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ROSSBY WAVES

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Large-scale undulations in the westerly winds are related to an ideal form of motion known as the 'Rossby' or 'planetary' wave. These waves owe their existence to the rotation and spherical shape of the Earth. Weather patterns and the general circulation are mostly much wider than the depth of the atmosphere: viewed from the side, a weather system is 100 to 1000 times thinner (vertically) than its width. This extreme thinness, beyond reminding us of the fragile nature of the atmosphere, causes horizontal winds to be stronger than vertical winds in such weather systems. Stable layering of the air, with its great variations in density reinforce this inequality. Waves then become possible, which are dominated by nearly horizontal wind patterns, in many ways unlike the familiar waves on the sea, or sound or light.

In their most general form, Rossby waves have an important bearing on what we call 'weather' and on the form of the general circulation of Earth's atmosphere, its oceans, and the atmospheres of other planets. Indeed, the form of the general circulation is in part shaped by Rossby waves. In their most general form, Rossby waves occur widely in fluid flows of many kinds (for example, in hurricanes). To understand these waves completely requires a challenging amount of mathematics and physics, but many of their properties can nevertheless be appreciated through ideas, observations and experiments that could be found in a high-school physics course. Because the language of science can be artificially complex we provide translations of some unfamiliar terms inside brackets {•}. Some mathematical equations are included, but these can be skipped over by those unfamiliar with them.

Horizontal propagation. If a westerly wind {one that blows eastward} passes over a mountain range, undulations develop downstream, meandering north and south with regularity. This is somewhat like a brook flowing rapidly over a stony bottom, where small ripples and longer gravity waves are seen on the surface. The uneven bed of the fluid disturbs the flow, sending out waves. Waves trying to 'stem the current' and propagate upstream are particularly strong: these waves are held fixed in space. They 'resonate' with the rocks and build up in amplitude (yet, more subtly, their energy does propagate away from the stony source, upstream for ripples and downstream for gravity waves). Such a pattern, calculated for the much larger scale flow appropriate to Rossby waves, is seen in figure 1. The waves are here excited by flow of a single layer of fluid over an isolated mountain; the 'mountain' is idealized in the form of a circular cylinder of finite height. For a westerly wind speed of 30 meters per second, at latitude 40^0 on the Earth, the wavelength {the distance from one northward undulation to the next} is about 8200km. Rossby waves are primarily horizontal motion, rather than vertical. Unlike waves at the surface of the sea, but somewhat like longer gravity waves on a stream, they involve the entire fluid, and help to shape its circulation. If the mean winds were reversed in Figure 1, they would flow over the mountain with very little disturbance: Rossby waves cannot then be generated.

In large-scale atmospheric flow, isobars {lines along which pressure is constant} nearly coincide with streamlines {lines along which the wind is directed}; thus weather maps show the atmospheric pressure. Notice at once that the meandering flow in figure 1 has a cyclonic low-pressure center just downstream of the mountain, with relatively high pressure upstream. This indicates that the wind is *pressing* eastward on the mountainous solid Earth, and in response the Earth is pressing westward on the atmosphere {with a horizontal force found by adding up everywhere the pressure multiplied by the slope of the topography}. It seems that in generating a Rossby wave, the mountain is also exerting, over time, a westward force on the atmosphere. In fact, this 'wave-drag' is a key part of the angular-momentum balance of the atmospheric circulation. With the passing of the seasons, fluctuations in this balance cause the speed of rotation of the Earth to fluctuate, and the length of the day to change by about 1.5 milleseconds.

The observed northern hemisphere flow in the upper troposphere is shown as an average over the winter season, in Figure 2a. The deviation from circular streamlines is the 'stationary' or 'standing' wave field of the atmospheric general circulation. The flow is westerly throughout most of the region, which does not extend south into the trade winds. Where the streamlines squeeze close together the winds are stronger: this occurs downwind of the Himalayan plateau at the western side of the Pacific Ocean, and downwind of the Rocky Mountains, at the western side of the Atlantic Ocean. Of course at any given moment the winds will not look like this figure, for transient waves and eddies, some of them associated with the jet stream, are as strong as the mean winds. Between two and three waves fill a latitude circle, in this pattern.

Vertical propagation. Complementing this purely horizontal view of the circulation, a vertical cross-section of the atmosphere (Figure 2b), running east and west, shows that in a broad-brush sense the wind patterns vary very little with altitude, though they grow in strength as one moves up to the top of the troposphere (where the jet streams are strongest). This means that a 'barotropic' model, {such as one with a single layer of uniform-density fluid, as in Figure 1}, should have some validity. Notice, as in the horizontal map of the same wind field (Figure 2a) and the theoretical calculation behind Figure 1, major low pressure regions lie in the lee of the Himalayan Plateau in Asia, near 45^0N, 90^0E (middle panel) and in the lee of the Rocky Mountains of North America at 45^0N, 90^0W longitude. Because a geostrophic {large-scale} wind moves with high pressure to its right (in the Northern Hemisphere, conversely in the Southern Hemisphere), there are northerly winds in the lee of the mountain ranges, as in Figures 1 and 2a. In winter these carry cold air across the Great Plains of the US.

A more subtle feature however is that the winds and temperature are organized in a pattern that tilts to the west as one moves upward through the atmosphere (Figure 2b). This tilt is a signature of several important things. It signifies upward propagation of energy and of easterly momentum. The easterly {westward} pressure force exerted by the solid Earth is transmitted upward through the atmosphere by 'wavy' layers of air that exert pressure forces similar to those on solid mountain slopes. These forces alter the winds aloft. Upward propagation of a given amount of energy into regions of thinner {less dense} atmosphere leads to increasing velocity with height (because the kinetic energy is the product of air density and squared velocity). For this reason Rossby waves that reach the stratosphere can be accompanied by strong winds, and can drive strong changes in the mean westerly winds.

The tilted pattern also indicates that north-south wind, v, and temperature, T, have a systematic correlation. Their product, vT, is the poleward {toward the North or South Pole depending on which hemisphere one is in} transport of heat. Recall (Chapter xx) that most of the significant motions of the atmosphere and ocean are driven by solar radiation, heating the tropics, with the heat radiating back to space most strongly near the poles. Rossby waves, or motions with some of their properties, contribute to the north-south "heat pipe" in this giant heat engine. This discussion fills out the fully three-dimensional picture of the two-dimensional problem shown in figure 1.

Because Rossby waves exist throughout the depth of the atmosphere, they are not 'superficial oscillations', but rather an expression of forces acting on the entire circulation. An isolated cyclone, if it is large enough in size and weak enough in wind velocity, will 'burst apart' into Rossby waves, forming new, elongated cyclones and anticyclones that gradually fill the fluid with motion (Figure 3). In this experiment the fluid itself oscillates gently, moving only slightly compared with the movement of the wind patterns. In the figure we have no mean westerly wind, so that most of the fluid is motionless. Stronger, more realistic cyclones do not radiate waves so efficiently, but the forces involved in the waves are still at work, for example nudging hurricanes out of the tropics toward the poles.

Examples: blocking patterns in the southeast Pacific and teleconnections over North America. Observations of winds at the 300 HPa level in the Pacific

(Figure 4) show a structure that illustrates the nature of Rossby wave propagation. Cumulus convection in the western tropical Pacific provides a large-scale pattern of divergent winds aloft. A train of cylones and anticyclones appears south of Australia and veers southeastward toward Chile, where it creates a lasting circulation cell strong enough to be called a 'blocking' pattern. The figure shows correlations of the winds with a time series that expresses this blocking: the winds themselves form a similar pattern. The waves arrive quickly (the speed of individual cyclones and anticyclones being about 5^0 of longitude per day, which appears to be slower than the southeastward development of the pattern as a whole). They decay slowly, influencing a vast region of the South Pacific and reaching into the South Atlantic. In a simpler fluid than the atmosphere, convection in the tropics would stimulate a more local response: here, waves cause a 'teleconnection' round half of the circumference of the Earth.

When the close-in views of theoretically solved Rossby waves (Figs. 1 and 3) are expanded to the whole sphere, the waves tend to propagate along great-circle paths. While theory predicts their structure in the simplest circumstances, computer models, which solve the mathematical equations approximately, must be employed if realistic mean wind patterns and land topography are included. The modelled Rossby-wave field generated by a similar pattern of equatorial heating by the ocean, Figure 5, has two branches propagating southeastward and northeastward from the western Pacific. The train of waves crossing North America is similar to the 'PNA pattern' (Chapter xx) which is associated with ENSO events (yet more will be said about this pattern below).

Some specific, and rather technical results. What features of the atmosphere are explained in some way by Rossby waves? Begun a century ago as an exploration of weak oscillations of the atmosphere and oceans (for example, the tides raised by moon and sun) the theory of Rossby waves now provides insight into the very heart of atmospheric (and oceanic) circulation dynamics. These waves are related to the meandering north and south of the westerly winds and, less directly, to the synoptic eddies that shape our weather. Rossby waves contribute to understanding the global pattern of these westerly winds, the enhancement of cyclonic disturbances in the lee of major mountain chains, the location and shape of storm tracks in the western Atlantic and western Pacific, some forms of blocking and stagnation of air masses, the propagation of energy in long waves upward to the stratosphere, the transport of east-west momentum with these waves and the attendant deceleration and 'sudden warming' of the wintertime vortex that sits above the North Pole.

Along the Equator, oceanic heat and water-vapor drive cumulus towers which heat the larger scale atmosphere. The winds converge below and diverge above such a heat source, and air pulled into the pattern creates a pattern of circulation extending both east and west from the heat source. Rossby waves propagating westward from the region of forcing control the sahpe of this pattern to the west, while Kelvin waves describe the movement east of the heating. At a yet larger scale, the atmosphere signals the onset of el Nino in the tropical Pacific by sending a train of waves across North America: in simplest idealization these are Rossby waves (meanwhile, in the sea below, Rossby waves move westward along the Equator to reinforce the recurrence of el Nino). In the lower troposphere in summer great anticyclones fill the North Atlantic and Pacific oceans, and these are established by monsoon forcing {warming of the land surface } yet organized and shaped by westward propagation of low-frequency Rossby waves. More distant relatives of the Rossby wave account for the basic instability of the primary, east-west atmospheric circulation: *baroclinic* instabilities which are the model of cyclonic storm development, tapping the potential energy of the atmosphere, and *barotropic* instabilities which tap the kinetic energy of the mean atmospheric flow. In the stratosphere, very large scale Rossby waves describe the undulations of the vortex sitting over the wintertime pole. They are excited by upward propagation of Rossby wave energy from the intense winter circulation below. The restoring force that gives us Rossby waves also inhibits mixing of fluid north and south, and in this way makes possible the ozone hole.

Before being completely carried away by the potency of this idea, however, we have to warn that Rossby waves are in competition with other forms of flow, particularly with turbulent, large-amplitude winds which are not waves at all. At the scale of the larger weather systems, the ‘flow’ dynamics and the ‘wave’ dynamics are nearly equal in importance.

Basic principles. The conservation principle for *potential vorticity* (see **Dynamic Meteorology**:Balance flows, and **Potential Vorticity**) helps to simplify the notion of Rossby waves, and also unifies them with the ‘flow’ dynamics just mentioned. Potential vorticity (‘pv’) combines the dynamical effects of the spin of the Earth about its axis with the much smaller scale spin of elements of the fluid about their centers. The spin of the Earth is, in effect, inherited by the fluid atmosphere, and concentrated into small, spinning storms. PV also incorporates effects of the sloping isentropic surfaces, and the shape of the atmosphere’s lower boundary. PV is thus a combination of small-scale fluid properties and large-scale environmental properties, written using the time derivative following the motion of the fluid (and with PV denoted by the symbol q)

$$\frac{Dq}{Dt} = 0 \quad where \ q = \frac{f+\zeta}{h}, \qquad (1)$$

describes conservation of q following an ideal fluid element as it moves. We have neglected friction, heat sources and effects of small, unobserved, turbulence here. PV is related to, yet more general than, the conservation of angular momentum encountered in the physics of spinning bodies. Here f is known as the Coriolis frequency. It is twice the vertical component of the Earth’s rotation vector, or

$$f = 2\Omega \sin\phi$$

where Ω is the Earth's rotation rate and ϕ is the latitude. Rather like a Foucault pendulum, the horizontal flow of the atmosphere picks out the *vertical* component of the rotation vector. ζ is known as relative vorticity (also resolving just the vertical component) which is twice the average rate of spin of small fluid elements about their centers; finally h is the thickness of the layer of fluid, measured along the local vertical. As the fluid moves it may trade off its small-scale spin (ζ) for its large-scale 'planetary spin' (f) by moving north or south: this is the essence of the Rossby wave. Variations in the thickness h of the fluid layer may also be involved in the trade-off, helping or hindering it. One can see a tendency for this balance in the upper troposphere in winter, Figure 2a. Here the dashed curves showing ζ+f nearly coincide with streamlines (solid curves), showing a tendency for absolute vorticity to be constant following the mean circulation. The difference between them (which is important to the dynamics) can be attributed primarily to variation of layer thickness h ('baroclinity' and 'divergence') and to lesser degree to the cumulative effect of transient eddies---weather.

A key quantity suggested by this description is the rate of change of the planetary spin, f, with latitude. It is known as β {beta}, defined as

$$\beta = 2\Omega\ \cos(latitude)/a$$

where Ω is the Earth's rotation rate in radians per second, and a is the Earth's radius, about 6367 km (± 22 km, owing to the equatorial 'bulge'). In SI units, (meters x seconds)$^{-1}$, β = 2.28 x 10^{-11} multiplied by the cosine of the latitude. The 'beta-effect' is the name given this systematic gradient {variation} of pv provided by the spherical shape of the planet.

For a single layer of homogeneous fluid like ordinary water, h in the formula (1) for pv represents the full depth of the layer: in this case we can have 'topographic Rossby waves' due to a slope of the solid bottom (variation of h), instead of the spherical shape of the Earth (variation of f). For a fluid layered with significant density variation (like either ocean or atmosphere), the conservation of pv can be applied to a small fluid element with h being the vertical thickness of the layer, bounded above and below by surfaces of constant potential density. PV thus has a dual nature: it gives a conserved quantity at each point in the fluid and yet it also has a vertically-averaged sense of being conserved, for an entire layer of fluid {this known as 'barotropic' pv}. Remarkably, many aspects of atmospheric Rossby waves can be largely understood in terms of the latter, simpler, barotropic pv. A map of pv throughout a fluid can be mathematically 'inverted' to give much of the velocity and density field (though a part of the flow for which PV=0 is 'invisible' to pv analysis), and a map of the showing curves of constant pv for the time-averaged state of the atmosphere (with mean winds and mean temperatures) describes restoring effect for the waves. Indeed, fluid should flow rather freely along such curves pv = constant (known as 'geostrophic contours' or just mean-pv contours) if dissipative turbulence and external forcing are weak. The persistent variation of f with latitude tells us that, east-west winds are favored on a rotating planet, and north-south winds may often lead to waves.

It is exceptionally handy that the fundamental dynamical quantity (pv) for the atmosphere is nearly unchanging, like the concentration of a trace chemical, following the circulation of the air. This adds great intuitive resource because we can *see* tracers move, distort and mix, and we can *see* pv in models and observations behaving in many of the same ways. Rossby waves are the shimmering of the mean pv contours of the atmosphere.

Barotropic Rossby waves. For the case when the layer thickness, h, is effectively constant, we have

$$\zeta + f$$

as the active part of pv (known as the absolute vorticity). An ideal situation would be a single layer of incompressible fluid (like water) of constant depth. This idealization is in fact immensely powerful, providing approximate wave solutions for the more complex environment of the stratified atmosphere.

Carl-Gustav Rossby, working at MIT in 1939, introduced the useful approximation for middle latitudes, known as the ‘beta-plane’. It approximates the spherical Earth locally by a plane tangent to it, allowing the simpler mathematics using Cartesian coordinates to replace the full spherical coordinates. Far from the tropics, the Coriolis frequency can be approximated as

$$f = 2\Omega\,\sin(latitude) \approx f_0 + \beta y$$

where y is the north-south position, measured from some mean latitude y_0

Rossby waves in an atmosphere at rest. Let us now use these ideas to construct a basic Rossby wave for an atmosphere otherwise at rest (without the usual east-west mean winds). Newton’s second law, the conservation of momentum, gives us equations in both horizontal directions, x (eastward) and y (northward), for the corresponding velocity components u and v. If we set up the wave with purely north-south motion, u=0, the momentum equations express an east-west force balance between the pressure gradient and the Coriolis force (known as geostrophic balance):

$$-fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

and a north-south force balance between acceleration per unit mass, and pressure gradient:

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

Eliminating the pressure, p, between these two equations gives a wave equation for v:

$$\frac{\partial^2 v}{\partial x \partial t} + \beta v = 0 \qquad (2)$$

where $\beta = \partial f/\partial y$, approximated as a constant in the equation. Assuming a wave of the form $v = A\cos(kx - \omega t)$ we substitute in the wave equation to find

$$\omega = -\beta / k$$

This key relation between the *wavenumber* k {which is 2π divided by the wavelength} and the frequency ω tells us that the longer the wave, the higher frequency. The propagation speed, c, (the phase speed) is westward, with magnitude β/k^2. In more familiar wave systems, for example *non-dispersive* sound waves, light waves or waves on a vibrating string, the frequency varies directly with wavenumber and the propagation speed c is a constant. The character of dispersive waves is that a localized disturbance breaks into sine-wave components of gradually varying length (as with a pebble thrown into a pond). By contrast, non-dispersive waves like sound and radio waves preserve the properties of isolated pulses, making possible communication to a great distance.

The restoring force for Rossby waves. The force balance above shows that the pressure gradient is the restoring force for the waves, and that this north-south gradient arises indirectly from the east-west force balance. Large-scale winds, slowly varying in time (relative to a day) are nearly *geostrophic* (see Chapter xx), with pressure gradient balancing the horizontal Coriolis force (which is at right angles to the wind). Because the Coriolis frequency and hence the Coriolis force on the air parcels increase with latitude, so too do the north-south pressure variations, and these provide the needed restoring force, driving the acceleration of the wind.

The principle of conservation of PV gives a clearer description of the workings of the Rossby wave. An air mass that moves northward in a standing wave pattern, conserving the sum $\zeta + f$ will have to develop negative spin or vorticity, ζ, as it encounters smaller values of f found at high latitude. This anticyclonic spin matches with the northward velocity, west of the parcel, and the southward velocity to its east (Figure 6), enforcing the basic wave pattern. In downwind regions where the wave has not yet penetrated, this spin will extend the pattern downwind at a rate twice the mean westerly wind speed.

When the wave is oriented in an arbitrary direction, it has two wavenumbers or, more succinctly, a *wave-vector* $\vec{k}$ with components k (east-west) and l (north-south). The corresponding equation (for a fluid without mean east-west winds, written in terms of the *stream function,* ψ) is now

$$\frac{\partial}{\partial t}\nabla^2\psi + \beta\frac{\partial \psi}{\partial x} = 0 \qquad (3)$$

The quantity ψ is a close approximation to the pressure or geopotential height field, as well as giving the horizontal velocities ($u = -\partial\psi/\partial y$, $v = \partial\psi/\partial x$), and $\nabla^2\psi$ is the horizontal Laplacian, $\partial^2\psi/\partial x^2+\partial^2\psi/\partial y^2$. The equation is a form of conservation of pv, rewritten as $PV = \nabla^2\psi + \beta y$. The frequency relation, found by substituting a wave of the form $\psi = \exp(ikx+ily-i\omega t)$ in the equation, is now

$$\omega = \frac{-\beta k}{k^2+l^2} = \frac{-\beta\cos\alpha}{|\vec{k}|} \qquad (4)$$

where α is the direction of $\vec{k}$ with respect to east. This relationship is plotted in Figure 7. A key property of dispersive waves is the velocity of energy propagation, known as the *group velocity*. This vector has magnitude

$$\beta / |\vec{k}|^2 \qquad (5)$$

equal to the westward component of phase speed, and it points in a direction 2α. The group velocity is perpendicular (pointing inward) to the circles of constant frequency in Figure 7. A remarkable property of Rossby waves is that their wavecrests always move westward relative to the air, with westward speed ω/k also given by (5), even though their energy can propagate in any direction. If a steadily oscillating force is exerted at a point in the fluid, it will radiate a Rossby waves in all directions (Figure 8). The theory gives the solution as a form of Bessel function { known as the Hankel function of the 2d kind }, multiplied by a westward traveling sine-wave, written explicitly as $\psi = \exp(i\beta x/2\omega - i\omega t)\, H_0^{(2)}(\beta r/2\omega)$. The wave crests form parabolas on the horizontal plane, which sweep westward while collapsingclosing in on the western part of the x-axis.

Effect of mean zonal winds. An east-west wind, U, adds to the intrinsic propagation speed of the waves. The frequency equation (4) has a new Doppler-shift term, -kU on the lefthand side. If we restrict our interest to *standing* or *stationary waves,* for which $\omega = 0$, we replace time derivatives with x-derivatives, or equivalently replace ω by -kU in the frequency relation. This gives instead of (4)

$$kU = k\beta/(k^2+l^2)$$

and instead of the wave equation (3),

$$\nabla^2\psi + \frac{\beta}{U}\psi = 0 \qquad (6)$$

Evidently a westerly wind generates stationary Rossby waves, all of the same wavelength

$$2\pi/\sqrt{k^2+l^2} = 2\pi\sqrt{U/\beta}\,.$$

We see from this formula that faster winds make longer waves, and in regions of easterly wind ($U<0$) there are no simple waves at all. The group velocity is found by adding the intrinsic group velocity of the Rossby wave to the mean wind velocity. Using relation (5), the intrinsic group velocity has magnitude which is thus equal to U. Taking account of the direction of energy propagation, this tells us that the waves' energy propagates with velocity $2U(1+\cos\theta)$, in a direction θ measured with respect to east. They fill an ever-expanding circle downstream of the source of waves. In the more general situation of propagating (rather than stationary) waves on a westerly wind, the two above analyses combine to give an east-west wavespeed ω/k which is

$$U - \frac{\beta}{|\vec{k}|^2}.$$

This is Carl-Gustav Rossby's 'trough formula'.

Forcing and β-plumes. There is one important signal missing from the above discussion. Waves with crests (and winds) running nearly east and west have $k \ll l$ have significant energy velocity even though their intrinsic frequency is small. At very low frequency, keeping the wavelength constant, the direction of energy propagation is due westward, and is sufficiently fast to overcome a westerly wind. This produces what can be called 'β-plumes', which are nearly steady cells of circulation reaching westward from their point of generation. The wavenumbers k and l are the reciprocals of the length scales of the wave in east and north directions. The group velocity (5) carries a plume of circulation westward of a mountain with north-south width L; it advances with speed βL^2 relative to the mean wind. In the case of flow over a mountain range, this plume can reduce the flow upwind of the mountains, expressing a *blocking* of the wind by the mountain. With a little friction added, the β plume can become a steady, closed circulation. A solution for the streamlines due to a point-source of pv, Figure 9, complements the oscillating Rossby-wave point source in Figure 8.

Here the 'twisting' force would produce a circular vortex in absence of the β effect (and indeed, near the origin the streamlines are circular). But, Rossby-wave propagation makes the vortex lop-sided, extending far to the west.

The β-effect is particularly strong in the tropics, and together with gravity-wave and Kelvin wave dynamics, helps to shape circulations there. An idealized steady heat source at the Equator, Figure 10, causes a low-level convergence of the winds which rise into the heating region. This is for an atmosphere initially at rest. Yet this convergence forms a double cell of circulation west of the heating region, which is again a β-plume. The winds are drawn in from east of the heating in another 'plume' which is shaped by Kelvin wave dynamics (Chapter xx). Motions that involve strong horizontal temperature variations and corresponding vertical velocity variations (through the thermal-wind balance, Chapter xx) are termed *baroclinic*, whereas winds with little vertical variation in the pattern of the velocity are termed *equivalent-barotropic.* The winds produced by heating here are quite baroclinic, yet with a significant equivalent-barotropic contribution. When the observed mean atmospheric winds are added to this model, the very different pattern of nearly stationary Rossby waves appear downwind of the forcing, Figure 5: yet, we can still see the β-plume upwind.

The β-plume form of Rossby-wave propagation is active in summer. Important monsoon circulations develop due to continental heating, which supply fresh water for about half of the world's population. Compensating subsidence produces, in the lower troposphere (~850 HPa), anticyclones above all major mid-latitude oceans. Mountains block the zonal flow, closing off the circulations rather as closed gyres form in the oceans below (driven by the winds, with the same sense). The vorticity in both atmosphere and oceans involves the Sverdrup balance $\beta v = f\partial w/\partial z$, yet it is Rossby wave dynamics that communicates the continental forcing westward and closes off the extensive anticyclones.

Horizonal propagation: refraction, waveguides and instability. Rossby waves follow propagation pathways ('rays') in an approximate sense, and these rays are bent by variations in large-scale pv from the time-averaged winds, the thermal structure, and topography of the solid Earth. Preferred paths ('waveguides' or 'ducts') of the waves are created in this way, for example in the core of the westerly winds, along the Equator, and in the upper polar troposphere.

We have seen some of the topographically induced standing wave patterns of the atmosphere which at least in part are attributable to Rossby wave dynamics. Consider now the kind of perturbation {alteration, often weak or slight} to the mean standing-wave circulation arising from an additional source of waves, as in Figure 5. For example, during el Nino events in the Pacific, the extraordinarily warm sea-surface temperature can excite waves in the atmosphere passing above. Yet it is found that the wave pattern generated is sensitive to the location (east and west) of the forcing region; this would not be the case for a simple Rossby wave problem. We must generalize the restoring effect for Rossby waves to include pv gradients in the mean winds themselves. In equation (6) β is replaced by $dq/dy = \beta - \partial^2 U/\partial y^2$ for this barotropic model. Now consider what this does. The *curvature* of the U(y) profile (which is the gradient of relative vorticity) is subtracted from β. For an easterly jet, the q(y) profile now has a 'flat spot' with small pv gradient cut like a plateau in the 'β hillside'. A westerly jet gains a concentrated gradient at the core of the jet. The concentration of vorticity in the mean flow augments the β-effect for the case of the westerly jet: it is the simple sum of the planet-scale PV and the vorticity of the time-averaged winds that counts. The ray paths describing propagation of groups of short Rossby waves will bend toward from regions of large β^*, defined by

$$\beta^* = \left[\frac{dq/dy}{U}\right]^{1/2}$$

In this way waves will be deflected away from an easterly jet and trapped inside a westerly jet, which thus acts as a wave-guide.

Three panels in Figure 11 show the observed mean winter westerly wind, the barotropic pv gradient, dq/dy, and the effective restoring term β^*. Using similar, but three-dimensional fields from observations, linear Rossby waves were generated by a stationary source of vorticity {a twisting force in a small part of the fluid } as an exploratory computer experiment (Figure 12). The size of the forced region is about 30^0 of latitude. The waves indeed follow the westerly Northern Hemisphere jet. Preferred propagation into and out of the tropics occurs (Figure 11) where the zonal winds are weak or westerly, rather than the more usual easterly winds. Lines along which U=0 lead to infinite values of β^*, and these 'critical lines' tend to reflect Rossby waves, after a certain amount absorption of their wave activity and momentum. Computer models are sensitive to the way such regions are handled, and to the levels of frictional damping assumed in their formulation, leading to lingering uncertainty about many features of the circulation.

While the idea of Rossby waves having preferred wave-guides is attractive, a slightly different interpretation of this experiment is that the jetstream itself is prone to meandering. It has a strongly concentrated pv gradient, and when disturbed it develops stable but intense oscillations, which have both stable and unstable components. This is not quite the same thing as a Rossby wave-guide, and perhaps better describes the 'capture' of Rossby-wave energy by the underlying circulation.

This idea has been extensively developed, and it is found that the time-averaged winter winds can actively contribute to the wave field, exhibiting barotropic instability which can resemble a simple train of Rossby waves. Thus, in the PNA pattern, describing the atmospheric response to a warm tropical Pacific ocean (Chapter xx), energy can be added to the wavetrain, transferred from the large-scale circulation, *en route* to North America. The weakly unstable modes are not easy to sort out because of the more rapidly growing baroclinic instabilities of the system.

Energetic eddies and breaking Rossby waves. The atmosphere is made more complex, however, by the great strength of the winds and the large temperature range. With winds as strong as those typically observed, nonlinear effects (neglected in the simplest theory of Rossby waves) are strong, and lead to a large-scale form of turbulence ('*geostrophic turbulence').* We have just described how Rossby waves *interact* with the general circulation. It goes both ways: the waves induce new and important arteries of general circulation and the circulation, through something like linear instability, generates waves and eddies.

A view of the great energy level of the atmosphere is readily seen in daily satellite images. The potential vorticity field at the 320K isentropic surface, on

May 14, 1992 , Figure 13, cuts through the tropopause, showing the highly convoluted path of the jet stream. Obviously this is not a ‘small perturbation to a westerly wind’.

And, general circulation aside, the eddies interact among themselves in ways that are not subject to the propagation rules of waves. Geostrophic turbulence {the strong interaction among eddies which are of the scale of storms and weather, and larger} obeys none of the rules of classical turbulence observed in Nature: energy cascades {flows, moves} predominantly to large horizontal scale and into barotropic {tall motions with flow that is similar over a wide range of altitude} eddies. The merging of two eddies of the same sign {both either cyclonic or anticyclonic} is a part of this cascade, and can lead to concentration of the flow into a few, sparse, intense eddies. The ‘*life-cycle’* of intensifying cyclonic storms in the atmosphere is a manifestation of this cascade toward tall eddies with reduced vertical shear and hence reduced potential energy, then followed by horizontal propagation: barotropic Rossby waves propagate rapidly, and the turbulence cascade feeds energy into them. Typically the growing storm develops cut-off {closed patterns of} temperature and pv fields whereupon it is more an *eddy* than a wave.

More generally, the interaction of the transient and stationary waves, as calculated from observations, is an expression of the complex mix of Rossby waves and eddies (stationary and traveling) found in the general circulation.

The chaotic development of geostrophic turbulence has similarities to the pairing of cyclones and their north-south movement, which reorganizes the larger-scale circulation. Purely barotropic turbulence also coexists with Rossby waves, and can feed energy into them: either through jostling of a pair of adjacent eddies or through distortion of an eddy by the larger scale circulation.

When the fluid motion is very energetic, *pv mixing* occurs, and it directly forces changes in the large scale atmospheric circulation by shifting momentum about. In some circumstances this can be described as a ‘breaking’ Rossby wave, in which lines of constant pv fold over sideways(still with nearly horizontal motion) and curl up.

More about vertical propagation.   As we have seen, the defining properties of Earth’s atmosphere and ocean (and indeed, the atmospheres of other planets) arise from the underlying *rotation* of the planet, its form as an irregular *sphere*; to these we now add the extreme thinness and their nearly *horizontal layering* of air and water, with gradation from large density below to small density above. This density varies significantly over one scale height of the atmosphere, defined as $H_s = RT/g$ (R being the gas constant, T the temperature and g the gravitational acceleration) it is typically 8 kilometers. The layered stratification allows vertical variation of the horizontal wind, according to the thermal wind relation (Chap xx). This introduces a new class of *baroclinic* Rossby wave which propagates in all three dimensions. We now have a basis for understanding Figure 2b.

The extreme gravitational stability of these fluid layers is expressed in the *buoyancy frequency,*

$$N = \left(\frac{g}{\theta}\frac{\partial\theta}{\partial z}\right)^{1/2}$$

which is the frequency of bobbing of a small air parcel, after it is given an upward or downward impulse (see Chap XX). In the troposphere, the period $2\pi/N$ is typically 10 to 20 minutes, and in the lower stratosphere it is about 5 minutes. Here we use a local Cartesian coordinate system, (x,y,z) = (east, north, up).

The form of the potential vorticity equation (1) including vertical propagation is

$$\frac{D}{Dt}(\nabla^2\psi + \rho^{-1}\frac{\partial}{\partial Z}\frac{\rho f^2}{N^2}\frac{\partial\psi}{\partial Z}) + \frac{dq}{dy}\frac{\partial\psi}{\partial x} = 0$$

Here $Z = H_s \ln(p_0/p)$ is a slightly modified vertical coordinate, proportional to the log of the pressure. For waves with simple sine-wave variation east and west, and in oscillating like a sine wave in time, we find a Helmholtz equation related to equation (3) but now with respect to y and z (north and vertical). For stationary waves with mean westerly wind U(y,z), and simplifying by considering N^2 to be constant, we let $\psi = \exp(ikx)\exp(Z/2H)\,\Phi(y,z)$, giving

$$\frac{\partial^2\Phi}{\partial y^2} + \frac{f^2}{N^2}\frac{\partial^2\Phi}{\partial Z^2} + \nu^2\Phi = 0$$

$$\nu^2 = \left[-\lambda^2 - k^2 + U^{-1}\frac{dq}{dy}\right];\ \lambda = f/2NH$$

Here ν^2 is the index of refraction for the waves; simple refraction occurs, in which rays bend toward regions of large index of refraction, and their paths can be sketched on a plot of ν^2. Solutions of this equation are wave-like where ν^2 is greater than zero, and waves cannot penetrate regions where ν^2 is negative; only a small tail of the disturbances reach inside such regions. In particular, the waves can penetrate only where U falls in the range

$$0 < U < \frac{dq/dy}{k^2 + \lambda^2}.$$

Jule Charney and Philip Drazin showed, in 1961, how in this analysis the zonal wind must be westerly and yet not too strong for stationary waves to propagate upward toward the stratosphere. The signature of this upward propagation is a three-dimensional wave-vector $\vec{k}$ that points downward and to the west, which means that the wave-crests are sloped upward to the west, just as in Figure 2. Upward propagation happens in a window of time in mid-winter. It is remarkable that the very dispersive Rossby wave should, in the case of a single wave in x and t, be governed by a simple, single index of refraction (as if it were non-dispersive). Tracing of the rays along which the waves propagate in the y-z plane (northward, upward) can be done immediately and easily (see model simulation inFigure 14). Errors due to east-west variations in the large-scale 'mean' atmosphere can be quite important, and tracing of rays in all 3 dimensions would seem important, yet the very long wavelength east and west invalidates simple ray-tracing theory.

The upward propagation of Rossby waves into the stratosphere is particularly energetic in the northern hemisphere in winter, and the forces they exert act to decelerate the strong winds of the (radiationally driven, cyclonic) polar vortex. The important waves are very long, with just one or two wavelengths around a latitude circle. The dynamics of a breaking Rossby waves and potential vorticity mixing by the turbulence that follows, is a crucial part of the dynamics of the stratosphere (see Theory (0000)). Confinement of air over the wintertime pole by the strong potential vorticity gradient makes possible the chemically induced ozone hole.

Rossby-wave forcing of the general circulation. Transport of momentum and energy by Rossby waves and by geostrophic turbulence is a key to the 'shaping' of the general circulation. For example the north-south circulation, averaged east and west, of the stratosphere, known as the Brewer-Dobson circulation, is related to strong radiative forcing, but is largely enabled by Rossby waves' transporting easterly momentum upward from the troposphere. The lower atmosphere thus 'pushes on the stratosphere above' and on our rapidly rotating planet this leads to both a deceleration of the polar vortex and flow at right angles (poleward). The Rossby-wave force that does this has been diagnosed from observations (Figure 15).

The transmission of energy and momentum in Rossby waves has an elegant theory, with deep relations to classical physics. Potential vorticity, being conserved following air parcels (in an ideal sense, ignoring dissipation and external forcing), provides remarkable connections between the 'stirring' of fluid (as seen by marking it with colored smoke) and forces and momentum. Such forces are at work in intensifying the jet streams, redistributing zonal momentum iN the vertical, as well as driving the meridional circulation of the stratosphere.

Concluding remarks. There are many more manifestations of Rossby waves in the atmosphere than have been described here, both ultra-long modes with rapid easterly propagation, modes related to tidal forcing, and particularly the large derivative class of potential-vorticity waves involved in baroclinic growth of storms. Waves, instability, wave-induced acceleration of the mean circulation, life cycles of geostrophic turbulence, and potential vorticity mixing fill out the dynamics of the atmosphere, and Rossby waves are just the first step toward their understanding.

Finally, we must remark that ideas in science should be tested when possible by physical experiments. On a rotating platform rather like Rossby's (Figure 16), we can easily produce a 'polar β-plane' in which the steady paraboloidal free surface of water in a cylinder provides a potential vorticity gradient very like that in the middle and high latitudes of Earth. By oscillating a small glass cylinder up and down, squashing vortex lines, we generate an energetic Rossby wave (Figure 17), most visible east of the wavemaker. The short waves seen there have phase propagation *toward* their energy source, rather like the theoretically derived waves in Figure 3. The waves also transport momentum, driving easterly flow at most latitudes (as predicted by theory), and a westerly jet at the latitude of the forcing. They coexist with turbulent eddies. Furthermore, the orange polar cap region, despite strong wave activity, does not mix with the lower latitudes. This is the 'ozone hole effect', by which potential vorticity gradients inhibit mixing just as they promote Rossby waves.

Historical notes. Large-scale waves on a rotating sphere were predicted with mathematical theory at the end of the 19th Century by Hough and Margules. Bernard Haurwitz (Figure 18), after leaving Germany in the early 1930s, derived the essential properties of these modes in a 1937 paper. His status as 'enemy alien' in the USA in 1942 did not prevent the Army Air Corps from asking him to direct a research program on weather forecasting. Carl-Gustav Rossby (fig. 16) developed an elegant approximation known as the β-plane, with which the derivation of the waves is greatly simplified. As so often happens in science, the full force of the earlier theory did not become apparent until long after its discovery. Rossby's influential first paper on the subject appeared in the Journal of Marine Research in 1939 (reminding us that these waves do exist in both ocean and atmosphere). By emphasizing the simple propagation formula Rossby successfully brought the ideas to bear on observations of the circulation at MIT in important papers in 1939, and through the war years. Before the era of computer simulation of the atmosphere, Rossby waves provided a foothold of dynamical theory in aid of weather forecasting. Much later, in the last quarter of the 20th Century, Rossby wave dynamics has filled out like a powerful flood light our understanding of the dark corners of atmospheric dynamics.

Acknowledgments. This review has drawn heavily on works of Brian Hoskins and his collaborators. The author's work in this area is supported by National Science Foundation.

Further Reading

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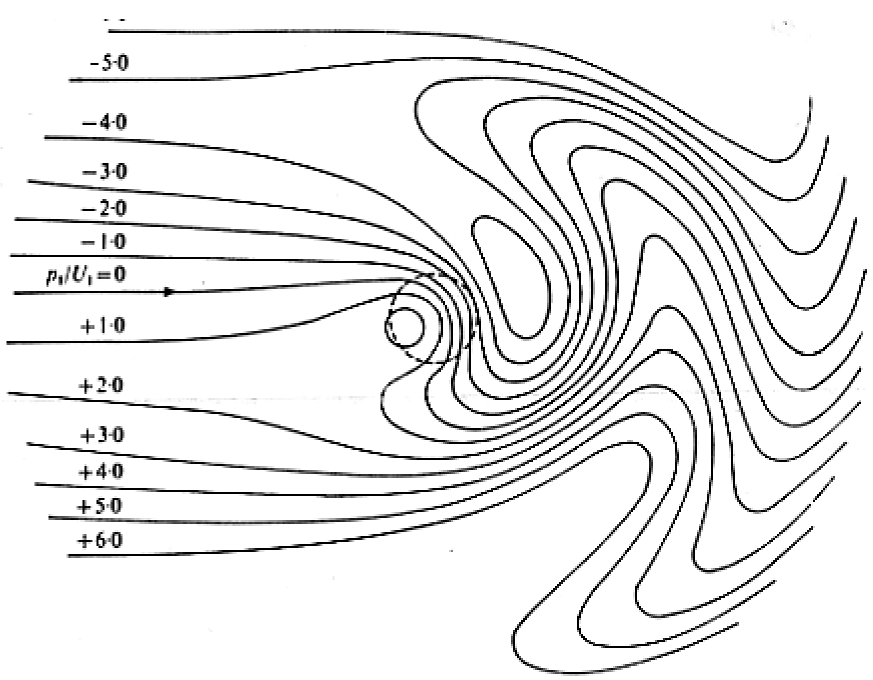


Figure 1. Rossby waves in westerly flow over an idealized mountain (a circular cylinder, shown at the center). (Figure from McCartney, Journal of Fluid Mechanics, 1976). Shown are the streamlines, a stationary pattern, along which the wind is directed.

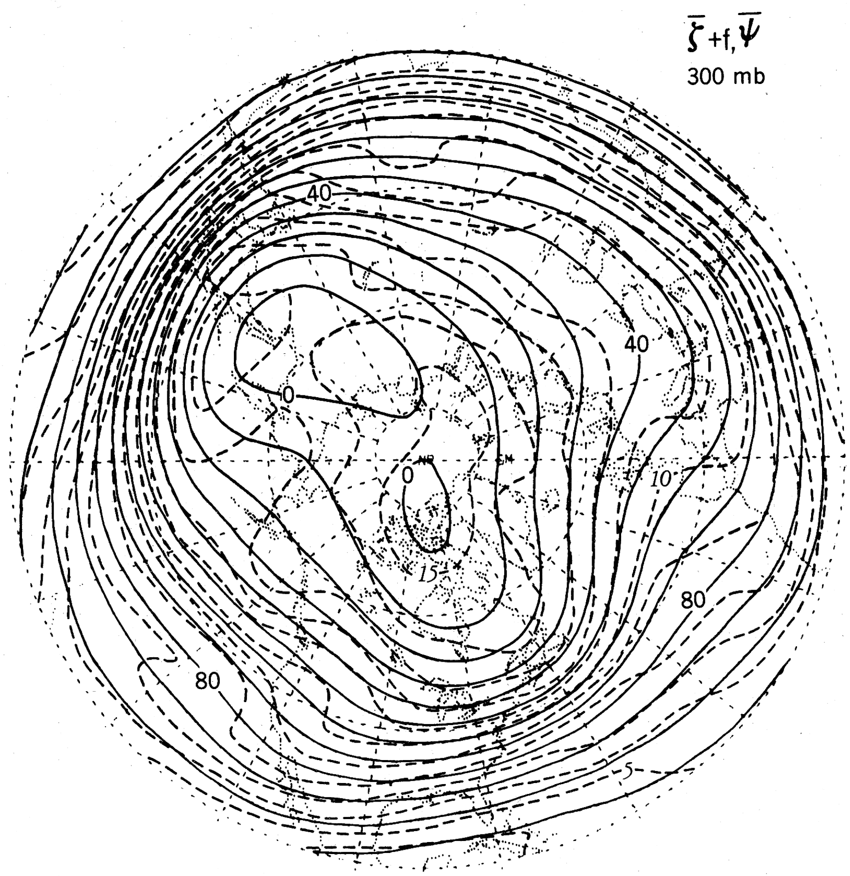


Figure 2a. Streamlines (solid) and absolute vorticity, $\zeta + f$, averaged at 300 HPa level in the upper troposphere (figure from Lau, Journal of the Atmospheric Sciences, 1979). The deviation of the streamlines from circles is a wave pattern analogous to Figure 1. If this were simply barotropic Rossby waves, the dashed and full lines would coincide. Continents are shown with dotted curves; North America is at the bottom of the figure.

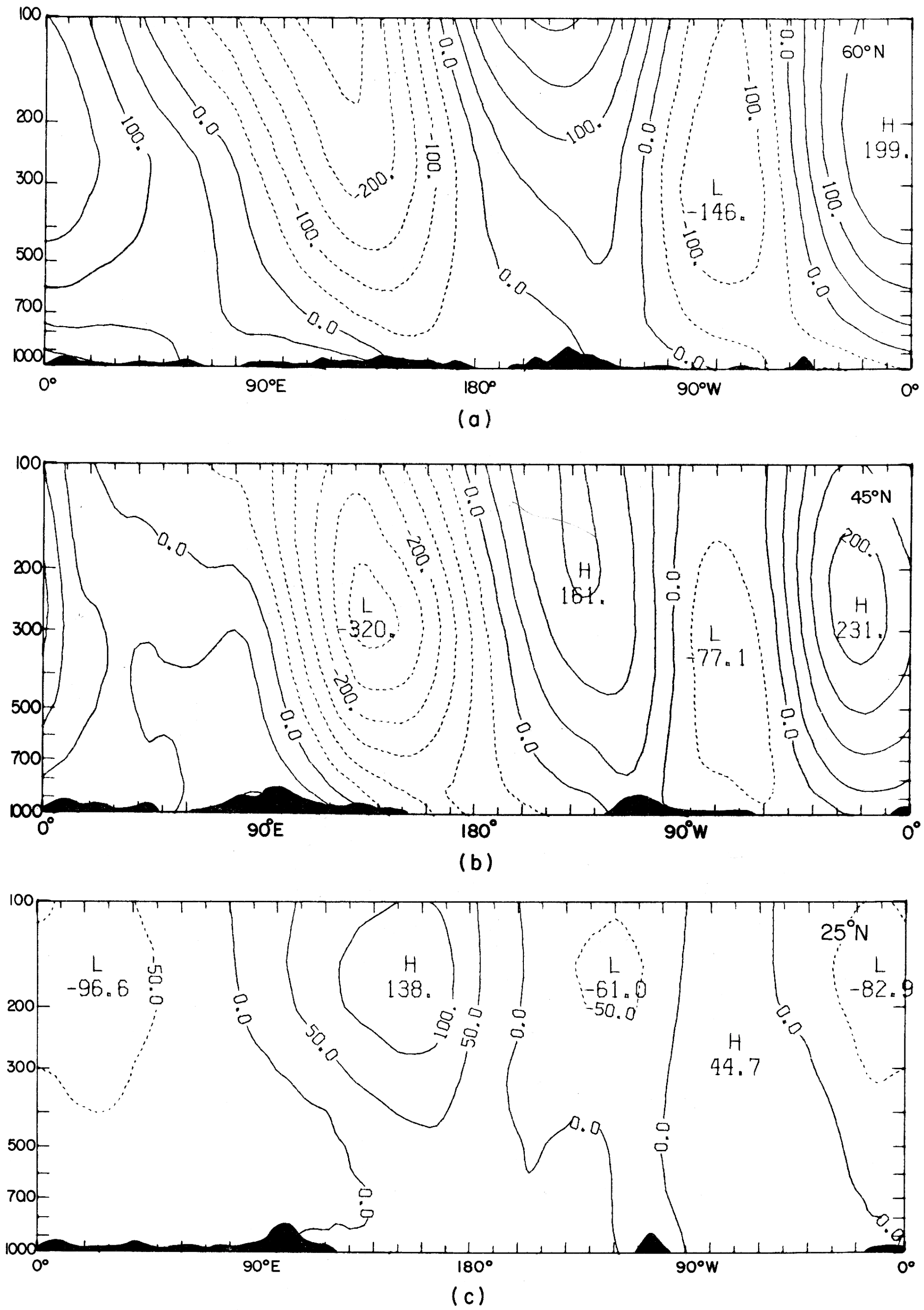


Figure 2b. Cross-sections of the geopotential height (in meters) of the atmosphere at 60^0N (a, upper); 45^0N (b, middle) and 25^0N (c, lower). This is the flow averaged in time during the winter season. Here the vertical axis is pressure-level and horizontal is longitude. To a good approximation this can be viewed as a plot of pressure with respect to actual altitude and longitude (figure from Lau, Journal of Atmospheric Sciences, 1979). The flow in Figure 2a cuts through this figure at the 300 Hpa level.

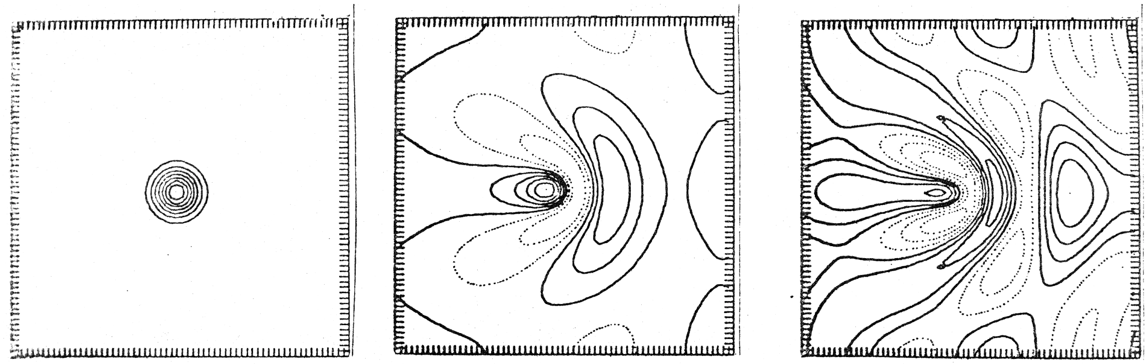


Figure 3. The breakup of an initial weak vortex (left panel). Shown are streamlines or constant pressure curves. The ‘banana’ shaped patterns are high- and low-pressure cells associated with Rossby waves. This is a single layer of fluid *without* a mean westerly wind, using a β-plane approximation to the rotating Earth. This numerical solution is periodic east and west, with waves entering the domain from the east after exiting in the west (Rhines, The Sea, V.6, Wiley Interscience, 1977).

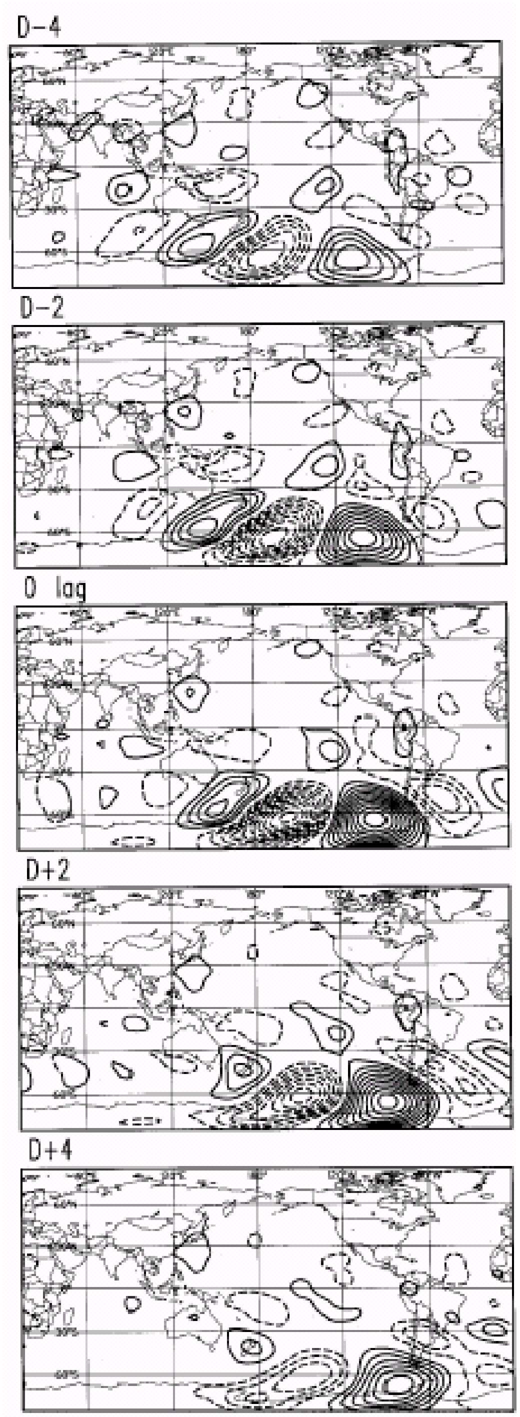


Figure 4. From Renwick and Revell Monthly Weather Review, 1999. A map showing the correlation between the north-south winds at 300 HPa level with the time series of southeast Pacific blocking. The panel labeled D – 4 represents the correlation pattern 4 days before maximum blocking, and the sequence proceeds in time to 4 days after peak blocking. The apparent wave train propagates from Australia southeastward across the Pacific. 'Blocking' here means a period of at least 5 days when the 500 HPa pressure is at least 0.5 standard deviation above the norm. The source of the wave train is thought to be cumulus convection in the western equatorial zone, and hence there is a strong correlation of warm el Nino periods with blocking patterns at higher latitude (SOI index:blocking index correlation reaches –0.8 during the past 15 years).

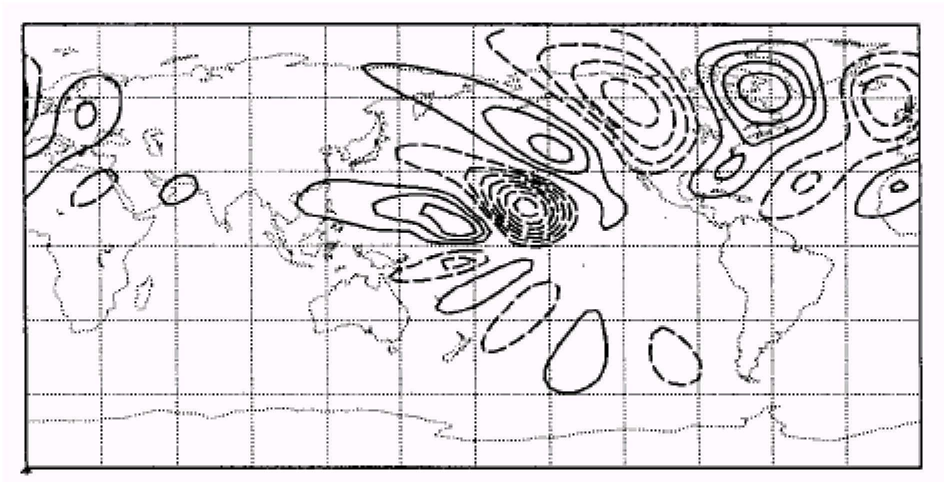


Figure 5. Computer models of the atmospheric circulation play a key role, not just in weather forecasting, but in developing ideas and testing theories of the underlying dynamics. Here, trains of Rossby waves propagate in both hemispheres from a source of cumulus-convective heating in the western equatorial Pacific. The model takes the observed, fully three-dimensional structure of the circulation, and calculates the change in the winds arising from tropical heating by the warm ocean (figure from Jin and Hoskins, Journal of Atmospheric Sciences, 1995). The waves all emanate from the western equatorial Pacific, moving eastward along (very approximately) great-circle paths. Plotted contours show the north-south wind (not including that of the time-averaged winds) in the upper troposphere, contour interval 0.5 meters per second.

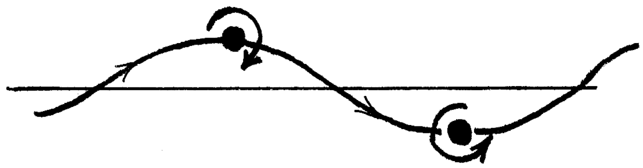


Figure 6. Showing how a parcel of air moved northward in a standing wave, conserving its potential vorticity, $\zeta + f$, develops negative (anticyclonic) spin, ζ, which affects fluid to the east and west, reinforcing the north-south motion of the wave itself.

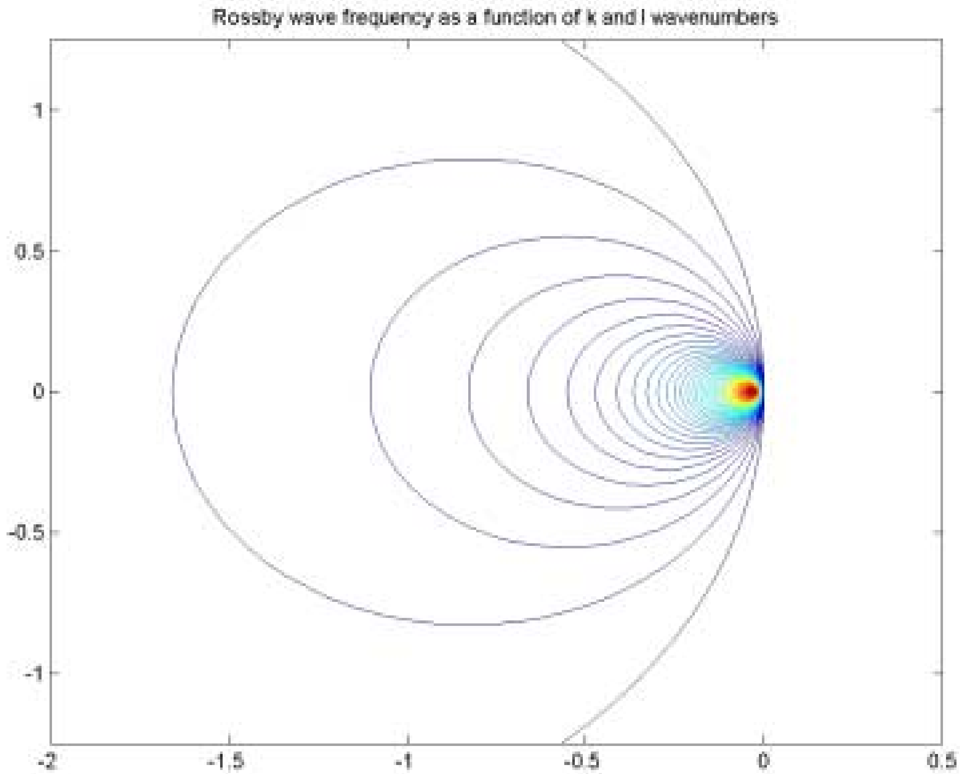


Figure 7. Frequency-wavenumber diagram for Rossby waves in a fluid without mean east-west winds. Here we have wavenumbers k and l in the east and north directions, respectively, as horizontal axes. Curves of constant frequency, ω, are plotted, with frequency increasing toward the origin. The waves reverse the normal property of non-dispersive waves, having higher frequency for longer wavelength. Energy propagation for a given wave is directed perpendicular (and inward) to these curves. Thus, for example, a wavevector pointing from the origin upward/leftward (northwestward) has wave crests and winds directed northeast-southwest, and energy propagating southward.

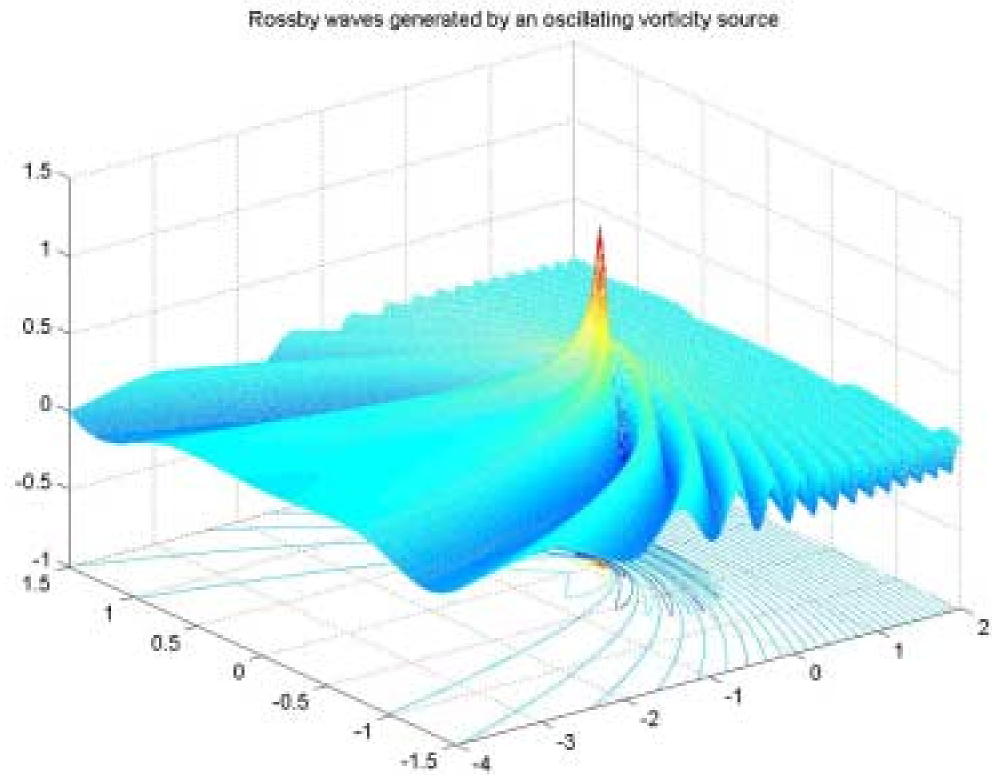


Figure 8. Rossby waves generated in a fluid initially at rest, by an oscillating force at where the field has its highest peak. This is a perspective view of the pressure, or streamfunction, seen from the southwest. Short Rossby waves have eastward energy velocity, and hence appear to the east, while long waves with nearly east-west velocity extend west of the forcing. On the plane beneath the contour plot of the same field shows the parabolic shape of the wave-crests. With time they sweep westward and 'collapse' on the latitude line emanating westward from the forcing.

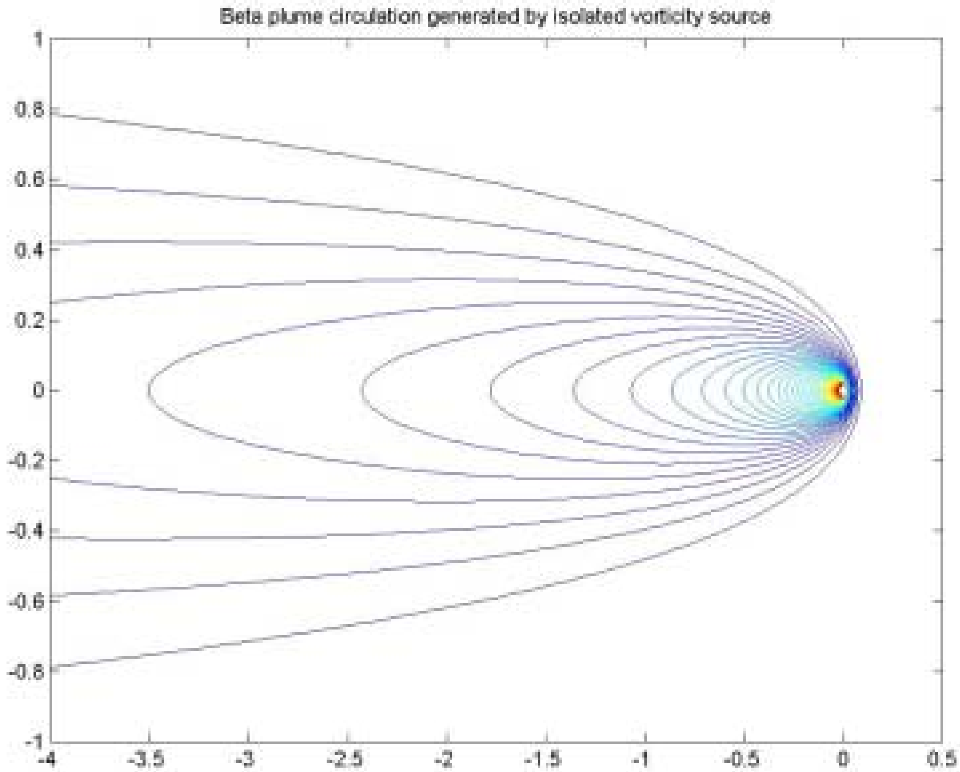


Figure 9. The β plume is a circulation generated by a small region of forcing (for example, by heating the atmosphere with convective clouds). Here, in the middle latitudes, on a β-plane, the circulation reaches west of the forcing forming an elongated gyre. Rossby waves of very low frequency are active in setting up this western extension of the circulation. The forcing at the origin could be a small region of heating by cumulus convection (driving divergent flow in the upper troposphere, and hence anticyclonic circulation and convergent winds, with cyclonic circulation in the lower troposphere), or a mechanical 'twisting' force concentrated at the origin. This is a plot of the theoretical result, which is a Bessel function with imaginary argument multiplied by an exponential.

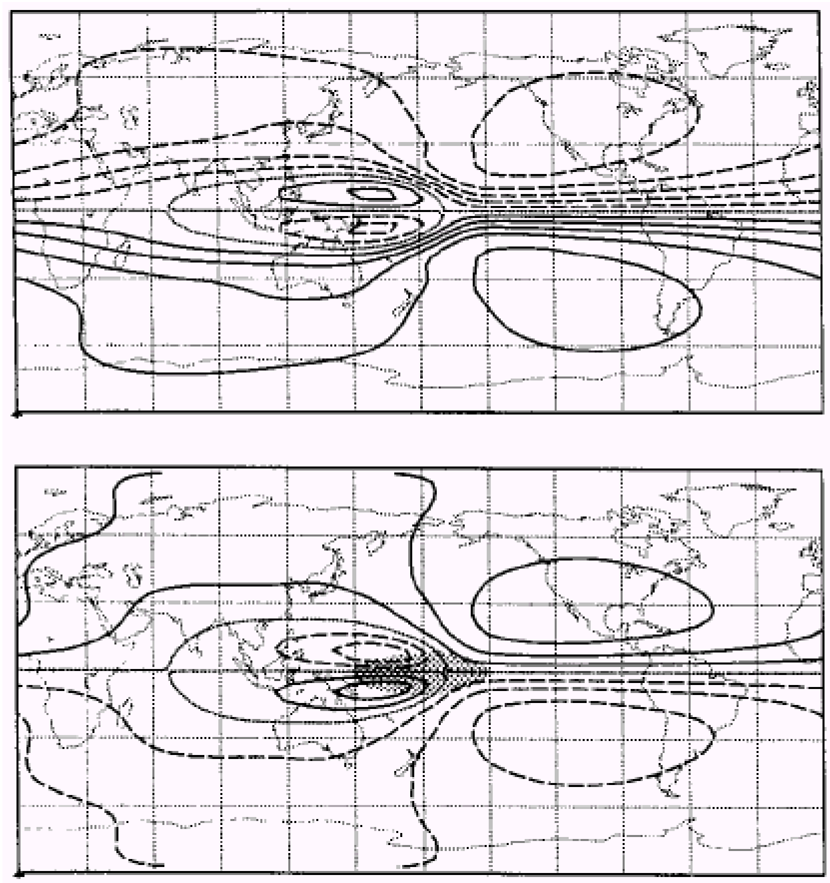


Figure 10. (From Jin and Hoskins, 1995) On the Equator, the pressure response of an atmosphere without mean winds, to heating by cumulus convection, (shaded region) involves both Rossby- and Kelvin waves. Here the β-plume draws air in from the west to feed the low-level convergence, while drawing in from a 'Kelvin plume' to the east. The top panel shows the upper troposphere, where the updrafts diverge outward, and the bottom panel is the lower atmosphere, where convergent winds feed the updraft. Because the sign of f changes across the Equator, we have a double-celled pattern rather than the single circulation cell in Figure 8 (cyclonic wind cells below, anticyclonic above in the β-plume west of the forcing).

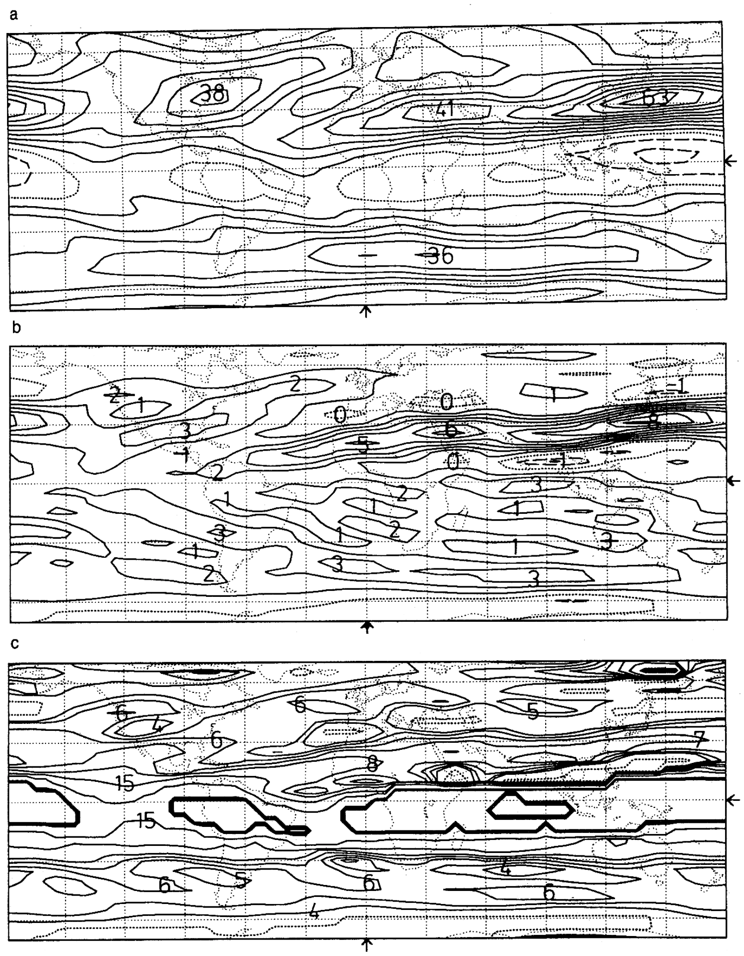


Figure 11 (A): Mean westerly wind speed in northern-hemisphere winter. (B): mean north-south barotropic potential vorticity gradient, $\beta-\partial^2U/\partial y^2$. Note the large values in the mean westerly jets. (C): effective restoring force for Rossby waves, β^*. Heavy line in lower panel is a concentration of high values of β^* at critical lines. Zero lines are dotted, negative are dashed. The Greenwich meridian is marked with an arrow. (From Hoskins and Ambrizzi, Journal of the Atmospheric Sciences, 1993).

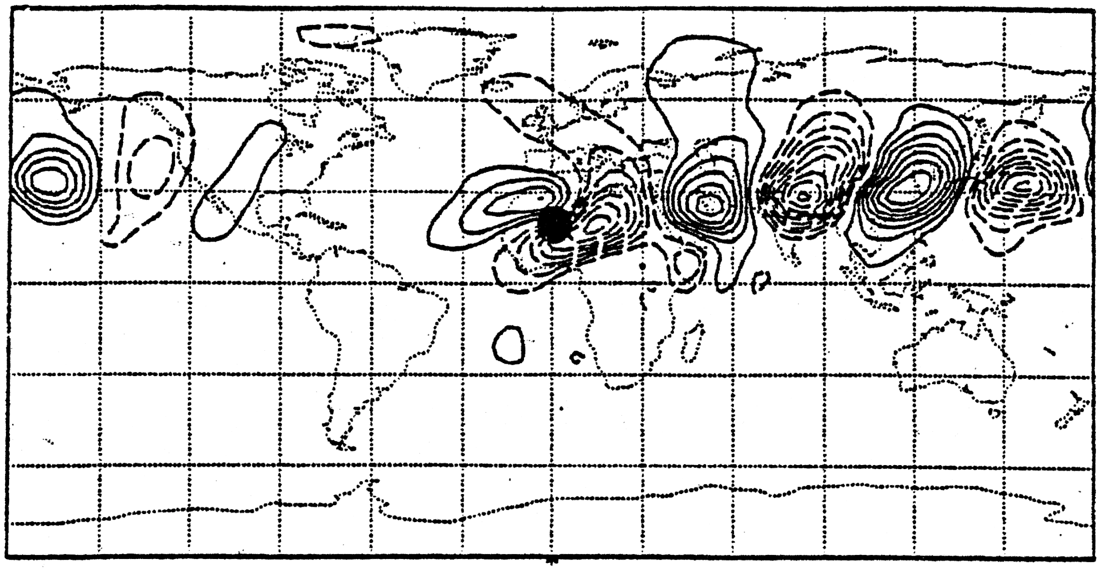


Figure 12. (From Ambrizzi and Hoskins, Quarterly Journal of the Royal Meterological Society, 1997). Meridional wind anomaly at day 10 for circular heat source at 20^0N, 0^0E, with December-January mean flow. This is a three-dimensional model calculation, but with a wave environment dominated by the barotropic pv, Figure 11.

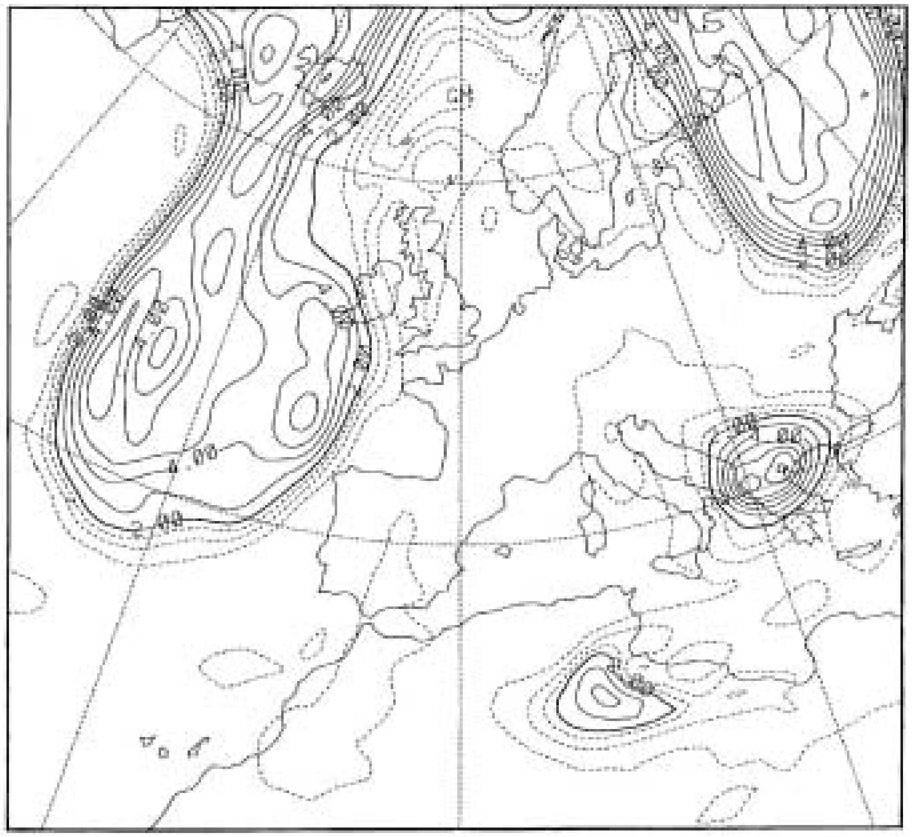
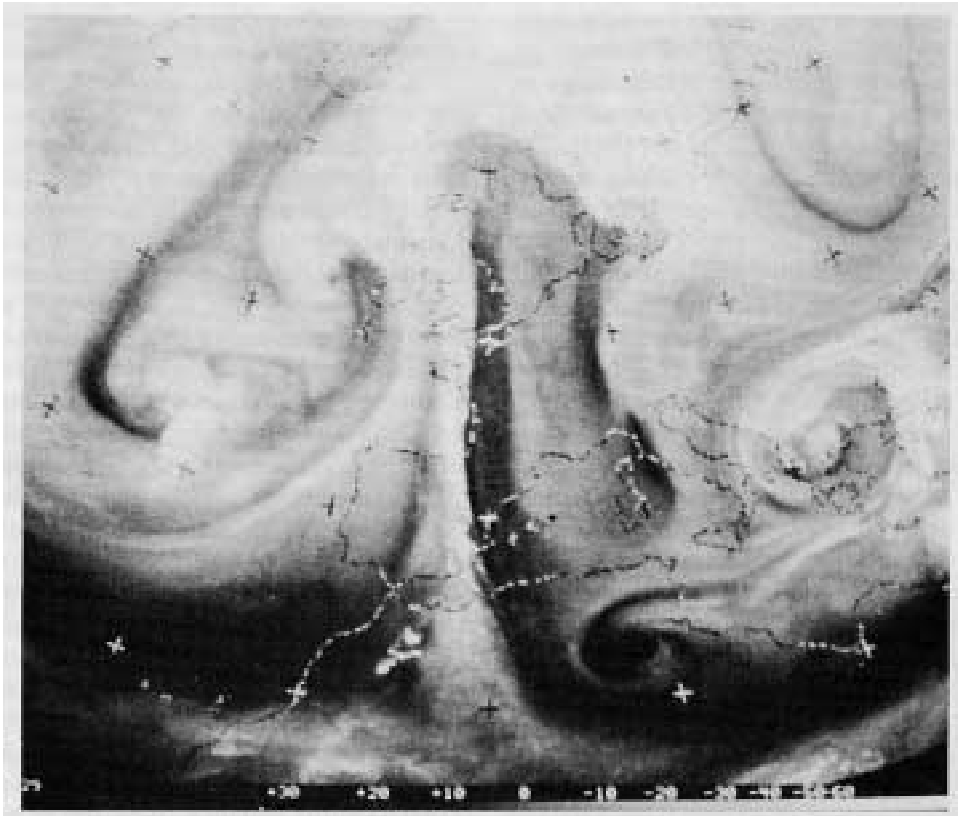


Figure 13. (A): Potential vorticity (PV) over Europe on the 320^{0}K surface at 1200UT on 14 May 1992, from ECMWF analysis. The tropopause, dividing troposphere and stratosphere, cuts through the figure along the 0.5 to 1.5 contours. (B): Meteosat image of water vapor (5.7 to 7.1 micron wavelength radiation) at the same time as (A) (from Appenzeller and Davies, Nature 359, 570-572, 1992).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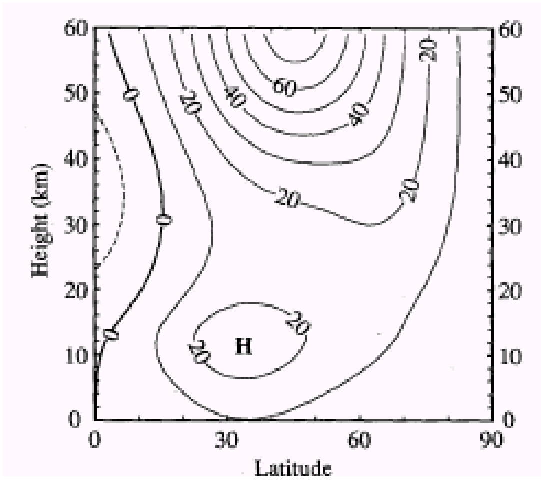
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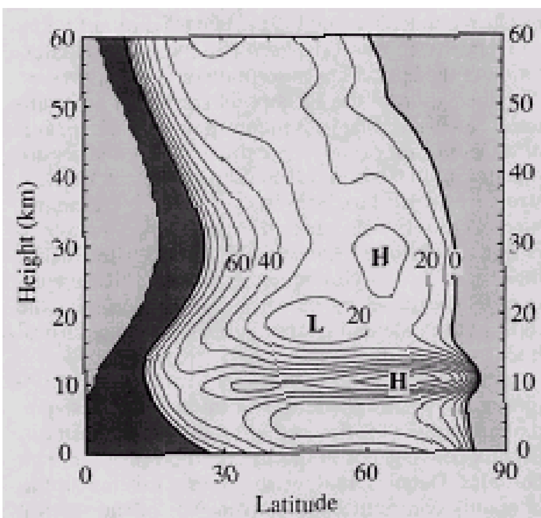
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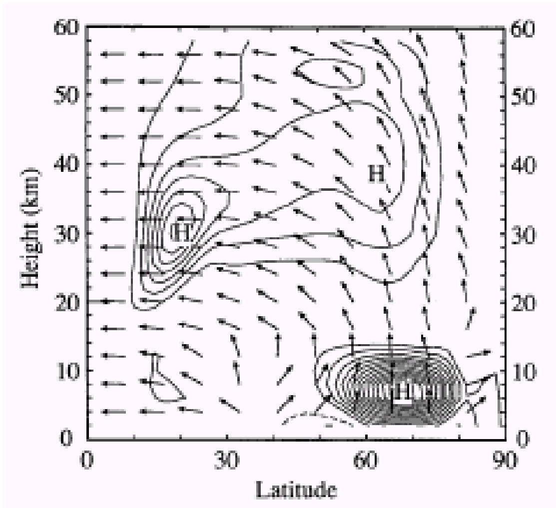

Figure 14. (Numerical model experiments from Chen and Robinson, Journal of the Atmospheric Sciences, 1993). Upper panel: mean east-west wind used in the model. Middle panel: refractive index for steady zonal wavenumber 2. Dark band has high positive values. Lower panel: propagation paths for idealized Rossby waves, refracting toward the Equator (toward large refractive index, or large wavespeed) with height. Contours show the convergence of the Elliasen-Palm flux, which exerts a westward force on the general circulation; it is particularly strong in the tropics where U goes to zero.

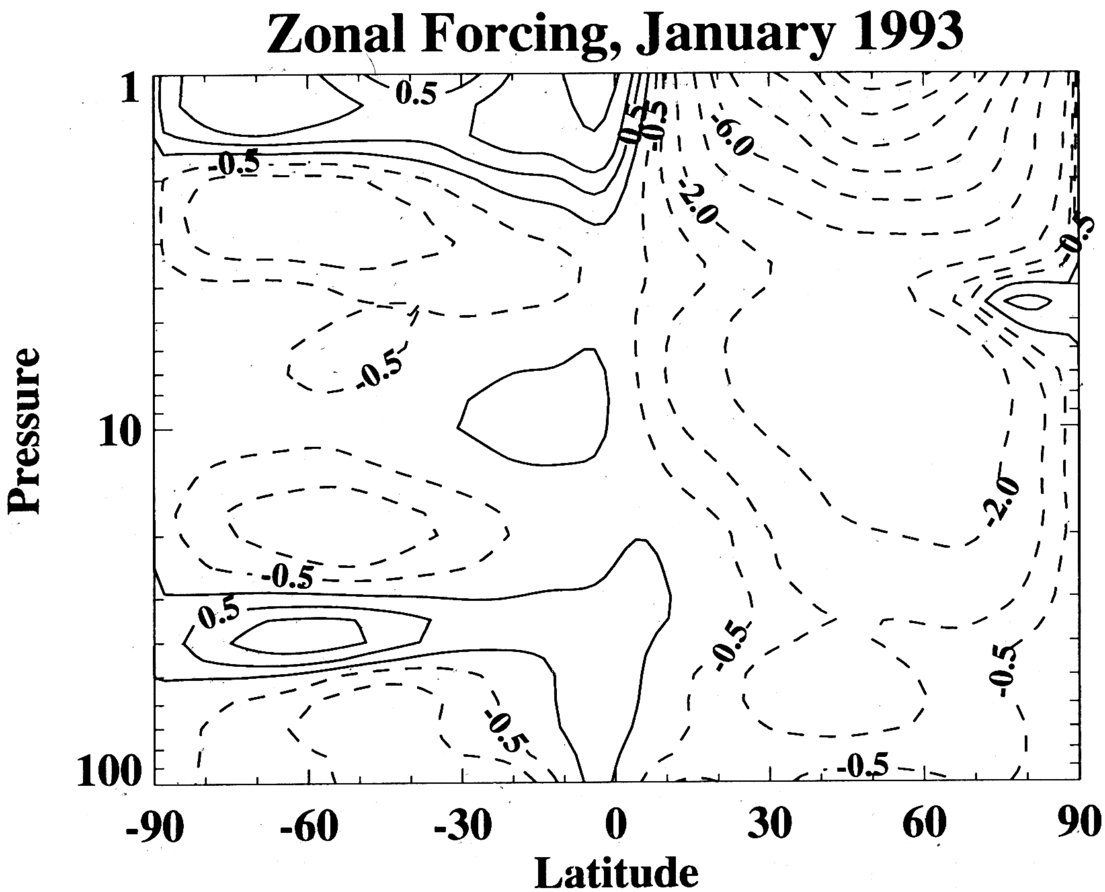

Figure 15. Wave-induced zonal force per unit mass, G, in units of meters per second, per day, for Jan 1993. Derived indirectly from the zonal momentum balance using radiatively derived meridional circulation and observed zonal mean wind. The dashed curves indicate the negative values expected from upward flux of east-west momentum transported by long Rossby waves. Figure after Rosenlof and Holton, Journal of Geophysical Research, 1993.

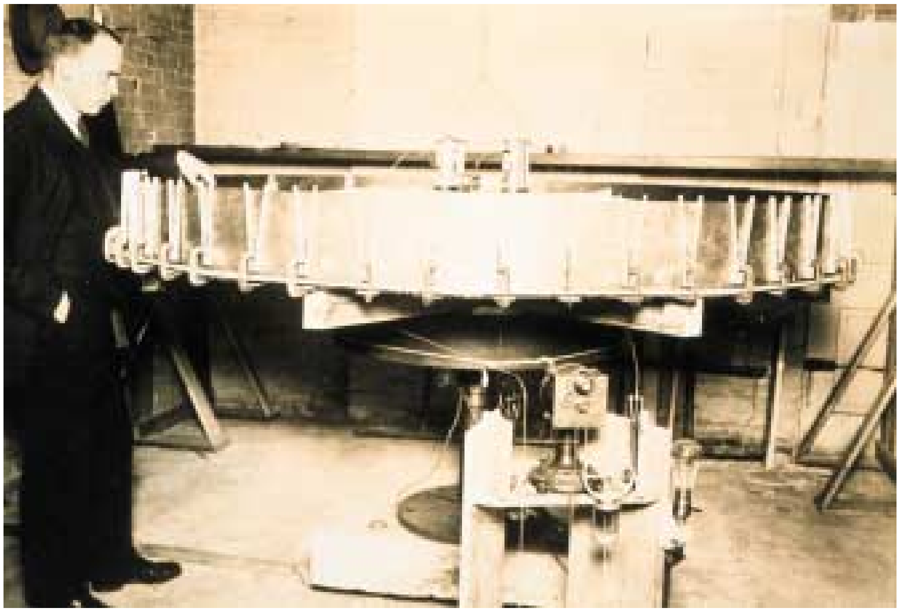

Figure 16. Carl-Gustav Rossby in 1926 or 1927, with a rotating platform designed to simulate the Earth's rotation and produce waves and 'weather' (NOAA historic photo archive, http://www.photolib.noaa.gov/historic)

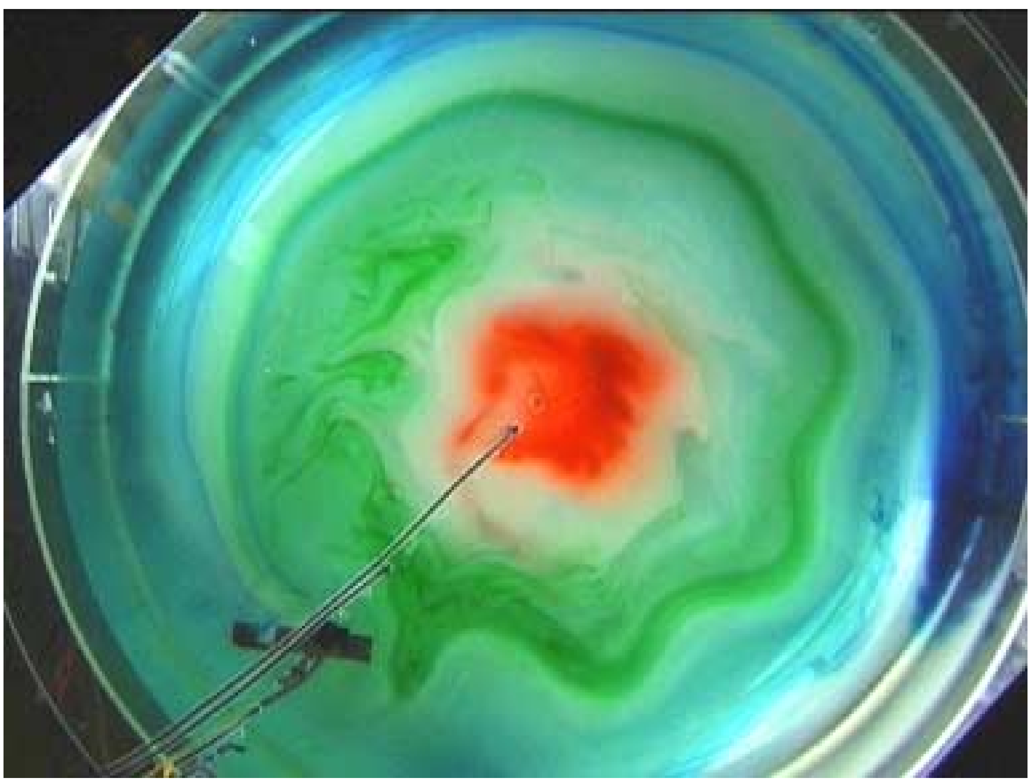

Figure 17. Rossby waves in the laboratory, as if viewed by a satellite above the North Pole. The wave source is at the lower left, and oscillating body. There is no pre-existing circulation, but the waves induce easterly flow at most latitudes, and westerly flow at the latitudes near the forcing (as seen in the dye drawn into circles). At the North Pole, the red dye remains unmixed by the strong wave activity. (Courtesy of Geophysical Fluid Dynamics Laboratory, University of Washington.)

Figure 18, Bernard Haurwitz, one of the pioneers of Rossby wave theory.