Lasing without inversion

2000 J. Opt. B: Quantum Semiclass. Opt. 2 R7

(http://iopscience.iop.org/1464-4266/2/3/201)

J. Opt. B: Quantum Semiclass. Opt. **2** (2000) R7–R24. Printed in the UK PII: S1464-4266(00)10684-6

REVIEW ARTICLE

Lasing without inversion

J Mompart and R Corbalán

Departament de Física, Universitat Autonoma de Barcelona, E-08193 Bellaterra, Spain

Received 6 January 2000, in final form 20 March 2000

Abstract. This review paper is devoted to amplification and lasing without population inversion involving atomic transitions in gas media. We start by discussing the main motivation in inversionless lasing research, namely, the generation of short-wavelength laser light. Then, we review the basic physics of inversionless lasing in two-level and, eventually, in three- and multilevel atomic configurations. Finally, we summarize the current state of the art of LWI experiments and indicate the main difficulties with respect to short-wavelength laser generation.

Keywords: Atomic coherence effects, lasing without population inversion

1. Introduction

Atomic coherence effects such as coherent population trapping (CPT), electromagnetically induced transparency (EIT), high index of refraction with vanishing absorption, amplification and lasing without population inversion (AWI and LWI) and population inversion without amplification or lasing (IWA and IWL) have been intensively investigated over recent years due to their multiple potential applications, ranging from laser cooling to isotope separation and from ultrahigh-sensitive magnetometers to the generation of giant pulses of laser light (see, for instance, [1–3] and references therein). Among all these phenomena, AWI and LWI† have received considerable attention for their potential to obtain laser light in spectral domains, e.g. the x-ray range, where conventional methods based on population inversion are not available or are difficult to implement. Although, in general, AWI and LWI are well understood phenomena, there are a few cases in which the lack of understanding of the basic physical processes is hidden in explanations of the type 'LWI due to quantum interference'. This paper attempts (i) to review LWI in atomic gas media while clarifying basic theoretical aspects on LWI, and (ii) to summarize the current state of LWI.

In section 2, the key points relating to the motivation of LWI research, namely the generation of short-wavelength laser light, are discussed. Thus, it is shown that the well known population inversion condition for lasing results in the fact that the pumping power required for lasing scales at least with the fourth power of the transition frequency which, in turn, explains the present difficulty of generating continuous-wave x-ray laser light in conventional inverted systems. Fortunately, as will be shown in this paper, the population inversion condition for lasing can be strongly violated by breaking the symmetry between stimulated emission and absorption which opens new possibilities in order to reach the interesting and challenging goal of continuous-wave x-ray lasing.

The basic physics of LWI is reviewed in section 3. Section 3.1 discusses the first proposals and experiments involving two-level systems. We first consider the possibility of LWI due to the asymmetry between the recoil-induced frequency shift for stimulated emission and absorption, and then we concentrate on coherently driven two-level systems. Although two-level systems do not offer good perspectives on the progress to short-wavelength lasing, they remain interesting in the sense that simple theoretical explanations, such as the dressed-atom approach, allow us to understand the origin of gain. Section 3.2 focuses on the most interesting three- and multilevel systems. Thus, the possibility of LWI involving transitions to autoionizing (AI) states is explored. Fano-type interferences are introduced in order to account for the asymmetry between stimulated emission and absorption. Several schemes are reviewed, demonstrating that, up to now, there is no experimental evidence of LWI in these kinds of systems. Finally, we focus on coherently driven three- and multilevel schemes. LWI is discussed in these systems in terms of the appearence or not of an inversion in a hidden basis. Three different possibilities are explored: (i) inversion in the so-called CPT basis, (ii) inversion in the dressed-state basis and, finally, (iii) LWI without hidden inversion. In this last case, the origin of gain is analysed both in terms of quantum interference resulting from atomic coherence between the dressed states and also by using the quantum jump technique. The last technique allows us on the one hand to elucidate the particular physical processes responsible for gain, and on the other hand to relate the asymmetry between stimulated emission and absorption to the quantum Zeno effect. In addition, generalized Einstein B coefficients for one- and two-photon gain/loss processes are introduced which yield simple quantitative expressions for the asymmetry between stimulated emission and absorption

† For early reviews on AWI and LWI see [4–7]; see also two articles in *Science* [8, 9].

in terms of the incoherent processes present in each particular scheme.

Section 4 summarizes the current state of LWI experiments, pointing out that in all these experiments the frequency of the inversionless generated laser was equal to or smaller than that of the external laser used to prepare the atoms and that, therefore, they must be considered only as proof-of-principle experiments. The main theoretical and experimental challenges raised by short-wavelength LWI are then discussed: Doppler broadening, propagation effects, and the particular relationships between rates related to spontaneous emission and incoherent pumping.

Finally, in section 5 the main conclusions of this review paper are established.

2. Motivation. Why LWI?

The combination of Bohr's atomic model with a stochastic conception of the light–matter interaction allowed Einstein at the beginning of the century to establish the existence of three basic processes in the interaction of light with matter: absorption, stimulated emission, and spontaneous emission. In the context of black-body radiation and from thermodynamical arguments, Einstein determined—in terms of the so-called Einstein A and B coefficients—the relationship between the rates of absorption and stimulated emission of light by a gas molecule possessing discrete energy levels and the spontaneous emission rate—given by the Einstein A coefficient. For a two-level system with ground level $|g\rangle$ and excited level $|e\rangle$ with unperturbed transition frequency ω probed by an electromagnetic field, the relationships between the Einstein coefficients read

$$B_{\mathrm{abs}} = B_{\mathrm{st\,em}} (\equiv B), \tag{1a}$$

$$\frac{A}{B} = \frac{\hbar\omega^3}{\pi^2 c^3}, \tag{1b}$$

with

$$A = \frac{\omega^3}{3\pi\varepsilon_0\hbar c^3}|\mu|^2, \tag{2}$$

where μ is the electric dipole moment of the two-level transition, $\hbar$ Planck's constant, ε_0 the electric permittivity, and c the speed of light in vacuum. Thus, the rates of absorption and stimulated emission of light by an atomic/molecular medium will be proportional to the corresponding Einstein B coefficient times the population $\rho_{ii} (i = g, e)$ of the initial state of the process. Then,

$$\frac{\text{absorption rate}}{\text{stimulated emission rate}} = \frac{B_{\mathrm{abs}}}{B_{\mathrm{st\,em}}} \cdot \frac{\rho_{gg}}{\rho_{ee}}. \tag{3}$$

As is shown in any standard laser textbook, from the above equations it can be deduced straightforwardly that (i) since $B_{\mathrm{abs}} = B_{\mathrm{st\,em}}$ then population inversion, i.e. $\rho_{ee} > \rho_{gg}$, is a necessary condition for light amplification and (ii) it is not possible to invert a two-level system with, for instance, resonant light since atomic excitation is always accompanied by de-excitation through the stimulated emission process. As a consequence, in order to create the required population inversion, laser systems operate on three or more level configurations (e.g., the three-level configuration of the Ruby laser or the four-level configuration of the He–Ne laser), where the inversion is created by pumping to an excited level which rapidly decays to the upper level of the lasing transition. Although more than two levels are involved, we will refer to these configurations, throughout this review paper, as *two-level schemes* since only two levels are directly coupled with coherent light.

In these schemes, in order to reach laser action the incoherent pump power P should be strong enough to create a population inversion such that the associated gain overcomes cavity losses. A simple estimation of the lower bound P_{th} for the pump power required to reach laser oscillation gives [10]

$$P_{\mathrm{th}} = \frac{\hbar\omega\Delta N_{\mathrm{th}}}{\tau}, \tag{4}$$

where $\Delta N_{\mathrm{th}} > 0$ is the population difference at the first laser threshold and τ the lifetime of the upper level of the lasing transition. This threshold population difference can be written [11] as $\Delta N_{\mathrm{th}} = 2\kappa/\hbar\omega B(\omega)$, κ being the cavity losses and $B(\omega)$ the Einstein B coefficient at frequency ω. For a broadened atomic system with normalized spectral line given by $g(\omega)$ such that $B(\omega) = Bg(\omega)$ the above expression becomes

$$P_{\mathrm{th}} = \frac{2\kappa}{g(\omega)}\frac{A}{B} = \frac{2\kappa\hbar\omega^3}{\pi^2 c^3 g(\omega)}, \tag{5}$$

where use of relation (1b) has been made. In general and due to the normalization of $g(\omega)$ one has $g(\omega)\cdot\Delta\omega \sim 1$. On the other hand, for natural broadening and using the Heisenberg principle one has $\Delta\omega \sim 1/\tau$ with $1/\tau = A$, while for Doppler broadening it is well known that $\Delta\omega$ scales with ω and does not depend on the electric dipole moment [10]. Therefore, according to equation (5) the threshold pump power scales with ω^6 and $|\mu|^2$ for natural broadening, while for Doppler broadening it scales with ω^4 and does not depend on the dipole moment [10]. All these arguments clearly show that as we increase the frequency of the laser transition it becomes harder to attain the required population inversion. Therefore, the main obstacle in the achievement of coherent oscillation in the short-wavelength domain, e.g. continuous-wave x-ray lasing, is the required pump power†.

LWI differs from conventional lasing in the fact that the reciprocity between absorption and stimulated emission is broken. In LWI the absorption of light is reduced or even cancelled, and lasing is possible with a small fraction of the atoms in the upper level (less than in the lower level) of the lasing transition. Therefore, LWI is not subject to the limitations of conventional lasing, i.e. the required incoherent pump power can be drastically reduced, and, in this sense, LWI opens a new way towards the generation of continuous-wave short-wavelength lasers.

† Note that, in general, it is not possible to state that a larger spontaneous emission rate corresponds to a larger frequency since this rate depends not only on the frequency but also on the electric dipole moment (equation (2)). Indeed, it is easy to find examples in which the spontaneous emission does not increase with the frequency. For instance: (i) compare the infrared transition $5s\,^2S_{1/2}$–$5p\,^2P_{1/2}$ at 794.8 nm with lifetime $\tau = 28.1$ ns with the blue transition $5s\,^2S_{1/2}$–$6p\,^2P_{1/2}$ at 421.6 nm with lifetime $\tau = 110$ ns of the ^{85}Rb atom; or (ii) a less specific example is the n^{-3} dependence on the principal quantum number n (for high n values) of the spontaneous decay rate of Rydberg atoms.

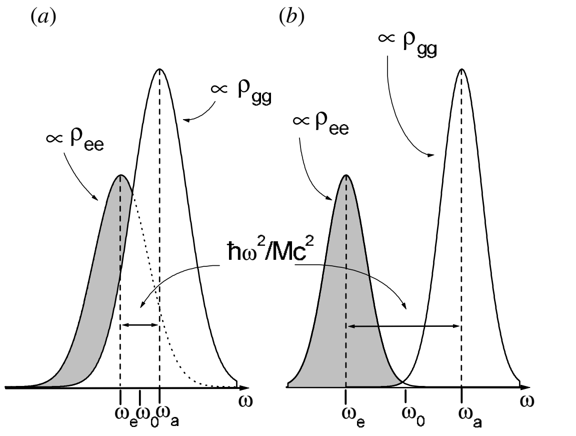

Figure 1. Frequency shift between estimulated emission ω_e and absorption ω_a from the two-level transition frequency ω_0 due to the recoil effect for (*a*) the optical domain and (*b*) the x-ray domain.

3. Basic physics of lasing without inversion

We now review the different ideas and attemps to achieve LWI in atomic transitions in gas media. We start by describing the different proposals involving two-level systems and then we consider three-level and more complex multilevel configurations†.

3.1. Two-level systems

3.1.1. Recoil-induced lasing. The first proposals of LWI in two-level systems made use of the asymmetry between the recoil-induced shift for stimulated emission and absorption processes [14, 15]. A simple energy and momentum balance shows that due to atomic recoil the difference between the resonance frequency for stimulated emission and absorption is given by $\hbar\omega^2/Mc^2$, $\hbar\omega$ being the photon energy and Mc^2 the rest mass of the atom under consideration. Due to this shift there is a frequency range in which stimulated emission overcomes absorption even if the excited population is smaller than the corresponding one of the ground level. Figure 1 illustrates qualitatively this situation for the optical and x-ray domains [7]. Note that, although in the optical domain this splitting is very small, it can be larger than the Doppler broadening in the x-ray range since the frequency splitting scales with ω^2 while Doppler broadening scales with ω‡. Indeed, a few years ago Ritsch, Walls, and co-workers [16, 17] proposed laser-cooled metastable (e.g. helium) atoms to achieve continuous-wave recoil-induced lasing in the deep ultraviolet domain.

3.1.2. Coherently driven two-level systems. The analysis of inversionless amplification of a probe field in strongly driven two-level systems was initiated by Rautian and Sobel'man [18], Mollow [19], and Haroche and Hartmann [20]. Figure 2 shows a typical gain/absorption spectrum of a probe field with frequency ω in interaction with a two-level system (figure 2(*a*)) with transition frequency ω_0 strongly driven by a coherent field with frequency ω_L and detuning $\Delta = \omega_L - \omega_0$§. The coupling between the driving field and the two-level system is characterized by the resonance Rabi frequency Ω_0. The parameter setting used in figure 2(*b*) is $\Omega_0 = 20\gamma$, $\Delta = -5\gamma$, γ being the spontaneous population decay rate from the excited to the ground state. The two resonances at $\omega = \omega_L \pm \Omega$, where $\Omega = \sqrt{\Omega_0^2 + \Delta^2}$ is the generalized Rabi frequency, are the so-called Rabi sidebands. As is clearly seen in the figure, gain appears (i) in the left Rabi sideband $\omega = \omega_L - \Omega$, and (ii) on the right side of the dispersive-like structure at line centre. As we will show next, the origin of the probe field resonances can be easily traced back by using the dressed-atom picture (see [22] and references therein).

Consider the two-level system of figure 3(*a*) in interaction with the quantized driving field $|N\rangle$ of figure 3(*b*), N being the driving field photon number. The coupling of the driving field with the two-level system gives rise to the dressed-atom energy spectrum shown in figure 3(*c*) where $|i, N + j\rangle$ with $i = 1, 2$ and $j = 0, \pm 1, \ldots$ are the well known dressed states (see, for instance, [22, 23]). States $|i, N + j\rangle$ are superposition states of the unperturbed states $|e\rangle|N + j - 1\rangle$ and $|g\rangle|N + j\rangle$. This spectrum consists of a series of doublets split by Ω and separated by ω_L. The asymptotic decomposition (i.e. for large detunings) of the dressed states $|i, N\rangle$ in terms of the atom-driving product states is also shown. The thickness of the curves representing the energy levels in (*c*) provides a qualitative description of the relative population of the corresponding dressed state. The gain/absorption lineshape of figure 2(*b*) can be explained in terms of transitions between the dressed states stimulated by the probe field. In figure 2(*b*) we choose $\Delta < 0$ and, therefore, we will pay attention to transitions occurring between dressed states located in the left-hand side of figure 3(*c*). In the dressed-atom picture, the absorbing peak in the right Rabi sideband of figure 2(*b*) is explained as a probe resonance between the dressed states of two neighbouring doublets separated by $\omega_L + \Omega$ (dashed up-pointing arrow) while the amplifying resonance in the left Rabi sideband of figure 2(*b*) is assigned to population inversion between the dressed-states of two neighbouring doublets separated by $\omega_L - \Omega$ (dashed down-pointing arrow). In the asymptotic limit $\Delta \to -\infty$, the absorbing Rabi resonance involves a transition between states $|g\rangle|N-1\rangle$ and $|e\rangle|N-1\rangle$, therefore it corresponds to the one-photon probe resonance slightly light-shifted by the non-resonance driving field. Note that for $|\Delta| \gg \Omega_0$, $\omega_L + \Omega \approx \omega_0$. On the other hand, the amplifying Rabi resonance corresponds, in the asymptotic limit, to a transition between states $|g\rangle|N+1\rangle$ and $|e\rangle|N-1\rangle$. This transition can be explained as the hyper-Raman process between the states $|g\rangle$ and $|e\rangle$ shown in figure 4(*a*) involving the absorption of two driving photons and the amplification of the probe field at frequency $\omega = 2\omega_L - \omega_0$ which corresponds

† Although we limit ourselves to atomic transitions in gas media, LWI has been discussed in a wide range of different physical contexts. In fact, LWI has been predicted in semiconductor media although it still remains to be observed (see [3] and references therein). Also, LWI seems to be the most promising approach to lasing using nuclear transitions [12]. And, as a last example, LWI has been also dicussed in the context of the micromaser [13].
‡ For helium atoms the recoil effect gives a splitting of a fraction of MHz in the visible, a few GHz in the ultraviolet, and a hundred or so GHz in the x-ray domain.

§ A detailed density-matrix calculation of the probe response shown in figure 2(*b*) can be found in [21].

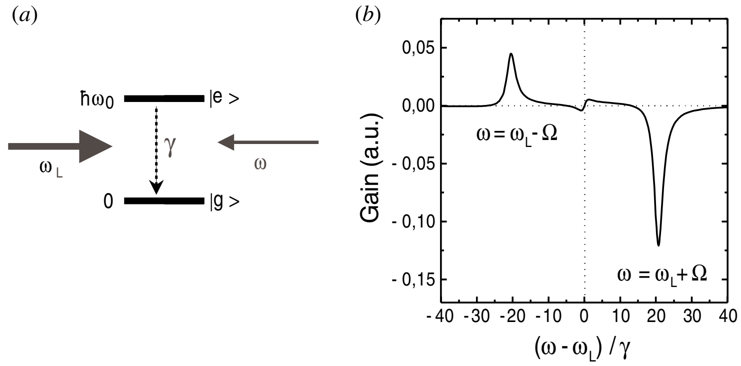

Figure 2. (a) Two-level system with transition frequency ω_0 driven by a strong field with frequency ω_L and probed by a weak field of tunable frequency ω. The coupling between the two-level system and the driving field is characterized by the resonance Rabi frequency Ω_0. γ is the spontaneous population decay rate from the excited state to the ground state. (b) Gain/absorption spectrum of the probe field in interaction with the strongly driven two-level system for the following parameter setting: $\Omega_0 = 20\gamma$ and $\Delta = \omega_L - \omega_0 = -5\gamma$. $\Omega = \sqrt{\Omega_0^2 + \Delta^2} \approx 20.6\gamma$ is the generalized Rabi frequency.

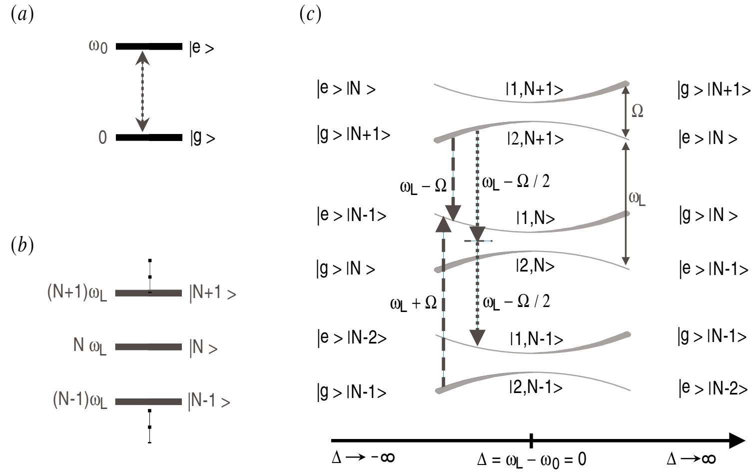

Figure 3. (a) Two-level system with transition frequency ω_0. (b) States of the quantized driving field $|N\rangle$ where N is the driving field photon number. (c) Dressed-atom picture of the probe response. $|i, N + j\rangle$ with $i = 1, 2$ and $j = 0, \pm 1, \ldots$ are the well known dressed states defined, for instance, in [22]. States $|i, N + j\rangle$ are superposition states of the unperturbed states $|e\rangle|N + j - 1\rangle$ and $|g\rangle|N + j\rangle$. As indicated in the figure, for $\Delta \to \infty$ $(-\infty)$ then $|1, N + j\rangle \to |g\rangle|N + j\rangle$ $(|e\rangle|N + j - 1\rangle)$ and $|2, N + j\rangle \to |e\rangle|N + j - 1\rangle$ $(|g\rangle|N + j\rangle)$. (Note that $\hbar = 1$).

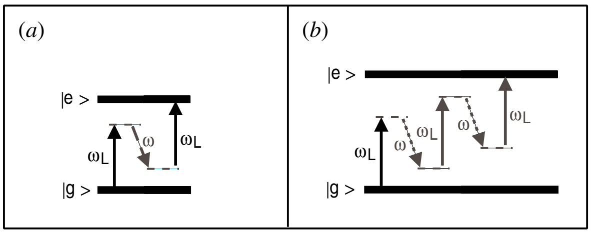

Figure 4. Asymptotic ($\Delta \to -\infty$) diagrams responsible for (a) the amplifying Rabi sideband at $\omega = \omega_L - \Omega$; and ($b$) two-photon gain at $\omega = \omega_L - \Omega/2$.

to $\omega = \omega_L - \Omega$ for $|\Delta| \gg \Omega_0$ [24]. Absorption at one of the Rabi sidebands and gain at the other one was first observed by Wu *et al* [25] while inversionless laser action in the amplifying Rabi sideband was later obtained by Khitrova *et al* [26].

Mossberg and co-workers [27] have suggested the possibility of laser emission involving two probe photons of frequency $\omega = \omega_L - \Omega/2$ (dotted arrows in figure 3(c)) coupling the inverted dressed-states of two non-consecutive doublets. In the asymptotic limit, this process couples state $|g\rangle|N+1\rangle$ to $|e\rangle|N-2\rangle$ and the corresponding hyper-Raman process between bare atomic states (figure 4(b)) involves three photons of the driving field. Two-photon LWI at $\omega_L - \Omega/2$ was observed a few years ago by Gauthier *et al* [28]†. It is worth remarking that lasing at $\omega = \omega_L - \Omega$ and at $\omega = \omega_L - \Omega/2$ occurs without inversion in the bare atom basis but with inversion in the dressed-atom basis. These are examples of the so-called lasing with 'hidden' inversion.

Finally, LWI to the right of the dispersive-like structure at $\omega \simeq \omega_L$ was first obtained by Grandclément *et al* [32] and the corresponding explanation in the dressed-atom picture was later given by Grynberg and Cohen-Tannoudji [33]. Since the dressed-atom energy spectrum has a periodicity

† A theoretical analysis of the particular properties, such as stability, photon statistics, phase diffusion and squeezing, of one- and two-photon dressed-state lasers in coherently driven two-level systems can be found in [29–31].

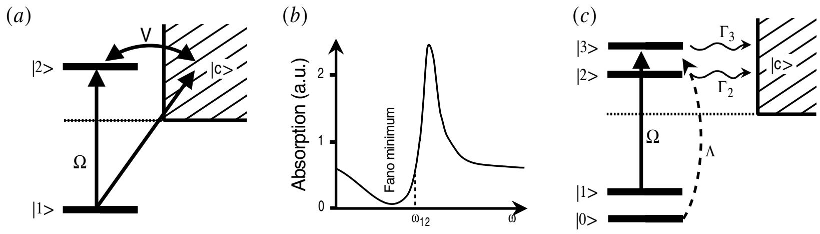

Figure 5. (a) Two-level system with the excited level embedded in a continuum probed by an electromagnetic field. (b) One of the possible Fano absorption profiles of the probe field corresponding to the configuration shown in (a). (c) Four-level system with the two excited levels decaying to the same continuum with relaxation rates Γ_i ($i = 2, 3$). Λ accounts for an incoherent pumping process.

ω_L, in this case gain cannot originate from an inversion between dressed states and it should be assigned to quantum interference of various scattering channels of the driving and probe fields [33].

Summarizing, except for the dispersive-like structure at line centre, the different probe resonances in strongly driven two-level systems can be understood straightforwardly by considering the population distribution in the dressed states. Lasers based on inversionless gain in these systems have indeed worked although at frequencies very close to that of the driving laser (differences of the order of some tens of MHz). It is important to note that, as probe and driving lasers operate on the same transition, coherently driven two-level systems do not offer good perspectives for obtaining LWI in the short-wavelength domain.

3.2. Three- and multilevel systems

3.2.1. Quantum interference through the continuum: autoionizing states. The first proposal concerning inversionless gain in multilevel systems based on the asymmetry between absorption and stimulated emission in transitions to AI states was made by Arkhipkin and Heller [34]. Consider the energy-level diagram shown in figure 5(a) which consists of a ground state $|1\rangle$ and an excited state $|2\rangle$ (the AI state) embedded in a continuum of states $|c\rangle$. The $|1\rangle$–$|2\rangle$ transition is probed by an electromagnetic field of Rabi frequency Ω. Those atoms that are initially in state $|1\rangle$ experience the well known Fano-type interference in their absorption profile since there are two quantum paths to the AI state: the direct transition $|1\rangle \to |2\rangle$ and the transition via the continuum $|1\rangle \to |c\rangle \to |2\rangle$. Thus, the coupling between the AI state and the continuum (the so-called configuration interaction and denoted by V in figure 5(a)) produces, as illustrated in figure 5(b), some Fano minimum in the absorption profile [35]†. Then, by pumping some small amount of population into the excited state $|2\rangle$ and tuning the probe field to a Fano minimum, it is possible to have net gain in the $|1\rangle \to |2\rangle$ transition even without population inversion in this transition.

Harris [36] extended the idea to the configuration shown in figure 5(c). The four-discrete-energy-level scheme consists of two ground states $|0\rangle$ and $|1\rangle$, and two excited states $|2\rangle$ and $|3\rangle$ that decay to the same continuum, e.g. through an Auger or AI process, with relaxation rates Γ_2 and Γ_3, respectively. Atoms that are initially in state $|1\rangle$ have two different quantum paths to the continuum: $|1\rangle \to |2\rangle \to |c\rangle$ and $|1\rangle \to |3\rangle \to |c\rangle$. The absorption profile of a probe field coupling the ground state $|1\rangle$ to the excited states exhibits Fano-type minima due to destructive interference between the amplitudes of these two quantum paths. Atoms which are initially in state $|2\rangle$ or $|3\rangle$ do not manifest this type of destructive interference in the stimulated emission profile, although they do display unusual properties [36]. In the Harris proposal, the additional state $|0\rangle$ was used to pump some amount of population into the excited states. As in the Arkhipkin and Heller model [34], the non-reciprocal gain–loss profiles can give rise to inversionless gain when the excited state is populated and the probe field is applied near to a Fano minimum. Different proposals of inversionless gain via AI states have been further analysed in [37–41].

Soon afterwards, Boller *et al* [42] observed a reduction in the absorption of a probe field ($\lambda_P = 337.1$ nm) applied between the ground state 5s5p $^1P_1$ and the AI state 4d5d $^1D_2$ in Sr using a slightly modified version of the original Harris proposal (figure 5(c)). In this case, the two excited states were built up by an external laser field ($\lambda_C = 570.3$ nm) applied between an intermediate state 4d5p $^1D_2$ and the AI state 4d5d $^1D_2$ that split the AI state into the two corresponding dressed states. This reduction of absorption is an example of the so-called EIT since absorption vanishes only in the presence of the external electromagnetic field‡. Nevertheless, there is still no experimental evidence of inversionless gain and lasing involving AI states.

Very recently, Zambon [44] analysed the probe response of a coherent field tuned to an AI state in the individual atomic evolution formalism, i.e. the so-called quantum trajectories [45–47], instead of using a density-matrix approach. From this analysis it has been found that to achieve gain without population inversion a strong departure from thermal equilibrium conditions is required. Thus, a strong pumping power is needed which, as has been mentioned previously, notably restricts the applicability of these schemes to the generation of short-wavelength laser light.

3.2.2. Coherently driven three- and multilevel systems. As early as 1957, Javan showed, in the context of the theory of a three-level maser, the possibility of inversionless amplification involving discrete energy levels in the so-called V-type three-level system [48]. In Javan's scheme, an intense maser field saturated the transition of one of the arms of a V

† A clear and detailed analysis of the Fano-type absorption profiles when a discrete state is coupled to a continuum can be found in [22].

‡ For a review on EIT see [43].

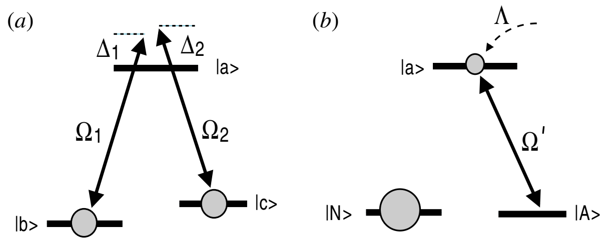

Figure 6. (*a*) Λ-type three-level system in interaction with two coherent fields of Rabi frequencies Ω_1 and Ω_2 and detunings Δ_1 and Δ_2. (*b*) The same configuration in the absorbing and non-absorbing state basis (CPT basis). Under the two-photon resonance condition inversionless amplification in the bare atomic basis can be understood straightforwardly in the CPT basis as an inversion between the upper state $|a\rangle$ and the absorbing state $|A\rangle$. Λ accounts for an incoherent process that populates the upper level $|a\rangle$.

scheme while another microwave coherent field of smaller frequency probed the adjacent transition in the other arm. As pointed out by Javan: '... the lineshape at the amplifying frequency, and its dependence on the intensity and frequency of the saturating power as derived in this paper, at first sight seem to contradict the results obtained from a treatment based on the population differences alone'. In fact, it was shown by Javan that one can obtain a net induced emission at some portions of a resonant line and a net absorption at other frequencies within the line width due to the contribution of an interference term arising from the overlap between the Autler–Townes doublet built up by the driving field.

Unfortunately, LWI in coherently driven discrete three-level systems was not further investigated until three decades later. The seminal papers of Kocharovskaya and Khanin [49] and Scully and co-workers [50] marked the starting point of the current research on LWI in coherently driven discrete three- and multilevel systems. Up to now, several experiments have shown transient and continuous-wave AWI although only a few have reported laser oscillation without inversion [51–53]. In some cases, performing a basis transformation, LWI can be understood as arising from a hidden inversion as in the experiments reported in [52, 53]. In what follows, LWI based (a) on inversion in the CPT basis and (b) on inversion in the dressed-state basis will be analysed. However, as in the first experimental demonstration of LWI [51], lasing without hidden inversion with respect to any meaningful basis† transformation is also possible, and will be discussed in (c).

(a) Inversion in the CPT basis. At the end of the 1980s Kocharovskaya and Khanin [49, 54] analysed transient inversionless amplification of ultrashort pulses in a Λ-type three-level system in connection with the well known phenomenon of CPT‡. To illustrate this phenomenon, let us consider the Λ-type three-level system shown in figure 6(*a*) with two coherent fields of Rabi frequencies Ω_1 and Ω_2 and detunings $\Delta_1 = \omega_1 - \omega_{ab}$ and $\Delta_2 = \omega_2 - \omega_{ac}$, respectively. In the semiclassical framework, the Hamiltonian of this system reads

$$H = H_0 + H_1, \qquad (6)$$

where H_0 is the atomic Hamiltonian:

$$H_0 = -\hbar\omega_{ab}|b\rangle\langle b| - \hbar\omega_{ac}|c\rangle\langle c|, \qquad (7)$$

and H_1 the electric dipole interaction:

$$H_1 = \frac{\hbar}{2}[\Omega_1 e^{-i(\omega_1 t+\phi_1)}|a\rangle\langle b| + \Omega_2 e^{-i(\omega_2 t+\phi_2)}|a\rangle\langle c|] + \text{c.c.} \qquad (8)$$

Here ω_i, ϕ_i $(i = 1, 2)$ are the angular frequency and the initial phase, respectively, of the corresponding laser field. Let us consider at $t = 0$ the following orthonormal superposition states:

$$|N(t=0)\rangle \equiv \frac{1}{\Omega'}[\Omega_2|b\rangle - \Omega_1 e^{i(\phi_2-\phi_1)}|c\rangle], \qquad (9a)$$

$$|A(t=0)\rangle \equiv \frac{1}{\Omega'}[\Omega_1 e^{i(\phi_1-\phi_2)}|b\rangle + \Omega_2|c\rangle], \qquad (9b)$$

where $\Omega' \equiv \sqrt{\Omega_1^2 + \Omega_2^2}$. The free time evolution given by the atomic Hamiltonian reads

$$|N(t)\rangle = \frac{1}{\Omega'}[\Omega_2 e^{i\omega_{ab}t}|b\rangle - \Omega_1 e^{i(\omega_{ac}t+\phi_2-\phi_1)}|c\rangle], \qquad (10a)$$

$$|A(t)\rangle = \frac{1}{\Omega'}(\Omega_1 e^{i(\omega_{ab}t+\phi_1-\phi_2)}|b\rangle + \Omega_2 e^{i\omega_{ac}t}|c\rangle). \qquad (10b)$$

Then, the strength of the interaction of these two states with the fields will be given by

$$|\langle a|H_1|N(t)\rangle| = \frac{\hbar}{2}\frac{1}{\Omega'}\Omega_1\Omega_2[e^{-i(\Delta_1-\Delta_2)t} - 1], \qquad (11a)$$

$$|\langle a|H_1|A(t)\rangle| = \frac{\hbar}{2}\frac{1}{\Omega'}[\Omega_1^2 e^{-i(\Delta_2-\Delta_1)t} + \Omega_2^2], \qquad (11b)$$

which for $\Delta_1 = \Delta_2$ reduces to

$$|\langle a|H_1|N(t)\rangle| = 0, \qquad (12a)$$

$$|\langle a|H_1|A(t)\rangle| = \frac{\hbar}{2}\Omega'. \qquad (12b)$$

This means that, under the two-photon or Raman resonance condition, i.e. $\Delta_1 = \Delta_2$, population trapped in state $|N\rangle$ remains there since it no longer interacts with the coherent fields. In this sense, this state is called non-absorbing or dark state [55–57]. On the other hand, population in state $|A\rangle$ does indeed absorb light and the strength of this interaction is given by the 'effective' Rabi frequency Ω'. As sketched in figure 6, the Λ-type three-level system driven by two coherent fields (figure 6(*a*)) reduces, in the CPT basis, to a driven two-level system with coupling Ω' (figure 6(*b*)). Thus, population in state $|A\rangle$ absorbs light of both laser fields which results in an optical pumping process from the absorbing state $|A\rangle$ to the non-absorbing state $|N\rangle$ (figure 6(*b*)). The population transfer from $|a\rangle$ to $|N\rangle$ can be established by, e.g., spontaneous emission. If all the population ends in the non-absorbing state, the medium becomes transparent for both fields even with all population in the atomic ground states

† In the LWI literature, *meaningful basis* refers to either the bare atomic, the CPT or the dressed-state bases. Nevertheless, in some cases it is possible to define a time-dependent basis transformation such that for each time *t* probe gain arises from an inversion in this new basis.
‡ For a review of CPT see [1].

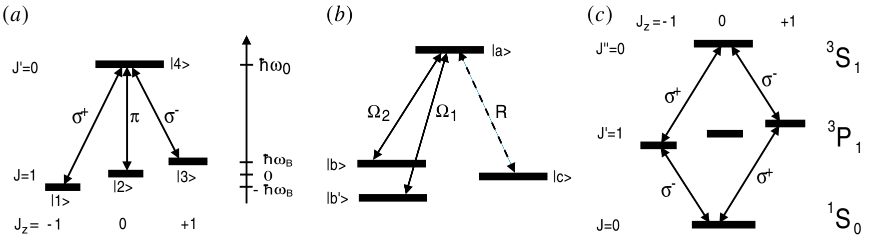

Figure 7. Transient AWI experiments: (*a*) in a $J = 1$ to $J' = 0$ Λ scheme in Sm (Nottelmann *et al* [58]), (*b*) in a Λ scheme in the D_1 line of Na (Fry *et al* [59]) and (*c*) in a $J' = 1$ to $J'' = 0$ Λ scheme in Cd (van der Veer *et al* [60]).

$|b\rangle$ and $|c\rangle$. Furthermore, the introduction of an incoherent pumping process that populates the upper state $|a\rangle$ (denoted Λ in figure 6(*b*)) can give rise to gain for both coherent fields when $\rho_{aa} > \rho_{AA}$. This is an example of what is now currently termed gain with hidden inversion.

The population of the absorbing state ρ_{AA} relates to the bare atomic populations as follows:

$$\rho_{AA} = \sum_{i,j=a,b,c} \langle A|i\rangle\langle i|\rho|j\rangle\langle j|A\rangle$$
$$= \frac{1}{(\Omega')^2}(\Omega_1^2\rho_{bb} + \Omega_2^2\rho_{cc} + 2\Omega_1\Omega_2 \mathrm{Re}\,\tilde{\rho}_{bc}), \qquad (13)$$

where $\tilde{\rho}_{bc} = \rho_{bc}\mathrm{e}^{\mathrm{i}[(\omega_{ab}-\omega_{ac})t+\phi_1-\phi_2]}$ is the atomic coherence between states $|b\rangle$ and $|c\rangle$ in the so-called rotating frame. In order to create the population inversion in the CPT basis, i.e. $\rho_{aa} > \rho_{AA}$, one needs to depopulate the absorbing state. The value of the ground state coherence that completely depopulates state $|A\rangle$, i.e. for which $\rho_{AA} = 0$, reads from equation (13) as

$$(\mathrm{Re}\,\tilde{\rho}_{bc}) = -\frac{1}{2}\left[\frac{\Omega_1}{\Omega_2}\rho_{bb} + \frac{\Omega_2}{\Omega_1}\rho_{cc}\right]. \qquad (14)$$

In general, due to the presence of dissipative processes such as spontaneous emission, incoherent pumping and collisions it is difficult to create large macroscopic atomic coherences. Nevertheless, if the two lower states $|b\rangle$ and $|c\rangle$ correspond to two long-lived states of a rarefied atomic gas medium then it is possible to obtain a value of $\mathrm{Re}\,\tilde{\rho}_{bc}$ close to the value given above.

The CPT phenomenon was first observed in 1976 by Alzetta and co-workers in a Λ-type three-level system in Na [55]. In their experiments, the ground-state Zeeman hyperfine sublevels of sodium were used as the equivalent of the ground states $|b\rangle$ and $|c\rangle$ in figure 6(*a*). The magnitude of their splitting was controlled by a magnetic field with a gradient along the axis of propagation of the two laser fields. A very strong and narrow dip in the fluorescence, the so-called 'black line', was observed when the frequency difference between the two coherent fields matched the splitting between two Zeeman hyperfine sublevels of the ground state. The explanation of the phenomenon in terms of population trapping in a superposition state was first given by Arimondo and Orriols [56, 57].

Stimulated by the proposals of Kocharovskaya and Khanin [49, 54], in 1993 three different experiments of inversionless amplification based on the CPT phenomenon in a Λ-type three-level system were performed in the transient regime [58–60]. In the experiment of Nottelmann *et al* [58] a train of linearly polarized picosecond pulses (PSPs) of $\lambda = 570.68$ nm (the σ^+ and σ^- polarizations of figure 7(*a*)) coupled the Zeeman sublevels $J_z = \pm 1$ (states $|1\rangle$ and $|3\rangle$) to the excited state $J' = 0$ ($|4\rangle$) in atomic samarium. This train of pulses created a superposition state between the Zeeman sublevels $|1\rangle$ and $|3\rangle$. Due to the presence of a magnetic field, the corresponding Zeeman coherence oscillated with the double Larmor frequency $2\omega_B$. To achieve the largest possible values of the ground state coherence the rf period of the train of pulses was taken resonant with the oscillation period of the Zeeman coherence. After the last PSP of the train, another PSP, π-polarized, was sent to pump some population from the Zeeman sublevel $J_z = 0$ ($|2\rangle$) to the upper level $J' = 0$ ($|4\rangle$). Finally, a third PSP tested the $J_z = \pm 1$ to $J' = 0$ transition at the particular times where most of the population in the Zeeman sublevels $|1\rangle$ and $|3\rangle$ was trapped in the non-absorbing state. For this last pulse, Nottelmann *et al* observed inversionless amplification with respect to the Zeeman sublevels although with inversion with respect to the CPT basis.

In the experiment by Fry *et al* [59] the D_1 line of atomic sodium was driven by two co-propagating right circularly polarized laser beams of Rabi frequencies Ω_1 and Ω_2 coupled to the transitions ${}_3S_{1/2}(F = 1) \rightarrow {}_3P_{1/2}(F' = 2)$ and ${}_3S_{1/2}(F = 2) \rightarrow {}_3P_{1/2}(F' = 2)$, respectively. The basic physics of the experiment can be easily understood with the help of the level diagram shown in figure 7(*b*). The two laser beams Ω_1 and Ω_2 produced two effects: (i) optically pumped population into the ground state $m_F = 2$ of the hyperfine level $F = 2$ (state $|c\rangle$ in figure 7(*b*)); and (ii) populated some non-absorbing superposition states of the $F = 1$ and $F = 2$ ground sublevels (which corresponds to the non-absorbing superposition of $|b\rangle$ and $|b'\rangle$ in figure 7(*b*)). Finally, a left circularly polarized beam was used to pump population from the $m_F = 2$ ground state to the upper level $F' = 2$ (R in figure 7(*b*)). At the switching-on times of the pumping beam the two laser beams Ω_1 and Ω_2 experienced transient inversionless amplification.

Finally, van der Veer *et al* [60] reported inversionless amplification in a cell containing low-pressure cadmium vapour in the level scheme of figure 7(*c*). A first linearly polarized laser pulse coupling level $5s^2\,{}^1S_0$ to $5s5p\,{}^3P_1$ was used to create a superposition state between the $J_z' = \pm 1$ states of the ${}_3P_1$ level. Then, a co-propagating second light pulse orthogonally polarized with respect to the first one was used to pump population into the upper state $5s6s\,{}^3S_1$. As in the Nottelmann *et al* experiment, a magnetic field controlled the splitting between the $J_z' = \pm 1$ states of the ${}_3P_1$ level

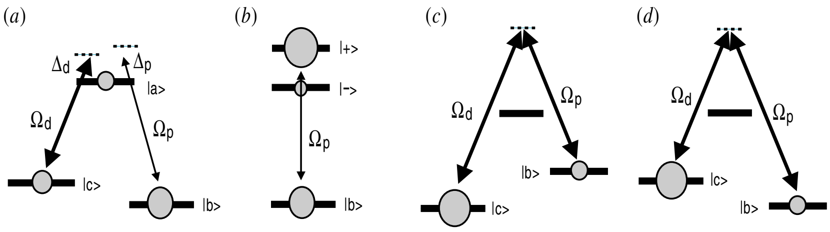

Figure 8. (*a*) Λ-type three-level scheme driven by a coherent field (Ω_d) in one transition and probed by a weak field (Ω_p) in the other one. Δ_d (Δ_p) is the driving (probe) field detuning from atomic resonance. (*b*) The same configuration in the dressed state basis. Inversionless gain in (*a*) can be traced back as gain due to the inversion between the dressed state $|+\rangle$ and the lower probed state $|b\rangle$. (*c*) Stokes Raman configuration. (*d*) Anti-Stokes Raman configuration.

producing a temporal evolution of the coherence between these two states. Finally, a third weak field polarized parallel with respect to the first one probed the Λ-configuration between the $J_z' = \pm 1$ states and the $J'' = 0$ excited level 5s6s ^{3}S$_1$. This last pulse experienced inversionless amplification at the particular times in which the population of the $J_z' = \pm 1$ states was trapped into the non-absorbing superposition state.

Soon afterwards, Padmabandu *et al* [52] observed amplification and continuous-wave LWI in a Λ-type system within the D_1 line in a sodium atomic beam where the Doppler width was drastically reduced. By using a weak probe laser they demonstrated, first, inversionless gain and then, after a laser cavity was installed and aligned, laser oscillation without population inversion. More recently, as an extension of the pulsed experiment by van der Veer *et al* [60], de Jong *et al* [53] have been able to obtain, in the same level structure of cadmium vapour shown in figure 7(*c*), a single-pass inversionless amplification by more than a factor of ten which has made it possible to observe synchronously pumped inversionless laser action.

Finally, the so-called double-Λ scheme has also been proposed in order to achieve LWI based on the CPT phenomenon [61–66]. In the double-Λ scheme, two laser fields (or, for instance, the two circularly polarized components of a linearly polarized field) in a Λ-type arrangement create the ground state coherence while two other coherent fields couple the same ground states to an excited level defining a second Λ scheme. In a modified double-Λ configuration in a He–Ne laser, Peters and Lange [67] observed laser action below threshold inversion at 611.8 nm with a coherence generating field at 826.9 nm. Although laser action took place with inversion in the lasing transition, it is important to note that the conventional threshold inversion for lasing at frequency ω was reduced by applying a driving field of frequency $\omega' < \omega$.

(b) Inversion in the dressed-state basis. As has been shown previously for two-level systems, in order to determine the response to a weak probe field of a laser-driven system it is very useful to diagonalize the atomic Hamiltonian plus the interaction with the driving field. In the semiclassical framework, the eigenstates of the diagonalized Hamiltonian are usually called semiclassical dressed states. For instance, let us consider the Λ-type three-level system of figure 8(*a*) where an intense coherent field of Rabi frequency Ω_d and detuning $\Delta_d = \omega_d - \omega_{ac}$ drives transition $|a\rangle$–$|c\rangle$ while a weak laser field of Rabi frequency Ω_p and detuning $\Delta_p = \omega_p - \omega_{ab}$ probes the adjacent transition $|a\rangle$–$|b\rangle$. Let us denote $|+\rangle$ and $|-\rangle$ the semiclassical dressed states obtained after the diagonalization of the corresponding Hamiltonian with $|a\rangle$ the zero energy level. These dressed states are given by [21]

$$|+\rangle = a_+|a\rangle + c_+|c\rangle, \tag{15a}$$

$$|-\rangle = a_-|a\rangle + c_-|c\rangle, \tag{15b}$$

where

$$a_\pm = -\frac{\Delta_\mp}{[\Delta_\mp^2 + (\Omega_d/2)^2]^{1/2}}, \tag{16a}$$

$$c_\pm = \frac{\Omega_d/2}{[\Delta_\mp^2 + (\Omega_d/2)^2]^{1/2}}, \tag{16b}$$

with $\Delta_\pm = \frac{\Delta_d}{2} \pm \frac{1}{2}\sqrt{\Omega_d^2 + \Delta_d^2}$ being the eigenvalues of states $|+\rangle$ and $|-\rangle$ and, therefore, the corresponding dressed-state energy ($\hbar = 1$) with respect to state $|a\rangle$. The dressed-state populations are related to the bare atomic populations ρ_{aa} and ρ_{cc}, and the coherence ρ_{ac} as follows:

$$\rho_{\pm\pm} = \frac{1}{\Delta_\mp^2 + (\Omega_d/2)^2}(\Delta_\mp^2\rho_{aa} + (\Omega_d/2)^2\rho_{cc} - \Omega_d\Delta_\mp \mathrm{Re}\,\rho_{ac}). \tag{17}$$

Depending on the explicit values of the parameters of the system it is possible to accumulate population in one of the dressed states such that an inversion between this dressed state and the remaining probed state occurs. This situation is pictured in figure 8(*b*) where probe gain at $\Delta_p \approx \Delta_+$ is possible since $\rho_{++} > \rho_{bb}$, in spite of the fact that there is no inversion in the bare atomic states $\rho_{bb} > \rho_{aa}, \rho_{cc}$. As in the CPT case, this is another example of gain with hidden inversion.

Let us now consider the particular case in which the driving field is far detuned from atomic resonance and all the population remains in the ground states $|b\rangle$ and $|c\rangle$, i.e. $\Delta_d \gg \Omega_d$ and $\rho_{aa} = 0$. In this case, we have $\Delta_+ \approx \Delta_d$, $\Delta_- \approx 0$ and, from equation (17), $\rho_{++} \approx \rho_{cc}$ and $\rho_{--} \approx 0$. Therefore, amplification of the probe field is possible at $\Delta_p \approx \Delta_+$ for $\rho_{++} \approx \rho_{cc} > \rho_{bb}$. This situation corresponds to the well known Raman amplification/lasing process where probe/laser gain takes place at the Raman (or two-photon) resonance condition, i.e. $\Delta_p = \Delta_d$, whenever Raman (or two-photon) population inversion occurs, i.e. $\rho_{cc} > \rho_{bb}$. In Raman lasers two-photon processes are the physical processes responsible for both gain (stimulated emission of one lasing photon simultaneously with absorption of

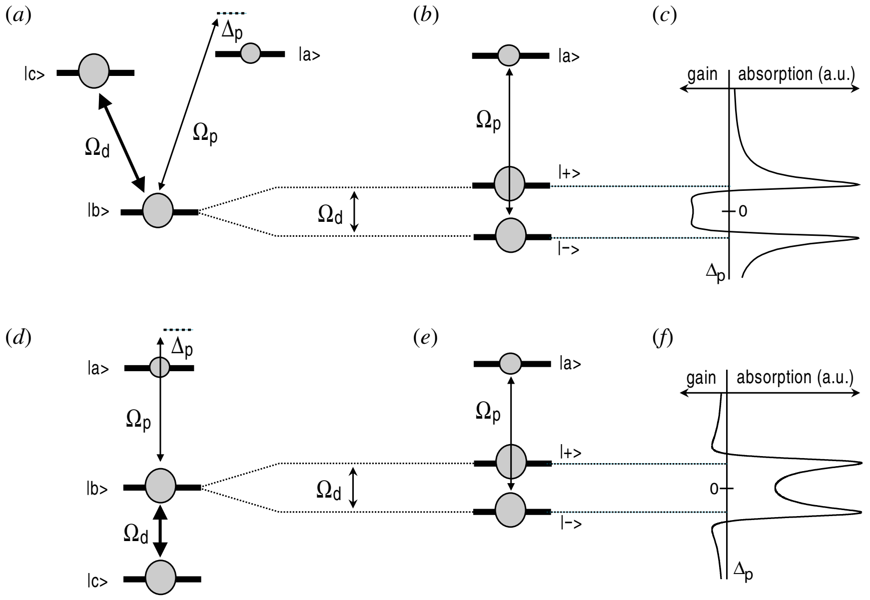

Figure 9. (*a*) Resonantly driven V scheme without inversion in the bare atomic basis. (*b*) The same V scheme without inversion in the dressed-state basis. (*c*) Probe response showing gain between the dressed-state resonances. (*d*) Resonantly driven cascade scheme without inversion in the bare atomic basis. (*e*) The same cascade scheme without inversion in the dressed-state basis. (*f*) Probe response showing gain outside the dressed-state resonances.

one driving photon) and losses (absorption of one lasing photon simultaneously with stimulated emission of one driving photon). Indeed, Raman lasers operate even with no population in the upper state $|a\rangle$. Figures 8(*c*) and (*d*) show the so-called Stokes and anti-Stokes Raman configurations for which $\omega_d > \omega_p$ and $\omega_d < \omega_p$, respectively. Notice that the anti-Stokes Raman configuration can be used to amplify a probe field with larger frequency than the driving one although it requires inversion in the reservoir. Raman lasers are commonly used as tunable coherent sources in the infrared domain since in this frequency region the Doppler effect plays a minor role which allows the two-photon resonance condition for all atoms irrespective of their velocity. Finally, note that although Raman lasers operate without inversion in the lasing transition they are not usually included in the LWI family for at least two reasons: (i) they indeed require inversion in the bare atomic system (Raman or two-photon inversion) and (ii) Stokes Raman lasers do not extract the energy from the atomic medium but, on the contrary, there is an energy exchange between both coherent fields.

In coherently driven three-level systems it is possible to define generalized Einstein B coefficients for one- and two-photon processes [68]. In the particular case of a Raman laser (figures 8(*c*) and (*d*)), the attenuation or amplification of the generated laser field will be determined by the ratio between the rates of two-photon loss and gain processes:

$$\frac{\text{absorption rate}}{\text{stimulated emission rate}} = \frac{B_{bc}}{B_{cb}} \cdot \frac{\rho_{bb}}{\rho_{cc}}, \tag{18}$$

where B_{cb} and B_{bc} denote the generalized Einstein B coefficients for two-photon gain and loss processes, respectively. Clearly, the two-photon population inversion condition for Raman amplification results from a symmetry between the generalized Einstein B coefficients for two-photon processes, i.e. $B_{bc} = B_{cb}$.

(c) LWI without hidden inversion. Thus far we have analysed several cases in which gain without inversion in the lasing transition can be understood as a hidden inversion in the CPT basis, or as inversion in the dressed-state basis or, finally, as Raman (or two-photon) inversion in the bare atomic basis. However, there are cases where probe amplification occurs without inversion in any meaningful basis. For instance, let us consider the V and cascade three-level schemes shown in figures 9(*a*) and (*d*), respectively. In both cases a resonant coherent field drives transition $|b\rangle$–$|c\rangle$ while a weak field probes the adjacent transition $|a\rangle$–$b\rangle$. In the dressed-state picture (see figures 9(*b*) and (*e*)), the driving field splits state $|b\rangle$ in two dressed states $|+\rangle$ and $|-\rangle$ with opposite detunings $\Delta_+(= -\Delta_-) = \Omega_d/2$ from state $|b\rangle$. Let us take a strong enough driving field such that it saturates transition $|b\rangle$–$|c\rangle$, i.e. $\rho_{cc} \simeq \rho_{bb}$, and therefore, $\rho_{++} = \rho_{--} = \rho_{bb} = \rho_{cc}$. In this case, population inversion in the probed transition of the bare atomic system implies population inversion in the dressed-state basis and vice versa. Therefore, one should expect no probe gain for $\rho_{bb} > \rho_{aa}$. Surprisingly, a simple density-matrix calculation shows that even in this case probe amplification is possible, as illustrated in figures 9(*c*) and (*f*)† by the probe response. For the parameter values used in these figures the dressed-states resonances contribute to probe field absorption since $\rho_{aa} < \rho_{++} = \rho_{--}$, even though inversionless amplification occurs between the dressed-state resonances for the V scheme and outside the dressed-state resonances for the cascade scheme.

Agarwal [70] analysed the origin of AWI in both the bare and dressed state bases in the Λ-type three-level system

† The explicit parameter values used in these figures can be found in [68,69]. It is worth noting that for both schemes it is assured that, for the parameter setting used, there is neither steady-state one-photon inversion nor steady-state two-photon inversion in the bare atomic basis.

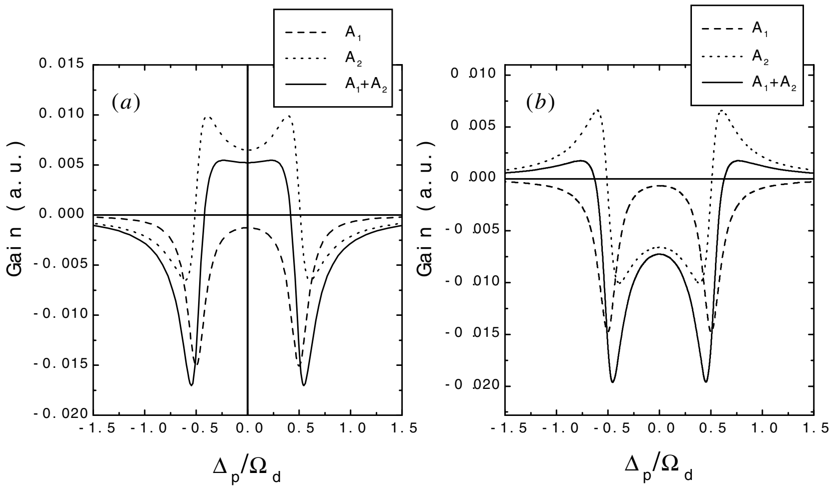

Figure 10. Gain for a weak probe field as a function of its detuning (solid curve) with an on-resonance driving field for (*a*) a V scheme and (*b*) a cascade scheme. The total gain profile (solid curve) is split into the independent contribution of the two dressed-state resonances, A_1 (dashed curve), and the contribution of the quantum interference between these resonances, A_2 (dotted curve). The parameters are (*a*) $\gamma_{ab} = 0.1\gamma_{cb}$, $\Lambda = \gamma_{cb}$, $\Omega_d = 10\gamma_{cb}$, and $\Omega_p = 0.002\gamma_{cb}$, and (*b*) $\gamma_{ab} = 0.1\gamma_{bc}$, $\Lambda = \gamma_{bc}$, $\Omega_d = 10\gamma_{bc}$, and $\Omega_p = 0.002\gamma_{bc}$. γ_{ij} is the spontaneous population decay rate from $|i\rangle$ to $|j\rangle$ and Λ is the rate of a bidirectional incoherent pumping process acting on the probed transition.

proposed by Imamoglu *et al* [39], showing that gain arises from atomic coherence between the dressed states. In fact, due to this coherence the probe response corresponds approximately to two independent Lorentzian contributions at the dressed-states position plus an interference term consisting in two dispersive-like structures also located at the dressed-states position. Thus, figures 10(*a*) and (*b*) show the same probe response plotted in figures 9(*c*) and (*f*), but now split into two separate contributions: the independent dressed-states resonances (term A_1) and the interference between the dressed-states resonances (term A_2). In both cases, term A_1 contributes to probe absorption since there is no inversion between the upper level of the probed transition and the dressed states, i.e. $\rho_{aa} < \rho_{++} = \rho_{--}$. For the V scheme the interference term is destructive ($A_1A_2 < 0$) between the dressed-states resonances which gives rise to inversionless gain at line centre. For the cascade scheme, the interference term is constructive ($A_1A_2 > 0$) between the dressed-states resonances but destructive outside this region which gives rise to inversionless gain at the wings of the dressed-states resonances. A detailed analysis of the character of this interference term in folded and cascade schemes has been performed very recently (see [71, 72]).

In addition, Agarwal [70] and Fleischhauer *et al* [73] have explained the origin of this inversionless gain in terms of Fano-type interferences in the dressed-state basis. This interpretation is based on the fact that a density-matrix analysis of the Λ and V schemes in the dressed-state basis resembles the density-matrix analysis corresponding to the Harris proposal [36] based on Fano-type interferences. For the V scheme, with both fields detuned far from resonance, AWI in any meaningful basis has also been explained by Grynberg *et al* [24] in terms of interference between Feynman diagrams in the dressed-state basis. Finally, note that the first experimental demonstration of laser oscillation without inversion in any dressed basis was reported by Zibrov *et al* [51] in a V-type three-level system in a Rb vapour cell within the D_1 line.

The quantum-trajectory or quantum-jump formalism [45, 46, 74, 75] has been used in order to calculate the individual contribution of the different physical processes (one-photon and two-photon gain/loss processes) responsible for inversionless amplification. This formalism gives the same results as the density-matrix formalism but provides new insights into the underlying physical mechanisms [45, 46]. In this formalism, the time evolution of the atom plus lasers system is pictured as consisting of a series of coherent evolution periods separated by quantum jumps occurring at random times, i.e. the so-called quantum trajectories. The quantum jumps are determined by dissipative processes, such as spontaneous emission or incoherent pumping, while the continuous evolution is governed by the coherent laser fields. Thus, a one-photon gain (loss) process is a coherent evolution period between two consecutive quantum jumps such that the probe field photon number increases (decreases) by one with no change in the driving photon number. A two-photon gain process corresponds to a coherent evolution period for which the probe field photon number increases by one and the driving field photon number decreases by one (V and Λ schemes) or increases by one (cascade schemes). Using this technique, Arimondo [74] and Cohen-Tannoudji *et al* [75] revealed that AWI in folded schemes results from the fact that, for appropriate parameter values, two-photon gain processes overcome one-photon and two-photon loss processes even without one-photon or two-photon inversion. By contrast, the quantum jump formalism shows that one-photon gain is the physical process responsible for inversionless gain in cascade schemes [71, 72]†. Figures 11(*a*) and (*b*) show, through a quantum jump analysis, the individual contribution of the different physical processes to the total probe absorption spectrum corresponding to the V and

† These results refer to the usual case in which the driven transition is not inverted. For the opposite case, the quantum jump formalism shows that inversionless gain in folded (cascade) schemes is due to the predominance of one-photon (two-photon) gain processes (see [71, 72]).

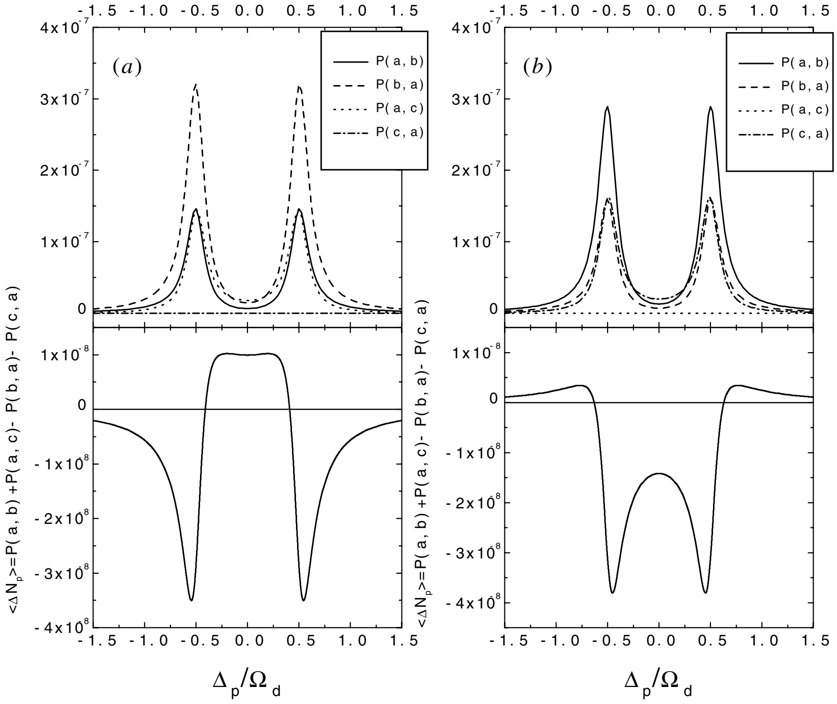

Figure 11. (*a*) V scheme. (*b*) Cascade scheme. Upper plots show the probability that a random choice among all coherent evolution periods gives a one-photon gain process $P(a, b)$, a one-photon loss process $P(b, a)$, a two-photon gain process $P(a, c)$ and, finally, a two-photon loss process $P(c, a)$ as a function of the probe field detuning. The lower plots show the mean change of the probe field photon number per coherent evolution period. Curves in the lower parts of (*a*) and (*b*) are identical to the solid curves in figures 10(*a*) and (*b*), and in figures 9(*c*) and (*f*), respectively.

cascade schemes of figures 9 and 10. On the one hand, for the V-scheme configuration, two-photon gain processes predominate at line centre and therefore they are the physical processes responsible for probe gain. On the other hand, inversionless gain in the wings of the dressed-state resonances in the cascade scheme is due to the predominance in this region of one-photon gain processes. One of the benefits of the quantum jump formalism is the fact that the knowledge of the particular physical processes responsible for inversionless gain allows us to select appropriate probe and driving field detunings to stimulate these processes while at the same time preventing the unfavourable ones. Thus, for the V scheme being investigated one should detune both fields out from their respective atomic resonances but while fulfilling the two-photon resonance condition in order to enhance the probability of two-photon processes while reducing the probability of one-photon processes. For the cascade scheme, one should detune the driving field out from its atomic resonance while maintaining the probe field tuned close to one-photon resonance. Finally, in the framework of the quantum trajectory formalism, Carmichael [76] has presented an extensive analysis of the origin of inversionless gain in the V scheme where LWI was observed for the first time [51].

As has been discussed in relation to equations (3) and (18), the population inversion requirement for conventional lasers to operate is a direct consequence of the symmetry between the Einstein B coefficients for one-photon processes in two-level lasers, and for two-photon processes in Raman lasers. The quantum jump formalism has been used very recently to define generalized Einstein B coefficients for one- and two-photon gain and loss processes [68]. Thus, it has been shown that in three-level systems coherently driven close to resonance, there is a symmetry breaking between the Einstein B coefficients for, on the one hand, one-photon processes and, on the other hand, two-photon processes. For instance, let us consider again the V and cascade schemes shown in figures 9(*a*) and (*b*) with N_p and N_d the photon number of the probe and driving field, respectively. Moreover, let us call B_{ab} and B_{ba} the generalized Einstein B coefficients for one-photon gain ($\Delta N_p = +1$, $\Delta N_d = 0$) and loss ($\Delta N_p = -1$, $\Delta N_d = 0$) processes, and B_{ac} and B_{ca} the corresponding coefficients for two-photon gain ($\Delta N_p = +1$ with $\Delta N_d = -1$ for the V scheme and $\Delta N_d = +1$ for the cascade scheme) and loss ($\Delta N_p = -1$ with $\Delta N_d = +1$ for the V scheme and $\Delta N_d = -1$ for the cascade scheme) processes, respectively. The meaning of these generalized Einstein B coefficients can be easily understood with the help of figure 12 which refers to the cascade scheme. For instance, the term involving the B_{ab} coefficient accounts for the rate corresponding to the summation of all processes starting in state $|a\rangle$ through a quantum jump given by the incoherent process $R_{ia}(i = a, b, c)$, and ending the coherent interaction in state $|b\rangle$ through a quantum jump given by $R_{bj}(j = a, b, c)$. In all these one-photon gain processes the energy exchange between the atom and the two coherent fields satisfies $\Delta N_p = +1$ with $\Delta N_d = 0$ irrespective of the explicit number of driving photons involved in the interaction. Figure 12(*b*) illustrates schematically the meaning of the B_{ac} coefficient accounting for two-photon gain processes.

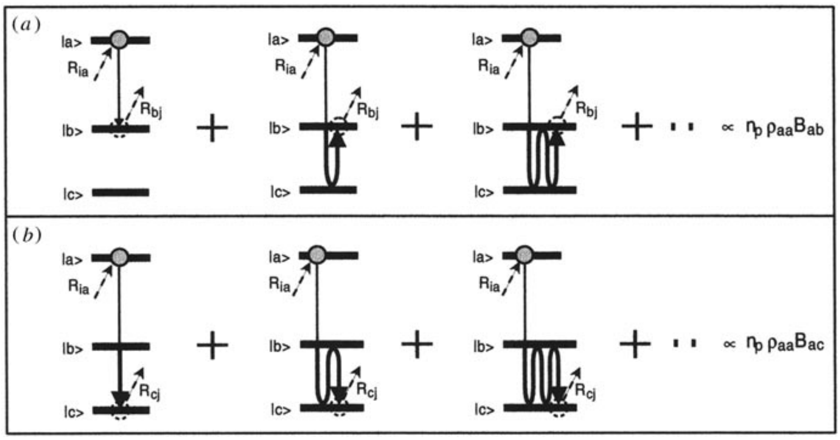

Figure 12. Schematic representation of (a) one-photon gain processes and (b) two-photon gain processes in a cascade scheme. The thin (thick) line accounts for the interaction with the probe (driving) field. In all three processes shown in (a) the atom undergoes a transition from state $|a\rangle$ to state $|b\rangle$ with an energy exchange with the two laser fields such that $\Delta N_p = +1$; $\Delta N_d = 0$. R_{ia} accounts for a quantum jump that brings the atom into state $|a\rangle$ and R_{bj} for a quantum jump that takes the atom out from state $|b\rangle$. (b) The same for two-photon gain processes with R_{cj} for a quantum jump out from $|c\rangle$. The atom undergoes a transition from $|a\rangle$ to $|c\rangle$ such that $\Delta N_p = +1$; $\Delta N_d = +1$. $n_p\rho_{aa}B_{ab}$ and $n_p\rho_{aa}B_{ac}$ account for the rates of one- and two-photon gain processes, respectively. n_p is the number of photons per unit volume in the probe mode, ρ_{aa} is the steady-state population of the upper level, and, finally, B_{ab} and B_{ac} are the generalized Einstein B coefficients for one- and two-photon gain, respectively.

In both V and cascade schemes the amplification/attenuation of the probe field will be given by

$$\frac{\mathrm{d}}{\mathrm{d}t}n_p = \hbar\omega_p n_p(\rho_{aa}B_{ab} + \rho_{aa}B_{ac} - \rho_{bb}B_{ba} - \rho_{cc}B_{ca}), \quad (19)$$

where n_p is the number of photons per unit volume in the probe mode. $n_p\rho_{aa}B_{ab}$ and $n_p\rho_{bb}B_{ba}$ account for the rates of one-photon gain and loss processes, while $n_p\rho_{aa}B_{ac}$ and $n_p\rho_{cc}B_{ca}$ account for the rates of two-photon gain and loss processes, respectively. Assuming a strong and close-to-resonance driving field and that in both cases transition $|a\rangle$–$|c\rangle$ is electric dipole forbidden, it has been demonstrated [68] that the generalized Einstein B coefficients satisfy

$$\frac{B_{ab}}{B_{ba}} \equiv 1 + \Delta B_{1p} = 1 + \frac{R_{bc} - R_{cb}}{R_{ba} + R_{cb}}, \quad (20a)$$

$$\frac{B_{ac}}{B_{ca}} \equiv 1 + \Delta B_{2p} = 1 + \frac{R_{cb} - R_{bc}}{R_{bc}}. \quad (20b)$$

Clearly, there is a symmetry breaking between, on the one hand, the generalized Einstein B coefficients for one-photon processes and, on the other hand, the generalized Einstein B coefficients for two-photon processes. The explicit value of this symmetry breaking is determined by the incoherent processes such that $\Delta B_{1p}\Delta B_{2p} \leqslant 0$. Note also that the sign of $R_{bc} - R_{cb}$ determines whether there is inversion or not in the driven transition. A detailed analysis of the properties of these generalized Einstein B coefficients can be found in [68].

Finally, de Jong *et al* [77, 78] have discussed LWI in the V-type system in terms of the quantum Zeno effect, i.e. the inhibition of a coherent evolution due to a frequent measurement process. To illustrate this phenomenon let us consider the propagation of a vertically polarized photon through a polarization rotating (chiral) medium such that it produces a 90° polarization rotation (figure 13(a))†. During

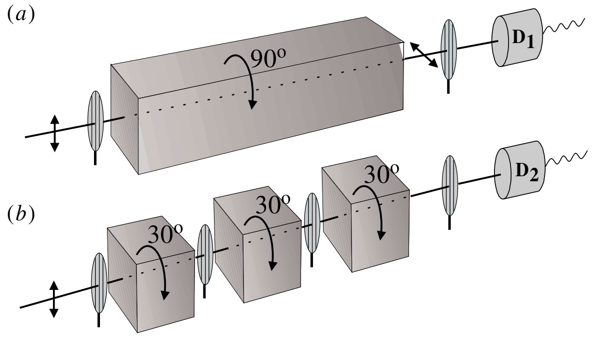

Figure 13. Quantum Zeno effect. (a) An ideal polarization rotating medium rotates the photon polarization from vertical to horizontal. No light is observed by detector D_1 after the second vertical polarizer. (b) The photon propagation through the polarization rotating medium is interrupted by vertical polarizers. After the last polarizer there is a non-vanishing probability that detector D_2 observes the photon.

its propagation the photon polarization evolves continuously from vertical to horizontal such that if a vertical polarizer is introduced after the medium the photon is absorbed by the polarizer. Suppose now that the photon propagation through the medium is interrupted by a vertical polarizer after each 30° polarization rotation as shown in figure 13(b). Thus, the first vertical polarizer forces the photon to choose between two complementary outcomes: (a) transmission of the photon with a probability $\cos^2 30° = 0.75$, and (b) absorption of the photon with a probability 0.25. If the photon is transmitted the photon wavefunction collapses to the vertical polarization state. Clearly, after the last vertical polarizer there is a probability $(\cos^2 30°)^3$ to detect the photon. In the limit of $n \to \infty$ vertical polarizers the absorption probability given by $1 - \cos^{2n}(90°/n)$ vanishes and the photon always emerges from the last vertical polarizer. This example shows clearly that it is possible to inhibit a nonlinear continuous evolution process, e.g. the photon polarization rotation,

† This example has been extracted from [78] and [79]. For a clear introduction to the classical Zeno paradox and the quantum Zeno effect see chapter 1 in [78].

Table 1. Experiments on pulsed and continuous-wave (CW) AWI and LWI, and lasing below threshold inversion (LBT). $R \equiv \omega_p/\omega_d$ is the frequency up-conversion ratio.

Category	Authors	Medium	Driving (nm)	Probe (lasing) (nm)	R
Pulsed AWI	Nottelmann *et al* (1993) [58]	Sm vapour cell (Λ)	570.68	570.68	1
Pulsed AWI	Fry *et al* (1993) [59]	Na vapour cell (Λ)	589.86 558.43	589.86 558.43	1
Pulsed AWI	van der Veer *et al* (1993) [60]	Cd vapour cell (Λ)	326	479	0.68
CW AWI	Kleinfeld and Streater (1994, 1996) [80, 81]	K vapour cell (4-level)	766.5	769.9	1
CW AWI	Zhu *et al* (1996) [82, 83]	Rb vapour cell (Λ)	780	780	1
CW AWI	Sellin *et al* (1996) [84]	Ba atomic beam (cascade)	554	821	0.67
CW AWI	Fort *et al* (1997) [85]	Cs vapour cell (V)	852	894	0.95
CW AWI	Shiokawa *et al* (1997) [86]	Laser-cooled Rb atoms (Λ)	780	780	1
CW AWI	Hollberg *et al* (1998, 1999) [87, 88]	Laser-cooled Rb atoms (V)	780	795	0.98
CW LWI	Zibrov *et al* (1995) [51]	Rb vapour cell (V)	780	795	0.98
CW LWI	Padmabandu *et al* (1996) [52]	Na atomic beam (Λ)	589.76	589.43	1
Pulsed LWI	de Jong *et al* (1998) [53]	Cd vapour cell (Λ)	326	479	0.68
CW LBT	Peters and Lange (1996) [67]	Ne vapour cell (double-Λ)	824.9	611.8	1.35

by a frequent measurement process, e.g. the wavefunction collapse produced by the vertical polarizers.

A similar situation occurs in the interaction of an intense coherent field with an atomic system. In this case the population evolution governed by the Rabi flopping plays the role of the nonlinear continuous evolution while irreversible processes, e.g. spontaneous emission, collisions and incoherent pumping, account for the wavefunction collapses or measurement processes. As shown by de Jong *et al* [77, 78] for the V scheme, it is possible to use the quantum jump formalism to discuss LWI in terms of the quantum Zeno effect. An extension to the rest of three-level systems of the role of the quantum Zeno effect in AWI can be found in [72].

4. LWI with frequency up-conversion

Table 1 summarizes the state of the art in the AWI/LWI experiments in coherently driven three- and multilevel systems. These experiments must be considered as a *proof of principle* of the feasibility of the LWI idea [51–53, 58–60, 67, 80–88]. Note, however, that up to now all AWI and LWI experiments have operated with the probe (lasing) frequency equal or smaller than the driving frequency, i.e. $R \equiv \omega_p/\omega_d \leqslant 1$. Only in the double-$\Lambda$ scheme experiment by Peters and Lange [67] was the frequency of the generated laser field larger than that of the driving field ($R = 1.35$) although, in this case, there was indeed inversion in the lasing transition. Therefore, LWI with frequency up-conversion is still a very interesting and challenging problem. Several papers have recently addressed the frequency up-conversion LWI issue [89–96] and, next, we will briefly indicate the main difficulties with this regime of lasing.

(a) Doppler broadening. In a gas medium, atomic motion leads unavoidably to Doppler broadening due to the velocity-dependent shift of probe and driving frequencies given, respectively, by $\omega_{p,d}(\vec{v}) = \omega_{p,d}(\vec{v} = 0) - \vec{k}_{p,d} \cdot \vec{v}$ where $\vec{k}_{p,d}$ accounts for the corresponding wavevector and $\vec{v}$ for the atomic velocity. In the optical domain, typical values of the Doppler broadening are 1 GHz for a vapour cell, a few MHz for a collimated atomic beam, and a few hundred kHz for an atomic trap.

As has been indicated previously, LWI in folded (Λ and V) schemes results, in general, from the contribution of the two-photon (Raman) gain processes and, therefore, the gain condition is fulfilled whenever the detunings of probe and driving fields from their corresponding atomic transitions are equal, i.e. $\Delta_p(\vec{v}) = \Delta_d(\vec{v})$. As a consequence, most AWI/LWI experiments operating in folded schemes have used almost a Doppler-free configuration, i.e. $\omega_p(\vec{v} = 0) = \omega_d(\vec{v} = 0)$, such that all atoms simultaneously satisfy the two-photon resonance condition irrespective of their velocity. On the other hand, in cascade schemes with a non-inverted driven transition, probe gain results from one-photon gain processes and only appears for certain domains of probe detuning (see figure 9(*f*)). Clearly, Doppler broadening notably restricts the applicability of LWI in the frequency up-conversion regime. Theoretically, it has been shown for the optical domain that a vapour cell can support LWI up to probe(lasing)-to-driving frequency ratios of the order of $R \sim 2$ [93, 94, 96]. A possible solution to this problem consists in using an atomic beam configuration or laser cooled atoms in an atomic trap as in the experiments reported in [52, 84] and [86–88], respectively. In this case, as has been shown in [96], the frequency of the generated field can be substantially larger than that of the driving field, e.g. $R \sim 10$. However, the use of an atomic beam or an atomic trap considerably reduces the number and density of active atoms and, then, the maximum achievable gain.

(b) Propagation effects. As has been described by Lukin *et al* [89–91], propagation effects should be taken into account in order to move towards the up-conversion LWI regime. In particular, let us consider the homogeneously broadened V scheme displayed in the inset of figure 14(*a*). γ_{ij} is the spontaneous population decay rate from $|i\rangle$ to $|j\rangle$ and Λ denotes an incoherent pump process used to populate the upper level of the probed transition. Figures 14(*a*) and (*b*) show, respectively, the driving and probe field attenuation/amplification during their propagation through the active medium for the following parameter values: $\gamma_{cb} =$

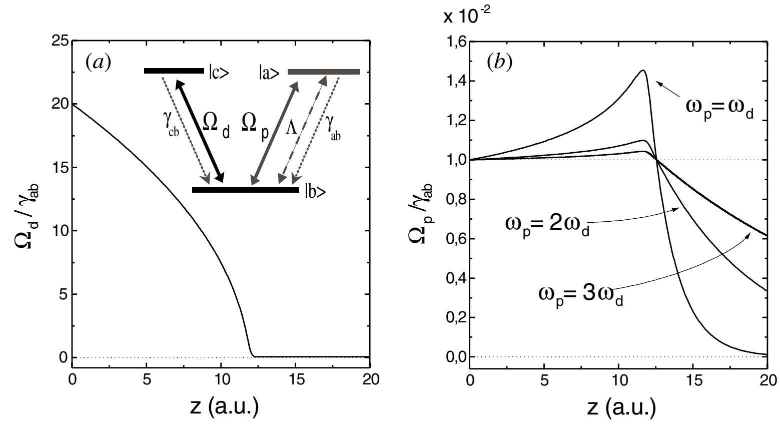

Figure 14. Attenuation/amplification of (a) the driving and (b) the probe field during the propagation in a homogeneously broadened V scheme. For a fixed driving field frequency three different cases are considered: (i) $\omega_p = \omega_d$, (ii) $\omega_p = 2\omega_d$ and (iii) $\omega_p = 3\omega_d$. For other parameters, see the text.

$2\gamma_{ab}$, $\Lambda = 6\gamma_{ab}$, $\Omega_p(z=0) = 0.01\gamma_{ab}$, $\Omega_d(z=0) = 20\gamma_{ab}$, $\Delta_p = \Delta_d = 0$, and the same electric dipole moments and atomic density for the three cases shown: (i) $\omega_p = \omega_d$, (ii) $\omega_p = 2\omega_d$ and (iii) $\omega_p = 3\omega_d$. As the incoherent pumping process is taken to be bidirectional it is assured that the probe transition is not inverted during the propagation although, for small enough driving Rabi frequencies, in our case $\Omega_d < 5.8\gamma_{ab}$, there is indeed Raman inversion, i.e. $\rho_{aa} > \rho_{cc}$. A numerical analysis of the density-matrix equation also shows that the probe field is amplified up to $\Omega_d = 2.7\gamma_{ab}$ for the three cases corresponding to $z \sim 12$ au. Between $z \sim 12$ and 12.5 au the probe field starts to be attenuated and, eventually, for $z > 12.5$ au the accumulated gain becomes negative. Clearly, as we increase the probe-to-driving frequency ratio the maximum probe amplification decreases. This clearly shows that propagation effects can notably restrict the applicability of LWI to the frequency up-conversion regime.

In fact, Lukin *et al* [89–91] have shown that during the propagation, probe and driving fields satisfy the inequality

$$\frac{\text{probe gain per unit length}}{\text{driving absorption per unit length}} = \left|\frac{\lambda_p^2\gamma_{ab}\text{Im}\,\rho_{ab}}{\lambda_d^2\gamma_{cb}\text{Im}\,\rho_{cb}}\right| \leqslant \frac{1}{R^2}\frac{\gamma_{ab}}{\gamma_{cb}} \qquad (21)$$

where λ_p (λ_d) is the probe (driving) field wavelength, and Im ρ_{ab} (Im ρ_{ab}) is the imaginary part of the coherence in the probed (driven) transition. In our notation, $\Omega_p\text{Im}\,\rho_{ab} > 0$ ($\Omega_d\text{Im}\,\rho_{cb} > 0$) means absorption of the probe (driving) field. Then, as we want to move towards the up-conversion regime we will be interested in spontaneous population decay rates fulfilling the condition $\gamma_{cb} \ll \gamma_{ab}$. In this case, even for large R values it is possible to have a significant probe amplification factor per single pass through the active medium before the driving is completely absorbed. In order to overcome the propagation problem, Yelin *et al* [91] have suggested the possibility of driving a weakly allowed transition with a microwave field while probing an adjacent transition with an optical field. Finally, the role of propagation effects on frequency up-conversion LWI in a Doppler-broadened V-type three-level system in atomic rubidium has been analysed in detail in [97].

(c) Decay rates. Most papers dealing with LWI in folded schemes have considered the completely resonant homogeneously broadened case. Neglecting dipole dephasing processes, it has been demonstrated that a necessary condition for continuous-wave AWI and LWI in Λ- and V-type three-level systems is that the spontaneous population decay rate in the driven transition must be larger than that of the probed transition [39, 98], e.g. $\gamma_{cb} > \gamma_{ab}$ in the V scheme shown in the inset of figure 14(a). Similar inequalities between the relaxation rates also apply for the resonantly driven cascade schemes [92]. These conditions restrict the number of possible suitable atomic systems for frequency up-conversion LWI and also, as has been indicated previously, reduce the maximum achievable gain factor during propagation.

As has been shown in [92], the above-mentioned conditions between the relaxation rates can be substantially relaxed in folded schemes for detuned driving and probe fields provided that the two-photon resonance condition $\Delta_p = \Delta_d$ is fulfilled. As a consequence, a Doppler-free configuration in a vapour cell or alternatively an atomic beam or an atomic trap must be used to achieve AWI/LWI with a population relaxation rate in the probed transition larger than in the driven transition.

(d) Incoherent pumping. As for the decay rates, it is possible to show [68] that in all schemes there is an incoherent pump threshold value below which AWI/LWI is not possible. For instance, for the V scheme of figure 14(a) with $\Delta_p = \Delta_d = 0$ the threshold value for the incoherent pump rate reads $\Lambda_{\text{th}} = \gamma_{ab}^2/(\gamma_{cb} - \gamma_{ab})$. Therefore, in general, in order to reduce the required pumping power we will be interested in schemes satisfying $\gamma_{cb} \gg \gamma_{ab}$ which again, from the propagation perspective, notably reduces the maximum probe gain per pass through the active medium. As has been shown in [92], it is possible to decrease the threshold value for the incoherent pump rate by taking both fields relatively far from resonance but fulfilling the two-photon resonance condition in folded schemes, and tuning the driving field far from resonance (*a*) (*b*)

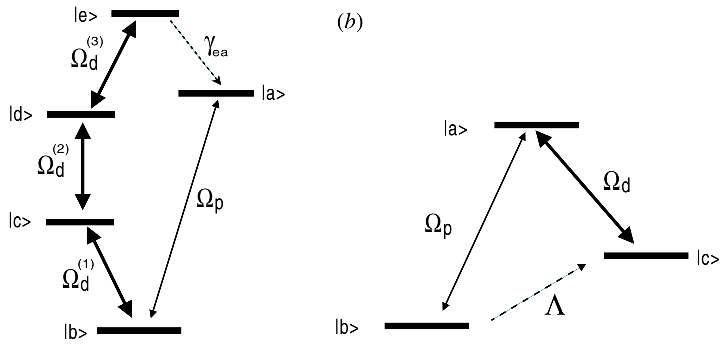

Figure 15. Possible LWI schemes for frequency up-conversion with respect to both the driving field and the pumping process. (*a*) V scheme where three laser fields pump population into an excited level $|e\rangle$ that decays, through spontaneous emission, to the upper level $|a\rangle$ of the probed transition. (*b*) Λ scheme where the rate Λ accounts for an incoherent pumping mechanism.

and maintaining the probe close to resonance in cascade schemes†.

Obviously, the ideal situation for lasing in the short-wavelength domain is that in which the frequency of the probe (lasing) field is larger than both the driving and the incoherent pump field frequencies. Figure 15 shows possible candidates in order to move towards this regime of LWI. Figure 15(*a*) shows a V-scheme configuration where three laser fields ($\Omega_d^{(i)}\ i = 1, 2, 3$) are used to pump population into the upper level $|e\rangle$ which rapidly decays into the upper probed level $|a\rangle$. Note that a careful choice of the levels, taking into account their parity, must be performed. Another possibility sketched in figure 15(*b*) consists in using an incoherent process denoted by Λ in order to populate state $|c\rangle$ in an anti-Stokes configuration in a Λ-scheme [95].

Finally, it is worth mentioning that short-wavelength coherent light can also be generated by nonlinear optical methods such as second-harmonic generation or four-wave mixing. In these methods, in contrast with the LWI method, there is no extraction of energy from the nonlinear medium; as pointed out by Mandel [6] the medium serves only as a support to the scattering process that results in the transfer of coherence and energy from the pump field(s) to the signal field. Harris *et al* [99] proposed using EIT as a method for achieving high conversion efficiency by nonlinear optical mixing. In fact, Stoicheff and co-workers [100] showed theoretically and experimentally that EIT induced by dc electric field coupling of the 2s and 2p states in atomic hydrogen leads to resonantly enhanced second-order susceptibility with reduced absorption at the second-harmonic wavelength (in the vacuum ultraviolet region), and exact phase matching at the centre of the Stark-split components. More recently, they have extended these studies to the production of extreme-ultraviolet radiation by four-wave frequency mixing in atomic hydrogen (see [101, 102] and references therein). Very interesting also are the pressure-induced extra resonances in four-wave mixing‡, discovered by Bloembergen and co-workers [104, 105], which arise because the destructive interference between two possible coherent pathways is eliminated by the collisions.

† These tunings hold for the usual case in which the driven transition is not inverted.

‡ For a review on pressure-induced extra resonances see [103].

5. Conclusions

We have reviewed the basic physics and the first experiments on AWI and LWI involving atomic transitions in gas media. Different theoretical approaches to LWI have been discussed. Thus, we have analysed in detail the origin of inversionless gain in coherently driven three- and multilevel atomic systems with special enphasis on the appearance or not of a hidden inversion. For this purpose we have made use of the quantum jump formalism which allows us to elucidate the origin of gain even though there is not inversion in any basis. In addition, we have reviewed the current state of the art of LWI pointing out that AWI/LWI experiments performed so far have still not operated in the frequency up-conversion regime. We have also summarized the main difficulties regarding this regime of frequency up-conversion LWI. Thus, the experimental realization of short-wavelength LWI is still a very interesting and challenging issue.

Finally, we would like to highlight other aspects related to LWI that have not been discussed in this paper such as, for instance, self-pulsing LWI near first lasing threshold [106–109], giant pulse lasing [110], multi-photon processes in the presence of a standing-wave drive [111], transverse effects [109, 112] and pattern formation [113].

Acknowledgments

Support by the DGICYT (Spanish Government), under contracts PB95-0778-C02-02 and PB98-0935-C03-03, and by the DGR (Catalan Government), under contract 1999SGR00096, is acknowledged. We would also like to thank the following people for their help and fruitful discussions on the present topic: V Ahufinger, E Arimondo, F B de Jong, O Kocharovskaya, W Lange, H B van Linden van den Heuvell, M Lukin, G Orriols, C Peters, F Prati, L Roso, E Roldán, M Scully, R Shuker, F Silva, G J de Valcárcel and R Vilaseca.

References

[1] Arimondo E 1996 Coherent population trapping in laser spectroscopy *Progress in Optics* vol 35, ed E Wolf (Amsterdam: Elsevier)

[2] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)

[3] Alam S 1998 *Lasing without Inversion and Electromagnetically Induced Transparency* (Bellingham, WA: SPIE Optical Engineering Press)
[4] Kocharovskaya O 1992 Amplification and lasing without inversion *Phys. Rep.* **219** 175
[5] Scully M O 1992 From lasers and masers to phaseonium and phasers *Phys. Rep.* **219** 191
[6] Mandel P 1993 Lasing without inversion: a useful concept? *Contemp. Phys.* **34** 235
[7] Gawlik W 1993 Lasing without inversion and quantum interference *Comment. At. Mol. Phys.* **29** 189
[8] Scully M O and Fleischhauer M 1994 Lasers without inversion *Science* **263** 337
[9] Taubes G 1995 Researchers build novel laser by putting a lock on atoms *Science* **270** 737
[10] Svelto O 1989 *Principles of Lasers* 3rd edn (New York: Plenum)
[11] Shimoda K 1986 *Introduction to Laser Physics* 2nd edn (Berlin: Springer)
[12] Baldwin G C and Solem J C 1997 *Rev. Mod. Phys.* **69** 1085
[13] Masiak P, Rzazewski K and Roso L 1995 *Phys. Rev.* A **51** 3267
[14] Marcuse D 1963 *Proc. IEEE* **51** 849
[15] Holt H K Gain without population inversion in two-level systems 1977 *Phys. Rev.* A **16** 1136
[16] Gheri K M, Ritsch H, Walls D F and Balykin V I 1995 How large photon recoil can make atoms to lase *Phys. Rev. Lett.* **74** 678
[17] Horak P, Gheri K M and Ritsch H 1995 Recoil-induced lasing in cold atoms *Phys. Rev.* A **52** 554
[18] Rautian S G and Sobel'man I I 1962 Lineshape and dispersion within the absorption band when forced transitions are taken into account *Sov. Phys.–JETP* **14** 328
[19] Mollow B R 1972 Stimulated emission and absorption near resonance for driven systems *Phys. Rev.* A **5** 2217
[20] Haroche S and Hartmann F 1972 Theory of saturated absorption lineshapes *Phys. Rev.* A **6** 1280
[21] Boyd R W 1992 *Nonlinear Optics* (San Diego, CA: Academic)
[22] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1992 *Atom–photon interactions—Basic Processes and Applications* (New York: Wiley)
[23] Cohen-Tannoudji C and Reynaud S 1977 Dressed-atom description of resonance fluorescence and absorption spectra of a multilevel atom in an intense laser beam *J. Phys. B: At. Mol. Phys.* **10** 345
[24] Grynberg G, Pinard M and Mandel P 1996 Amplification without inversion in a V three-level system: a physical interpretation *Phys. Rev.* A **54** 776
[25] Wu F Y, Ezekiel S, Ducloy M and Mollow B R 1977 Observation of amplification in a strongly driven two-level atomic system at optical frequencies *Phys. Rev. Lett.* **38** 1077
[26] Khitrova G, Valley J F and Gibbs H M 1988 Gain-feedback approach to optical instabilities in sodium vapour *Phys. Rev. Lett.* **60** 1126
[27] Lezama A, Zhu Y, Kankar M and Mossberg T W 1990 Radiative emission of driven two-level atoms into the modes of an enclosing optical cavity: the transition from fluorescence to lasing *Phys. Rev.* A **41** 1576
[28] Gauthier D J, Wu Q, Morin S E and Mossberg T W 1992 Realization of a continuous-wave, two-photon optical laser *Phys. Rev. Lett.* **68** 464
[29] Zakrzewski J, Lewenstein M and Mossberg T 1991 Theory of dressed-state lasers I. Effective Hamiltonians and stability properties *Phys. Rev.* A **44** 7717
[30] Zakrzewski J, Lewenstein M and Mossberg T 1991 Theory of dressed-state lasers II. Phase diffusion and squeezing *Phys. Rev.* A **44** 7732
[31] Zakrzewski J, Lewenstein M and Mossberg T 1991 Theory of dressed-state lasers III. Pump-depletion effects *Phys. Rev.* A **44** 7746
[32] Grandclément D, Grynberg G and Pinard M 1987 Observation of continuous-wave self-oscillation due to pressure-induced two-wave mixing in sodium *Phys. Rev. Lett.* **59** 40
[33] Grynberg G and Cohen-Tannoudji C 1993 Central resonance of the Mollow absorption spectrum: physical origin of gain without inversion *Opt. Commun.* **96** 150
[34] Arkhipkin V G and Heller Yu I 1983 Radiation amplification without population inversion at transitions to autionizing states *Phys. Lett.* A **98** 12
[35] Fano U 1961 Effects of configuration interaction on intensities and phase shifts *Phys. Rev.* **124** 1866
[36] Harris S E 1989 Lasers without inversion: interference of lifetime-broadened resonances *Phys. Rev. Lett.* **62** 1033
[37] Agarwal G S, Ravi S and Cooper J 1990 dc-field-coupled autoionizing states for laser action without population inversion *Phys. Rev.* A **41** 4721
[38] Agarwal G S, Ravi S and Cooper J 1990 Lasers without inversion: Raman transitions using autoionizing resonances *Phys. Rev.* A **41** 4727
[39] Imamoglu A, Field J E and Harris S E 1991 Lasers without inversion: a closed lifetime broadened system *Phys. Rev. Lett.* **66** 1154
[40] van Enk S J, Zhang J and Lambropoulos P 1995 Coherent effects through the continuum: transparency, population trapping and amplification without inversion through autoionizing resonances *Appl. Phys.* B **60** 141
[41] Khurgin J B and Rosencher E 1996 Balance equations and threshold conditions for the inversionless laser in an autoionizing system *Phys. Rev.* A **54** 2451
[42] Boller K-J, Imamoglu A and Harris S E 1991 Observation of electromagnetically induced transparency *Phys. Rev. Lett.* **66** 2593
[43] Harris S E 1997 Electromagnetically induced transparency *Phys. Today* **50** 36
[44] Zambon B 1998 Inversionless gain versus efficient gain: the autoionizing states configuration *Phys. Rev.* A **58** 690
[45] Carmichael H J 1991 An open systems approach to quantum optics *(Université Libre de Bruxelles, Bruxelles, Belgium, Autumn 1991)*
[46] Dalibard J, Castin Y and Mølmer K 1992 Wavefunction approach to dissipative processes in quantum optics *Phys. Rev. Lett.* **68** 580
[47] Dum R, Zoller P and Ritsch H 1992 Monte Carlo simulation of the atomic master equation for spontaneous emission *Phys. Rev.* A **45** 4879
[48] Javan A 1957 Theory of a three-level maser *Phys. Rev.* **107** 1579
[49] Kocharovskaya O and Khanin Y I 1988 Coherent amplification of an ultrashort pulse in a three-level medium without population inversion *JETP Lett.* **48** 630
[50] Scully M O, Zhu S-Y and Gavrielides A 1989 Degenerate quantum-beat laser: lasing without inversion and inversion without lasing *Phys. Rev. Lett.* **62** 2813
[51] Zibrov A S, Lukin M D, Nikonov D E, Hollberg L, Scully M O, Velichansky V L and Robinson H G 1995 Experimental demonstration of laser oscillation without population inversion via quantum interference in Rb *Phys. Rev. Lett.* **75** 1499
[52] Padmabandu G G, Welch G H, Shubin I N, Fry E S, Nikonov D E, Lukin M D and Scully M O 1996 Laser oscillation without population inversion in a sodium atomic beam *Phys. Rev. Lett.* **76** 2053
[53] de Jong F B, Mavromanolakis A, Spreeuw R J C and van Linden van den Heuvell H B 1998 Synchronously pumped laser without inversion in cadmium *Phys. Rev.* A **57** 4869
[54] Khanin Ya I and Kocharovskaya O A 1990 Inversionless amplification of ultrashort pulses and coherent population trapping in a three-level medium *J. Opt. Soc. Am.* B **7** 2016

[55] Alzetta G, Gozzini A, Moi L and Orriols G 1976 An experimental method for the observation of RF transitions and laser beat resonances in oriented Na vapour *Nuovo Cimento* B **36** 5
[56] Arimondo E and Orriols G 1976 Nonabsorbing atomic coherences by coherent two-photon transitions in a three-level optical pumping *Nuovo Cimento Lett.* **17** 333
[57] Orriols G 1979 Nonabsorption resonances by nonlinear coherent effects in a three-level system *Nuovo Cimento* B **53** 1
[58] Nottelmann A, Peters C and Lange W 1993 Inversionless amplification of picosecond pulses due to Zeeman coherence *Phys. Rev. Lett.* **70** 1783
[59] Fry E S, Li X, Nikonov D, Padmabandu G G, Scully M O, Smith A V, Tittel F K, Wang C, Wilkinson S R and Zhu S-Y 1993 Atomic coherence effects within the sodium D_1 line: lasing without inversion via population trapping *Phys. Rev. Lett.* **70** 3235
[60] van der Veer W E, van Diest R J J, Dönszelmann A and van Linden van den Heuvell H B 1993 Experimental demonstration of light amplification without population inversion *Phys. Rev. Lett.* **70** 3243
[61] Kocharovskaya O and Mandel P 1990 Amplification without inversion: the double-Λ scheme *Phys. Rev.* A **42** 523
[62] Kocharovskaya O A, Mauri F and Arimondo E 1991 Gain without inversion and coherent trapping *Opt. Commun.* **84** 393
[63] Karawajczyk A, Zakrzewski J and Gawlik W 1992 Dressed-atom model of lasing without inversion in the double-Λ configuration *Phys. Rev.* A **45** 420
[64] Karawajczyk A and Zakrzewski J 1994 Semiclassical study of double-lambda laser without inversion *Opt. Commun.* **107** 145
[65] Fleischhauer M and Scully M O 1994 Lasing without inversion versus optical pumping and lasing without inversion assisted by optical pumping *Opt. Commun.* **105** 79
[66] Fleischhauer M, Lukin M D, Nikonov D E and Scully M O 1994 Influence of pump-field phase diffusion on laser gain in a double-Λ non-inversion laser *Opt. Commun.* **110** 351
[67] Peters C and Lange W 1996 Laser action below threshold inversion due to coherent population trapping *Appl. Phys.* B **62** 221
[68] Mompart J 1999 Lasing without inversion *PhD Thesis* Universitat Autonoma de Barcelona, Bellaterra
[69] Mompart J and Corbalán R 1999 Lasing without inversion: realities and prospects *3rd Iberoamerican Optics Meeting and 6th Latin American Meeting on Optics, Lasers and Their Application* ed A M Guzmán (Bellingham, WA: SPIE Optical Engineering Press) pp 3572
[70] Agarwal G S 1991 Origin of gain in systems without inversion in bare or dressed states *Phys. Rev.* A **44** R28
[71] Mompart J and Corbalán R 1998 Inversionless amplification in three-level systems: dressed states quantum interference and quantum-jump analyses *Opt. Commun.* **156** 133
[72] Mompart J, Ahufinger V, Silva F, Corbalán R and Vilaseca R 1999 Quantum interference and quantum Zeno effect in amplification without inversion *Laser Phys.* **9** 844
[73] Fleischhauer M, Mc Illrath T and Scully M 1995 Coherent population trapping and Fano-type interferences in lasing without inversion *Appl. Phys.* B **60** S123
[74] Arimondo E 1992 Mechanism in laser without inversion *Proc. SPIE* **1726** 484
[75] Cohen-Tannoudji C, Zambon B and Arimondo E 1993 Quantum-jump approach to dissipative processes: application to amplification without inversion *J. Opt. Soc. Am.* B **10** 2107
[76] Carmichael H J 1997 Coherence and decoherence in the interaction of light with atoms *Phys. Rev.* A **56** 5065
[77] de Jong F B, Spreeuw R J C and van Linden van den Heuvell H B 1997 Quantum Zeno effect and V-scheme lasing without inversion *Phys. Rev.* A **55** 3918
[78] de Jong F B 1998 Lasing without inversion *PhD Thesis* The Van der Waals–Zeeman Institute, Amsterdam
[79] Bennet C H 1994 Night thoughts, dark sight *Nature* **371** 479
[80] Kleinfeld J A and Streater A D 1994 Observation of gain due to coherence effects in a potassium–helium mixture *Phys. Rev.* A **49** R4301
[81] Kleinfeld J A and Streater A D 1996 Gain and coherence effects induced by strong cw laser coupling in potassium–rare-gas mixtures *Phys. Rev.* A **53** 1839
[82] Zhu Y and Lin J 1996 Sub-Doppler light amplification in a coherently pumped atomic system *Phys. Rev.* A **53** 1767
[83] Zhu Y, Lin J and Sanchez P 1996 Mesurement of light amplification in coherently pumped rubidium atoms *Opt. Commun.* **128** 254
[84] Sellin P B, Wilson G A, Meduri K K and Mossberg T W 1996 Observation of inversionless gain and field-assisted lasing in a nearly ideal three-level cascade-type atomic system *Phys. Rev.* A **54** 2402
[85] Fort C, Cataliotti F S, Hänsch T W, Inguscio M and Prevedelli M 1997 Gain without inversion on the caesium D_1 line *Opt. Commun.* **139** 31
[86] Shiokawa N, Torii Y, Hirano T and Kuga T 1997 Light amplification without population inversion by using laser-cooled rubidium atoms *Conf. on Quantum Electronics and Laser Science 1997 (OSA Technical Digest Series vol 12)* (Washington, DC: Optical Society of America) QPD2-2
[87] Hollberg L, Zibrov A S, Kitching J, Lukin M, Robinson H G, Scully M O and Velichansky V L 1998 Optical coherence effects in laser driven atoms: concepts and experiments *16th Int. Conf. on Coherent and Nonlinear Optics (Technical Digest)* (URSS Publishers) p 215
[88] Kitching J and Hollberg L 1999 Interference-induced optical gain without population inversion in cold, trapped atoms *Phys. Rev.* A **59** 4685
[89] Lukin M D, Scully M O, Welch G R, Fry E S, Hollberg L, Padmabandu G G, Robinson H G and Zibrov A S 1996 Lasing without inversion: the road to new short-wavelength lasers *Laser Phys.* **6** 436
[90] Lukin M D 1997 Quantum coherence and interference in optics and laser spectroscopy *PhD Thesis* Texas A & M University, College Station
[91] Yelin S F, Lukin M D, Scully M O and Mandel P 1998 Gain without inversion in the frequency up-conversion regime *Phys. Rev.* A **57** 3858
[92] Mompart J, Corbalán R and Vilaseca R 1998 Lasing without inversion in the V-type three-level system under the two-photon resonance condition *Opt. Commun.* **147** 299
[93] Karawajczyk A and Zakrzewski J 1995 Lasing without inversion in a Doppler-broadened medium *Phys. Rev.* A **51** 830
[94] Boon J R, Zekou E, Fulton D J and Dunn M H 1998 Experimental observation of a coherently induced transparency on a blue probe in a Doppler-broadened mismatched V-type system *Phys. Rev.* A **57** 1323
[95] Pinard M, Wasik G, Gawlik W and Zachorowski J 1999 Amplification without inversion in a medium with collisional dephasing *Phys. Rev.* A **59** 848
[96] Ahufinger V, Mompart J and Corbalán R 1999 Lasing without inversion with frequency up-conversion in a Doppler-broadened V-scheme *Phys. Rev.* A **60** 614
[97] Mompart J, Ahufinger V, Corbalán R and Prati F Propagation effects on lasing without population inversion *J. Opt. B: Quantum Semiclass. Opt.* at press
[98] Zhu Y 1992 Lasing without inversion in a closed three-level system *Phys. Rev.* A **45** R6149
[99] Harris S E, Field J E and Imamoglu A 1999 Nonlinear

optical processes using electromagnetically induced transparency *Phys. Rev. Lett.* **64** 1107
[100] Hakuta K, Marmet L and Stoicheff B P 1991 Electric-field-induced second-harmonic generation with reduced absorption in atomic hydrogen *Phys. Rev. Lett.* **66** 596
[101] Zhang G Z, Tokaryk D W and Stoicheff B P 1997 Nonlinear generation of extreme-ultraviolet radiation in atomic hydrogen using electromagnetically induced transparency *Phys. Rev.* A **56** 813
[102] Tokaryk D W, Zhang G Z and Stoicheff B P 1999 Nonlinear optical generation in a hydrogen discharge *Phys. Rev.* A **59** 3116
[103] Agarwal G S 1992 *Adv. At. Mol. Phys.* **29** 113
[104] Prior Y, Bogdan A R, Dagenais M and Bloembergen N 1981 Pressure-induced extra resonances in four-wave mixing *Phys. Rev. Lett.* **46** 111
[105] Bogdan A R, Downer M W and Bloembergen N 1981 Quantitative characteristics of pressure-induced degenerate frequency resonance in four-wave mixing with continuous-wave laser beams *Opt. Lett.* **6** 348
[106] Sánchez-Morcillo V, Roldán E and de Valcárcel G J 1995 Lasing without inversion via a self-pulsing instability *Quantum Semiclass. Opt.* **7** 889
[107] Mompart J, Peters C and Corbalán R 1998 Lasing without inversion in three-level systems: self-pulsing in cascade schemes *Phys. Rev.* A **57** 2163
[108] Vladimirov A G, Mandel P, Yelin S F, Lukin M D and Scully M O 1998 Nonlinear dynamics in a single-mode three-level laser without inversion *Phys. Rev.* E **57** 1499
[109] Prati F, Mompart J and Corbalán R 1999 Transverse profile effects on lasing without inversion *J. Opt. B: Quantum Semiclass. Opt.* **1** 146
[110] Mompart J, Corbalán R and Vilaseca R 1999 Giant pulse lasing in three-level systems *Phys. Rev.* A **59** 3038
[111] Silva F, Ahufinger V, Mompart J and Corbalán R 1999 Inversionless amplification with standing wave drive *Laser Phys.* **9** 858
[112] Zhang H-Z and Gao J-Y 1997 Effects of transverse profile of probe and driving fields on the gain in a Raman-driven four-level system *Phys. Rev.* A **56** 2225
[113] García-Ojalvo J, Torrent M C and Vilaseca R 1998 Spatiotemporal dynamics of dressed broad-area lasers *Quantum Semiclass. Opt.* **10** 809