbs (1991), Fullerton (1995), Nakamura (1997)).

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All models that embody one or more recycling processes require conditions that reflect actual economic processes. Ordinarily, these model have to fulfill the technological feasibility conditions of social production processes. Although general equilibrium models may allow the production possibility frontier not to appear in the non-negative quadrant, input-output models have to fulfill the net-production condition that possible levels of production for all commodities are non-negative, and at least one commodity can be produced with a strictly positive level.

These feasibility conditions are not special conditions but are generally required. In recycling models, we require additional technological conditions. If the social recycling process embodied in the model can produce a larger amount of recycled material resources than the amount of virgin materials put into the economic system, this situation would be paradise for material resources and is clearly impossible. Of course, it is often difficult to compare the reproduced resources with virgin material resources. Here for simplicity, both types of resources will be measured by the amount of primary molecular material content, e.g. carbon or iron, that are substitutable for one another.

This material paradise is certainly an extreme case. We can imagine a weaker case in which a type of resource can be completely recycled. In other words, material resources are reproduced by recycling processes to the extent that the system needs no input of virgin material resources. This situation is also practically impossible. Complete recycling is extremely unrealistic when we regard the recycling process as a social process with interdependencies between economic subjects.

This proposition was powerfully advocated by Nicholas Georgescu-Roegen (Georgescu-Roegen (1979a, 1979b)). He expressed it as "Matter Matters, Too." This means that matter deteriorates in utilization processes in the same way that energy deteriorates as expressed in the second law of thermodynamics. Georgescu-Roegen tried to apply the second law of thermodynamics to matter. He said that "The fact that recycling cannot be complete proves that matter, just like energy, continuously and irrevocably dissipates. Matter is not lost; it only ultimately becomes *unavailable* to us. Briefly, matter, too, is subject to entropic degradation." (Georgescu-Roegen (1979b, p.20)).

However, Georgescu-Roegen's proposition has some important exceptions, one of which appears in ecosystems. Although ecosystems constitute a closed system for matter, they are not necessarily a closed system for energy. For example, Y.Kurihara (Kurihara (1960, 1978a, 1978b)) proved using experiments in a materially closed flask that some species can continue to live perpetually through the interdependency between biological subjects. Complete recycling

is actually realized in the flask.

Another exception appears in hunting and gathering societies. For example, until two thousand years ago, the inhabitants of Japan lived in such a society for a period of ten thousand years (the Jomon era). Japanese archaeological research has demonstrated that the society was sustainable and had achieved a relatively high level of affluence. Although stone implements and earthen vessels caused some irreversible dissipation of matter, these effects can be regarded as almost negligible.

These two examples suggest that there are exceptions to the applicability of Georgescu-Roegen's proposition. These exceptions are, however, not essential for this paper. This is because the recycling models are primarily used for capturing the technological process of economic systems in industrial societies. Recycling activities inevitably and irreversibly disperse some material, that is, complete recycling is impossible in industrial societies.

Although Georgescu-Roegen intended to pose an additional law of thermodynamics, the proposition of the incomplete recycling is a different issue with more practical ramifications. It is undeniable that complete recycling is theoretically possible under the condition of sufficient energy. The proposition we will employ in this paper is the impossibility of complete recycling for practical purposes. Since recycling is a social process which includes many physical process supported by many human activities, the dissipation of matter is unavoidable (Söllner (1997)).

Another important question is the relationship between the impossibility condition of complete recycling and the feasibility condition which have to be included in economic recycling models. These two conditions are intuitively independent of each other. If this is true, these two conditions must be checked independently when an economic recycling model is constructed. Although previous research has looked at whether economic models fulfill the feasibility condition, it has not looked at whether these models satisfy the impossibility conditions of complete recycling.

The first purpose of this paper is to represent the material dissipative conditions that ensure the impossibility of complete recycling in a simple input-output type model that uses a recycling process and the joint production of waste matter. Second, the paper discusses how the conditions affect the value system, especially focusing on the price of waste matter being potentially recycled. The result is that material dissipative conditions are necessary conditions for the value of the waste matter to be positive. Third, the paper discusses a new system which directly represents material flow in an economic system. At first glance, this system resembles
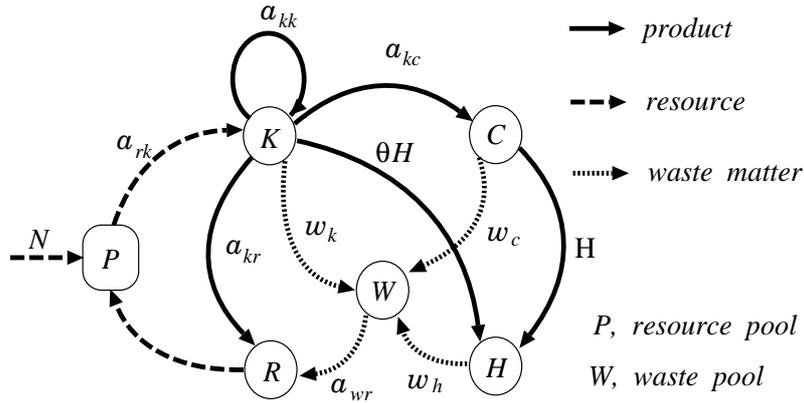
Figure 1: Basic Structure of the Model

the value system mentioned above. The crucial differences lie in the fact that the values of commodities in the new system are measured by the amount of physically contained matter, and the production processes are expressed in terms of physical processes that convert raw materials into products. We can then define the rate of dissipation of matter in the production processes. Owing to these features of this new system we can directly introduce the material dissipative conditions. Fourth, the paper reveals the exact relationship between the equations that define the material dissipative conditions. This inquiry will lead to a more effective definition of the material dissipative conditions.

2. The model

Let us use a three sector model with an industrial sector (K), an agricultural sector (C) and a recycling sector (R). We assume that the agricultural sector does not produce pure biological goods but produces processed agricultural or biological products. Therefore it may include the material of industrial products e.g. containers. For simplicity, we will assume that one sort of virgin material resource is utilized by this economic system and that the material is completely substitutable with the recycled material produced by the recycling sector. The basic input-output structure of the model is shown in Figure 1.

The industrial sector inputs the material resource and the products of itself while the agricultural sector inputs industrial products. Both sectors produce their products and waste matter jointly. The recycling sector inputs industrial products and pooled waste matter and produces the recycled material. The technological relationship between input and output for all sectors is assumed to be expressed by fixed coefficients. a_{ij}'s (> 0) $(i = k, r : j = k, c, r)$ mean the

amount of products for the i's sector required to produce one unit of the primary products for j's sector. w_i's (> 0) $(i = k, c, h)$ mean the amount of waste matter jointly produced by one unit of production and consumption for i's subjects. H shows the amount of consumption for agricultural products and θH shows that for industrial products. $\theta (> 0)$ is assumed to be given previously. We assume that only one sort of waste matter is produced by each economic subject. Furthermore, we assume the waste matter is recyclable and the amount of the matter discharged into incinerators or landfill facilities does not explicitly appear in this model. Theoretically speaking, this type of matter is the residual of the inputted virgin material subtracted by the reproduced material by the recycling sector. In the stationary state, the virgin material is at the same time the discharged matter not being able to be recycled, for if completely recyclable, there is no use to input the virgin material.

Here we have to decide how to measure the amount of waste matter. Forms of waste matter for each economic subject are usually different. In the recycling process, however, the amount of matter contained in the waste is exclusively significant. Therefore it is convenient and rational to measure the waste matter in the amount of contained matter that can be reproduced as a material resource. This enables us to define the input coefficient of the recycling process as a_{wr} (> 0) that is the amount of waste matter required to reproduce one unit of the material resource.

3. Feasibility conditions

First we have to show the feasibility conditions that ensure a sustainable and steady state within the economy. Since this is a narrow sustainability condition, we are only required to check the balance of supply and demand for each factor. Let x with a suffix for a subject be the amount of product for each sector. The balance can then be expressed as follows.

$$x_k \geq a_{kk}x_k + a_{kc}x_c + a_{kr}x_r + \theta H \tag{1}$$

$$x_c \geq H \tag{2}$$

$$N + x_r \geq a_{rk}x_k \tag{3}$$

$$w_k x_k + w_c x_c + w_h H \geq a_{wr}x_r \tag{4}$$

$$x_k, x_c, x_r \geq 0 \tag{5}$$

where (1)~(4) express the balance of the industrial product, the agricultural product, the material resource and the waste matter respectively. The symbol N represents the inflow of the

virgin material resource. These equations show that the demand (right-hand side) should not be strictly larger than the supply (left-hand side). (5) is the non negativity condition for each variable.

The first feasibility condition is quite clear and simple, and is as follows.

$$1 - a_{kk} > 0 \tag{6}$$

Since this is the fundamental feasibility condition, we assume this condition is satisfied throughout this paper. If the stationary inflow of the virgin material resource can be sufficiently large, this is the unique condition of the feasibility. In other words, if this condition is satisfied under sufficiently large N, a certain amount of consumption $H > 0$ can be realized without any additional conditions. There is then no point in activating the recycling sector. This situation is ignored in our analysis.

We now have to investigate the feasibility condition with the recycling sector. For simplicity, we assume temporarily that the production level of the agricultural sector is equal to the consumption H. This assumption, though, is not essential and does not affect the results of this section.

The feasibility condition with recycling is a technological condition that can ensure the social net production with positive activity of the recycling sector. The possible net production (V) can be expressed by the following equation.

$$V \equiv x_k - a_{kk}x_k - a_{kr}x_r$$

This simply shows the net production of the industrial product. Moreover, if V is strictly positive $(V > 0)$, then we can produce a certain amount of the agricultural product. However, $V > 0$ cannot be induced by (6) because of the dependence upon x_r, which is given by two factors N and $a_{rk}x_k$ in (3). Since we have assumed (6), using (3) we have the following equation.

$$V \leq \frac{1 - a_{kk}}{a_{rk}} N + \frac{1 - a_{kk} - a_{kr}a_{rk}}{a_{rk}} x_r$$

The first term on the right-hand side shows the maximum net production induced by the virgin material resource and the second term shows the maximum net production induced by the activity of the recycling sector. This clearly shows that the numerator of the coefficient of x_r has to be positive for recycling activities to be effective and reduce virgin material inputs. Therefore, the feasibility condition is expressed as follows.
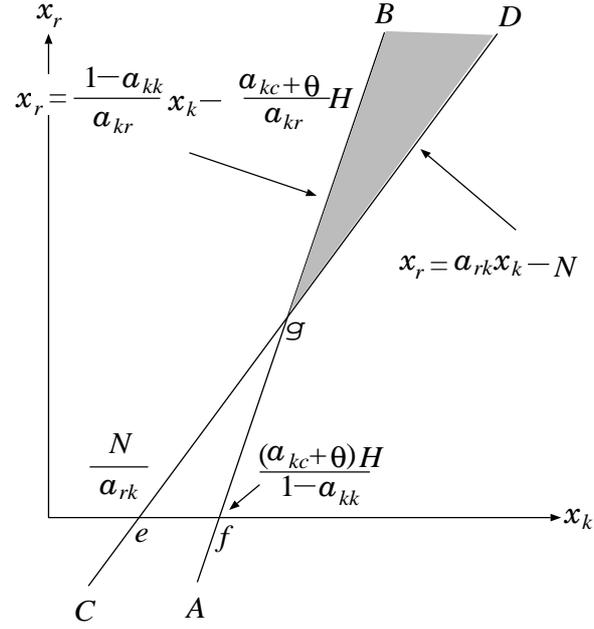
$$1 - a_{kk} - a_{kr}a_{rk} > 0 \tag{7}$$

Figure 2: Feasibility of Recycling

Equivalently, this condition can be obtained by investigating the domains that satisfy (1), (3) and (5). If there exists a domain that satisfies those inequalities, it can be expressed as Figure 2. The domain that satisfies (1) is the right-hand side of the line AB and that for (3) is the left-hand side of the line CD. In this figure, the insufficiency of the virgin material resource is expressed by the fact that the point e is located to the left-hand side of the point f. In other words, if the situation is different then we inevitably have points (x_r, x_k)'s with $x_r = 0$ and $x_k > 0$ which satisfy those inequalities, that is, the positive activities of the recycling sector are not required to produce necessary consumption goods in this economic system.

If the slopes of the two lines are given as shown in the figure, there appears the domain BgD that satisfies those inequalities simultaneously and x_r is strictly positive. Therefore the feasible condition of recycling is,

$$a_{rk} < \frac{1 - a_{kk}}{a_{kr}}.$$

This can be transformed into (7).

Clearly, if the condition (7) is satisfied, condition (6) is automatically satisfied. The converse is not true. a_{kk} is the amount of industrial product required to produce one unit itself in the industrial sector. On the other hand, $a_{kr}a_{rk}$ is the amount of industrial product required to produce one unit of itself mediated by the recycled resource. This is a recursive process as the
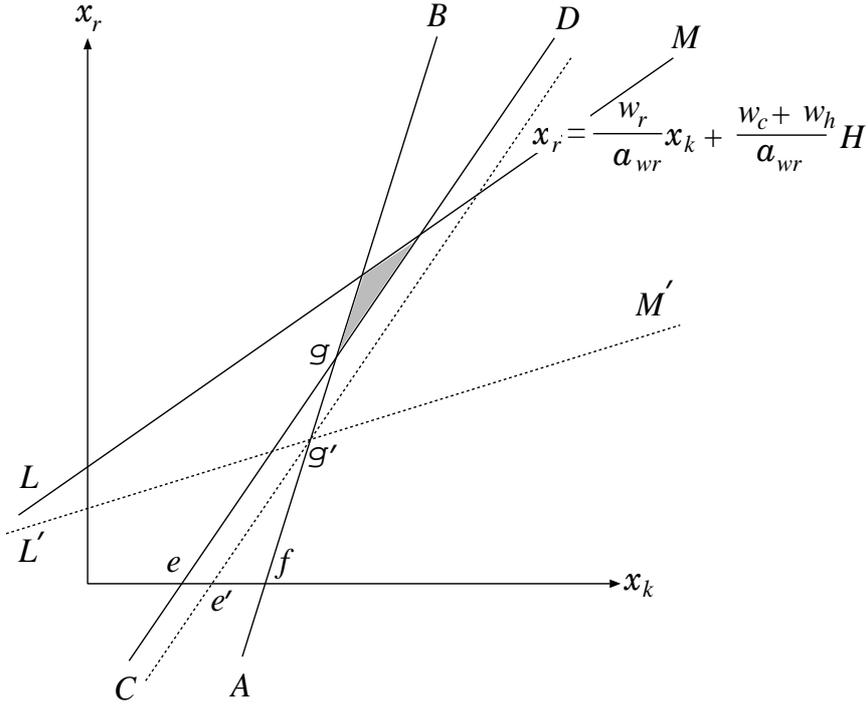
Figure 3: Upper limit of Recycling

recycled material is required to produce the industrial product (a_{rk}), which is also required to produce the recycled material (a_{kr}).

We must pay attention to the fact that condition (7) is not simply a technological condition for the recycling sector, but it express the technology for a social process and is dependent on the recycling sector, industrial sector and their interactions.

If condition (7) is satisfied, it is always worth while to activate the recycling sector, because we can reduce the input of the virgin material resource by this recycling activity. This reduction can be expressed by the shift of the point e to the left-hand side. This shift does not make the domain BgD disappear. In other words, some value (x_r, x_k) that satisfy both (1) and (3) can be found for any shift. However, this shift is limited by the supply of waste matter. This limit is given by the inequality (4). Figure 3 shows an example. The points that satisfy (4) is the domain under the line LM. In this example, the economic system can reduce the virgin material input N as long as the shaded area in Figure 3, where points can satisfy inequalities (1), (3) and (4) simultaneously, exists.

Conversely, if the supply of the recyclable waste matter is insufficient as expressed by the line $L'M'$, then the input of the virgin material has to be increased. This is expressed by the shift

of the line CD toward the right-hand side and new crossing points at e' and g'.

The above consideration suggests that if the coefficients w's are sufficiently large the virgin material resource seems to be reduced until $N = 0$ i.e. zero virgin material input. This is, however, impossible as discussed in the introduction. Therefore, there must be an upper limit of the supply of the waste matter before N reaches zero. The conditions that give this upper limit must be the material dissipative conditions. Let us examine the conditions in more detail in the next section.

4. The material dissipative conditions

Our objective in this section is to derive the material dissipative conditions that are expressed by the interrelations among coefficients implemented in the model.

First we have to note that the impossibility of sustaining $N = 0$ means that the minimum input of the virgin material resource is positive under (1) $\sim$ (5). This type of optimization is not familiar when constructing an economic model. This is due to the fact that the material dissipative conditions are necessary technological conditions for the social economic system. They are not economic conditions. It means that even if we have the best coordination of the economic system, we cannot escape from the dissipation of the material resource.

Let us begin with drawing the conditions of (1) $\sim$ (5). Figure 4 shows a possible situation. It may, at a first glance, look rather complex. Thus let us relate a portion on the figure to each equation. The simplest one is for (2). The domain that satisfies the inequality is the side of the area divided by the plane RHD. The domain that satisfies (1) is the right-hand side of the area divided by the plane that includes LKC. The domain that satisfies (4) is the lower area divided by RMP. Finally, for (3), first we have to consider a mobile plain that consists of the points $SGIJ$ which satisfy (3) with an equality symbol. As N becomes smaller, the plane approaches the origin O. Therefore the configuration that attains the smallest input of the virgin material is given at the final point where, as the plane shifts to the left-hand, the plane contacts with the domain in which (1), (2), (4) and (5) are all satisfied.

Based on this figure, let us introduce the conditions where the matter inevitably dissipates into unavailable forms in economic activities.

The relatively basic condition is related to (3) and (4). This states that the slope of the line RF must be smaller than that of the line GS. If this condition is not satisfied, the point A that connect the plane $SGIJ$ with the feasible domain does not appear and we can necessarily make N zero. This corresponds to the situation in Figure 3 where the slope of LM should be smaller
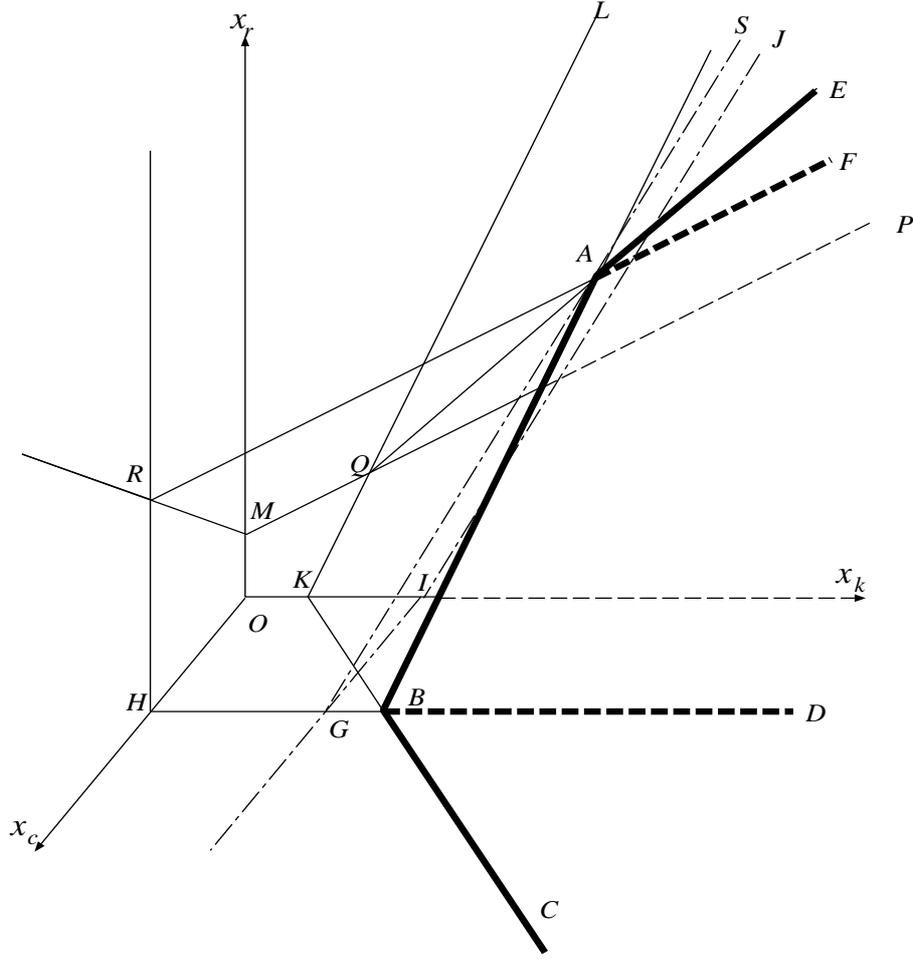
Figure 4: Feasible Area

than the line eg. If this condition is satisfied, the slope of RF also become smaller than that of BA. This is because the slope BA should be larger than GS from the condition (7). Since the slope of RF is w_k/a_{mr} and that of GS is a_{rk}, we finally have,

$$\frac{w_k}{a_{wr}} < a_{rk}.$$

This conditions can be transformed into the following equations.

$$a_{wr}a_{rk} - w_k > 0. \tag{8}$$

Since this condition has a simple form, it is not so difficult to catch its meanings. The first term of the left-hand side shows the amount of waste matter indirectly required through the recycling sector to produce one unit of industrial product. Therefore the condition shows that the amount of waste matter, which is indirectly required to produce one unit of industrial product, must be

larger than the amount of the waste matter which is jointly produced by one unit of industrial production. In other words, there should not be complete recycling between industrial activities and recycling activities. However, we must pay attention to the fact that this does not mean the rejection of the complete recycling for the whole system, for the matter inputted into the industrial sector is not transfered completely into waste matter and a certain amount of it is transfered into industrial products. This depends upon the assumption $w_c, w_h > 0$.

The next condition is related to the comparison between the slope QE and that of GS. The line QE is formed by the intersection of the plane LKC that shows the demand and supply balance for the industrial products and the plane RMP for the balance of the waste matter. GS is the line that constructs the plane which is related to the balance between the demand and supply of the material resource. The slope of the line QE must be smaller than that of the line GS. Because if this is not satisfied, it becomes possible to be $N \leq 0$.

It is then necessary to compare the slope of QE with the slope of GS on the same plane. Since the line IG is parallel with the x_c axis, we can compare the two slopes by projecting two lines on the plane $O - x_r x_k$. The slope of GS is directly given by the projection on the plane $O - x_r x_k$ and that is a_{rk}. The slope of QE is given by the relation between x_r and x_k on the line generated by the intersection of the two planes. This is done by erasing x_c from the two equations given by changing the inequality symbols for (1) and (4) into equality symbols. The condition that the slope of this process is smaller than a_{rk} can be expressed as follows.

$$\frac{w_c(1 - a_{kk}) + w_k a_{kc}}{a_{kr} w_c + a_{wr} a_{kc}} < a_{rk}$$

where the left-hand side of the equation is the slope of QE. This can be simply transformed into the following equation.

$$\frac{a_{wr} a_{rk} - w_k}{1 - a_{kk} - a_{kr} a_{rk}} > \frac{w_c}{a_{kc}} \tag{9}$$

We can interpret this equation as follows. First, the denominator of the left-hand side is the same as the left-hand side of the feasible condition (7), that is, the net production from producing one unit of the industrial product. The numerator is the same as the left-hand side of (8), that is, the amount of material resource either embodied in one unit of the industrial product or dissipated in the industrial production process. Therefore the left-hand side as a whole show the amount of the material resource embodied or dissipated and not included in the waste matter for one unit of net-production in the industrial sector. On the other hand, the right-hand side shows the amount of the waste matter, strictly speaking the material resource embodied in the waste matter, jointly produced in the agricultural sector for one unit input of industrial products.

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Therefore the condition (9) shows that the amount of the material resource required to produce the agricultural product with the industrial product is larger than the amount of the material resource embodied in the waste matter jointly produced in the agricultural sector. In other words, the material resource is not completely embodied in the waste matter. Of course, since a part of the material resource is embodied in the agricultural product e.g. as its container, the material dissipative conditions are not finally expressed by this equation.

It is worth mentioning the relationship between this condition and a previously derived condition (8). If the condition (9) is satisfied, this shows not only the dissipation of the material resource in the agricultural production process but also the dissipation in the industrial production process. This can be shown in the fact that if (9) holds, the left-hand side must be positive and also the denominator is positive from (7), therefore the numerator also has to be positive. Thus if (9) holds, (8) also holds. This relationship between the conditions is due to the fact that since $w_c > 0$, the complete recycling between the industrial sector and the recycling sector are automatically rejected depending upon only (9).

If the feasible condition (7) and the material dissipative condition (9) are simultaneously satisfied, the solution of this resource minimizing problem is given by the point like A in Figure 4. It is not sure, however, that N becomes positive only with these conditions. Therefore let us derive the final conditions that make N positive. This is not difficult. By changing inequality symbols of (1) $\sim$ (4) into equality symbols we can erase x_k, x_r, x_c and get N as follows.

$$N = \frac{(a_{wr}a_{rk} - w_k)(\theta + a_{kc}) - (1 - a_{kk} - a_{kr}a_{rk})(w_h + w_c)}{(1 - a_{kk})a_{wr} - w_k a_{kr}}$$

We have to check the positivity condition of the right-hand side of the above equation. The denominator becomes positive because of (7) and (8). Thus we have to check only the numerator. The positive condition of the numerator is expressed as follows.

$$\frac{a_{wr}a_{rk} - w_k}{1 - a_{kk} - a_{kr}a_{rk}} > \frac{w_c + w_h}{a_{kc+\theta}} \tag{10}$$

The meanings of this condition may be easily imagined from previous remarks. This condition shows that there should be material dissipation not only in the agricultural sector through the industrial sector but also in the process of consumption by households. If this equation holds with an equality symbol, this means that complete recycling is achieved through all processes. This can be understood by $N = 0$ in this case. (10) means that the material resource is dissipated in one or more processes.

We should note that even if (10) holds, it does not directly mean that (9) holds. However, when (10) is satisfied, even if (9) does not hold, the minimum N has to be positive. Though

(9) seems not to be essential, this view is superficial. (9) has independent meanings. We will investigate this problem in Section 7.

The relationship among the conditions (7), (9) and (10) may be rather difficult to understand. Therefore, I will propose another model which can express the meanings of these conditions directly and help to understand these conditions in Section 6.

5. Positivity for the value of the waste

In this section we show that both the values of the material resource and the waste matter become positive under the situation that both the feasible condition and the material dissipative conditions are satisfied. The values are given by a dual system which is consistent with the primary system that has been discussed previous sections. Although these values have no direct relationship with market prices, it is worth investigating these value. The reason is that these values can indicate the direction towards which we can reduce the virgin material resource by changing production technology or household baskets of consumption. If we can reduce the usage of relatively high value products, this reduces the use of virgin material.

First let me show the dual problem. The values of products, waste matter and the material resource are denoted by p with corresponding suffixes.

$$Max. \quad (p_k\theta + p_c - p_w w_k)H$$

s.t.

$$p_k + p_w w_k \leq p_k a_{kk} + p_r a_{rk} \tag{11}$$

$$inevitinevitablyablyp_c + p_w w_c \leq p_k a_{kc} \tag{12}$$

$$p_r \leq p_k a_{kr} + p_w a_{wr} \tag{13}$$

$$p_r \leq 1 \tag{14}$$

$$p_k, \ p_c, \ p_r, \ p_w \geq 0 \tag{15}$$

The objective function to be maximized expresses the total cost of the consumption basket. When the value of waste matter is possitive ($P_w > 0$), $p_w w_k$ in the objective function means income earned by supplying waste matter for recycling, thus the sign of this term is negative. The first constraint equation (11) shows that the total value of one unit of the product and the jointly produced waste matter have to be less than or equal to the total value of input materials for the corresponding unit of production. (12) and (13) are similar conditions for the

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agricultural sector and the recycling sector respectively. (14) seems to be strange. It shows the value produced from one unit of input of the virgin material. This ensures the consistency with the primary problem.

We can confirm that (14) has to be satisfied with an equality symbol under the feasibility condition and the material dissipative conditions. First, owing to the duality theorem of linear programming problems, the sectors with positive output in the original problem can completely preserve the total value of input factors in the products (See Dorfman et al. (1961) and Gale (1960)). We have already shown that the minimum material input N is necessarily positive under the feasibility condition and the material dissipative conditions. Therefore the equation has to be satisfied with an equality symbol and the price of the material resource is 1. In other words, we can use the material resource as the standard product for value.

Thus let us rewrite (11) $\sim$ (13) with $p_r = 1$ as follows.

$$p_k + p_w w_k \leq p_k a_{kk} + a_{rk} \tag{16}$$

$$p_c + p_w w_c \leq p_k a_{kc} \tag{17}$$

$$1 \leq p_k a_{kr} + p_w a_{wr} \tag{18}$$

Now we can show the domain that satisfies those three constraint equations. If both the feasible condition (7) and the material dissipative condition (9) are satisfied then we have such a domain as enclosed by $ABCD$ described in Figure 5. First, the domain below the plane sustained by $GHIJ$ satisfies the condition (16). The domain above the plane sustained by $KLMN$ satisfies (18). The domain beyond the plane sustained by EOF satisfies (17). Moreover if the value of the objective function is given by V, then $V = (p_k \theta + p_c - p_w w_k)H$ expresses a plane which is shown by $PQRS$ in Figure 5. Since the increase in V corresponds to the shift of the plane toward this side, the point that maximizes the value of the objective function has to be A. At this point, all values are strictly positive. Therefore the value of the waste is positive.

Moreover, since owing to the condition (10) the minimum input of N is positive, the positivity of the objective function V in the dual problem is ensured by the duality theorem.

However, if (7) is not satisfied then the point I is inverted by the point M and the feasible domain spreads across the bottom plane. This means $p_w = 0$ immediately. On the other hand, if (8) is not satisfied then the point H is inverted by the point L and the feasible domain disappears. Furthermore, if (9) is not satisfied then the slope of OE becomes smaller than that of OD. This also makes the feasible domain disappear. It is worth mentioning the last problem.
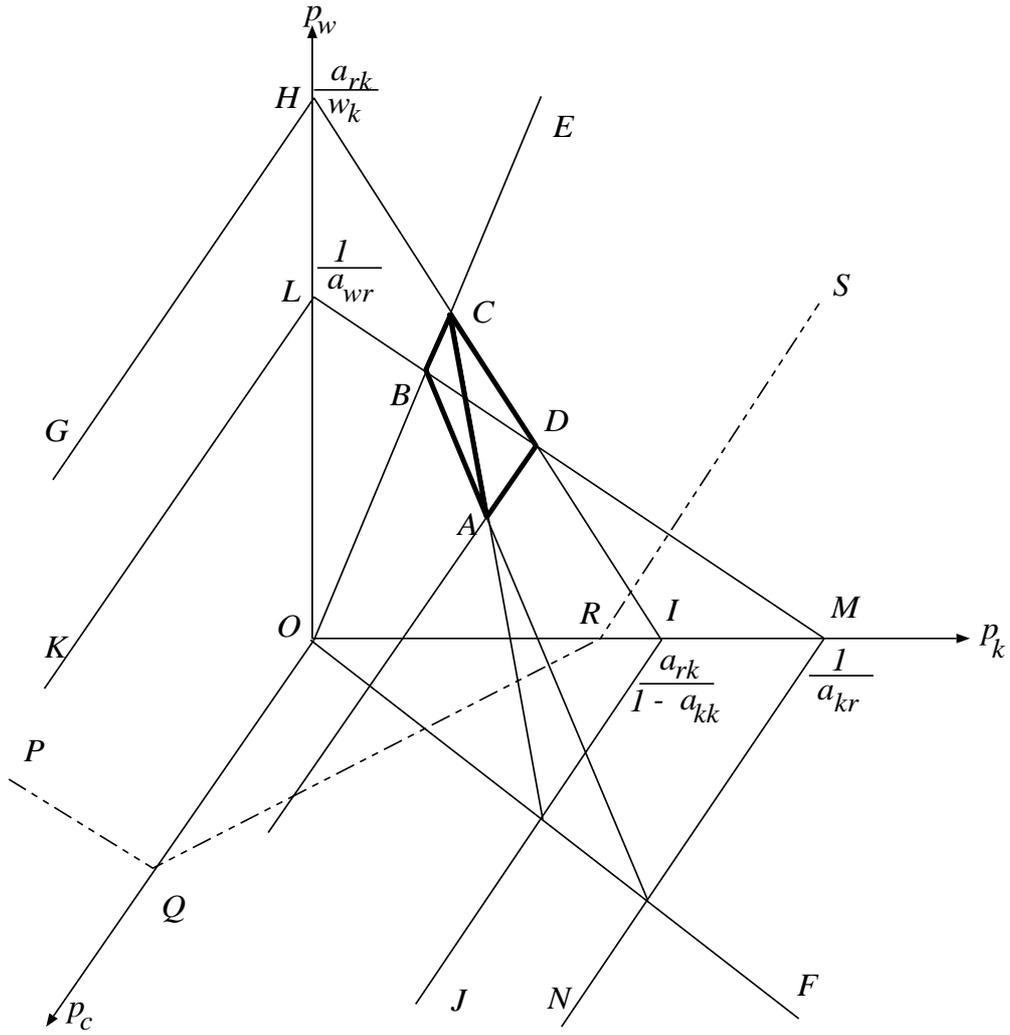
Figure 5: Feasible area in the dual problem

When the feasible domain disappears without (9) it seems to be asymmetrical to the primary problem. This is due to the fact that the feasible domain is constructed only by the conditions for the production processes, i.e., the industrial sector, the agricultural sector and the recycling sector. On the other hand, (9) also has the same feature, that is, it rejects complete recycling only through these sectors and it is not related to the dissipation in the consumption processes.

To conclude the above discussions, we have seen that the values of both the waste material and the material resource are positive under the feasible condition and the material dissipative condition (9).

6. The material transferability system

The material dissipative conditions appear rather complex in Section 4. We have another way to introduce the conditions which explain them more directly and clearly.

We have two types of products, industrial and agricultural, that possibly contain the material resource, but we have so far no exact information about how much each of them contains the material resource. On the other hand, the waste material is directly measured by the content of the material resource. Now let λ_k and λ_c be the content of the material resource in one unit of the industrial product and the agricultural product respectively. These λ's are not any kind of economic values but the terms been physically measurable.

Then let δ's with suffixes for each sector be the rates of preservation of material resource to products and waste for one unit input of material resource in the processes of production. These are also physical terms that have direct measurability. Therefore it shows that the material resource dissipates into the impossible form for recycling by the rate of $1 - \delta$. This δ is also applied to the process of consumption in the household.

With these notations, we can describe the transfer process of the material resource for each sector as follows.

$$\delta_k(a_{rk} + \lambda_k a_{kk}) = \lambda_k + w_k \tag{19}$$

$$\delta_c \lambda_k a_{kc} = \lambda_c + w_c \tag{20}$$

$$\delta_r(a_{wr} + \lambda_k a_{kr}) = 1 \tag{21}$$

$$\delta_h(\lambda_c + \lambda_k \theta) = w_h \tag{22}$$

$$0 \leq \delta_i \leq 1 (i = k, c, r, h) \tag{23}$$

These are the material transferability system. (19), (20), (21) and (22) are the condition equations of material transfer for the industrial sector, the agricultural sector, the recycling sector and the household. (23) shows that all δ's have to be nonnegative and less than one. It is plausible to think that these equations determine δ's. This is because λ's and w's can be measured directly and the others are technological coefficients or consumption coefficients, all of which were previously determined.

Applying (23), we can rewrite (19) $\sim$ (22) as follows.

$$a_{rk} + \lambda_k a_{kk} \geq \lambda_k + w_k \tag{24}$$

$$\lambda_k a_{kc} \geq \lambda_c + w_c \tag{25}$$

$$a_{wr} + \lambda_k a_{kr} \geq 1 \qquad (26)$$

$$\lambda_c + \lambda_k \theta \geq w_h \qquad (27)$$

Let us introduce the material dissipative condition by those equations. First by (24) we have,

$$a_{rk} \geq \lambda_k (1 - a_{kk}) + w_k.$$

(26) multiplied by a_{rk} is,

$$a_{wr} a_{rk} + \lambda_k a_{kr} a_{rk} \geq a_{rk}.$$

By these two equations, we have,

$$a_{wr} a_{rk} + \lambda_k a_{kr} a_{rk} \geq \lambda_k (1 - a_{kk}) + w_k$$

Therefore,

$$a_{wr} a_{rk} \geq \lambda_k (1 - a_{kk} - a_{kr} a_{rk}) + w_k \qquad (28)$$

Since $1 - a_{kk} - a_{kr} a_{rk} > 0$ by (7) and $\lambda_k \geq 0$, we have,

$$a_{wr} a_{rk} \geq w_k,$$

where the symbol of equality holds if and only if $\delta_k = \delta_r = 1$ and $\lambda_k = 0$. However, if $\lambda_k = 0$ then $w_c = w_h = 0$ by (25) and (27), and this contradicts our assumptions. Thus (8) must hold under the material transferability system.

Next, by transforming (25) it follows that

$$\frac{a_{kc}}{w_c} \geq \frac{\lambda_c}{w_c \lambda_k} + \frac{1}{\lambda_k}.$$

By transforming (28), we also have,

$$\frac{1}{\lambda_k} \geq \frac{1 - a_{kk} - a_{kr} a_{rk}}{a_{wr} a_{rk} - w_k}. \qquad (29)$$

Thus,

$$\frac{a_{kc}}{w_c} \geq \frac{\lambda_c}{w_c \lambda_k} + \frac{1 - a_{kk} - a_{kr} a_{rk}}{a_{wr} a_{rk} - w_k}. \qquad (30)$$

Further, since $\lambda_k > 0$ and $\lambda_c \geq 0$, we have,

$$\frac{a_{kc}}{w_c} \geq \frac{1 - a_{kk} - a_{kr} a_{rk}}{a_{wr} a_{rk} - w_k}.$$

By inverting both sides of the above equation, we have,

$$\frac{w_c}{a_{kc}} \leq \frac{a_{wr} a_{rk} - w_k}{1 - a_{kk} - a_{kr} a_{rk}}.$$

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In this case, considering the fact that we use (28), the symbol of equality holds if and only if $\delta_k = \delta_r = \delta_c = 1$ and $\lambda_c = 0$. The important point is that $\lambda_c = 0$ does not contradict our assumption. This is different from the case for λ_k. Though $\lambda_c = 0$ means that the agricultural products do not contain the material resource, it is possible that $w_h > 0$ because of $\lambda_k \theta H > 0$, that is, the consumption of the industrial product produce the waste that contain the material resource. Therefore, the material dissipative condition (9) is equivalent to the situation that at least one δ among $\delta_k, \delta_r, \delta_c$ is less than one or $\lambda_c > 0$ in the material transferability system.

Let us move on to the final condition. By (30), we have,

$$a_{kc} - \frac{(1 - a_{kk} - a_{kr}a_{rk})w_c}{a_{wr}a_{rk} - w_k} \geq \frac{\lambda_c}{\lambda_k}. \tag{31}$$

By transforming (27), it follows that

$$\frac{\lambda_c}{\lambda_k} + \theta \geq \frac{w_h}{\lambda_k}.$$

Applying (29) and (31) to this equation, we have,

$$a_{kc} + \theta - \frac{(1 - a_{kk} - a_{kr}a_{rk})w_c}{a_{wr}a_{rk} - w_k} \geq \frac{\lambda_c}{\lambda_k} + \theta \geq \frac{(1 - a_{kk} - a_{kr}a_{rk})w_h}{a_{wr}a_{rk} - w_k}.$$

Comparing the right-hand side and the left-hand side, we finally have,

$$\frac{w_c + w_h}{a_{kc} + \theta} \leq \frac{a_{wr}a_{rk} - w_k}{1 - a_{kk} - a_{kr}a_{rk}}.$$

The equality symbol holds if and only if $\delta_k = \delta_r = \delta_c = \delta_h = 1$. This means that the material dissipative condition (10) is equivalent to the situation that at least one δ in $\delta_k, \delta_r, \delta_c, \delta_h$ is less than one in the material transferability system.

We have now introduced all of the material dissipative conditions. Moreover, it is closely related to the actual processes of the dissipation of the material resource. We can conclude that the material transferability system can play an effective role in analyzing the material dissipative conditions.

7. Refinement of the material dissipative conditions

Now we are in the position to be able to investigate the interrelationship between (9) and (10). Since (8) is necessarily satisfied under our presumptions of the model, we have to exclude it from the material dissipative conditions.

First we can confirm that even if (9) is satisfied (10) is not always satisfied. Conversely, even if (10) is satisfied (9) is not always satisfied. Therefore it seems that these two conditions are mutually independent. This is, however, not the complete truth.

(9) expresses the impossibility of complete recycling through the industrial sector, the agricultural sector and the recycling sector. However, it does not explicitly take into account the dissipation in the consumption processes by the household. Therefore, if we differentiate production processes from consumption processes and give some specific meanings to the impossibility of complete recycling in social production processes, (9) is independently effective. If it is not so, the condition is effective only in the case of $H = 0$. Because the condition cannot capture the dissipation in the consumption processes.

On the other hand, (10) cannot include (9). This is because (10) rejects complete recycling in the social process inclusive of the consumption process. Take the situation where $\delta_k = \delta_r = \delta_c = 1$ and $\delta_h < 1$, that is, the material dissipation occurs only in the consumption process, and $\lambda_c = 0$, that is, the agricultural goods do not contain the material resources. In this case, (10) holds but (9) does not hold. The dissipation occurs in consuming the industrial products.

Therefore if we interpret the material dissipative conditions as the material dissipation that has to occur somewhere in the social economic process and there is no distinction between production process and consumption process, (10) is only essential and (9) is not. In the other cases, we have to admit the independent role of (9).

8. Concluding remarks

We have shown that the material dissipative conditions could be introduced by the material transferability system. Furthermore, our inquiries have brought us a diversity of meanings of the material dissipative conditions. In other words, we have to make it clear in what processes the material dissipation inevitably occurs, e.g. a simple production process, or social production processes, or all economic processes inclusive of consumption process. Depending on the difference among meanings of the material dissipative conditions, we can define slightly different conditions for dissipation of matter. This question will have to be investigated further.

On the other hand, the material dissipative conditions introduced in this paper may have some specific features depending upon our specification of the model and our assumptions. Thus, the conditions have both general features and specific features. Efforts to discriminate the latter from the former will be required.

Acknowledgments

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