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ANCIENT AND MEDIEVAL VALUES FOR THE MEAN SYNODIC MONTH

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In a paper published shortly before his death, Neugebauer reviewed the transmission of the standard Babylonian value for the mean synodic month (hereafter: M).[1] He restricted his attention to this value for M and did not consider its derivation from the observational record. In other articles and books that he wrote over the years Neugebauer discussed a variety of values for M, but he did not gather these data together in one place. Recently, I argued that the Babylonian values for the lengths of the various months could have been derived from relatively crude observations by simple counting and arithmetic manipulation. The only value for which I was unable to find a simple derivation was the standard length of the mean synodic month, although I came upon a method that yielded a value close to it.[2] I believe that this gap in the argument can now be filled by a much more plausible conjecture, based in part on information that has just come to my attention.

In this paper I will focus on a few selected values for M that occur in sources prior to A.D. 1500, starting with the Babylonians. In general, the precision of the parameters far exceeds their accuracy, but I will ignore problems of accuracy and report the evidence to the number of sexagesimal places given in the texts themselves (or that otherwise seems appropriate). For purposes of this discussion, it may be useful to distinguish 'ghosts' from 'phantoms': by a 'ghost' I simply mean a value unintended by the author that is ascribed to him as a result of a copyist's error or, in modern times, a printer's error; by a 'phantom' I mean a value correctly derived from rounded data that yields an unintended value. Both ghosts and phantoms may occur in ancient and modern literature. As a rule, I have tried to avoid both ghosts and phantoms, but for an example of a phantom, see Appendix 1.

The standard Babylonian value for M is $29;31,50,8,20^{d}$ and it is associated with System B where it appears in the form 29d $191;0,50^{\textit{uš}}$ (*uš* correspond to time-degrees: $360^{\textit{uš}} = 1^{d}$).[3] In medieval texts, it is also given equivalently as 29^{d} $12;44,3,20^{h}$ and 29^{d} 12^{h} 793^{p} (where 'p' stands for 'parts', and $1080^{p} = 1^{h}$). This value is close to what one would get from taking the 18-year period of 223 synodic months (called the Saros) to be $6585;20^{d}$, for $6585;20^{d}/223 = 29;31,50,18,...^{d}$.[4] Another attested Babylonian value for the Saros of 223 months is $6585;19,20^{d}$ which yields $M = 29;31,50,8,4,...^{d}$.[5] Since the same tablet has the standard value for M, it could be argued that $6585;19,20^{d}$ was curtailed from $6585;19,20,58,...^{d} = 223 \cdot 29;31,50,8,20^{d}$, i.e., that $M = 29;31,50,8,4,...^{d}$ is a phantom. But before reaching this conclusion, further discussion is needed (see below).

In a Babylonian text recently discovered by John Steele (BM 45861), there is a

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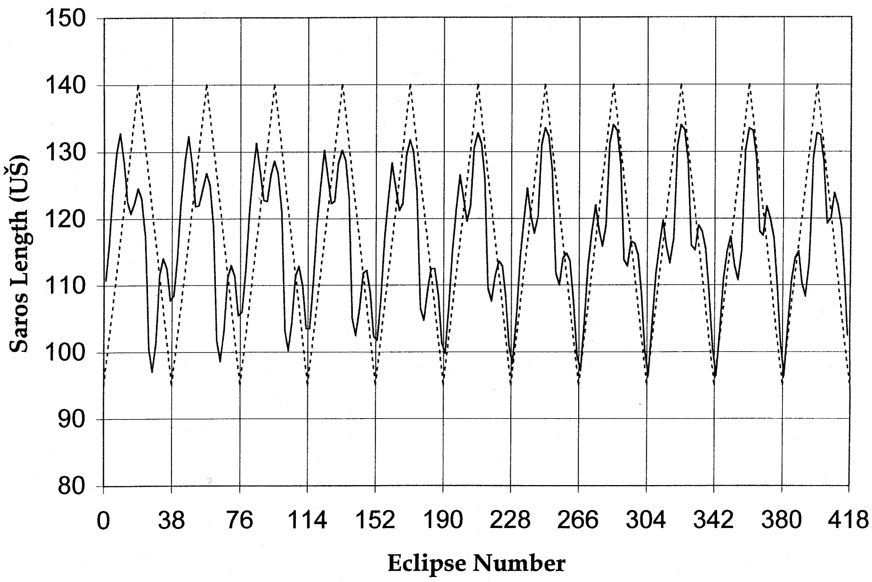

FIG. 1. The variation of the Saros length for the odd numbered eclipse possibilities, beginning with the lunar eclipse of –746 Feb. 6. The solid line gives the excess of the Saros length over 6585 days measured in *uš* associated with the odd numbered eclipse possibilities, and the dashed line represents the zigzag function on the Babylonian tablet, BM 45861. Courtesy of J. M. Steele.

zigzag scheme that, he argued, represents the excess of the Saros over 6585 days, measured in *uš*.[6] This suggests that one should consider the length of 223 months (the Saros) as the primary datum rather than the length of 1 month that was probably derived from it. Moreover, the surplus over 6585^d should be considered in the first instance in *uš*, rather than in sexagesimal fractions of a day. In fact, the Babylonians recorded the time of eclipses in *uš* (before or after sunrise or sunset) and, in the early period (*c.* –750 to *c.* –570), these times were given in units of $5^{uš}$. Hence, $6585;20^d$ should be recast as 6585^d $120^{uš}$. There are good reasons to believe that the Babylonians had begun a systematic record of eclipse observations in –746, although the early years are poorly represented in the extant documents.[7]

The Babylonian scheme is deceptively simple. This zigzag has the following characteristics (see the dashed line in Figure 1): its period is 18 lines, each step (i.e., the line-by-line difference) is $5^{uš}$, the minimum is $95^{uš}$, the maximum is $140^{uš}$, the amplitude (i.e., the difference between maximum and minimum) is $45^{uš}$ or $9 \cdot 5^{uš}$, and the mean value is $117;30^{uš}$ (= $0;19,35^d$) which implies that M = $29;31,50,12,...^d$. In Figure 1, this zigzag is compared to what one finds by modern computation for a set of eclipse possibilities with the following conditions. In each Saros of 223 months there are 38 eclipse possibilities, and to each of them is assigned the length of the following 223 months in excess over 6585 days. If the eclipse possibilities are assigned an indexing number, they form two sets of 19 eclipse possibilities (one set with odd numbers, and the other with even numbers). If these two sets are separated, the resulting graphs are similar to one another, i.e., the curve for one set is very nearly the reflection of the other about the mean value. The function displayed as a solid line in Figure 1 is for the odd-numbered set of 19 eclipse possibilities in a series of Saroi, beginning in –746 and extending for about 200 years. The period of 18 for the zigzag seems to have been chosen for arithmetic simplicity, instead of the accurate value of 19 corresponding to half the number of eclipse possibilities, and Steele has not determined how the Babylonians took account of this discrepancy. Despite all the indications of arithmetic simplicity and 'nice' numbers, the graph of the zigzag function is very close to the recomputed data. Steele remarks that about 100 years of data would have been sufficient to determine the parameters for the zigzag function and, as he also notes, the recomputed data indicate that the minimum of the zigzag is reasonably good whereas the maximum is a little too high, perhaps to maintain the simplicity of the amplitude. In units of $5^{u\check{s}}$, a maximum of $135^{u\check{s}}$ looks like a better value than the attested $140^{u\check{s}}$, and it yields a mean value of $115^{u\check{s}}$ or M = $29;31,50,5,...^{d}$.

The zigzag scheme, even as modified, yields a length of the mean synodic month that is close to the standard value, but not identical to it. I then realized that to arrive at '8' in the third sexagesimal place, one would need a mean value of about $116^{u\check{s}}$ (which cannot be derived directly from a minimum and a maximum expressed in units of $5^{u\check{s}}$). Such a value might arise, however, if one decided that a maximum of $135^{u\check{s}}$ was slightly too low and that a maximum of $140^{u\check{s}}$ was too high (keeping the minimum at $95^{u\check{s}}$). The decision in favour of $116^{u\check{s}}$ as the mean value is not explained by this argument, but the choice would have been limited to the interval between $115^{u\check{s}}$ and $117;30^{u\check{s}}$. Taking $116^{u\check{s}}$ as the mean value, $223 \cdot M = 6585;19,20^{d}$, and M = $29;31,50,8,4,...^{d}$. (To get the standard value for M exactly, one would need an excess over 6585^{d} of $116;5,50^{u\check{s}}$.) Note that this value for the Saros, $6585;19,20^{d}$, appears in a Babylonian text where we thought it might be a result of rounding, but here it seems to be a plausible number in its own right. To be sure, the text in which it appears was written long after the early stages in the development of System B, and so it is not clear if its occurrence there provides relevant historical data. Also, Britton has argued (in a personal communication) that the fourth sexagesimal place ('20' in the standard value) was chosen by the Babylonians for arithmetic convenience in the scheme used in System B, and that it is sufficient to account for the '8' in the third place. The derivation proposed here can only be considered 'possible' for there is no textual evidence that directly supports it. On the other hand, the Babylonians never derive their parameters explicitly, and one is left with possibilities of varying degrees of plausibility, to be judged by their compatibility with the interests of the Babylonians and the results they achieved. Further, it seems more likely that the Babylonians did not directly seek mean values by averaging lots of data; rather, they probably focused their attention on minimum and maximum values that occur in relatively short time intervals from which they derived the mean.[8]

In the *Almagest*, Ptolemy gives the Babylonian value for M, $29;31,50,8,20^{d}$, and associates it with a cycle of about 345 years: $4267^m = 126007^d\ 1^h$.[9] In fact, $4267 \cdot 29;31,50,8,20^d \approx 126007^d\ 1;5^h$. Aaboe and others have noticed that Copernicus has $M = 29;31,50,8,9,20^d$ which results from dividing $126007^d\ 1^h$ by 4267^m, the two values given in the *Almagest*.[10] The variant, $29;31,50,8,9,20^d$, already appeared in al-Ḥajjāj's Arabic translation of the *Almagest* in the ninth century and then in many medieval texts in Arabic, Hebrew, and Latin.[11] So, in his *Yesod ᶜolam*, Isaac Israeli (*c*. 1310) reports the value $29;31,50,8,9,20^d$ as that of Hipparchus cited by Ptolemy, and al-Bīrūnī (d. *c*. 1050) cites the value $29;31,50,8,9,20,13^d$.[12] Ibn Yūnus (*c*. 1000) and al-Biṭrūjī (*c*. 1200) have $29;31,50,8,9,24^d$ and, as Neugebauer noted, this variant also appears in some Ethiopic texts.[13] It would seem that the variant $29;31,50,8,9,20^d$ was a phantom in the ninth century in the sense that it was ascribed to Ptolemy who did not intend it, particularly if it resulted from a rounding of $1;5^h$ to 1^h in the length of 4267 months.

The mean synodic month continued to serve as a fundamental astronomical parameter in the Middle Ages. Although a great variety of values are recorded in this period, we will discuss only a few of those that occur in Arabic, Hebrew, and Latin texts. The origins of these values are diverse but, as far as I can determine, there is only one case where the value is derived from specific observational data.

In contrast to the value for M that ultimately came from Babylon, some medieval authors depended on Hindu sources. Of particular interest is the value, $29^d\ 12;44,2,17,21,12^h$ (= $29;31,50,5,43,23^d$), supposedly based on observations but suspiciously close to Hindu values, that al-Bīrūnī ascribes to the Banū Mūsā (ninth century).[14] Yaᶜqūb ibn Ṭāriq (eighth century), as reported by al-Bīrūnī, gives the correlation that 57,753,300,000 revolutions of the Moon take place in $1,577,916,450,000^d$ and in 4,320,000,000 revolutions of the Sun, which implies that $M = 29;31,50,5,43,24,...^d$ (= $29\ {}^{189005}/_{356222}{}^d$, according to al-Bīrūnī), and these numbers derive from the *Brāhmasphuṭasiddhānta* [*BSS*] of Brahmagupta (seventh century).[15] Ibn al-Muthannā (tenth century) ascribes to Ptolemy the value $29;31,50,5,\underline{44},33^d$ that clearly belongs to the Indian tradition and al-Hāshimī (late ninth century) has a similar value, $29;31,50,5,\underline{43},33^d$, which is also ascribed to Ptolemy.[16]

The Jewish tradition for M presents a number of problems. The value for M in the Jewish calendar is $29^d\ 12^h\ 793^p$, and it is equivalent to the standard value for M. The earliest source is a passage in the Babylonian Talmud that ascribes a value of M to Rabban Gamaliel (second century) of $29\frac{1}{2}$ days and $\frac{2}{3}$ hour and 73 parts (it is generally assumed that in this text 'part' means what it does in later treatments of the Jewish calendar where $1080^p = 1^h$):

> Our Rabbis taught: Once the heavens were covered with clouds and the likeness of the moon was seen on the twenty-ninth of the month. The public were minded to declare New Moon, and the Beth din [court] wanted to sanctify it, but Rabban Gamaliel said to them: I have it on the authority of the house of my father's father that the renewal of the moon takes place after not less than twenty-nine days and a half and two-thirds of an hour and seventy-three parts.[17]

The earliest evidence for the standard value in the form, $29^d\ 12^h\ 793^p$, comes from an Arabic text by al-Khwārizmī (ninth century) discussed by Kennedy.[18] It is also reported by al-Bīrūnī, and then by such Jewish scholars as Ibn Ezra, Bar Ḥiyya, and Maimonides, all of whom lived in the twelfth century.[19] Two early Jewish sources have $\frac{2}{3}^h$ and 73^p and they have been called into question as possibly corrupt due to later interventions, but the arguments are not persuasive;[20] note that they do not present M in the same form as in the later tradition. The unit called a 'part' (Heb. *ḥeleq*) is peculiar and Neugebauer argued that it represents the Babylonian unit called *še* (barleycorn), used in the theory of lunar latitude, where 180 *še* = 1 cubit of 2;30°.[21] Since an hour corresponds to 15°, we can set 6 cubits = 1^h, and then 1080 *še* = 1^h (as in the Jewish calendar). The problem is that for measuring time the Babylonians generally used *uš* (time-degrees) and *bēru* (double-hours or 30°). Although seasonal hours are found in some Babylonian texts, I am not aware of any equinoctial hours.[22] So the use of equinoctial hours seems to point to a Greek source. Moreover, there is a word for barleycorn in Hebrew, and a unit called a barleycorn (*shaᶜīra*) was still used in Arabic for linear measurement in Iraq in the ninth century.[23] In the Jewish tradition, 'part' as a unit of time only appears in calendrical contexts.[24]

Levi ben Gerson (d. 1344), also known as Gersonides, cites the traditional Jewish value both as $29;31,50,8,20^d$ and $29^d\ 12^h\ 793^p$, and contrasts it with the value $29;31,50,8,9,20^d$ that he ascribes to Ptolemy and Hipparchus, presumably based on the Hebrew version of the *Almagest* that, in turn, was based on the Arabic version of al-Ḥajjāj.[25] He then determined a new value based on two lunar eclipses, one observed by Ptolemy and one that he observed himself (see Appendix 2): $29;31,50,7,54,25,3,32^d$.[26] Levi's value for M was adopted, with slight modification, by Jacob ben David Bonjorn (fourteenth century) in his eclipse tables that were composed in Hebrew and then translated into Latin and Catalan. From the cycle he introduced, namely, that $11324^d\ 23;34,11^h$ corresponds to 383;30 synodic months, it follows that M = $29;31,50,7,53,39,49^d$.[27] It is likely that in deriving his cycle Bonjorn rounded (or curtailed) Levi's value for M, or intermediate computations based on it, for the difference between Bonjorn's value and Levi's only accumulates to about $0;0,2^h$ in 383;30 months or about 31 years.

The daily motion in lunar elongation from the Sun that appears in the Parisian version of the Alfonsine Tables (*c*. 1320), namely $12;11,26,41,37,51,50,39^{o/d}$, implies that M = $29;31,50,7,37,27,8,25^d$ which is close to Levi's value but, as usual, there is no indication of the way it was determined.[28] The Parisian version of the Alfonsine Tables was already in place when Levi derived his value for M, but there is no evidence to suggest that he was aware of the work by his Parisian contemporaries. On the other hand, in the Toledan Tables that were widely diffused in medieval Europe, the table for syzygies is based on Ptolemy's value for M, $29;31,50,8,20^d$.[29]

The data are summarised in Appendix 3, and it is readily seen that the standard Babylonian (System B) value was by no means universally accepted in the Middle Ages. As far as I can determine, Levi ben Gerson provides the earliest explicit derivation of a value for M from the circumstances of observed eclipses.

APPENDIX 1

In Ptolemy's *Planetary hypotheses*, Book 1,[30] we find the correlation:

$$8523 \text{ trop. yrs} = 8528^{y} + 277;20,24^{d} = 105416^{m} \text{ (where } 1^{y} = 365^{d}\text{).}$$

As Neugebauer noticed, it follows that

$$M = 29;31,50,8,48,...^{d} = [(8528 \cdot 365) + 277;20,24]^{d}/105416^{m}$$

which is otherwise unattested.[31] Is this an unintended value (i.e., a "phantom")? It seems to be the result of rounding, for:

$$(29;31,50,8,20^{d} \cdot 105416^{m})/365;14,48^{d} = 8522;59,57,... \text{ trop. yrs}$$
$$\approx 8523 \text{ trop. yrs.}$$

Curiously, Ptolemy himself transforms the number of tropical years into a number of days (with two places of sexagesimal fractions) and this leads to a false sense of accuracy. Ptolemy should have said that 105416 synodic months are approximately equal to 8523 tropical years, and left it at that.

APPENDIX 2

Given the times of eclipse-middle of two lunar eclipses, M may be determined from the corresponding mean oppositions. This procedure requires the following corrections: (1) a correction for the time difference between the two places of observation; (2) corrections for the equation of time; (3) a small correction for the difference between eclipse-middle and true opposition; and (4) a correction of the time interval from true opposition to mean opposition that, in turn, depends on the solar and lunar anomalies. Condition (4) can be ignored only if there is a return in both solar and lunar anomaly (as noted by Ptolemy in *Almagest*, iv.2). Levi ben Gerson's *Astronomy*, Chapter 82, illustrates this procedure.[32]

Levi cites two lunar eclipses, separated by 14854 synodic months: one observed by Ptolemy (20/21 Oct. 134: Goldstine no. 14035) and one that he observed himself (2/3 Oct. 1335: Goldstine no. 28889).[33] Note that this is not a period of return for lunar anomaly: Ptolemy gives the lunar anomaly at the time of the eclipse he observed as 64;38°, and Levi gives the lunar anomaly at the time of his eclipse as 143;31°.[34] Levi indicates that the time difference between Alexandria and Orange (where he lived) is $1;56^{h}$, based on Ibn Ezra's claim that Montpellier is $2;16^{h}$ to the west of Jerusalem together with the assumptions that Jerusalem is $0;20^{h}$ east of Alexandria and that Orange and Montpellier lie on the same meridian, very nearly.[35] Levi takes the time interval between the two eclipses to be 1201 Eg. yrs + 282^{d} + $6;35^{h}$, and then adds a correction of $3;24^{h}$ to yield the time interval between the corresponding two mean oppositions: $438647;24,57,30^{d}$ (= 1201 Eg. yrs + 282^{d} + $9;59^{h}$).

Levi appeals to Ptolemy's observation of a lunar eclipse in *Almagest*, iv.6, and reports that the mean solar longitude was Libra 26;42° with a solar equation of –1;32° or a true longitude of Libra 25;10°; whereas the mean longitude of the Moon was Aries 29;30°, i.e., the mean elongation was 182;48°. Levi then gives the time of eclipse-middle since the epoch of Era Nabonassar as 881 Eg. yrs + 91^{d} + $10;30^{h}$ after mean noon (taking the equation of time to be $-0;30^{h}$). Levi's own eclipse took place, he tells us, on 2 Oct. 1335, 15;9^h after mean noon in Orange, or 2083 Eg. yrs + 8^d + 17;5^h after mean noon in Alexandria, since the epoch of Era Nabonassar (taking into account the time difference between Orange and Alexandria). Hence the time interval between the two eclipses is 1201 Eg. yrs + 282^d + 6;35^h. The mean elongation at the time of the Levi's eclipse was 181;4,18° (with Heb.; Latin: 181;<u>14</u>,18°); hence the difference in elongation between the two eclipses is 1;43,42° (= 182;48° – 181;4,18°). The correction, 3;24^h, was probably computed from the following equation:

$$\Delta t = \Delta\eta/(v_m - v_s) \tag{1}$$

where Δt is the correction in time corresponding to $\Delta\eta$, the difference in elongation, and v_m and v_s are the mean motions of the Moon and Sun, respectively. Then, substituting $\Delta\eta$ = 1;43,42°, v_m = 0;32,56$^{\circ/h}$, and v_s = 0;2,28$^{\circ/h}$, it follows that

$$\Delta t = \Delta\eta/(v_m - v_s) = 1;43,42°/0;30,28^{\circ/h} \approx 3;24^h. \tag{2}$$

Levi's tables 40–43 are not invoked here, but they yield the interval to be added to mean opposition in order to obtain the time of true opposition.[36] With Oct. 2 and anomaly 143;31° (for Levi's eclipse), I compute 2;27^h (= 9;6^h + 0;35^h + 15;48^h + 0;58^h – 24^h); and with Oct. 20 and anomaly 64;38° (for Ptolemy's eclipse), I compute 5;39^h (= 9;37^h + 0;39^h + 19;1^h + 0;22^h – 24^h). Therefore, the difference, 5;39^h – 2;27^h = 3;12^h (i.e., $m_2 = t_2$ – 2;27^h, and $m_1 = t_1$ – 5;39^h; hence, $m_2 - m_1 = t_2 - t_1$ – 2;27^h + 5;39^h, where m and t are the times of mean and true opposition, respectively), which is close to the value in the text.

If 438647;24,57,30^d corresponds to 14854 synodic months then, according to Levi, M = 29;31,50,7,54,25,3,32^d. Accurately, this division yields M = 29;31,50,7,54,25,3,35,...^{d}.[37]

APPENDIX 3

Mean Synodic Month	Source
29;31,50,5,43,23^d	Banū Mūsā
5,43,24,...	Brahmagupta, *BSS*; Ya^{c}qūb ibn Ṭāriq
5,43,33	al-Hāshimī
5,44,33	Ibn al-Muthannā
7,37,27,8,25	Alfonsine Tables (Paris)
7,53,39,49	Bonjorn
7,54,25,3,32	Levi ben Gerson
8,4,...*	*ACT*, No. 210, line 10
8,9,20	al-Ḥajjāj, Copernicus, etc.
8,9,20,13	al-Bīrūnī
8,9,24	Ibn Yūnus, al-Biṭrūjī
8,20	System B, Ptolemy (*Alm.*), Jewish cal., etc.
8,48,...*	Ptolemy (*Plan. hyp.*)
18,...	Geminus

*phantom?

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2. B. R. Goldstein, "On the Babylonian discovery of the periods of lunar motion", *Journal for the history of astronomy*, xxxiii (2002), 1–13, espec. pp. 10–11 (note that on p. 11, line 3, '4267' is a mistake for 6247 synodic months).
3. For the mean value of column G (in *uš*) in System B that yields the length of the mean synodic month, see O. Neugebauer, *Astronomical cuneiform texts* [henceforth *ACT*] (3 vols, London, 1955), i, 78. The standard value, 29;31,50,8,20d, occurs in this form in Text No. 210, line 6 (*ibid*., i, 272) but, as far as I can tell, it does not occur in any other published Babylonian text. Neugebauer also identified a Greek papyrus that contains a fragment of an ephemeris according to System B (including column G) which implied, among other things, that the System B value for M was available in the Greek world in its proper context: *idem*, "A Babylonian lunar ephemeris from Roman Egypt", in *A scientific humanist: Studies in memory of Abraham Sachs*, ed. by E. Leichty *et al*. (Philadelphia, 1988), 301–4. For more recent treatment of this text, see A. Jones, "A Greek papyrus containing Babylonian lunar theory", *Zeitschrift für Papyrologie und Epigraphik*, cxix (1997), 167–72.
4. Geminus, xviii.3 (Geminos, *Introduction aux phénomènes*, ed. and transl. by G. Aujac (Paris, 1975), 94), says that $19756^{d} = 669^{m}$ (equivalent to $6585;20^{d} = 223^{m}$), and this relationship implies that $M = 29;31,50,18,...^{d}$. Ptolemy also reports an 18-year period of $6585;20^{d}$ and ascribes it to "the even more ancient [astronomers]": *Almagest*, iv.2 (G. J. Toomer, *Ptolemy's Almagest* (London, 1984), 175). See also Goldstein, "Periods" (ref. 2), 1–2.
5. Neugebauer, *ACT* (ref. 3), i, 272: Text No. 210, line 10.
6. J. M. Steele, "A simple function for the length of the Saros in Babylonian astronomy", in *Under one sky: Astronomy and mathematics in the ancient Near East*, ed. by J. M. Steele and A. Imhausen (*Alter Orient und Altes Testament*, ccxcvii (Muenster, 2003), 405–20). I am most grateful to Dr Steele for providing me with a preprint of his paper and for allowing me to summarize part of it here. A fuller discussion of the analysis will appear in a forthcoming paper by Lis Brack-Bernsen and John Steele.
7. For early lunar eclipse observations by the Babylonians (including the eclipse of –746 Feb. 6), see J. P. Britton, "Scientific astronomy in pre-Seleucid Babylon", in *Die Rolle der Astronomie in den Kulturen Mesopotamiens*, ed. by H. D. Galter (Graz, 1993), 61–76, espec. p. 63, n. 4; *cf*. J. M. Steele, *Observations and predictions of eclipse times by early astronomers* (Dordrecht and Boston, 2000), 43–45.
8. *Cf*. J. P. Britton, "Lunar anomaly in Babylonian astronomy", in *Ancient astronomy and celestial divination*, ed. by N. M. Swerdlow (Cambridge, MA, 1999), 187–254, espec. p. 207.
9. Ptolemy, *Almagest*, iv.2; Toomer, *Almagest* (ref. 4), 175–6. G. J. Toomer ("Hipparchus' empirical basis for his lunar mean motions", *Centaurus*, xxiv (1981), 97–109) offers a reconstruction of Hipparchus's methods and the eclipses he might have used to confirm the Babylonian parameter.
10. N. Copernicus, *De revolutionibus*, iv.4 (Nuremberg, 1543), 101v; *cf*. A. Aaboe, "On the Babylonian origin of some Hipparchian parameters", *Centaurus*, iv (1955), 122–5; and N. M. Swerdlow

and O. Neugebauer, *Mathematical astronomy in Copernicus's* De revolutionibus (New York and Berlin, 1984), 199.

11. For al-Ḥajjāj's translation, see Leiden, MS Or. 680, 50b:6. As noted by J. L. Mancha ("The Provençal version of Levi ben Gerson's tables for eclipses", *Archives internationales d'histoire des sciences*, xlviii (1998), 269–352, espec. p. 309), this variant appears in Gerard of Cremona's translation of the *Almagest*, ed. 1515, 36r; and in George of Trebizond's Latin translation of the *Almagest* from Greek, ed. 1528, 33r. Copernicus annotated a copy of Gerard of Cremona's translation of the *Almagest* (ed. 1515): see P. Czartoryski, "The library of Copernicus", *Studia copernicana*, xvi (1978), 355–96, espec. p. 372. See now J. L. Mancha, "A note on Copernicus' 'correction' of Ptolemy's mean synodic month", to appear in *Suhayl*.
12. Isaac Israeli, *Liber jesod olam seu Fundamentum mundi*, ed. by B. Goldberg and L. Rosenkranz (2 vols, Berlin, 1846–48), i, 49b, col. 1; al-Bīrūnī, *al-Qānūn al-mas*[c]*ūdī* (3 vols, Hyderabad, 1954–56), ii, 730. Abraham Bar Ḥiyya (*Sefer ha-*[c]*ibbur*, ed. by H. Filipowski (London, 1851), 37) also says that Ptolemy's value for the synodic month is 29;31,50,8,9,20d.
13. For Ibn Yūnus, see Leiden, MS Or. 143, p. 20; and for al-Biṭrūjī, see B. R. Goldstein, *Al-Biṭrūjī: On the principles of astronomy* (2 vols, New Haven, 1971), i, 145; *cf.* O. Neugebauer, *Ethiopic astronomy and computus* (Vienna, 1979), 18.
14. Al-Bīrūnī, *Chronology of ancient nations*, transl. by C. E. Sachau (London, 1879), 143, 147; *cf.* 408, 410.
15. D. Pingree, "The fragments of the works of Ya[c]qūb ibn Ṭāriq", *Journal of Near Eastern studies*, xxvii (1968), 97–125, espec. p. 99; al-Bīrūnī, *India*, transl. by C. E. Sachau (2 vols, London, 1888), ii, 15–16; *cf.* i, 350, 368.
16. B. R. Goldstein, *Ibn al-Muthannā's commentary on the astronomical tables of al-Khwārizmī* (New Haven, 1967), 18; E. S. Kennedy, D. Pingree, and F. I. Haddad, *The book of the reasons behind astronomical tables by* [c]*Alī ibn Sulaymān al-Hāshimī* (Delmar, NY, 1981), 105, 234–6.
17. Babylonian Talmud, *Seder mo*[c]*ed*, Tractate *Rosh ha-shanah*, 25a; transl. by I. Epstein (London, 1938), 110. See also, *Pirḳê de Rabbi Eliezer,* transl. by G. Friedlander (London, 1916), 43: "The total of the days of the lunar month is $29\frac{1}{2}$ days, 40 minutes, and 73 parts." The date of this Hebrew text is disputed, but a version of it was probably composed in the eighth century.
18. E. S. Kennedy, "Al-Khwārizmī on the Jewish calendar", *Scripta mathematica*, xxvii (1964), 55–59; reprinted in *idem*, *Studies in the Islamic exact sciences* (Beirut, 1983), 661–5.
19. Al-Bīrūnī, *Chronology* (ref. 14), 143; Abraham Ibn Ezra, *Sefer ha-*[c]*ibbur*, ed. by S. Z. H. Halberstam (Lyck, 1874), 3a; Abrahm Bar Ḥiyya, *Sefer ha-*[c]*ibbur* (ref. 11), 45; Maimonides, *Sanctification of the new moon*, transl. by S. Gandz, with supplementation and an introduction by J. Obermann, and an astronomical commentary by O. Neugebauer (New Haven, 1956), 27, 33, 114.
20. See, e.g., S. Stern, *Calendar and community: A history of the Jewish calendar, 2nd century BCE–10th century CE* (Oxford and New York, 2001), 201–4.
21. O. Neugebauer, "Studies in ancient astronomy, VII: Magnitudes of lunar eclipses in Babylonian astronomy", *Isis*, xxxvi (1945) 10–15, espec. pp. 12–13; *idem*, *ACT* (ref. 3), i, 39, 47. For an extensive recent study of Babylonian metrology, see M. A. Powell, "Masse und Gewichte", in *Reallexikon der Assyrologie und Vorderasiatischen Archäologie*, ed. by D. O. Edzard (9 vols, Berlin, 1932–), vii (1987–90), 457–517: on the barleycorn, see pp. 458, 478–9.
22. E. Reiner and D. Pingree, "A Neo-Babylonian report on seasonal hours", *Archiv für Orientforschung*, xxv (1977), 50–55; F. Rochberg-Halton, "Babylonian seasonal hours", *Centaurus*, xxxii (1989), 146–70.
23. D. A. King, "Too many cooks: A new account of the earliest Muslim geodetic measurements", *Suhayl*, i (2000), 207–41, espec. pp. 223, 235:4.
24. *Cf.* Stern, *Calendar* (ref. 20), 204. Note also that $1^p = 0;0,0,8,20^d$, i.e., 1^p is the difference between $29;31,50,8,20^d$ and $29;31,50^d$ (the value for M in the Muslim calendar).
25. M. Zonta, "La tradizione ebraica dell'*Almagesto* di Tolomeo", *Henoch*, xv (1993), 325–50, espec. p. 332; *cf.* Mancha, "Provençal version" (ref. 11), 307; Levi ben Gerson's *Astronomy*, chap. 64:

Paris, Bibliothèque nationale de France, MS Heb. 724, 127a:21 ff.

26. Levi ben Gerson's *Astronomy*, chap. 82: Paris, Bibliothèque nationale de France, MS Heb. 724, 155a; *cf*. B. R. Goldstein, *Levi ben Gerson's astronomical tables* (New Haven, 1974), 106.
27. J. Chabás, "The astronomical tables of Jacob ben David Bonjorn", *Archive for history of exact sciences*, xlii (1991), 279–314, espec. pp. 283–4.
28. *Tabulae astronomice illustrissimi Alfontij regis castelle* , ed. by E. Ratdolt (Venice, 1483), d7r; *cf*. N. M. Swerdlow, "The length of the year in the original proposal for the Gregorian calendar", *Journal for the history of astronomy*, xvii (1986), 109–18, espec. pp. 115–16.
29. G. J. Toomer, "A survey of the Toledan tables", *Osiris*, xv (1968), 5–174, espec. p. 80.
30. Ptolemy, *Upotheseôn tôn planômenon* [*The Planetary hypotheses*], ed. and transl. by J. L. Heiberg and L. Nix, in J. L. Heiberg, *Claudii Ptolemaei Opera astronomica minora* (Leipzig, 1907), 70–145, espec. p. 79.
31. O. Neugebauer, *A history of ancient mathematical astronomy* (3 vols, Berlin and New York, 1975), ii, 902.
32. For the Hebrew text, see Levi ben Gerson's *Astronomy* (ref. 26); for the Latin text, see Mancha "Provençal version" (ref. 11), 308.
33. H. H. Goldstine, *New and full moons 1001 B.C. to A.D. 1651* (Philadelphia, 1973).
34. Toomer, *Almagest* (ref. 4), 204; B. R. Goldstein, "Medieval observations of solar and lunar eclipses", *Archives internationales d'histoire des sciences*, xxix (1979), 101–56, espec. p. 143.
35. In fact, according to *The Times atlas of the world* (2nd edn revised, Boston, 1971), the longitude of Orange is 4;48°E and the longitude of Alexandria is 29;55°E, for a difference of 25;7° ≈ 1;40h. The longitude of Montpellier is 3;53°E, and so it is not on the same meridian as Orange.
36. Goldstein, *Astronomical tables* (ref. 26), 229–38.
37. *Cf*. Mancha, "Provençal version" (ref. 11), 308.