## Book Review

# The Archimedes Codex

*Reviewed by J. L. Berggren*

**The Archimedes Codex: How a Medieval Prayer Book is Revealing the True Genius of Antiquity's Greatest Scientist**
*Reviel Netz and William Noel*
*Da Capo Press, 2007*
*US$27.50, 352 pages*
*ISBN-13: 978-0306815805*

This book tells the story of one of the most famous codices in the world, a medieval prayer book that turned out to contain not only prayers but also text from lost treatises of Archimedes.

The authors of this book have been closely involved in the study of the codex since an anonymous buyer bought it for US$2.2 million dollars at Christie's auction house in New York in October of 1998, and both are uniquely qualified to write on the topic. Reviel Netz, professor of philosophy and classics at Stanford University, is a historian of mathematics who has been engaged in publishing an English translation of the works of Archimedes. He has also, almost since the purchase of the codex, been directing the scholarly study of its Archimedean text. The other author, William Noel, is curator of manuscripts at the Walters Art Museum in Baltimore, where the codex is currently housed, and director of the Archimedes Palimpsest Project. Together they have written a book that, although aimed at a "lay" audience, is one that any mathematician interested in a fascinating chapter in the history of mathematics will surely enjoy.

The reviewer first met Archimedes in 1972 in a small seminar run by Asger Aaboe at Yale University, where we read the Greek master in Greek and English. The Greek text was that of the edition of the Danish philologist J. L. Heiberg, who devoted much of his scholarly life to editing the extant texts of the great Greek mathematicians (with Latin translations for those who could not read the Greek!).

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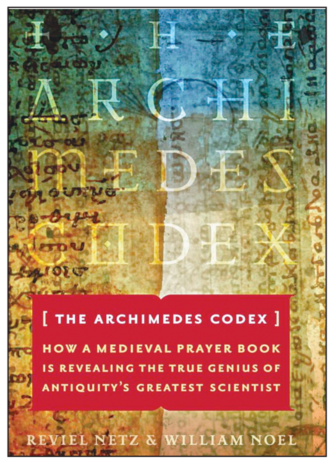


We learned, of course, something of the curious story of the oldest (by 400 years) surviving manuscript of some of Archimedes' works, collected in a Byzantine codex that contained the end of *Equilibrium of Planes*, great parts of *The Sphere and the Cylinder* and *Measurement of the Circle*, a sizable portion of *On Floating Bodies*, one folio of *The Stomachion*, and a large portion of *The Method*. This codex had not been easy for Heiberg to read because it was a palimpsest. It seems that at some point early in the year 1229, a Greek priest, Ioannes Myronas, needing parchment for a prayer book, took a collection of Archimedes' works that had been written on parchment about 250 years earlier and erased, as best he could, the Archimedean text from those of its pages that were still usable. He then used these pages to write the text of a collection of Greek prayers at right angles to the much fainter Archimedean text underneath. Myronas in fact needed more parchment than the old Archimedes codex could provide, so he also recycled other important, now lost, texts in the same way. (But that is another story.) Despite the obstacles to reading the text, Heiberg felt it was more than worth the effort, especially since the Greek texts of *Floating Bodies*, the *Stomachion,* and the *Method*, were unknown. *Floating Bodies* existed only in an incomplete Latin translation, only a short Arabic account of the *Stomachion* was known, and the *Method* was known only from its mention in a Byzantine lexicon and three brief citations, of results only, in Heron's *Metrika*. Moreover, since the one extant folio of the *Stomachion* was almost impossible to read, while a large part of the *Method* was readable (at least to Heiberg's practiced eye), it was the *Method* that attracted the most attention.

When Heiberg published it in his revised edition of Archimedes' works during the years 1910–1915, it forced scholars to revise completely their understanding of Archimedes and Greek mathematics. For the first time we had a Greek mathematician—and the greatest one of all, at that!—explaining a powerful, heuristic method he had discovered for finding areas, volumes, and centers of gravity of such figures as segments of parabolas, spheres, and conoids.

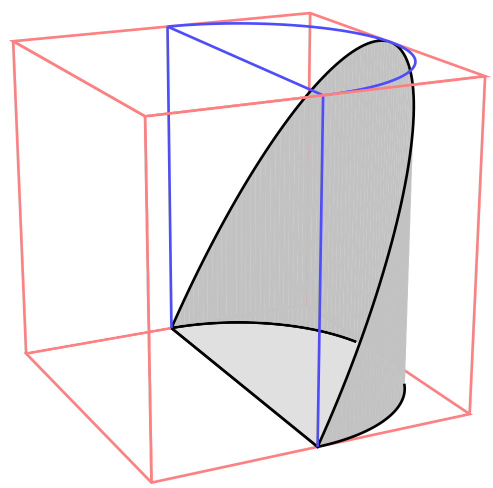

**Figure 1.**

This is what the reviewer, and most historians of Greek mathematics, knew in the early 1970s. But, after that, the manuscript of the *Method* dropped off the radar. There were rumors that it was with a private owner in France and, later, that an American scholar was trying to interest an American university in purchasing it. But we believed that Heiberg had read what could be read, Archimedes' method was well understood, and, although we might like to have some of the gaps filled in, they would, if we could read them, probably not tell us much that we didn't already know.

Thus, when the manuscript appeared in an auction at Christie's in New York and was purchased for US$2.2 million by a private collector in a spirited bidding war, many historians of ancient mathematics would have agreed with what Netz records as his opinion at the time (p. 188): "In terms of the traditional concerns of the history of mathematics I doubted that the Palimpsest could teach us much that was new. Perhaps we would be able to read something, perhaps not. But it would not be of much consequence to the history of mathematics."

Indeed, as Noel points out, the manuscript had been in private hands for a number of decades before its sale and was in significantly worse condition than when Heiberg first saw it in 1906. Amongst other problems, mold was rampant, some leaves were so fragile one dared not touch them, and parts of the text Heiberg saw in 1906 had been covered by forged Byzantine art (doubtless to increase the value of the codex).

This perceived lack of scholarly importance and abysmally poor condition may be one of the reasons why public institutions that were offered the book before the sale, at a price well below US$2.2 million, all declined. To those who will lament this fact, Noel has some advice: "If you think this is a shame then it is a shame that we all share. We live in a world where value translates into cash. If you care about what happens to world heritage, get political about it, and be prepared to pay for it." Yet, in this case, it is probably fortunate that the palimpsest ended up in private hands—at least in the particular private hands it did. For it is very hard to imagine that a university, responsible to its board of governors, or a government library, responsible to a legislative body, would even have dreamed of spending the kind of money the purchaser has spent on the purchase, preservation, and restoration of this ancient ruin.

But those of us who thought Heiberg had read what could be read reckoned without knowledge of the remarkable advances in imaging technology, which the authors explain at considerable length. And we reckoned without either the generosity of the purchaser or the determination of Noel and Netz to explore possibilities in the use of this technology. Thus, as a result of much money being spent and devoted labors by a large team of specialists, much that is new has been read, and many things have been learned about Archimedes since 1998. (The authors repeatedly credit by name the dedicated individuals whose specialties range from paleography to combinatorial mathematics and nuclear physics.) What has been read so far will influence how we look at the role of the visual in the history of mathematics, our view of Archimedes' techniques for dealing with infinity, and our view of the history of combinatorics.

As for the role of the visual, it was not the text that Netz initially thought would be the most instructive but (p. 22) the diagrams accompanying the text. These he hoped might be closer to the originals than the diagrams in our other two main sources for Archimedes' works. Netz makes a convincing case that Greek geometry was about diagrams, in the sense that they played the same role there that equations play in modern mathematics, and thus it would be important to try to get as near as possible to the diagrams Archimedes actually drew. And, as Netz demonstrates in Chapter 4 ("Visual Science"), the diagrams in the codex probably are closer to the ones Archimedes drew than any we had earlier. Looking at the diagrams Netz found, one sees they are far from self-explanatory: as with equations, one has to learn to interpret them. Netz, naturally, focuses on the diagrams of the *Method*, where chords of circles are occasionally represented as concave arcs and arcs are sometimes represented as straight lines. The diagrams are not "realistic" but "schematic", as—for a further striking example—are those representing the geometry of a surface of a sphere in the ancient manuscripts of Euclid's treatise on spherical astronomy, his *Phænomena*.[1]

The chapters in the book alternate between those written by Netz on—broadly— Archimedes' mathematics and its historical significance and those written by Noel, on the history of the manuscript and what it takes to make a badly decayed palimpsest readable once again. This device works well, and the alternation of "voice" and focus keeps the reader's interest alive.

Netz begins in Chapter 2 with a discussion of Archimedes' family background and circle of friends, but he occasionally treats intriguing conjecture as fact. For example, of Netz's three claims that "The grandfather [of Archimedes] was an artist and the father was…an astronomer who turned to the new religion of beauty and order in the cosmos," only the father's profession is more than conjecture based on slender evidence. Readers will, however, enjoy Netz's account of Archimedes' mathematical style and a joke Archimedes played on his mathematical colleagues.

Netz's discussions of several pieces of Archimedes' mathematics convey both his style and the depth and beauty of his work very well. And anyone who has studied Archimedes' works and their historical influence will know that both topics invite superlatives. However, this reviewer must dissent from Netz's paraphrase of Whitehead's remark about the European philosophical tradition and Plato that "The safest general characterization of the European scientific tradition is that it consists of a series of footnotes to Archimedes."

In support of this astonishing claim Netz cites Galileo's approximation of curves (by rectilineal figures) and his use of the proportions of times and motions, both ideas that Netz credits to Archimedes. Of course, the importance of Archimedes' works for the scientific revolution is without doubt. However, not only Archimedes but also Aristotle in his *Physics* (equally well-known to the European Renaissance) wrote about the proportionality of motions and times. And Archimedes' idea of approximating curves by rectilineal figures was not too great a step from the use of regular polygons to approximate the areas of circles that we find in *Elements* XII, 2. Nor was it too far from the idea of Eudoxus, who, as Archimedes tells us in the preface to his *Method*, "first proved" that a cone is a third part of the circular cylinder containing it. (This is not to deny that Archimedes' idea was an important step, but only to point out that that it was an extension of ideas that proved to be equally important in the scientific revolution.)

Netz also claims that (p. 28) "The two principles that the authors of modern science learned from Archimedes are: 'The mathematics of infinity' and 'the application of mathematical models to the physical world'."

As to the former, it has long been accepted that Greek mathematicians, in the words of Aristotle, neither used infinity nor did they need to. When the "infinite" entered Greek mathematics it was as the potentially infinite, i.e., as the possibility of always bisecting a line one more time, extending a line by a fixed length once more, or always finding a whole number greater than any given one. For that reason, Netz is justifiably excited about what seems to be a use of an actual infinity in Archimedes' mathematics, one that he and a colleague, Ken Saito, found in the text of Proposition 14 of the *Method*.

Here Archimedes studies the volume of the solid shown in gray in Figure 1. The base of the solid is a semi-circle tangent to the sides of the outer square prism. The surface of this solid has become known as the "cylinder hoof". In Proposition 14 Archimedes shows that this solid is 1/6 of the whole prism.

The infinite sets involved are (1) two families of triangles formed by the set of all planes perpendicular to a given line and cutting two solid figures, and (2) the two families of parallel straight lines formed by the same set of planes cutting two plane figures. At a certain point in the argument Archimedes shows that to each pair of triangles, $T_1$ and $T_2$, formed by the plane sections of the two solids, corresponds a pair of lines, $a_1$ and $a_2$, such that $T_1 : T_2 = a_1 : a_2$. This much of the text Heiberg read, but there was a large gap he could not read, and here Netz and Saito feel they have read enough of the much-mutilated text to be confident that at this point something very surprising happens. Archimedes passes from this proportion, involving areas and lines, to another, involving solids and areas. And he does so by arguing that, since the sets of corresponding magnitudes (the $T$'s and the $a$'s) are "equal in multitude" and the members of corresponding pairs are equal magnitudes, it follows that solids formed by the parallel planes on one side of the proportion and the areas formed by the parallel lines on the other side of the proportion must also be in the same proportion.

[1] *See J. L. Berggren and R.S.D. Thomas,* Euclid's *Phaenomena*: A translation and study of a Hellenistic Treatise in Spherical Astronomy *(second printing: AMS-London Mathematical Society in the series History of Mathematics: Sources, Vol. 29, 2006).*

This, of course, does not follow, and if Archimedes did write this he was in error. For it was long ago pointed out that in triangles *ABD* and *ACD* (see Figure 2) pairs of lines $x, x'$; $y, y'$; etc., are parallel and equal, i.e., in the proportion of 1 to 1. However, although the triangles *ABD* and *ABC* are composed of an equal multitude of such equal lines, they are not themselves in the proportion of 1 to 1. So, perhaps it was fortunate that the Archimedean treatises that Galileo, Kepler, and Newton knew contained no notion of infinity that went beyond what they could equally well find in *Elements*, XII.

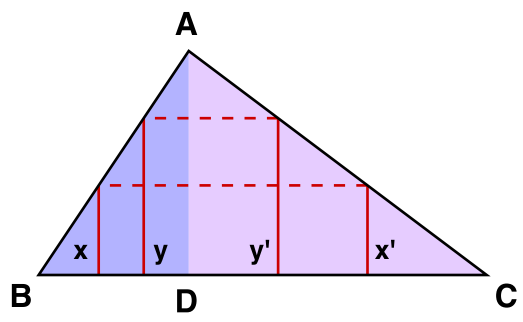


**Figure 2.**

As for Netz's claim concerning the application of mathematical models to the study of the physical world, one must point out that Euclid's application of geometrical models to the study of vision in his *Optics* was a very influential application of mathematics to the study of the physical world.[2] And, a good argument can be made that Archimedes neither invented the concept of center of gravity nor was he the first to prove the law of the lever. (Indeed, the proof Netz refers to, found in Propositions 6 and 7 of *Equilibrium of Planes*, *I*, has in Proposition 7 just the kind of tacit assumption that one can hardly believe Archimedes would make.[3])

A highly interesting chapter of Netz's work is Chapter 10, dealing with Archimedes' *Stomachion*. Only a fragment of this was known prior to Netz's study of the work, and no one understood what the treatise was about. Netz makes a convincing case that the treatise, in fact, sets the problem of counting the number of different ways a square can be assembled from a set of 14 polygonal pieces. Archimedes clearly liked problems involving counting, since he wrote both *The Sand-Reckoner*, in which he establishes an upper bound for the number of grains of sand that would fill the cosmos, and a problem known as The Cattle of the Sun God, which demands the calculation of a (very large) solution to a system of Diophantine equations. But, here again, Netz overstates his case by claiming that Archimedes was the first to tackle nontrivial counting problems with very large numbers as answers. For example, Xenocrates (396–314 BC) of Chalcedon, born about 120 years before Archimedes and head of the Platonic school, is said to have determined that a total of 1,002,000,000,000 syllables could be formed from the letters of the Greek alphabet.[4]

In Chapters 3, 5, and 7 Noel gives an engrossing account of the history of the manuscript. Particularly fascinating is his account of its fate after Heiberg studied it in the library of the Metochion of the Monastery of the Holy Sepulcher in Constantinople. Noel begins with the standard account, which is that at some unknown date, after Heiberg had studied the codex, portraits of the four evangelists were painted over portions of different pages of its text, and he shared the general assumption that some greedy Greek monk had forged the portraits to increase the value of this ancient prayer book. Then, at the end of the First World War, amidst the confusion of the collapsing Ottoman Empire, a French soldier, Marie Louis Sirieix, obtained the book and spirited it away to France. There, in Paris, it came into the care of his daughter, Anne Guersan, where it survived the Nazi occupation. It was eventually sold by Christie's in 1998.

However, this story of wars and human greed began to unravel as a result of the Canadian Conservation Institute's meticulous study, commissioned by the purchaser, of the condition of the book. Among its discoveries was that a green pigment used in the forged illustrations was one that became commercially available (in Germany) only in 1938! The Greek monks were thus exonerated and suspicion now fell on Europeans—presumably Sirieix or another member of his family. However, a letter was then discovered, written in 1934 by a Parisian antiquities dealer, who offered the book for US$6,000 to a specialist in palimpsests at the University of Chicago. Thus, Noel concludes, in 1934 the book had not yet come into Louis Sirieix's possession. But here—just as things are getting really interesting!—a curtain falls over the history of the work. Noel has, however, concocted a story to fill this gap, a story that he calls "The Casablanca Hypothesis". It is, he admits, "just as short on hard facts as the movie and should be

[2] *One assumes here that the key word in Netz's claim is "physical", in the sense of Aristotle's* Physics, *and in opposition to the celestial realm. Otherwise, not only Greek but also Babylonian astronomers were applying mathematical models of considerable sophistication well before Archimedes wrote.*

[3] *For a detailed consideration of these claims see J. L. Berggren, "Spurious Theorems in Archimedes'* Equilibrium of Planes, Book I*",* Archive for History of Exact Sciences ***16*** *(2), 1976, pp. 87–103.*

[4] *According to Plutarch, cited in T. L. Heath,* A History of Greek Mathematics, *Vol. I. Oxford: Clarendon Press, p. 319.*

similarly understood as fiction.” It is a fascinating story, though, and I shall not spoil the mystery by revealing Noel’s hypothesis here.

Throughout this book the authors are generous with their praise of the large, multidisciplinary team of experts who have devoted so much time and energy to the study of the codex. (Indeed, the “Acknowledgements” collects, under five areas of responsibility, a list of names reminiscent of the credits that one sees rolling by on the screen at the end of a movie!) Singled out for special praise are “The Patron” and “The Philologist”. This first is a gracious acknowledgment of the generosity of the purchaser and contains some apt thoughts about public and private patronage of scholarship. The second, written by Netz, contains the following: “…we have been critical towards [Heiberg] throughout this book—the gaps he left, the false guesses he made, the diagrams with which he never bothered. Now is the time to admit the truth. Without Heiberg, we could have never made it. We would look at the text and see just a jumble of meaningless traces. We would interpret a few of them. We would conjecture a sense. Then we would check Heiberg and, low [sic] and behold, he had already made sense of it. He had even read further! Only then, looking back at the page, do we see those traces that provided Heiberg with his reading. And then, finally, based on Heiberg’s foundations, we can go further and add to his readings.” This passage is not only a gracious acknowledgment of pioneering work but describes perfectly the process of reading an ancient manuscript. And, indeed, the whole book could well be used as “required reading” in a course on the restoration and study of ancient manuscripts.

In the end, of course, it will be the words (and diagrams!) of Archimedes that matter. Puzzling these out of the mutilated text, publishing them in a critical edition, and accompanying them by a good translation will put “paid” to the authors’ debt to “The Patron”, “The Philologist”, and the dedicated team that has given them such help. And it will, one hopes, bring back to life for a new generation of mathematicians the mathematical imagination of one of the greatest mathematicians who ever lived.