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A FORMULA FOR $C(T)$ IN GUPTA'S PAPER

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§1. In the preceding paper*, Hansraj Gupta tells how I brought to his attention the problem which he discusses, and how we worked on it independently and solved it. In this short note, I say briefly what the problem was and what remained to be done about it

Suppose that we have a circle whose circumference is divided into n equa arcs, the points of division being marked by dots. Any k of these points may be joined by straight lines to form a convex k-gon. The arc-lengths of the sides of this k-gon sum up to n. If rotations and reflections are considered to be redundant, Gupta finds in his paper, an enumeration function $R(n, k)$ giving the number of possible k-gons, different from each other in the sense that none of these can be obtained from any other by rotation or/and reflection. In finding the formula for $R(n, k)$, use was made of functions $C(T)$ which represent the contribution to $R(n, k)$ of any partition of n of the type

$$T = (t_1, t_2, \ldots, t_j), t_1 + t_2 + \ldots + t_j = k$$

into k parts, i.e. one which has t_1 parts each equal to b_1; t_2 parts each equal to b_2; ...; and t_j parts each equal to b_j. Without loss of generality, we may take

$$t_1 \leqslant t_2 \leqslant \ldots \leqslant t_j.$$

Recall that $C(T)$ does not depend on the size of the b's (all distinct of course) but only on their frequencies i.e. t's.

Beyond giving a few 'easy to prove' rules (see section 3.3 of Gupta's paper), which do not cover all the cases that arise, Gupta does not say anything about the evaluation of $C(T)$ for any given T.

The object of this note is to give a general formula for $C(T)$ which will be as pretty as Gupta's for $R(n, k)$. While I am certain that the formula is correct, I must leave the proof to the reader or to Professor Gupta, for I simply do not know how to do it, my excuse being that I am not really a mathematician.

*See pages 964-999 of this issue of the Journal.

§2. *The formula for* $C(T)$ — Let g.c.d. $(t_1, t_2, \ldots, t_l) = g$, then we have

$$2\,C(T) = S(T) + C'(T),$$

where

$$S(T) = u.\,\frac{([t_1/2] + [t_2/2] + \ldots + [t_l/2])!}{[t_1/2]!\,[t_2/2]!\ldots[t_l/2]!}$$

with $u = 0$ if at least three of the t's are odd;

$= 1$ otherwise;

and

$$C'(T) = \frac{1}{k}\sum_{d\mid g}\frac{\phi(d).(k/d)!}{(t_1/d)!\,(t_2/d)!\ldots(t_l/d)!}.$$

As the reader will readily guess, my cue came from the rules given by Gupta in his paper and the formula

$$2\,R(n, k) = S(n, k) + \frac{1}{k}\sum_{d\mid(n,k)}\phi(d)\left(\frac{n}{d} - 1;\frac{k}{d} - 1\right).$$

Examples

(i) $$C'(6, 6) = \frac{1}{12}\left\{\frac{12!}{6!\,6!} + \frac{6!}{3!\,3!} + 2.\frac{4!}{2!\,2!} + 2.\frac{2!.}{1!\,1!}\right\}$$

$$= \frac{(924 + 20 + 12 + 4)}{12} = 80;$$

and $$S(6, 6) = \frac{6!}{3!\,3!} = 20.$$

Hence $C(6, 6) = 50.$

(ii) $$C'(3, 3, 3) = \frac{1}{9}\left\{\frac{9!}{3!\,3!\,3!} + 2.\frac{3!}{1!\,1!\,1!}\right\}$$

$$= \frac{(1680 + 12)}{9} = 188;$$

and $S(3, 3, 3) = 0;$

so that $C(3, 3, 3) = 94.$