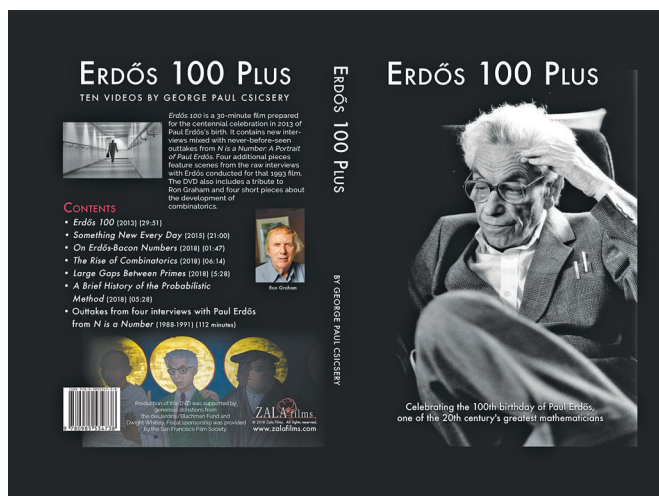




## Erdős 100 Plus

*Reviewed by Dan Goldston*



*Erdős 100 Plus* is a DVD follow-up to the 1993 movie *N is a Number: A Portrait of Paul Erdős*. Erdős died in 1996 at the age of 83 at a combinatorics conference in Warsaw, but his legacy and popularity continue to increase. The film *N is a Number* is still popular and its producer-director George Csicsery notes in promoting the new DVD: “Perhaps the best testament to its durability is the frequency with which it is pirated on YouTube.” To celebrate Erdős’s 100th birthday, in 2013 Csicsery prepared a half-hour film, *Erdős 100*, which is the first of 10 pieces that comprise the DVD *Erdős 100 Plus*. What is not included in this DVD is the original movie *N is a Number*, which should be rented or purchased for streaming at the [zalafilms.com](http://zalafilms.com) website.

The first video of this collection is *Erdős 100*, a half-hour film made in 2013 for the centennial of Erdős’s birth. The video contains interviews with many well-known

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mathematicians interspersed with footage of Erdős. Next, there is a 21-minute video, *Something New Every Day*, from 2015, which is about Ron Graham, who managed Erdős’s affairs for many years and sent checks to solvers of problems for which Erdős offered money. Much of the footage is from AT&T (formerly Bell) Labs in 1999, when Graham was the director of information sciences. To outward appearances there couldn’t be a greater contrast than between Ron Graham, the head of a large division at AT&T Labs, and Erdős. But this video shows that the joy of play and discovery—or to put an adult face on it, research—connects all mathematicians. And the video shows that Graham and Erdős are kindred spirits in many ways. One can only wish that more academic leaders would embrace Graham’s principles for management: people first, discipline second, and company third.

Next, there are four shorter pieces on mathematical problems of interest. The first is on Erdős-Bacon numbers, which is not related to what Erdős ate for breakfast. The



**Figure 1.** Paul Erdős, Ronald Graham, and Fan Chung Graham in Japan (1986).

second is *The Rise of Combinatorics*. The third has James Maynard talking about the solution to the record \$10,000 Erdős problem on large gaps between primes—solved in 2014 by Maynard and independently by the team of Ford, Green, Konyagin, and Tao, and then improved further by all five authors. The fourth is on the probabilistic method.



**Figure 2.** Paul Erdős teaching Terence Tao in 1985.

Finally, there are four interviews made of Erdős for the movie *N is a Number*, providing almost two hours of viewing Erdős close up. These took place in Philadelphia in 1988, San Francisco in 1988, Budapest in 1989, and Cambridge in 1991. These interviews are fascinating to watch. As a bonus, for those interested in the controversy around the elementary proof of the Prime Number Theorem, the 1991 Cambridge interview is must viewing.

What will you learn from watching these 10 videos? First, they provide a fairly complete introduction to Erdős's life, often as told by Erdős or other mathematicians. He was born in 1913 in Hungary, his Jewish parents were high school math teachers. He had two sisters who at age three and five died of scarlet fever within 24 hours of each other just a few days before Erdős was born. This horrible tragedy led to Erdős growing up in a protective environment at home and not going to school. He was recognized as a mathematical prodigy at an early age, and he "met" other talented math students through solving difficult math problems posed in a monthly Hungarian mathematical magazine for high school students, where the solver's names were published in later editions along with some of the solutions. In 1930, at age 17, Erdős entered the University of Budapest, where he met in person other high school mathematicians he knew through problem-solving including Turán and Szekeres. Soon he was publishing research with them as a teenager. In 1934, when Erdős was 21, he obtained his PhD and left Hungary for Manchester, England, to work with Davenport and Mordell. The political situation in Europe was becoming untenable, and Erdős spent most of the next four years in England with yearly trips back to Hungary. In 1938, the situation in Hungary became hopeless, and he left for the Institute for Advanced Study in Princeton. He

lived in the United States for the next 12 years becoming a permanent resident. In 1954, he applied for and was refused a re-entry visa to give an invited lecture at the 1954 International Congress of Mathematicians in Amsterdam. This was during the McCarthy period, but for Erdős principles always came first, "I don't let Sam and Joe tell me when and where I travel," so he left and was only able to return freely to the US in 1962. Also at this time, Erdős could not go to Hungary because he might have not been allowed to leave again, thus preventing him from visiting his mother who lived in Budapest. By the 1960s, conditions improved so that Erdős could travel relatively freely to both the US and the rest of the world. His mother (now in her 80s) was also allowed to travel, and thus began a period where Erdős often traveled with his mother to math conferences around the world. Erdős's life stabilized into what is seen in the videos, traveling from math conferences to talks to visits with mathematicians all over the world.

The videos present many personal details of Erdős at various stages of his life. We learn that when he first left Hungary for Manchester the people there described him as eccentric and requiring lots of help, a description often repeated throughout his life. Erdős had never had to butter his own bread until he was an adult, and it was in England that he learned how to do this, and "found it wasn't so difficult." Later at the Institute for Advanced Study, the teenage Peter Lax, also from Hungary, found the now nearly 30-year-old Erdős unworldly: "He was the most eccentric person I've known. He didn't live in the real world." Erdős, always gregarious, introduced Lax to Einstein. Erdős also tried but failed to interest Einstein in prime numbers. Erdős never obtained a permanent position at the Institute and spent semesters visiting various other universities. According to Lax, this was partly because at the time Erdős was viewed as too easily distracted by problems which were not significant enough for the IAS. Throughout his life, Erdős continued to work on these unfashionable pursuits, applying probability and randomness to graph theory, combinatorics, and number theory. Time has shown that some of Erdős's interests have now become fundamental in theoretical computer science, and some have led to new fields within mathematics such as additive combinatorics. While recognizing the importance of his work in these areas, Erdős didn't himself appear to attach any importance to the applicability or popularity of the mathematics he liked.

A constant activity of Erdős was fostering mathematical talent, especially young prodigies. Bollobas says he was "like a hen looking out for other mathematicians." Before email and the internet he traveled the world keeping informed on the latest developments and communicating between mathematicians. One aspect of his mentoring is his unmatched collaborations with other mathematicians: his 1500 publications most of which were collaborative. His collaborative spirit is still memorialized in the famous



Erdős number which demonstrates his overwhelming influence in the mathematical world.<sup>1</sup>

Unlike the stereotype of introverted mathematicians, Erdős was always strongly human-oriented. He allowed others to take care of many life details as he navigated the world, and no doubt his reputation of being a demanding guest was earned. But the videos demonstrate over and over that here is a man who always plays well with others. For Erdős, each mathematical topic is always also about the history and personality of the people who worked on the subject.

The interviews in this collection are fascinating. Here we get to see Erdős doing mathematics over the phone and making speaking arrangements for upcoming trips. Even to us older mathematicians, this appears like a lost historical age before the internet and cell phones changed our lives. One irony is to see AT&T/Bell Labs in 1999, a large enterprise previously supported by huge telephone revenue, and know that its support of academic-style mathematical research was about to become a dispensable luxury. Erdős often tells jokes about getting old or dying. To repeat one, according to Erdős, the best way for a mathematician to leave (die): Prove the Riemann Hypothesis and give a talk where you present the proof. After the talk, when asked the inevitable question, “Does this proof apply to  $L$ -functions?,” say “This is for the younger generation to decide because I am dying.” Then fall over dead. The frequent jokes and random comments in the videos contradict the general public’s notion of mathematicians as boring professors. There are the obvious characters like Erdős, Graham, and Persi Diaconis, but most of the other mathematicians are also interesting people, especially when compared to many of our current multimillion-followed social media stars who are not very interesting (but rich) people.

Much has been made of Erdős’s eccentricities: never having a house or permanent job, a family or private life. But this gave him extraordinary freedom. As mathematicians with families to support and careers to build, one often is forced to make judgments and rankings of whose work is better than whose, and which fields are more important than others. This is expected on grant panels, but it can easily extend beyond this. Erdős was free to never do this. He was quick to like work, and call it very clever or interesting, and he was perfectly able to point out the opposite, but his interests were always directed towards improving the areas of mathematics he worked in, without judging the fields he didn’t work in.

It seems widely agreed on by most mathematicians that Erdős’s eccentricities are so extreme that he is an outlier among mathematicians, but I am not so sure the public finds Erdős significantly weirder than any other mathematician who spends a lot of time doing mathematics. Some

<sup>1</sup> One also learns in one of these videos how Lax has the seemingly impossible Erdős number of 1.5.

evidence for this view can be found in the 233 pages of FBI files on Erdős obtained by the Freedom of Informations Act in 2014. <https://www.muckrock.com/news/archives/2015/jul/21/nothing-indicate-nothing-indicate-subject-had-any-/>

FBI agents kept tabs on Erdős from time to time from 1950 until 1975. From the start the FBI concluded he was not a communist agent and harmless. Nevertheless, they followed him around to universities reporting on his talks and movement even as late as 1975. For example, the last FBI report follows Erdős on a trip in September 1975 giving talks at the University of Wyoming in Laramie, the University of Colorado in Boulder, Fort Lewis State College in Durango, Colorado, and then returning to UCLA. From Fort Lewis State, the FBI reported:

“Source stated the subject arrived on campus on a Friday morning and gave a talk to a group of students and professors from the Mathematics Department. He then gave a talk to another group at noon, and then again on the evening of the same day. The subject remained in Durango overnight, and the next day, Saturday, the subject visited the Mesa Verde National Park. Source further stated he believed subject departed Durango, Colorado, Saturday evening, but does not recall the mode of transportation. Source said that the subject appears to be quite elderly and is very obviously an extremely intellectual and highly educated person. Source further stated he observed nothing to indicate the subject had any interest in any matter other than mathematics, and the subject’s activities were completely within the intended scope of the visit.”

Throughout the FBI files the agents seemed to perceive Erdős as a perfectly normal example of a successful and renowned mathematician.

To sum up, the *Erdős 100 Plus* DVD makes the life of Paul Erdős visible and is a time capsule of 20th-century mathematics and mathematicians and how the world changed during that century. One strong impression it left on me was how spending time forgetting the real world and entering the world of mathematics on a daily basis is similar to kindergarteners eagerly going off to play at the playground. Erdős lived through both terrible times and wonderful times, but neither prevented him from spending many hours every day thinking about whatever math questions interested him and working with other people all over the world to find answers that led to even more questions. To do this as an adult is a lot harder than it looks, and there is a disciplined nobility in leading such a life.

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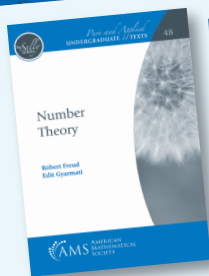
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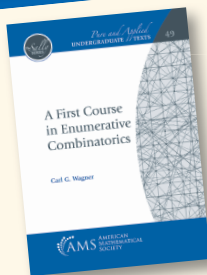


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