

# 2017 Frank Nelson Cole Prize in Number Theory



Henri Darmon

HENRI DARMON was awarded the 2017 Cole Prize in Number Theory at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017.

### Citation

Henri Darmon of McGill University is awarded the Cole Prize in Number Theory for his contributions to the arithmetic of elliptic curves and modular forms. The prize recognizes, in particular, the papers

“Generalized Heegner cycles and  $p$ -adic Rankin  $L$ -series” (with Massimo Bertolini and Kartik Prasanna and with an appendix by Brian Conrad), published in 2013 in the *Duke Mathematical Journal*, and “Diagonal cycles and Euler systems, II: The Birch and Swinnerton-Dyer Conjecture for Hasse-Weil-Artin  $L$ -functions” (with Victor Rotger), published in 2016 in the *Journal of the American Mathematical Society*. These works, which are themselves only high points of a long sequence of influential papers, prove  $p$ -adic analogues of the Gross-Zagier formula, thus relating the value of a  $p$ -adic  $L$ -function to a cohomology class constructed from the geometry of modular curves. In certain situations, these cohomology classes can be used to control the Mordell-Weil group of an elliptic curve, thus establishing new cases of the conjecture of Birch and Swinnerton-Dyer.

### Biographical Sketch

Henri Darmon was born in Paris, France, in 1965, before moving to Canada in 1968, first to Quebec City and then to Montreal at the age of eleven. He earned a bachelor of science degree with joint honours in mathematics and computer science at McGill University in 1987 and a PhD in mathematics from Harvard University in 1991, under the supervision of Benedict Gross. After a postdoctoral instructorship at Princeton University under the mentorship of Andrew Wiles, he returned to his undergraduate alma mater in 1994, where he is currently a James McGill Professor in the Department of Mathematics and Statistics. He has delivered an invited lecture in the number theory section of the 2006 ICM in Madrid, the Earle Raymond

Hedrick lectures of the MAA in 2003, and two plenary AMS lectures at the annual joint meetings in Orlando (1996) and San Antonio (2015).

### Response from Henri Darmon

I am tremendously honored to receive the Frank Nelson Cole Prize of the American Mathematical Society, as well as humbled by the thought of the many close friends, towering influences, and fortuitous events that were instrumental for the articles mentioned in the prize citation (referred to henceforth as [BDP] and [DR]).

Above all, I thank my collaborators, Massimo Bertolini, Kartik Prasanna, and Victor Rotger, who deserve this recognition as much as I do. Massimo and I have known each other since our graduate student days almost thirty years ago, and since then have shared many mathematical dreams and written twenty-five papers together. Without Massimo's friendship my career would have been very different: less successful for sure and also far less enjoyable. We started working with Kartik in 2006, at a time when our earlier ideas had largely played themselves out and we were eager for fresh perspectives. We learned a great deal from Kartik, who made us venture outside our comfort zone and expand our horizons. Our collaboration lasted roughly four years and culminated in the  $p$ -adic Gross-Zagier formula described in [BDP]. Over a memorable summer in Barcelona in 2010, Victor encouraged me to extend this formula to the setting, originally explored by Gross, Kudla, and Schoen in the early 1990s, of diagonal cycles in the triple product of modular curves. I was a bit reluctant at first to embark on this project, fearing it would interfere with my two main preoccupations at the time: questions surrounding elliptic curves and the Birch and Swinnerton-Dyer conjecture, and the search for a counterpart of the theory of complex multiplication for real quadratic fields. It is fortunate that Victor prevailed, because a few months later we discovered close connections with both topics, thanks to the extra “miracle ingredient” of  $p$ -adic variation of modular forms and associated cohomology classes. One of the contributions of [DR] is a proof of the weak Birch and Swinnerton-Dyer conjecture in analytic rank zero for elliptic curves over  $Q$  twisted by ring class characters of real quadratic fields.

Although this is hardly a mainstream result, proving it had become something of a personal obsession since the late 1990s, when I realised it would follow from a conjectural extension of Heegner points to the setting of real

## FROM THE AMS SECRETARY

quadratic fields that I proposed back then and on which I was—and continue to be—quite stuck.

I can hardly do justice in a short paragraph to all the mathematical giants on whose shoulders Massimo, Kartik, Victor, and I have stood, but let me at least try. Much of my work with Massimo over the years has been guided by Barry Mazur's grand vision of the Iwasawa theory of elliptic curves. Some  $p$ -adic variants of the influential results of Benedict Gross, Don Zagier, and Jean-Loup Waldspurger, in the spirit of Leopoldt's  $p$ -adic analogue of Dirichlet's analytic class number formula, are explored in [BDP]. The seminal ideas of John Coates and Andrew Wiles originally used to study the arithmetic of elliptic curves with complex multiplication, along with the spectacular refinements and variations that arose in the work of Victor Kolyvagin, Francisco Thaine, Karl Rubin, and Kazuya Kato, are a cornerstone of [DR]. Both [BDP] and [DR] exploit the notion of  $p$ -adic families of modular forms pioneered by Haruzo Hida and his school and rest on an approach towards  $p$ -adic  $L$ -functions that grew out of the work of Coates–Wiles, as systematized and vastly extended by Robert Coleman, Kazuyo Kato, and Bernadette Perrin-Riou. Out of such excellent ingredients, even a mediocre cook can make a good stew!

Lady Luck has played an inordinate role in my career and deserves a paragraph of her own. I was fortunate to be the PhD student of Dick Gross from 1987 to 1991, in the heady days when the Gross–Zagier formula was still fresh but starting to assert its profound and lasting influence on number theory through its role in such breakthroughs as the 1989 work of Kolyvagin on the Birch and Swinnerton-Dyer conjecture. As a postdoc in Princeton from 1991 to 1994, I had the privilege of witnessing first hand Andrew Wiles's momentous announcement of his proof of Fermat's Last Theorem and the Shimura–Taniyama conjecture. If there is one merit I can claim with some confidence, it is the knack for being at the right place at the right time, which served me well in my formative years. I also thank my family, most of all my parents, my wife, and my daughter, for their love and support, and my colleagues at McGill University, Concordia University, and the Centre de Recherches Mathématiques in Montréal for providing the most pleasant, stimulating, and supportive environment, bar none, that a research mathematician could ask for.

### About the Prize

The Cole Prize in Number Theory is awarded every three years for a notable research memoir in number theory that has appeared during the previous five years. The awarding of this prize alternates with the awarding of the Cole Prize in Algebra, also given every three years.

These prizes were established in 1928 to honor Frank Nelson Cole (1861–1926) on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The endowment was made by Cole and has received contributions from Society members and from

Cole's son, Charles A. Cole. The Cole Prize carries a cash award of US\$5,000.

The Cole Prize in Number Theory is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2017 prize the members of the selection committee were the following individuals:

- Guy Henniart
- Michael J. Larsen (Chair)
- Akshay Venkatesh

The complete list of recipients of the Cole Prize in Number Theory follows:

- 1931 H. S. Vandiver
- 1941 Claude Chevalley
- 1946 H. B. Mann
- 1951 Paul Erdős
- 1956 John T. Tate
- 1962 Kenkichi Iwasawa, Bernard M. Dwork
- 1967 James B. Ax, Simon B. Kochen
- 1972 Wolfgang M. Schmidt
- 1977 Goro Shimura
- 1982 Robert P. Langlands, Barry Mazur
- 1987 Dorian M. Goldfeld, Benedict H. Gross, Don B. Zagier
- 1992 Karl Rubin, Paul Vojta
- 1997 Andrew J. Wiles
- 2002 Henryk Iwaniec, Richard Taylor
- 2005 Peter Sarnak
- 2008 Manjul Bhargava
- 2011 Chandrashekhara Khare and Jean-Pierre Wintenberger
- 2014 Yitang Zhang, Daniel Goldston, János Pintz, Cem Y. Yildirim
- 2017 Henri Darmon

### Photo Credit

Photo of Henri Darmon is courtesy of Henri Darmon.