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The "Wide Influence" of Leonard Eugene Dickson

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ABSTRACT. Saunders Mac Lane has referred to "the wide influence" exerted by Leonard Dickson on the mathematical community through his 67 PhD students. This paper considers the careers of three of these students—A. Adrian Albert, Ko-Chuen Yang, and Mina Rees—in order to give shape to our understanding of this wide influence. Somewhat surprisingly, this influence extends to contemporary issues in academia.

Introduction

This paper raises the question: How do we, as a mathematical community, define and measure success? Leonard Dickson produced sixty-seven PhD students over a forty-year career and provides many examples of successful students. We explore the careers of just three of these students: A. Adrian Albert, Ko-Chuen Yang, and Mina Rees. Albert made important advances in our understanding of algebra and promoted collaboration essential to a flourishing research community. Yang returned to China with ideas and problems from the mathematical frontier and helped build the educational structures necessary to begin a research focus in his homeland. Rees played a fundamental role in creating the interface between government and academic research that has been crucial to the United States' preeminence in mathematics since World War II. And yet Albert is typically celebrated as Dickson's most noteworthy student. The lives of these three students combine with contemporary issues in hiring and diversity in education to suggest that the time is ripe to expand our understanding of success beyond traditional measures. It seems unlikely that Leonard Dickson had an intentional diversity agenda for his research program at the University of Chicago. Yet this contemporary theme of diversity adds a new dimension to our understanding of Dickson as a role model/mentor.

Leonard Dickson produced 67 PhD students over a forty-year career.

A. Adrian Albert (1905–1972)

When Albert arrived at Chicago in 1922, the theory of algebras was among Dickson's main research interests. As Irving Kaplansky observed, Dickson's "considerable" influence on Albert manifested itself in his 1927 master's thesis—where he determined all two-, three- and four-dimensional associative algebras over a nonmodular field—as well as in his 1928 dissertation "Algebras and their radicals and division algebras" [2, p. 246]. In his dissertation Albert proved that every central division algebra of dimension 16 is not necessarily cyclic but is always a crossed product. Albert's thesis placed him at the center of activity in the field of linear associative algebras. In particular, he, along with the German mathematicians Richard Brauer, Helmut Hasse, and Emmy Noether, sought to determine all central division algebras. In 1931 the German trio established the principal theorem that every central division algebra over an algebraic number field of finite degree is cyclic. One year later, Albert and Hasse published a joint work that gave the history

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of the theorem and described Albert's near miss. Although Albert would go on to make significant contributions to the theory of Riemann matrices and to introduce singlehandedly the American school of nonassociative algebras, he maintained an interest in associative division algebras throughout his more than forty-year-long career. He wrote more than one hundred forty papers and eight books, was invited to deliver a plenary lecture at an International Mathematical Congress, and received the AMS Cole Prize in 1939.

Mac Lane referred to Albert as Leonard Dickson's best student

The scope of Albert's talents extended beyond the production and publication of mathematical results. He, like Dickson and E. H. Moore, made significant contributions to both the University of Chicago and the AMS. During Albert's tenure as a faculty member at the University of Chicago, he participated in a variety of committees, organized conferences, chaired the mathematics department, and served as dean of the Division of Physical Sciences. While chair he "skillfully" found support to maintain a steady flow of visitors and research instructors. Albert used his influence to persuade the university to make an apartment building available, affectionately known as "the compound," to house visitors. Kaplansky claims that the compound became the "birthplace of many a fine theorem" [2, pp. 251–2]. Albert realized that the infiltration of new ideas frequently encouraged a fresh perspective on mathematics.

Albert's career reflected a strong commitment to the mathematical community at large. He served the AMS in a variety of capacities: as a committee member, as an editor of the *Bulletin* and the *Transactions*, and, like Dickson and Moore, as president of the Society, in 1965–66. The concerns of American mathematicians in the middle two quarters of the twentieth century were, however, somewhat different from those in the early years when Moore and Dickson made their contributions, and Albert's service quite naturally addressed the changing needs of American mathematicians. In particular, Albert helped establish government research grants for mathematics comparable to those existing in other areas of science. He apparently found satisfaction in this nationally oriented work for "he was always pleased to use his influence in Washington to improve the status of mathematicians in general, and he was willing to do the same for individual mathematicians whom he considered 'worthy'" [4, p. 665]. This latter category included his students.

A. Adrian Albert served the AMS in a variety of capacities, including as president in 1965–1966.

Beyond his service to Chicago, the AMS, and the mathematical community at large, Albert exerted considerable influence in mathematics through his students. In his memorial article in *Scripta Mathematica*, I. N. Herstein observed, "Adrian was extremely good at working with students. This is attested by the thirty mathematicians who took their PhDs with him. In their number are many who are well known mathematicians today. His interest in his students—while they were students and forever afterwards—was known and appreciated by them." Daniel Zelinsky, in particular, described Albert as an advisor who treated his PhD students "almost like members of his family" [4, p. 665].

A. A. Albert's success clearly reflects the influence of Dickson. He pursued a vigorous research program that emphasized collaboration, helped launch the careers of numerous students, and served the profession and his institution in varied and important ways. Any advisor would consider a student like Albert an unqualified success. In fact, Saunders Mac Lane referred to Albert as Leonard Dickson's best student [3, p. 131]. But are there other ways of measuring influence and success? Would a career dedicated solely to bringing the next generation of researchers to maturity or striving to bridge the gap between academia and the larger world be worthy of this description?

Ko-Chuen Yang (1896–1973)

In addition to Albert, four other students completed their PhDs under Dickson's guidance in 1928. Yet we rarely hear of them. Dickson had recently turned his mathematical attention, in part, to Waring's Problem. Ko-Chuen Yang (克純 楊) earned his PhD that year with a dissertation on "Various Generalizations of Waring's Problem." This thesis not only represented the first of many dissertations that reflected Dickson's recent investigations in this area, but it also marked the first and only Chinese student to earn a PhD under Dickson.

Yang's journey to Dickson actually began as a small boy at the very beginning of the twentieth century, when China was still in the Qing Dynasty. In June 1900 the Boxer uprising spread to Beijing and, in particular, to the area where foreign diplomats lived and worked. On June 21, 1900, the Qing declared a war on all foreign nations with diplomatic ties to China. This Boxer Rebellion was suppressed in August by a coalition army of soldiers from eight countries (Russia, Britain, Germany, France, the United States, Italy, Japan, and Austria-Hungary). Consequently, in 1901 the Qing government was forced to sign the Boxer Protocol, which demanded that China pay an indemnity valued at about US$337 million at the time to the eight foreign governments over the course of thirty-nine years. About US$24–$25 million was paid to the United States.

This amount was not only deemed excessive by many American government officials, but it also exceeded the actual expense for losses incurred. President Theodore Roosevelt proposed to Congress that the United States return the indemnity funds to China with the stipulation that the money be used for Chinese students to study in the United States. The proposal was implemented in 1908 with about US$12 million returned to China in this manner. These students subsequently became known as Boxer Scholars. According to *The Cambridge History of China*, this sum "created a potent mechanism for support of Chinese higher education."

The Chinese government also used the Boxer Indemnity Funds to create a college preparatory school in 1911 to prepare students for study in the United States. This preparatory school, known as Tsing-hua School, ultimately grew into the National Tsing-hua University and, finally, Tsing-hua University in 1928. The initial department of mathematics at Tsing-hua University had four core faculty, three of them Boxer Scholars, including Ko-Chuen Yang. In this position, Yang would influence many Chinese mathematicians, including Hua Luo-geng (1910–1985), who would go on to become an important mathematician, leader, scholar, and teacher in China in the mid- to late-twentieth century.[1]

Ko-Chuen Yang left a lasting imprint on Chinese mathematics. His son, Franklin Yang, would go on to win a Nobel Prize in physics.

Ko-Chuen Yang's dissertation improved existing bounds for certain cases of Waring's Problem. The only manageable aspect of Waring's Problem is its statement. Waring's Problem concerns $g(k)$, the smallest integer such that any positive integer can be expressed as the sum of at most $g(k)$ nonnegative kth powers. The case for squares, $g(2) = 4$, has been known since Lagrange and was conjectured by Diophantus. Following Dickson, Yang extended known results using polynomials. In particular, he improved Edmond Maillet's 1896 result that showed every integer greater than 19271 is a sum of twelve pyramidal numbers. He also proved that every positive integer is a sum of at most nine pyramidal numbers.

While at Tsing-hua University, Yang introduced the young Hua Luo-geng to Waring's Problem. Inspired by Yang, Hua worked on Waring's Problem using summands of polynomial functions with odd power. Hua went on to study with G. H. Hardy at Cambridge, where he published more than ten papers, most of them related to Waring's Problem.

Hua was a visiting member at the Institute for Advanced Study in 1946–48 and then secured a position at the University of Illinois at Urbana-Champaign. He returned to China in March 1950, where he devoted his attention, in part, to educational reform, in particular to the organization of graduate-level education in mathematics. He helped establish the Mathematical Institute of the Academia Sinica and, in July 1952, was appointed its first director.

Hua Luo-geng emerged as an important mathematician, leader, scholar, and teacher in China in the mid- to late twentieth century.

Through it all, Hua emerged as an important mathematician, leader, scholar, and teacher in China in the mid-to late twentieth century. As the British number theorist Heini Halberstam put it, "If many Chinese mathematicians nowadays are making distinguished contributions at the frontiers of science and if mathematics in China enjoys high popularity in public esteem, that is due in large measure to the leadership Hua gave his country, as scholar and teacher, for 50 years."

Hua Luo-geng offers a compelling example of the influence a teacher can have, not just on one individual, but on an entire country. Although only a handful of mathematicians may recognize the name of Ko-Chuen Yang, he left a lasting imprint on Chinese mathematics, especially through his early influence on Hua Luo-geng.

Mina Rees (1902–1997)

In 1929, a year after Albert and Ko-Chuen Yang earned their PhDs, Mina Rees took a leave of absence from her position at Hunter College to pursue a doctorate at Chicago under Dickson. Rees had studied Dickson's celebrated *Algebras and Their Arithmetics* and had fallen in love with the topic. Dickson had evolved into a notably successful advisor for women pursuing PhDs in mathematics in the United States in the early decades of the twentieth century. Dickson's sixteen women PhD students between 1900 and 1940 meant that he advised 8 percent of all women PhDs in mathematics in the United States and 40 percent of those at Chicago.[2] Dickson's line of inquiry on division algebras was constructive, with an eye towards classification issues. This constructivist approach becomes unwieldly as the number of generators increases, so it was not generalizable. Since Rees had specifically asked to work in division algebras, Dickson assigned her the task of constructing an associative division algebra with four generators, the limit of what his method could realistically achieve, thus closing that line of research. This topic proved beneficial to both Dickson and Rees: it gave closure to Dickson's line of inquiry and provided Rees with a ready-made research topic.

Rees completed her PhD in December 1931 and returned to Hunter as an assistant professor. However, when Warren Weaver assumed his position as director of the wartime Applied Mathematics Panel (AMP) of the National Defense Research Committee in 1943, at the suggestion of Richard Courant he invited Rees to serve as a technical aide and then as his executive assistant. The AMP consisted of government-appointed mathematicians and

[1] *Halberstam describes Hua as "the leading mathematician of his time, and, with S. S. Chern, also a student of Yang, the most eminent mathematician of his generation" [1, p. 137].*

[2] *Data from Green and LaDuke,* Pioneering Women in Mathematics: The Pre-1940s PhDs, 2009.

engineers, including Courant, Marston Morse, and Oswald Veblen. The panel received priority problems from the military and assigned them to research groups at universities across the country. As Weaver's proxy, Rees traveled the country throughout the war to determine how to assign particular problems to the most appropriate research group. In the process, she became intimately acquainted with the state of mathematical and scientific research in the nation, its personalities, and its future. She gained unique understanding of the needs of researchers. The panel also oversaw contracts between the government and universities. These negotiations required tact and finesse, and Rees emerged as an effective bridge between research and academia on the one hand and the government and military on the other. The connections, skills, and insights she gained from her work on the AMP made Rees one of the most informed persons in the country on the pulse of academic scientific research.

With the end of hostilities in 1946, Rees once again tried to return to Hunter. But within a year the call came for her to head the Mathematical Sciences Division of the newly formed Office of Naval Research (ONR) and ultimately assume the role of deputy science director. While at ONR, Rees made funding decisions and set policy for mathematical research. As F. Joachim Weyl, son of Hermann Weyl, described it in *Science*, "ONR made [Rees] the architect of the first large-scale, comprehensively planned program of support for mathematical research; she pioneered its style, scale, and scope." Thus Rees's initiatives set the norms for funding research in the United States for the rest of the century.

Despite these valuable contributions to mathematics and the American mathematical community, Rees is more widely known for her influence on early advances and uses of computers. As just one example of her insight and influence, Rees emphasized the development of visual displays for computers. With output originally via ticker tape, the general opinion at the time was that the nation's needs for computers would be limited to fewer than a dozen machines in total. Rees clearly foresaw that multiple inputs and a more robust form of output and memory would prove essential and lead to the growth of computers.

A recommendation from Richard Courant landed Mina Rees on the Applied Mathematics Panel during World War II. Here Rees and Courant share a laugh at Courant's retirement party from NYU in 1965.

"Can we have excellence and equality or must we choose between them?"

With the founding of the National Science Foundation in 1950, the ONR's role in funding began to diminish (although the ONR supports a program of external research grants to this day), and Rees returned to Hunter in 1953 as professor and dean of the faculty. She aimed to create a graduate program at Hunter and the larger CUNY system that kept pace with an increasingly changing society. She became the first president of the Graduate School and University Center of the City University of New York. As such, her influence continued to shape academia in the United States, this time in the realm of graduate education. Rees posed this meaningful question in an essay that argued for equal access to higher education as a way to redress inequities in our society: "Can we have excellence and equality or must we choose between them?" This question guided much of her work and remains relevant to contemporary discussions in academia.

Rees reminding President and Lady Bird Johnson of the importance of science funding at the time of Rees's nomination to the National Science Board of the NSF, 1965.

As indications of her impact and influence, the Mathematical Association of America awarded Mina Rees the first Award for Distinguished Service to Mathematics in 1962 and the American Association of Science elected her its first female president in 1969.

Concluding Thoughts

George David Birkhoff described Leonard Dickson as "dogged," and his influence on the mathematical community supports that characterization. As Saunders Mac Lane put it, "One can contemplate with amazement the wide influence exerted by Dickson" [3, p. 133]. The careers of just three of his students exemplify this "wide influence." A. Adrian Albert, with his algebraic legacy and key leadership roles in the American mathematical community; Ko-Chuen Yang, with his work on establishing early links between Chinese and American mathematicians and his influence on one of the foremost Chinese mathematicians of the twentieth century; and Mina Rees, with her keen ability to build bridges between mathematicians at universities and the government, her wartime efforts, and her broad views of education.

Even more, these students, who earned their degrees in 1928 and 1931—one Jewish, one Chinese, and one female—show Dickson was willing to take a broad view of who could earn a doctorate in mathematics in this country at that time. In contemporary terms, Dickson modeled diversity, at least as a graduate advisor, long before diversity became a matter of concern, or even awareness, in academia. Long before his student Mina Rees gave voice to the thought, Dickson merged excellence and equality in graduate education in mathematics. That commitment was only a beginning. This analysis of one aspect of his career offers promising insight into a diverse group of graduate students who, in turn, led to diverse populations of faculty and researchers who, indeed, have had a wide influence.

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Photo of Richard Courant and Mina Rees courtesy of the Graduate School and University Center Archives, CUNY.

Photo of President and Lady Bird Johnson with Mina Rees courtesy of the Graduate School and University Center Archives, CUNY.

For more on Mina Rees, see the article on "The World War II Origins of Mathematics Awareness," Michael Barany, in the April 2017 issue of *Notices.*

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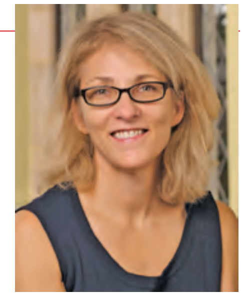
Della Dumbaugh's research focuses on the history of mathematics, especially in the early twentieth century. Her co-authored book *Emil Artin and Beyond: Class Field Theory and L-Fuctions* appeared in 2015.



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