Flatland: The Movie

Reviewed by Ian Stewart

Flatland: The Movie—A Journey of Many Dimensions
Flat World Productions
Directed by Jeffrey Travis

"It must be that a square… moving *somehow parallel to itself…can make something else—like a…* supersquare *that represents three to the third power, or 27 units…That's so hard to imagine, but—wouldn't that be amazing? A supersquare in the third dimension! I wonder what's so special about Area 33H? I bet it has a clue to explain all this!"*

And so the intrepid little Hex, a small orange hexagon living in a Euclidean plane, defies her well-meaning grandfather, Arthur Square, and comes into conflict with the arrogant priesthood by dabbling in heresy. And her world is changed, forever.

It's *Flatland*—I don't think I'm giving much away here—updated for a modern audience, animated using modern techniques, and packaged into a 45-minute movie. The DVD includes an interview with Tom Banchoff, geometer and *Flatland* authority *par excellence*, but I'll confine this review to the main story.

Flatland, first published in 1884, was written by the clergyman, schoolmaster, and Shakespearean scholar Edwin Abbott Abbott—*two* Abbotts, to distinguish him from his father, Edwin Abbott, and to commemorate his mother, Jane Abbott, who was his father's cousin. *Flatland* is a much-loved classic, the founding member of a tiny but select sub-genre of "Math Fiction", most of whose other members are sequels by modern writers or short science fiction stories with an overtly mathematical flavour. Its main competitors are Charles Howard Hinton's 1907 *An Episode of Flatland* and Kee Dewdney's 1984 *The Planiverse*. Both of these writers pay close attention to the physics and engineering of two-dimensional structures, whereas Abbott is more interested in telling the story of "A. Square" and his revelation of the Third Dimension.

Anyone who tries to reinterpret or extend *Flatland* faces a number of obstacles. The original was written in an elegant but old-fashioned style. It managed to combine a masterly exposition of the "dimensional analogy" between the Flatland view of 2-space and the human view of 3-space with some biting social criticism of the Victorian treatment of two disadvantaged groups: women and the poor. It relied for its effect on a default view of society that mercifully has largely vanished. And it started from a level of scientific and mathematical understanding that is now 120 years out of date. So the choice facing any would-be writer or animator is simple but stark. They can pander to the *Flatland* fans' prejudices, and follow the original faithfully for both content and style…but then they will without doubt offend a number of pressure groups, because satire and political correctness are uneasy bedfellows. Alternatively, they can bring both style and content up to date, and risk the wrath of those who dislike anyone tampering with holy writ.

Abbott lived and worked in Victorian England, which we tend to think of as an inhibited, hypocritical, and exceedingly snobbish society in which everyone knew their place, and everyone except criminals and other low-life conformed

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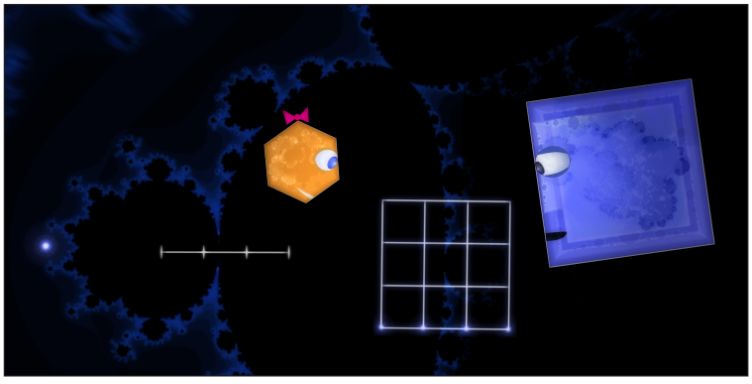
Distribution of *Flatland: The Movie*

The "Special Educator Edition" of *Flatland: The Movie* (US$120.00) is ready for purchase on the website `http://store.flatlandthemovie.com`. This edition is primarily intended for educators, teachers, schools, and institutions that will use the movie as part of classes, lectures, and courses. The purchase of the DVD includes a school site license, a bonus featurette discussing the 4th dimension, behind-the-scenes footage, and math worksheets for use in the classroom.

At the time of this writing, Flat World Productions was also taking pre-orders for a private home-use DVD (US$29.95), to be shipped starting in fall 2007.



Arthur Square and his granddaughter Hex imagine the geometric progression of dimensions.

to the prevailing social norms. The Church (of England, thanks to the marital problems of Henry VIII) was the guardian of the nation's morals, the Law regulated behaviour, and the Monarchy set policy—ordinary people did not have the vote and Parliament mainly expressed the views and wishes of the wealthy. The Queen supposedly had to be protected from such erotic sights as the legs of a piano, which were covered up to avoid exciting wild passions.

Well, yes, maybe...but it wasn't quite that simple. Victoria collected paintings of male nudes, one being presented to her by her husband Albert, and had a healthy enjoyment of sex. More significantly, despite its apparent social conformity, Victorian England also gave the world some of its greatest innovations. Darwin's *Origin of Species*, first published in 1859, made a persuasive case against supernatural creation by pointing out the mechanism of natural selection. Charles Lyell's 1830 *Principles of Geology*, following pioneering work of James Hutton, made it clear that our planet is far older than had previously been assumed. Advances began to be made in the humane treatment of prisoners and the poor. Women began to free themselves from centuries of subservience. In fact, many basic rights and ideas that intelligent, informed people now take for granted emerged from the Victorians' struggle to reconcile their rigid social structure with a changing understanding of the natural world.

Mathematics, too, changed. When Victoria came to the throne in 1837, mathematics was almost entirely concrete, its concepts mostly modelled on the natural world. Analysis had embraced complex numbers and progressed as far as elliptic functions, but its logical foundations remained obscure, and it was really just calculus. Geometry remained rooted in human perceptions of physical space, although it was now projective as well as Euclidean. Algebra was largely about the solution of polynomial equations with numerical coefficients. There were revolutions waiting in the wings—Bernhard Bolzano was pioneering logical rigour in calculus; Janós Bolyai and Nicolai Ivanovich Lobachevsky had invented non-Euclidean geometries, though these were still neither welcomed nor understood; Evariste Galois had begun to free algebra from the shackles of numerical interpretation. But these revolutions were at best incipient.

By 1901, when Victoria died, mathematics had become almost unrecognisable. Abstractions were replacing the concrete. Non-Euclidean geometry was part of the mainstream, joined by differential geometry and even the beginnings of topology. Karl Weierstrass and his successors had reformulated analysis, and infinite processes were no longer a puzzle. Georg Cantor had dazzled the world, and baffled it, with set theory and transfinite numbers. Algebra now included finite fields, rings of algebraic integers, and groups. William Rowan Hamilton had struggled his way to the invention of quaternions, Sophus Lie had invented his eponymous groups, Henri Poincaré had encountered the obstacle that we now call his conjecture, and David Hilbert had produced his hit-list of twenty-three unsolved problems. Almost everything that was taken for granted at the start of Victoria's reign was up for grabs when it ended. Allegedly "self-evident" propositions proved to be false, allegedly universal imperatives were revealed as parochial conventions, and alleged impossibilities were not only possible, but unavoidable.

The late Victorian era, in short, was a period of remarkable progress and free thinking. Science and the Church came to a gentleman's agreement not to tread on each others' toes, and many a country clergyman became the world expert on six types of beetle or the reproductive habits of slugs. Scientific advances were discussed along with the price of sugar and the increasingly parlous state of the British Empire at garden parties and polite social gatherings.

Victorians, in particular, were fascinated by "the" fourth dimension. Mathematicians had come to recognise that the dimensionality of our own familiar space did not necessarily impose constraints on the dimensionality of any other structure. Mathematics was littered with "spaces"

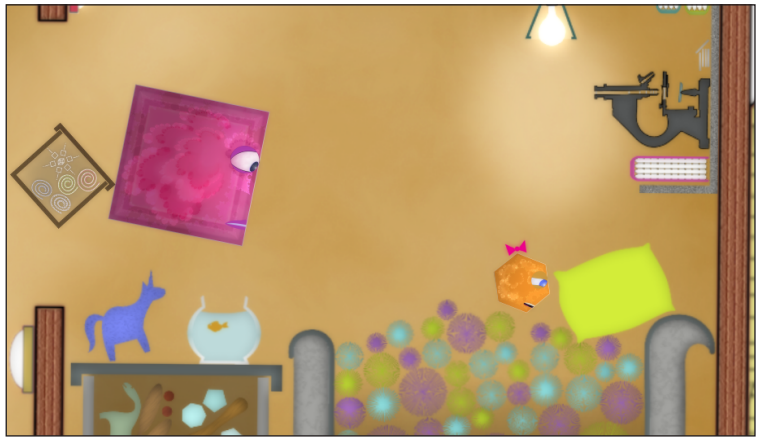
of dimension four, or ten, or a hundred. Many of these spaces accurately represented aspects of the physical world—for instance, as "degrees of freedom" of a mechanical system. Spiritualism, another flourishing Victorian interest, latched on to the fourth dimension as a convenient location for the spirit world. Ghosts could enter our world "sideways" along a dimension that mere humans could not observe or experience. Hyperspace theologians seized on the fourth dimension as an excellent place in which to put God and His angels, though they quickly realised that the fifth, sixth, and seventh dimensions were even better, and the infinitieth dimension added a satisfactory element of closure.

While well-meaning people and charlatans of every kind were appealing to the fourth dimension to justify their beliefs and scams, the mathematics of four or more dimensions was changing from an obscure and esoteric collection of ideas into something straightforward and ubiquitous. And it was at this juncture that Abbott produced his curious and highly original book. *Flatland* has a timeless appeal, and it remains in print to this day; as I write it is the bestselling mathematics book on Amazon. It has spawned several sequels and two animations, the most recent being the subject of this review, to which I now return.

To appreciate the problems faced by the animators, and how they handle them, let me briefly recall the plot. Flatland is a Euclidean plane, inhabited by polygonal beings whose lives resemble our own closely enough for social satire. The main character, A. Square, lives in a pentagonal house together with his wife and children. His wife, like all Flatland women, is a mere line segment. His four sons are pentagons, his two grandsons are hexagons. Flatland heredity produces an extra side in each generation, except in the isosceles triangles of the lower classes, where each generation approaches more closely the equilateral ideal that admits the family into the middle classes. Irregular offspring are destroyed. Intelligence (allegedly) and social standing (indubitably) increase with the number of sides, and the pinnacle of society comprises the priests, which are circles—or, more accurately, polygons with several hundred sides.

The most serious heresy in Flatland's totalitarian theocracy is belief in the Third Dimension. Every new millennium a Sphere from this nonexistent realm visits Flatland and causes trouble, and the priests suppress public knowledge of such incidents to preserve their own power. Poor Mr. Square (Abbott never tells us what "A" stands for, because his protagonist is merely *a* square) gets caught up in these events and is set up for the book's main mathematical theme, a dimensional analogy. Our own difficulties in contemplating the fourth dimension are analogous to those of A. Square contemplating the third.

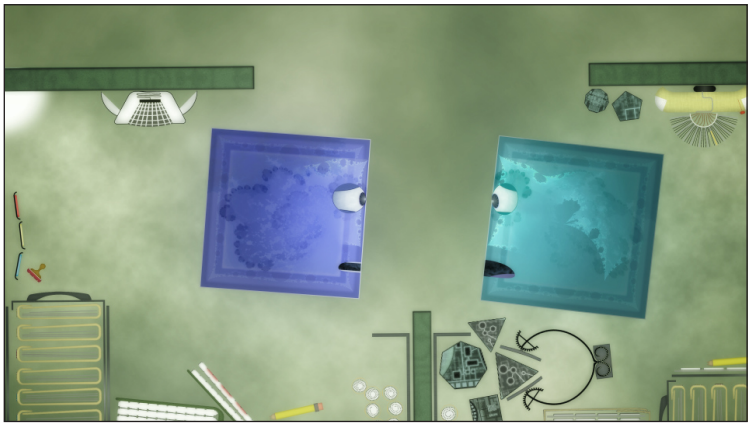
Arthur and Arlene Square in their Flatland Home.



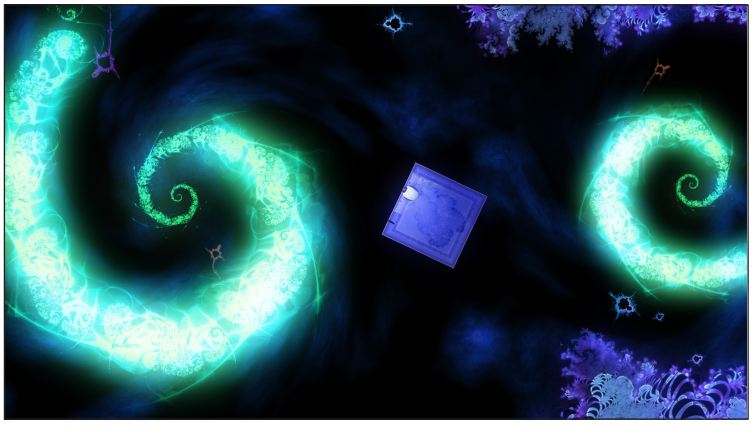
Hex is comforted by her grandmother, Arlene Square.

Abbott uses this analogy to explain four-dimensional space to his readers, while appearing to be explaining three-dimensional space to A. Square. It seems likely that he got the basic ideas from Charles Howard Hinton, an incorrigible rogue and accomplished geometer who wrote widely about four dimensions. In 1880 Hinton published an article "What is the Fourth Dimension" in the *Dublin University Magazine*, and it was reprinted in the *Cheltenham Ladies College Gazette* a year later. In 1884 it was reprinted yet again as a pamphlet, with the subtitle "Ghosts explained". While there is no documentary evidence that the two men ever met, the similarities between Hinton's article and plot elements of *Flatland* are extensive.

It also seems inconceivable that they did not meet. Abbott's interest in advancing the education of women brought him into contact with Dorothea Buss, headmistress of Cheltenham Ladies College, and Hinton was appointed to teach at the college in 1875. In 1884 Hinton moved to Uppingham School as science master, where Abbott's lifelong friend Howard Candler was mathematics master. Abbott and his wife were regular visitors to the Candlers. *Flatland* is dedicated to "The Inhabitants of SPACE IN GENERAL and H. C. IN PARTICULAR", and H. C.



Arthur Square shocks his brother and officemate Abbott Square.



Arthur awakens in a mysterious part of Flatland.

is Candler. Finally, Abbott's theological book *The Kernel and the Husk* of 1887 refers to "a very able and original work by C. H. Hinton" about "a being of Four Dimensions".

In 1907 Hinton produced his own book, the aforementioned *An Episode of Flatland*, subtitled *How a Plane Folk Discovered the Third Dimension*. The geometry of Hinton's world "Astria" is closer to our own: not an infinite Euclidean plane, but the surface of a circular planet in a planar (actually, slightly curved) universe. Hinton was probably influenced by Herbert George Wells, who used four dimensions to justify time travel in his 1895 novel *The Time Machine*, and Wells picked up his ideas from the wider scientific community.

At any rate, the Sphere convinces A. Square that the third dimension exists, but he succeeds only when he takes the drastic step of pushing A. Square out of his plane into the wider realm of Space. Even the most ardent disbeliever in the fourth dimension might well change their minds in similar circumstances. But the ending is dark and tragic. A. Square, on his return to Flatland, proclaims the Truth of the Third Dimension and ends up in jail, having failed to convince anybody else, and at times doubts his own sanity: "It is part of the martyrdom which I endure for the cause of the Truth that there are seasons of mental weakness, when Cubes and spheres flit away into the background of scarce-possible existences; when the Land of Three dimensions seems almost as visionary as the Land of One or none; nay, when even this hard wall that bars me from my freedom, these very tablets on which I am writing, and all the substantial realities of Flatland itself, appear no better than the offspring of a diseased imagination, or the baseless fabric of a dream."

Fans of the original will wonder how much of this survives in the animation. (The dangers are evident: my friend Terry Pratchett once sold an option for the movie rights to his book *Mort*, which is all about Death's apprentice. Death, in the Discworld series of which this one is a part, is a skeletal figure with a scythe. The gentlemen from Hollywood contemplated their purchase and approached Terry, saying "Is it okay if we leave this Death guy out? He's a bit depressing.") The good news is: more than you might expect. The price to be paid (I don't think it's bad news but some of you will) is that the story has been modernised.

Arthur Square (so much for my theory that his name had to be Albert, for reasons you will find in the preface to my sequel *Flatterland*) and his wife are living in a house much the shape of any normal three-floor building, with their granddaughter Hex, who is a hexagon. Equal opportunities have clearly arrived in Flatland, and the women are no longer one-dimensional and mindless. Little Hex is bright as a button, and has a deep desire to find out what happened to her parents. Arthur knows, but insists that she waits until she is old enough to understand.

Social standing still depends upon how many sides you have, as we are reminded when Hex is going over her homework with Grandpa, but she is sceptical, pointing out that she doesn't feel any more intelligent than her four-sided progenitor. The top dogs are still the priests, which are circular. So are the senior civil servants, notably Miss Helios, who is some sort of Big Cheese in the Ministry of Regularity where Arthur works. The circles are portrayed as arrogant and strident. Their attitude seems to be more a parody of Soviet-era communism than anything else, except for the Circle of Circles, Pantocyclus, who runs the whole country and makes a credibly nasty high priest.

The Ministry employees assemble, and Pantocyclus addresses them on the subject of the New Millennium, warning them not to talk about the Third Dimension. This topic always surfaces at such times, mainly because that is when a visitor from the Third Dimension has a habit of appearing and proclaiming that he and his world do, in fact, exist. But this cannot be so, because "such a notion is of course absurd, and furthermore illegal." In particular, the law-abiding citizens of

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Flatland are forbidden to approach Area 33H, where foolish conspiracy theorists have long claimed some artifact from the Third Dimension is being kept.

This proclamation leads Hex to ask "what is a dimension?", and the dimensional analogy can then be wheeled out. A point, a line three units long, and a square with sides three units long, serve to illustrate the progression 3^0, 3^1, 3^2. This reminds Hex of some pictures she once saw, supposedly from Area 33H, and prompts her to wonder about the next term in the series, and conjecture the existence of a supersquare. Grandpa tells her off and justifies his harshness to his wife: "Hex is going down a dangerous path. The circles don't tolerate curiosity." To which his wife responds "Apparently neither do you," and goes off to comfort Hex, giving her a box of documents that once belonged to her (mysteriously vanished) mother.

Arthur now has a dream in which he visits Pointland and Lineland, sequences that are fairly faithful to the original. The solipsistic King of Pointland acknowledges no other thing than himself in the entire zero-dimensional universe. The King of Lineland has a Queen on either side and is shocked when Arthur intersects his linear world, so that a new line segment suddenly appears from nowhere. Arthur's explanation that he comes from *above* the line is met with incredulity: "Above…uh? What do you mean by this? You jest, Arthur of the above. The youngest child knows that space consists of the two directions—left and right." In vain does Arthur tell him that real space also has *width*—the vast two-dimensional plane of Flatland.

At the dawn of the New Millennium, Arthur receives a visit from Spherius the sphere, an extradimensional being from the mysterious world of Space. But this is no dream. A quick run through the dimensional analogy fails to convince Arthur that Space can possibly exist, so Spherius bumps him out of Flatland. Now he sees the entire plane spread out before him. "I can see *inside* everything…I can see inside everyone's bodies and I'm going to be sick." He even sees the artifact in Area 33H, which is a slowly spinning cube. Now Arthur realises that Hex was right—a supersquare does exist. And back to Flatland he goes, to spread the gospel of the Third Dimension.

No sooner has he returned than Hex goes off on some unspecified errand, and the isosceles security staff detect an intruder in Area 33H, a discovery that is announced to all the civil servants in the Ministry of Regularity. Putting two and two together, Arthur and his colleague Abbott Square head off to rescue Hex…Well, that's about as much of the plot as I can give away without spoiling the story, but you get the idea.

I think it works. The graphics, with their Mandelbrot-ish decorations, are wonderful, and so is the soundtrack music. The characters work pretty well, for polygons, and the dialogue stays on the right side of sentimentality, though occasionally it gets too close for comfort. And Hex's voice reminds me of Babe, the pig that became a sheepdog. But then, Dorothy Parker said much the same about Winnie-the-Pooh ("Tonstant weader fwowed up"), and all that did was prove she didn't have a clue what was going on. Pooh Bear (the one portrayed by A. A. Milne, not the Disneyfied one in the movies) is a blunt, matter-of-fact type. Anyway, *Babe* was a brilliant movie. The portrayal of a two-dimensional world is played for amusement rather than physical realism, which is a sensible choice; otherwise the story would get hung up trying to explain a realistic design for an internal combustion engine in 2D or how a hexagon eats peas when it has to hold the fork in its mouth. The technology is visibly more modern than anything in Abbott's version.

The ending (which I mustn't divulge, but we do find out about Hex's mother's suppressed work on cubic equations) is a bit abrupt, but we're not talking social realism here. There is a marvellously subtle hint about—no, I can't even reveal *that*. Damn. The animation of *Flatland* stays close to the original in spirit, reinterpreting the tale for today's audiences. You won't learn a lot about either geometry or the fourth dimension just by watching the movie, but I can imagine young people being intrigued by the ideas. And a teacher or parent with moderate skills can draw a lot of useful lessons from this animation, as Banchoff's interview makes clear. The main objective is to have fun playing around with the dimensional analogy, and I'd say the animation does that very well indeed. I only wish it had been a bit longer, but I imagine the budget didn't stretch that far, and it's always better to leave your audience wanting more.

Maybe we'll get the Director's cut next year.