VARIABLE OORT CLOUD FLUX DUE TO THE GALACTIC TIDE

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Abstract: We review the subject of the time dependence of the component of Oort cloud comet flux due to the adiabatic Galactic tide, including the possibility of detecting such a signal in the terrestrial cratering record.

1. Introduction

Over long time scales the flux of new comets coming from the outer Oort cloud is likely to be dominated by the near-adiabatic tide due to the Galactic matter distribution (Heisler, 1990). As the Solar System moves in its Galactic orbit, this tide is substantially modulated for all models of the Galactic mass distribution that are consistent with stellar dispersion studies. Therefore a quasi-periodic variability of the tidally induced component of the Oort cloud flux having significant amplitude is to be expected (Matese *et al.*, 1995). If Shoemaker *et al.* (1990, 1998) was correct in his estimate that 80% of terrestrial craters having diameter > 100 km are produced by long-period comets (and 50% of craters > 50 km), then the phase and period of the Solar System oscillation about the Galactic disk should be consistent with the ages of the accurately dated largest craters. The phase is well constrained, but the dynamically predicted plane crossing period has been sufficiently uncertain (30-45 Myr) to preclude a meaningful comparison with the best fit measured cratering period of 34-37 Myr. If the mean plane crossing period is ultimately determined to exclude this interval we will be able to confidently reject Shoemaker's hypothesis of the dominance of cometary impacts in the production of the largest terrestrial craters.

2. Tidal Dynamics

We are primarily interested in understanding how the Galactic tide makes an Oort cloud comet observable. A widely accepted model (Duncan *et al.*, 1987) of the formation of the Solar System comet cloud considers unaccreted comets in the giant planetary region being gravitationally pumped by these planets into more extended orbits. When semimajor axes grow to values $a \approx 5000$ AU Galactic tidal torques can sufficiently increase the angular momentum of a comet, $\mathbf{H} = \mathbf{r} \times \mathbf{v}$, and therefore its perihelion distance $q = a(1-e)$,

$$H = \left[\mathrm{GM}_\odot a(1-e^2)\right]^{1/2} = \left[\mathrm{GM}_\odot (2q - q^2/a)\right]^{1/2}, \quad (1)$$

so that it becomes detached from the planetary zone. The near-adiabatic nature of the Galactic tide will keep a essentially constant during this stage. Episodically, passing molecular clouds and stars impulsively pump comet energies as well as effectively randomize the phase space of semimajor axis orientations and angular momenta, thus forming the Oort cloud (Bailey, 1986).

To make a comet observable it must be injected into the inner planetary region so that it is sufficiently insolated to form a coma. Here we discuss the dynamical mechanism that is predominantly responsible for doing so, the quasi-adiabatic tidal interaction with the Galaxy. The same mechanism that increased angular momentum and detached the comet orbit from the planets can also decrease angular momentum and bring it back into the planetary zone. But now the planets Saturn and Jupiter provide a dynamical "barrier" to the migrating perihelia of Oort cloud comets.

Semimajor axes $a > 10000$ AU denote the energy range commonly refered to as the outer Oort cloud. As we shall see, comets with smaller semimajor axes are inefficiently torqued by the tide so that they are dynamically captured by Saturn or Jupiter before becoming observable. Only if the tidal interaction is sufficiently strong to make the perihelion distance migrate from beyond this "loss cylinder" barrier at ≈ 15 AU to the observable zone interior to ≈ 5 AU in a single orbit, will we recognize the comet as having originated in the outer Oort cloud - a "new" comet is observed.

If we ignore modest mass inhomogeneities due to local molecular clouds and voids (Frisch and York, 1986), we can take the gravitational potential generated by the smoothed Galactic mass density to be axisymmetric (the analysis presented here is adapted from Heisler and Tremaine(1986)),

$$\nabla^2 U(R,Z) = \frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial U}{\partial R} + \frac{\partial^2 U}{\partial Z^2} = 4\pi \mathrm{G}\rho(R,Z), \tag{2}$$

where $R = 0$ defines the Galactic center and $Z = 0$ the Galactic midplane. From this potential we can obtain the R-dependent azimuthal velocity, Θ, and angular velocity, Ω. Values at the Solar location (Merrifield, 1992) are denoted by the subscript $\circ$, and the present epoch is $t \equiv 0$,

$$R_\circ(0) \approx\ 8\mathrm{kpc},\ \ \Theta_\circ(0) = \left[\frac{\partial U}{\partial \ln R}\bigg|_{R_\circ}\right]^{1/2} \approx\ 200\mathrm{kms}^{-1} \tag{3}$$

with uncertainties of approximately 10% (Kuijken and Tremaine, 1994).

The Galactic tidal field in an orbiting, but non-rotating Solar reference frame O is

$$\mathbf{F}_O(t) = -\nabla U(\mathbf{R}_\circ + \mathbf{r}) + \nabla U(\mathbf{R}_\circ) = -\left(\mathbf{r}\cdot\nabla\right)\nabla U(R_\circ(t), Z_\circ(t)). \tag{4}$$

Here $\mathbf{r}$ is the comet position vector relative to the Sun with components (x, y, z) in frame O, and components

$$(x', y', z') \equiv (x\cos\Omega_\circ t + y\sin\Omega_\circ t, y\cos\Omega_\circ t - x\sin\Omega_\circ t, z) \tag{5}$$

in a rotating frame with x' axis pointing toward the Galactic center, y' axis opposite to Θ and z' along the Galactic normal.

In this notation the tidal force is expressible as

$$\mathbf{F}_O(t) = {\Omega_\circ}^2(1-2\delta)\mathbf{x}' - {\Omega_\circ}^2\mathbf{y}' - ({\Omega_z}^2 + 2\delta{\Omega_\circ}^2)\mathbf{z}'. \tag{6}$$

Here $\Omega_\circ = \frac{\Theta}{R}\Big|_\circ$ and $\delta \equiv \frac{d\ln\Theta}{d\ln R}\Big|_\circ$ are simply related to the Oort constants A and B (Heisler and Tremaine, 1986), while ${\Omega_z}^2 \equiv 4\pi\mathrm{G}\rho_\circ$. It can be shown (Binney and Tremaine, 1987) that the perigalactic radial frequency of the Solar motion is larger than its orbital frequency by a factor $(2+2\delta)^{1/2}$. The time dependence of $\mathbf{F}_O(t)$ is contained not only implicitly in the components (x, y, z), but explicitly in the transformation to (x', y', z') of Eq.(5) and in Ω_z, $\Omega_\circ$, and δ through their dependencies on $\rho_\circ = \rho(R_\circ(t), Z_\circ(t))$.

Our Galaxy has a nearly flat rotational velocity curve at the Solar location (Merrifield, 1992), $\delta \approx\ -0.1$. The various time scales are then the comet orbital period ($P \approx 5$ Myr), the oscillatory Solar Z period about the Galactic midplane ($P_z \approx\ 2\pi/\Omega_z \approx$ 60-90 Myr), the Solar azimuthal period about the Galactic center ($P_\circ = 2\pi/\Omega_\circ \approx 240$ Myr), and the radial period of the Solar orbit $\approx P_\circ/\sqrt{2} \approx 180$ Myr. The larger value of P_z corresponds to the no-dark-disk-matter case while the smaller period is obtained in model calculations (Matese *et al.*, 1995) with modest amounts of CDDM (dark disk matter distributed over compact scale heights, comparable to the interstellar medium).

The angular momentum evolves in accord with the Newtonian equation

$$\frac{d\mathbf{H}}{dt} = \mathbf{r} \times \mathbf{F}_O \equiv \tau_O. \tag{7}$$

Also of interest is the Laplace-Runge-Lenz eccentricity vector which instantaneously points to the osculating perihelion point

$$\mathbf{e} = \frac{\mathbf{v} \times \mathbf{H}}{\mathrm{GM}_\odot} - \frac{\mathbf{r}}{r}, \quad \frac{d\mathbf{e}}{dt} = \frac{\mathbf{v} \times \tau_O - \mathbf{H} \times \mathbf{F}_O}{\mathrm{GM}_\odot}. \tag{8}$$

Since we are primarily interested in the tidal mechanism as it relates to making a comet observable during a single orbit, we consider the various terms in $\mathbf{F}_O$ in the context of a time average over a single comet period. Comparing terms, we see that $(\Omega_\circ/\Omega_z)^2 \approx 0.1$ so that setting $\delta \to 0$ introduces only a modest formal error of $\approx$ 2%.

To proceed further in the spirit of doing an orbital average of the equations of motion, two distinct approximations could be made. If we set $\Omega_\circ \to 0$, which formally introduces errors of $\approx 10\%$, the problem simplifies substantially and a complete analytic solution to the orbital averaged equations of motion can be obtained for all of the orbital elements in the adiabatic limit (Matese and Whitman, 1989; Breiter *et al.*, 1996).

Alternatively, we can concentrate on near-parabolic comets since they are most easily made observable by the Galactic tide in a single orbit. In this approximation, the coordinates in $\mathbf{F}_O$ are replaced by those of a comet freely falling along its semimajor axis with $e \to 1$ and position vector $\mathbf{r} = -r\hat{\mathbf{q}} = -r\hat{\mathbf{e}}$, *i.e.*,

$$(x', y', z') \to -r\,(\cos b\ \cos(l - \Omega_\circ t),\ \cos b\ \sin(l - \Omega_\circ t),\ \sin b) \tag{9}$$

where b, l are the Galactic latitude and longitude of perihelion. In this case we introduce relative errors of order $1-e = q/a \approx 10^{-3}$ in the equations of motion. Further, from Eq. (8), eccentricity changes during an orbit are of order $\Delta\mathbf{e} \approx PF_O H/\mathrm{GM}_\odot \approx 4\pi\sqrt{2q/a}(P/P_z)^2 \approx 10^{-3}$, so we can treat the perihelion angles $b,\ l$ as constants during the averaging of the torque to the same level of approximation. Note that the conventional orbital angles, the longitude of the ascending node (Ω), the argument of perihelion (ω), and the inclination (i), are all rapidly changing for a near-parabolic comet and thus cannot be held constant in a perturbative orbital average analysis.

Defining an azimuthal unit vector, $\hat{\phi} \equiv \hat{\mathbf{z}} \times \hat{\mathbf{q}} / \cos b$, we construct a near-constant set of orthogonal unit vectors $(\hat{\mathbf{q}}, \hat{\phi}, \hat{\theta} \equiv \hat{\phi} \times \hat{\mathbf{q}})$. Since $\mathbf{H} \perp \hat{\mathbf{q}}$, it has two components in this basis (H_ϕ, H_θ) which can be changed by the torque. Performing the time average over an orbit, $\langle r^2 \rangle = a^2(4+e^2)/2 \rightarrow 5a^2/2$, we obtain

$$\langle \tau_{\mathbf{O}} \rangle = \frac{5}{2} a^2 \cos b \left[\hat{\phi} \sin b \left({\Omega_z}^2 + {\Omega_\circ}^2 \cos 2(l - \Omega_\circ t) \right) + \hat{\theta} \left({\Omega_\circ}^2 \sin 2(l - \Omega_\circ t) \right) \right]. \tag{10}$$

We are not presently interested in the long term evolution of an individual comet orbit, but in the single-orbit evolution of all near-parabolic comets, so we set $t = 0$. The relative error introduced in replacing the torque by its time average over a single orbit is of magnitude $\approx \left[\frac{{\Omega_\circ}^2 P}{\Omega_z} \right]^2 < 10^{-2}$ when we set $t = 0$ in the sinusoidal functions. The largest error is made when we treat ${\Omega_z}^2 = 4\pi \mathrm{G} \rho(R_\circ(t), Z_\circ(t))$ as adiabatically constant. A standard analysis in perturbation theory shows that in using the adiabatic approximation we make an error of order

$$Max \left[\left(\frac{\dot{\Omega}_z P}{\Omega_z} \right)^2, \frac{\ddot{\Omega}_z P^2}{\Omega_z} \right] \approx \frac{1}{6} \left(\frac{Z_{max} \Omega_z P}{Z_\rho} \right)^2. \tag{11}$$

Here $Z_{max}/Z_\rho = Order(1)$ is the ratio of the Solar amplitude to the scale height of the disk density. Today $Z_\circ \approx 10$ pc $< Z_{max} \approx 80$ pc (Reed, 1997) and the acceleration term dominates. This a-dependent error is $\approx 5\%$ for $a = 30000$AU.

When both torque components are included we find that the tidal induced change in angular momentum during a single orbit is

$$\Delta \mathbf{H} \approx \frac{5}{2} {\Omega_z}^2 P a^2 \cos b \left(\hat{\phi} \sin b [1 + \varepsilon \cos 2l] + \hat{\theta} \varepsilon \sin 2l \right) \tag{12}$$

where $\varepsilon \equiv (\Omega_\circ / \Omega_z)^2$. However, the assumption of azimuthal symmetry in the local tidal field is probably in error by an amount comparable to ε so that a more nearly self consistent final result, onto which we should append an $\approx$ 20% uncertainty in the l, b dependence, would be

$$\Delta \mathbf{H} \approx \hat{\phi} \frac{5}{2} {\Omega_z}^2 P a^2 \sin b \cos b. \tag{13}$$

The minimum value of the semimajor axis that can enable a comet to leap the loss cylinder barrier in a single orbit is $\approx$ 25000AU for the observed no-dark-matter disk density of $\rho_\circ \approx 0.1 \mathrm{M}_\odot \mathrm{pc}^{-3}$ (Flynn and Fuchs, 1994) which is in good agreement with the observed inner edge of the Oort cloud energy distribution when we account for uncertainties in the determination

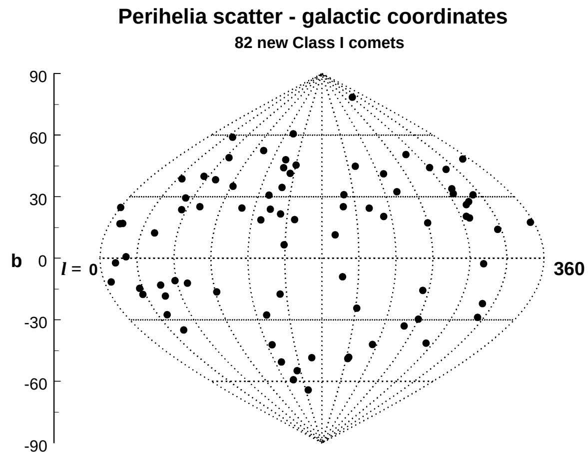

Figure 1. An equal area scatter on the celestial sphere of the perihelion directions of 82 new Class I comets. Results are distributed in Galactic latitude, b, and longitude, l.

of the original value of the cometary semimajor axis, prior to its entry back into the planetary region.

The change in $\mathbf{H}$, and so the change in q, is predicted to be minimal at $\sin b \cos b = 0$, *i.e.*, at the Galactic poles and equator, once we recognize that b is essentially constant during an orbit for near-parabolic comets. Therefore if the tide dominates in making Oort cloud comets observable we should see minima in the distributions at these values. Randomly oriented perihelia would be uniformly distributed on the celestial sphere. In Figure 1 we show an equal area scatter in perihelia of new Class I Oort cloud comets (Marsden and Williams, 1996) which illustrates this characteristic signature of the Galactic tide.

Byl(1983) was the first to recognize the importance of the adiabatic Galactic tide in making Oort cloud comets observable. He also noted the prediction of smallest changes in q for perihelia near the poles and equator and pointed out that the observed distributions had such depletions. But Byl modeled the Galactic interaction as a point mass and several others, most notably Heisler and Tremaine (1986), noted that the Galactic disk dominates the core interaction and performed the appropriate analysis.

The first comprehensive modeling of observed Oort cloud comet orbital element distributions was given later (Matese and Whitman, 1989; Matese and Whitman, 1992). A Monte Carlo procedure was employed in which the *in situ* Oort cloud population was modeled to have an energy distribution determined by Bailey (1986), an angular momentum distribution which is empty inside the loss cylinder (but otherwise random) and a $\hat{\mathbf{q}}$ distribution that is also random. Elements describing comets as they left the planetary region on their prior orbit were randomly selected from this population, *i.e.*, $(a,\ l,\ b, H_{\phi}{}^{prior}, H_{\theta}{}^{prior})$ are chosen. $\Delta\mathbf{H}$ is then computed from Eqs.(12-13), and comet orbital properties recorded if the change made the comet "observable". Distributions of orbital elements are then obtained for the theoretically "observable" population, and correlations between orbital elements were studied. Predicted distributions were found to compare reasonably with those actually observed. These results have been recently confirmed (Weigert and Tremaine, 1999).

The most statistically significant evidence that the Galactic tide dominates over stellar impulses in making Oort cloud comets observable during the present epoch is the three-fold correlation between orbital elements that is predicted by tidal theory, and is observed (Matese *et al.*, 1999). The correlations are embodied in Eq.(13) and we leave the reader to investigate the discussion there.

3. Time Dependent Oort Cloud Comet Flux

In a similar manner Matese *et al.* (1995) have modeled the Solar motion $\mathbf{R}_{\circ}(t)$ to estimate the time dependence of the tidal-dominated Oort cloud flux over time scales of hundreds of Myr. In Eq.(13) the time dependence of the tidal strength was obtained by replacing $\Omega_z \rightarrow \Omega_z(\mathbf{R}_{\circ}(t - P/2))$ so that the predicted observations at time t were appropriately retarded.

TABLE 1. Large Accurately Dated Craters

Crater	D(km)	T(Myr BP)	$\Delta T(Myr)$
Chesapeake	85	35.5	0.5
Popigai	100	35.7	0.8
Montagnais	45	50.5	0.8
Chicxulub	170	64.98	0.05
Kara	65	73	3
Manson	35	73.8	0.3
Mjölnar	40	142	3
Morokweng	100	145	3
Manicouagan	100	214	1

In Figure 2 we show the modeled flux for a single case with a mean plane crossing period of 36 Myr, the period that best fits the observed periodicity

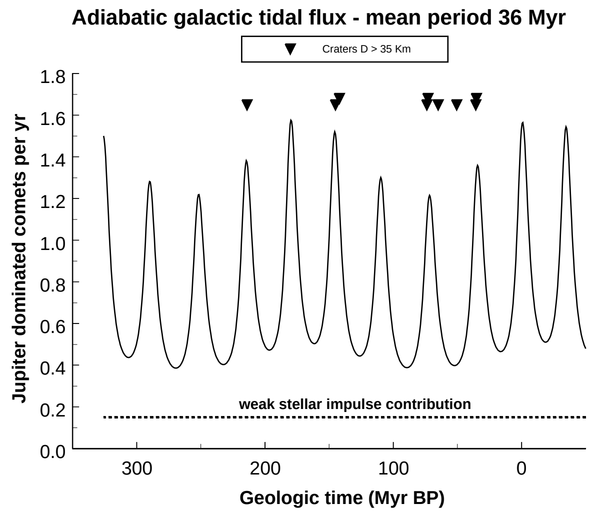

Figure 2. A model of the variable Oort cloud comet flux as modulated by the adiabatic Galactic tide with a mean plane crossing period of 36 Myr. Past time is positive here. Markers for the 9 accurately dated large craters listed in Table 1 are shown.

of large craters (Matese *et al.*, 1998). The peak flux times lag the Galactic plane crossing times by ≈ 2 Myr and are not precisely periodic because of decreasing Galactic density as the Sun recedes from the Galactic core. The phase of the oscillations is restricted by observations which place the last previous plane crossing at ≈ 1.5 Myr in the past (and the next flux peak ≈ 1 Myr in the future). A perigalactic period of ≈ 180 Myr is observable in the data. The background shown is a combination of the adiabatic tidal effects of the large scale height old star population as well as an assumed steady state contribution from stellar impulses of the outer Oort cloud affecting the phase space not accessible to the Galactic tide. We do not show the larger but rarer contributions due to random stellar-induced showers from the inner Oort cloud which are estimated to add ≈ 20% to the total but dominate only 2% of the time (Heisler, 1990).

The peaks above the background are attributable to the adiabatic tide of the compact component of the disk composed of molecular clouds, dust, young stars and a modest amount of CDDM. Random molecular cloud impulses of the Oort cloud are not included, but will have their probabilities modulated is a manner similar to that shown. The standard deviations of the peaks are ≈ 4-5 Myr, but if the background is included the formal standard deviation of a complete cycle is closer to 7-8 Myr. That is, roughly 2/3 of the model's flux occurs in a time interval of ≈ 15 Myr. Random strong stellar impulses will broaden this interval while stochastic strong molecular cloud impulses will narrow it, but not below a value of ≈ 9 Myr. Phase jitter in the Solar motion due to impulses should be substantially less than these widths in a 240 Myr interval.

Also shown in Figure 2 are markers for the 9 largest accurately dated craters. The data, listed in Table 1, are taken from Grieve and Pesonan (1996) as modified by Shoemaker (1998) and Rampino and Stothers (1998). The crater name, diameter (D), age (T) and dating uncertainty ($\Delta T \leq 3$ Myr) are given. Figure 3 shows the results for oscillation models having a range of plane crossing periods from 25-45 Myr. Statistical analyses have been performed (Matese *et al.*, 1998; Rampino and Stothers , 1998). It is concluded that only if the mean Solar plane crossing period is ultimately found to be in the interval 34-37 Myr will we be able to say that there is a statistically significant correlation between the Galactic oscillation cycle of the Solar System and periodicity in the formation of large impact craters. The statistical significance ($> 2\sigma$) would hold independent of whether one considers only the largest 5 craters or all 9 craters. Including smaller accurately dated craters will degrade the significance level.

A recent analysis using *Hipparcos* observations of A and F star distributions (Holmberg and Flynn, 2000) is the first that effectively sheds light on the question of the Solar period. They conclude that there is no evidence for enough CDDM to significantly reduce the Solar oscillation plane crossing period below 45 Myr. Prior analyses could not definitively exclude CDDM and therefore could not effectively constrain this period.

4. Summary

The Galactic tide dominates in making Oort cloud comets enter the planetary region during the present epoch, and likely over long time scales. Substantial modulation of the tidally induced comet flux must occur independent of the existence or non-existence of CDDM. This is due to the adiabatic variation of the local disk density during the Solar cycle. A Galactic oscillations model in which the Solar cycle is manifest in the cratering record will only be sustainable as a working hypothesis if the mean cycle period is found to include the interval 35.5 ± 1.5 Myr. Should the Holmberg-Flynn result be confirmed, we can reject Shoemaker's suggestion that impacts from long-period comets dominate large terrestrial crater formation.

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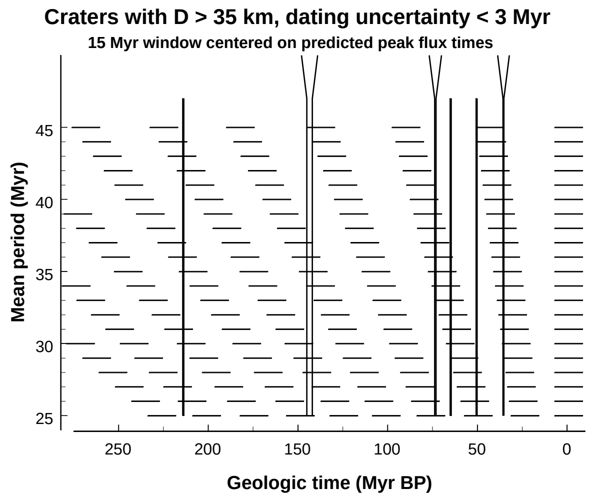

Figure 3. A comparison of 9 accurately dated large crater ages with a sequence of peak flux times for various mean plane crossing periods ($\frac{1}{2}\bar{P}_z$) as predicted by the Galactic oscillations model. A 15 Myr window is centered on each peak flux time as a reference.

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