C. R. Rao: A life in statistics

B. L. S. Prakasa Rao

C. R. Rao.

When I think of modern statistics, Dr C. R. Rao features on the top of the list. He once said that statistics is the technology of finding the invisible and measuring the immeasurable.

– Abdul Kalam, *Bharat Ratna*

Calyampudi Radhakrishna Rao or C. R. Rao needs no introduction to statisticians, mathematicians, scientists or engineers. In the volume *Glimpses of Indian Statistical Heritage*[1], Rao wrote an autobiographical account highlighting the circumstances and influences that led him to a career in statistics and probability. He titled his autobiographical account as *Statistics as a Last Resort*. It is appropriate to mention that he came into statistics by chance. By spending a life time, putting chance to work, he has built an inspiring legacy.

Early life as a student

Rao was born on 10 September 1920 in Huvvina Hadagalli, then in the integrated Madras Province, but now in the state of Karnataka. His father C. Doraswamy Naidu was an Inspector of Police with Reputation in CID work and his mother was A. Laxmikanthamma. Rao grew up in a family environment. He was admitted to Class 2 in 1925 when he was only five years old and was able to memorize multiplication tables up to 16 by 16 (needed for monetary transactions during the British rule with monetary denominations of Rupees, 16 annas for a rupee, 12 kanis for an anna and 4 dammidies for a kani). Naidu's job required the family to move from place to place every three years. Rao completed classes 2 and 3 in Gudur, classes 4 and 5 in Nuzvid and the first and second forms in Nandigama, all in the present state of Andhra Pradesh. After retirement, Rao's father decided to settle down in Visakhapatnam, a coastal city in Andhra Pradesh. Rao finished his high school and obtained his first college degree B A (Hons), with a first class and first rank, in Visakhapatnam. His early childhood involved frequent moves, but that did not affect him. His parents provided him guidance and environment conducive to excellence in studies and installed work ethics that enabled him to achieve higher goals in life. As a student, his ambition was to keep on learning. He said that he has inherited his father's analytical ability and his mother's zeal and industry. His mother was instrumental in installing a sense of discipline in him. In *Statistics and Truth: Putting Chance to Work*[2], Rao acknowledges her contributions to his life with the dedication: 'For instilling in me the quest for knowledge, I owe to my mother, A. Laxmikanthamma, who, in my younger days, woke me up every day at four in the morning and lit the oil lamp for me to study in the quiet hours of the morning when the mind is fresh.'

Rao developed research interest in mathematics when he was a student of the B A (Hons) degree course at the age of 17 in the Andhra University. He used to solve problems posed in the journal *Mathematics Student* and was happy to see that his name was mentioned as one of those who solved the problem. His most inspiring teacher was a Cambridge-trained mathematician, Vommi Ramaswami, who was the head of the department of mathematics. Rao obtained the B A (Hons) degree at the age of 19 and wanted to pursue a research career in mathematics. With a first class and first rank in the degree examination, Rao thought he would qualify for a scholarship for doing research in mathematics. However, he did not get the scholarship for bureaucratic reasons. He decided to search for a job and saw an advertisement for a mathematician for the army survey unit to work in North Africa during the Second World War. He went to Calcutta and appeared for an interview for the job which eluded him. During his stay in Calcutta, he met one Subramanian who was employed in Bombay, but had been sent to Calcutta for training in statistics at the Indian Statistical Institute (ISI). This chance encounter led Rao to join the training programme in statistics at ISI hoping that with some additional qualification he could get some job.

40 Years at the Indian Statistical Institute

Rao joined ISI in 1941 at the age of 20 and started doing research by himself and publishing papers. He received M A degree in Statistics from Calcutta University in 1943 with a first class, first rank and a high percentage of marks which remains unbroken up to now. With two Master's degrees, Rao was given the position of a research scholar at ISI in 1943 and a part time job in Calcutta University to teach a course in statistics. He continued to do research by himself on a variety of topics in combinatorics and estimation of parameters and publish papers.

A request from the Department of Anthropology, Cambridge University was sent to ISI in 1946 to send a person to analyse measurements made on human skeletons brought from Jebel Maya in North Africa by the University Museum of Archaeology and Anthropology to trace the origin of the people who lived there using the method of Mahalanobis D-square statistic. The analysis of multiple measurements was not well developed at that time. Rao was sent to Cambridge by Mahalanobis as he had the required expertise. Rao worked in Cambridge for two years (1946–48) as a visiting scholar at the Cambridge University Museum of Archaeology and Anthropology and developed some new methods

of analysis of multiple measurements and used them to analyse the data. The results of his work on the skeletal material were published in the book *Ancient Inhabitants of Jebel Maya*[3]. During this period, while working in the museum, Rao also registered for Ph D degree under the supervision of Ronald A. Fisher, a distinguished statistician. Rao received his Ph D in 1948 based on the new multivariate methodology, 'Multivariate Analysis of Variance (MANOVA)' generalizing (ANOVA), and other multivariate tests, were developed by him while analysing skeletal data. Cambridge University awarded him the Sc D degree in 1974 based on a peer review of his publications. In 1974 he was made an Honorary Life Fellow of King's College, Cambridge, which is a rare honour.

He returned to ISI in 1948 after two years' stay in Cambridge and was appointed as Professor at the young age of 28 'in recognition of his creativity'. He worked in ISI in various positions as the Head of Research and Training School (RTS), Director of RTS, Director of ISI, Jawaharlal Nehru Professor and National Professor for a period of 40 years and took mandatory retirement at the age of 60 and he was still active in research. He had published 201 research papers during the 40 years of employment at ISI. He wanted a suitable job in India without any administrative responsibilities to continue his research work. As this was not possible, he accepted positions of distinguished professorships offered to him with minimal teaching responsibilities in USA. He worked for another 30 years as University Professor at the University of Pittsburgh for 8 years and as Eberly Chair Professor of Statistics at The Pennsylvania State University for 13 years continuing his research in diverse areas of statistics. He retired from Penn State at the age of 81, but continued doing research as Director of the Center for Multivariate Analysis at Penn State until 2008. At present, at the age of 93, he has the position as a Research Professor at the University at Buffalo. He published 201 research papers while working in ISI and 274 while working in USA.

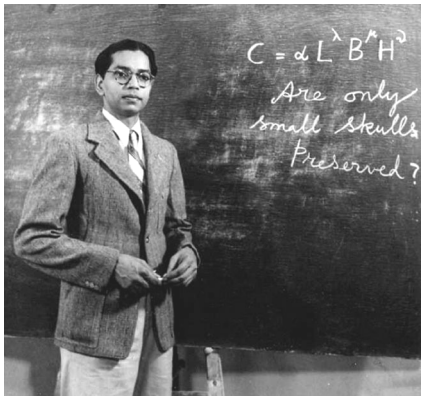
Development of statistical education and training at ISI

> *For its (ISI) educational programs, the institute needs not only leaders of mathematical thought like Professor Rao, who can uphold and maintain the high place in the world opinion that Indians have already won.*
>
> – Sir Ronald A. Fisher,
> the Father of Modern Statistics

At ISI, Rao developed a variety of courses to train statisticians to work in different applied areas. The students and trainees, who were deputed by research, government and industrial organizations to study at ISI, were given, in addition to formal lectures, on the job experience in design of experiments, biostatistics, industrial quality control and other areas. Rao established research units in ISI to work on special projects in subjects such as economics, sociology, psychology, genetics, anthropology, geology and related areas. The idea of establishing these applied research units is to provide interaction between statisticians and scientists to promote the application of statistical methods in research in other areas and to develop new statistical methods motivated by real problems. The ISI served as a meeting place for scientists from all over the world to do collaborative and interdisciplinary research with statistics as the common bond. Some of the famous visitors who spent a few weeks at the ISI are Norbert Wiener (USA), Academician Kolmogorov (Russia), R. A. Fisher (UK), Ragnar Frisch (Norway) and Y. Linnik (Russia). They gave lectures and interacted with research scholars.

Degree courses at ISI

Rao developed numerous courses in statistics at ISI over the years which were later converted into bachelor's and master's degree, when ISI was declared as an Institute of National Importance by an act of Parliament in 1959 and empowered to offer courses of study leading to degrees in statistics. Rao worked out programmes for undergraduate and post graduate degree courses leading to B Stat and M Stat degrees. He also initiated the Ph D programme in theoretical statistics and probability. The late Prof. D. Basu, who is well known for his seminal contributions to statistics was the first Ph D in theoretical statistics guided by Rao. Over the years of his professorship at universities, Rao guided the research work of over fifty students for Ph D, who in turn have produced about 450 Ph D up to now. The training and research activi-



Rao giving a talk based on research work done at the Museum of Archeology and Anthropology in Cambridge.

C. R. Rao and research scholars of ISI with Academician A. N. Kolmogorov on his 60th birthday 25 April 1963.

ties developed by Rao earned for ISI a place 'not far from the centre of the statistical map of the world'.

Development of National Statistical System

Under the direction of P. C. Mahalanobis, Rao helped in establishing the State Statistical Bureaus. The Indian National Statistical System, with the Central Statistical Organization and State Statistical Bureaus, is considered to be one of the best in the world, thanks to the efforts of Mahalanobis and Rao. During the early years, when Rao was at ISI, there was no ministry for statistics. Problems related to the development of statistics were under the administrative control of the Prime Minister. Mahalanobis was appointed as honorary statistical advisor to the cabinet in 1949. Pandit Jawaharlal Nehru, who was the Prime Minister at that time was greatly interested in development of statistics for economic planning. He visited ISI a number of times at the invitation of Mahalanobis and Rao had the opportunity of discussing with him the national statistical system and training of statisticians to work in state statistical bureaus.

Rao was a member of several Government committees for development of national statistical systems, statistical education and research in India. Some of them are, Chairman of the Committee on Statistics (1962–1969), Chairman of the Demographic and Communication for Population Control (1968–1969), Chairman of the Committee on Mathematics, Atomic Energy Commission (AEC) (1969–1978), Member of COST (Committee on Science and Technology, 1969–1971), Member of Justice Sarkar Committee to enquire into the overall functioning of CSIR.

D. Basu, the first Ph D student of C. R. Rao.

P. C. Mahalanobis and Nirmal Kumari Mahalanobis (sitting); C. R. Rao and Bhargavi Rao.

Initiation of research in econometrics in India

Rao published a paper in *Econometrica* in 1947 answering a problem raised by Ragnar Frisch, an econometrician[4]. He continued his research in statistics with applications to problems in econometrics. He developed the Delhi branch of the ISI as a centre for research in econometrics with emphasis on economic planning. He founded the Indian Econometric Society in 1971 and developed it over a period of 5 years.

International Statistical Education Centre

Rao played a significant role in establishing an International Statistical Educational Center (ISEC) at ISI for training students and statisticians deputed from developing countries. During the last few years he has been functioning as Chairman of the Board of Directors of ISEC.

Research at ISI and USA

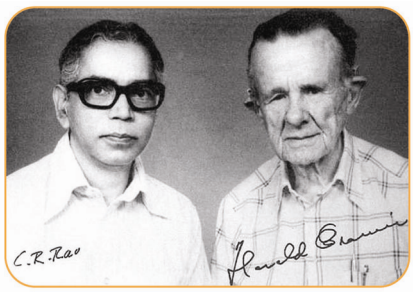
C. R. Rao is a great name from the golden age of statistics. His work was done in India and his intellect shaped statistics worldwide.

– Julian Champkin
Editor of the journal *Significance*

Rao has authored 14 books and 475 research papers. Two of his books have been translated into several European, Japanese, Taiwan, Mainland Chinese and Turkish languages.

Research during the period 1945–1950 at ISI

Rao's career in statistics is dotted with remarkable achievements. Two of his papers, written during the forties were reproduced[5], in the book *Breakthroughs in Statistics, 1890–1990*. One was published[6] at the age of 25 and another was published[7] at the age of 27. The first paper opened up several new areas of research and generated several technical terms bearing his name such as Cramer–Rao inequality and Rao–Blackwellization, which are basic to estimation theory and appear in text books on statistics, engineering and econometrics. Cramer–Rao inequality is listed as a technical term in the McGraw-Hill *Dictionary of Scientific and Technical Terms*, Fifth edition, 1994. In a recent book[8], the author B. Roy says, 'The Heisenberg Uncertainty Principle is an expression of Cramer–Rao Inequality of classical measurement theory, as applied to position determination.' The quantum physicists derived what is termed as Quantum



C. R. Rao and H. Cramer.

C. R. Rao and D. Blackwell.

J. Neyman and C. R. Rao.

C. R. Rao and R. A. Fisher.

Cramer–Rao Bound (1998) which provides a sharper version of Heisenberg Principle of Uncertainty. Rao–Blackwellization provides a method by which an unbiased estimator can be improved in efficiency, when a sufficient statistics exists. Results obtained by other authors based on Rao's paper and named after Rao are Global (Bayesian) Cramer–Rao Bound (1968), Complexified and Intrinsic Cramer–Rao Bound (2005), Rao–Blackwellized Particle Filters (1996), Stereological Rao–Blackwell Theorem (1995), Rao–Blackwell versions of cross validation and nonparametric bootstrap and Cramer–Rao Function.

In the same 1945 paper, Rao proposed a differential geometric foundation for statistics by introducing a quadratic differential metric in the space of probability measures. The idea of connecting statistics and differential geometry was too early at that time. However, half a century later, his idea has been developed to become one of the most active and important topics in information science connecting statistics, information theory, control theory and statistical physics. The concept of distance between two probability measures introduced by Rao, using differential geometric concepts is known as Rao distance. The metric is known as Fisher–Rao metric as Rao used Fisher information matrix in defining the quadratic differential metric. Some technical terms arising out of papers by others and named after Rao are Rao Measure and, Cramer–Rao Functional. The article received accolades from various sources.

The article focuses on an important of the world renowned Indian statistician, Calyampudi Radhakrishna Rao. In 1945, C.R. Rao (25 years old then) published a path breaking paper, which had profound impact on subsequent statistical research. It opens up a novel paradigm by introducing differential geometric modeling ideas to the field of statistics. In recent years, this contribution has lead to the birth of a flourishing field of information geometry.

– Frank Nielsen
Ecole Poly technique, France

Some technical terms bearing Rao's name in research papers by others are 'Rao Measure and Cramer–Rao functional'. The second breakthrough paper published in the *Proc. Cambridge Philos. Soc.* introduced a new asymptotic test, termed as Rao's Score Test, as an alternative to the likelihood ratio and Wald tests, the three known as holy trinity. The test appears in books on econometrics and its merits are discussed in various conferences. Some features of this test are discussed in a paper by Chandra and Joshi[9]. Several papers appeared from 1983, describing some good features of this test.

The combinatorial arrangements called orthogonal arrays were developed in a series of papers in the 1940s by Rao. A general formulation of orthogonal arrays and their use as experimental designs was given by Rao, which was accepted by the editor 'as a fresh and original piece of work'[10]. Orthogonal arrays have received wide applications in industrial experimentation to determine the optimum mix of factors using observations on small number of factor combinations. Taguchi, who learnt about it during his visit to ISI, made extensive use of orthogonal arrays in what is now known as the Taguchi methods in industry for determining an optimum combination of factors, which gives a high output and is robust to environmental changes. An article in *Forbes Magazine*[11] refers to orthogonal arrays as a new mantra in a variety of industrial establishments in USA.

In three papers, i.e. 'Tests with discriminant functions in multivariate analyses'[12] on the choice of minimum set of measurements for analysis; 'Utilization of multiple measurements in problems of biological classification'[13], which was the first attempt to represent high-dimensional data in a two- or three-dimensional graph and 'Tests of significance in multivariate analysis'[14], Rao laid the foundation of modern theory of multivariate methodology. All these papers contributed to the development of statistics as an independent discipline.

The 1940s were ungrudgingly C. R. Rao's. His 1945 paper, which contains the Cramer–Rao Inequality, Rao–Blackwell Theorem, and the beginning of differential geometry of parameter space will guarantee that, even had he done nothing else – but there was much else.

– Terry Speed
The Walter and Eliza Hall Institute of Medical Research, Melbourne

The first half of the 20th century was the golden age of statistical theory, during which our field grew from ad hoc origins similar to the current state of computer science to a firmly grounded mathematical science. Men of the intellectual caliber of Fisher, Neyman, Pearson, Hotelling, Wald, Cramer, and Rao were needed to bring statistical theory to maturity.

– Brad Efron
Stanford University, USA

Research during the period 1950–1980 at ISI

An estimator is said to be first-order efficient if its asymptotic variance attains the Cramer–Rao lower bound. Under some conditions, the first order efficiency holds for a large class of estimators. In order to choose a sub-class of first order efficient estimators which are better than others, Rao introduced a criterion called second-order efficiency[15]. This is the first paper, which initiated studies on higher order asymptotics.

Rao used the idea of canonical correlations in estimating the dominant factors which explain the correlation between measurements[16]. This method is known as Rao's canonical factor analysis.

Rao made significant contributions to results on characterization of probability distributions. These results are described in the book, *Characterization Problems of Mathematical Statistics*[17]. Some of the technical terms arising out of characterization of probability distributions are Rao's Damage Model (1963), Rao–Rubin Theorem (1964), Kagan, Linnik and Rao Theorems (1963). Research in this area was continued during his stay in USA.

Research during 1980–2000 at USA

Rao in collaboration with Jacob Burbea, introduced a series of new measures of information and diversity measures and studied their properties[18]. Rao developed analysis of diversity (ANODIV), generalizing analysis of variation (ANOVA). He introduced what is termed as Rao's Quadratic Entropy, as a general measure of variance, which is used by ecologists[19].

Rao's quadratic entropy fulfills all a priori criteria and it surpasses other proposed indices, because it includes species abundance and more than one trait. Therefore, it seems to be an improvement compared to other measures of functional diversity that are currently available.

– Z. Botta-Dukat
Hungarian Academy of Science

Rao developed the concept of cross entropy[20]. He continued his research in USA on characterization of probability distributions in collaboration with Khatri and Shanbhag. The results are summarized in the book, *Choquet–Deny Functional Equations with Applications to Stochastic Models*[21]. In the area of functional equations in mathematics, he introduced a new equation called the Integrated Cauchy functional equation. This equation provided a general technique for characterizing probability measures and solving problems of stochastic modelling of data for statistical analysis[22]. Matrix theory is another branch of mathematics he used in the discussion of statistical problems, which in turn gave impetus to research on matrices. His most important contribution to the theory of matrices is the concept of generalized inverse of a matrix (singular or not). This has become a valuable tool in developing unified theory for linear stochastic models used in prediction problems. A very general definition of inverse of a matrix, singular or not, was discussed in ref. 23.

Using generalized inverse of a matrix, Rao provided a general technique for characterizing probability measures and solving problems of stochastic modelling of data for statistical analysis. He provided a unified theory of least squares for the linear model

$$Y = X\beta + \varepsilon \text{ with } E(\varepsilon) = 0,$$

and covariance matrix V of ε, where X and V may be rank-deficient. The regression coefficients β are estimated by minimizing the quadratic form

$$(Y - X\beta)'M^{-}(Y - X\beta),$$

where M is any generalized inverse of the matrix, as described in ref. 24. Rao also generalized what are known as Kantorovich inequalities on matrices for use in statistics which opened a new area of research in matrix algebra[25,26].

Rao's area of research covered wide fields in statistical theory and practice and some aspects of matrix theory needed to express statistical results in their generality.

A few other technical terms which promoted research by others are, Rao's paradox in sample surveys, Rao's paradox in multivariate analysis, Rao's covariance Structure, Geary–Rao Theorem, Khatri–Rao product of matrices and Khatri–Rao subspace.

A place in the history of statistics

When Rao joined the ISI in early forties, statistics was not considered as an independent subject and no university offered courses at the Masters level. Rao's contributions in the forties earned for him a place in history of statistics as one who contributed to the development of statistics as an independent discipline.

C. R. Rao is among the worldwide leaders in statistical science over the last five decades. His research, scholarship and professional service had a profound influence in the theory and applications of statistics and are incorporated into standard references for statistical study and practice. Rao's contributions to statistical theory earned him a place in the history of statistics.

– Samuel Karlin
US National Academy of Sciences

Rao is the only Asian listed in all websites on history of statistics, giving lists of persons with photos and a summary of their contributions: Figures from the *History of Probability and Statistics* contributed by Aldrich (UK) giving a list of 35 major contributors from 16th century, *Chronology of Probabilists and Statisticians*, University of Texas, USA, giving a list of 57 major contributors from 16th to 20th centuries and *Statisticians in History* by American Statistical Association giving a list of 52 contributors.

Highest awards given to a statistician

Samuel Wilks Medal of American Statistical Association 1989, the highest award given to a statistician in USA, 'for major contributions to the theory of multivariate statistics and applications of that theory to problems of biometry; for worldwide activities as advisor to national and international organizations; for long time conscientious as a teacher, editor, author, and founder of academic institutions; and for the great influence he has had on the application of statistical thinking in different scientific disciplines, embodying over a career of more than 40 years the spirit and ideals of Samuel S. Wilks.'

National Medal of Science 2003, the highest award given to a scientist in USA: Awarded by the president of USA with the citation 'as a prophet of new age

Mahalanobis Prize, International Statistical Institute 'for life time achievement'.

for his contributions to the foundations of statistical theory and multivariate statistical methodology and their applications, enriching the physical, biological, mathematical, economic, and engineering sciences'.

India Science Award 2009, the highest recognition given to a scientist in India, 'for major contribution(s) of a path-breaking nature.' The award, given by the Prime Minister of India, carries a gold medal and cash prize of Rs 25 lakhs.

Guy Medal in Gold of the Royal Statistical Society 2011, the highest award given to a statistician in UK: This award is given once in 3 years to 'those who are judged to have merited a significant mark of distinction by reason of their innovative contribution to theory or application of statistics.' This is the first time the medal was given to an Asian and second time given to a non-British citizen during the last 118 years since the inception of the medal.

Mahalanobis Birth Centenary Gold Medal 1996, awarded by the Indian Science Congress Association.

Bhatnagar Award 1963, of the Indian Council of Scientific and Industrial Research for contributions to science

International Mahalanobis Prize 2003, 'for lifetime achievement in statistics and the promotion of best statistical practice' awarded by the International Statistical Institute.

The Ministry of Statistics and Programme Implementation, The Government of India has instituted a National award in honour of C. R. Rao, the renowned statistician of the country.

Professional Awards

Membership in Academies

Rao received recognition from all statistics societies for his pioneering contributions to statistical theory and applications. He was elected to Royal Society (FRS, UK Academy of Sciences); the National Academy of Sciences (USA); American Academy of Arts and Science; Indian National Science Academy; Indian Academy of Sciences; National Academy of Sciences, India; Lithuanian Academy of Sciences, and Third World Academy of Sciences. He was made an Honorary Member of the European Academy of Sciences, the International Statistical Institute, International Biometric Society, Royal Statistical Society (UK), Finnish Statistical Society, Portuguese Statistical Society, Institute of Combinatorics and Applications, American Statistical Association, and World Innovation Foundation. He is a Life Fellow of Kings College, Cambridge. He has been the president of the International Statistical Institute, the first one from Asia; Institute of Mathematical Statistics, USA, the first one from outside USA and the International Biometric Society, the first one from Asia.

Honorary doctorate degrees

Rao's early research in the forties of the last century on statistical theory and practice brought him international recognition. He had the opportunity of visiting several countries to attend conferences, give lectures and collaborate with noted statisticians for joint research.

Rao was awarded 38 Honorary Doctorate degrees from universities in 19 countries, spanning six continents: Europe (10 countries, 11 degrees): Germany, Russia, Switzerland, Poland, Serbia, Spain, Finland, Portugal, Greece and Cyprus; North America (2 countries, 7 degrees): USA and Canada; South America (2 countries, 2 degrees): Brazil and Peru; Asia (3 countries, 15 degrees): India, Sri Lanka and Philippines; Australia (1 degree) and Africa (1 degree).

Rao is purely an Indian product who had all his education in India and who did all his research by himself without any guidance by others. The booklet on some famous scientists of modern India by TIFR lists Rao as 'one of those who have been instrumental in building up the vast and rich scientific culture of modern India with no infrastructure and with little support from the government'.

He is not just a statistician. He has a lot of other interests. He has interest in music and dance and pursues his hobbies of photography and gardening. When Rao moved from Calcutta to Delhi in 1970 to be at the Indian Statistical Institute, Delhi, he was surprised to find there was no dance school to teach Kuchipudi dance style and Kuchipudi did not receive the same status as Bharatanatyam, Kathak, Kathakali and Odissi. He started the Kuchipudi Dance Academy at Delhi and was its first president. The academy organized regular performances in Kuchipudi dance style. He was the president of the academy until he left for USA in 1979.

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